

Shopping Effort in Self-Insurance Economies^{*}

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Abstract

How are income fluctuations transmitted to consumption decisions in the presence of price dispersion? I propose a novel and tractable framework to study search for consumption as part of the optimal savings problem. Due to frictions in the retail market, households have to exert some effort to purchase the consumption good. This effort has two components: 1. effort to search for price bargains; 2. effort required to purchase consumption of a given size. These two motives are necessary to replicate two seemingly contradictory shopping patterns observed in the data, namely: higher time spent shopping by the unemployed and retirees and (conditioned on being employed) the positive elasticity of shopping time with respect to labor income. The former is well known in the literature, while the latter is new and I document it using data from the American Time Use Survey. The model reconciles the traditional savings theory with households' shopping behavior in a quantitatively meaningful way. As I show frictions in the purchasing technology generate important macroeconomic implications for modeling inequality and, in general, household consumption.

KEYWORDS: Consumption, Price Search, Inequality

JEL CLASSIFICATION: D11, D31, D91, E21, E30

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I. INTRODUCTION

HOUSEHOLD CONSUMPTION accounts for nearly 70% of the GDP in the US. A good understanding of how household income fluctuations are passed on consumption expenditures is crucial for credible quantitative analysis in a vast class of economic models, when household consumption plays an important role¹. This paper provides a theory of the pass-through of shocks into consumption when the law of one price does not hold, *i.e.* different retailers charge different prices for exactly the same good. The theory is motivated by recent strong evidence for price dispersion and heterogeneity in household shopping behavior. None of those findings have been integrated into standard consumption models yet and as I show they generate some important implications for modeling household inequalities and, in general, the aggregate consumption.

“Big
Picture”

The standard incomplete-markets models with heterogenous households (henceforth, SIM) in the tradition of [Bewley \(1986\)](#), [Aiyagari \(1994\)](#), and [Huggett \(1993\)](#), where consumers insure against future income fluctuations accumulating a risk-free bond, are a workhorse for quantitative analysis of consumption from both macroeconomic and microeconomic standpoints². Nonetheless, they underestimate the level of risk sharing present in the economy and given income process observed in data they have serious problems with generating enough inequality³. Especially, the former is of particular importance. As [Heathcote, Storesletten, and Violante \(2014\)](#) point out quantifying existing risk sharing is crucial to evaluate the welfare consequences of counterfactual policy experiments⁴. In addition to this, in their current form the SIM models completely abstract from any kind of price dispersion and assume that the law of one price always holds, namely all households pay the same competitive price.

SIM
and
Price
Dispersion

¹Just to name but a few: optimal capital income taxation ([Aiyagari, 1995](#)), the benefits of insuring unemployed people ([Hansen and İmrohoroğlu, 1992](#)), effects of fiscal stimulus payments in a recession ([Kaplan and Violante, 2014](#)), the redistributive role of monetary policy ([Auclert, 2016](#); [Kaplan, Moll, and Violante, 2016](#)), effects of a credit crunch on consumer spending ([Guerrieri and Lorenzoni, 2015](#)), the role of household debt and bankruptcy filing rates ([Chatterjee, Corbae, Nakajima, and Ríos-Rull, 2007](#)).

²See survey articles by [Heathcote, Storesletten, and Violante \(2009\)](#), [Guvenen \(2011\)](#), and [Attanasio and Weber \(2010\)](#).

³Admittedly, there are some models that manage to capture the right level of inequalities, but they either assume the very risky income process as in ([Castañeda, Díaz-Giménez, and Ríos-Rull, 2003](#)) or substantial heterogeneity in time preference as in ([Krusell and Smith, 1998](#)).

⁴In a similar way, [Kaplan and Violante \(2010\)](#) argue that replicating the level of consumption smoothness, a measure of risk-sharing proposed by [Blundell, Pistaferri, and Preston \(2008\)](#), should be one of the central goals in quantitative macroeconomics.

Substantial and systematic price dispersion is a fact observed in data. A growing literature documents many examples of this phenomenon on both sides of the market, between different households and between different retailers. (i.a) [Aguiar and Hurst \(2007\)](#) show that retirees pay approximately 4% less for the same goods than households with heads in their working age. (i.b) A similar price differential is observed between non-employed and employed households. [Kaplan and Menzio \(2015\)](#) used the same dataset and found that on average non-employed consumers pay between 1 and 4% less for the same consumption baskets. (ii) Moreover, price dispersion is also present from retailers' perspective. [Kaplan and Menzio \(2016\)](#) and [Kaplan, Menzio, Rudanko, and Trachter \(2016\)](#) observed the average standard deviation of prices for the same goods amounts to 15.3%. In addition to this, the authors identified that only at most 15% of the price variance is due to variation in the expensiveness of the stores at which a good is sold.

Empirical
Price
Dispersion

Second, individual shopping effort, which is measured as time spent obtaining goods, varies significantly between households depending on their labor status. Using time diaries [Aguiar and Hurst \(2007\)](#) documented that retirement-age people spend on average 33% more time shopping than households aged 25-29. Similarly, [Krueger and Mueller \(2012\)](#) show that the unemployed people spend between 15 and 30% more time shopping than the employed. Traditionally, higher shopping effort is rationalized by search for lower prices. Thus, both empirical patterns, heterogeneity in shopping intensity and in average prices paid by households, are connected by the price comparison mechanism.

Empirical
Shopping
Heterogeneity

On the empirical side, I contribute to studies on the consumer shopping behavior. Using the American Time-Use Survey I find that unemployed and retired consumers spend on average more time shopping. These observations are consistent with the aforementioned findings made by [Aguiar and Hurst \(2007\)](#) and [Krueger and Mueller \(2012\)](#). What is new in my analysis is that conditioned on being employed, households from top earnings deciles spend about 12% more time shopping than poor employed households. This observation together with the fact that rich households pay higher prices⁵ seem to stay at odds with the traditional mechanism relying on the price comparison motive. This suggests that apart from price hunting there must be another motive driving shopping effort in such a fashion that rich people spend more time purchasing goods. High earners are also households that consume more. Therefore, the observed increase in their shopping

New Empirical
Shopping Pattern

⁵[Aguiar and Hurst \(2007\)](#) documented households earning more than \$70,000 a year pay 2.1 percent more than households earning less than \$30,000 a year.

time can be driven not by the price search intensity, but rather by willingness to increase their consumption. The findings obtained in the empirical part are used for disciplining the behavior of my quantitative model.

In the theoretical part, I integrate search for consumption into a life-cycle version of the SIM model due to [İmrohoroğlu, İmrohoroğlu, and Joines \(1995\)](#); [Huggett \(1996\)](#); [Ríos-Rull \(1996\)](#). The household's income is driven by idiosyncratic productivity shocks. Every household makes the decision about level of savings, that are used to insure against the future income fluctuations and to smooth the future consumption. The remaining disposable resources of the household are spent on consumption. I extend the benchmark SIM economy by adding frictions in the purchasing technology. Households have to exert effort to purchase goods. This effort can be decomposed into two components: 1. *price search intensity* – effort to search for price bargains, 2. *purchase effort* – effort required to purchase consumption of a given size⁶. Both retailers and households' shopping come together at random through a frictional meeting process. Households that search for low price more intensively are able to find lower prices more often. Households exhibiting higher purchase effort are able to obtain more consumption. Retailers set their prices in response to the distribution of household search intensity. Sellers charging relatively high (low) prices sell less (more) often but with higher (lower) markups. In an equilibrium every seller yields the same profits, but for different prices the profit comes from a different combination of appropriation of consumer surplus and stealing customers of other competing retailers. To the best of my knowledge, the proposed model is the first to combine the optimal savings problem and search for consumption in a quantitatively meaningful way.

Finally, I juxtapose the aggregate consumptions of two versions of calibrated economies, the SIM economy without product-market frictions and the “shopping” economy with frictions in the purchasing technology. I use two alternative approaches to study their properties: 1. consumption responses to idiosyncratic income shocks; and 2. cross-sections of households' decisions (consumption expenditures) and endogenous states (net wealth). Consumption responses reflect the dynamic character of the aggregate demand, while cross-sections of decisions and endogenous states determine the initial state of any

⁶A size of consumption might be understood in three ways: quantity, variety, and quality. In the model I focus on quantity but it can be extended to variety very easily, if preferences of households are modeled as in [Wolinsky \(1986\)](#).

counterfactual experiments. Thus, an accurate measure of both is of particular interest for quantitative analyses. Using simulated panels, I show that in the SIM economy without product-market friction households overreact to income changes and 80% of permanent shocks are translated to consumption⁷, while in the shopping economy only 60% of permanent shocks are transmitted into consumption expenditures. The responses generated in the shopping model are much closer the empirical counterpart of 64% documented by [Blundell, Pistaferri, and Preston \(2008\)](#). This effect can be explained by the fact that marginal disutility for the shopping effort partially offsets utility of consumption, which makes consumption responses smoother. Even if the shopping generates smoother consumption responses, it also amplifies wealth and consumption inequality. Consumption expenditures are more dispersed in the shopping economy due to disentangling consumption from consumption expenditures. Poor households with lower consumption exert higher price search intensity. As a result, they pay lower prices, which leads to a lower share in aggregate consumption expenditures accrued to poor groups. On the other hand, net wealth inequality is increased by rich working households, who detain from increasing current consumption due to the high utility cost of additional purchases.

The rest of the paper is structured as follows. Section [II](#) reviews connections to the existing literature. In section [III](#), I present empirical patterns of shopping time by American households. In section [IV](#), I present the building blocks of the quantitative model and characterize the equilibrium. Subsequently in section [V](#) I carry out the calibration to match moments observed in the US data. In section [VI](#) I highlight the nature of price dispersion at play for the calibrated version of the shopping model. In section [VII](#) I deconstruct aggregate demands generated by two artificial economies, the standard incomplete-markets economy without frictions in the purchasing technology and the shopping economy with product-market frictions. Section [VIII](#) concludes.

II. RELATED LITERATURE

The idea that consumer search might have important macroeconomic implications for modeling aggregates is relatively new. [Kaplan and Menzio \(2016\)](#) propose a theory of amplification of shocks driven by changing shopping behavior of households. To this purpose they build a model that combines consumer search ([Burdett and Judd, 1983](#); [But-](#)

⁷This finding is consistent with the “excess smoothness” of empirical consumption documented in aggregate data by [Campbell and Deaton \(1989\)](#) and in individual data by [Attanasio and Pavoni \(2011\)](#).

ters, 1977) with labor search (Mortensen and Pissarides, 1994). Despite some similarities in modeling product-market frictions between my framework and theirs, it is important to note some remarkable differences. First, in my setup price search intensities are determined endogenously by household decisions, while the authors calibrate them to exogenous values. Second, Kaplan and Menzio (2016) assume that agents are hand-to-mouth, they can neither save or borrow and are not allowed to smooth their marginal utility of consumption over their lifecycle. Consequently, my model allows addressing important questions related to household consumption, which are beyond interests of the aforementioned article.

Huo and Ríos-Rull (2013) and Bai, Ríos-Rull, and Storesletten (2011) offer an alternative model of search for consumption. In their models the authors employ directed search due to Moen (1997). In directed search retailers are divided into locations that provide goods at different prices and lengths of queues. Household with different earnings and net wealth visit different locations. I argue that random search is a more natural choice for modeling consumption decisions for two reasons. Affluent households are able to obtain goods in locations with short queues and high prices. In this sense they substitute shopping effort with higher prices. This behavior is common for both protocols, but in directed search there is no limit for such a substitution while in random search households can substitute effort with prices up to the limit where they decide to be captive in all transactions where prices are drawn at random. Consequently, without limits of substitution between prices and shopping effort consumption expenditure responses in economies with directed search for consumption can be even higher than in the SIM model without shopping frictions, which problems with generating smoothness of consumption observed in data is well documented (Attanasio and Pavoni, 2011; Kaplan and Violante, 2010). Furthermore, the recent empirical literature due to Kaplan, Menzio, Rudanko, and Trachter (2016) and Kaplan and Menzio (2015) shows that only 15% of the variance of prices is due to variation in the expensiveness of the stores at which a good is sold. This finding suggests to use the random search rather than directed search, where the market is split into locations with different prices.

The search protocol used in my paper also relates to the classical model of random search for consumption due to Burdett and Judd (1983). The household problem is the aspect in which my approach departs from that model substantially. In the economy of Burdett and Judd (1983) all households buy a unit of good and make a decision on their

price search. They can either draw one price from the equilibrium dispersion or by paying an additional cost they can draw two prices and choose the lower one. In my model households make a decision about the quantity of consumption and the *probability* of drawing two prices. The former is important for introducing consumption search into the optimal savings problems, while the latter has implications for properties of the equilibrium, stability and multiplicity. I study and compare equilibria of both models in great details in the companion paper (Pytka, 2016).

The potential effect of search for price bargains on aggregates was recently studied by Krueger, Mitman, and Perri (2016). The authors consider an economy, in which output does not depend only on capital and labor inputs but also on consumption. This effect is obtained in a reduced form simply by introducing consumption as another input in the production function. In my shopping economy this aggregate demand externality has intuitive micro foundations. Any transfer targeted to poor households with high search intensity increases the aggregate price search intensity. Retailers respond to this change by charging lower prices and this results⁸ in a hike in consumption spendings for all households, not only the recipients of the transfer. As a result, the shopping friction magnifies the effect of stimulus transfers.

The model developed in this paper relates also to concerns raised by Petrosky-Nadeau, Wasmer, and Zeng (2016). The authors used time diaries to document the average decline in shopping time in the Great Recession compared to 2005–2007. This finding was used to call into question whether the shopping effort can be used as an amplifier of shock propagation. They argue that in a contraction households should increase the shopping effort, if the theory of bargain hunting is correct. However, this argument does not have to be true, if the shopping effort is constituted not only by search for price bargains but also by the purchase effort associated with the size of consumption. Therefore, I claim that introducing a new margin of shopping effort proposed in this paper is necessary to reconcile the pattern observed by Petrosky-Nadeau, Wasmer, and Zeng (2016) and the theory of macroeconomic implications of the price dispersion.

⁸Under the assumption that consumption is a normal good.

III. EMPIRICAL PATTERNS

I start by characterizing the shopping effort observed in the data. I follow the literature (e.g., [Aguiar and Hurst, 2007](#); [Aguiar, Hurst, and Karabarbounis, 2013](#); [Kaplan and Menzio, 2016](#)) and I use time spent shopping as a proxy for the shopping effort. For this purpose I study time diaries, which document the allocation of time of the American households. In particular, I am interested in the relationship between shopping and labor market status, i.e. unemployment, retirement, and the level of labor earnings. I show that conditioned on being employed, the level of shopping effort exerted by households is positively correlated with the level of their earnings. Next, the findings from this section are used to discipline the quantitative model outlined in the subsequent section, [IV](#).

Data. In the analysis I use data from the 2003–2015 waves of the American Time-Use Survey. The ATUS is conducted by the U.S. Census Bureau and individuals are randomly selected from a subset of households from the Current Population Survey. Each wave is based on 24-hour time diaries where respondents report the activities from the previous day in specific time intervals. Next the ATUS staff categorizes those activities into one of over 400 types. The 2003 wave includes over 20,000 respondents, while the later waves consist of around 13,000 respondents.

Identification Strategy. To assess how shopping effort is reallocated across different households i with different levels of labor earnings I estimate the following regression:

$$\log shopping_i = \alpha + \sum_j \beta_j earn_i^j + \gamma X_i + \varepsilon_i. \quad (1)$$

To abstract from the discussion on the functional specification, I regressed the dependent variable on dummies. In addition to this I logarithmized⁹ $shopping_i$ to include possible multiplicative interactions between covariates.

The variable $shopping_i$ measures cumulative daily time (in minutes) spent obtaining goods or services (excluding education, restaurant meals, and medical care) and travels related to these activities. Some examples of activities captured by this variable are: grocery shopping, shopping at warehouse stores (e.g., WalMart or Costco) and malls, doing banking, getting haircut, reading product reviews, researching prices/availability, and online

⁹Due to the fact that there are observations with zero values, I use the inverse hyperbolic sine function $\text{arsinh}(x) = \ln(x + \sqrt{1 + x^2})$ as an approximation of the logarithmic function.

shopping.

There are three types of variables associated with the labor force status: 1. nine categorical variables $earn_i^j$ where each of them represents j -th decile of weekly labor income with the bottom decile as the referential category (Table: 12), 2. unemployment status (both on lay-off and looking), and 3. retirement. To control other sources of heterogeneity I introduce some demographic variables: the (quadratic) age trend, gender (woman as reference), and race (white as reference).

Aguilar and Hurst (2007) suggest controlling for ‘shopping needs,’ which stem from differences in the family composition. For this reason, I add dummies indicating: 1. if the respondent has a partner (both spouse and unmarried), 2. whether the partner is unemployed, and 3. the presence of children.

Results. I estimated the model using the pooled regression. All observations were weighted to ensure that each stratification group is correctly represented in the population. I restricted the sample only to households aged 22-74 and excluded top-percentile households with respect to earnings and shopping. Three specification are considered: with labor market variables only (I); with controls for labor market status and shopping needs (II); with controls for market status, shopping needs, and year/day dummies (III). The estimated results of model (1) are presented in Table 1 and Figures 1 and 2. The conducted analysis leads to the following observations on the relationship between the shopping behavior and the labor market status:

Pattern 1 (Shopping effort and labor market status) *In the ATUS 2003-2015 the following patterns are observed:*

- i the unemployed people spent on average $\exp(.321) = 37.85\%$ more time shopping than the referential earning group;*
- ii the retired people spent on average $\exp(.165) = 17.94\%$ more time shopping than the referential earning group;*
- iii top deciles of the labor earnings spent on average more time shopping than the referential earning group.*

Observation 1.i and 1.ii do not differ qualitatively from results present in the literature. Kaplan and Menzio (2016) show that the unemployed people spend between 13% and

Table 1: Regression results

	Dependent variable <i>log(shopping)</i>		
	(I)	(II)	(III)
Earnings dummies (Fig. 1)	■	▲	●
Retired	0.147*** (0.035)	0.161*** (0.035)	0.165*** (0.035)
Unemployed	0.302*** (0.032)	0.314*** (0.032)	0.321*** (0.032)
Male	-0.484*** (0.013)	-0.466*** (0.013)	-0.470*** (0.013)
Age	0.007** (0.003)	-0.002 (0.003)	-0.003 (0.003)
<i>Age</i> ²	-0.0001* (0.00004)	0.00004 (0.00004)	0.00005 (0.00004)
Black	-0.151*** (0.020)	-0.128*** (0.020)	-0.127*** (0.020)
Single		-0.125*** (0.016)	-0.124*** (0.016)
Unemployed Partner		-0.170*** (0.019)	-0.170*** (0.018)
Child		0.041*** (0.015)	0.041*** (0.015)
Constant	1.979*** (0.070)	2.182*** (0.073)	2.217*** (0.078)
Shopping needs	No	Yes	Yes
Year and day dummies	No	No	Yes
<i>N</i>	132,131	132,131	132,131
F Statistic	100.574***	90.876***	71.039***

Notes:

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

20% more time on shopping than the employed. The difference obtained by [Krueger and Mueller \(2012\)](#) is larger and amounts to 28%. My finding is of the same sign but is also quantitatively higher. The reason for this can be attributed to the fact that I used the bottom decile of labor earners as the referential category in my study, whereas in the aforementioned articles the unemployed are compared with the whole population of the employed. Regarding the shopping behavior of retirees, [Aguiar and Hurst \(2007\)](#) compare the cell of retirement-age¹⁰ people with households aged 25-29 and show the older spend on average 32.7 log-points more on shopping. This effect is twice as large as observation 1.ii. However, if instead of regressing on the quadratic age trend I use age bins in the way [Aguiar and Hurst \(2007\)](#) did, then the people who are in the oldest cell *and* are retired spend 30.7 log-points more on shopping. In the further considerations I stick to estimate 1.ii, which disentangles the retirement state from the age effect.

Patterns 1.i and 1.ii are well known and rationalized by the search for price bargains. Households with low resources pay more attention to expenditures and are more patient to get lower prices. They are able to decrease their prices by increasing the search effort embodied by such activities as visiting more stores for comparison shopping, clipping coupons, or waiting for sales. All of them require some additional amount of time though. Those observations led to a traditional view that equalizes shopping effort with the search for price bargains.

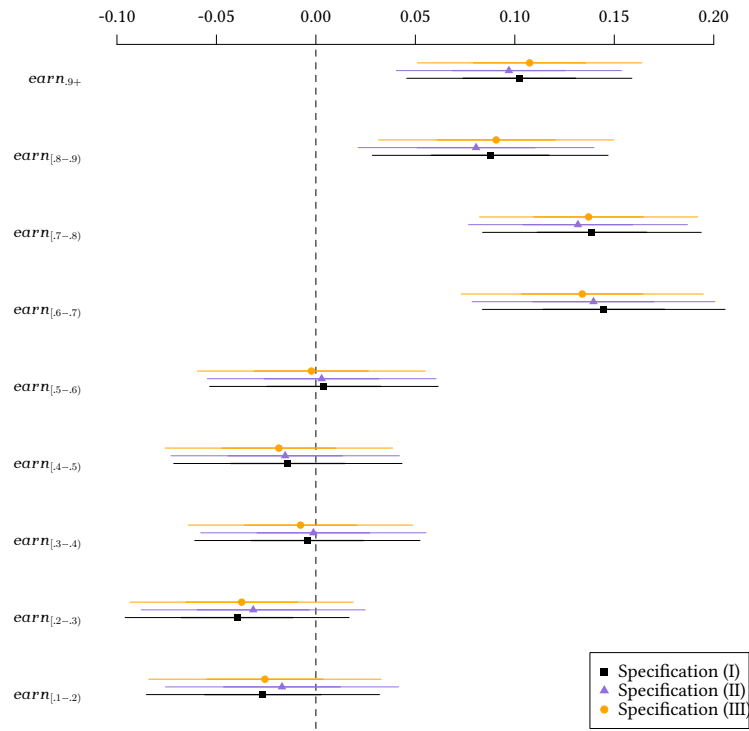
In this sense, observation 1.iii, which to the best of my knowledge is novel, seems to be paradoxical. Conditioned on being employed, affluent households from top deciles spend significantly more time shopping than poor households (see Figure 1). Nonetheless, according to the reasoning above we should rather observe the opposite effect¹¹. This finding suggests that apart from price hunting there must be another motive driving shopping effort in such a fashion that rich people spend more time shopping. I claim that an effort accompanying the size of consumption is a good candidate for such a motive that rationalizes fact 1.iii. The argument for this is that households with a high level of consumption have to visit more stores. Every shopping trip takes additional amount of time¹².

¹⁰The authors do not use an explicit dummy for the retirement state.

¹¹In this claim I implicitly assume that leisure and consumption are normal goods.

¹²Admittedly, households can purchase many units of consumption in one store. This concern is discussed thoroughly in section IV. Without going too much into details, such a shopping strategy of households increases the market power of retailers and makes customers captive for a bigger fraction of purchases.

Figure 1: Regression estimates for dummies of earning deciles.

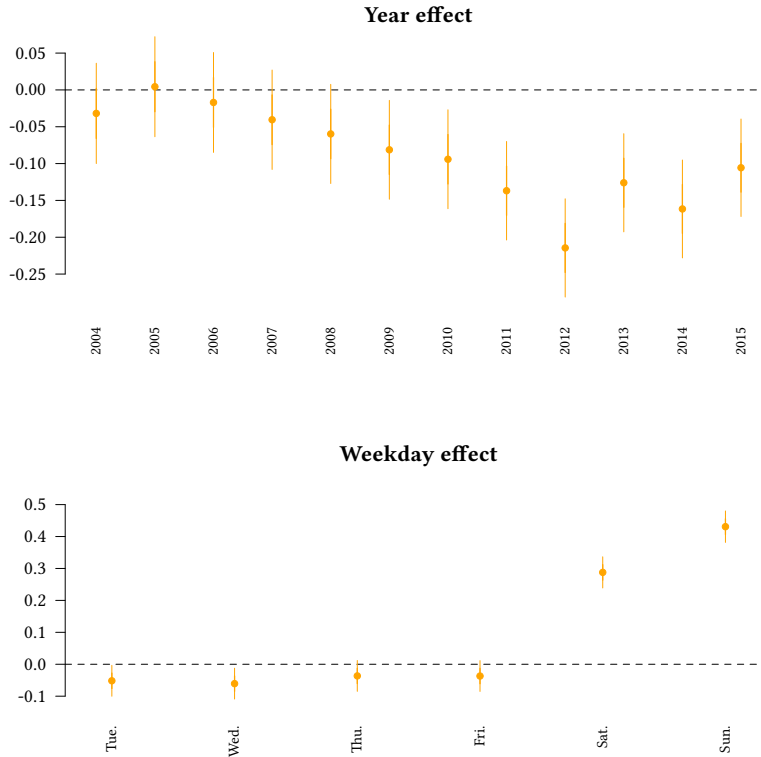


The remaining estimates of variables controlling for shopping needs are consistent with the intuition. Single households spend less time shopping due to lower variety and amount of needed consumption. Respondents with a unemployed partner also spend less time on purchasing goods. This can be explained with delegation of non-market work to unemployed members of a family, who have more time. Having children increases shopping needs too¹³.

Last but not least, Figure 2 shows how the shopping effort varied over years and weekdays. Unsurprisingly, households spend more time shopping in weekends. It is worth noting, there is a visible downward trend from 2003 through 2015. Every year households spent less time shopping, on average 1.28% less every year. One reason for this phenomenon can be the profound improvement of purchasing technology. New technologies developed recently such as online purchases, comparison shopping engine, mobile

¹³In an auxiliary analysis, I verified if a number of children is important. It does not seem so as the increase of shopping time is of about 4% for every number of children (treated as dummies).

Figure 2: Regression estimates for year and weekdays dummies.



payments, and many more may caused the decline in the magnitude of product market frictions. Consequently, it might have given rise to lower time required for obtaining consumption goods.

IV. A LIFE-CYCLE MODEL OF SHOPPING EFFORT

My framework integrates random search for consumption into a life-cycle incomplete markets model with heterogenous agents (e.g., [İmrohoroğlu, İmrohoroğlu, and Joines, 1995](#); [Huggett, 1996](#); [Ríos-Rull, 1996](#)). Household income is driven by idiosyncratic productivity shocks. Every household makes the decision about level of savings, that are used to insure against future income fluctuations and to smooth the future consumption. The remaining disposable resources of household are spent on consumption. On top of the economy I introduced the frictional transactions technology. Households have to exert effort to purchase goods. This effort has two components: 1. effort to search for price

bargains, 2. purchase effort required to purchase consumption of a given size. The former accounts for increasing probability that household during a single purchase samples a lower price, while the latter relates to the assumption that more consumption is possible by increasing a number of purchases¹⁴. The price search is present and documented in the literature (e.g., [Kaplan and Menzio, 2016](#)) and the purchase effort is new and explained in more details later.

I first describe the setup of the economy. Next I characterize the model equilibrium and present some examples to shed some light on the shopping mechanism at work.

A. Building Blocks of the Economy

Demographics. The model period is one year. The stationary economy is populated by a continuum of households living T periods. Consumers work for T_{work} periods and next go into retirement for $T - T_{work}$ periods.

Preferences. Households exhibit preferences defined over stochastic sequences of consumption and overall shopping effort $\{c_t, f_t\}_{t=1}^T$ represented by the instantaneous utility function:

$$u(c_t) - v(f_t), \quad (2)$$

and the discount factor β . Households are expected utility maximizers. The utility from consumption, $u(c_t)$ is additively separable from the disutility from shopping effort, $v(f_t)$. Both functions are assumed to be increasing and $u(c_t)$ is concave while $v(f_t)$ is convex. Overall shopping effort f_t is a function of two shopping margins, a number of purchases m_t and search intensity s_t . It increases in both margins, i.e. $\frac{\partial f_t}{\partial m_t} > 0$, $\frac{\partial f_t}{\partial s_t} > 0$. Besides, both shopping margins affect each other's impact on f_t as follows $\frac{\partial^2 f_t}{\partial s_t \partial m_t} > 0$. It means that higher shopping effort m_t increases the marginal cost of searching for price bargains, s_t , and vice versa, the higher search intensity leads to a higher marginal cost of shopping effort.

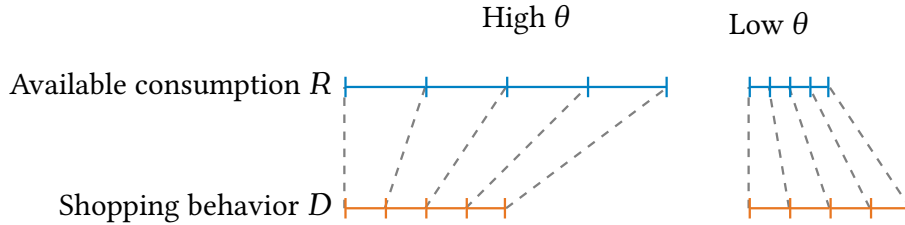
Purchases (m_t). In order to consume goods c_t , households must spend some time for visiting stores. They make many repeated purchases (shopping visits) m_t in a given

¹⁴In this regard, there is an important difference with the story of long queues with low prices and short queues with high prices offered by the directed search (e.g., [Moen, 1997](#); [Bai, Ríos-Rull, and Storesletten, 2011](#)). The recent empirical literature due to [Kaplan, Menzio, Rudanko, and Trachter \(2016\)](#); [Kaplan and Menzio \(2015\)](#) shows that only 15% of the variance of prices is due to variation in the expensiveness of the stores at which a good is sold. This finding suggests to use the random search instead.

period. The level of the required effort is strictly increasing with consumption. Consumers make purchases, which are matched with goods offered by the retailers. Let D be the aggregate level of shopping effort (yet to be defined) of all households, R be the total amount of consumption purveyed by the retailers and $\theta = \frac{R}{D}$ be the market tightness of the consumption market. Both sides come together through a constant return to scale Cobb-Douglas function, $M(D, R) = D^\alpha R^{1-\alpha}$. A single shopping visit allows a household to purchase $\frac{M(D,R)}{D} = \theta^{1-\alpha}$ units of consumption. Thus, given an equilibrium market “tightness” θ , there is a linear relationship between consumption and the level of required shopping effort, *viz.*

$$c_t = m_t \theta^{1-\alpha}. \quad (3)$$

Figure 3: Matching shopping effort with retailers



Note, that the efficiency of purchase $\theta^{1-\alpha}$ does not necessarily have to be less than one. This statistics tells us about the level of feasible consumption for a single purchase¹⁵. Suppose that a household wants to consume a certain amount of goods. In an economy with high θ she has to make fewer shopping trips to be able to purchase it (Figure 3).

Price Search (s_t). Apart from the number of purchases (which directly translates to the level of consumption), each household makes a decision on the intensity of search for price bargains, s_t . Suppose prices quoted by retailers are distributed according to a cdf $G(p) = Pr(x \leq p)$ with a lower bound \underline{p} , such that $G(\underline{p}) = 0$ and an *exogenously*¹⁶ set upper bound \bar{c} , such that $G(\bar{c}) = 1$. For a single purchase the price is sampled independently. Depending on the search intensity s_t , the purchase receives with probability of s_t two

¹⁵In this regard, the interpretation of the efficiency of shopping effort differs from the probability that an unemployed worker matches with a vacancy used in the labor search literature. It is due to the fact that consumption is intuitively divisible while jobs are not.

¹⁶The relevance of this assumption is discussed more thoroughly later.

independent offers drawn from $G(p)$ and the lower one is paid, or with complementary probability of $1 - s_t$ one price is sampled and the customer is captive for this specific transaction. Thus the distribution of the effective price of a single purchase is a result of the compound lottery:

$$F(p; s_t) = (1 - s_t)G(p) + s_t \left(1 - [1 - G(p)]^2\right). \quad (4)$$

The first term, $(1 - s_t)G(p)$ tells us the probability that the purchase is captive *and* the effective price will be lower than p , while the second term is the probability that two prices are drawn and the minimum of those offers are lower than p ¹⁷. A household can decrease the expected value of the price drawn from the lottery by increasing its search intensity s_t , but on the other hand, there is a trade-off since it increases the disutility from shopping visits¹⁸.

The cost of the consumption bundle. The price of every purchase constituting the overall shopping effort (m_t) is sampled independently. It means that the overall cost of the consumption bundle $c_t = m_t\theta^{1-\alpha}$ is the realization of continuum of lotteries, *i.e.*

$$\int_0^{m_t\theta^{1-\alpha}} p(i)di, \quad (5)$$

where prices $p(i)$ are drawn from the cdf $F(p; s_t)$. Lemma 1 states that, while the cost of a single purchase is random and ex-ante unknown, the cost of many purchases is certain with probability one.

Lemma 1 (Cost of consumption bundle) *Let the effective price of a purchase be distributed according to the cdf $F(p; s_t)$. Then the cost of consumption c_t given search intensity converges almost surely:*

$$\int_0^{m_t\theta^{1-\alpha}} p(i)di \xrightarrow{\text{a.s.}} \underbrace{m_t\theta^{1-\alpha}}_{c_t} \mathbb{E}(p|s_t), \quad (6)$$

where the effective price of consumption is equal to $\mathbb{E}(p|s_t) = \int p dF(p; s_t)$.

¹⁷Clearly, $\Pr(x \geq \min\{p', p''\}) = (1 - G(p))^2$, so the cdf of the minimum of two prices is given by $\Pr(x \leq \min\{p', p''\}) = 1 - [1 - G(p)]^2$.

¹⁸It is a consequence of assuming the positive cross partial derivative, $\frac{\partial f_t}{\partial m_t \partial s_t} > 0$.

Proof The lemma is an immediate result of applying the weak law of large numbers for random continuum in a version proposed by [Uhlig \(1996, Theorem 2\)](#). ■

It is convenient to make a decomposition of $\mathbb{E}(p|s_t)$ to disentangle the marginal effect of increasing search intensity on the effective price.

Lemma 2 (Linearity of the effective price function) *For given distribution of the quoted prices $G(p)$ the effective price paid by households is a linear function with respect to search intensity s :*

$$\mathbb{E}(p|s_t) = p^0 - s_t MPB, \quad (7)$$

where:

- i. $p^0 = \int_{\underline{p}}^{\bar{p}} x dG(x)$ is the price for the fully captive consumer;
- ii. $MPB = \mathbb{E} \max\{p', p''\} - p^0 (\geq 0)$ is the marginal (price) benefit of increasing the search intensity s_t , where $\mathbb{E} \max\{p', p''\}$ is the expected maximum of two independent draws of prices.

Proof To derive (7) I use the fact that the expected value of any non-negative random variable x distributed according to a cdf $H(x)$ can be computed integrating over its survival function ([Billingsley, 1995, p. 79](#)), namely:

$$\mathbb{E}(x) = \int_0^{\infty} (1 - H(x)) dx. \quad (8)$$

The price of the consumption bundle is then a result of applying this property to equation (4):

$$\mathbb{E}(p|s_t) = \int_0^{\infty} 1 - G(x) - s_t (G(x) - [G(x)]^2) dx,$$

where $\int_0^{\infty} 1 - G(x) dx$ is the expected value for the captive offer and, using an analogous reasoning from [Lemma 1](#), is also the price of consumption for the fully captive household that decides not to make any search for prices.

The residual part is equal to:

$$\int_0^{\infty} (G(x) - [G(x)]^2) dx =: MPB, \quad (9)$$

and which is clearly always positive as $\forall_x G(x) \geq [G(x)]^2$. For better interpretation it is convenient to reformulate equation (9):

$$\int_0^{\infty} (G(x) - [G(x)]^2) dx = \underbrace{\int_0^{\infty} 1 - [G(x)]^2 dx}_{\mathbb{E} \max\{p', p''\}} - \underbrace{\int_0^{\infty} 1 - G(x) dx}_{p^0}.$$

The expected maximum of two independent draws, $\max\{p', p''\}$ is distributed according to $[G(x)]^2$. It can be easily shown by the fact that $Pr(\max\{p', p''\} \leq x) = Pr(p' \leq x, p'' \leq x)$. Assuming independence of p' and p'' we get $Pr(p' \leq x) \cdot Pr(p'' \leq x) = [G(x)]^2$. Therefore, $\mathbb{E} \max\{p', p''\} = \int_0^{\infty} 1 - [G(x)]^2 dx$. ■

Lemma 1 shows that thanks to the fact that the cost of consumption is a sum of many repeated purchases the overall cost of the consumption basket can be pinned down deterministically. Each purchase is a result of different lottery price. Lemma 2 goes even further. It says that only two statistics of the price distribution, p^0 and MPB are needed to be known by households for making the optimal decision.

Productivity process. While being active in the labor market ($t \in \overline{1, T_{work}}$) every household faces the idiosyncratic wage risk. Log productivities follow an exogenous stochastic process:

$$\ln y_t = \kappa_t + \eta_t + \varepsilon_t,$$

$$\eta_t = \eta_{t-1} + \nu_t,$$

where $\varepsilon_t \sim_{\text{iid}} \mathcal{N}(0, \sigma_\varepsilon^2)$ and $\nu_t \sim_{\text{iid}} \mathcal{N}(0, \sigma_\nu^2)$. The deterministic part κ_t is a lifecycle component common to all households. The martingale part η_t and the serially uncorrelated part ε_t account for the permanent and transitory components of the productivity, respectively. While being employed all households receive the labor income wy_t .

Retirement. Households older than T_{work} receive a deterministic retirement that is a

function of their income in the last working-age period with replacement rate $repl$:

$$\log y_t = \log(repl) \cdot \{\kappa_{T_{work}} + \eta_{T_{work}} + \varepsilon_{T_{work}}\}.$$

Budget constraint. Households can hold a single risk-free asset which pays a net return, r . Let a_{t+1} be the amount of asset carried over from t to $t + 1$. Every household faces the sequence of intertemporal budget constraints:

$$\mathbb{E}(p|s_t)c_t + a_{t+1} \leq wy_t + (1 + r)a_t, \quad \forall t \in \overline{1, T}. \quad (10)$$

The effective price of consumption is a function of search intensity and is given by equation (7). It is worth noting that the intensity of search for prices s does not affect, at least directly, the level of consumption, but only the price of consumption. The shopping effort m_t affects the cost of consumption bundle only by the level of consumption expressed by the upper limit of the integral in formula (5). In addition to this every household faces the exogenous borrowing constraint $a_{t+1} \geq \underline{B}$.

Households' Decision Problem. The dynamic problem of a household of age t whose state is $x = (a, \varepsilon, v, \eta)$ is:

$$\mathcal{V}_t(a, \varepsilon, \eta) = \max_{c, f, m, s, p, a'} u(c) - v(f) + \beta \mathbb{E}_{\eta'|\eta} \mathcal{V}_{t+1}(a', \varepsilon', \eta') \quad (11)$$

s.t.

$$\begin{aligned} pc + a' &\leq (1 + r)a + wy, \\ c &= m\theta^{1-\alpha}, \\ f &= f(m, s), \\ p &= p^0 - sMPB, \\ a' &\geq \underline{B}, \\ s &\in [0, 1], \\ \log y &= \begin{cases} \kappa_t + \eta + \varepsilon, & \text{for } t \leq T_{work}, \\ \log(repl) \cdot \{\kappa_{T_{work}} + \eta_{T_{work}} + \varepsilon_{T_{work}}\}, & \text{for } t > T_{work}, \end{cases} \\ \eta' &= \eta + v', \end{aligned}$$

The problem is not convex due to bilinearity in controls s and c in the budget constraint.

This may cause that the first order conditions do not suffice and might lead to local solutions. However, this issue is solved by using envelope convexification of the bilinear constraint, which was proposed by [McCormick \(1976\)](#).

Retailers' problem. Sellers buy consumption goods at the cost standardized to one and quotes her price in every period conditioned on being matched with households' purchases. She maximizes the sales revenue:

$$S(p) = \theta^{-\alpha} \sum_{t=1}^T \int \frac{\theta^{1-\alpha} m_t(x)(1 + s_t(x))}{D} \underbrace{\left(1 - \frac{2s_t(x)}{1 + s_t(x)} G(p)\right)}_{\text{Business Stealing}} \underbrace{(p - 1)}_{\text{Surplus Appropriation}} d\mu_t(x), \quad (12)$$

where $\mu_t(x)$ is the distribution of households of age t over the individual states $x = (\alpha, \varepsilon, v, \eta)$. In the problem of sellers there are two opposite motives. First, the net revenue $(p - 1)$ from a single purchase is increasing with the set price. The second motive is generated by lack of information whether the matched buyer has the alternative offer for this purchase. The probability that the household has an alternative that with a better price than p amounts to $\frac{2s_t(x)}{1+s_t(x)} G(p)$. Thus, the probability of acceptance a given price price is the complementary event with probability $\left(1 - \frac{2s_t(x)}{1+s_t(x)} G(p)\right)$. Higher prices decrease the probability that the offer will be accepted by the buyer. Thus, these two motive can generate a price dispersion, in which there are retailers that have higher markups but their prices are rejected more often and retailers that cut their prices to increase the probability of the successful transaction. In an equilibrium the sellers are indifferent¹⁹.

Relevance of exogenous reservation price ζ . A question that arises from the exogenous price ζ is about the commitment of households to pay sampled prices for all purchases. The repeated purchases can be interpreted as consumption in different subperiods of the year. If the subperiods are long enough it is reasonable to say that households agree to pay the lowest offered (but still high) price in order to avoid starving to death due to the lack of consumption. On the other hand, if subperiods are short enough households might prefer setting their own endogenous reservation price \bar{p} and deferring from paying above this price. In this case, the model should be augmented by an additional control, \bar{p} . However, this extensions leads to some issues. First, lemma 2 does not hold and the

¹⁹In this sense, the mechanism is similar to the theory of homogenous hotel rooms with different prices given by [Prescott \(1975\)](#).

constraint for the effective price is not linear with all controls. This is due to the fact that \bar{p} replaces exogenous ζ . Second, there is no clear distinction between two motives, shopping effort m_t and search intensity s_t anymore. An additional increase in shopping effort accompanied by a decrease in \bar{p} plays the same role as an increase in s_t ²⁰.

Equilibrium. Having outlined the building blocks of the economy, I am in the position to define an equilibrium of the economy.

Definition 3 (Rational Stationary Equilibrium) *A stationary equilibrium is a sequence of consumption and shopping plans $\{c_t(x), m_t(x), s_t(x)\}_{t=1}^T$, and the distribution of quoted prices $G(p)$ and paid prices $F(p; s_t(x))$, distribution of households $\mu_t(x)$ and interest rate r such that:*

1. $c_t(x), m_t(x), s_t(x)$ are optimal given $r, w, G(p), \underline{B}$, and θ ;
2. individual and aggregate behavior are consistent:

$$D = \sum_{t=1}^T \int (1 + s_t(x)) m_t(x) d\mu_t(x); \quad (13)$$

3. retailers post prices p to maximize the sales revenues taking as given households' behavior;
4. the private savings sum up to an exogenous aggregate level \bar{K} :

$$\sum_{t=1}^T \int a_t(x) d\mu_t(x) = \bar{K}; \quad (14)$$

5. $G(p)$ and $F(p; s_t(x))$ are consistent given the household distribution $\mu_t(x)$;
6. $\mu_t(x)$ is consistent with the consumption and shopping policies.

B. Characterization of the Equilibrium

The dispersed distribution of posted prices is consistent with the solution to the maximization of the retailers' net sales revenue, (12). Lemma 4 presents properties of an equilibrium of this kind. The proof of the lemma is similar to ones used in [Burdett and Judd \(1983\)](#) and [Kaplan and Menzio \(2016\)](#).

²⁰However, search intensity s_t is still necessary for generating price dispersion.

Lemma 4 (Characterization of the Equilibrium Price Dispersion) *The c.d.f. $G(p)$ exhibits following properties:*

- i. $G(p)$ is continuous.*
- ii. $\text{supp } G(p)$ is a connected set.*
- iii. the highest price charged by retailers is equal to ζ ,*
- iv. all retailers yield the same profit, $\forall p \in \text{supp } G(p) S(p) = S^*$,*

where $\text{supp } G(p)$ is the smallest closed set whose complement has probability zero.

Proof The two first properties are an immediate result of Lemma 1 from (Burdett and Judd, 1983). Suppose that $G(p)$ has a discontinuity at some $p' \in \text{supp } G(p)$. The retailer posting an infinitesimally smaller price $p' - \epsilon$ would increase its profit as the probability of making a sale would change by a discrete amount. Furthermore, $\text{supp } G(p)$ is a connected set. Suppose there is a gap of zero probability between p' and p'' . The seller's gain would be strictly higher at p'' as $p'' > p'$, and $G(p') = G(p'')$. This cannot occur in an equilibrium.

Next, suppose that (iii) is not true. Then $\max \text{supp } G(p) =: \bar{p} \leq \zeta$.²¹ Moreover, we know that $G(\bar{p}) = G(\zeta) = 1$. If we substitute values of the c.d.f. for both prices into (12) all firms will have incentives to set higher price for higher demand, which leads us to contradiction. As a result, $\max \text{supp } G(p) = \zeta$. Fact (iv) is an equilibrium condition. If there would be such a price p that would yield higher profit, each individual retailer would have incentives to set this price. ■

It is convenient to decompose the aggregate shopping effort D defined in (13) into two components:

²¹Recall that there is the exogenous upper-bound for prices ζ , so $\bar{p} \geq \zeta$ is not considered.

$$\Psi_{(-)} := \sum_{t=1}^T \int m_t(x)(1 - s_t(x))d\mu_t(x), \quad (15)$$

$$\Psi_{(+)} := \sum_{t=1}^T \int m_t(x)2s_t(x)d\mu_t(x), \quad (16)$$

$$D = \sum_{t=1}^T \int m_t(x)(1 + s_t(x))d\mu_t(x) = \Psi_{(-)} + \Psi_{(+)}. \quad (17)$$

Notice that $\Psi_{(-)}$ in (15) is an aggregate measure of visits where customers are captive and $\Psi_{(+)}$ in (16) where households draw two prices and choose the lower one. D from (17) is the measure of the aggregate shopping defined in (13) and is a sum of $\Psi_{(-)}$ and $\Psi_{(+)}$. Consequently, $\frac{\Psi_{(-)}}{D}$ and $\frac{\Psi_{(+)}}{D}$ are probabilities that a single draw is captive or matched with an alternative offer, respectively. By construction all offers of $\Psi_{(-)}$ are effective for the reason that buyers are captive during these purchases. On the other hand, only half of $\frac{\Psi_{(+)}}{D}$ is accepted by buyers and the remaining part is rejected. It is so because for this measure of offers consumers get two price offers and choose the lower one.

Properties from Lemma 4 can be used to derive the formula for an equilibrium price dispersion.

Theorem 5 (Equilibrium Price Dispersion) *Given aggregate statistics of households' shopping decisions $\{\Psi_{(-)}, \Psi_{(+)}, D\}$, (where $\Psi_{(-)}, \Psi_{(+)} > 0$), the equilibrium price dispersion can be expressed in a closed form:*

$$G(p) = \begin{cases} 0, & \text{for } p < \underline{p}, \\ \frac{D}{\Psi_{(+)}} - \frac{\Psi_{(-)}}{\Psi_{(+)}} \cdot \frac{\zeta - 1}{p - 1}, & \text{for } p \in [\underline{p}, \zeta], \\ 1, & \text{for } p > \zeta, \end{cases} \quad (18)$$

where the lower bound of $\text{supp}G(p)$ is:

$$\underline{p} = \frac{\Psi_{(+)}}{D} + \frac{\Psi_{(-)}}{D}\zeta. \quad (19)$$

Proof The proof is relegated to Appendix B. ■

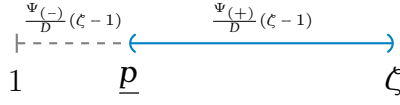
Discussion of Theorem 5. Given p , the equilibrium price dispersion $G(p)$ is a linear

function decreasing in: 1. the inverse odds ratio²² of being matched with a non-captive customer, $\left(\frac{\Psi_{(+)} }{\Psi_{(-)}}\right)^{-1}$ and 2. the probability that a visiting buyer draws an alternative offer, $\frac{\Psi_{(+)}}{D}$. Suppose that there are two economies with the same aggregate shopping effort D and different level of search intensity, $\Psi'_{(+)} > \Psi''_{(+)}$. Due to the fact that $\frac{\partial G(p)}{\partial \frac{\Psi_{(+)}}{D}} > 0$ for every p from the the interior of $\text{supp } G(p)$, the price lottery of the economy with higher search intensity $\Psi'_{(+)}$ first-order stochastically dominates the price lottery of the economy with lower search intensity $\Psi''_{(+)}$. This observation leads to an immediate remark that economies with higher search intensity exhibit the lower expected value of the price lottery. The result is consistent with economic intuition. The higher fraction of buyers with alternative offers is, the stronger competition between retailers is observed. For a better understanding how the price equilibrium changes in $\Psi_{(+)}$ consider three cases:

1. $\Psi_{(+)} = 0$ – the business stealing motive from (12) embodied by $\left(1 - \frac{2s_t(x)}{1+s_t(x)} G(p)\right)$ disappears and only the surplus appropriation motive occurs. Every customer is captive and this leads to a degenerate Diamond (1971)-type equilibrium, where all retailers charge the monopolistic price, ζ ;
2. $\Psi_{(+)} = D$ (*hypothetical*) – every consumer draws two prices and chooses the lower one. Consequently, all retailers start playing a Bertrand game and the only price equilibrium is a degenerate competitive one, $p = 1$. Nonetheless, it is a purely hypothetical case since an equilibrium from Definition (3) with $\Psi_{(+)} = D$ never exists. If all prices are set competitively, then none of households have incentives to make any search. To them it pays off to be captive all the time but then $\Psi_{(-)} = D$ and $\Psi_{(+)} \neq D$, which contradicts the constituting assumption of the case that $\Psi_{(+)} = D$;
3. $\Psi_{(+)} \in (0, D)$ – there occurs a tug of war between two motives, 1. the appropriation of consumers' surplus and 2. business stealing. In every point of the support of the equilibrium price dispersion $\text{supp } G(p) = [p, \zeta]$ retailers yield the same profit S^* . However, for each price there is a different composition of sources of this profit. The business stealing motive is the only motive for retailers charging p , while the surplus appropriation only rationalizes the behavior of sellers that set ζ . Prices from the interior of $\text{supp } G(p)$ are supported by a combination of both. As the aggregate

²²Notice that $\frac{\Psi_{(+)}}{\Psi_{(-)}} = \frac{\frac{\Psi_{(+)}}{D}}{1 - \frac{\Psi_{(+)}}{D}}$.

Figure 4: The equilibrium support of $G(\mathbf{p})$.



search $\frac{\Psi(+)}{D}$ increases, retailers set lower prices and the lowest quoted price, \underline{p} gets closer to the competitive pricing.

The lower bound \underline{p} of $\text{supp } G(\mathbf{p})$ also depends on the aggregate search intensity in the economy. Interestingly, it is a convex combination of the competitive price (normalized to 1) and the monopolistic price ζ , where $\frac{\Psi(+)}{D}$ and $\frac{\Psi(-)}{D}$ are weights. The higher $\frac{\Psi(+)}{D}$ is, the further \underline{p} is from the monopolistic price and closer to the competitive price (see Figure 4).

Equilibrium price moments. Finally, \mathbf{p}^0 and $\mathbb{E} \max\{\mathbf{p}', \mathbf{p}''\}$ from Lemma 2 can be pinned down using the closed form solution from Theorem 5.

Proposition 6 *Given households' aggregate shopping efforts $\Psi_{(-)}$ and $\Psi_{(+)}$, the price for captive customers (\mathbf{p}^0) and the expected maximum of two independent draws ($\mathbb{E} \max\{\mathbf{p}', \mathbf{p}''\}$) can be expressed in a closed form:*

i. *price of the captive customer:*

$$\mathbf{p}^0 = \underline{p} + \frac{\Psi_{(-)}}{\Psi_{(+)}}(\zeta - 1) \log \left(\frac{\zeta - 1}{\underline{p} - 1} \right) + \left(1 - \frac{D}{\Psi_{(+)}} \right) (\zeta - \underline{p}); \quad (20)$$

ii. *the expected maximum of two independent draws:*

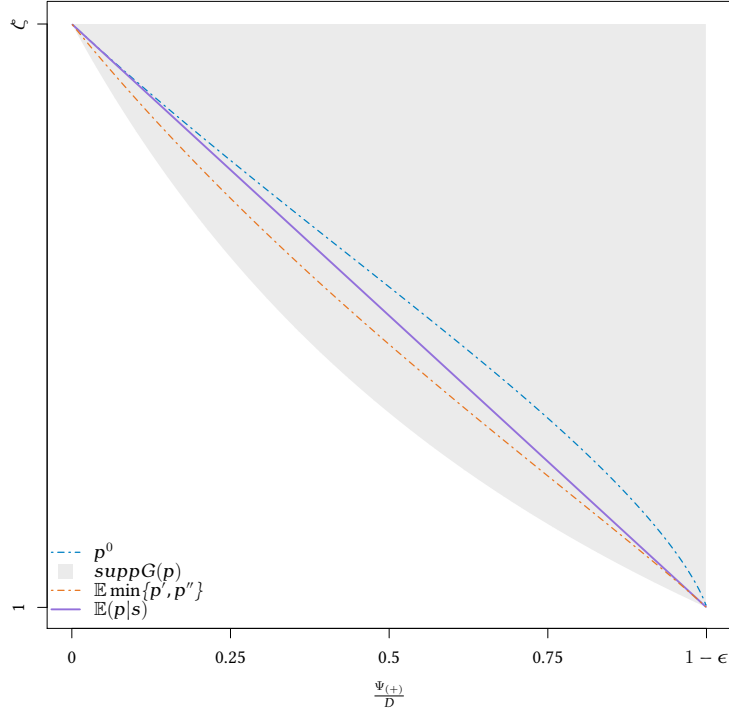
$$\begin{aligned} \mathbb{E} \max\{\mathbf{p}', \mathbf{p}''\} = & \zeta - \left(\frac{D}{\Psi_{(+)}} \right)^2 (\zeta - \underline{p}) + 2 \frac{D \Psi_{(-)}}{\Psi_{(+)}} (\zeta - 1) \log \left(\frac{\zeta - 1}{\underline{p} - 1} \right) - \\ & - \left(\frac{\Psi_{(-)}}{\Psi_{(+)}} \right)^2 (\zeta - \underline{p}) \frac{\zeta - 1}{\underline{p} - 1}. \end{aligned}$$

Proof *The proof is relegated to Appendix B.* ■

The moments from Proposition 6 are “sufficient” price statistics²³ that are required in the

²³Recall that $MPB = \mathbb{E} \max\{\mathbf{p}', \mathbf{p}''\} - \mathbf{p}^0$.

Figure 5: Moments of equilibrium price distribution and the aggregate search intensity.



Note: The figure depicts summary shopping moments for $\frac{\Psi(+)}{D} \in [0, 1)$.

household's problem (11). For gaining a better insight into the mechanics of the equilibrium it is helpful to conduct the comparative statics with respect to the search intensity. Without loss of generality, in this exercise I focus on the representative consumer framework. For this case there is a one-to-one mapping between the individual search intensity of the consumer and the aggregate search intensity, *i.e.* $\frac{\Psi(+)}{D} = \frac{2s}{1+s}$ ²⁴. Figure 5 shows how the key price characteristics change in the probability of being matched with a non-captive customer, $\frac{\Psi(+)}{D}$. First, the average effective price $\mathbb{E}(p|s)$ varies between the price of the fully captive customer p^0 and the expected minimum of two draws $\mathbb{E} \min\{p', p''\}$. Even though prices are sampled from a whole interval $\text{supp } G(p) = [p, \zeta]$, the (unit) cost of

²⁴Besides, notice that in this case the equilibrium cdf is distributed according to $G(p) = \frac{1+s}{2s} - \frac{1-s}{2s} \frac{\zeta - 1}{p - 1}$. Only the search intensity s matters, while the number of purchases m cancels out. In the heterogenous-agent framework it is analogous. The latter margin plays only a weighting role for purchases made by various households.

the consumption bundle is the average price $\mathbb{E}(p|s)$ drawn from $F(p)$ and given by (7). As mentioned before, for $\Psi_{(+)} = 0$ there exists only the degenerate [Diamond \(1971\)](#)-type equilibrium, where $\mathbb{E}(p|s) = p^0 = \zeta$. An increase in $\frac{\Psi_{(+)}}{D}$ makes $\mathbb{E}(p|s)$ further from the captive price p^0 and closer to the expected minimum $\mathbb{E} \min\{p', p''\}$. In the limit case you can observe²⁵:

$$\lim_{s \rightarrow 1^-} \mathbb{E}(p|s) = 2p^0 - \mathbb{E} \max\{p', p''\} = \mathbb{E} \min\{p', p''\}. \quad (21)$$

Higher search intensity in the economy fosters higher competition between retailers. As a result, all price statistics (p^0 , $\mathbb{E} \min\{p', p''\}$, $\mathbb{E} \max\{p', p''\}$, $\mathbb{E}(p|s)$) tend towards the competitive solution, which in the model is normalized to unity.

A natural concern that arises here is the assumption on the exogeneity of the upper bound ζ of $\text{supp}G(p)$. The minimum price quoted by retailers responds to the level of search intensity, while the maximum price is constant all the time. However, it is not a problem. As [Figure 6.a](#) shows top percentiles decrease in search intensity. Effective price ($\mathbb{E}(p|s)$ in [Figure 6.b](#)) decreases even faster. For instance, 97th percentile in a low search economy is close to the upper bound, ζ . In fact, the whole support is concentrated in this neighborhood. On the contrary, the same percentile is much closer to the competitive price in a high search economy. This observation is true especially for the paid prices ([Figure 6.b](#)). In fact, in spite of the exogeneity of ζ , prices paid by consumers can be successfully reduced by increasing search intensity, s .

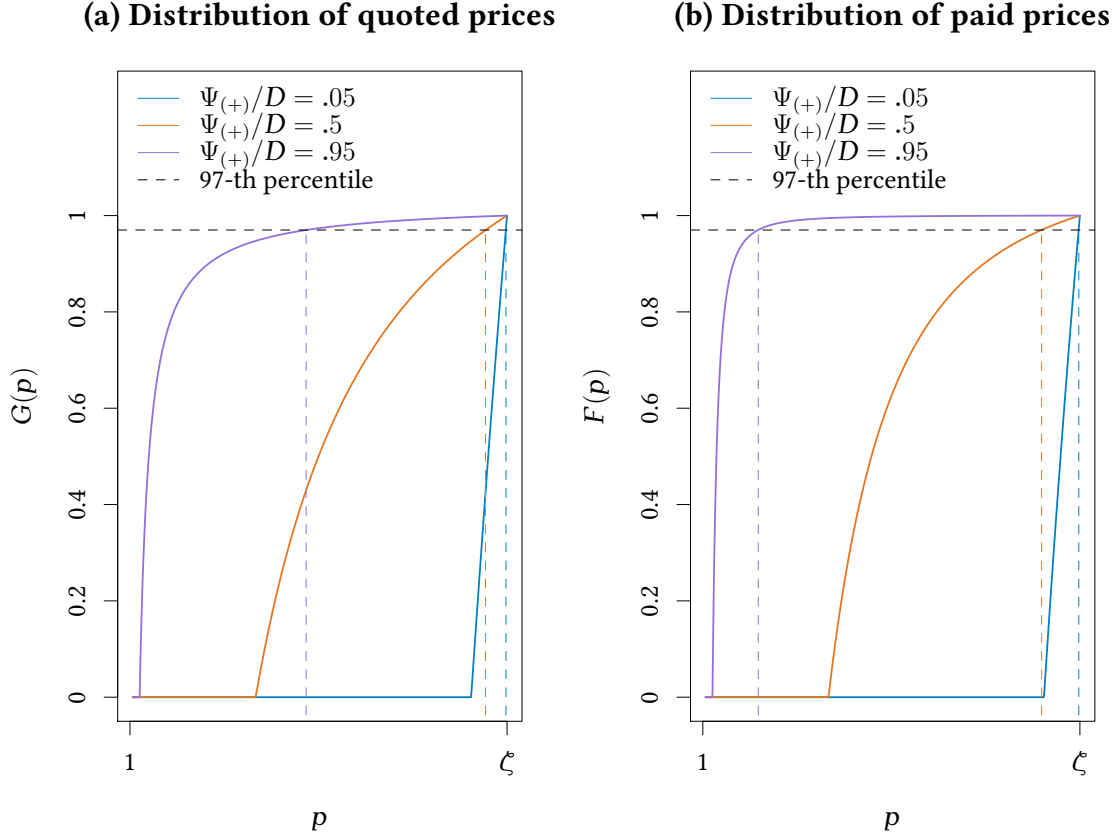
Solution to the household's problem. Finally, I am in the position to write the first order conditions that constitute the solution to the households' problem (11). The intertemporal decision is determined by:

$$\frac{u'(c)\theta^{(1-\alpha)} - v'(f) \frac{\partial f}{\partial m}}{p^0 - sMPB} \geq \beta(1+r)\mathbb{E}_{x'|x} \frac{u'(c')\theta^{(1-\alpha)} - v'(f') \frac{\partial f'}{\partial m'}}{p^0 - s'MPB}, \quad (22)$$

and $\alpha' \geq \underline{B}$, with complementary slackness. The main departure from the textbook Euler equation is the additional convex cost, $v(f_t)$ and varying price, $p = p^0 - sMPB$, which is a function of the control, s in the considered case. For the CRRA specification the

²⁵Note $\max\{p', p''\} = \frac{p'+p''}{2} + |p' - p''|$ and $\min\{p', p''\} = \frac{p'+p''}{2} - |p' - p''|$. Then $\mathbb{E} \min\{p', p''\} + \mathbb{E} \max\{p', p''\} = \mathbb{E}p' + \mathbb{E}p'' = 2p^0$, which gives the latter equality in (21).

Figure 6: The equilibrium price dispersion for the different aggregate search intensities, $\frac{\Psi_{(+)} }{D}$.



household makes also an intratemporal decision on its shopping behavior:

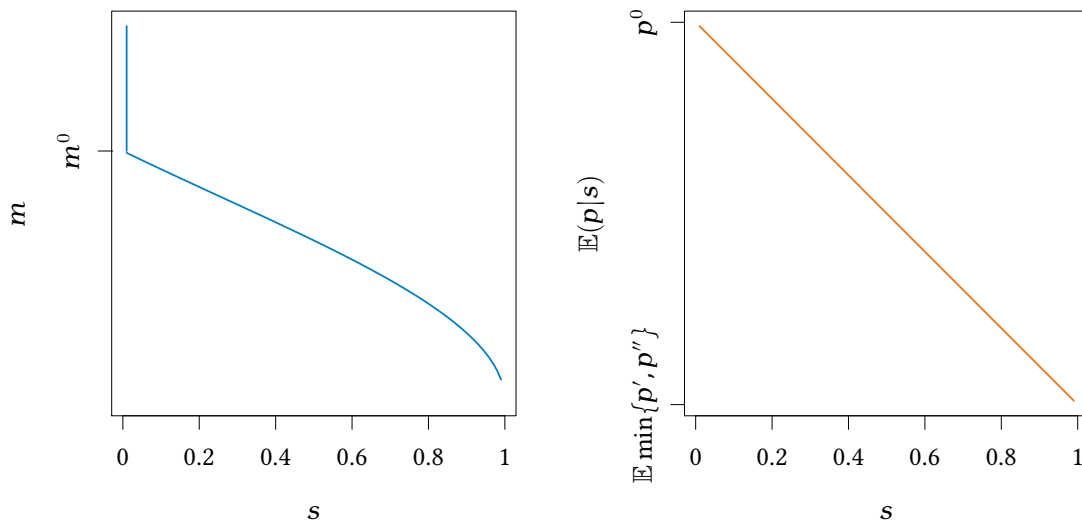
$$m \geq \left\{ \frac{\theta^{(1-\sigma)(1-\alpha)}}{\left(\frac{1+s}{1-s}\right)^\phi \left(1 + \frac{2p^0}{(1-s)MPB}\right)} \right\}^{\frac{1}{\sigma+\phi}} \quad (23)$$

and $s \geq 0$, with complementary slackness²⁶. First, as in the standard model, consumption goes along with the level of wealth. By construction, it affects m_t in the same way due to the linear relationship, $c_t = m_t \theta^{1-\alpha}$. Second, both shopping margins are Frisch complements to each other in the disutility function. Consequently, households with higher

²⁶Condition (23) is not defined in $s = 1$. However, the assumed functional specification meets an Inada-like condition, $\lim_{s \rightarrow 1^-} v(m_t, s_t) = \infty$, which guarantees that such a search intensity is never chosen.

consumption exert lower search for prices, s_t . There is also a certain number of purchases m^0 (which translates directly into $c^0 = m^0\theta^{1-\alpha}$), above which households decide to be captive in every purchase, $\mathbb{E}(p|s = 0) = p^0$ (see Figure 7). However, as mentioned before it does not make them to pay ζ all the time because there is a positive externality generated by households with high search. This is embodied by $p^0 < \zeta$.

Figure 7: Optimal relationship between the number of purchases (m_t), price search (s_t) and the effective consumption price $\mathbb{E}(p|s)$.



V. TAKING THE MODEL TO DATA

In this section I present my strategy for parametrization of the model and computation of the equilibrium. Model parameters are divided into two groups. Values of the first group (Table 2) are preset exogenously to standard values drawn from the literature. Values of crucial parameters which account for the shopping technology (Table 4) are determined internally using the method of simulated moments.

Demographics. The model is annual. Households enter the labor market when they are 25, they retire at age of 60 and die at age 90. This implies $T_{work} = 35$ and $T = 65$.

Preferences. The preferences over consumption are represented by a CRRA specifica-

tion, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. The elasticity of relative risk aversion parameter $\frac{1}{\sigma}$ was set to .5. The disutility from overall shopping effort is modeled by an isoelastic function:

$$v(f) = \begin{cases} \frac{f^{1+\phi}}{1+\phi}, & \text{for } t \in 1, \dots, T_{work}, \\ \chi^{ret} \frac{f^{1+\phi}}{1+\phi}, & \text{for } t \in T_{work} + 1, \dots, T. \end{cases} \quad (24)$$

Factor $\chi^{ret} (< 1)$ is supposed to capture a lower opportunity cost of shopping time for the retired consumers. The function of overall shopping effort f is chosen to meet assumptions on increasing in both margins and mutual complementarity. It is represented by a functional specification $f = \frac{1+s}{1-s}m$. Besides this form is convenient in the computational procedure, which is described and explained more carefully in the end of the section. Finally the discount factor β was chosen to replicate an aggregate wealth-income ratio of 2.5.

Interest rate and assets. I calibrate the discount factor β to generate an aggregate wealth-income ratio of 2.5. Following the RBC literature (Cooley and Prescott, 1995), the interest rate r was set to .04. Household debt contributes very little to wealth distribution. In aggregate it poses less than 1% of the total wealth and a median quarterly credit limit reported by households from the SCF amounts to merely 74% of quarterly labor income, which is not much in comparison to the mean net worth equal to over 900% of the labor income. For this reason, I assume households can save but cannot borrow, $\underline{B} = 0$ as modeled in Carroll (1997) or more recently in Krueger, Mitman, and Perri (2016). The sales tax, τ_c was set to the average US sales tax.

Income process. The income process is a combination of two components, transitory $\{\varepsilon_t\}$ and permanent $\{\eta_t\}$. Following the literature, the log variances of those shocks were set to $\sigma_\varepsilon^2 = .05$, $\sigma_\eta^2 = .01$. The age-dependent deterministic component, κ_t is approximated by a quadratic regression using the PSID data as in Kaplan and Violante (2010). In retirement, households receive a social security income payment that is a function of their income in the last working-age period with replacement rate $repl$ (Guisar and Smith, 2014; Berger, Guerrieri, Lorenzoni, and Vavra, 2015).

Shopping parameters. The key shopping parameters are determined internally using a simulated method of moments. There are six parameters to be pinned down: the discount factor (β), curvature of the disutility from shopping (ϕ), matching efficiency of a single

purchase²⁷ ($\theta^{1-\alpha}$), wage (w), the maximum price quoted by retailers (ζ), and lower disutility from shopping of retired households (χ^{ret}). The quantitative behavior of the model is disciplined by seven internal targets, which can be divided into three categories: shopping effort, price dispersion and aggregate state. The calibration consists in simulating artificial panels of data for shopping economies and the final parametrization is chosen to set values of simulated moments as close as possible to their empirical counterparts.

Shopping effort. The shopping effort targets are matched using indirect inference (Gourieroux, Monfort, and Renault, 1993). First, I estimate an auxiliary regression model using the ATUS data, which captures the empirical findings 1.i-iii presented in Section III:

$$\log shopping_i = \alpha + \beta earn_i^{\frac{2}{3}+} + \delta_u unemp_i + \delta_r retir_i + \delta_a Age_i + \gamma X_i + \varepsilon_i, \quad (25)$$

where $earn_i^{\frac{2}{3}+}$ is a dummy accounting for the top labor income tertile. In this regard, one single dummy variable replaces nine dummies ($\sum_j \beta_j earn_i^j$) of the baseline specification (1). This change makes calibration more straightforward and at the same time not much information is lost²⁸. Table 3 shows the results of the estimation. The regression of the

²⁷ Admittedly, θ is an equilibrium object. However, it is easy to show that every $\theta^{1-\alpha}$ can be rationalized either by fixing the measure of consumption available in aggregate or by setting a fixed entry cost for retailers. In partial equilibrium both approaches are tantamount. Rationalization with a fixed entry cost is used in a work-in-progress paper that studies shopping in general equilibrium (Kaplan and Pytka, 2016).

²⁸ Recall that only top deciles of the earnings spend more time shopping.

Table 2: External choices

Parameter	Interpretation	Value	Source
T_{work}	Age of retirement	35	–
T	Length of life	65	–
σ	Risk aversion	2.0	–
$repl$	Retirement replacement rate	.45	Guvenen and Smith (2014)
σ_ε^2	Variance of the transitory shock	.05	Kaplan and Violante (2010)
σ_η^2	Variance of the permanent shock	.01	Kaplan and Violante (2010)
r	Interest rate	.04	Cooley and Prescott (1995)
τ_{cons}	Consumption tax	.08454	Reuters
κ_t	Deterministic life-cycle income profile	–	Kaplan and Violante (2010)
\underline{B}	Borrowing constraint	0	–

reduced specification exhibits estimated values that are similar to the baseline version. Finally, estimates of variables associated with retirement, age, and being in the top earning tertile were used for disciplining the structural model.

Table 3: Coefficients of interest for the auxiliary model

	$\log(\textit{shopping})$
Retired	0.245*** (0.035)
Unemployed	0.368*** (0.031)
Age	0.0005 (0.003)
$\textit{earn}_{\frac{2}{3}+}$	0.110*** (0.017)
Shopping needs	Yes
Year and day dummies	Yes
Demographic controls	Yes

Notes: ***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.

Price dispersion. In the parametrized model I want to capture certain cross-sectional price characteristics of the US economy. For this reason, I targeted two price differentials: 1. between high earners and low earners, 2. between employed and retired. Both moments were observed by [Aguiar and Hurst \(2007\)](#). The authors using scanner data showed that retirement-age households pay on average 3.9 percent less than young households and that households earning more than \$70,000 a year pay 2.1 percent more than households earning less than \$30,000 a year. These moments are differences in average prices paid by households with different characteristics. In the model this is embodied by heterogeneity in price search intensity, s amongst households. Consequently, this gives rise to variety in average prices, $\mathbb{E}(p|s)$. Apart from heterogeneity in first moments I use also range statistics of prices paid by households. [Kaplan and Menzio \(2016\)](#) used the same dataset as [Aguiar and Hurst \(2007\)](#) to measure price ranges of *transactions* for various goods and markets. They observed that the average 90-to-10 percentile ratio of paid prices varies between 1.7 and 2.6. The heterogeneity in marginal cost, which is not

present in the model, is very likely to contribute to a higher price dispersion²⁹. Thus, similarly to the aforementioned paper I decide to target the price ratio of 1.7. It is noteworthy that the authors use the consumer panel dataset which collects data on prices of *effective* transactions. This observation has an important implication as it means the percentile ratio should be computed not from the distribution of prices set by retailers, $G(p)$, but rather the distribution of prices *accepted* by households³⁰.

Table 4: Calibration targets and model values

Target	Data Value	Source	Model Value
Shopping effort:			
Shopping time of retired relative to the referential group	1.245	This paper	1.251
Shopping time of the top earn. tercile relative to the referential group	1.11	This paper	1.112
Age trend for shopping time	0	This paper	.010
Price dispersion:			
95 th decile / 5 th decile of paid prices	1.7	Kaplan and Menzio (2016)	1.369
Price differential between high earners and low earners	.021	Aguiar and Hurst (2007)	.011
Price differential between retirees and working-age households	-.039	Aguiar and Hurst (2007)	-.051
Aggregate state:			
Aggregate wealth-income ratio	2.5	Kaplan and Violante (2010)	2.498

Computation. An equilibrium allocation and equilibrium prices are determined by solutions to the household’s dynamic problem (11) given prices set by retailers and solutions to the retailer’s problem (12) given households’ consumption and shopping decisions. The allocation is computed iteratively. The household’s problem is solved given an initial guess on pricing strategy of retailers. Then, the retailer’s problem is solved given the solution from the previous step. Next, the pricing strategy of retailers is updated and used for solving the household’s problem. The process is repeated until convergence to a fixed point, in which the household’s decisions generate the retailers’ pricing and vice versa.

²⁹This conjecture relies on theoretical results about the impact of firm productivity differentials on the wage dispersion (e.g., [Burdett and Mortensen, 1998](#); [Mortensen, 2003](#)).

³⁰In aggregate they are distributed according to $(1 - \frac{\Psi(+)}{D})G(p) + \frac{\Psi(+)}{D}[1 - (1 - G(p))^2]$, where $\frac{\Psi(+)}{D}$ is the probability that a transaction is non-captive.

Table 5: Calibrated parameters

Parameter	Value	Description
ϕ	.104	curvature of disutility from shopping
$\theta^{1-\alpha}$.113	matching efficiency
w	14.041	wage
ζ	84.605	maximum price
χ^{ret}	.588	lower disutility from shopping for retirees
β	.951	discount factor

The algorithm is intuitive and relatively fast (it takes 10-15 iterations to converge). A drawback of this approach is a risk of computing a degenerate [Diamond \(1971\)](#)-type equilibrium³¹. Given a “wrong” initial guess the algorithm might converge to an allocation where all households want to exert the maximum search intensity, $s = 1$. In response to this all retailers set monopolistic prices, $p = \zeta$, and in the next iteration all households become captive all the time, $s = 0$. The decision to make the minimum price search effort supports degenerate pricing, $p = \zeta$, in subsequent iterations. Nonetheless, this problem can be tackled by imposing an Inada-like assumption, $\lim_{s \rightarrow 1^-} v(m_t, s_t) = \infty$, which guarantees that a search intensity ($s = 1$) is never chosen by any household. The functional specification ($f = \frac{1+s}{1-s}m$ together with (24)) used in the calibration meets this condition.

Calibration results. The parameter values are presented in [Table 5](#) and the targeted data moments with their model counterparts are summarized in [Table 4](#). The model matches the shopping effort statistics and the aggregate wealth-income ratio very well. On the other hand, the price range and the price differential between high earners and low earners are too low while the price differential between retirees and working-age households is slightly overestimated. However, in the calibration there are seven targets and six parameters, so the system of targeted moments is overdetermined. It makes exact identification impossible. Overall, the simulated moments are pretty close to the data targets.

³¹In a companion paper I show that this type of equilibrium is unstable for this model setup ([Pytko, 2016](#)). This property stays in contrast with the classical model of search for consumption due to [Burdett and Judd \(1983\)](#).

VI. PRICE DISPERSION(S) AT PLAY

In the equilibrium allocation the price dispersion can be characterized in four dimensions, that is:

- i. the distribution of prices *quoted* by retailers,
- ii. the distribution of prices *accepted* by households,
- iii. the distribution of prices of *individual purchases* for a household exerting a certain search intensity, $s_t(x)$,
- iv. the distribution of *average* prices paid by households with different search intensities $s_t(x)$ distributed according to the distribution of types given by $\mu_t(x)$.

Table 6 presents moments of the equilibrium price dispersion for the calibrated version of the model. Aggregate search intensity is equal to $\frac{\Psi(+)}{D} = .271$. From retailers' perspective this number can be interpreted as the probability that a visiting customer received an alternative offer from another retailer. This probability constitutes the equilibrium price dispersion given by (18). Consequently, the offered prices are a connected set $[\underline{p}, \zeta]$, where the equilibrium lower bound \underline{p} amounts to about 70% of the monopolistic price. If a seller is matched with a consumer who is also matched with a lower alternative price, then the offer with the higher price is rejected. Only offers of measure $\frac{\Psi(+)}{2D} = .1355$ drawn with an alternative competitive offer come into force. As a result, 86.45% offers³² quoted by retailers are accepted, while the complementary 13.55% is rejected. The accepted offers are distributed according to:

$$\left(1 - \frac{\Psi(+)}{D}\right) G(p) + \frac{\Psi(+)}{D} \left\{1 - [1 - G(p)]^2\right\}. \quad (26)$$

Both distributions, quoted prices and accepted prices, are depicted in Figure 8. Intuitively, lower prices have higher probability for being accepted by customers. This property is embodied by the fact that $G(p)$ is first-order stochastically dominated by formula (26).

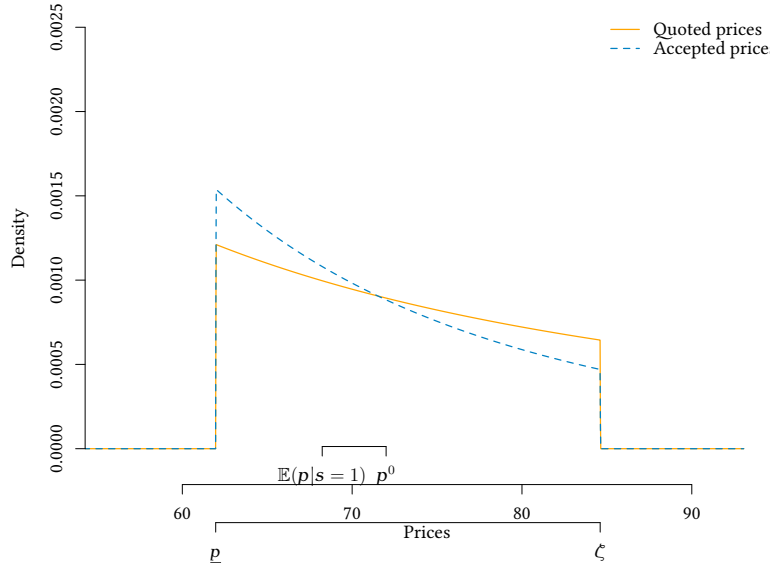
Given the distribution of prices quoted by retailers, households decide on their individual search intensity, s . Consequently, every consumer draws prices from her individual price lottery generated by equation (4), which is illustrated in Figure 9. Households with

³²This number is a sum of 13.55% offers matched with competitors with higher prices and 72.9% transactions with captive customers.

Table 6: Moments of the Equilibrium Price Dispersion

Moment	Value	Description
Price moments:		
\underline{p}/ζ	.703	Min-max ratio of quoted prices
$\Psi_{(+)} / D$.271	Aggregate search
p^0 / ζ	.851	Captive price-max ratio
MPB / p^0	.052	Marginal price benefits
Shopping moments:		
$\mathbb{E}s$.201	Average search intensity
$\mathbb{E}(s s > 0)$.683	Average price search conditioned on being non-captive

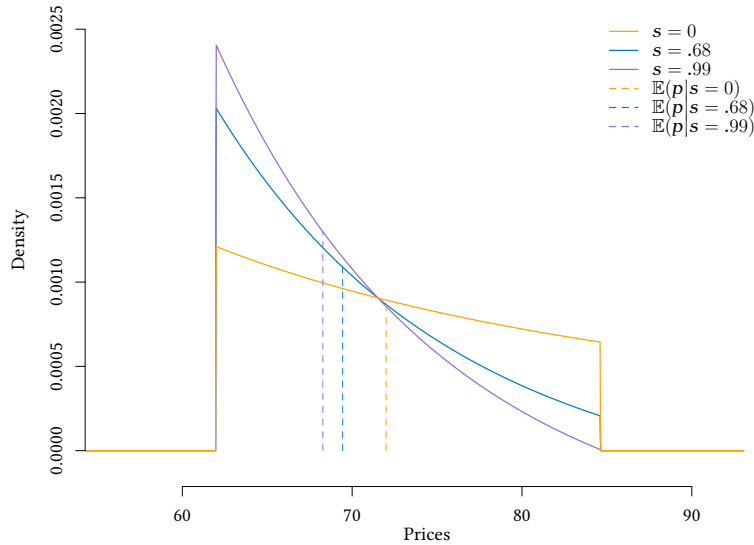
Figure 8: Distribution of quoted and accepted prices.



low search intensity sample prices from a price lottery with a higher expected value. Recall that a consumption bundle, c consists of continuum of shopping lotteries. Thanks to Lemma 1 the overall cost of consumption is pinned down deterministically. Therefore, the cost of a unit of consumption belongs to an interval, $(p^0 - MPB, p^0]$. Households that decide to be captive in every purchase ($s = 0$) pay p^0 , while consumers with positive search intensity ($s > 0$) spend $p^0 - sMPB$ on every unit of consumption. In the limit case, for $s = 1$ they would pay the minimum possible price $p^0 - MPB$, which is equal

to the expected minimum from two draws³³, $\mathbb{E} \min\{p', p''\}$. The fact that retailers cannot distinguish between captive and non-captive transactions, the expected value of a single draw (p^0) is 15.5% lower than the monopolistic pricing. The marginal price benefit from increasing search intensity allows to reduce prices up to 5.2% compared with prices paid by captive customers, p^0 .

Figure 9: Individual price lotteries.



The equilibrium price dispersion characterizing the economy is supported by non-captive households that exert positive price search intensity ($s > 0$). As shown in Table 7, less than 30% households decide to make some search effort to draw (with some probability) two prices. Households exerting a positive price search effort draw two prices and choose the lower one with the probability of 68.3%. Interestingly, if households are broke down into two groups, those in the working-age and retirees, then it turns out that more than half of retirees and merely 12.9% of workers are non-captive. In intensive margins there is also a substantial difference. The average search intensity of non-captive retirees equals to 76.5%, whilst the average search intensity of non-captive amounts to 41.7%. This result stems directly from lower opportunity cost modeled by χ^{ret} . Finally, the last type

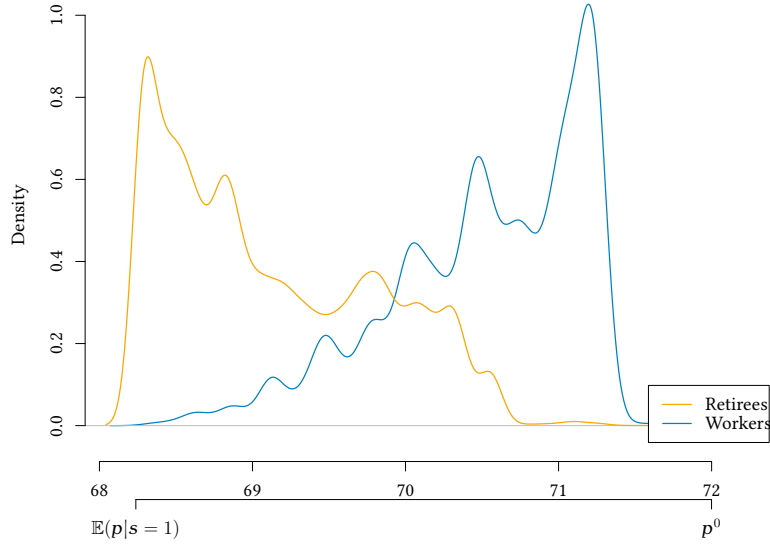
³³However this shopping strategy is never chosen for the assumed utility function. In the calibrated economy maximum effort is set to $s = .979$ for the employed households and $s = .998$ for the retirees.

Table 7: Captive and non-captive households

Type of households	Non-captive	Captive	$\mathbb{E}(s s > 0)$
Working-age	.129	.871	.417
Retired	.485	.515	.765
Overall	.293	.707	.683

of price dispersion, heterogeneous average prices are generated by different search intensities, presented in Figure 10.

Figure 10: Distribution of average prices for non-captive households.



VII. DECONSTRUCTING THE AGGREGATE CONSUMPTION

The aggregate demand generated by artificial economies is characterized using two alternative approaches, consumption responses to idiosyncratic income shocks and cross-sectional distributions of households' decisions (consumption expenditures) and endogenous states (net wealth positions). All results stem from simulating the invariant distribution of two versions of the economy, the standard incomplete-markets model *without* frictions in the purchasing technology (SIM) and the incomplete-markets economy augmented with the search friction described in Section IV. Both economies are calibrated to

replicate the same level of aggregate savings (wealth-income ratio of 2.5). The remaining non-shopping parameters are set at the same level for both models (see Table 2). Next, simulated statistics are compared with their empirical counterparts. It is worth pointing out that none of statistics presented in this section were used in the calibration procedure. In this sense, results of this section provide natural yardsticks to measure which model is better for the quantitative analysis.

A. Consumption Responses to Shocks

To study consumption responses to income shocks I employ an identification strategy proposed by [Blundell, Pistaferri, and Preston \(2008\)](#). The authors, using data on non-durable consumption from the Panel Study of Income Dynamics and making imputations basing on food demand estimates from the Consumer Expenditure Survey, assessed the transmission of income shocks into consumption. To this end they used the log-linear approximation of the Euler equation and run the regression:

$$\Delta \ln c_{it} = \alpha + MPC^\varepsilon \varepsilon_{it} + MPC^\eta \eta_{it} + \xi_{it}, \quad (27)$$

where MPC^ε and MPC^η are the pass-through coefficients of income shocks into consumption. Intuitively, they can be interpreted as marginal propensity to consume out of different types of shocks, permanent (η) and transitory (ε).

In the data the distinction between different types of shocks is difficult. The authors offer an estimator of MPC that is consistent under two assumptions: short history dependences and no advanced information. This estimator is as follows:

$$\widehat{MPC}^x = \frac{\text{cov}(\Delta(p_{it}c_{it}), g(x_{it}))}{\text{var}(g(x_{it}))}, \quad (28)$$

where:

$$g(\varepsilon_{it}) = \Delta y_{i,t+1}, \quad g(\eta_{it}) = \Delta y_{i,t-1} + \Delta y_{i,t} + \Delta y_{i,t+1}.$$

It is worth presenting values of the coefficients for some notable examples:

1. complete markets (with separable labor supply): $MPC^\varepsilon = MPC^\eta = 0$ – households are able to smooth the marginal utility of consumption fully and all shocks are insured away,
2. autarky with no storage technology: $MPC^\varepsilon = MPC^\eta = 1$,

3. the classical version of the permanent income-life cycle model: $MPC^\eta = 1$ and the response to transitory shocks MPC^ε depends on the time horizon. For a long horizon it should be very small and close to zero, while for a short horizon it tends to one.

In their empirical study [Blundell, Pistaferri, and Preston \(2008\)](#) documented that on average 64% of permanent shocks are translated directly into consumption. [Kaplan and Violante \(2010\)](#) applied an analogous procedure to an artificial panel generated from a calibrated version of the life-cycle SIM model. In their simulation they reported that in the artificial economy between 78 and 93% of permanent shocks are passed on consumption, depending on the borrowing limit.

In the shopping economy I depart from the law of one price so the price component is not constant and does not cancel out. Hence, I have to modify the baseline specification to the following form:

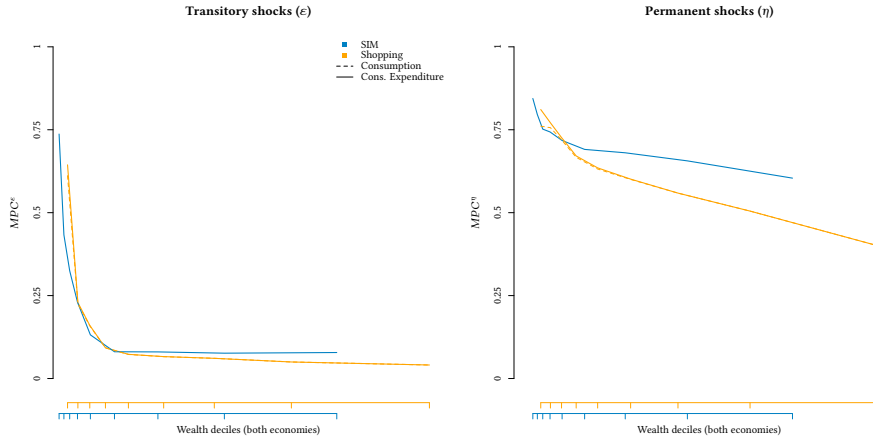
$$\Delta \ln(p_{it}c_{it}) = \alpha + MPC^\varepsilon \varepsilon_{it} + MPC^\eta \eta_{it} + \xi_{it}. \quad (29)$$

I estimate equation (29) using artificial panels of two versions of the economies and I compare the results with the empirical counterparts. Table 8 presents the obtained estimates. In the shopping economy average consumption responses are smoother and are closer to the empirical counterparts than in the SIM economy without product market-frictions. This phenomenon can be explained by the fact that marginal disutility for the shopping effort partially offsets utility of consumption, which makes consumption responses smoother. This effect increases in the level of consumption, which is conformed by the values of the pass-through coefficients for different wealth groups depicted in Figure 11. The interesting implication of the model is also higher heterogeneity in consumption responses. Households with low wealth exhibit similar willingness to consume in both economies. In the presence of the shopping friction they are even slightly higher since households decide to search less intensively for lower prices in response to permanent shocks. The discrepancy between two economies is larger for consumers from the wealthiest groups. For those households the cost of obtaining additional units of consumption is so high that they refrain from consuming more.

Table 8: BPP MPC

Economy	\widehat{MPC}^η	\widehat{MPC}^ϵ
USA (BPP 2008)	.64	.05
Shopping	.602	.152
SIM	.8	.280

Figure 11: Distribution of MPCs.



B. Cross-sectional Distribution

The cross-sectional distributions of consumption expenditures and net wealth are another dimension describing the aggregate demand. For this exercise I generate simulated moments from both artificial economies and compare with data. Following the macroeconomic literature of inequalities (e.g., [Castañeda, Díaz-Giménez, and Ríos-Rull, 2003](#); [Krueger, Mitman, and Perri, 2016](#)), distributions are compared with the use of Gini indices and share of the total value held by chosen groups of households. The data counterparts were calculated using the 2006 wave of the Panel Study of Income Dynamics. I focus on households aged between 25 and 90 to make computed statistics compatible with the calibration of the models. Following [Blundell, Pistaferri, and Saporta-Eksten \(2016\)](#) I dropped observations with extremely high net wealth (>\$ 20 millions). In the theoretical framework I do not model the household decision to purchase durables, so I focus on non-durables and services.

Table 9 presents the distributions of consumption expenditures in both economies and

Table 9: Consumption (expenditure) distribution

Economy	GINI	Quintile					Top Percentiles		
		First	Second	Third	Fourth	Fifth	90-95	95-99	99-100
USA (PSID 2006)	.353	.051	.113	.165	.224	.440	.087	.088	.121
Shopping	.402	.053	.112	.163	.208	.457	.073	.061	.200
SIM	.234	.100	.150	.190	.235	.330	.083	.078	.025

observed in the data. The shopping economy mirrors inequalities remarkably better than the SIM economy without product-market frictions. The Gini indices for consumption in the baseline SIM and in the shopping economy amount to .234 and .401, respectively. The empirical counterpart computed from the PSID is equal to .353. This effect is generated mainly by groups exerting high search for price bargains, households with low consumption and retirees. First, households with low consumption search more intensively, which leads to lower effective prices paid by them. Consequently, the fraction of aggregate consumption expenditures is smaller than in the benchmark SIM model. Second, a drop in consumption expenditures after retirement is higher in the shopping economy as can be seen in Table 10. This result is generated by the lower opportunity cost of time for retirees and is consistent with findings made by [Aguiar and Hurst \(2005, 2007\)](#).

Table 10: Gini consumption: working-age households vs retirees

Economy	GINI working-age	GINI retirees	overall GINI	$\frac{\mathbb{E}(pc retired)}{\mathbb{E}(pc working)}$
USA (PSID, 2006)	.330	.383	.353	.701
Shopping	.380	.381	.402	.742
SIM	.214	.243	.235	.809

The distributions of net wealth are presented in Table 11. As can be seen the shopping friction amplifies the wealth inequalities as well. The Gini indices in the baseline SIM and in the shopping economy amount to .569 and .667, respectively. The empirical counterpart computed from the PSID is equal to .771. If we look at the fine print, the higher Gini index comes from the higher share of the total wealth held by the top quintile. In the shopping economy households from the top quintile own nearly 70% of total wealth, while in the data 82.6% is observed. In the SIM model only 55% is owned by households from

the top quintile. The improvement in this moments is generated in the analogous way to consumption responses from the previous subsection. For this group of households increasing the current consumption is too costly. Instead it is beneficial to them to save more and increase consumption during retirement when the opportunity cost of time is lower. There is still discrepancy between inequalities generated by the shopping economy and observed in data. Admittedly, there are models outperform the shopping economy in this regard. Nonetheless, recall that those statistics were not used in the calibration process, while for instance in [Castañeda, Díaz-Giménez, and Ríos-Rull \(2003\)](#) moments describing wealth inequalities were targets. Moreover, the shopping economy presented in this paper abstracts from important motives for wealth accumulation, such as bequests.

Table 11: Wealth distribution

Economy	GINI	Quintile					Top Percentiles		
		First	Second	Third	Fourth	Fifth	90-95	95-99	99-100
USA (PSID 2006)	.771	-.015	.005	.042	.142	.826	.157	.256	.244
Shopping	.667	.011	.031	.065	.198	.696	.1636	.211	.114
SIM	.569	.014	.052	.128	.258	.549	.138	.149	.056

VIII. CONCLUDING COMMENTS

The article advances a novel theory of the search for consumption as part of the optimal savings model. Motivated by recent empirical findings on price dispersion and systematic heterogeneity in shopping time I use the model to address the question how income fluctuations are passed on consumption expenditures when the law of one price does not hold. I show that frictions in the purchasing technology generate important implications for the aggregate consumption. The shopping effort increases the level of risk sharing and brings predictions of the model much closer to the empirical counterparts than the standard incomplete-markets model without frictions in purchasing technology. Moreover, the level of consumption and wealth inequalities are amplified as well and in this sense the theory contributes to the literature of inequality as well.

More broadly, the model is a first step to understand macroeconomic implications of search for consumption. It can provide interesting structural insight into some recent

empirical findings on household consumption. For instance, [Stroebel and Vavra \(2016\)](#) documented reactions of retail prices to changes in local house prices, with elasticities of 15–20%. Using my model it can be rationalized by an increase in share of consumption of homeowners, who are most likely captive customers. Consequently, the aggregate price search decreases and retailers adjust their pricing strategy to the new distribution of consumption by charging on average higher prices.

The model can be extended to a more general setup. In another paper ([Kaplan and Pytka, 2016](#)), which is being developed, we use the proposed search protocol in an economy with both idiosyncratic and aggregate risk. The model is tailored to quantitatively address policy-related questions, in which household consumption plays an important role. The framework allows studying the relationship between the retail market and the production sector from a macroeconomic standpoint. In addition to this, the shopping effort generates a real effect on output. Two possible applications are suggested. First, our model is likely to give some insight into the source of transition of unskilled workers from the production sector to the retail market observed in the data. Second, the model can be used for studying monetary policy in a framework where price stickiness is a result, not an assumption. We claim that product market frictions in the transactions technology can replace the price-setting rigidities. Moreover, unlike existing Keynesian models, our economy does not require firms or households to be off their optimality conditions.

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Table 12: Deciles of weekly earnings

10%	20%	30%	40%	50%	60%	70%	80%	90%
250.00	360.00	461.53	570.00	675.00	807.69	961.53	1192.30	1538.46

A. DATA

B. PROOFS FOR SUBSECTION IV.B

A. Proof of Theorem (5).

Proof Lemma (4) implies that every retailer yields the same level of profits and that the highest quoted price is equal to ζ . The profit of retailers charging ζ comes only from captive consumers. The probability that a visiting buyer is captive is equal to $\int \frac{m_t(x)(1+s_t(x))}{D} \left(1 - \frac{2s_t(x)}{1+s_t(x)}\right)$. Then the profit is equal to:

$$S(\zeta) = \theta^{-\alpha} \sum_{t=1}^T \int \frac{\theta^{1-\alpha} m_t(x)(1+s_t(x))}{D} \left(1 - \frac{2s_t(x)}{1+s_t(x)}\right) (\zeta - 1) d\mu_t(x). \quad (30)$$

Profits of retailers charging different prices p must be equal to $S(\zeta)$ (otherwise those prices were not be chosen):

$$\theta^{-\alpha} \sum_{t=1}^T \int \frac{\theta^{1-\alpha} m_t(x)(1+s_t(x))}{D} \left(1 - \frac{2s_t(x)}{1+s_t(x)} G(p)\right) (p - 1) d\mu_t(x) = S(\zeta).$$

The distribution of quoted prices given by (18) is the unique dispersed equilibrium given household search strategies. ■

C. STATISTICAL TABLES

Table 13: Statistical table for $G(p)$

$G(p)$.01	.1	.3	$\frac{\Psi(+)}{D}$.5	.7	.9	.99
.99	1.000	.998	.991	.980	.958	.847	.337
.975	.999	.994	.981	.953	.892	.694	.166
.95	.999	.989	.958	.912	.809	.531	.095
.9	.998	.978	.921	.832	.684	.359	.045
.85	.997	.967	.888	.771	.592	.273	.035
.8	.996	.958	.855	.717	.518	.215	.025
.75	.995	.947	.823	.670	.459	.187	.025
.7	.994	.938	.795	.623	.418	.158	.015
.65	.993	.928	.772	.589	.376	.139	.015
.6	.992	.919	.744	.556	.351	.120	.015
.55	.991	.910	.720	.529	.326	.110	.015
.5	.990	.901	.702	.502	.301	.100	.015
.45	.989	.892	.678	.475	.276	.091	.015
.4	.988	.882	.660	.455	.260	.081	.015
.35	.987	.873	.641	.434	.251	.081	.015
.3	.986	.866	.627	.414	.235	.072	.005
.25	.985	.857	.608	.401	.226	.072	.005
.2	.984	.849	.594	.387	.210	.062	.005
.15	.983	.840	.580	.374	.201	.062	.005
.1	.982	.833	.566	.360	.193	.062	.005
.05	.981	.826	.552	.347	.185	.053	.005
.025	.981	.822	.543	.340	.176	.053	.005
.01	.980	.820	.543	.333	.176	.053	.005
\underline{p}	.980	.818	.538	.333	.176	.053	.005

Table 14: Statistical table for $G(p)$

$F(p)$	$\frac{\Psi(+)}{D}$						
	.01	.1	.3	.5	.7	.9	.99
.99	1.000	.998	.986	.960	.875	.464	.055
.975	.999	.994	.972	.912	.750	.321	.035
.95	.999	.987	.944	.845	.626	.225	.025
.9	.998	.976	.897	.744	.493	.167	.015
.85	.997	.965	.855	.677	.418	.139	.015
.8	.996	.954	.818	.623	.376	.120	.015
.75	.995	.943	.786	.576	.335	.100	.015
.7	.994	.934	.758	.542	.310	.100	.015
.65	.993	.923	.734	.515	.293	.091	.015
.6	.992	.914	.711	.488	.276	.081	.015
.55	.991	.904	.688	.468	.260	.081	.015
.5	.990	.895	.669	.448	.243	.072	.015
.45	.989	.886	.655	.428	.235	.072	.015
.4	.988	.879	.636	.414	.226	.072	.005
.35	.987	.870	.622	.401	.218	.062	.005
.3	.986	.862	.608	.387	.210	.062	.005
.25	.985	.855	.594	.380	.201	.062	.005
.2	.984	.848	.580	.367	.193	.062	.005
.15	.983	.838	.571	.360	.193	.062	.005
.1	.982	.833	.557	.347	.185	.053	.005
.05	.981	.826	.548	.340	.185	.053	.005
.025	.981	.822	.543	.340	.176	.053	.005
.01	.980	.820	.538	.333	.176	.053	.005
p	.980	.818	.538	.333	.176	.053	.005