Identification in Structural Vector Autoregressive models with structural changes, with an application to U.S. monetary policy*

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Abstract

A growing line of research makes use of structural changes and different volatility regimes found in the data in a constructive manner to improve the identification of structural parameters in Structural Vector Autoregressions (SVARs). A standard assumption made in the literature is that the reduced form unconditional error covariance matrix varies while the structural parameters remain constant. Under this hypothesis, it is possible to identify the SVAR without needing to resort to additional restrictions. With macroeconomic data, the assumption that the transmission mechanism of the shocks does not vary across volatility regimes is debatable. We derive novel necessary and sufficient rank conditions for local identification of SVARs, where both the error covariance matrix and the structural parameters are allowed to change across volatility regimes. Our approach generalizes the existing literature on 'identification through changes in volatility' to a broader framework and opens up interesting possibilities for practitioners. An empirical illustration focuses on a small monetary policy SVAR of the U.S. economy and suggests that monetary policy has become more effective at stabilizing the economy since the 1980s.

Keywords: Heteroskedasticity, Identification, Monetary policy, Structural VAR.

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1 Introduction

Structural Vector Autoregressions (SVAR) are widely used for policy analysis and to provide stylized facts about business cycle. As is known, it is necessary to identify the structural shocks to run policy simulations. Magnusson and Mavroeidis (2014) have recently shown how structural changes, which are pervasive in the macroeconomy, can be used constructively to identify structural relations which are time invariant. In this paper, we focus on the identification of SVARs characterized by changes in the error covariance matrix, allowing for changes also in the structural parameters.

The identification of structural dynamic macro models through heteroskedasticity was originally proposed by Rigobon (2003), who formalized the intuition that the information that there exist different volatility regimes in the data represents an 'additional' identification source that can be exploited to identify the shocks without the need to resort to other type of restrictions.[1] Lanne and Lütkepohl (2008) have extended this idea to the case of SVARs, see also Lanne and Lütkepohl (2010), Lanne *et al.* (2010) and Ehrmann *et al.* (2011).[2] However, this literature is exclusively based on the idea that the structural parameters remain constant across volatility regimes. This assumption appears reasonable in certain applications, but is in general questionable with macroeconomic data, where there is widespread evidence of parameter instability. It is well recognized that structural breaks may have marked consequences on both the transmission and propagation mechanisms of the shocks.

This paper shows that the identification approach suggested by Rigobon (2003) and Lanne and Lütkepohl (2008) can be generalized to a broader framework, opening up interesting possibilities for SVAR's practitioners. By applying the seminal identification rules of Rothenberg (1971), we derive novel necessary and sufficient rank conditions for local identification which apply when discrete permanent (not recurring) breaks occur simultaneously in the reduced form VAR error covariance matrix and in the (structural) parameters which define the relationships between the VAR disturbances and the structural shocks. The results in Rigobon (2003) and Lanne and Lütkepohl (2008) obtain as special cases of our analysis. Unlike Rigobon (2003) and Lanne and Lütkepohl (2008), in our setup the patterns of SVAR impulse response functions may vary across volatility regimes.

As is known, structural changes offer identifying power only if some parameters do not change. The difficult open question is what these parameters are. In our approach, different structural models are imposed on different volatility regimes through a balanced combination of the statistical information provided by the data, and 'conventional' linear restrictions. It is economic reasoning that provides indications about which are the structural parameters likely

[1] In the recent literature, Sentana (1992) and Sentana and Fiorentini (2001) have introduced similar ideas in the context of factor models, Klein and Vella (2010) and Lewbel (2010) in the context of simultaneous systems of equations. See also Keating (2004) for the case of SVARs.

[2] Other examples include Caporale *et al.* (2005a), Dungey and Martin (2001), King *et al.* (1994), Caporale *et al.* (2005b), Rigobon and Sack (2003, 2004) and Normandin and Phaneuf (2004).

to change across the volatility regimes, and which are the structural parameters which are likely to remain unchanged.

Our approach opens up interesting possibilities for practitioners. We discuss new identification schemes that stem from our analysis, using examples taken from the empirical monetary policy literature. SVARs which would typically be unidentified in the case of constant structural parameters, can be identified or overidentified and hence tested against the data, when also the structural parameters are allowed to vary across volatility regimes.

To illustrate the usefulness of our approach, we identify and estimate a small monetary policy SVAR for the U.S. economy, using quarterly data and two regimes of volatility. Our empirical evidence suggests that in spite of a general reduction in the volatility of the shocks, the response of U.S. monetary policy changed in the move from the 'pre-Volcker' period to the 'Great Moderation' period.

The remainder of this paper is organized as follows. In Section 2 we discuss the background and motivations of our paper. In Section 3 we derive our main result by providing the necessary and sufficient rank conditions for local identification. In Section 4 we present an empirical illustration where we estimate a small monetary policy SVAR of the U.S. economy, using quarterly data and two volatility regimes. Section 5 contains some concluding remarks. An online Technical Supplement complements the results of the paper in several dimensions.[3]

Throughout the paper we use the following notation, matrices and conventions, most of which are taken from Magnus and Neudecker (2007). K_n is the $n^2 \times n^2$ *commutation matrix*, i.e. the matrix such that $K_n vec(M) = vec(M')$ where M is $n \times n$, and D_n is the *duplication matrix,* i.e. the $n^2 \times \frac{1}{2}n(n+1)$ full column rank matrix such that $D_n vech(M) = vec(M)$, where $vech(M)$ is the column obtained from $vec(M)$ by eliminating all supra-diagonal elements. Given K_n and D_n, $N_n := \frac{1}{2}(I_{n^2} + K_n)$ is a $n^2 \times n^2$ matrix such that rank$[N_n] = \frac{1}{2}n(n+1)$ and $D_n^+ := (D_n' D_n)^{-1} D_n'$ is the Moore-Penrose inverse of D_n. Finally, when we say that the matrix $M := M(v)$, whose elements depend (possibly nonlinearly) on the elements of the vector v, 'has rank r evaluated at v_0', we mean that v_0 is a 'regular point', i.e. that rank$(M)=r$ does not change within a neighborhood of v_0.

2 Background and motivations

In this section, we present the basic econometric framework upon which our analysis will be developed. To fix main ideas and notation, we first review the standard approach to the identification of SVARs (Sub-section 2.1), and then move to the mechanics of the 'identification via changes in volatility' method (Sub-section 2.2). Finally, we anticipate our contribution in this literature (Sub-section 2.3).

[3] The Technical Supplement is available at http://www.rimini.unibo.it/fanelli/TS_Bacchiocchi_Fanelli.pdf

2.1 Standard identification approach

Let Z_t be the $n \times 1$ vector of observable variables. Our reference reduced form model is given by the VAR system with constant parameters:

$$Z_t = A_1 Z_{t-1} + \ldots + A_k Z_{t-k} + \Psi D_t + \varepsilon_t \quad , \quad t = 1, ..., T \tag{1}$$

where ε_t is a n-dimensional White Noise process with positive definite time-invariant covariance matrix $\Sigma_\varepsilon{:=}E(\varepsilon_t \varepsilon_t')$, A_j, $j = 1, ..., k$ are $n \times n$ matrices of time-invariant coefficients, k is the VAR lag order, D_t is an m-dimensional vector containing deterministic components (constant, trend and dummies), and Ψ is the $n \times m$ matrix of associated coefficients. T is the sample length.

We compact the VAR system (1) in the expression

$$Z_t = \Pi W_t + \varepsilon_t \quad , \quad t = 1, ..., T \tag{2}$$

where $W_t{:=}(Z_{t-1}', ..., Z_{t-k}', D_t')$ and $\Pi{:=}(A, \Psi)$. The matrix Π is $n \times f$, $f{:=}\dim(W_t){:=}nk + m$, and the VAR reduced form parameters are collected in the p-dimensional vector $\theta{:=}(\pi', \sigma_+')'$, where $\pi{:=}vec(\Pi)$ and $\sigma_+{:=}vech(\Sigma)$, $p{:=}nf + \frac{1}{2}n(n+1)$.

The SVAR we are interested in this paper is defined by

$$\varepsilon_t{:=}Ce_t \quad , \quad E(e_t e_t'){:=}I_n \quad , \quad \Sigma_\varepsilon{=}CC' \tag{3}$$

where C is a non-singular $n \times n$ matrix of structural parameters and e_t is a n-dimensional i.i.d. vector of structural shocks with covariance matrix normalized to I_n.[4] As is known, the system (2)-(3) is unidentified without any restriction on the elements of the C matrix. The standard way to achieve identification is to include a set of linear restrictions on C that we write in 'explicit form'

$$vec(C){:=}G_C\ \gamma + g_C. \tag{4}$$

In Eq. (4), G_C is a $n^2 \times a_C$ selection matrix, γ is $a_C \times 1$ and contains the 'free' elements of C, and g_C is a $n^2 \times 1$ vector. The information required to specify the matrix G_C and the vector g_C usually comes from the economic theory or from structural and institutional knowledge related to the problem under study. The condition $a_C{:=}\dim(\gamma) \leq n(n+1)/2$ is necessary for identification. Necessary and sufficient condition for identification is that the $n(n+1)/2 \times a_C$

[4]We consider the formulation in Eq. (3) of the SVAR (the 'C-model', using the terminology in Amisano and Giannini, 1997) because it is largely used in empirical analysis, although our approach is consistent with the alternative specification

$$K\varepsilon_t{:=}e_t \quad , \quad E(e_t e_t'){:=}I_n$$

(termed 'K-model' in Amisano and Giannini, 1997) where $K{:=}C^{-1}$.

matrix

$$2D_n^+(C \otimes I_n)G_C \tag{5}$$

has full column rank evaluated at C_0, where C_0 denotes the counterpart of C that fulfills the restriction $vec(C_0){:=}G_C\gamma_0 + g_C$, and γ_0 is the 'true' value of γ, see e.g., Giannini (1992), Hamilton (1994) and Amisano and Giannini (1997).[5]

To avoid confusion, throughout the paper we call 'reduced form parameters' the elements in the vector θ, and 'structural parameters' the elements of the vector γ and, possibly, the variances of the structural shocks e_t when these are not normalized to one. If the rank condition in Eq. (5) holds, the orthogonalized impulse response functions (IRFs) are taken from the matrices $\Xi_h{:=}[\psi_{lm,h}]{:=}\Phi_h\breve{C}{:=}(J'\mathring{A}^hJ)\breve{C}$, $h = 0, 1, 2, ...$, where

$$\mathring{A}{:=}\begin{pmatrix} & A & \\ I_{n(k-1)} & , & 0_{n(k-1)\times n} \end{pmatrix} \tag{6}$$

is the VAR companion matrix, $J{:=}(I_n, 0, \ldots, 0)$ and $\breve{C}$ denotes a specification of C such that the matrix in Eq. (5) has full column rank. The coefficient $\psi_{lm,h}$ captures the response of variable l to a one-time impulse in variable m, h periods before.[6]

2.2 Identification through heteroskedasticity

In a seminal contribution, Rigobon (2003) proposed an alternative way to solve the identification problem in simultaneous systems of equations that can be extended to the case of SVARs. The distinctive feature of Rigobon's (2003) approach is that when the data are characterized by (at least) two different regimes of volatility, the identification of the shocks can be achieved without linear constraints of the type in Eq. (4).

Without any loss of generality, we consider a bivariate SVAR model for the vector $Z_t = (Z_{1t}, Z_{2t})'$ and assume that the data generating process belongs to the class of models described by the system (2)-(3). We further assume that at time $t{=}T_B$, where $1 < T_B < T$, the variance

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[5] Giannini (1992) and Amisano and Giannini (1997) derive the necessary and sufficient identification rank condition by referring to linear restrictions in 'implicit form'. The necessary and sufficient rank condition in Eq. (5) can be checked ex-post at the ML estimate but also prior to estimation at random points drawn uniformly from the parameter space, see e.g. Giannini (1992). Iskrev (2010) applies the same idea to check the identification of DSGE models. Lucchetti (2006), instead, has shown that Eq. (5) can be replaced with a 'structure condition' which is independent on the knowledge of the structural parameters but is still confined to the local identification case. Rubio-Ramirez *et al.* (2010) have established novel sufficient conditions for global identification in SVARs and necessary and sufficient conditions for exactly identified systems.

[6] The identification of C can also be achieved by complementing the symmetry restrictions $\Sigma = CC'$ with a proper set of constraints on the matrix

$$\Xi_\infty{:=}(I_n - A_1 - \cdots - A_k)^{-1}C{:=}\sum_{h=0}^{\infty}\Phi_h C{:=}J'(I_{nk} - \mathring{A})^{-1}JC$$

which measures the long run impact of the structural shocks on the variables (Blanchard and Quah,1989). Constraints on Ξ_∞ can be used in place of, or in conjunction with, the 'short run' restrictions in Eq. (4).

of the data changes in the sense that the two sets of observations $Z_1,...,Z_{T_B}$ and $Z_{T_B+1},...,Z_T$ are characterized by the two (distinct) VAR covariance matrices $\Sigma_{\varepsilon,1}$ and $\Sigma_{\varepsilon,2}$, respectively, where

$$\Sigma_{\varepsilon,i}:=\begin{pmatrix} \sigma_{11.i} & \sigma_{12.i} \\ & \sigma_{22.i} \end{pmatrix} \quad , \quad i=1,2.$$

Consider the relationship $\varepsilon_t:=Ce_t$, where e_t is the vector of structural shocks. Rigobon's (2003) identification approach is based on the joint use of the moment conditions

$$\Sigma_{\varepsilon,1}=C\Lambda_1C' \quad , \quad \Sigma_{\varepsilon,2}=C\Lambda_2C' \tag{7}$$

where C, Λ_1 and Λ_2 are defined by

$$C:=\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \quad , \quad \Lambda_1:=\begin{pmatrix} \lambda_{11.1} & 0 \\ 0 & \lambda_{22.1} \end{pmatrix} \quad , \quad \Lambda_2:=\begin{pmatrix} \lambda_{11.2} & 0 \\ 0 & \lambda_{22.2} \end{pmatrix}$$

and the matrices Λ_1 and Λ_2 collect the variances of the structural shocks in the two volatility regimes. Eq. (7) links the reduced form coefficients $\sigma_+:=(vech(\Sigma_{\varepsilon,1})', vech(\Sigma_{\varepsilon,1})')' = (\sigma_{11.1}, \sigma_{22.1}, \sigma_{21.1}, \sigma_{11.2}, \sigma_{22.2}, \sigma_{21.2})'$ to the structural form parameters in the matrices C, Λ_1 and Λ_2. If, as is standard in the SVAR literature, Λ_1 is normalized to be the identity matrix, I_2, the six structural parameters $\vartheta:=(c_{11}, c_{12}, c_{21}, c_{22}, \lambda_{11.2}, \lambda_{22.2})'$, can be recovered uniquely from σ_+ by solving system (7).

This identification approach has been extended by Lanne and Lütkepohl (2008) to the case of SVARs, where $\dim(Z_t)=n>2$. Lanne and Lütkepohl (2008) exploit the algebraic result in Horn and Johnson (1985, Corollary 7.6.5), according to which the condition $\Sigma_{\varepsilon,1}\neq\Sigma_{\varepsilon,2}$ guarantees the simultaneous factorization

$$\Sigma_{\varepsilon,1}=PP' \quad , \quad \Sigma_{\varepsilon,2}=PVP' \tag{8}$$

where P is a $n\times n$ non-singular matrix and $V:=\text{diag}(v_1,...,v_n)\neq I_n$ is a diagonal matrix with distinct positive elements $v_i>0$, $i=1,...,n$. Identification in this setup is achieved by setting $C:=P$ and $\Lambda_2:=V$, where the choice $C:=P$ is unique except for sign changes if all v_i's are distinct.

2.3 Our contribution

The 'purely statistical' approach to the identification of SVARs put forth by Rigobon (2003) and Lanne and Lütkepohl (2008) has important implications for the transmission mechanisms of the shocks. In their framework, the structural break at time T_B affects only the VAR error covariance matrix, hence the IRFs computed on the sub-samples $Z_1,...,Z_{T_B}$ and $Z_{T_B+1},...,Z_T$ remain unchanged, i.e. they have the same time patterns across the two volatility regimes.

Our paper contributes to this literature by relaxing the restrictive assumption that the changes in the volatility of the data have no impact on the transmission mechanisms of the shocks. In the next sections, we develop a new theoretical framework where also the structural parameters contained in the matrix C may vary across volatility regimes, still contributing to the identification of the SVAR. When it is known that SVAR parameters, including the elements of C, change at time T_B, VAR practitioners typically deal with two distinct SVARs: one for the sub-sample $Z_1, ..., Z_{T_B}$ and the other for the sub-sample $Z_{T_B+1}, ..., Z_T$. Interestingly, we show that our approach leads to new identification schemes, which can be fruitfully implemented empirically, without the need to recur to distinct SVARs.

In the Technical Supplement, we show in detail that the results in Rigobon (2003) and Lanne and Lütkepohl (2008) can be obtained as special cases of our approach.

3 Identification analysis

Consider the SVAR summarized in Eq.s (2)-(3) and assume that at time T_B, $1 < T_B < T$, the unconditional reduced form covariance matrix Σ_ε changes. We denote with $\Sigma_{\varepsilon,1}$ and $\Sigma_{\varepsilon,2}$ the covariance matrix before and after the break, respectively. Without any loss of generality, we focus on the case of a single break, i.e. two volatility regimes. Results are extended to the case of a finite number $s \geq 2$ of breaks in the Technical Supplement. In this section, we discuss the representation of the reference SVAR (Sub-section 3.1), and then introduce our main necessary and sufficient conditions for local identification in Proposition 1 (Sub-section 3.2). Finally, we present some new identification schemes, consistent with Proposition 1, by considering two examples taken from the empirical monetary policy literature (Subsection 3.3).

3.1 Representation

The reference reduced form VAR is given by:

$$Z_t = \Pi(t)W_t + \varepsilon_t \quad , \quad \Sigma_\varepsilon(t) := E(\varepsilon_t \varepsilon_t') \quad , \quad t = 1, ..., T \tag{9}$$

where

$$\Pi(t) := \Pi_1 \times \mathbb{1}(t \leq T_B) + \Pi_2 \times \mathbb{1}(t > T_B) \quad , \quad t = 1, ..., T \tag{10}$$

$$\Sigma_\varepsilon(t) := \Sigma_{\varepsilon,1} \times \mathbb{1}(t \leq T_B) + \Sigma_{\varepsilon,2} \times \mathbb{1}(t > T_B) \quad , \quad t = 1, ..., T, \tag{11}$$

$\mathbb{1}(\cdot)$ is the indicator function, $\Pi_1 := (A_1, \Psi_1)$ and $\Pi_2 := (A_2, \Psi_2)$ are the $n \times f$ matrices containing the autoregressive coefficients before and after the break, respectively. As it stands, the specification in Eq.s (9)-(11) covers the case in which the structural break affects both the autoregressive coefficients and the error covariance matrix. The changes in the latter are of crucial importance in our approach. Thus we consider the following assumption.

Assumption 1 [Change in the reduced form error covariance matrix] Given the VAR system in Eq.s (9)-(11), $T_B \geq f$ and $T - (T_B + 1) \geq f$, and it holds the condition

$$\sigma_{+1}{:=}vech(\Sigma_{\varepsilon,1}) \neq \sigma_{+2}{:=}vech(\Sigma_{\varepsilon,2}).$$

The main implication of Assumption 1 is that the two sub-samples $Z_1, ..., Z_{T_B}$ and $Z_{T_B+1}, ..., Z_T$ are characterized by two distinct regimes of volatility (and there are sufficient observations to estimate the VAR in each regime). In our framework, the autoregressive coefficients in Eq. (10) may change or may be equal at time T_B, i.e. both cases $\Pi_1 \neq \Pi_2$ and $\Pi_1{=}\Pi_2$ are consistent with the identification analysis of the SVAR presented below. The break date, T_B, can be either known, a common assumption in macroeconomic analysis in which permanent (not recurring) events that lead to relevant institutional or behavioral changes are typically identified ex-post, or can be inferred from the data by applying any method suited to detect changes in the covariance matrix of the disturbances in multivariate regression models, see e.g. Qu and Perron (2007) and references therein.

Given Assumption 1, we consider the following counterpart of the structural specification in Eq. (3):

$$\varepsilon_t = Ce_t \qquad t \leq T_B \tag{12}$$

$$\varepsilon_t = (C + Q)\, e_t \qquad t > T_B \tag{13}$$

where C and Q are two $n \times n$ matrices of structural parameters, and $E(e_t e_t'){:=}I_n$. The non zero elements of the matrix Q capture the changes in the structural parameters, if any, across the two regimes.

The relationships in Eq.s (12)-(13) lead to the system of equations

$$\Sigma_{\varepsilon,1} = CC' \tag{14}$$

$$\Sigma_{\varepsilon,2} = (C + Q)\,(C + Q)' \tag{15}$$

that links the reduced to the structural form parameters of the SVAR. The parametrization in Eq.s (14)-(15) is more general that one might expect. Consider, for instance, the alternative one

$$\begin{aligned} \Sigma_{\varepsilon,1} &= CC' \\ \Sigma_{\varepsilon,2} &= (C + Q)\,\Lambda\,(C + Q)' \quad , \end{aligned}$$

in which the difference $\Sigma_{\varepsilon,1} \neq \Sigma_{\varepsilon,2}$ is (apparently) explained either by the change in the variance of the structural shocks e_t before ($\Lambda_1 = I_n$) and after ($\Lambda_2 = \Lambda$) the break, or by the different impact of the shocks across volatility regimes if $Q \neq 0_{n \times n}$, or, possibly, by a combination of these two factors. However, for every Q and any given $\Lambda \neq I_n$, one can always find a matrix Q^* such that the equality $(C+Q)\Lambda(C+Q)' = (C+Q^*)(C+Q^*)'$ is respected, or $Q^* = (C+Q)\Lambda^{1/2} - C$.[7] This means that any situation with apparent change in the volatilities of the structural shocks across regimes, can be rewritten as one in which the volatility of the structural shocks remain constant (and normalized to the identity matrix I_n), and only the structural parameters change (via the Q matrix). This also explains why the parametrization in system (14)-(15) allows us to nest the frameworks of Lanne and Lütkepohl (2008) and Rigobon (2003), see the Technical Supplement for details.

The $n(n+1)$ symmetry restrictions provided by Eq.s (14)-(15) are not in general sufficient alone to identify the $2n^2$ elements in the matrices C and Q, hence it is necessary to add at least $2n^2 - n(n+1) = n(n-1)$ restrictions on these matrices. Linear restrictions on the $2n^2 \times 1$ vector ϑ:=$(vec(C)', vec(Q)')'$ can be imposed through a specification analogue to that in Eq. (4), i.e.

$$\underset{\vartheta}{\begin{pmatrix} vec\,(C) \\ vec\,(Q) \end{pmatrix}} := \underset{G}{\begin{pmatrix} G_C & G_I \\ 0_{n^2 \times a_C} & G_Q \end{pmatrix}} \underset{\psi}{\begin{pmatrix} \gamma \\ q \end{pmatrix}} + \underset{g}{\begin{pmatrix} g_C \\ g_Q \end{pmatrix}} \tag{16}$$

where the $a \times 1$ vector ψ:=$(\gamma', q')'$ contains the $a = a_C + a_Q$ free elements of C and Q, respectively. The matrix G and the vector g, of suitable dimensions, summarize the linear restrictions on C and Q and G_I is a selection sub-matrix through which it is possible to impose cross-restrictions on the elements of these two matrices.

The next sub-section shows that the relationships in Eq.s (14)-(15) combined with the restrictions in Eq. (16) can be used to identify the reference SVAR.

3.2 Main result

Consider the SVAR introduced in Sub-section 3.1, Assumption 1, and the set of restrictions in Eq.s (14)-(16). We denote with γ_0 and q_0 the 'true' values of γ and q respectively, and with C_0 and Q_0 the matrices obtained from Eq. (16) by replacing γ and q with γ_0 and q_0. Our main result is summarized in the next proposition.

Proposition 1 [Identification of C and Q] Assume that the data generating process belongs to the class of SVARs in Eq.s (9)-(11) and Eq.s (12)-(13), and that the matrices C and $(C+Q)$ are non-singular, where C and Q are subject to the restrictions in Eq. (16). Under Assumption 1, a necessary and sufficient rank condition for the SVAR to be locally

[7] We are indebted with a referee for this observation.

identified is that the $n(n+1)\times a$ matrix

$$(I_2 \otimes D_n^{+})\begin{pmatrix} (C\otimes I_n) & 0_{n^2\times n^2} \\ (C+Q)\otimes I_n & (C+Q)\otimes I_n \end{pmatrix}\begin{pmatrix} G_C & G_I \\ 0_{n^2\times a_C} & G_Q \end{pmatrix} \tag{17}$$

has full column rank a evaluated at $C{:=}C_0$ and $Q{:=}Q_0$; necessary order condition is

$$a{:=}(a_C + a_Q) \leq n(n+1). \tag{18}$$

Proof. See Appendix.

When $a{=}n\,(n+1)$ and the rank condition holds, the SVAR is 'exactly identified', while is overidentified when $a < n\,(n+1)$.

When the specified matrices C and Q meet the requirements of Proposition 1, the log-likelihood of the SVAR can be maximized as described in the Technical Supplement. Moreover, if the SVAR is overidentified, the $n\,(n+1){-}a$ overidentifying restrictions can be validated/rejected by computing a (quasi-) likelihood ratio (LR) test that compares the log-likelihood of the structural form and the log-likelihood of the reduced form. The (normalized) impulse responses implied by the identified SVAR are given by

$$\Xi_{1,h}{:=}\left[\psi_{1,lm,h}\right]{:=}J'(\mathring{A}_1)^h J\breve{C}\ , \quad h=0,1,2,... \text{ 'pre-change' regime} \tag{19}$$

$$\Xi_{2,h}{:=}\left[\psi_{2,lm,h}\right]{:=}J'(\mathring{A}_2)^h J(\breve{C}+\breve{Q})\ , \quad h=0,1,2,... \text{ 'post-change' regime} \tag{20}$$

where $\mathring{A}_1$ and $\mathring{A}_2$ are the companion matrices in the two volatility regimes, as described in Eq. (6), and $\breve{C}$ and $\breve{Q}$ denote counterparts of C and Q such that the rank condition implied by Proposition 1 is fulfilled. Note that irrespective of whether $\mathring{A}_1 = \mathring{A}_2$ or $\mathring{A}_1 \neq \mathring{A}_2$, the two sets of population impulse responses in Eq. (19) and Eq. (20) differ across volatility regimes when $Q \neq 0_{n\times n}$. The coefficient $\psi_{i,lm,h}$ captures the response of variable l to a one-time impulse in variable m, h periods before, in the volatility regime i.

3.3 Identification schemes: some examples

Proposition 1 opens up new identification schemes for SVARs that we discuss with some examples taken from the monetary policy literature. The models presented below would not be identified through the 'standard' identification approach summarized in Sub-section 2.1, or by considering two distinct SVARs on the sub-samples $Z_1,...,Z_{T_B}$ and $Z_{T_B+1},...,Z_T$.

Example 1 ['DSGE-consistent SVAR'] Consider the three-variable monetary policy SVAR in which $Z_t{:=}(\tilde{y}_t, \pi_t, R_t)'$ $(n{:=}3)$, where $\tilde{y}_t$ is a measure of the output gap, π_t the inflation rate and R_t a nominal policy interest rate. Imagine that a structural break changes the error covariance matrix at time T_B, and that the structural specification in Eq.s (12)-(13) specializes to

$$\underbrace{\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix}}_{\varepsilon_t} := \left\{ \underbrace{\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}}_{C} + \underbrace{\begin{pmatrix} q_{11} & 0 & 0 \\ 0 & q_{22} & 0 \\ 0 & 0 & q_{33} \end{pmatrix}}_{Q} \times \mathbb{1}\left(t > T_B\right) \right\} \begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \end{pmatrix}. \quad (21)$$

We interpret $e_{3,t}$ as the 'monetary policy shock', $e_{1,t}$ as the 'output shock' and $e_{2,t}$ as the 'inflation shock'. Apparently, the SVAR in Eq. (21) is 'close' to the one based on the factorization in Eq. (8). However, despite the C matrix is full in both cases, in Eq. (21) the instantaneous impact of the shock $e_{j,t}$ on the variable $Z_{j,t}$, $j = 1, 2, 3$ varies from c_{jj} in the first volatility regime to $c_{jj} + q_{jj}$ in the second volatility regime. The specification in Eq. (21) is interesting because it can be to some extent related to the debate about the consistency between SVAR analysis and dynamic stochastic general equilibrium (DSGE) modeling, see Bacchiocchi *et al.* (2014). As is known, small-scale new-Keynesian DSGE models of the type discussed, among many others, in e.g. Lubik and Schorfheide (2004) and Carlstrom *et al.* (2009), typically admit an immediate reaction of output and inflation to monetary policy impulses, while 'conventional' triangular (Cholesky-based) SVARs feature a lag in such reactions. The existing empirical evidence seems to suggest that monetary policy shocks exert a non-zero instantaneous impact on macroeconomic variables like prices and output. For instance, by employing their 'DSGE-VAR' approach, Del Negro *et al.* (2007) find Cholesky-based SVARs to be implausible due to the very likely immediate reaction of output to a policy shock. Likewise, Faust *et al.* (2004) show that the zero response of prices to a monetary policy shock imposed by Cholesky-based SVARs is not supported by the data when disturbances are inferred using high frequency futures data in a two-step procedure. Thus, under the null of a valid DSGE model, triangular SVARs offer a misspecified representation of monetary policy shocks and their propagation, and can produce price puzzles and muted responses of inflation and the output gap to monetary shocks, see Castelnuovo and Surico (2010), Castelnuovo (2012*b*) and Bacchiocchi *et al.* (2014). In other words, given $\varepsilon_t := Ce_t$, the C matrix must be full with highly restricted non-zero coefficients for the SVAR to be consistent with the predictions of a DSGE model. Eq. (21) suggests that an identified SVAR featuring a full matrix C can be obtained on condition that the response on impact of $Z_{j,t}$ to $e_{j,t}$ varies across volatility regimes. It is worth remarking, however, that in our framework the SVAR does not embody the whole set of restrictions on the VAR lag structure implied by DSGE models under rational expectations, and that the matrix C does not feature the highly nonlinear cross-equation restrictions implied by these models. With a slight abuse of language, we denote the system defined by Eq. (21) as 'DSGE-consistent SVAR', where by this term we mean a SVAR which, aside from the cross-equation restrictions, features a full matrix C, so that all shocks hitting the modeled economy are allowed to affect all variables contemporaneously, as typically predicted by DSGE models.

Example 2 [SVAR with changing policy reaction function] Consider the same three-variable monetary policy SVAR of the previous example, and the structural specification

$$\underbrace{\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix}}_{\varepsilon_t} := \left\{ \underbrace{\begin{pmatrix} c_{11} & 0 & c_{13} \\ 0 & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}}_{C} + \underbrace{\begin{pmatrix} q_{11} & 0 & 0 \\ 0 & q_{22} & 0 \\ q_{31} & q_{32} & q_{33} \end{pmatrix}}_{Q} \times \mathbb{1}\left(t > T_B\right) \right\} \underbrace{\begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \end{pmatrix}}_{e_t} \tag{22}$$

This model describes an heteroskedastic SVAR in which, in addition to the non-triangular (non-recursive) structure similar to that in the Example 1, the last row of the specified matrix Q accommodates changes in the parameters governing the response of the policy reaction function to the structural shocks. In particular, the response on impact of the nominal short term interest rate R_t to $e_{1,t}$, $e_{2,t}$ and $e_{3,t}$, is postulated to change from the levels c_{3j} in the 'pre-break' regime, to the levels $c_{3j} + q_{3j}$ in the 'post-break' regime, $j = 1, 2, 3$. Moreover, this system shares with the 'DSGE-consistent SVAR' presented in the Example 1 the fact that the monetary policy shock may have an instantaneous impact on the macroeconomic variables $\tilde{y}_t$ and π_t in both volatility regimes, as implied by the non-zero specification of the last column of the matrix C. As it stands, however, the specification in Eq.s (22) meets the necessary order condition of Proposition 1 (indeed a_C:=dim(γ)=7, a_Q:=dim(q) = 5, so that there are a=12 structural parameters and $n(n+1)$=12 estimable moments in the covariance matrices $\Sigma_{\varepsilon,1}$ and $\Sigma_{\varepsilon,2}$), but does not satisfy the necessary and sufficient rank condition in Eq. (17). (Over)identification is achieved by imposing e.g. the (testable) restriction $q_{33} = 0$ in the Q matrix. This restriction maintains that the response of the short term interest rate to monetary policy shocks is invariant across the two volatility regimes. The specification in Eq. (22) will be estimated and tested in the next section, using U.S. quarterly data.

4 Empirical illustration

In this section, we apply the identification rules derived in Section 3 to estimate a small monetary policy SVAR using U.S. quarterly data. As in Lanne and Lütkepohl (2008), we identify the shocks by exploiting the change in volatility that occurred across several macroeconomic time series in the transition from the 'pre-Volcker' (or 'Great Inflation') to the 'Great Moderation' regimes, documented, among many others, in McConnell and Perez-Quiros (2002) and Boivin and Giannoni (2006). Our SVAR is based on the vector $Z_t{:=}(\tilde{y}_t, \pi_t, R_t)'$ ($n{:=}3$), where $\tilde{y}_t$ is a measure of the output gap, π_t the inflation rate and R_t a nominal policy interest rate. We deal with quarterly data, sample 1954.q3-2008.q3 (including initial values). The measure of real activity, $\tilde{y}_t$, is the Congressional Budget Office (CBO) output gap, constructed as percentage log-deviations of real GDP with respect to CBO potential output. The measure of inflation, π_t, is the annualized quarter-on-quarter GDP deflator inflation rate, while the policy instrument, R_t, is the Federal funds rate (average of monthly observations). The data were collected from the website of the Federal Reserve Bank of St. Louis.

We discuss the identification and estimation of the SVAR for $Z_t{:=}(\tilde{y}_t, \pi_t, R_t)'$ in Sub-section 4.1, and summarize some robustness checks in Sub-section 4.2.

4.1 A small monetary policy SVAR

In line with the empirical literature on the 'Great Moderation', we divide the postwar period 1954.q3-2008.q3 into two sub-samples: the 'pre-Volcker' period, 1954.q3-1979.q2, and the 'Great-Moderation' period, 1979.q3-2008.q3. This choice is consistent with Boivin and Giannoni (2006).[8] In our notation, $T_B{:=}1979.q2$, and hereafter this date will be treated as known. Our statistical tests, presented below, confirm that the two sub-periods 1954.q3-1979.q2 and 1979.q3-2008.q3 can be regarded as two periods characterized by different volatilities. The modeled reduced form VAR is a system with six lags ($k{:=}6$) and a constant. The VAR lag order is obtained by combining LR-type reduction tests with standard information criteria. Table 1 reports the estimated covariance matrices of the VAR and some (multivariate) residual diagnostic tests relative to the entire period and the two sub-periods, respectively. The Technical Supplement motivates our choice of treating the VAR for $Z_t{:=}(\tilde{y}_t, \pi_t, R_t)'$ as an approximately stationary system on both sub-periods, and checks the robustness of our specification to a different choice of the VAR lag order.

We test for the occurrence of a break at time $T_B{:=}1979.q2$ in the reduced form coefficients $\theta{:=}(\pi', \sigma_+')'$ of the VAR, in particular in the error covariance matrix σ_+. We first apply a standard Chow-type (quasi-)LR test for the (joint) null H_0: $\theta_1{=}\theta_2{:=}\theta$ against the alternative H_1: $\theta_1 \neq \theta_2$, where $p{:=}\dim(\theta){:=}63$. The results (Table 1) suggest that H_0 is strongly rejected because the (quasi-)LR test is equal to $LR{:=}$-2[719.98-(322.86+488.96)]=183.68 and has a p-value of 0.000 (taken from the $\chi^2(63)$ distribution). We then test the null $H_0^{(1)}{:}\sigma_{+1} = \sigma_{+2} = \sigma_+$ vs $H_1^{(1)}{:}\sigma_{+1} \neq \sigma_{+2}$ while maintaining the assumption that $\pi_1{=}\pi_2{=}\pi$. The results of the

[8] We are aware that many other choices for T_B are equally possible: an alternative would be to start the second period in 1984.q1 as in e.g. McConnell and Perez-Quiros (2002). It is worth observing that the period 1979.q3-2008.q3 includes the three-year window, 1979-1982, known as the 'Volcker experiment', during which the Federal Reserve implemented monetary policy actions by dealing with non-borrowed reserves, more than with the federal fund rate. As a result, the IRFs we compute on the 'post-Volcker' period (see Figure 1 and Figure 2 below) might be affected by the course of this 'non-standard' policy. Unfortunately, the sub-period 1979.q3-1984.q1 is not long enough to allow for the consideration of two break dates and three potential volatility regimes in the empirical analysis.

computed (quasi-)LR test also lead us to strongly reject the null $H_0^{(1)}$. We find formal support to the hypothesis that our 'pre-Volcker' and 'Great moderation' samples are characterized by two distinct volatility regimes.

We next move to the identification of the structural shocks, considering the schemes discussed in the two examples of Sub-section 3.3. We denote with $\mathcal{M}_1$ the exactly identified 'DSGE-consistent SVAR' in the Example 1, see Eq. (21), and with $\mathcal{M}_2$ the 'SVAR with changing policy reaction function' in the Example 2, see Eq. (22). Given the vector of structural shocks $e_t{:=}(e_t^{\tilde{y}},\, e_t^{\pi},\, e_t^{R})'$, we call e_t^R the 'monetary policy shock', $e_t^{\tilde{y}}$ the 'output shock' and e_t^{π} the 'inflation shock'.

The (quasi-)ML estimates of the structural parameters of $\mathcal{M}_1$ and $\mathcal{M}_2$ are summarized in Table 2, which also reports the log-likelihood associated with each model and the (quasi-)LR test for the overidentifying restriction for $\mathcal{M}_2$. Many studies based on SVARs typically find that U.S. monetary policy shocks have had a much smaller impact on output gap and inflation since the beginning of the 1980s. Overall, the results in Table 2 seem to confirm such evidence. In addition, we detect significant changes to the structural parameters in the move from the 'pre-Volcker' to the 'Great Moderation' period, because the specified elements in the Q matrices are found to be highly significant in both estimated SVARs. However, we also notice that as concerns model $\mathcal{M}_1$, the sign of the estimated parameters relative to the 'Great Moderation' period (i.e. the elements of the matrices $\hat{C}+\hat{Q}$, last column of Table 2) are not consistent with what a small monetary policy DSGE model would predict. This result admits at least two explanations. First, the estimated SVAR $\mathcal{M}_1$ does not feature the cross-equation restrictions implied by a monetary DSGE model (see the discussion in the Example 1). Second, the system might omit, as it stands, important transmission mechanisms, see Bacchiocchi *et al.* (2014).

The results stemming from the SVAR $\mathcal{M}_2$ are more interesting. Table 2 shows that model $\mathcal{M}_2$ is strongly supported by the data by the (quasi-)LR test for the overidentifying restriction, which has a p-value equal to 0.34. Figure 1 displays, for both volatility regimes, the IRFs implied by $\mathcal{M}_2$ relative to the monetary policy shock, e_t^R, with associated 95% (asymptotic) confidence interval, over a horizon of 20 periods. To improve the comparability of the IRFs, we have normalized the quarterly response on impact of the Federal funds rate R_t to a monetary policy shock e_t^R at the value 0.25 in both volatility regimes. The pattern of the two sets of impulse responses reveals the change in the monetary policy conduct. The key result from the comparison of the 'pre-Volcker' period (left column) and the 'Great Moderation' period (right column) in Figure 1 is that the effect of a monetary policy shock was stronger before the 1980s.[9] Figure 2 displays instead the response of the Federal funds rate to the shocks $e_t^{\tilde{y}}$ and e_t^{π},

[9]A further remarkable fact that emerges from the impulse responses in Figure 1 is the absence of the 'price puzzle' in the 'post-Volcker' period. This evidence, which is also documented in e.g. Barth and Ramey (2001), Hanson (2004), Boivin and Giannoni (2006) and Castelnuovo and Surico (2010), supports the view that the 'price puzzle' phenomenon is far more evident in situations in which the central bank responds weakly to inflationary dynamics.

respectively, for both samples. While the sensitivity of the short term nominal interest rate to the two shocks seems to be weak prior to the 1980s, the Fed's responsiveness to these two shocks is clear cut in the 'Great Moderation' period. According to a large (but much debated) strand of the literature, this evidence reflects the switch to a more aggressive ('active') policy intended to rule out the possibility of sunspot fluctuations induced by self-fulfilling expectations, see e.g. Clarida *et al.* (2000).

4.2 Robustness of the results

We conduct some experiments to check the robustness of our results to the specification of a different VAR lag order, the use of output growth in place of the output gap, and the inclusion of monetary balances. The details of these analyses are reported in the Technical Supplement, and unequivocally indicate that the core results discussed in the previous sub-section are indeed robust. We briefly summarize the main findings.

Different lag order. We consider a different dynamic specification based on a VAR with four lags, as suggested by many empirical contributions in the literature, see, among many others, Christiano *et al.* (2005). We strongly reject the null hypothesis of constant covariance matrices before and after time T_B:=1979.q2, and the set of IRFs implied by the SVAR $\mathcal{M}_2$ does not differ qualitatively from the IRFs reported in Figure 1 and Figure 2.

Output growth. The output gap is largely used in the empirical literature and should be preferred, in our context, to other measures of economic activity for reasons discussed in e.g. Giordani (2004). However, it might reasonably be affected by measurement errors. The natural alternative is to replace the output gap with real output growth, Δy_t (under the hypothesis that y_t is integrated of order one and π_t and R_t are stationary). The estimation of the SVAR $\mathcal{M}_2$ based on $Z_t^{*}:=(\Delta y_t, \pi_t, R_t)'$ shows that there are still two distinct regimes of volatility before and after T_B:=1979.q2, and that the implied IRFs do not differ qualitatively from the IRFs reported in Figure 1 and Figure 2.

Omitted variables: the role of money. A variety of recent empirical studies suggests that omitting money balances in the analysis of the monetary transmission mechanisms can produce severely distorted inference, see, *inter alia*, Canova and Menz (2009), Favara and Giordani (2009) and Castelnuovo (2012*a*). A thorough investigation of the role of monetary aggregates in the dynamics of U.S. business cycle and the actual transmission mechanisms at work goes well beyond the scopes of our paper. We limit our attention to check the robustness of our results to considering the identification and estimation of a SVAR based on $Z_t^{**}:=(\tilde{y}_t, \pi_t, R_t, \Delta mr_t)'$, where Δmr_t is the growth rate of real money balances. The variable $mr_t:=m_t - p_t$ is constructed by taking the log of the M2 money stock divided by the GDP deflator (source: website of the Federal Reserve Bank of St. Louis). Also in this case, we strongly reject the null hypothesis of constant covariance matrices for the VAR for $Z_t^{**}:=(\tilde{y}_t, \pi_t, R_t, \Delta mr_t)'$ before and after time T_B:=1979.q2.

To identify the SVAR, we exploit part of the (non-triangular) identifying restrictions already used for the SVAR $\mathcal{M}_2$, see the Technical Supplement. In addition, we postulate that the response on impact of real money growth to the output gap shock is non-zero on the 'pre-Volcker' period and is zero on the 'Great Moderation' period, as a consequence of the increasing attention of firms towards financial markets. Overall, our empirical results show that even controlling for real money growth, the IRFs obtained with our baseline three-equation SVAR $\mathcal{M}_2$ in Figure 1 and Figure 2 remain qualitatively unchanged.

5 Conclusions

A recent strand of the literature makes use of the heteroskedasticity found in the data to identify SVARs. The maintained assumption in this literature is that the structural parameters remain constant when the VAR covariance matrix changes. This approach reflects the intuition that structural changes may offer identifying power if some parameters do not change, see Magnusson and Mavroeidis (2014) for a comprehensive discussion.

In this paper, we have relaxed the assumption that all structural parameters are invariant to volatility regimes. We have derived novel necessary and sufficient rank conditions more general than the identification conditions discussed by other authors for heteroskedastic SVARs. We have illustrated the usefulness of our approach by focusing on a small-scale monetary policy SVAR model estimated using U.S. quarterly data. Overall, our results support the view that monetary policy has become more effective at stabilizing the economy since the 1980s.

Appendix: Proof of Proposition 1

(a) We write the mapping between the reduced- and structural-form parameters in Eq.s (14)-(15) in the form

$$\sigma_{+}^{*} = h(\psi)$$

where $\sigma_{+}:=(\sigma_{+1}', \sigma_{+2}')'$, $\psi:=(\gamma', q')'$, and $h(\cdot)$ is a nonlinear differentiable vector function. Given the constraints in Eq. (16) and following Rothenberg (1971), necessary and sufficient condition for local identification is that the $n(n+1) \times a$ Jacobian matrix

$$\frac{\partial \sigma_{+}^{*}}{\partial \psi'} := \frac{\partial \sigma_{+}^{*}}{\partial \vartheta'} \times \frac{\partial \vartheta}{\partial \psi'} = \frac{\partial \sigma_{+}^{*}}{\partial \vartheta'} \times G \tag{23}$$

has full column rank $a:=a_C + a_Q$, evaluated a ψ_0. The necessary order condition $a \leq n(n+1)$ is therefore obvious.

To compute the $n\,(n+1) \times \left(2n^2+n\right)$ matrix $\frac{\partial \sigma_{+}^{*}}{\partial \vartheta'}$, it is convenient to apply the first differential and standard matrix algebra rules to the system in Eqs. (14)-(15), obtaining

$$2N_n\,(C\otimes I_n)\,vec\,\mathrm{d}C = 0_{n^2\times 1} \tag{24}$$

$$2N_n\,((C+Q)\otimes I_n)\,vec\,\mathrm{d}C + 2N_n\,((C+Q)\otimes I_n)\,vec\,\mathrm{d}Q = 0_{n^2\times 1}. \tag{25}$$

However, $N_n = D_n D_n^+$, implying that among the set of n^2 equations of the system (24)-(25), only $n\,(n+1)\,/2$ are linearly independent, and in particular, those given by

$$2D_n^+\,(C\otimes I_n)\,vec\,\mathrm{d}C = 0_{\frac{1}{2}n(n+1)\times 1}$$

$$2D_n^+\,((C+Q)\otimes I_n)\,vec\,\mathrm{d}C + 2D_n^+\,((C+Q)\otimes I_n)\,vec\,\mathrm{d}Q = 0_{\frac{1}{2}n(n+1)\times 1}.$$

It turns out that up the the multiplicative scalar "2", the derivative $\frac{\partial\sigma_+^*}{\partial\vartheta'}$ can be written as

$$\frac{\partial\sigma_+^*}{\partial\vartheta'} = \begin{pmatrix} D_n^+ & 0_{\frac{1}{2}n(n+1)\times n^2} \\ 0_{\frac{1}{2}n(n+1)\times n^2} & D_n^+ \end{pmatrix} \times \begin{pmatrix} (C\otimes I_n) & 0_{n^2\times n^2} \\ ((C+Q)\otimes I_n) & ((C+Q)\otimes I_n) \end{pmatrix} \tag{26}$$

and coming back to Eq. (23), the result is obtained. ■

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TABLE 1

Estimated covariance matrices and diagnostic tests from the VAR with six lags. Break date T_B:=1979.q2

Overall period	1954.q3-2008.q3 (T=211)	dim(θ):=63
LM_{AR5}:= 1.064 [0.363]	$\hat{\Sigma}_{\varepsilon}:=\begin{pmatrix} 0.762 & -0.012 & 0.037 \\ & 0.273 & 0.007 \\ & & 0.191 \end{pmatrix}$	log-Likelihood:=719.98
JB_N:=186.27 [0.000]		
'pre-Volcker' period	1954.q3-1979.q2 (T=94)	dim(θ_1):=63
LM_{AR5}:= 0.838 [0.753]	$\hat{\Sigma}_{\varepsilon,1}:=\begin{pmatrix} 0.989 & -0.021 & 0.036 \\ & 0.314 & 0.007 \\ & & 0.152 \end{pmatrix}$	log-Likelihood:=322.86
JB_N:= 14.525 [0.0234]		
'Great Moderation' period	1979.q3-2008.q3 (T=117)	dim(θ_2):=63
LM_{AR5}:= 1.69 [0.010]	$\hat{\Sigma}_{\varepsilon,2}:=\begin{pmatrix} 0.562 & 0.010 & 0.045 \\ & 0.204 & 0.007 \\ & & 0.194 \end{pmatrix}$	log-Likelihood:=488.96
JB_N:= 50.91 [0.000]		

NOTES: LM_{AR5} is the Lagrange Multiplier vector test for the absence of residuals autocorrelation against the alternative of autocorrelated VAR disturbances up to lag 5; JB_N is the Jarque-Bera multivariate test for Gaussian disturbances. Number in brackets are p-values.

TABLE 2.

Estimated SVARs with break date T_B:=1979.q3 on U.S. quarterly data $Z_t:=(\tilde{y}_t, \pi_t, R_t)'$, $e_t:=(e_t^{\tilde{y}}, e_t^{\pi}, e_t^{R})'$

Model: $C(t):=C + Q \times \mathbb{1}\,(t > T_B)$, $t = 1, ..., T$

Model	$\hat{C}$			$\hat{Q}$			$(\hat{C}+\hat{Q}),\ t > T_B$		
$\mathcal{M}_1$	0.876	-0.045	-0.105	-0.374			0.502	0.045	0.105
	(0.065)	(0.055)	(0.054)	(0.073)					
	-0.020	0.255	-0.115		-0.402		-0.020	0.147	0.115
	(0.022)	(0.030)	(0.038)		(0.052)				
	0.049	0.072	0.105			-0.260	0.049	-0.072	0.155
	(0.012)	(0.027)	(0.018)			(0.032)			
	Log-Likelihood = 811.81			exact identification					
$\mathcal{M}_2$	0.883		-0.058	-0.373			0.510		-0.058
	(0.064)		(0.102)	(0.073)					
		0.263	-0.100		-0.105			0.158	-0.100
		(0.031)	(0.051)		(0.027)				
	0.042	0.067	0.112	0.044	0.042		0.086	0.108	0.112
	(0.020)	(0.028)	(0.018)	(0.025)	(0.023)				
	Log-Likelihood =811.37			LR test = 0.90 [0.34]					

NOTES: Standard errors in parenthesis, p-values in squared brackets. The columns of the matrix $(C+Q)$ have been normalized such that the elements on the main diagonal are positive. Empty entries correspond to zeros. The reduced form associated with the estimated SVAR is a VAR with six lags.

FIGURE 1. Impulse responses of the variables in Z_t to a same-size monetary policy shock e_t^R with 95% (analytic) confidence bands, based on the SVAR model $\mathcal{M}_2$ estimated in Table 2.

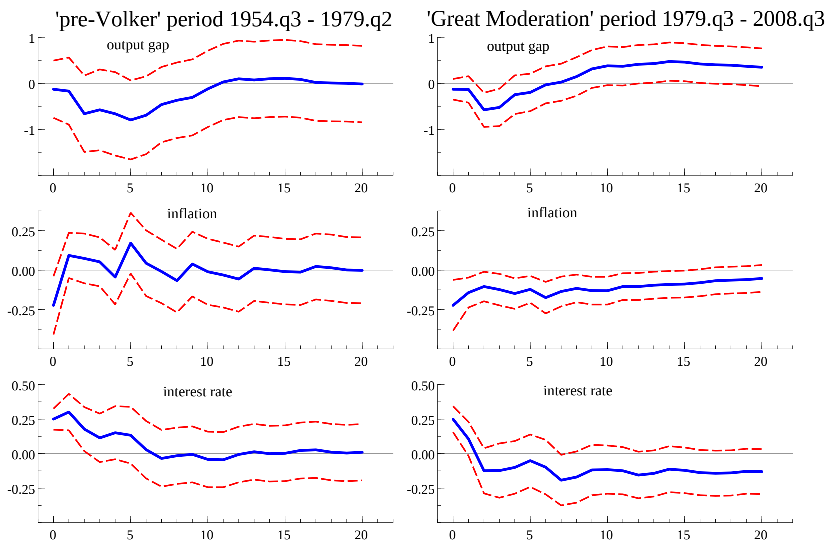

FIGURE 2. Impulse responses of R_t to same-size output and inflation shocks $e_t^{\tilde{y}}$ and e_t^{π} with 95% (analytic) confidence bands based on the SVAR model $\mathcal{M}_2$ estimated in Table 2.