On Prepayment & Rollover Risk in the U.S. Credit Card Market

Lukasz A. Drozd and Ricardo Serrano-Padial

April 7, 2014

ABSTRACT

The balance transfer rate in the U.S. credit card market has increased dramatically over the last two decades, far outpacing the growth of the credit card market itself. Recently, estimates put it at a whopping 17% annual rate, rising from negligible levels in the early 1990s. Here we aim to study this phenomenon by developing a model in which balance transfers arise in equilibrium as a result of an entry game between lenders. Using our model, we propose a comparative statics exercise that rationalizes the observed rise of balance transfers and other aspects of the U.S. credit card market, such as the increased dispersion of interest rates. Our analysis shows that the very mechanism behind the rise of balance transfers leads to excessive debt rollover risk borne by consumers, which we study in light of the most recent financial crisis.

JEL: E21, D91, G20

Keywords: credit cards, deleveraging, financial crisis, credit crunch, default, non-exclusivity, unsecured credit, balance transfers, refinancing
1 Introduction

One of the key features of the U.S. credit card market is that contracts are subject to a large risk of prepayment in the form of balance transfers. During the 1990s, the volume of repriced debt rose rapidly and, according to industry studies, in the early 2000s, as much as 17% of credit card balances were transferred annually by consumers seeking better terms (Evans and Schmalensee, 2005). Today almost every credit card company tries to ‘poach’ customers of other companies by offering them better terms. Yet very little is known about the equilibrium impact of this peculiar phenomenon. Existing theories of unsecured consumer credit typically assume that credit contracts are exclusive and are only repriced at contract maturity.

To close this gap, here we develop a model of dynamic competition between lenders in an environment where borrowers are allowed to reprice their time-varying credit risk at a later date. Repricing is endogenous, and takes place when an improvement in borrower’s overall creditworthiness is observed. Since repricing cuts into the revenue stream of incumbent lenders, while leaving the default risk borne by them largely intact, it leads to a distortion of ex-ante credit terms. As we show, this distortion is manifested by excess interest rate dispersion, overly generous credit limits, and shortened ‘effective’ maturity of contracts. Notably, the latter feature can lead to macroeconomic fragility of the market to credit supply disruptions, such as the one that arose during the recent financial crisis. This is because it introduces debt roll-over risk on the borrower side, despite the fact that contracts per se are of long-maturity.

Specifically, in our theoretical framework lenders have access to a signal extraction technology that reveals changes in consumers’ creditworthiness. For this reason, lenders dynamically compete to extend credit in rounds of competition that take place across differed information sets regarding borrower’s time-varying creditworthiness. This results in ‘poaching’ of existing credit relations by future lenders. The second key feature is that contracts in the model are nonexclusive. That is, consumers can hold multiple credit cards, while lenders cannot condition the extension of credit on the cancellation of prior credit agreements – leading to competition that is effectively ‘capacity constrained’.

We characterize the kind of distortions that arise in equilibrium. We show that they generally take two forms, manifested either through ex-ante interest rate or credit limit,
respectively. The interest rate distortion is brought about by the fact that incumbent lenders may follow the strategy of raising their interest rate in order to front-load payments before repricing occurs. This is possible because the entry of future lenders is delayed in our model. We refer to this type of credit contracts as ‘rollover contracts,’ as in this case borrowers accept relatively high initial interest rate contracts in expectation of future repricing. They key feature of these contracts is that they effectively exhibit shortened maturity, as borrowers would find the terms either unattractive or unsustainable absent repricing. Alternatively, distortion may lead to excessive credit limits due to capacity constraint induced by non-exclusivity. In this case, incumbent lenders may choose to expand their offered credit limits in a way that crowds out future lenders. Such strategy works by pushing borrowers closer to strategic default, thereby leaving little space for future lenders. The interesting feature of the model is that, while rollover contracts lead to full balance transfers in equilibrium, crowding out contracts completely suppress them.

In the context of our model, the high volume of balance transfers can only be rationalized by strong presence of roll-over contracts. This has far reaching consequences, as these contracts can render the market vulnerable to short-lived credit supply shocks. We explore this feature of the model quantitatively, and ask the question to what extent this particular vulnerability might have contributed to the deleveraging observed in the credit card market during the recent 2007-2009 financial crisis. As is well known, the credit card market has been significantly affected by the crisis. The default rate on credit card debt increased at least by a factor of two, and there has been a significant decline in credit card debt outstanding, both due to elevated default rates as well as consumers actually reducing their debt.

To analyze how our model fares in light of the data, we embed the aforementioned novel features in a fairly standard life-cycle environment similar to Livshits, McGee and Tertilt (2007). We calibrate our model by requiring that it to be consistent with the key characteristics of the U.S. credit card market such as: 1) gross level of unsecured debt equal to 9% relative to average disposable income, 2) charge-off rate on outstanding balances equal to 5.5%, and 3) a high volume of balance transfers (20% per annum) – the data pertains to year 2004, which is the last year the precedes major changes in bankruptcy law and later financial crisis. The model perfectly matches all three characteristics and, most importantly,
it suggests that the third feature of the data can only be consistent with a substantial presence of rollover contracts in equilibrium. Notably, their prevalence is also consistent with the growing dispersion of interest rates over the same time period. In particular, our model is consistent with a fatter right tail of the interest rate distribution due to the prepayment risk premium charged in rollover contracts, as well as the higher default premium of crowding out contracts, and at the same time gives rise to a fatter left tail brought by the low introductory rates associated to balance transfer offers. Figure 1 shows that, in parallel to the exponential rise of balance transfers, this is precisely what happened in the U.S. data.

![Figure 1: Rate Distribution: 1995 (left) and 2004 (right) [Survey of Consumer Finances].](image)

In our quantitative exercise, we model the impact of the crisis on the credit card market by (unexpectedly) increasing the probability that consumers fall into financial distress (macroeconomic shock), and by partially disrupting entry of future lenders offering repricing opportunities (credit crunch). The macroeconomic shock is calibrated to match the spike in the charge-off rate, i.e., the ratio of defaulted to outstanding credit card debt. The credit crunch, on the other hand, which in our model is short-lived and only disrupts repricing opportunities for one period, explains the change in outstanding interest rates and causes a further drop in debt-to-income of about 16% of the total deleveraging in the model. Incidentally, while our model implies an increase in the average interest rates, the effect is asymmetric across agents. Specifically, only about 11% of revolvers are affected by interest rate hikes but, in those cases, the increase in interest rates typically varies between 10 and 20 percentage points (p.p., thereafter). Such an asymmetric effect amplifies the impact of the
credit crunch on consumer welfare.

While in our exercise we assume that the supply side is the key driver of deleveraging, two features of the data support this approach, as opposed to a demand-driven story. First, it is difficult to explain how the latter can lead to an increase in interest rates, given that consumers pay down the most expensive debt first. Second, the data also shows a dramatic decline in promotional balance transfers as well as in overall credit card solicitations, without an accompanying drop in the response rate to solicitations.¹

A remarkable aspect of our results is that, as Figure 2 illustrates, the observed deleveraging and the increase in interest rates early on during the crisis is almost fully matched by the spike of defaults due to the macroeconomic shock and by disrupting repricing opportunities, without affecting (new) consumers’ access to credit. In contrast, a model with exclusive contracts and no prepayment risk, which we study for comparison, falls short of matching the deleveraging and misses the increase in interest rates since the above disruption of credit has no effect in the selection of contracts under exclusivity. We believe this to be one of the main appeals of our approach given the long maturity of credit card contracts, and the fact that banks after 2009 were no longer allowed to force repayment nor increase interest rates on preexisting debt. In this context, it does not seem very plausible that a short-lived credit shock would severely affect access to pre-existing credit cards. Accordingly, by showing how

¹During the crisis new credit card accounts offering promotional balance transfers drop by 70%, while credit card mail solicitations fell by 58%, despite a simultaneous increase in response rates (the response rates to direct mail solicitations went from 0.467% in 2006 to 0.575% in 2009). Sources: Synovate Mail Monitor, Argus, Information & Advisory Services, LLC (files.consumerfinance.gov/f/2011/03/Argus-Presentation.pdf), and Mintel Media (www.comperemedia.com).
the nature of credit card competition leaves the unsecured credit market prone to deleveraging in response to credit shocks, our paper studies a potentially important transmission channel that might have exacerbated the impact of the crisis on the U.S. households.

Independently, our analysis highlights how the structure of credit market competition, which is a function of specific regulations in place, can have unintended perverse effects. In this context, it contributes to a better understanding of important aspects of the current regulatory framework governing the market for consumer credit in the US, such as non-exclusivity.

In terms of related literature, existing theories of competition featuring non-exclusive lending only consider static entry — see DeMarzo and Bizer (1992), Petersen and Rajan (1995), Kahn and Mookherjee (1998), Parlour and Rajan (2001), Bisin and Guaitoli (2004), Hatchondo and Martinez (2007), and also the information sharing model of Bennardo, Pagano and Piccolo (Forthcoming). These models are not quantitative, and they often do not explore these features in the context of consumer markets. Existing quantitative models of unsecured borrowing typically build on the work by Livshits, McGee and Tertilt (2007) and Chatterjee et al. (2007) and assume full exclusivity of contracts for the duration of one period. A notable exception is the model by Rios-Rull and Mateos-Planas (2007), which exhibits long-term credit lines, and in which consumers can switch to a new credit line, while still assuming exclusivity. In contrast, we focus here on the impact of a nonexclusive arrangement in which the effective maturity of debt is endogenous.

2 The Model

We begin from a two-period environment to analytically derive and illustrate the effect of prepayment. We later extend this particular setup and nest it in a multiperiod life-cycle environment that we calibrate to the U.S. data. While several aspects of the setup laid out here are intentionally simplified, the life-cycle model is more general.

2.1 Environment

The economy is comprised of a large number of risk-neutral lenders and risk-averse consumers, and there are two periods. Lenders commit unsecured credit lines to consumers.
Consumers use credit extended by lenders to smooth consumption intertemporally and/or absorb a financial distress shock that hits them in the second period. Lenders compete in two rounds of Bertrand competition in the first period. The first round takes place ex-ante, i.e., before consumers know whether they are under financial distress, while the second round takes place after both consumers and lenders learn the true realization of the shock—implying that second-round lenders have an informational advantage over the first-round lenders. Contracts are pre-committed but non-exclusive, as is the case in the U.S. credit card market. That is, during the second round consumers do not have to cancel their initial credit lines, if they wish so. This feature allows them to smoothly roll over their debt onto a possibly cheaper credit line by executing a transfer of existing balances. It also implies that neither second-round lenders can force first-round credit lines to be cancelled upon acceptance of their offer nor borrowers can credibly commit to do so after transferring their balances. This aspect gives rise to a notion of credit carrying capacity in our model, which turns the relation between first- and second-round lenders into a strategic one. This key aspect of our model is intended to capture the reality of the U.S. credit card market. In this market contracts are non-exclusive, and consumers hold multiple cards with varying degrees of utilization. Furthermore, a highly developed credit reporting system has been developed over the years, which provides a wealth of information about borrower’s changing default risk to lenders. This system, on one hand facilitates competition by stripping incumbent lenders from informational advantages (Bertrand competition in each round), but on the other hand gives rise to frequent repricing of existing debt.

2.2 Consumers

Consumer preferences are given by $u(G(c_1, c_2))$, where $c_i$ denotes consumption in period $i$, $u$ is a concave utility function and $G$ is an intertemporal consumption aggregator. Such recursive specification conveniently separates preferences for intertemporal smoothing from risk attitudes.

The timing of events from the consumer’s point of view is as follows. After accepting the first credit contract, denoted by $C$, and characterized by a committed fixed interest rate $R$ and a credit limit $L$, consumers learn whether they are in financial distress or not (e.g. face a major
life-cycle shock such as: medical bills, divorce or unwanted pregnancy). This information is relayed, possibly with some delay, to lenders and the second round of competition opens up. In this new round borrowers can accept a balance transfer offer \( C' \), given by a rate \( R' \) and limit \( L' \). With both contracts in hand, consumers choose how much to borrow \( b \) and how much to consume in the first period, and any potential balance transfers are executed.\(^2\) At the beginning of period two, consumers decide whether they want to borrow more from their credit lines and default or, instead, pay back their debts, which determines their second period consumption. It is assumed that as soon as consumers learn their shock, they can cash out their credit line, making it impossible for initial lenders to retract it.\(^3\)

The above setup introduces two assumptions that help us gain analytic tractability and illustrate the effects of repricing in a more transparent manner. Compared to the more standard case of consumers choosing how much to borrow before receiving imperfect information updates about the future shock realization, here we let consumers to borrow after they perfectly learn about the shock realization. Such timing makes the model more tractable by avoiding taking expectations to pin down the optimal borrowing policy, while having perfect information leads to second-round contracts exhibiting risk-free interest rates. Since the probability of distress is typically low in relevant applications, such simplifying assumptions should not make a significant difference qualitatively. Moreover, the timing in our model allows consumers to exhibit different utilization levels depending on the shock realization, reflecting the flexibility that credit cards provide over standard loan contracts. In what follows next, we only consider interest rates satisfying \( R > R' \). Given our assumptions below and lender behavior in the second round this is without loss of generality.

Interest on borrowing is distributed among lenders proportionally to the delay associated to the second round, which we parameterize by \( \zeta \in [0, 1] \). Specifically, the accrued interest on transferred debt that corresponds to the first round line is proportional to \((1 - \zeta)\). In the absence of delay \((\zeta = 1)\), all revenue on transferred debt is undercut, while contracts are fully

\(^2\)Our specification assumes that consumers accept one contract per round. The following simple game can rationalize this particular assumption: Within each round borrowers shop sequentially for lines while lenders observe all contracts being accepted in the process and can change the terms on unaccepted contract. Lenders only commit not to change terms in case borrowers do not apply for more credit from other lenders within a round. It is easy to show that merging two lines from the same round into one can better for borrowers because it reduces the marginal interest rate while providing the same total credit limit.

\(^3\)This is consistent with the U.S. law.
exclusive and thus not subject to repricing when $\zeta = 0$. In the latter case, our model features no prepayment risk, and is thus similar to existing theories of unsecured credit.

In each period, consumers receive deterministic income $y > 0$ and, as already mentioned, face an i.i.d. binary expense shock $\kappa \in \{0, x\}$, $x > 0$, which hits consumers in period two with probability $p$. They start with some pre-existing debt $B > 0$ (endogenous in the life-cycle model), and smooth consumption across periods by resorting to credit card borrowing and an option to default. Default involves an exogenous pecuniary penalty.

Intertemporal preferences are represented by the aggregator function $G$, here given by:

$$G(c_1, \kappa, c_2, \kappa) = c_1 + c_2 - \mu(c_2 - c_1)^2,$$

where $c_i, \kappa$ is consumption in period $i$, given realization of the shock $\kappa$, and $\mu > 0$ is a curvature parameter. The quadratic function makes the model more tractable. In particular, it implies that the policy function is linear in the marginal interest rate.$^4$

The borrowing constraint that we impose on our consumers captures the predatory nature of balance transfers. Specifically, we assume that consumers cannot borrow more than $L$ in the first period, and so the initial line $C$ is essential to rollover pre-existing debt $B$ into the period. The second round line only reduces interest, but effectively does not provide credit to our consumers. Consumer can max out and default on both cards in the second period, thereby exposing both lenders to default risk.

Formally, the consumers solve the following problem. After observing the state $\kappa$, and with contracts in hand, they strategically plan their future default decision $\delta(\kappa, C, C') \in \{0, 1\}$, with $\delta = 1$ implying default. This choice maximizes their indirect utility function given by

$$V(\kappa, C, C') \equiv \max_{\delta \in \{0, 1\}} V^\delta(\kappa, C, C').$$

where $V^\delta(\kappa, C, C') = u(G(c_1, c_2))$ stands for the indirect utility function conditional on default decision $\delta$, which we characterize next.

**Repayment.** In case of no default ($\delta = 0$), the budget constraints are, respectively,

$$c_1, \kappa + B + \rho(b, C, C')/2 = y + b, \text{ with } b \leq L,$$

$^4$Qualitatively, results are similar when $G$ is CES. These results are available from the authors upon request.
and
\[ c_{2,\kappa} + \rho(b, C, C')/2 + \kappa + b = y, \]
where \( \rho(b, C, C') \) denotes interest payments made to lenders, here evenly spread across the two periods.\(^5\) Such payments take into account balance transfers that are aimed to reduce the overall interest burden.

The crucial assumption of the model is that balance transfers are subject to delay \( 1 - \zeta \). That is, the budget constraint assumes that the first round lender receives a flow of interest for a fraction \( (1 - \zeta) \) of the period on total borrowing \( b \), while during the remainder of the period the second round lender is present and the first round lender only collects interest rate on the residual balance \( \max\{0, b - L'\} \). Accordingly, interest payments, given borrower beliefs about \( C' \), are given by

\[
\rho(b, C, C') = \begin{cases} 
(1 - \zeta)Rb + \zeta[R(\max(b - L', 0) + R' \min(L', b)] & b > 0 \\
0 & b \leq 0,
\end{cases}
\]  

(3)

where \( b < 0 \) implies saving. (Here note that, given that \( \text{wlog } R > R' \), the decision to transfer balances is mechanical: the consumer will transfer as much debt from \( C \) to \( C' \) as possible, i.e., up to \( L' \).

To simplify our notation, let \( b_\kappa \) denote \( b(\kappa, C, C') \). By the first order condition, applicable whenever \( L \) is non-binding, borrowing is given by

\[
b_\kappa = \frac{B - \kappa - \rho_b(C, C')/4\mu}{2}
\]  

(4)

where \( \rho_b \) stands for the partial derivative of \( \rho \) with respect to \( b \), i.e., the marginal interest rate. In this case, the marginal interest rate is equal to \( R \) whenever the balance transfer is incomplete, and \( (1 - \zeta)R + \zeta R' \) when the initial line is subject to a full balance transfer. The borrowing policy implies an interest rate distortion of intertemporal consumption smoothing given by the quadratic term \( \mu(c_{2,\kappa} - c_{1,\kappa})^2 = \mu \left( \frac{\rho_b}{4\mu} - \kappa \right)^2 \).

\(^5\)This is done for tractability and has no substantive bearing on our results.
**Default.** Upon default, the consumer can discharge both the principal and interest. Thus, the first period budget constraint takes the form

\[ c_{1,\kappa} + B = y + b_{\kappa}; \]

while the second period budget constraint reflects the fact that, subject to penalty for defaulting proportional to income and given by \((1 - \theta_{\kappa})y\), the consumer maxes out on both credit lines prior to discharging all credit card debt. The borrower can also discharge fraction \(1 - \phi\) of the distress shock. This implies:

\[ c_{2,\kappa} + b_{\kappa} = \theta_{\kappa}y - \phi\kappa + L + L'. \]

Here we also make a natural assumption that income after defaulting, net of shocks, is higher in normal times than in distressed times, i.e. \(\theta_0 y \geq \theta_x y - \phi x\). As we comment below, such restriction does not affect our results qualitatively.

Our setup implies that the default decision is effectively governed by a comparison of intertemporally aggregated consumption along each path (default/repayment), both endogenously determined in equilibrium. For this reason, it is not possible to fully characterize it. Nevertheless, we can show that, given any fixed set of contracts, punishment for defaulting induces a well-defined bound on consumers’ aggregate access to credit \(L + L'\), which we refer to as borrower’s *credit capacity*. Above this bound, the consumer decides to default even if she is non-distressed, and below this level she repays when she is not hit by the shock. Notice that a low capacity, e.g., due to small default penalties, can severely constrain credit. On the other hand, it can also serve as a protection against prepayment risk. This is because it mitigates the exposure of initial contracts to entry by a second round lender. It does so by limiting the residual capacity that the second round lenders can utilize, without tipping the borrower into (strategic) default. The existence of a finite credit capacity is irrespective of whether default penalties are pecuniary (our focus on pecuniary penalties is for analytical convenience). Furthermore, as the next lemma shows, credit capacity (when \(\kappa = 0\)) decreases w.r.t. \(L\) at least one-to-one, implying that a higher credit limit \(L\) ‘crowds out’ future balance.
transfers \((L')\) more than one-to-one. This feature will be important in the rest of the paper.

**Definition 1.** (credit capacity) Given \((L, R, R') \geq 0\), \(\mathcal{L}_{\text{max}}(\kappa; L, R, R')\) represents the total credit limit such that \(V^0(\kappa, C, C') < V^1(\kappa, C, C')\) for all \(L' + L > \mathcal{L}_{\text{max}}(\kappa; L, R, R')\) and \(V^0(\kappa, C, C') \geq V^1(\kappa, C, C')\) otherwise.

**Lemma 1.** \(\mathcal{L}_{\text{max}}(\kappa; L, R, R')\) is bounded for all \(L\), and decreasing in \(L\) for all non-binding \(L\).

Unless otherwise noted, all proofs are in the Appendix. To ease notation, we will write \(\mathcal{L}_{\text{max}}(L, R)\) to denote the capacity associated to \(\kappa = 0\) and \(R' = 0\).

### 2.3 Lenders

Lenders maximize expected profits, and compete in a Bertrand fashion. They have deep pockets and, for simplicity, their cost of funds is normalized to zero. When a consumer defaults, because she maxes out on all available credit lines, lenders incur a loss equal to the credit limit they granted to this consumer. Specifically, for any set of contracts \(C\), and \(C'\) obeying \(R > R'\), the profit function of the first round lender is given by:

\[
\pi(\kappa, C, C') = (1 - \delta(\kappa, C, C'))R[\zeta \max\{b_\kappa - L', 0\} + (1 - \zeta) \max\{b_\kappa, 0\}] - \delta(\kappa, C, C')L, \quad (5)
\]

while the profit function of second round lenders is given:

\[
\pi'(\kappa, C, C') = (1 - \delta(\kappa, C, C'))\zeta R' \min\{L', b_\kappa\} - \delta(\kappa, C, C')L'. \quad (6)
\]

The profit functions highlight an important implication of nonexclusivity of contracts in our environment. Namely, higher credit limit \(L\) here reduces the size of future balance transfers, given by \(\mathcal{L}_{\text{max}}(.) - L\). As a result, high credit limits can shield first round lenders from the threat of entry.\(^6\) Importantly, this crowding out feature is only present under an incomplete balance transfer, i.e., in the case of contracts satisfying the condition \(b_0 > \mathcal{L}_{\text{max}} - L\). In any other case, a (small) increase in \(L\) no longer crowds out future balance transfer offers that may be extended to these consumers. This distinction will be important.

---

\(^6\)Recall that, by Lemma 1, \(\mathcal{L}_{\text{max}}\) is decreasing in \(L\).
for our results. It will lead us to conclude that there are two possible types of credit lines exposed to default in equilibrium. Since competition is dynamic, and takes place in two rounds, in two distinct information sets, the equilibrium in the credit market is required to be subgame perfect. Applying backward induction, we define first the problem of second round lenders and proceed to discuss the problem of the first round lender.

**Ex-post lending (second round).** Bertrand competition implies that equilibrium contracts must maximize consumer’s indirect utility subject to zero (expected) profits. At this point, the realization of the distress shock is known to lenders. Accordingly, their choice is contingent both on $\kappa$ and on $C$, and formally involves the following maximization problem:

$$\mathcal{C}'(\kappa, C) = \arg \max_{C'} V(\kappa, C, C'), \text{ subject to } \pi'(\kappa, \cdot) \geq 0.$$ 

It should be clear from the above equation that the second round lenders best respond to $C$, and satisfy the requirements of the Bertrand competition, only if they charge interest rate equal to their cost of funds, and only if they extend credit up to the residual credit capacity $(L_{\max} - L)$.

Lemma 2. $\mathcal{C}'(\kappa, C) = (0, \max\{0, L_{\max}(\kappa; L, R, 0) - L\})$ for all $C = (L, R)$.

*Proof.* Omitted.

Remark 1. *It is important to stress out that the identity of the lender offering a balance transfer is irrelevant. In particular, it is possible to have initial lenders react to a balance transfer offer by improving the terms on the existing line. In both cases, balance transfer and line modification, the revenue of existing lines is undercut in the same fashion.*

---

7This best response may not be unique. If the agent transfers all her borrowing to $C'$ but does not fully utilize the line, any line with $L'$ between consumer’s borrowing level and $L_{\max} - L$ is also a best response. Since borrowing is given by (4), it does not depend on her beliefs about $L'$ as long as $L'$ is not binding, and all these contracts are equivalent. Thus, it is without loss to focus on this particular best response, which is unique when the balance transfer line is fully utilized.
Ex-ante lending (first round). Anticipating \( C'(\cdot, s) \), lenders choose first round contract \( C^* = (R^*, L^*) \) to solve

\[
C^* = \arg \max \limits_C EV(\kappa, C, C'(\kappa)), \text{ subject to } E\pi \geq 0. \tag{7}
\]

Since the above problem may not always have a solution with \( L^* > 0 \), we shall define a notion of a feasible first round credit limit. We will use this notion throughout.

**Definition 2.** We say that \( L > 0 \) is feasible if there exists an interest rate \( R \) such that the first round lender’s profits are non-negative when \( C \) satisfies Lemma 2 and consumers optimally choose borrowing and default given \( C = (L, R) \) and correct beliefs about \( C' \).

Our main result contrasts the properties of equilibrium in the presence of prepayment risk versus its properties without such risk, namely under the assumption of exclusivity of contracts. To that end, we denote the equilibrium contract and its associated borrowing level \( b_0 \) under exclusivity by \( C^0 = (L^0, R^0) \) and \( b^0 \), respectively. Incidentally, such contract is constrained efficient in the sense that it solves the problem of a benevolent planner who is restricted by the same market incompleteness as lenders are.\(^8\) Furthermore, we define \( L_{\text{max}} \) as the upper bound on \( L \) that can be feasibly offered under exclusivity; that is, the highest \( L \) that does not trigger default in normal times. Such upper bound is determined by the credit capacity and is thus the solution to a fixed point.\(^9\)

**Definition 3.** \( L_{\text{max}} \) is the credit limit satisfying \( L_{\text{max}} = L_{\text{max}}(L_{\text{max}}, R_{\text{max}}) \) when \( \zeta = 0 \), where \( R_{\text{max}} \) is the lowest interest rate yielding zero expected profits at \( L = L_{\text{max}} \).

Finally, to focus our analysis on an interesting case, we introduce two simplifying assumptions. The first assumption warrants that consumers always want to default when distressed; it also assures that contracts exposed to default risk are feasible under exclusivity. A sufficient condition for the former is that default penalties under distress, i.e. \( (1 - \theta)\pi_y \), are lower than the discharged portion of the shock, that is \( (1 - \phi)\kappa \). This assumption implies that our analytical results pertain to the case when lenders and borrowers opt for a contract that

\(^8\)Specifically, it can be shown that \( C^0 \) and \( C' = (0, 0) \) solve \( \max_{C, C'} EV(\kappa, C, C') \), s.t. \( E\pi \geq 0 \) and \( \pi' \geq 0 \).

\(^9\)Such a fixed point exists, given that \( V^0(0, \cdot) \) is continuous in \( L \) and \( R \) and bounded while \( V^1(0, \cdot) \) is increasing and continuous in \( L \), with \( V^0 > V^1 \) at \( L = 0 \).
exposes the lenders to a positive risk of default. We refer to such credit lines as *risky* lines, as opposed to a *risk free* credit lines – which in principle can also be offered in equilibrium.

**Assumption 1.** $\mathcal{L}_{\text{max}}(x, \cdot) = 0$. There exists a feasible $L$ when $\zeta = 0$.

The second assumption warrants that there exists a range of feasible credit limits under which consumers would not be credit constrained under exclusivity. If this was not the case, lenders would always offer the highest credit limit possible to relax the borrowing constraint under both exclusivity and non-exclusivity, making the problem trivial. Since full intertemporal smoothing under $\kappa = 0$ is achieved when the agent borrows $B/2$, the following condition assures that there exist non-binding credit limits in our model.

**Assumption 2.** $B/2 < L_{\text{max}}$.

As already mentioned, these assumptions allow us to focus our attention on the case when there is action in the model. In the case of risk free contracts, or binding borrowing constraints, the features that we introduce simply have no bite.

### 3 Characterization of Equilibrium

We next characterize the equilibrium of our model. Throughout, we use the exclusivity case as a benchmark, which, recall, corresponds to the case of $\zeta = 0$. This allows us to assess the impact of prepayment risk, which is the central focus of our paper.

The first proposition characterizes the types of contracts that may arise in equilibrium. The first type of contracts, referred to as *rollover* contracts, feature a complete balance transfer, yet break even thanks to the existence of a positive delay ($\zeta > 0$). The second contract type aims at ‘strategically’ crowding out the credit capacity of the borrower in order to prevent entry all along. Interestingly, such *crowding out* contracts can be sustained even without any delay, as future lenders cannot extend any credit without tipping the borrower into a ‘strategic default zone’, i.e., part of the state space when the borrower defaults regardless the realization of the shock. This, however, requires a not too large credit capacity, so as to limit the losses of first round lenders in case of default, given by $L$.

**Definition 4.** A zero-profit credit line $\mathcal{C}$ is a *rollover contract* if $b_0 \leq \mathcal{L}_{\text{max}}(L, R) - \mathcal{L}$. We say that $\mathcal{C}$ is a *crowding out* contract if $\mathcal{C} = (L_{\text{max}}, R_{\text{max}})$.
We now state the main analytical result. This result states that, whenever delay frictions are small but positive, the first round contracts are of the aforementioned two types.

**Proposition 1.** There exists $\zeta < 1$ such that for all $\zeta \in (\zeta, 1)$ the following is true:

(i) if $L^0 \leq L_{\text{max}}(L^0, \frac{R^0}{1-\zeta}) - b^0$ the first round contract is a rollover contract with $(L^*, R^*) = (L^0, \frac{R^0}{1-\zeta})$;

(ii) if $L^0 \in \left(L_{\text{max}}(L^0, \frac{R^0}{1-\zeta}) - b^0, L_{\text{max}}\right)$ the first round contract is either a rollover contract with $L^* = L_{\text{max}}(L^*, R^*) - b^0 < L^0$ or a crowding out contract;

(iii) if $L^0 = L_{\text{max}}$ then $(L^*, R^*) = (L^0, R^0)$.

Moreover, if $\zeta = 1$ then equilibrium involves either no credit provision ($L^* = 0$) or a crowding out contract.

The intuition why positive delay is critical for the sustainability of rollover contracts is fairly straightforward, although subtle. With delay, first round lenders can charge an interest rate high enough to fully make up for the lost revenue due to a later balance transfer. The subtle point is that, since entry is certain, the excess interest rate they need to charge has no effect on borrowing. This is what makes it possible always. Formally, this follows from the fact that, according to (4), borrowing $b$ is linear in $(1 - \zeta)R$ in the presence of a complete balance transfer. Now, since first period lenders’ profit is also proportional to $(1 - \zeta)R$, lenders can always front-load all the revenue that they would otherwise collect in the absence of prepayment risk by raising $R$. Since $(1 - \zeta)R$ is all that matters for the borrower, scaling up rates by $1/(1 - \zeta)$ leaves both borrowing levels and profits unaffected and, hence, is always feasible.\(^{10}\) In contrast, in the absence of any delay, revenue cannot be front loaded this way. In such a case, risky credit lines can only be sustained is by limiting the overall exposure to prepayment. The reason why the sustainability of crowding out contracts requires credit capacity that is not too large is easy to see. By definition, crowding out requires that the initial credit limit is close to or at the credit capacity. Since sufficiently high credit limits result in high default losses, this strategy may not be sustainable as there may be no interest rate to sustain such contract (as at such high interest rates consumers borrow too little to cover expected default losses).

\(^{10}\)When $C$ is subject to a full transfer, $E\pi = (1 - p)(1 - \zeta)Rb_0 - pL$, and $b_0 = \frac{B}{2} - \frac{(1-\zeta)R}{8\mu}$. 

15
In addition to the above characterization of contracts, Proposition 1 establishes that rollover contracts may actually implement the equilibrium allocation under exclusivity, which is constrained efficient in this environment. This is the case whenever setting \( L = L^0 \) leads to a full balance transfer. The intuition is similar to the one given above regarding the sustainability of debt. As long as the balance transfer is full, initial lenders can mimic the contract that would have been optimal under exclusivity. This is accomplished by simply scaling \( R^0 \) up by a factor \( 1/(1-\zeta) \). Such adjustment does not affect the borrowing choice of the consumer, as the marginal rate on debt that she faces is equal to \( R^0 \). While this invariance is particular to the quadratic functional form of intertemporal preferences, it illustrates how, in general, lenders can accommodate future balance transfers by front-loading revenue.\(^{11}\)

Proposition 1 also shows that when \( L^0 \) leads to an incomplete balance transfer, then either a rollover contract with an inefficiently low credit limit or a crowding out contract exhibiting the highest credit limit possible (\( L_{\text{max}} \)) will be offered in equilibrium. Which one is chosen depends on consumer borrowing needs (\( G \) and \( B \)) and risk aversion (\( u \)). As we show in the example below, high borrowing needs and/or high insurance needs typically select crowding out contracts, whereas low borrowing and insurance needs lead to rollover contracts.

Importantly, when balance transfers are incomplete, crowding out contracts always prevail, i.e., \( L < L_{\text{max}} \) will never be offered in equilibrium (unless it is a rollover contract). The intuition is as follows. Under an incomplete balance transfer, the marginal interest rate faced by the consumer is \( R \), rather than \( (1-\zeta)R \). In this case, as we explain below, lenders have an incentive to increase the credit limit all the way to \( L_{\text{max}} \), even though this may sound counterintuitive. This is because the benefit from crowding out future balance transfers outweighs the loss caused by consumers defaulting under distress on a higher credit limit, as long as delay frictions are small. This allows lenders to lower the marginal interest rate \( R \), thereby reducing the distortion of intertemporal smoothing in the case of \( \kappa = 0 \). At the same time, the increase in \( L \) raises consumption levels under \( \kappa = x \). Overall, the consumers like such a change, as it gives them more generous insurance against the distress shock and reduces the distortion. Notably, this negative relationship between interest rates and credit

\(^{11}\)One could think of quadratic \( G \) as a Taylor approximation of standard preferences. In fact, we calibrate it in the quantitative model to approximate a CES utility function.
limits contrasts with the exclusivity case, in which higher default losses must necessarily be offset by higher interest rates. In general, crowding out may be incomplete, but only when defaulting on $L = L_{\text{max}}$ would imply higher consumption under distress than under no distress, which here we rule out by the aforementioned assumption that income after defaulting, net of shocks, is higher in normal than in distressed times.

To better illustrate lenders’ incentive to crowd out future entry, focus for a moment on the case of $\zeta = 1$. In such a case, observe that the benefit of increasing $L < L_{\text{max}}(L, R)$ comes from the fact that $L'$ declines at least one-to-one by Lemma 1. As we can see from (5), the increase in revenue caused by raising $L$ by $\Delta$ is proportional to the product of the interest rate $R$ and the probability of repayment $(1 - p)$. Specifically, since $L'$ goes down by at least $\Delta$ and $b_0$ only depends on $R$, the increase in revenue is at least $(1 - p)R\Delta$. The cost of such an increase in $L$ is, on the other hand, proportional to the probability of defaulting $p$ and given by $p\Delta$. Thus, the benefit outweighs the cost whenever $R(1 - p) > p$, implying the above result. It turns out that this condition is always true since $R$ must be higher than the lowest possible rate at which initial lenders could possibly break even. In the case of a frictionless entry, i.e. $(\zeta = 1)$, we know that $(1 - p)R(b_0 - L') \geq pL$ for lenders to break even, or equivalently $(1 - p)R(b_0 - L')/L > p$. Finally, since $b_0 \leq L$, we must have that $(1 - p)R > p$. Hence, lenders can offer a higher $L$ and a lower $R$ as long as $b_0 > L_{\text{max}}(L, R) - L$.

Finally, the crowding out motive applies as long as the delay friction is not too high. Actually, the bound on the delay friction is a function of pre-existing debt and credit capacity and is typically quite slack – as we show in the proof of Proposition 1, $\Delta \leq B_{L_{\text{max}}}$. To conclude the discussion of Proposition 1, we next present an example that illustrates the properties of equilibrium contracts in our model.

Leading example: utility $u$ is CES with elasticity $\sigma = 2$ and the curvature of inter-temporal aggregator $G$ is $\mu = .6$, which is approximately equivalent to a CES aggregator with elasticity of 2. Income $y$ is normalized to 1 and default costs are 35% of income in case of no distress and 5% of income in the case of distress ($\theta_0 = 0.65, \theta_x = 0.95$). The probability of the distress shock is 5%, and its size is equal to 40% of income ($x = 0.4$). 50% of this shock is defaultable ($\phi = .5$). The entry delay is set equal to 1/4 of a period ($\zeta = 0.75$).
In this example, the lower bound on delay frictions above which Proposition 1 applies is \( \zeta = 0.6 \) for a pre-existing debt of 40% of income \((B = 0.4)\). For such debt level the equilibrium contract is actually a crowding out contract exhibiting a credit limit \( L^* = \mathcal{L}_{max}(R^*, L^*) = 0.332 \), which is more than 25% higher than the (constrained efficient) limit under exclusivity \( L^0 = 0.266 \). This is also highlighted by the difference in utilization rates \((b_0/L)\): while 69% of line \( C^0 \) is utilized, in equilibrium the utilization rate is only 54%. The oversized credit line leads to an inefficiently high charge-off rate \((pL/(pL + (1-p)b_0))\) of 8.8% in equilibrium versus 7.1% under exclusivity.

The switch between rollover and crowding out contracts happens at a debt level of roughly 36.2% of income. That is, agents with \( B < 0.362 \) go for a rollover contract, while agents with \( B > 0.362 \) prefer a crowding out contract. For instance, a consumer with \( B = 0.25 \) will be offered a rollover contract here, characterized by credit limit \( L^* = L^0 = 0.180 \), which is much lower than the credit capacity of 0.339 and thus leaves enough room for a full balance transfer. Such line exhibits an utilization rate of 59%, implying that consumer insurance needs affect contract selection, since they are willing to pay a higher interest rate in order to max out on a bigger credit limit when under distress — after defaulting, (aggregated) consumption under distress is 91% of consumption in normal times, compared to only 73% in the absence of credit markets.

We next proceed discuss two important implications of Proposition 1. The first pertains to the distribution of interest rates in equilibrium of the market as a whole. It shows that increases in the intensity of competition are generally associated with a growing dispersion of interest rates, even if the pricing of default risk is perfectly precise (which it is the case in our model, given there is no ex ante asymmetric information). Such growth in dispersion is a feature of the US data, which has thus far been interpreted as evidence that lenders price default risk more precisely. While this still may be the case, here we show that the fact that balance transfers have become a staple of the credit card market may be a contributing factor to the observed growth in rate dispersion. The second implication is that, while rollover contracts greatly enhance sustainability of defaultable debt despite contract non-exclusivity and fierce ex-post repricing, this work-around so to speak comes with strings attached. Specifically, it creates a rollover risk that manifests itself as a hike in interest rates faced by consumers when
credit supply is disrupted in some way. As such, it may accelerate deleveraging in response to such shocks or push consumers into default.

**Interest Rate Dispersion.** Prepayment risk has important implications for the pricing of credit card debt. Specifically, as the next corollary states, defaultable debt exhibits higher interest rates compared to those under exclusivity. This, combined with the low introductory rates associated to balance transfer offers, implies a ‘spreading out’ of the distribution of interest rates as prepayment options became more prevalent, something that happened during the 90 and 00s as illustrated in Figure 1. Such spreading out follows a particular pattern, which is predicted by our model: The rise of a left tail about rates equal to costs of funds associated to balance transfer offers and a fatter right tail associated to interest rates of contracts exposed to prepayment risk. The latter follows from two different channels. First, revenue front loading associated with rollover contracts requires lenders to set interest rates that are proportional to \( \frac{1}{1 - \zeta} \). Hence, for small \( \zeta \), rollover contracts exhibit very high rates. Second, high exposure to default losses exhibited by crowding out contracts also needs to be priced in the interest rate, leading to \( R^* > R^\emptyset \). Both features lead to more dispersion, and the gap between crowding out and rollover contracts is proportional to \( \zeta \).

This feature is interesting, as it challenges the conventional view that growing interest rate dispersion solely represents better risk pricing. This point has been forcefully made in the literature, and there is even some empirical work that notes more pronounced spread between interest rates and risk characteristics of borrowers. In our case such spread can arise too, but for a very different reason: riskier customers are more likely to be approached by future lenders after a perceived decline in their risk, since they are the ones that stand to gain more from a balance transfer.

**Corollary 1.** If delay friction is small, then \( R' < R^\emptyset \leq R^* \), with strict inequality whenever \( L^\emptyset < L_{\text{max}} \).

Figure 3 illustrates the distribution of interest rates (i.e., the rate on the biggest credit card balance) in our leading example; this is derived for a population of agents among which 50% have pre-existing debt \( B = 0.25 \) and the other 50% have \( B = 0.4 \). In the case of balance transfers and rollover contracts the prevalence is weighted proportional to the delay \( \zeta = 0.75 \).
The figure shows that while the distribution of rates is concentrated around 8\% in the absence of prepayment risk ($\zeta = 0$), it becomes quite dispersed with a left tail at the balance transfer rate and a ‘fat’ right tail associated to the rate on rollover contracts (35.7\%), with the rate on crowding out contracts is somewhere in-between (9.7\%). The introduction of prepayment risk also raises the average interest rate on debt, which goes up from 8.25\% to 9.3\%.

![Figure 3: Rate Distribution with (dashed) and without Prepayment Risk (solid).](image)

**Macroeconomic Fragility.** Our leading example also highlights another important feature the model, namely, the macroeconomic fragility of the unsecured credit market. To see why, note that prepayment risk, from the consumer side, induces debt rollover risk, if entry of future lenders is not set in stone. Specifically, note that in the case of rollover contracts consumers borrowing is sustained by the mere anticipation of repricing. Consumers accept first round contracts only because of repricing, since they are characterized by onerous interest rates. However, what if, due to a major credit supply disruption, such opportunity does not materialize in the marketplace? The effect is not difficult to predict, and we explore it further in our quantitative analysis of the model. Consumers will be forced to deleverage, and if the model was extended to incorporate a continuous distress shock, some of them would opt
to default. This mechanism is clearly reminiscent of the issues that arose in the sub-prime mortgage market, namely a wave of defaults arguably exacerbated by contract terms that were simply rigged to force refinancing after a few years. When such opportunities did not arrive due to the ongoing credit crunch, a self-propelling wave of defaults followed. Independently, oversized credit limits associated with crowding out contracts lead to an excessive debt discharge by distressed consumers. Consequently, a higher occurrence of distress due to macroeconomic shocks may lead to inefficiently high charge-offs on these contracts, relative to what these consumers would obtain under exclusivity.

To illustrate the key mechanism behind this prepayment-induced macroeconomic fragility, consider again our leading example assuming that half of consumers hold rollover contracts and half crowding out contracts. The charge-off rate in such a case is 8.6% and the outstanding credit card debt, net of charge-offs, is 13.6% of income. Now, imagine the following scenario: after the first round contracts have been accepted, half of consumers with rollover contracts learn that they will not get a balance transfer offer (credit shock); and, simultaneously, the probability of the distress shock jumps from 5 to 7% (macroeconomic shock). Clearly, the macroeconomic shock by itself raises the charge-off rate to 11.9%. However, since the credit shock also leads to a reduction of borrowing levels for those with rollover contracts, it results in both substantial deleveraging and a further increase in the charge-off rate due to lower outstanding credit card balances. In particular, in our leading example consumers would reduce their balances from 10.6% to 5.1% of income, causing average outstanding debt to drop to 12.0% of income. This implies a deleveraging of more than 12% and leads to a final charge-off rate of 13.0%. In contrast, deleveraging under exclusivity would be just 2% and the charge-off rate would increase by just 2.9 percentage points (from 7.5% to 10.4%).

We conclude our discussion by pointing out that changes in the regulatory environment in our model that critically affect credit capacity change the nature of contracts in equilibrium and thus may lead to outcomes unintended by regulators. A relevant example is the recently introduced means testing regulation, introduced as part of the bankruptcy law reform of 2005. According to our model, the implied increase in default penalties can be beneficial if \( \theta_0 \) was inefficiently low so that it restricted borrowing and thus consumption smoothing. However, our model implies that it can also exacerbate the detrimental effects of balance transfers and
deepen the aforementioned market fragility. None of these predictions arise in the standard models. A more detailed analysis is warranted and we leave it for future research.

4 Quantitative Analysis

This section develops a quantitative version of our model. The goal is twofold. First, we want to demonstrate that under plausible conditions our model can account for the level of balance transfers and interest rate dispersion seen in the data. Second, using the calibrated model, we explore how much of the observed deleveraging during the financial crisis our model can explain. Specifically, we model the crisis by increasing the frequency of the distress shock, and assume that repricing opportunities unexpectedly dry up, quantitatively in consistency with the decline in credit card solicitations observed in the U.S. data (in 60% of cases the repriced contract does not arrive). By restricting the credit supply shock to only affect balance transfer opportunities, our goal is to quantify the contribution of the discussed above repricing channel alone.

Our quantitative results show that, first, a two year (partial) disruption of repricing opportunities can result in a hike in interest rates paid on credit cards that matches quite well the increase observed in the data. Second, we show that our model can come close to fully matching the extent of the deleveraging observed during the first two years of the recent financial crisis. In contrast, we show that none of these features can be matched by a counterfactual model that assumes exclusivity of credit card contracts and features no prepayment risk.

4.1 Multi-period Life-cycle Model

The quantitative extension is fairly standard and all life-cycle features of the model follow closely the model by Livshits, MacGee and Tertilt (2010). Within each period, the setup is almost identical to our earlier two period model. For this reason, we streamline the exposition and highlight only the less obvious aspects below.

In our specification of the model, consumers live for 27 (long) periods, each comprised of two subperiods. The first 22 (long) periods are working age periods. During this time consumers are subject to stochastic income $y$, which remains fixed within the period. Income
follows a Markov process, and in retirement periods $y$ is a deterministic function of realized income in the period right before retirement $y_{22+t} = f(y_{22})$, $t = 1, 5$. Relative to our earlier setup, $B$ is now endogenous.

Within each long period, we embed our two-period model with some changes which connect the periods. Specifically, we allow the consumer to carry over debt into the future. Formally, the second period budget constraint is now replaced by:

$$c' = Y + B' - b - \rho(b, C, C'),$$

where $B'$ is the debt that the consumer carries over into the following long period. The rest is the same. The value functions that describe consumers involve $B$ and $y$ as state variables. Specifically, we assume that a consumer who does not have a default flag on record at the interim stage (after she sees $\kappa$), and decides not to default in the current period, solves the following dynamic program:

$$V_0^0(B, y) = \max_{c_1, c_2, B'} \{u(G(c_1, c_2)) + E\beta V_{t+1}^0(B', y')\}$$

where $c_1, c_2$ satisfy the budget constraints implied by the equilibrium contracts $C, C'(C, \kappa)$ in interim state $(B, y, \kappa)$ and $V_{t+1}^0$ denotes the continuation value of an agent with no default on record.

In the above problem, we assume that consumers choose $c_1, c_2$ within each period, and face a budget constraint quite similar to the two-period model. The value function $V_1^1$ for a consumer who chooses to default is defined analogously, and therefore omitted. The ex-ante value function $V_t$ is given by equation (1).

The cost of defaulting involves a one period exclusion to autarky, in addition to the pecuniary cost of defaulting discussed earlier. In autarky, the consumer can save but cannot borrow. Furthermore, if the consumer experiences another distress shock when she is excluded from the market, she is able to rollover a fraction $\phi$ of the shock at a penalty interest rate $\bar{r}$ to the next period. The remainder of the shock, $(1 - \phi)\kappa$ must be absorbed in current consumption. In the next period the default flag is removed and the consumer starts fresh with $\delta = 0$ and $B = (1 + \bar{r})\phi\kappa$. 

23
4.2 Parameterization

We require our model to be consistent with some of the key moments in the US data for the year 2004. We have chosen 2004 as the baseline year because of a major bankruptcy reform in 2005 and the subsequent financial crisis.

The model period is two years long and each subperiod is one year long. Since default statistics reported in the data are annual, while default in our model can only happen once per two years, measurement must be appropriately adjusted. To this end, all model implied statistics that are flow variables, such charged-offs or default rates, are consequently divided by two. The only exception from this rule are balance transfers. This is because balance transfers pertain to the dynamics across all periods, whereas in our model consumers only have one such opportunity before the contract expires. This is obviously imperfect and it is a price we pay to include the repricing in a tractable manner.

We next describe our data targets and how we select the key parameters.

**Parameters selected arbitrarily.** Our model is complex and there are many parameters that we need to calibrate. To this end, we start from a set of parameters for which we lack a good data target. We choose values that we find reasonable. This set includes: the entry delay parameters \( \zeta \) and the penalty interest rate \( r \). In the benchmark model, we set \( \zeta = 0.75 \). That is, we assume that entry of ‘poachers’ is delayed on average by six months. We also consider the case of exclusivity \( (\zeta = 0) \). This counterfactual economy is parameterized analogously. Finally, the penalty interest rate is set equal to 35% per annum. We also assume that \( u \) is CES with risk aversion coefficient \( \sigma = 2 \).

**Parameters independently selected to match the data.** We calibrate the income process \( y \) and the distress shock \( \kappa \) using income and distress data reported by Livshits, MacGee and Tertilt (2010). Specifically, starting from the usual annual AR(1) process for income taken from Livshits, MacGee and Tertilt (2010), we reallocate any income drop of at least 25% to our distress shock. We then use the resulting income process to obtain a biannual Markov process using the Tauchen method, which gives us the 6x6 transition matrix \( P \) and values for the associated income grid points. We start our simulation from the
ergodic distribution of this Markov process.

Accordingly, the distress shock $\kappa$ absorbs negative income shocks of 25% or more. In addition, we augment it by including three major lifetime expense shocks singled out as important by Livshits, MacGee and Tertilt (2010). These include medical bills, the cost of an unwanted pregnancy, and divorce costs. We use their estimated values and appropriately adjust them to obtain a single biannual distress shock. The procedure gives us $x = 0.4$ (40% of median annual household income), and shock frequency of 7.7% on a biannual basis ($p = 0.077$).

We consider medical bills to be the only shock that household can directly default on, and consequently set $\phi = 0.24$. This approach departs from the usual practice of treating the distress shock as almost fully defaultable. In contrast to the literature, in our model only a small fraction of the shock is actually accounted for by medical bills, which results in a largely ‘non-defaultable’ distress shock. It is well known that such, arguably more realistic, approach makes it very difficult for this class of models to account for the relatively high frequency of default in the data. To account for default related statistics, we emulate the endogenous punishment implied by the mechanism in Drozd and Serrano-Padial (2013). These authors argue that, when the predominance of informal default over formal bankruptcy filings in the data is taken into account, the pecuniary cost of default is endogenously state contingent, allowing to sustain more gross debt that is still frequently defaulted on.

**Parameters jointly selected to match the data.** We calibrate the remaining parameters to match the key characteristics of the US credit card market in 2004, such as the charge-off rate, the debt-to-income ratio, the balance transfer rate, and the debt-defaulted-on-to-income ratio per defaulting household. The calibration is joint because most parameters affect several targets at the same time. The parameter values that we choose this way include: $r, \beta, \theta_0, \theta_1$. The associated targets are listed in Table 1, along with their calibrated values in the model. Since we only have data for the fraction of balances transferred in 2002 (17%), we use the average annual growth rate on balance transfers reported by Evans and Schmalensee (2005) in order to obtain the approximate value for 2004, which is set at 20%.
Table 1: Data moments characterizing the US credit card market

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data target</th>
<th>Model</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Non-</td>
<td>Exclusivity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Exclusivity</td>
<td>(ζ = 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(ζ = .75)</td>
<td></td>
</tr>
<tr>
<td><strong>A. Targeted moments</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC debt to disposable income</td>
<td>9.2%</td>
<td>9.2%</td>
<td>9.2%</td>
<td>9.2%</td>
</tr>
<tr>
<td>Net charge-off rate</td>
<td>4.7%</td>
<td>4.65%</td>
<td>4.7%</td>
<td></td>
</tr>
<tr>
<td>Debt discharged to income per defaulting hh(^a)</td>
<td>93%</td>
<td>93%</td>
<td>60%</td>
<td></td>
</tr>
<tr>
<td>Average interest rate on cc debt(^b)</td>
<td>10.95%</td>
<td>10.66%</td>
<td>10.93%</td>
<td></td>
</tr>
<tr>
<td>Balance transfer rate per annum(^c)</td>
<td>20%</td>
<td>20%</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td><strong>B. Endogenous moments</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual frequency of default per 1000 persons(^d)</td>
<td>8.6/1000</td>
<td></td>
<td>9/1000</td>
<td>9/1000</td>
</tr>
<tr>
<td>Interest rate dispersion (coef. variation)(^e)</td>
<td>64% in 2004, (\approx 15)% in 1990</td>
<td></td>
<td>56%</td>
<td>15%</td>
</tr>
</tbody>
</table>

Data values pertain to US data for year 2004, unless otherwise noted.

\(^a\) Target consistent with data for formal bankrupts in Sullivan, Westbrook and Warren (2001).

\(^b\) Debt weighted interest rate on revolving accounts assessing interest rate. Data from Federal Reserve Board.

\(^c\) Data from Evans and Schmalensee (2005) for 2002, extrapolated to 2004 based on approximate historic annual rate of growth.

\(^d\) Formal bankruptcy filings; includes Chapter 7 and chapter 13, per 1000 persons 20 years old or older. Data from American Bankruptcy Institute. May include some business filings, especially chapter 13. On the other hand, does not include informal bankruptcies, which our model also captures by targeting the net charge-off rate.

\(^e\) Authors' calculations using SCF data on interest rates on the main/most recent credit line of revolvers. Coefficient of variation in the data is not weighted by debt, in the model it is weighted. Results were similar when we did not weight by debt, and so we decided to report the weighted statistic. The number for 1990 is extrapolated using regression analysis based on the available time series 1995-2004.
4.3 Sample Simulation

The calibration of our model directly aims at delivering a balance transfer rate of about 20% per annum, among other targets. The ability of our model to deliver this target endogenously is remarkable. It is far from obvious that a mere presence of an option to transfer balances will lead to this option being exercised so frequently in equilibrium. The high balance transfers in the data, when interpreted through the lens of our model, simply implies that rollover contracts are quite prevalent.

To illustrate the workings of our calibrated model, Figure 4 shows a sample simulation of a single agent. The top panel shows the evolution of income of this agent, with “x” denoting distress shocks. This is a relatively low income agent, who experiences one distress shock during her lifetime. This shock triggers a default early on. The bottom panel plots the credit terms that this agent faces over the life-cycle. Specifically, the figure plots the actual credit limit $L$ offered in equilibrium, the balance transfer offer $L'$, as well as the counterfactual credit limit $L^\theta$ lenders would offer to this agent under exclusivity. The label “-” identifies cases when the underlying contract is exposed to a positive risk of default, i.e., the agent would have defaulted had she experienced a distress shock. As we can see from the figure, balance transfers (i.e., $L' > 0$) are quite prevalent here. In fact, as much as 22% of this agent’s risky credit contracts are exposed to prepayment risk. Incidentally, crowding out contracts are used even more frequently in this case (61% of risky contracts are crowding out contracts). In the overall population of consumers, crowding out contracts are less frequently used. Specifically, among all contracts exposed to default risk, 33% are rollover contracts and about 33% are crowding out contracts exhibiting credit limits at least 3% higher than $L^\theta$.\(^\text{12}\)

4.4 Cross-Sectional Implications

Table 1 reports the key endogenous implications of our model. The table compares two cases: Non-exclusivity featuring $\zeta = .75$ and the exclusivity benchmark ($\zeta = 0$). The latter regime is parameterized analogously to the baseline model.

As we can see, the non-exclusivity model matches well the dispersion of interest rates

\(^{12}\)The remaining 34% of contracts exposed to default risk are crowding out contracts with $L = L_{max} \approx L^\theta$. 

27
for year 2004. Moreover, the extrapolated value of interest rate dispersion for year 1990 in the data turns out remarkably close to the dispersion implied by the exclusivity case. We think of this result as reflecting the informational revolution in the unsecured credit market, allowing lenders to track almost in real time the creditworthiness of each borrower. With better technology both precision and availability of information improved, which through the lens of our model could be interpreted as an increase in $\zeta$. This view fits the evolution of the balance transfer rate data pretty well: according to Evans and Schmalensee (2005) balance transfers were almost nonexistent back then. In this vein, the comparison across regimes shows that the rise in interest rate dispersion in the US credit card market may be a by-product of intense competition through ex-post repricing.

Our model also accounts for the level of total bankruptcy filings in the US. However, an important caveat applies here. Namely, consumers often default informally in the data, which means that they do not pay their debt yet do not formally file in court. Consequently,
as a measure of the overall default rate, this data target is likely understating the facts. Nevertheless, we are quite confident that we could match a higher level of default, had we lowered our target for the average debt defaulted on per bankrupt. Existing studies suggest that informal bankrupts default on smaller amounts. At this point more precise data is required to get a good handle on the frequency and nature of informal bankruptcy in the US and so we decided to follow the conventional approach.

4.5 Default and deleveraging During the 2007-09 Crisis

One of the key implications of our theory is that balance transfers distort credit contracts offered in equilibrium. In particular, a key effect of balance transfers is that they introduce a rollover risk for consumers. This is because high balance transfer rate is only consistent with rollover contracts in our environment, and such contracts feature onerous interest rates that are expected to be repriced in the future. The rollover risk may materialize if some disruption of credit supply precludes lenders to offer these opportunities to existing borrowers. In such a case, the may face a hike in the interest rate charged on debt. Such hike can either increase the default rate or accelerate repayment of existing debt. In our model it is mostly manifested in the latter, since we only have two states, distress and no distress. If such a credit crunch is aggregate in nature, it may lead to a counter-cyclical deleveraging in the economy, despite the fact that maturity of credit card contracts is relatively long (5-6 years).

Given the above property of the model, we explore the potential of our model to explain some of the key changes in the credit card market observed during the recent financial crisis. In particular, we ask whether it can replicate the sharp decline in credit card debt to income and the increase in interest rates during the crisis. We do so in consistency with the dramatic decline in credit card solicitations and balance transfer offers, which in our model we link to a decline in repricing opportunities.

To set up our quantitative exercise, we target a change in three moments in the data between 2007 and 2010 (peak to trough change). We do so by introducing two unanticipated changes from the agent’s point of view: An increase in the frequency of the distress shock \( p \) and a positive probability \( 1 - \xi \) of disrupted entry of second round lenders in the middle of the period (i.e., the second round happens with probability \( \xi \)). This first parameter is chosen
to match the change in the charge-off rate; the second parameter is reduced from 1 to 0.42 to match the drop in credit card solicitations of 58% in the data. Both shocks shock only last one period.

Figure 2 summarizes the key results implied by this experiment. In the figure we similarly compare the predictions of the non-exclusivity case to the benchmark exclusivity case. Both are set up similarly in terms of calibration. The only parameter that is different across these two cases is the discount factor $\beta$ and the risk free interest rate $r$, as the exclusivity case generally leads to more debt-to-income in equilibrium due to cheaper credit (recall that $C^0$ exhibits lower marginal rates than crowding out contracts).

As Figure 2 shows, the model under exclusivity falls short of rationalizing the observed deleveraging and does not capture the increase in interest rates. This is because the credit shock only affects the availability of balance transfer offers, without having any impact on first round contracts. In contrast, the benchmark model fits the data almost perfectly. Specifically, debt-to-income falls from 9.2% to 8.4% in the model, compared to the drop from 9.2% to 8.3% in the data, whereas in the case of exclusivity the drop is only to 8.7%, as it is solely driven by the spike in charge-offs caused by the macroeconomic shock. The ability of our model to match the observed deleveraging is due to the added impact of some agents being ‘stuck’ with rollover contracts and thus borrowing less than expected. This channel represents about 16% of the overall deleveraging in the model, the rest being driven by the sharp increase in debt charged off due to higher default rates. Most importantly, entry disruption makes the average interest rate on existing debt jump by 2.4 percentage points, almost exactly matching the observed change in the data (2.5 p.p.). For obvious reasons, this is not the case under exclusivity.

The effect of disrupting repricing is also fairly asymmetric in our model. The 2.5 p.p. increase in the average interest rate stems from an interest rate hike faced by only about 12% of revolving contracts. Such rate hike is on average about 16 p.p., leading these agents to borrow 8% less on average.

These results highlight the importance of the rollover risk created by balance transfers and demonstrate that the induced market fragility can be quantitatively relevant given the observed level of balance transfers in the US. It is important to emphasize that the disruption
of credit supply here merely affects the ability of consumers to reprice credit card debt within just one period. In spite of this its performance is fairly remarkable. Note that we do not reduce access credit during the crisis period and in the period thereafter. We consider this as a realistic way of introducing credit supply shocks, given the relatively long maturity of credit card contracts in the data, and the fact that banks after 2009 were no longer allowed to force repayment nor increase interest rates on pre-existing debt. In such context, a change in households’ time preferences or the lack of continuation contracts beyond expiration of existing ones seem the only plausible remaining alternatives to the short-term disruption of debt repricing explored here.

5 Conclusions
We have developed here a model that helps understand the mechanism behind the high volume of balance transfers observed in the U.S. credit card market. Through the lens of our model, they can arise in equilibrium because lenders front-load interest revenue. As we point out, this has important consequences. In particular, on the consumer side, it effectively shortens the maturity of contracts. As a result, whenever credit supply is disrupted, a credit crunch arises despite the fact that contracts per se are of much longer maturity. We have demonstrated the effects of such a shock in the context of the recent financial crisis. Our model also implies a novel mechanism that generates equilibrium dispersion of interest rates. We also have provided an explanation for why balance transfers have increased so much over the last few decades, this is related to much better information technology lenders have access to, which among other things, intensifies ex-post repricing of the kind that we model here.
Appendix

A1. Proof of Proposition 1

Part (i) immediately follows from the argument in the text. Part (iii) is obvious: if the credit capacity constraints lenders under exclusivity then first round lenders offer the same contract when $\zeta > 0$.

We prove Part (ii) by laying out the proof argument using a series of Lemmas that are then proved below. The proof goes as follows.

First, focus on the aggregated consumption combinations $(c_0, c_x)$ that involve positive credit provision $(L > 0)$, where $c_\kappa = G(c_{1,\kappa}, c_{2,\kappa})$ are candidates for equilibrium aggregate consumption under exclusivity. We call such set of candidates the exclusivity consumption frontier (ECF). Since the agent defaults under distress by Assumption 1, $c_x$ is increasing in $L$ and independent of $R$, so the ECF is obtained by increasing the credit limit $L$ and adjusting the interest rate so that lenders make zero profits. The solution to (7) when $\zeta = 0$ must then maximize consumers’ expected utility, which is represented by convex indifference curves on the space of $(c_0, c_x)$, with slope $-\frac{1-p}{p} \frac{u'(c_0)}{u'(c_x)}$. Thus, if we pin down the properties of the ECF we should be able to identify the allocation under exclusivity and the contract implementing it. The first step towards characterizing the ECF is to show the relationship between credit limits and interest rates on the frontier. Let $\Delta R/\Delta L$ refer to the change, after an increase of $\Delta L$ in the credit limit, in the lowest interest rate lenders can feasibly charge at $L$.

**Lemma 3.** $\Delta R/\Delta L > 0$ along the ECF.

This positive relationship between $L$ and $R$ implies that, as $c_x$ is raised by increasing $L$, the marginal rate faced by consumers goes up and so does the distortion of intertemporal smoothing under $\kappa = 0$. This leads to an ECF with the following properties. Let $c_{L_{\text{max}}}$ denote the highest feasible aggregated consumption under distress, i.e., the one associated with contract $(R_{\text{max}}, L_{\text{max}})$.

**Lemma 4.** The ECF is continuous and concave. Furthermore, there exists $c_L < c_{L_{\text{max}}}$ such that the ECF is flatter than $-(1-p)/p$ at all points with $c_L \in [c_L, c_{L_{\text{max}}}]$.

Accordingly, since indifferent curves are convex with slope $-(1-p)/p$ on the diagonal $(c_0 = c_x)$, the optimal allocation will exhibit $c_x < c_{L_{\text{max}}}$, i.e., $L^0 < L_{\text{max}}$, except in the case that indifference curves are flatter than the ECF at $c_{L_{\text{max}}}$, in which case a corner solution arises and $L^0 = L_{\text{max}}$.

We then turn to the equilibrium allocation and proceed in a similar fashion. First, we identify the set of $(c_0, c_x)$ that are candidates for equilibrium, which we call the profit feasible
consumption frontier (PFCF) and show that, when $\zeta$ is high enough, interest rates and credit limits have a negative relationship at the PFCF.

**Lemma 5.** If $\zeta \geq \zeta^*$ then $\Delta R/\Delta L < 0$ for all non-binding $L$ satisfying $L > L_{\text{max}} - b_0$.

Lemma 5 has the following implications for the slope of the PFCF:

**Lemma 6.** If $\zeta \geq \frac{1}{2} B/L_{\text{max}}$ then the slope of the PFCF is steeper than $-(1 - p)/p$ at all points involving incomplete balance transfers.

This result then implies that, when equilibrium is implemented by a contract subject to incomplete transfers, it must exhibit a credit limit $L^* = L_{\text{max}}$. The reason is that consumption combinations on the frontier exhibit $c_x \leq c_0$. But then indifference curves at those points are flatter than $-(1 - p)/p$, leading to a corner solution in which $c_x = c_{L_{\text{max}}}$.

To see why $c_x \leq c_0$, notice that the highest $c_x$ and the lowest $c_H$ are associated to $L = L_{\text{max}}$. But at that point the agent is indifferent between defaulting or not in normal times. That is, $c_0$ equals aggregated consumption contingent on default when $\kappa = 0$. Since $L_{\text{max}}$ is not binding by Assumption 2, this means that, in the event of default, the agent fully smooths consumption intertemporally, yielding

$$c_0 = (1 + \theta_0)y + L_{\text{max}} - B \geq (1 + \theta_x)y - \phi x + L_{\text{max}} - B = c_x,$$

where the last inequality comes from our assumption that $\theta_0 y \geq \theta_x y - \phi x$.

To finish the proof of part (ii) we argue that if equilibrium is implemented by a rollover contract with $L^* < L^0$ then, by the concavity of the ECF, it must exhibit the highest credit limit among contracts subject to a full transfer. This follows directly from the fact that the exclusivity problem is isomorphic to the lenders’ restricted problem of choosing among rollover contracts and so the portion of the PFCF associated to rollover contracts coincides with the segment of the ECF exhibiting the same first round credit limits.

Finally, notice that when $\zeta = 1$ rollover contracts are not sustainable and thus equilibrium must exhibit either no credit provision or an incompletely poached contract, which must be a crowding out contract by the argument above. This completes the proof.

**A2. Remaining Proofs**

*Proof of Lemma 1.* We prove the result for $\kappa = 0$. The proof for $\kappa = x$ is analogous and therefore omitted. By definition, $L_{\text{max}}(0; L, R, R')$ is given by the lowest aggregate credit limit $L + L^*$ such that $V^0(0, (L, R), (L', R')) > V^1(0, (L, R), (L', R'))$ for all $L' > L^*$.

To show that $L_{\text{max}}(0; L, R, R')$ is bounded from above, note that for all $L \geq 0$ if $L + L' > (1 - \theta_0)y$ we must have $V^0(0, (L, R), (L', R')) < V^1(0, (L, R), (L', R'))$ as long as aggregated
consumption is increasing in $c_{1, \kappa}$ and $c_{2, \kappa}$. This is because second period income is higher after maxing out and defaulting on $L + L'$ while first period income and borrowing constraints are the same under repayment and default. This implies that the agent can enjoy higher consumption on both periods under default.

We next show that $\mathcal{L}_{\text{max}}(0; L, R, R')$ is decreasing in $L$ as long as $b_0 < L$. We can express aggregated consumption under repayment and default respectively as

\[
V^0(0, (L, R), (L', R')) = 2y - B - (1 - \zeta)Rb_0 - \zeta R \max\{b_0 - L', 0\} - \zeta R' \min\{b_0, L'\} - \mu(B - 2b_0)^2, \quad \text{(A1)}
\]

and

\[
V^1(0, (L, R), (L', R')) = (1 + \theta_0)y - B + L + L' - \mu(\max\{0, B - (1 - \theta_0)y + L' - L\})^2, \quad \text{(A2)}
\]

where the last term in (A2) comes from the fact that, when $L'$ is high enough, $L$ will be binding under default. In addition, borrowing $b_0$ in (A1) is given by

\[
b_0 = \begin{cases} 
\max\left\{0, \min\left\{L, \frac{B}{2} - \frac{R}{8\mu}\right\}\right\} & \text{if } L' \leq \min\left\{L, \frac{B}{2} - \frac{R}{8\mu}\right\} \\
\max\{0, L'\} & \text{if } \min\left\{L, \frac{B}{2} - \frac{R}{8\mu}\right\} < L' \leq \min\left\{L, \frac{B}{2} - \frac{(1 - \zeta)L'}{8\mu}\right\} \\
\max\left\{0, \min\left\{L, \frac{B}{2} - \frac{(1 - \zeta)L'}{8\mu}\right\}\right\} & \text{if } L' > \min\left\{L, \frac{B}{2} - \frac{(1 - \zeta)L'}{8\mu}\right\},
\end{cases}
\]

(A3)

and reflects the fact that the agent faces two constraints: the borrowing constraint $L$ and also, the balance transfer constraint $L'$. It is easy to see that (A1) does not change after an increase in $L$ given by $\Delta L$ when $b_0 < L$ whereas (A2) increases by at least $\Delta L$. Thus, since a reduction in $L'$ equal to $\Delta L$ exactly offsets the effect on $V^1$ of the increase in $L$, but such reduction in $L'$ lowers $V^0$, $\mathcal{L}_{\text{max}}(0; L, R, R')$ must have gone down after the increase in $L$. That is, $\mathcal{L}_{\text{max}}(0; L, R, R')$ is decreasing in $L$ as long as $b_0 < L$.

\[\Box\]

**Proof of Lemma 3.** First note that, along the ECF, lenders charge the lowest feasible interest rate, i.e., $R$ is the smallest solution to

\[(1 - p)Rb_0 = pL.\]

In addition, borrowing constraints will never be binding along the ECF when $\kappa = 0$ unless $L^0 = L_{\text{max}}$ is binding, which is ruled out by Assumption 2. This is because, whenever $L$ is binding, the interest rate that satisfies the zero profit condition $(1 - p)RL = pL$, is
independent of \( L \) and given by the full utilization rate \( R = p/(1-p) \). Hence, they can increase \( L \) without increasing \( R \), thereby relaxing the borrowing constraint on the repayment path while increasing consumption on the default path. This leads to higher aggregated consumption under both distress and non-distress.

Thus, for non-binding \( L < L_{max} \), interest revenue is given by \( R \left( \frac{B}{2} - \frac{R}{8\mu} \right) \), which is a continuous, concave function of \( R \), initially increasing (and equal to zero) at \( R = 0 \) and reaching a maximum at \( R = 2\mu B \). Thus, at each \( L \) lenders would choose the interest rate in \( (0, 2\mu B] \) satisfying the above zero profit condition. But then, since \( Rb_0 \) is strictly increasing for all \( R \in [0, 2\mu B) \) any increase of \( L \) on the ECF must be accompanied by an increase in \( R \).

\[ \text{Proof of Lemma 4.} \] As argued in the proof of Lemma 3, credit limits are not binding on the ECF when \( \kappa = 0 \). In addition, Assumption 2 also guarantees that there is a range of credit limits that are also non-binding under \( \kappa = x \). To see this is the case, recall that \( \theta_0 y \geq \theta_x y - \phi x \). Since \( L_{max} \) is bounded above by \( (1 - \theta_0)y \) we then must have that

\[
y - L_{max} \geq \theta_0 y \geq \theta_x y - \phi x.
\]

Now, since \( B < 2L_{max} \) by Assumption 2, adding \( 2L_{max} - B \) to the LHS of the above expression yields \( y - B + L_{max} > \theta_x y - \phi x \), i.e., consumers under distress are not credit constrained at credit limits close to \( L_{max} \).

We also know that, since profits are zero for any contract associated to a point on the ECF, interest payments satisfy \( Rb_0 = \frac{p}{1-p}L \). Using this expression and substituting \( b_0 = B/2 - R/(8\mu) \) into (A1) we can write aggregated consumption when \( \kappa = 0 \) as

\[
c_0 = V^0((L,R), (0,0)) = 2y - B - \frac{p}{1-p}L - \frac{R^2}{16\mu}.
\]

In addition, aggregated consumption when \( \kappa = x \) is given by

\[
c_x = \begin{cases} 
(1 + \theta_x)y - B - \phi \kappa + L - \mu (B - L - \phi \kappa - (1 - \theta_x)y)^2 & L < B - \phi \kappa - (1 - \theta_x)y \\
(1 + \theta_x)y - B - \phi \kappa + L & L \geq B - \phi \kappa - (1 - \theta_x)y,
\end{cases}
\]

where the first case reflects the fact that \( L \) may be binding. Given these expressions, the continuity of the ECF is straightforward. Also, it is easy to see that \( c_x \) increases one-to-one with \( L \) when \( L \) is not binding while, given Lemma 3, \( c_0 \) decreases by more than \( p/(1-p) \) when \( L \) goes up by one. Hence, the ECF must be flatter than \(- (1-p)/p \) at all points associated
with $L \in [B - \phi \kappa - (1 - \theta_x)y, L_{max}]$. But since $L_{max}$ is not binding and $c_x$ is increasing in $L$, this interval of credit limits maps onto an interval $[c_x, c_{L_{max}}]$ of consumption under distress for some $c_x < c_{L_{max}}$.

Finally, to show concavity, it suffices to show that $c_0$ and $c_x$ are concave in $L$, given that $c_0$ is strictly decreasing and $c_x$ is strictly increasing in $L$. If we differentiate twice the above expression of $c_0$ w.r.t. $L$ we get that

$$d^2c_0\over dL^2 = \frac{-dR}{dL} \frac{1}{8\mu} \frac{d^2R}{dL^2} \frac{R}{8\mu}.$$ 

Since $\frac{dR}{dL} > 0$ by Lemma 3, this expression is negative whenever $\frac{d^2R}{dL^2} > 0$. By implicitly differentiating the zero profit condition we find that

$$\frac{d^2R}{dL^2} = \frac{p}{1 - p} \frac{1}{4\mu(B/2 - R/(4\mu))^2} \frac{dR}{dL},$$

which is always positive for all $R < 2\mu B$. Similarly, $c_0$ is concave in $L$:

$$d^2c_0\over dL^2 = \begin{cases} 
-2\mu & L < B - \phi \kappa - (1 - \theta_x)y \\
0 & L \geq B - \phi \kappa - (1 - \theta_x)y.
\end{cases}$$

Proof of Lemma 5. The proof logic is to show that, when the condition in the lemma is satisfied, first round lender profits go up when $L$ is increased while keeping fixed the interest rate. Such rate is the lowest $R$ satisfying the zero profit condition $E\pi(0, (L, R), (0, L_{max}(L, R) - L)) = 0$, given that lenders must earn zero expected profits at all points on the PFCF. Accordingly, if profits go up with $L$ initial lenders can increase $L$ and lower the interest rate while earning zero profits.

Expected profits are given by

$$E\pi(0, (L, R), (0, L_{max}(L, R) - L)) = (1 - p)R [\zeta(b_0 - (L_{max}(L, R) - L)) + (1 - \zeta)b_0] - pL. \quad (A4)$$

Since $b_0$ does not depend on $L$ when borrowing constraints are non-binding, the change in profits after a change of $\Delta L$ is given by

$$\frac{\Delta E\pi(0, (L, R), (0, L_{max}(L, R) - L))}{\Delta L} = (1 - p)\zeta R \left(1 - \frac{\Delta L_{max}(L, R)}{\Delta L}\right) - p.$$
By Lemma 1 we know that \( \frac{\Delta L_{\text{max}}(L,R)}{\Delta L} < 0 \). Accordingly, a sufficient condition for profits to go up is

\[
\zeta R > \frac{p}{1-p}.
\]  

(A5)

From the above zero profit condition we can express \( \zeta R \) as

\[
\zeta R = \frac{p}{1-p} \frac{\zeta L}{\zeta (b_0 - (\mathcal{L}_{\text{max}}(L, R) - L)) + (1 - \zeta)b_0}.
\]

Accordingly, (A5) holds as long as

\[
\zeta L > \zeta (b_0 - (\mathcal{L}_{\text{max}}(L, R) - L)) + (1 - \zeta)b_0,
\]

which yields

\[
\zeta > \frac{b_0}{\mathcal{L}_{\text{max}}(L, R)}.
\]

But this condition must hold as long as \( \zeta \geq \frac{B}{2} \frac{1}{L_{\text{max}}} \), given that \( b_0 < B/2 \) for all \( R > 0 \) and \( \mathcal{L}_{\text{max}}(L, R) \geq L_{\text{max}} \) by Lemma 1.

**Proof of Lemma 6.** Since \( L' = 0 \) when \( \kappa = x \), \( c_x \) follows the same expression as under exclusivity and thus increases with \( L \) by at least one-to-one. Thus, to show that the slope of the PFCF is steeper we just need to show that \( c_0 \) decreases by less than \( -\Delta L p / (1 - p) \) after an increase in \( L \) equal to \( \Delta L \).

Consider first the case of \( L \) non-binding when \( \kappa = 0 \). Since lenders earn zero profits on the PFCF, we can write \( c_0 \) as

\[
c_0 = 2y - B - \frac{p}{1-p}L - \frac{R^2}{16\mu}.
\]

By Lemma 5 we know that \( \Delta R / \Delta L < 0 \) and thus the last term goes down as \( L \) goes up, implying that \( \Delta c_0 / \Delta L > -pL / (1 - p) \) on the PFCF.

Now consider the case of binding \( L \). Then, we must have \( B > 2b_0 = 2L \). In this case,

\[
c_0 = 2y - B - \frac{p}{1-p}L - \mu(B - 2L)^2.
\]

From this expression, it is clear that \( \Delta c_0 / \Delta L > -pL / (1 - p) \) also when \( L \) is binding.
References


Rios-Rull, José-Víctor and Xavier Mateos-Planas. 2007. “Credit Lines.” unpublished manuscript.