Inequality and the Value of Government Spending Multiplier*

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March 29, 2017

Abstract

This paper analyzes the impact of income and wealth inequality on effectiveness of fiscal expansions. First, I consider a stylized framework based on the model by Michaillat and Saez [2015], in which matching frictions on the product and labor markets allow for capturing three potentially important factors that may affect the value of government multiplier: economic slack, sticky prices and wages. I show analytically, that irrespectively of their intensity, the size of the multiplier does not exceed unity if households are identical. Next, I extend the model by introducing wealth and income heterogeneity across consumers and find that government multiplier can be larger than one if the amount of economic slack is sufficiently high. This theoretical result indicates that inequality substantially reinforces the effects of fiscal interventions. Second, I embed the analyzed framework into an otherwise standard Bewley-Huggett-Aiyagari model with two types of idiosyncratic risks faced by workers: time-varying labor productivity and changes in employment status, and study the transitional dynamics following an unexpected, transient increase in government purchases. I quantify the channels through which fiscal expansion affects private consumption and measure the degree of government multiplier's countercyclicality.

Keywords: Fiscal Stimulus, Heterogeneous Agents, Frictional Markets

JEL Classification: D30, E62, H23, H30, H31

*The views presented in this paper are those of the author, and should not be attributed to the National Bank of Poland. I would like to thank P. Beaudry, A. Ferriere, P. Rendahl, S. Schmitt-Grohe and L. Uren for helpful comments.

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1 Introduction

This paper explores the consequences of changes in government expenditures in environment characterized by frictional product and labor markets and inequality across households. It shows that under some conditions, that are related to income and wealth heterogeneity, the interplay between public purchases and private demand is sufficiently strong to give rise government spending multipliers that exceed unity.

First, I compare two tractable, static frameworks: environment with representative agent which is a version of the model with frictional product and labor markets by Michaillat and Saez [2015] and its extension that allows for income and wealth heterogeneity across agents. It turns out that in the former case the value of government spending multiplier is always below one. It happens because the only channel through which product market tightness influences private consumption is the so-called crowding-out channel. This implies that private absorption reacts negatively to a rise in product market tightness that follows from an increase in fiscal expenditures which means that the value of fiscal multiplier is always lower than one. In contrast to the representative agent framework, the value of government multiplier in the model with heterogeneous agents may exceed unity if market conditions are sufficiently poor (i.e., the amount of idle resources in economy is substantial). This occurs because apart from the crowding-out mechanism, product market tightness affects private demand through income effects: a decrease in slack on the product market boosts firms' profits which makes them raise employment. This leads to higher dividends and increased labor income which rises the amount of households' disposable resources and stimulates private demand. If economic conditions are poor, income effects dominate crowding-out which implies that private consumption increase when markets get tighter as a result of fiscal expansion. Income effects are absent in the representative agent version of the model as they are eliminated by general equilibrium effects.

Second, I develop a dynamic framework that embeds the discussed mechanism into a version of the Bewley-Huggett-Aiyagari model. This framework is used to conduct a quantitative assessment of the impact of changes in fiscal expenditures on private consumption which are crucial for the value of the government spending multiplier. I isolate and quantify several channels through which increased fiscal consumption affects private absorption. It turns out that increase in labor (capital) income and reduction in unemployment fears influence the response of private consumption the most. Moreover, I show that the value of government multiplier depends positively on unemployment rate and study cross-sectional responses of private consumption to fiscal expansion.

The remaining sections of the paper are structured as follows. Section 2 discusses the literature that is related to my work and presents my contributions. In Section 3 I consider the extended, heterogenous agent version of the model by Michaillat and Saez [2015] and show that departure from the representative agent paradigm has some important consequences for the effects of fiscal expansions. The model analyzed in Section 3 is then embedded into the standard framework based on works by Bewley, Huggett and Aiyagari. This fully-fledged macroeconomic model is presented in Section 4. Section 5 concludes.

2 Related literature and contributions

Let us first discuss the determinants of the size of the multipliers in closed economies that can be found in the literature.[1]

Economic slack. There are numerous works studying the influence of the state of the business cycle on effectiveness of fiscal stimuli. Auerbach and Gorodnichenko [2012] use regime-switching models and find large differences in the size of government multipliers in booms and recessions with the value of multiplier being significantly higher during economic downturns. In contrast to Auerbach and Gorodnichenko, Owyang et al. [2013] find no evidence that fiscal multipliers exhibit a countercyclical pattern in the United States. They document, however, increased multipli-

[1] I focus on the case of a closed economy in my analysis so I abstract from determinants that are associated with open economies, such as trade openness and the exchange rate regime (Ilzetzki [2013]). Since I study interventions that are financed with increased taxes, I do not discuss sovereign debt level as a factor affecting the value of the multiplier (Kirchner et al. [2010], Ilzetzki [2013]).

ers during periods of slack in Canada, with some multipliers exceeding unity. Ramey and Zubairy [2014] confirm the results regarding the multiplier in the USA presented by Owyang et al. [2013] using new quarterly U.S. data that cover multiple large wars and deep recessions. Michaillat [2014] constructs a model with frictional labor market and analyzes the impact of labor market slack on the value of "public employment multiplier". He finds that it exhibits a countercyclical pattern and it is always bounded by unity. I show that those two results remain valid for the government spending multiplier in the representative agent framework with labor market and product market frictions. Interestingly, once I depart from the assumption about agents' homogeneity the upper bound on the value of fiscal multiplier ceases to hold.

Price and wage rigidities. The stickiness of prices and wages tends to amplify the effects of increased government expenditures. Woodford [2011] argues that the response of monetary policy is crucial for efficacy of fiscal expansion. This is because higher government expenditures tend to reduce the output gap and raise inflation which induces tighter monetary policy. This in turn decreases private consumption and hampers the increase in aggregate demand initiated by fiscal stimulus. If prices are less flexible then the rise of inflation associated with government expenditures is limited and hence the reaction of monetary authority is less aggressive. Rendahl [2015] shows that nominal wage rigidities reinforce job creation resulting from higher aggregate demand and prices that are closely related to a rise in government purchases. This occurs because the drop in real wages provides incentives for firms to employ new workers.

Zero lower bound. As it has been already mentioned, monetary policy can dampen the impact of fiscal expansion on aggregate demand. This implies government multipliers can potentially be larger in times when the use of monetary policy is impaired. This occurs when monetary policy reaches the lower bound on nominal interest rates. Christiano et al. [2011] and Eggertsson [2010] emphasize the role of the timing of fiscal intervention during the ZLB episode. The former paper argues that implementation lags reduce the value of multiplier at the ZLB and the latter indicates that increased government expenditures should be reduced when the ZLB ceases to hold. Rendahl [2015] highlights the fact that fiscal policy outcomes are larger at the ZLB because output becomes demand driven in such circumstances.

Inequality/heterogeneity. Let us turn to works that document empirical facts about the impact of unexpected changes in fiscal spending on consumers that differ with respect to individual characteristics. Carroll et al. [2014] study the dependence between Marginal Propensity to Consume (MPC), wealth heterogeneity and income uncertainty in 15 European countries. They find that economies characterized by larger wealth inequality exhibit higher average level of MPC and larger disproportions of MPCs across consumers. Moreover, they document that increases in the variance of transitory shocks leads to higher levels of aggregate MPC. These findings have important consequences for the potential impact of fiscal expenditures on private absorption studied in this work. More precisely, higher levels of MPC across agents imply that government expenditures that affect household's budgets through disposable income have more pronounced effects in economy characterized by larger inequalities. Anderson et al. [2015] study the impact of fiscal expenditures on households that differ with respect to income and age levels. They find that responsiveness of the wealthiest consumers to increases in fiscal expenditures is negative and the opposite is true for the poor. The quantitative version of my model replicates those patterns successfully. My analysis bears substantial similarities to theoretical work by Ferriere and Navarro [2016] that studies government expenditures in the standard Aiyagari model with labor indivisibility. They found that only an increase in government spending that is accompanied by a rise in tax progressivity may generate a hike in aggregate consumption. The reason for this fact is intuitive: when tax progressivity increases, authorities are able to decrease the average tax level because they tax higher incomes at a higher rate. Main beneficiaries of the associated tax cuts are agents with low labor income who exhibit relatively high marginal propensity to consume. Similarly to Ferriere and Navarro, in the quantitative version of the model I assume that additional fiscal purchases are financed with taxes levied on the richest. I show, however, that a rise in tax progressivity is not the most important driver of the positive response in private consumption to government expenditures (as claimed by those authors): income effects and the reduction in unemployment risk are much more significant than changes in tax progressivity. Beaudry and Portier [2011] present a model that formalizes the idea that changes in observed macroeconomic activity are driven by shifts in the volume of trade between agents. They embed intra-temporal gains from trade (in a sense that not all agents are perfect substitutes in the production of goods from different sectors) into a standard macro setting and show that this modification provides new insights concerning the mechanisms that drive economic fluctuations and the effects of policy. In particular, they show that if labor markets are fully integrated and preferences of agents are identical (i.e. which is equivalent to representative agent framework) then government purchases cannot create multiplier greater than one. Although I do not include the intra-temporal trade in my model, I obtain an analogous result for the model with homogeneous agents. Moreover, fiscal multipliers exceeding unity emerge in both frameworks once heterogeneity across agents is introduced to the model. In case of my paper, it is inequality associated with income and wealth, in case of Beaudry and Portier [2011] it is heterogeneity in preferences and labor specialization. On the top of that, they show how taxing decisions affect the value of government multiplier: fiscal expenditures have a greater effect when sectoral spending is not offset by proportional sectoral taxes. Formula 2 in my paper shows that taxing decisions matter for efficacy of fiscal purchases in my case, too. More precisely, to give rise to higher multiplier government should finance additional purchases with taxes levied on agents with higher labor income. This is because poorer agents exhibit higher MPC which makes their individual demand highly vulnerable to changes in disposable income. To generate high government spending multipliers it is thus crucial to avoid taxing those agents.

To articulate the role of inequality as a determinant of the size of the multiplier, I consider a static model that captures the factors listed above: economic slack, price and wage rigidity.[2] I choose the framework proposed by Michaillat and Saez [2015] as a workhorse of my analysis because product and labor market frictions present in this model give a precise meaning to the notion of economic slack and allow for incorporating

[2]Since the model is static, I abstract from the ZLB in Section 3.

price and wage rigidities in a tractable way. I show that in the representative agent version of the model, the size of the multiplier is always below unity, even if prices and wages are perfectly rigid and markets are extremely slack. In contrast to the model in which agents are identical, the multiplier in the heterogeneous agent variant may exceed unity which indicates that inequality is an important determinant of fiscal policy effectiveness.

3 Forces at play: illustration using a static model

In this section I consider two versions of the model by Michaillat and Saez [2015]: the one with representative agent (henceforth RA) and the second with agents that differ with respect to income and asset holdings (i.e., the model with heterogeneous agents, HA). The Michaillat-Saez model describes the interaction between slacks in two frictional markets (for goods and for labor) and hence it is a natural modeling choice when analyzing the consequences of changes in the quantity of unemployed resources on the value of government multiplier. Moreover, this framework offers a tractable way of incorporating price and wage rigidities.

3.1 Equilibrium condition and the government multiplier

To structure our thinking about the determinants of the value of government spending multiplier, it is instructive to begin with the analysis of equilibrium condition which takes the following, compact form (for both HA and RA case):[3]

$$C\left(x_g, \tau(G)\right) + G = Y\left(x_g\right) \tag{1}$$

where C is aggregate private consumption, G is amount of goods purchased by government and Y is aggregate output. Observe that both C and Y depend on argument x_g which is defined as ratio between aggregate number of visits V_g (made by households and government when pur-

[3] Microfoundations that give rise to this condition are presented later. Condition 1 summarizes equilibrium in the model in which agents trade in three markets: for labor, products and nominal assets.

chasing goods from producers) and aggregate output capacity of firms T (defined later):

$$x_g = \frac{V_g}{T}.$$

In other words, the value of x_g is inverse to product market slack. Observe that x_g may affect Y through various channels. First of them is related to the fact that, in general, output is generated only if customers show up to purchase goods or place orders for them.[4] The other include: the positive impact of tightness of x_g on prices of goods (that makes firms increase their output and employment) and wages (since higher employment reduces labor market slack and thus puts an inflationary pressure on wages).[5] Similarly, changes in x_g affect aggregate consumption C. This occurs owing to two groups of effects. First group is associated with crowding-out of private consumption: higher x_g is equivalent to tighter markets which leads to higher prices of goods and lower probability of a successful match in the frictional product market. This rises the (effective) price of goods which leads to a drop in private demand. Second group includes income effects: larger values of product market tightness increase firms' profits (which translates into higher dividends) and labor income (through a rise in wages and employment). Finally, observe that C is influenced negatively by τ - taxes levied on households to finance government purchases.

The Implicit Function Theorem applied to 1 yields the following formula for government multiplier:

$$\frac{dY}{dG} = \frac{dY}{dx_g} \cdot \frac{dx_g}{dG} = \frac{1 + \frac{\partial C}{\partial \tau} \cdot \frac{d\tau}{dG}}{1 - \frac{\frac{\partial C}{\partial x_g}}{\frac{dY}{dx_g}}}. \tag{2}$$

Expression 2 summarizes the determinants of $\frac{dY}{dG}$ in a concise way. Let us first concentrate on $\frac{\partial C}{\partial \tau}$ and $\frac{dY}{dx_g}$. Both the RA and HA variants of the model

[4]This means that I depart from the neoclassical view according to which output is a function of production inputs (like capital and labor) only.

[5]Notice, that labor market conditions (labor market tightness) are not captured by 1 in an explicit way. This is because, as we shall see later, labor market tightness is a strictly increasing function of x_g and hence in can be omitted in 1.

predict that $\frac{\partial C}{\partial \tau} \cdot \frac{d\tau}{dG} \in (-1, 0]$ and $\frac{dY}{dx_g} > 0$. In other words, taxes always have an adverse influence on private absorption and increase in product market tightness tends to have an expansionary effect on GDP. Notice, that in such a situation, the value of $\frac{\partial C}{\partial x_g}$ has a decisive impact on whether $\frac{dY}{dG}$ exceeds unity or not. If it is negative (i.e., crowding-out effects dominate income effects) then $\frac{dY}{dG} < 1$. If, on the other hand, $\frac{\partial C}{\partial x_g} \in \left(0, \frac{dY}{dx_g}\right)$ and tax distortion $\frac{\partial C}{\partial \tau}$ is sufficiently small then $\frac{dY}{dG}$ may exceed unity. As we shall see, in the representative agent version of the model crowding out is the only channel through which x_g affects consumption and thus $\frac{\partial C}{\partial x_g} < 0$. Interestingly, agents' heterogeneity opens up a possibility that $\frac{\partial C}{\partial x_g} \in \left(0, \frac{dY}{dx_g}\right)$ which results from the fact that income effects become stronger than the crowding-out process. This in turn implies that $\frac{dY}{dG} > 1$ as the initial rise in x_g initiated by a growth in G is propagated even further by private demand (since $\frac{\partial C}{\partial x_g} > 0$) which leads to additional stimulation of aggregate output.

3.2 Households, government and aggregate demand

Households. The model is populated by households (customers) of measure one. Households derive utility from consumption of goods c (purchased at price p) and holdings of liquid wealth (money balances) m'. Goods and liquid assets are traded in frictional and Walrasian market, respectively. Preferences are described by functions $u(c)$ and $W(m')$: both u and W are strictly increasing, strictly concave and they satisfy the Inada conditions:

$$\lim_{c \to 0} u'(c) = \lim_{m' \to 0} W'(m') = +\infty.$$

$$\lim_{c \to +\infty} u'(c) = \lim_{m' \to +\infty} W'(m') = 0.$$

W can be thought of as discounted utility derived from future consumption streams. To purchase goods household makes visits v_h. A visit is successful (i.e., a unit of good is bought) with probability $q_g\left(x_g\right)$ (with $q_g' < 0$) and effort exerted to make v_h visits generates disutility $-\kappa_g v_h$

where $\kappa_g > 0$ is a parameter.[6] The value of x_g is taken as given by consumers. Each household consists of measure one of workers that supply labor inelastically on the frictional labor market. The number of active workers N is equal across households.[7] Each of them receives wage wz where w is average nominal wage in and z is a household-specific productivity level. Profits Π generated by firms are redistributed to agents in the following way: customer with productivity level z holds $s(z) \geq 0$ shares that entitle to $s(z)\Pi$ units of dividends. It is assumed that $s'(z) > 0$.[8] On the top of that, customers are endowed with money holdings $m \geq 0$ (that may differ across households) and chose its "future" level m'. The joint distribution of productivity and money across customers is denoted by $\pi(m, z)$. Household pays tax $p\tau_z$ (which satisfies $0 \leq p\tau_z < wzN$) that is dependent on his productivity. The maximization problem of customer that has m units of liquid assets and technology level z reads:

$$\max_{c>0,\, m'>0,\, v_h>0} u(c) - \kappa_g v_h + \beta W(m') \tag{3}$$

$$\textit{subject to}:$$

$$pc + m' + p\tau_z = wzN + s(z)\Pi + m$$

[6] First, I abstract from randomness at the individual level - i.e. it is assumed that each household purchases exactly $q_g(x_g)\, v_h$ of goods. Second, Michaillat and Saez [2015] specify search cost in terms of manufactured goods and not in terms of search effort. I depart from their approach because the model that contains their specification of search costs exhibits two equilibria for a positive amount of fiscal spending. The formulation of search costs, applied in this paper, allows for avoiding this problem. This is associated with certain cost, though: search costs specified in terms of manufactured goods offer a better portability, in the sense that they can be incurred not only by consumers but also by government. The formulation applied here does not have this property: it assumes that government purchases goods in frictional market with probability $q_g(x_g)$ but it does not incur any search costs associated with this transaction. To introduce economic costs associated with government purchases it is assumed that products bought by government are wasted (households do not derive utility from them). A specification of search costs that is similar to mine was used by Bai et al. [2012].

[7] This means that I abstract from idiosyncratic labor market risk. This assumption is relaxed in the quantitative version of the model.

[8] The intuition is that households earning higher income tend to have larger portfolios of shares and earn higher capital income. Function $s(z)$ will be chosen to match the SCF data when constructing the dynamic version of the model. To keep the numerical burden manageable, it is assumed that households do not maximize with respect to stock holdings (a similar approach was used by McKay and Reis [2016]).

$$c = q_g(x_g)\, v_h$$

where $\beta > 0$ is a parameter. First equation is the standard budget constraint and the second captures the impact of search frictions on consumption. Constraints are used to eliminate v_h and m' from the maximization problem. Optimization with respect to c yields the first order condition:

$$u'(c) = \frac{\kappa_g}{q_g(x_g)} + p\beta W'\left(wzN + s(z)\Pi + m - pc - p\tau_z\right). \tag{4}$$

Equation 4 states that the marginal utility from consumption of an additional unit of manufactured good is equal to the sum of marginal disutility from search effort exerted to obtain this unit and disutility from the associated drop of consumption "in the future". By $c(x_g; m, z)$ I denote the solution to equation 4 which, thanks to assumptions about u and W, exists and is unique.[9] Budget constraint is used to define the optimal choice of m' that is associated with $c(x_g; m, z)$:

$$m'(x_g; m, z) \equiv wzN + s(z)\Pi + m - pc(x_g; m, z) - p\tau_z. \tag{5}$$

Government. Government chooses the amount of goods G that it wants to purchase. It is assumed that households derive no additional utility from these goods. Government takes market tightness as given and adjusts the number of visits V_G needed to purchase the chosen amount G:

$$G = q_g(x_g) \cdot V_G. \tag{6}$$

Fiscal expenditures are financed with labor tax schedule $\tau = \{\tau_z\}$ levied on households which satisfies:

$$G = \mathbb{E}_z \tau_z. \tag{7}$$

Aggregate demand. We are in position to present the microfounded version of the LHS of equation 1. Private absorption reads:

[9] As we shall see, all the variables that are taken as given by households (i.e., w, N, Π, p) can be expressed (in equilibrium) as functions of x_g and hence their impact on the optimal choice of c can be summarized with a single variable - product market tightness.

$$C\left(x_g, \tau(G)\right) = \int c\left(x_g; m, z\right) d\pi\left(m, z\right). \qquad (8)$$

Observe, that since z determines τ_z and because from 7 tax rates $\tau = \{\tau_z\}$ are implicit functions of G then the integral in 8 is a function of $\tau(G)$. This implies that the aggregate demand in economy is given by:

$$C\left(x_g, \tau(G)\right) + G = \int c\left(x_g; m, z\right) d\pi\left(m, z\right) + G.$$

3.3 Firms and aggregate output

There is measure one of identical firms that produce consumption goods using a linear technology with labor n as the only input. Output is sold at price p with probability $f_g\left(x_g\right)$ (with $f_g' > 0$). It can be interpreted as rate at which customers show up to purchase goods (or place orders). To hire workers, firms post vacancies v_l that are costly in terms of output - each posted vacancy decreases the output capacity by κ_l. The probability that a vacancy is filled is denoted by $q_l\left(x_l\right)$ (and satisfies $q_l' < 0$) where x_l is labor market tightness that is given by:

$$x_l = \frac{V_l}{1}.$$

By V_l I denote the aggregate number of vacancies posted by firms. The unity in the denominator describes the total number of workers available in the economy. It is assumed that firms are not able to distinguish between high and low productivity workers during the recruitment process but after hiring them, they learn their levels of z and pay wages proportional to this value. This assumption allows for merging two important building blocks (labor income heterogeneity and labor market frictions) within a single framework in a tractable way. The zero profit from entry condition, that governs the decisions about vacancy posting:

$$pf_g\left(x_g\right)\kappa_l = q_l\left(x_l\right) \cdot \mathbb{E}_z\left[pf_g\left(x_g\right)z - wz\right]. \qquad (9)$$

Without loss of generality I set $\mathbb{E}_z z = 1$. Firm's profit reads:

$$\Pi = p f_g \left(x_g\right) \left[n - \kappa_l v_l\right] - wn.$$

Let us derive the microfounded formula for aggregate output that corresponds to $Y\left(x_g\right)$ in equation 1. First, notice that 9 defines an implicit, increasing relationship between x_g and x_l.[10] To continue the analysis I have to impose a specific functional form for q_l:[11]

$$q_l\left(x_l\right) = q_l\left(\frac{V_l}{1}\right) = \frac{1}{\left(1 + V_l^{\alpha_l}\right)^{\frac{1}{\alpha_l}}}$$

where $\alpha_l > 1$ is a parameter. It is shown in the Appendix that given this specification of q_l, the formula for $Y\left(x_g\right)$ takes the following form:

$$Y\left(x_g\right) \equiv f_g\left(x_g\right)\left[N - \kappa_l V_l\right] = \frac{w}{p} \cdot \left(1 - \left(\frac{\kappa_l \cdot f_g\left(x_g\right)}{f_g\left(x_g\right) - \frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}} \tag{10}$$

which is a strictly increasing function of x_g that is well-defined for $x_g \geq \underline{x_g} \equiv f_g^{-1}\left(\frac{w/p}{1-\kappa_l}\right)$ (with $Y\left(\underline{x_g}\right) = 0$).

3.4 Price-setting mechanism

When analyzing the static model, I follow Hall [2005] and Michaillat and Saez [2015] and abstract from the microfounded price-setting mechanisms (like Nash Bargaining or competitive search equilibrium) by making a simplifying assumption that price p and nominal wage w are fixed and become parameters of the model.

[10] This follows from the fact that $q_l' < 0$ and $f_g' > 0$ and enables to eliminate x_l from further considerations and to represent aggregate output as function of x_g only.

[11] This formula is induced by the matching technology introduced by Den Haan et al. [2000] and was used by Michaillat and Saez [2015] in their work.

3.5 Consistency conditions and resource constraints

Consistency in the labor market requires that:

$$n = N,\ v_l = V_l$$

The total number of visits V_g is given by:

$$V_g = V_G + V_h$$

where V_h (aggregate number of visits made by households) is given by:[12]

$$V_h = \frac{1}{q_g\left(x_g\right)} \cdot \int c\left(x_g; m, z\right) d\pi\left(m, z\right)$$

Aggregate capacity T is defined as:

$$T = N - \kappa_l V_l.$$

It is assumed that the number of successful matches in frictional markets is governed by the CRS functions $M_g\left(V_g, T\right)$ and $M_l\left(V_l, 1\right)$ in the product and in the labor market, respectively. They are specified as follows:

$$M_g\left(V_g, T\right) = \frac{V_g \cdot T}{\left(T^{\alpha_g} + V_g^{\alpha_g}\right)^{\frac{1}{\alpha_g}}},\ M_l\left(V_l, 1\right) = \frac{V_l}{\left(1 + V_l^{\alpha_l}\right)^{\frac{1}{\alpha_l}}}$$

where α_g, $\alpha_l > 1$. Consistency between matching technologies and probabilities q_g, f_g and f_l is guaranteed by:

$$q_g\left(x_g\right) = \frac{M_g\left(V_g, T\right)}{V_g},\ f_g\left(x_g\right) = \frac{M_g\left(V_g, T\right)}{T},\ q_l\left(x_l\right) = \frac{M_l\left(V_l, 1\right)}{V_l}. \quad (11)$$

The market clearing condition for labor is:

$$N = M_l\left(V_l, 1\right).$$

[12] It is derived by aggregating the constraints of maximization problem 3: $c = q_g\left(x_g\right) v_h$.

The resource constraint for liquid assets is:

$$\int m d\pi\left(m,z\right) = \int m'\left(x_g; m,z\right) d\pi\left(m,z\right)$$

and it is assumed that $\int m d\pi\left(m,z\right) > 0$. The resource constraint for the product market reads:

$$C\left(x_g, \tau(G)\right) + G = f_g\left(x_g\right) \cdot T. \tag{12}$$

Observe that not only is aggregate output $Y\left(x_g\right) \equiv f_g\left(x_g\right) \cdot T$ influenced by capacity T (which is determined by production capacity) but also its value is affected by rate at which customers visit producers $f_g\left(x_g\right)$. This is an important departure from the neoclassical paradigm that was made by Michaillat and Saez [2015] and Bai et al. [2012], among others.

3.6 Equilibrium

As it has been already mentioned, all aggregate variables in the model can be represented as functions of x_g.[13] The definition of a fixprice equilibrium (see Michaillat and Saez [2015]), in which w and p are parameters, is the following:

Definition 1. *A fixprice equilibrium is market tightness x_g such that given tax scheme $\tau = \{\tau_z\}$ and government purchases G: households and firms optimize, government budget constraint holds and markets clear.*

From now on it is assumed (without loss of generality) that $G = 0$ which means that $\forall_z\, \tau_z = 0$.

From what was said above it is clear that the value of product market tightness (satisfying $x_g \geq \underline{x_g}$) that characterizes equilibrium solves the

[13] Labor market tightness x_l is a well-defined function of x_g from 9. The formula for aggregate employment N as a function of product market tightness is presented in the proof of Proposition 1. Profits Π are a constant function of x_g in the static version of the model (see the proof of Proposition 1), p and w are fixed. All this means that the individual optimal choice c is a function of x_g, endowments m, z and tax τ_z. This in turn implies that C is a function of x_g and $\tau = \{\tau_z\}$. This shows that the LHS of 12 is a function of one endogenous variable: x_g. The RHS of 12 is equal to Y which is a function of x_g by 10.

following equation:[14]

$$C\left(x_g, 0\right) = Y\left(x_g\right). \tag{13}$$

The following proposition establishes existence and uniqueness of equilibrium:

Proposition 1. *If* $\frac{\kappa_l}{1-\frac{w}{p}} < 1$ *then the fixprice equilibrium exists and is unique.*

The proof of Proposition 1 is presented in the Appendix. Assumption about parameters κ_l, w and p guarantees that firms have sufficient incentives to manufacture goods. As we shall see in the next subsection, function $C\left(x_g, 0\right)$ can be potentially non-monotonic in x_g which is the main difficulty when establishing the result described by Proposition 1.

3.7 Government multiplier in the microfounded model

In this part, we will focus on the consequences of an infinitesimal increase of government expenditures from 0 to dG. Observe that since our model can be characterized by equation that is identical to formula 1 then the value of government multiplier (derived in Section 3.1) is given by:

$$\frac{dY}{dG} = \frac{1 + \frac{\partial C}{\partial \tau}\frac{d\tau}{dG}}{1 - \frac{\frac{\partial C}{\partial x_g}}{\frac{dY}{dx_g}}}. \tag{14}$$

In what follows I will be studying the determinants of $\frac{dY}{dG}$: $\frac{\partial C}{\partial \tau}\frac{d\tau}{dG}$, $\frac{\partial C}{\partial x_g}$ and $\frac{dY}{dx_g}$ in the model with representative agent and the model with heterogeneous agents, respectively. Observe that the supply side is identical in both cases (which from 10 satisfies $\frac{dY}{dx_g} > 0$). We shall concentrate on the properties of function $C\left(x_g, \tau\left(G\right)\right)$, then.

The RA case and the role of economic slack. Let us rewrite the condition that pins down the aggregate consumption as function of x_g (equation 4):

$$u'(c) = \frac{\kappa_g}{q_g\left(x_g\right)} + p\beta W'\left(wzN + s(z)\Pi + m - pc - p\tau_z\right).$$

[14] Recall, that I assumed that $G = 0$ and hence I omit it in the formulation of the aggregate demand.

It is easy to see, that from the market clearing condition:

$$m' = wzN + s(z)\Pi + m - pc - \tau_z = m > 0$$

which is a parameter. In other words:

$$C\left(x_g, \tau\left(G\right)\right) = \left(u'\right)^{-1}\left(\frac{\kappa_g}{q_g\left(x_g\right)} + p\beta W'\left(m\right)\right). \tag{15}$$

First, notice that $\frac{\partial C}{\partial \tau} = 0$: general equilibrium effects eliminate the impact of taxes on consumption decisions. Similarly, the second channel through which changes in tightness x_g affect private demand (the channel working through disposable income) is shut down, too. Formula 15 implies that the only way through which consumption is affected by changes in product market tightness is the crowding-out channel: a rise in x_g decreases the probability of a successful visit $q_g\left(x_g\right)$ which raises the effective price of consumption and leads to a drop in $C\left(x_g, \tau\left(G\right)\right)$. Putting it differently, in the RA case: $\frac{\partial C}{\partial x_g} < 0$. This coupled with $\frac{dY}{dx_g} > 0$ and with formula 14 implies:

Proposition 2. *The value of government multiplier in economy with a representative agent is bounded from below by* 0 *and from above by* 1.

Proposition 2 indicates that efficacy of fiscal stimulus in the economy with representative agent is limited (as $\frac{dY}{dG} < 1$) as it is always associated with crowding-out of private expenditures (see the left panel of Figure 1).

The RA case is useful for the isolation of the impact of economic slack on the value of government multiplier. In what follows, we will be considering RA economies that differ with respect to the value of β which shifts the demand curve $C\left(x_g, \tau\left(G\right)\right)$ and leaves the supply curve $Y\left(x_g\right)$ unchanged. Higher values of β can be interpreted as a preference shock that decreases private demand. Proposition 3 shows that fiscal policy becomes more effective in economy characterized by a significant amount of slack.

Proposition 3. *Assume that* $\lim_{c\to 0} u''\left(c\right) = -\infty$ *and* $u'''\left(c\right) > 0$*. The value of government multiplier* $\frac{dY}{dG}$ *increases in* β *and converges to its upper limit (i.e. the value of unity) as* $\beta \to +\infty$.

Assumptions about preferences are necessary to derive the result analytically and they are satisfied by utility functions which are commonly used in the literature. Proposition 3 describes the "countercyclical" behavior of the government multiplier.[15] The amount of idle capacity, measured by a low value of $f_g\left(x_g\right)$, is significant in the slack regime (characterized by high β) so the crowding-out of private consumption caused by an increase in government expenditures dG is relatively small. This leads to a large shift in rate at which customers and government visit producers which raises firms' incentives to expand output and create jobs. These incentives are enlarged by the fact slack in the product market is always accompanied by the slack in labor market (see equation 9). This lowers the effective cost of posting vacancies and magnifies the increase in job creation which is a reaction to increased probability of selling goods $f_g\left(x_g\right)$.

The HA case and the role of inequality. In this part we will concentrate on the impact of fiscal expansion in the situation when households differ with respect to their productivity levels z (share holdings $s(z)$) and endowments of liquid assets m. As it has been already mentioned (see Section 3.1), it is necessary that $\frac{\partial C}{\partial x_g} > 0$ and that tax distortions $\frac{\partial C}{\partial \tau} \leq 0$ are relatively small for the value of the government multiplier $\frac{dY}{dG}$ to exceed unity. To guarantee the former, in turn, $c\left(x_g; m, z\right)$ (individual demand function) has to be an increasing function of x_g for a sufficiently large proportion of agents. First, I provide an analytic, sufficient condition for $\frac{\partial}{\partial x_g} c\left(x_g; m, z\right) > 0$. It provides valuable insights about the determinants of the direction of changes in the aggregate private absorption that result from changes in x_g. Second, I present numerical examples that illustrate the impact of a rise in government purchases in the HA economy.

From what was said above, it is crucial to understand under what conditions $\frac{\partial}{\partial x_g} c\left(x_g; m, z\right)$ can be positive. The following, simple inequality

[15] An analogous pattern in the context of economy with a single friction (in the labor market) was observed by Michaillat [2014]. Similarly to the RA case, he finds that the multiplier of public employment in his model is bounded by unity (from above).

describes these circumstances:[16]

$$\underbrace{wzN'\left(x_g\right)+s(z)\Pi'\left(x_g\right)}_{income\,effects} > \underbrace{\frac{\kappa_g}{q_g^2\left(x_g\right)}q_g'\left(x_g\right)\cdot\frac{1}{\beta pW''\left(m'\left(x_g;m,z\right)\right)}}_{crowding-out\,effects} \tag{16}$$

which is summarized by the following Proposition.

Proposition 4. *If condition 16 is satisfied then* $\frac{\partial}{\partial x_g}c\left(x_g;m,z\right)>0$.

Proof is postponed to the Appendix. Condition 16 has a natural interpretation: the LHS of the inequality represents the impact of changes in the disposable income (resulting from a shift in x_g) on agent's consumption. Notice that it tends to be larger for agents with high productivity levels. The RHS describes the impact of crowding-out effects (associated with changes in the probability of successful transaction $q_g\left(x_g\right)$) on consumption. Observe that since policy m' is increasing in m and z then the RHS is larger for richer agents (their consumption decisions are more susceptible to crowding out effects).[17] It is thus hard to assess whether 16 is more likely to hold for more or for less productive/wealthy agents. Numerical simulations indicate that the latter is more probable. Proposition 5 relates condition 16 with amount of slack in the economy.

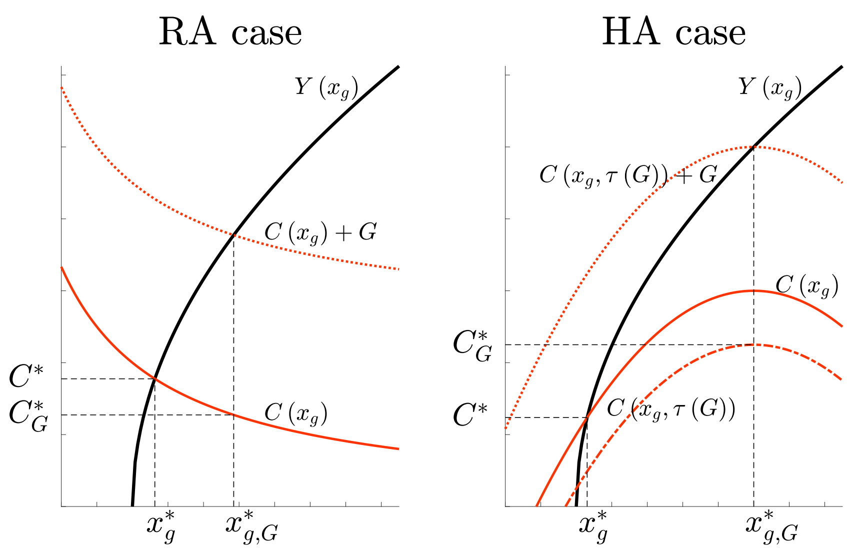
Proposition 5. *Assume that* $W'''>0$ *and* $\lim_{m'\to+\infty}W''\left(m'\right)=0$. *There exists a critical value* $\tilde{x}_g$ *such that inequality 16 holds for* $x_g\in\left[\underline{x_g},\tilde{x}_g\right)$ *and ceases to be satisfied for* $x_g\geq\tilde{x}_g$.

Assumptions about W are not restrictive and are satisfied by utility functions used in the literature. Putting it differently, Proposition 5 states, that individual demand is a hump-shaped function of product market tightness. This implies that a combination of economic slack and inequality is needed to give rise to a situation in which $\frac{\partial}{\partial x_g}c\left(x_g;m,z\right)>0$ which, as we have seen, is critical for the aggregate private absorption to satisfy $\frac{\partial C}{\partial x_g}>0$ - a necessary condition for the government multiplier to exceed unity.

[16]Observe that since equilibrium can be characterized by a single number x_g then all economic variables are functions of product market tightness.

[17]This result holds under an additional assumption about W_z: $W_z'''>0$ which is standard in the literature.

Figure 1: Increase in government purchases in RA/HA economy



To illustrate the consequences of Propositions 4 and 5 I use numerical simulations.[18] The right panel of Figure 1 illustrates the results of a government intervention in economy with heterogeneous agents and slack markets. Observe that the aggregate demand curve $C(x_g) \equiv C(x_g, 0)$ exhibits a hump-shaped pattern which is in line with predictions for the individual-level demand described by Proposition 5 and it is increasing at its intersection with aggregate supply curve $Y(x_g)$. We can see how equilibrium changes when $G > 0$: aggregate demand curve shifts upwards and private absorption $C(x_g, \tau(G))$ is slightly below its initial position $C(x_g)$ due to the negative impact of taxes on disposable income.[19] This negative movement is compensated by an increase in product market tightness x_g that stimulates aggregate private consumption. The latter occurs since x_g is relatively low and hence income effects dominate crowding-out effects.

[18] I assume that there are two types of households that differ with respect to their initial endowment of liquid assets.

[19] To guarantee that this shift is small it is assumed that taxes are levied on rich households (a form of progressive taxation).

4 The Bewley-Huggett-Aiyagari model with frictional product and labor markets

4.1 The model

In this section I develop a fully fledged macroeconomic model that embeds the mechanism discussed above into a version of the Bewley-Huggett-Aiyagari framework that preserves the structure of economy analyzed in Section 3. Moreover, the model is extended to capture unemployment fears which turns out to be an important factor affecting the response of private consumption to fiscal stimulus. First, I consider the stationary equilibrium of the model and then I study transition path resulting from a transitory change in government expenditures.

Households. The model is populated by a continuum of households (customers) of measure one who face uninsurable idiosyncratic income and labor status shocks that are driven by: exogenous changes in labor productivity z and endogenous shifts in the job-finding rate $f_l(x_l)$. As in the static model, I assume that individual shareholdings $s(z)$ are tightly connected to labor productivity level and entitle to profits Π generated by firms. The only asset that is actively traded in the economy are liquid assets/money balances. Agent preferences are given by the instantaneous utility function:

$$u(c) - \kappa_g v_h$$

where $u' > 0$, $u'' < 0$, u satisfies the Inada conditions, c is consumption and v_h is search effort (number of visits) exerted by a household and $\kappa_g > 0$ is a constant. Their values are related by constraint that is imposed by product market frictions:

$$c = q_g\left(x_g\right) \cdot v_h$$

where $q_g\left(x_g\right)$ is the probability at which a visit that is made by household ends with a purchase of a unit of consumption good. Each household supplies a unit of labor inelastically. Employed household earns nominal wage wz where w is the average nominal wage in economy and the unemployed receives unemployment benefits equal to νwz where $\nu \in (0,1)$ is the replacement rate. Idiosyncratic productivity shocks z follow a Markov process defined on space $Z = \{z_1, z_2, ..., z_N\}$. Employed households pay linear tax on labor income with rate $\tau \in (0,1)$.[20] Unemployed household becomes employed (in the next period) with probability $f_l'\left(x_l'\right)$. Employed ones lose their jobs with probability $\sigma \cdot \left(1 - f_l'\left(x_l'\right)\right)$ where $\sigma \in (0,1)$ is exogenous separation rate. Household's choice of next period balances m' is subject to the borrowing constraint:

$$m' \geq -\phi$$

where ϕ is a positive constant. The dynamic maximization problem of employed household with current balances m and productivity level z can be represented by the following Bellman equation:[21]

$$\begin{aligned} W_e\left(z,m\right) = \max_{c,\, v_g,\, m'} \Big\{ & u(c) - \kappa_g v_h + \beta \mathbb{E}_{z'|z}\left[\left(1-\sigma\cdot\left(1-f_l'\right)\right) W_e\left(z',m'\right)\right. \\ & \left.+\sigma\cdot\left(1-f_l'\right) W_u\left(z',m'\right)\right] \Big\} \end{aligned} \tag{17}$$

$$\textit{subject to}:$$

$$pc + m' = m + (1-\tau)\, wz + \Pi s\,(z)$$

$$c = q_g \cdot v_h$$

$$m' \geq -\phi$$

where W_e and W_u are value functions associated with the dynamic problem of employed and unemployed agent, respectively. Agents discount future utility streams with factor $\beta \in (0,1)$. First equation is household's budget constraint, second describes product market frictions and third is the borrowing constraint. Bellman equation of the unemployed household reads:

$$W_u\left(z,m\right) = \max_{c,\, v_g,\, m'} \Big\{ u(c) - \kappa_g v_h + \beta \mathbb{E}_{z'|z}\left[f_l' \cdot W_e\left(z',m'\right)\right.$$

[20] Notice, that since the number of hours worked is fixed for each household (i.e., equal to one) then labor income tax is not distortionary, i.e., it has no effect on labor supply.

[21] To simplify the notation I omit the dependence of q_g and f_l on their arguments.

$$+\left(1-f_l'\right) W_u\left(z', m'\right)\right]\Big\}$$

$$pc + m' = m + \nu w z + \Pi s\left(z\right)$$

$$c = q_g \cdot v_h$$

$$m' \geq -\phi.$$

Euler inequalities associated with dynamic problems of employed and unemployed households are:

$$\begin{gathered}
\frac{u'(c_e) - \frac{\kappa_g}{q_g}}{p} \geq \beta \mathbb{E}_{z'|z}\left[\left(1-\sigma\cdot\left(1-f_l'\right)\right)\cdot\frac{u'(c_e') - \frac{\kappa_g}{q_g'}}{p'} + \sigma\cdot\left(1-f_l'\right)\cdot\frac{u'(c_u') - \frac{\kappa_g}{q_g'}}{p'}\right], \\
\frac{u'(c_u) - \frac{\kappa_g}{q_g}}{p} \geq \beta \mathbb{E}_{z'|z}\left[f_l'\cdot\frac{u'(c_e') - \frac{\kappa_g}{q_g'}}{p'} + \left(1-f_l'\right)\cdot\frac{u'(c_u') - \frac{\kappa_g}{q_g'}}{p'}\right]
\end{gathered} \tag{18}$$

where c_e is consumption policy conditional on being employed and c_u corresponds to the unemployed household.

Firms. There is measure one of identical firms producing consumption goods. They use labor as the only production input and hire workers in frictional labor market by posting vacancies. A value of vacancy matched with a worker that has productivity z is described by the following recursive formula:[22]

$$J\left(z\right) = \left(p\cdot f_g\left(x_g\right) - w\right)\cdot z + \left(1-\sigma\right)\beta J\left(z\right). \tag{19}$$

Vacancy posting decision is governed by condition that equates the marginal cost of posting an additional vacancy with marginal benefit of doing so:

$$p f_g\left(x_g\right)\kappa_l = q_l\left(x_l\right)\cdot\mathbb{E}_z J\left(z\right). \tag{20}$$

The LHS of 20 is the marginal cost of posting a vacancy (in terms of foregone output) and the RHS is the expected benefit from a successful match in the market. It is assumed that firms are not able to distinguish between more productive and less productive workers while recruiting

[22]Observe that firms discount future profits at rate β and not with a stochastic discount factor dependent on agents' utilities. This is a simplification which makes the calibration exercise more tractable.

them. Once the worker is hired, firm learns about his productivity and pay wage that is proportional to his productivity level (see equation 19). As it has been already mentioned when analyzing the static model, the assumption about unobservability of z during the recruitment process is made to blend together the idiosyncratic income risk captured by changes in z with the Diamond-Mortensen-Pissarides specification of the frictional labor market. Firm's profits are:

$$\Pi = pf_g\left(x_g\right) \cdot \left[\mathbb{E}_z zn - \kappa_l v_l\right] - \mathbb{E}_z zwn \tag{21}$$

Average productivity is standardized to unity: $\mathbb{E}_z z = 1$. Individual law of motion for firm's employment is:

$$n = (1-\sigma)\, n_{-1} + q_l\left(x_l\right) v_l \tag{22}$$

where n_{-1} is firm's labor force in the previous period.

Government. Government runs a balanced budget:

$$N\tau w = pG + (1-N) \cdot \nu \cdot w \tag{23}$$

where N is the aggregate number of employed households and G is government consumption. By V_G I denote visits made by government to purchase goods G. Symmetrically to households, government faces product market frictions so the following formula holds:

$$G = q_g\left(x_g\right) \cdot V_G.$$

Price-setting. When analyzing the stationary equilibrium price p is standardized to 1 and value of w is fixed at the level that enables to match average unemployment rate in the calibration exercise.[23] When discussing the transitional dynamics, I use empirical data to estimate the laws of motion for prices and wages and embed them directly into the model.

[23]It can be checked that if nominal wage w is a certain proportion of recruiting costs $p\kappa_l f_g$ (which is assumed in the calibration exercise) then the value of p does not affect the stationary allocation.

Consistency conditions. By $\pi_e(m,z)$ let us denote the measure of employed agents with asset holdings m and productivity z. An analogous object applied to unemployed agents is denoted by $\pi_u(m,z)$. Aggregate number of visits made by households V_h satisfies:

$$V_h = \frac{1}{q_g\left(x_g\right)}\left(\int c_e\left(m,z\right)d\pi_e(m,z) + \int c_u\left(m,z\right)d\pi_u(m,z)\right)$$

where $c_e\left(m,z\right)$ and $c_u\left(m,z\right)$ are policy functions associated with dynamic problems of employed and unemployed households, respectively. Product market tightness is defined as:

$$x_g = \frac{V_h + V_G}{T}$$

where T denotes aggregate capacity which is defined later. Labor market tightness is:

$$x_l = \frac{V_l}{1-\left(1-\sigma\right)N_{-1}}$$

where $V_l = v_l$ and N_{-1} is aggregate employment in the previous period. Aggregate employment satisfies:

$$N = n.$$

Aggregate output Y and aggregate capacity T read:

$$Y = f_g\left(x_g\right)\cdot\left(N-\kappa_l V_l\right),\ T = N - \kappa_l V_l.$$

Probabilities f_g, q_g and f_l, q_l satisfy:

$$f_g = \frac{M_g\left(V_h+V_G,T\right)}{T},\ q_g = \frac{M_g\left(V_h+V_G,T\right)}{V_h+V_G},$$

$$f_l = \frac{M_l\left(V_l,1-\left(1-\sigma\right)N_{-1}\right)}{1-\left(1-\sigma\right)N_{-1}},\ q_l = \frac{M_l\left(V_l,1-\left(1-\sigma\right)N_{-1}\right)}{V_l}$$

where M_g and M_l are matching technologies specified as in the static model. The market clearing condition for liquid assets requires that:

$$\int m_e'(m,z)\, d\pi_e(m,z) + \int m_u'(m,z)\, d\pi_u(m,z) = \mu$$

where $m_e'(m,z)$ and $m_u'(m,z)$ are policy functions associated with dynamic problems of employed and unemployed workers, and $\mu \geq 0$ is the aggregate supply of liquid assets in the economy. The resource constraint for consumption goods is:

$$\int c_e(m,z)\, d\pi_e(m,z) + \int c_u(m,z)\, d\pi_u(m,z) + G = Y. \tag{24}$$

The law of motion of agents across states is characterized by two equations:

$$\pi_e(\mathcal{M}',z') = \left(1 - \sigma \cdot \left(1 - f_l'\right)\right) \cdot \int_{Z \times \{m: m_e'(m,z) \in \mathcal{M}'\}} \mathbb{P}(z'|z) d\pi_e(m,z) \tag{25}$$

$$+ f_l' \cdot \int_{Z \times \{m: m_u'(m,z) \in \mathcal{M}'\}} \mathbb{P}(z'|z) d\pi_u(m,z)$$

$$\pi_u(\mathcal{M}',z') = \sigma \cdot \left(1 - f_l'\right) \cdot \int_{Z \times \{m: m_e'(m,z) \in \mathcal{M}'\}} \mathbb{P}(z'|z) d\pi_e(m,z) \tag{26}$$

$$+ \left(1 - f_l'\right) \cdot \int_{Z \times \{m: m_u'(m,z) \in \mathcal{M}'\}} \mathbb{P}(z'|z) d\pi_u(m,z)$$

where $\mathcal{M}'$ is a Borel subset of $[-\phi, +\infty)$. Moreover, it is required that:

$$\int d\pi_e(m,z) + \int d\pi_u(m,z) = 1. \tag{27}$$

Stationary equilibrium. We are in position to define the stationary equilibrium of the model:

Definition 2. *A stationary equilibrium is: positive numbers x_l and x_g, value functions W_e and W_u, policy functions c_e, c_u, $v_{g,e}$, $v_{g,u}$, m_e', m_u' and probability distribution π such that given G, τ, ϕ and μ:*

(a) Value functions solve household maximization problems given x_l, x_g, τ, Π and c_e, c_u, $v_{g,e}$, $v_{g,u}$, m_e', m_u' are the associated policy functions,

(b) Given x_l, x_g equations 19-22 associated with firm's problem hold,

(c) Government budget constraint holds,

(d) Measures π_e and π_u are a fixed point of the dynamical system described

by equations 25-27,

(e) Consistency conditions and market clearing conditions hold.

4.2 Calibration

To analyze the model by numerical simulations, we need to specify functional forms of u, M_g, M_l and choose parameter values. The target of my calibration of the stationary equilibrium are the moments that characterize the U.S. economy. I divide the calibration exercise into 4 parts corresponding to parameters associated with: labor market, product market, fiscal policy and households. The time period is a quarter.

Parameters associated with labor market. Functional form of M_l is the same as in Section 3.5. There are 5 parameters that we need to assign values to: replacement rate ν, parameter α_l associated with function M_l, cost of a vacancy κ_l, separation rate σ and wage w. The value of replacement rate is taken from Shimer [2005]: $\nu = 0.4$. I adjust κ_l to match the steady state value of unemployment. I have experimented with different values of α_l and it has turned out that those changes have a negligible impact on moments generated by the model. I took the value of α_l from Den Haan et al. [2000]. Separation rate is taken from Shimer [2005]: $\sigma = 0.057$. Wage w is chosen to match the ratio between the recruiting costs and worker's wage reported by Michaillat [2012].

Parameters associated with product market. In this part I calibrate 3 parameters: price p, disutility from search activities (making visits) κ_g and parameter associated with matching technology in the product market α_g. Without loss of generality, the value of p is standardized to 1.[24] The value of κ_g is set to match the average value of marginal propensity to consume (MPC) documented by Blundell et al. [2008]. Simulations show that α_g does not have any significant impact on allocation. I use $\alpha_g = 2$ in my simulations.[25]

[24] It can be checked that choice of p does not affect the allocation if w is calibrated to match certain proportion of recruiting costs $p\kappa_l f_g$.

[25] Results of the simulation are robust to changes in α_g and α_l.

Table 1: Calibrated parameters: labor market and product market

Parameter	Description	Value	Target/Source
ν	Replacement rate	0.4	Shimer [2005]
α_l	Parameter of matching technology M_l	1.27	Den Haan et al. [2000]
κ_l	Vacancy cost	0.306	Unemployment rate of 5%
σ	Separation rate	0.057	Shimer [2005]
w	Nominal wage	0.189	Ratio between w and recruitment cost (equal to 0.32, Michaillat [2012])
p	Price of consumption goods	1	Standardization
κ_g	Disutility from a visit	13.5	Average MPC in the US economy (equal to 0.05, Blundell et al. [2008])
α_g	Parameter of matching technology M_g	2	Alternative values do not affected the results

Parameters associated with fiscal policy. There are 2 parameters that I calibrate here: government consumption G and tax rate τ. First of them is set to match the ratio between G and aggregate output that is equal to 0.15. Tax rate τ is assumed to balance the government budget.

Parameters associated with households. The last set of calibrated parameters concerns households. First, it is assumed that u has the following specification:

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}.$$

I calibrate the following parameters in this part: risk aversion θ, parameters associated with the discretized version of the process that governs idiosyncratic productivity shocks, share holdings $s(z)$, discount factor β, aggregate supply of liquid assets μ and the borrowing constraint ϕ. I follow the literature by setting $\theta = 2$. Similarly to Guerrieri and Lorenzoni [2016] I assume that evolution of logs of idiosyncratic productivity (for $z > 0$) follows an AR(1) process with autocorrelation ρ_z and variance ϵ_z chosen to match the evidence documented by Floden and Linde [2001]. Then I use the procedure constructed by Tauchen [1986] to approximate the AR(1) process by a discrete Markov chain. Share holdings $s(z)$ are calibrated to match the SCF data from 1992 to 2013 about stockholdings

Table 2: Calibrated parameters: fiscal policy and households

Parameter	Description	Value	Target/Source
G	Gov. consumption	0.028	The ratio between G and Y of 0.15
τ	Labor tax rate	0.178	Balances the budget
θ	Risk aversion	2	Literature
ρ_z	Persistence of productivity shock	0.967	Floden and Linde [2001] - persistence of wage process
ϵ_z	Variance of productivity shock	0.017	Floden and Linde [2001] - variance of wage process
$s(z)$	Share holdings	not reported here	SCF data 1992-2013
β	Discount factor	0.965	Top 10% share of liquid wealth (equal to 0.86, Kaplan et al. [2016])
μ	Supply of liquid assets	1.2	Positive liquid assets to GDP (equal to 1.78, Guerrieri and Lorenzoni [2016])
ϕ	Borrowing limit	0.4	Private debt to GDP (equal to 0.18, Guerrieri and Lorenzoni [2016])

across US households.[26] More precisely, I adjust the Lorenz curve implied by $s(z)$ and the invariant distribution across z to empirical evidence from the SCF. Discount factor β is calibrated to match the proportion of liquid assets held by top 10% of wealthiest households to entire supply of those assets (equal to μ). This proportion is taken from the work of Kaplan et al. [2016]. Aggregate supply of liquid assets μ is chosen to match the ratio between nominal assets and GDP which is reported by Guerrieri and Lorenzoni [2016] and equal to 1.6. Finally, the value of borrowing constraint ϕ is set to match the ratio between private debt and GDP from Guerrieri and Lorenzoni [2016].

4.3 Simulations: an unexpected increase in government expenditures

In this section I study the consequences of an unexpected increase in government consumption financed by an increase in labor taxes. First, I describe the effects of a transitory change in government consumption.

[26] Table 6 of the SCF Public Data Set.

Second, I study the determinants of fiscal policy efficacy by concentrating on factors that influence private absorption during the intervention. Third, I establish the countercyclical character of the government multiplier. Fourth, I study cross-sectional consumption responses to shock in fiscal purchases.

Transitory change in government purchases. It is assumed that an unexpected increase in government spending is financed with an additional tax $\tilde{\tau}$ levied on labor income of the richest (in terms of labor earnings).[27] More precisely, I assume that the deviation of government expenditures from its value in stationary equilibrium (i.e., from G) follows an AR(1) process with persistence rate equal to 0.8 and the initial value equal to 0.8% of GDP.[28] Since $\tilde{\tau}$ is used for financing this deviation then it satisfies:

$$\tilde{\tau}_t = \frac{p_t\left(G_t - G\right)}{wN\mathbb{P}\left(z > \underline{z}\right)\mathbb{E}\left(z|z > \underline{z}\right)}$$

where $\underline{z}$ is threshold value on labor income above which this additional tax is levied. All employed pay tax τ_t that evolves according to:

$$\tau_t = \frac{p_t G + \nu w_t\left(1 - N_t\right)}{w_t N_t} \tag{28}$$

which is analogous to 23 for the case of transition after an unexpected rise in government purchases. Prices and wages are assumed to be governed by the following laws of motion:

$$\frac{p_t - p}{p} = \gamma_{pp}\frac{p_{t-1} - p}{p} + \gamma_{py}\frac{Y_{t-1} - Y}{Y}$$

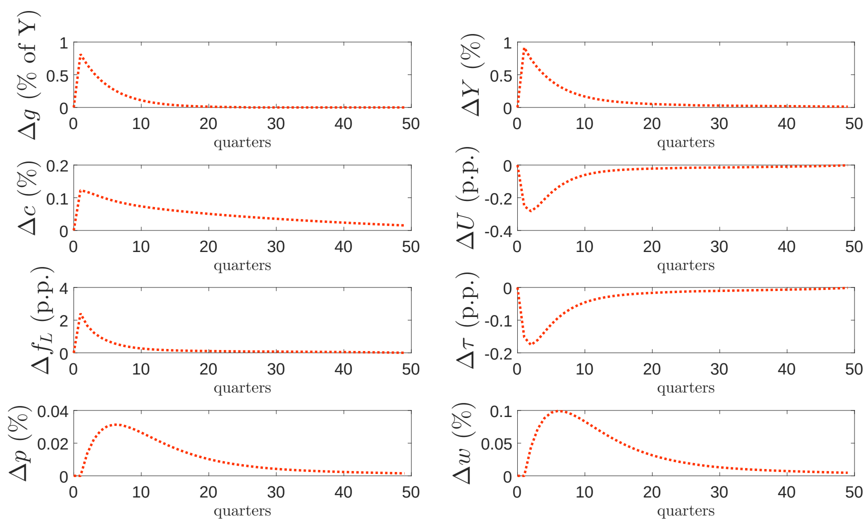
$$\frac{w_t - w}{w} = \gamma_{ww}\frac{w_{t-1} - w}{w} + \gamma_{wy}\frac{Y_{t-1} - Y}{Y}.$$

These formulations capture the stickiness of nominal prices and wages and their sluggish response to the value of output gap. Parameters γ_{pp}, γ_{py}, γ_{ww}, γ_{wy} are estimated using empirical observations on prices, nominal wages and output in the US. Estimated values are: $\gamma_{pp} = 0.83$, $\gamma_{py} =$

[27] This "progressive" tax scheme is applied in order to decrease the negative impact of taxes on private absorption $\frac{\partial C}{\partial \tau}$ defined when discussing the static model.

[28] I assume that government can credibly commit to a future increase in purchases.

Figure 2: Effects of a transitory change in government purchases



$0.015, \gamma_{ww} = 0.82, \gamma_{wy} = 0.048$.

Figure 2 shows the impulse response functions of the economy to a transitory change in government purchases. Fiscal expansion leads to a significant drop in unemployment and increase in job finding rates. Since output increases then (according to estimated laws of motion) prices and wages increase. In contrast to the IRF of GDP, the transitional dynamics of prices and wages exhibits a hump-shaped pattern due to the assumed specification of their evolution in time. There several opposite effects that affect the tax rate τ_t that is used to cover expenditures on unemployment benefits and the steady state amount of G (see equation 28). First effect is associated with lower unemployment $1 - N_t$ and higher employment N_t which imposes a downward pressure on τ_t. Second effect is associated with a rise prices p_t which increases the value of fiscal purchases $p_t G$ which tends to increase τ_t. Third effect, implied by a rise in wages w_t decreases τ_t.[29] This happens despite the fact that the replacement rate ν remains constant which implies an increase in the average unemployment benefit per capita. Figure 2 shows that the second effect is weaker than the remaining two and hence τ_t drops. Notice that response of output is larger than the rise in fiscal spending. It can be inferred from the LHS of

[29] A simple differentiation of τ_t with respect to w_t leads to this conclusion.

24 that this excessive response of output is driven solely by an increase in private consumption. It is thus crucial to understand what are the factors driving this reaction.

Private consumption: decomposition. In this part I decompose the response of private absorption to fiscal expansion with respect to several factors. One can use equation 17 to list them: changes in profits Π, wages w, taxes (τ and $\tilde{\tau}$), job finding rate f_l', price p and probability of a successful visit q_g. For tractability, I distinguish between five groups of effects shaping the response of private consumption: income effects (the joint impact of wages and profits), effects of taxes (the joint impact of τ and $\tilde{\tau}$), effects of changes in unemployment fears f_l, crowding-out effects (the joint influence of q_g and p) and the aggregate employment effect associated with the time path of N.[30] Observe that all effects affect the decision about accumulation of assets and hence have an impact on evolution of π over time.

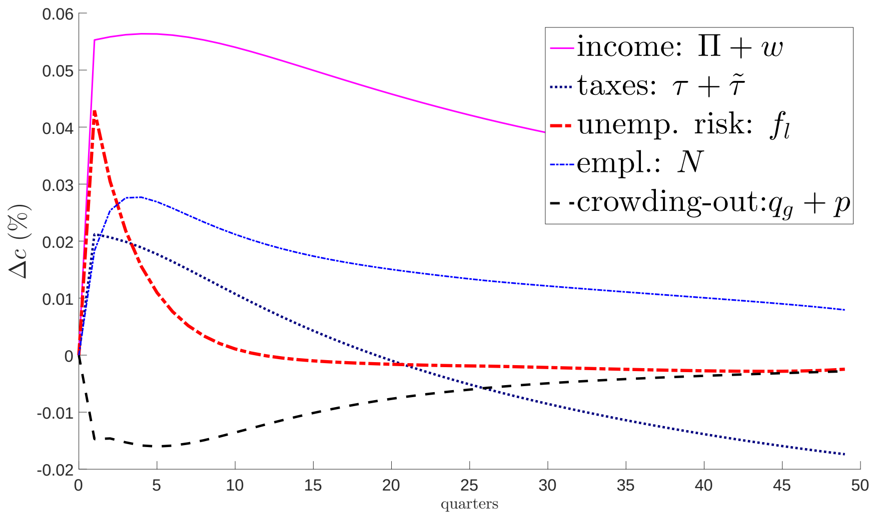
To separate those channels from each other, I proceed in the following way. First, I take the transition path of the effect that I want to separate (which was calculated when studying the impact of an unexpected increase in fiscal expenditures). Second, I set the values of all remaining effects at their levels in stationary equilibrium. Then I run the model and obtain the desired, isolated effect of a given factor on aggregate consumption.[31]

Results of the decomposition exercise are presented in Figure 3. I turns out that income effects have the largest positive impact on increase in private consumption. This result is in line with predictions of the stylized model presented in Section 3 in which government multiplier exceeds unity if income effects of fiscal expansions dominate crowding-out for a sufficiently large number of households (see condition 16). It turns out that a decrease in future unemployment risk (i.e., a rise in f_l) has a signif-

[30] To capture the effects of unemployment fears on aggregate consumption, I isolate the impact of f_l on consumption decisions from its effects on employment dynamics (that are described by the system 25-26). The latter force is called the "aggregate employment effect".

[31] To check whether my decomposition is correct (i.e., whether my set of factors is complete) I aggregate the transition paths of all partial effects and compare this sum to the IRF of aggregate consumption. It turns out that those two values are equal.

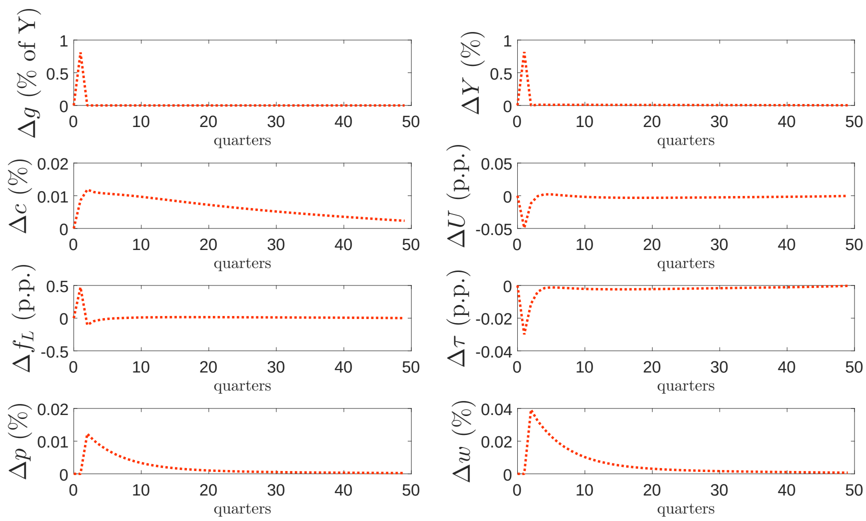
Figure 3: The response of private consumption: decomposition



icant impact on aggregate consumption: agents cut their current precautionary savings accumulated to self-insure against future drops in labor earnings experienced during unemployment spells. The joint change in taxes $\tau + \tilde{\tau}$ stimulates private absorption despite the negative impact of rise in $\tilde{\tau}$. This happens because τ drops and hence taxes paid by agents with lower income (i.e. those with productivity lower than $\underline{z}$) decrease. This implies that fiscal expansion in the model is accompanied by a rise in tax progressivity as in Ferriere and Navarro [2016]. Figure 3 shows that fiscal policy efficacy is not driven solely by a higher tax progressivity as claimed by those authors. The influence of higher employment on consumption is quite substantial and long lasting. It can be explained by a difference in consumption patterns of employed and unemployed workers presented in the left panel of Figure 5. The only factor that affects private absorption negatively is the crowding-out effect associated with increase in prices and with the drop in probability of a successful transaction.

Observe that there is an interesting dynamic interaction between the rise in aggregate consumption and employment. Since private consumption increases then firms find it easier to sell products which makes them expand employment. Since, due to the frictional labor market, changes

Figure 4: Effects of a one-time change in government purchases

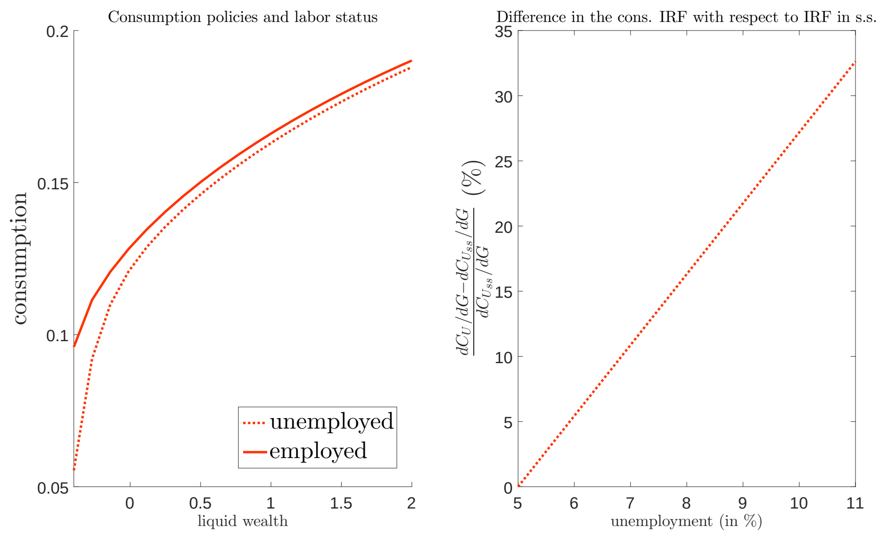


in employment persist over time and because employed workers consume more than unemployed agents, then aggregate demand increases and stimulates job creation in future periods. This leads to a drop in unemployment risk "today" which stimulates aggregate demand, employment (that persists over time) even further, etc.

Countercyclicality of the multiplier. In this part I will concentrate on the countercyclicality of government multiplier. To measure its value properly, I will study the impact of a one-time increase in government consumption (in contrast to a transient and persistent change described above). Results of such a single fiscal impulse are presented in Figure 4.

Before continuing the analysis of the countercyclicality of fiscal stimulus, it is instructive to compare Figures 2 and 4. Despite the fact that the magnitude of a fiscal impulse in period 1 is the same in both cases, the order of magnitude of response of private consumption is one time lower when fiscal expansion is not persistent. Moreover, employment returns to its steady state level very quickly. This means that the feedback loop between employment, unemployment risk and consumption analyzed before is shut down. The reason for this is change in job finding rate (that partly reflects the dynamics of job creation, too). Notice that a sharp spike in job finding-rate in period 1 is followed by a drop (below its value in

Figure 5: Consumption policies for different employment statuses and the countercyclicality of consumption reaction to a one-time fiscal expansion

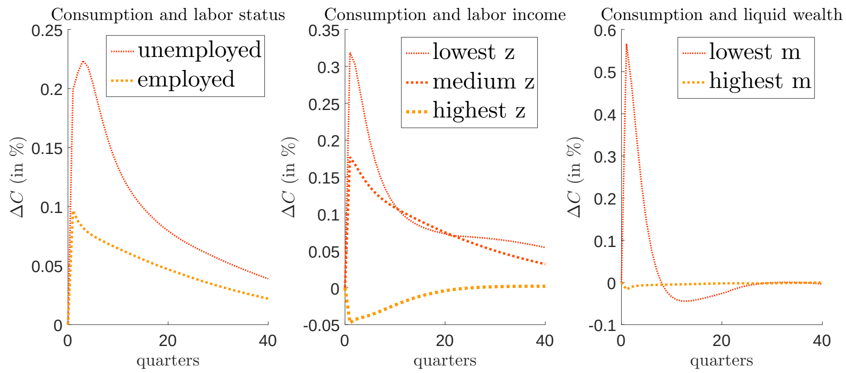


stationary equilibrium) in the following periods. This happens because firms that increased employment in reaction to fiscal expansion in period 1 face an abrupt decline in demand (as additional government purchases vanish) which implies that labor force "inherited" from period 1 (due to persistence in changes in employment) is excessively high. To adjust labor force to the optimal level, firms reduce job creation which boosts unemployment risk and has a negative impact on private consumption.

Let us return to the main issue of this part. Consider economies that differ with respect to the initial level of unemployment in period 1.[32] I do not specify the shock that could lead to such an increase in the number of

[32]Observe that the only state variable in economy is the distribution of agents across employment status, liquid wealth holdings and productivity. When analyzing the dependence of the fiscal multiplier on the unemployment level it is thus important to isolate this impact of change in marginal distribution of job status from changes along other dimensions of the distribution of households (productivity and liquid wealth). The situation is clear in case of distribution across productivity levels: it remains unaffected by transitional dynamics associated with exogenous changes in government expenditures or initial unemployment level as it is equal to ergodic distribution all the time. The situation can be potentially different in case of distribution of wealth holdings: higher unemployment may change agents' incentives to accumulate assets which in turn would affect the aggregate distribution of money balances. It turns out, however, that this one-time decrease in employment leads to negligible changes in the distribution of wealth holdings so no need for the isolation of changes in marginal distribution of money balances resulting from an increase in unemployment emerges.

Figure 6: Cross-sectional responses of private consumption



jobless workers. Instead, for each initial unemployment level I consider two transition paths that converge to the stationary equilibrium. First of them is undistorted by fiscal stimulus and second takes into account a one-time rise in government expenditures in period 1. The change in consumption resulting from the increase in fiscal expenditures between the distorted and undistorted path will be compared for various scenarios characterized by different initial levels of unemployment. I concentrate on private absorption because, as it has been argued, the reaction in consumption is decisive for efficacy of fiscal policy. The right panel of Figure 5 shows whats is the change in reaction of the aggregate consumption in period 1 to a one time rise in fiscal expenditures with respect to its response in the steady state (in which unemployment rate is 5%). Notice that e.g. for the initial unemployment level of 11%, the increase in consumption is 32% larger than in benchmark case (i.e., stationary equilibrium). This countercyclicality has analogous roots to those described when analyzing the static model: high unemployment is associated with slack in labor and product market (as unemployed workers consume less and hence aggregate demand is lower). This in turn implies that crowding-out effects associated with fiscal stimulus are less intense. Moreover, high unemployment decreases the effective cost of posting vacancies as q_l is higher. This stimulates job creation and leads to a larger drop in unemployment risk.

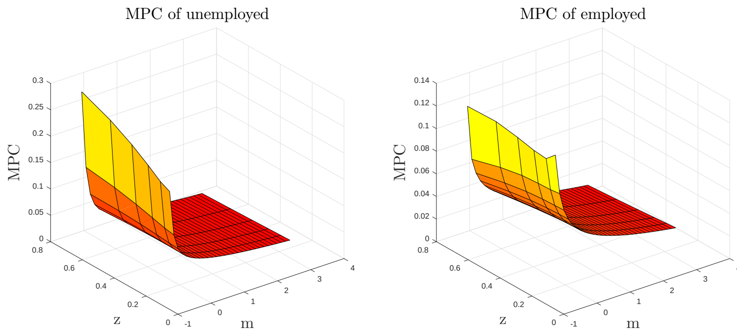
Cross-sectional responses to government purchases. Finally, let us analyze cross-sectional responses of private absorption to fiscal expansion analyzed in Figure 2. It can be seen that positive increase in aggregate consumption is driven by responses of unemployed, least productive and poorest workers (in terms of wealth). Figure 6 presents these results. Unemployed agents are vulnerable to changes in unemployment risk which explains the strong reaction of their consumption. Since least productive agents are likely to be liquidity constrained (and thus they exhibit high MPC - see Figure 7) then their response to changes in income is more dynamic than the one exhibited by workers with medium productivity. Negative response of consumption for agents with high productivity follows from the fact that they finance fiscal expansion with additional tax $\tilde{\tau}$. Agents with lowest levels of liquid wealth are liquidity constrained so, again, their response is strong due to high level of MPC.

5 Conclusions

This paper explores the consequences of changes in government expenditures in environment characterized by frictional product and labor markets and inequality. It shows that under some conditions, that are related to wealth (income) heterogeneity and amount of unemployed resources, the interplay between public purchases and private demand is sufficiently strong to give rise to government spending multipliers that exceed unity.

More specifically, I have used a version of the model by Michaillat and Saez [2015] to derive analytic results concerning the values of fiscal multipliers under different assumptions about agents heterogeneity and market slack. It has turned out, that: i) fiscal multiplier is bounded by unity if agents are identical (irrespectively of price rigidities and the level of economic slack), ii) the multiplier exceeds unity only if markets are sufficiently slack and agents are heterogeneous. This result highlights the important role of inequality in magnifying the outcomes of fiscal interventions. Next, I have extended the model to a standard, dynamic macroeconomic model with uninsured idiosyncratic risk to quantify mechanisms described by the stylized model. I have studied the determinants of the positive response of private absorption and found that income effects and drop in unemployment risk are the most important

Figure 7: Marginal Propensity to Consume



determinants of this reaction. I have shown that government multiplier exhibits a countercyclical pattern and it increases dynamically with a rise in unemployment. Finally, I have discussed cross-sectional responses of private consumption which indicate that the positive response in aggregate consumption is driven mainly by reactions of unemployed, liquidity constrained and low-income agents.

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Appendix

Derivation of the formula for aggregate output $Y\left(x_g\right)$

Let us rewrite the formula for $q_l\left(x_l\right)$:

$$q_l\left(x_l\right) = q_l\left(\frac{V_l}{1}\right) = \frac{1}{\left(1+V_l^{\alpha_l}\right)^{\frac{1}{\alpha_l}}}$$

This coupled with equation 9 and assumption $\mathbb{E}_z z = 1$ implies that the aggregate number of posted vacancies is:

$$V_l = \left(\left(\frac{f_g\left(x_g\right)-\frac{w}{p}}{\kappa_l f_g\left(x_g\right)}\right)^{\alpha_l}-1\right)^{1/\alpha_l}.$$

Aggregate output is:

$$\begin{aligned}
Y\left(x_g\right) &= f_g\left(x_g\right)\left[N-\kappa_l V_l\right] = f_g\left(x_g\right)\left[q_l\left(x_l\right)-\kappa_l\right]V_l \\
&= f_g\left(x_g\right)\left[\frac{\kappa_l f_g\left(x_g\right)}{f_g\left(x_g\right)-\frac{w}{p}}-\kappa_l\right]V_l \\
&= f_g\left(x_g\right)\left[\frac{\kappa_l\frac{w}{p}}{f_g\left(x_g\right)-\frac{w}{p}}\right]V_l \\
&= \frac{w}{p}\left[\frac{\kappa_l f_g\left(x_g\right)}{f_g\left(x_g\right)-\frac{w}{p}}\right]\cdot\left(\left(\frac{f_g\left(x_g\right)-\frac{w}{p}}{\kappa_l f_g\left(x_g\right)}\right)^{\alpha_l}-1\right)^{1/\alpha_l} \\
&= \frac{w}{p}\cdot\left(1-\left(\frac{\kappa_l\cdot f_g\left(x_g\right)}{f_g\left(x_g\right)-\frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}}
\end{aligned}$$

where I have used the fact that $N = q_l\left(x_l\right)V_l$ and condition 9.

Proof of Proposition 1

Proof. **Existence.** Let us first analyze the RHS of 13. Recall that under assumption about the specification of M_l (matching technology in the product market), it takes the following form:

$$Y\left(x_g\right) = \frac{w}{p} \cdot \left(1 - \left(\frac{\kappa_l \cdot f_g\left(x_g\right)}{f_g\left(x_g\right) - \frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}}.$$

It is clear that for $\underline{x_g} \equiv f_g^{-1}\left(\frac{w/p}{1-\kappa_l}\right)$ we have $Y\left(\underline{x_g}\right) = 0$ and that $Y' > 0$ for $x_g \geq \underline{x_g}$ (as $f_g' > 0$). Moreover, due to the assumed functional form of M_g, we have $\lim_{x_g \to +\infty} f_g\left(x_g\right) = 1$ which implies that $\lim_{x_g \to +\infty} Y\left(x_g\right) = \frac{w}{p} \cdot \left(1 - \left(\frac{\kappa_l}{1-\frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}}$ which is positive only if our assumption about κ_l, w and w holds. This means that, in our case, $\lim_{x_g \to +\infty} Y\left(x_g\right)$ is a strictly positive number.

Let us turn to aggregate demand now. I will investigate the behavior of $c\left(x_g; m, z\right)$ for each pair m and z separately. Recall that $c\left(x_g; m, z\right)$ solves (equation 4):

$$u'(c) = \frac{\kappa_g}{q_g\left(x_g\right)} + p\beta W'\left(wzN + s(z)\Pi + m - pc - p\tau_z\right).$$

Since (for the assumed specification of M_g) $\lim_{x_g \to +\infty} q_g\left(x_g\right) = 0$, $\lim_{c \to 0} u'(c) = +\infty$ and N, Π are bounded in the limit for $x_g \to +\infty$ (see derivations below) then the solution $c\left(x_g; m, z\right)$ to the equation above (that exists and is unique due to the Inada conditions for u and W) converges to 0 as $x_g \to +\infty$. This holds for all pairs of m and z so for aggregate consumption we have:

$$\lim_{x_g \to +\infty} \int c\left(x_g; m, z\right) d\pi\left(m, z\right) \to 0$$

Notice that:

$$\begin{aligned}
\Pi &= pf_g\left(x_g\right)\left[n - \kappa_l v_l\right] - wn \\
&= pY_g\left(x_g\right) - wN\left(x_g\right) \\
&= w \cdot \left(1 - \left(\frac{\kappa_l \cdot f_g\left(x_g\right)}{f_g\left(x_g\right) - \frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}} - wq_l\left(x_l\right)V_l \\
&= w \cdot \left(1 - \left(\frac{\kappa_l \cdot f_g\left(x_g\right)}{f_g\left(x_g\right) - \frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}} - w\frac{\kappa_l f_g\left(x_g\right)}{f_g\left(x_g\right) - \frac{w}{p}} \cdot \left(\left(\frac{f_g\left(x_g\right) - \frac{w}{p}}{\kappa_l f_g\left(x_g\right)}\right)^{\alpha_l} - 1\right)^{1/\alpha_l}
\end{aligned}$$

$$= w \cdot \left(1 - \left(\frac{\kappa_l \cdot f_g\left(x_g\right)}{f_g\left(x_g\right) - \frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}} - w \cdot \left(1 - \left(\frac{\kappa_l \cdot f_g\left(x_g\right)}{f_g\left(x_g\right) - \frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}} = 0.$$

where I have used the consistency condition $N = n$, the analytic formula for $Y\left(x_g\right)$, equation 9 and the formula for V_l presented when deriving the analytic expression for $Y\left(x_g\right)$. Additionally, notice that (from above):

$$N\left(x_g\right) = \left(1 - \left(\frac{\kappa_l \cdot f_g\left(x_g\right)}{f_g\left(x_g\right) - \frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}} \to 0$$

for $x_g \to \underline{x_g}$ which implies that the disposable income

$$wzN + s(z)\Pi + m - \tau_z$$

converges to m (recall that WLOG I have assumed that $G = 0$ so taxes satisfy $\tau_z = 0$ for all z) for any m and z as $x_g \to \underline{x_g}$. Notice that since I assumed that $\int m d\pi\left(m, z\right) > 0$ then there is a strictly positive measure of agents with strictly positive endowment of liquid assets m. It is clear that for them the solution $c\left(x_g; m, z\right)$to equation 4 is positive when $x_g \to \underline{x_g}$. For those with $m = 0$ (recall that I have assumed that $m \geq 0$ for all agents, so there are no households with negative holdings of liquid wealth), $c\left(x_g; m, z\right) \to 0$ as $x_g \to \underline{x_g}$ as their disposable income vanishes completly in that case. Summarizing:

$$\lim_{x_g \to \underline{x_g}} \int c\left(x_g; m, z\right) d\pi\left(m, z\right) > 0$$

Since both $\int c\left(x_g; m, z\right) d\pi\left(m, z\right)$ and $Y\left(x_g\right)$ are continuous in x_g for $x_g \geq \underline{x_g}$ then existence of equilibrium follows directly from what was said above about the properties of those functions in the limits of $\underline{x_g}$ and $+\infty$.

Uniqueness. Our plan is to show that:

$$\frac{\partial}{\partial x_g} \int c\left(x_g; m, z\right) d\pi\left(m, z\right) < Y'\left(x_g\right)$$

for all $x_g > \underline{x_g}$. Since we already know that the intersection of $\int c\left(x_g; m, z\right) d\pi\left(m, z\right)$ and $Y\left(x_g\right)$ exists then this condition will imply uniqueness. Observe, that from 4 and by the Implicit Function Theorem, for all pairs of m and z we have:

$$\frac{\partial c\left(x_g;m,z\right)}{\partial x_g}=\frac{\beta pW''\cdot\left(wzN'\left(x_g\right)+s(z)\Pi'\left(x_g\right)\right)}{u''+\beta p^2W''}-\frac{\frac{\kappa_g q_g'\left(x_g\right)}{\left(q_g\left(x_g\right)\right)^2}}{u''+\beta p^2W''}$$

$$<\frac{\beta pW''\cdot\left(wzN'\left(x_g\right)+s(z)\Pi'\left(x_g\right)\right)}{u''+\beta p^2W''}<\frac{wzN'\left(x_g\right)+s(z)\Pi'\left(x_g\right)}{p}$$

where I have used the fact that $u''<0$, $W''<0$, $q_g'\left(x_g\right)$ to establish the first inequality and that $u''<0$ (again) to establish the second. Since this is true for all pairs m and z then (after aggregation) the following approximation holds:

$$\frac{\partial}{\partial x_g}\int c\left(x_g;m,z\right)d\pi\left(m,z\right)<\frac{wN'\left(x_g\right)+\Pi'\left(x_g\right)}{p}$$

Observe that:

$$\frac{wN\left(x_g\right)+\Pi\left(x_g\right)}{p}=\frac{w}{p}N\left(x_g\right)+f_g\left(x_g\right)\left[N\left(x_g\right)-\kappa_l V_l\right]-\frac{w}{p}N\left(x_g\right)$$

$$=Y\left(x_g\right)$$

which implies:

$$\frac{\partial}{\partial x_g}\int c\left(x_g;m,z\right)d\pi\left(m,z\right)<Y'\left(x_g\right).$$

This completes the proof. □

Proof of Proposition 3

Proof. Recall that in the RA case:

$$\frac{dY}{dG}=\frac{1}{1-\frac{\frac{\partial C}{\partial x_g}}{\frac{dY}{dx_g}}}.$$

It is obvious that the equilibrium value of x_g converges monotonically to $\underline{x_g}$ as $\beta \to +\infty$. Observe that:

$$\frac{dY}{dx_g} = \frac{d}{dx_g}\left(\frac{w}{p}\cdot\left(1-\left(\frac{\kappa_l \cdot f_g\left(x_g\right)}{f_g\left(x_g\right)-\frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}}\right)$$

$$= \frac{w}{p}\cdot\left(\left(\frac{f_g\left(x_g\right)-\frac{w}{p}}{\kappa_l \cdot f_g\left(x_g\right)}\right)^{\alpha_l}-1\right)^{\frac{1-\alpha_l}{\alpha_l}}\cdot\frac{f_g'\left(x_g\right)\kappa_l\frac{w}{p}}{\left(f_g\left(x_g\right)-\frac{w}{p}\right)^2}$$

since $\frac{1-\alpha_l}{\alpha_l} < 0$ and since $\frac{f_g\left(x_g\right)-\frac{w}{p}}{\kappa_l \cdot f_g\left(x_g\right)}$ converges to 1 as $x_g \to \underline{x_g}$ (it is because $\underline{x_g} \equiv f_g^{-1}\left(\frac{w/p}{1-\kappa_l}\right)$) then $\frac{dY}{dx_g} \to +\infty$ for $x_g \to \underline{x_g}$. On the other hand:

$$\frac{\partial C}{\partial x_g} = \left(\left(u'\right)^{-1}\left(\frac{\kappa_g}{q_g\left(x_g\right)}+p\beta W'\left(m\right)\right)\right)'$$

$$= \frac{1}{u''\left(\underbrace{\left(u'\right)^{-1}\left(\frac{\kappa_g}{q_g\left(x_g\right)}+p\beta W'\left(m\right)\right)}_{=C}\right)}.$$

Observe that $q_g\left(x_g\right)$ is continuous and bounded so it does not play any role in our considerations as $\beta \to +\infty$. The latter implies that $\left(u'\right)^{-1}\left(\frac{\kappa_g}{q_g\left(x_g\right)}+p\beta W'\left(m\right)\right)$ converges to 0 as $\beta \to +\infty$ which implies that (under our assumption about u'') $\frac{\partial C}{\partial x_g} \to 0$. This means that $\frac{dY}{dG}$ converges to 1 as $\beta \to +\infty$.

Let us turn to the "countercyclicality" of government multiplier, i.e. its reaction to changes in β. Observe that not only shifts in β affect the aggregate consumption curve but also they affect the equilibrium value of x_g. It is clear that the latter is a decreasing function of β: a rise in β shifts the downward sloping aggregate demand curve down and leaves the upward sloping supply curve $Y\left(x_g\right)$ unaffected. This implies that $Y\left(x_g\right)$ decreases in β and so does C (from the resouce contraint). We can thus define two decreasing mappings $\beta \to x_g\left(\beta\right)$ and $\beta \to C\left(\beta\right)$ that assign equilibrium values of product market tightness and aggregate consumption to a given value of β. In other words: $C'(\beta) < 0$ and $x_g'(\beta) < 0$. We are in position to study the dependence of dY/dG on β. Observe that from what has been derived above:

$$\frac{dY}{dG}(\beta) = \frac{1}{1 - \frac{\frac{1}{u''(C(\beta))}}{Y'\left(x_g(\beta)\right)}}.$$

Notice that:[33]

$$\frac{d}{d\beta}\left(-u''(C(\beta)) \cdot Y'\left(x_g(\beta)\right)\right) = -u'''(C(\beta)) \cdot C'(\beta) \cdot Y'\left(x_g(\beta)\right)$$

$$-Y''\left(x_g(\beta)\right) \cdot x_g'(\beta) \cdot u''(C(\beta)) > 0,$$

which implies that $\frac{dY}{dG}(\beta)$ increases in β. □

Proof of Proposition 4

Proof. From the proof of Proposition 1 we know that:

$$\frac{\partial c\left(x_g; m, z\right)}{\partial x_g} = \frac{\beta p W'' \cdot \left(wzN'\left(x_g\right) + s(z)\Pi'\left(x_g\right)\right)}{u'' + \beta p^2 W''} - \frac{\frac{\kappa_g q_g'\left(x_g\right)}{\left(q_g\left(x_g\right)\right)^2}}{u'' + \beta p^2 W''}$$

since $u'' < 0$ and $W'' < 0$ then $\frac{\partial c\left(x_g; m, z\right)}{\partial x_g} < 0$ if and only if:

$$\beta p W'' \cdot \left(wzN'\left(x_g\right) + s(z)\Pi'\left(x_g\right)\right) - \frac{\kappa_g q_g'\left(x_g\right)}{\left(q_g\left(x_g\right)\right)^2} < 0$$

which is equivalent to 16 as $W'' < 0$. □

[33] $Y'' < 0$ as $\frac{dY}{dx_g}$ decreases in $f\left(x_g\right)$ (which is an increasing function of x_g) and:

$$f'\left(x_g\right) = \frac{1}{\left(1 + x_l^{\alpha_l}\right)^{1+\alpha_l}}$$

which is a decreasing function of x_g.

Proof of Proposition 5

Proof. Let us rewrite the condition 16 in the following form (notice that $q_g' < 0$):

$$\beta p W'' \left(m' \left(x_g; m, z\right)\right) \cdot \left(wzN' \left(x_g\right) + s(z)\Pi' \left(x_g\right)\right) \cdot \frac{1}{q_g' \left(x_g\right)} > \frac{\kappa_g}{\left(q_g \left(x_g\right)\right)^2}.$$

Now I use the fact that Π is a constant function of x_g (derived in the proof of Proposition 1) so that $\Pi' \left(x_g\right) = 0$. I use the Chain Rule to get:

$$N' \left(x_g\right) = \frac{dN}{df_g} \cdot f_g' \left(x_g\right)$$

so that I can write the inequality in the following form:

$$\beta p W'' \left(m' \left(x_g; m, z\right)\right) \cdot wz \cdot \frac{dN}{df_g} \cdot \frac{f_g' \left(x_g\right)}{q_g' \left(x_g\right)} > \frac{\kappa_g}{\left(q_g \left(x_g\right)\right)^2}$$

Let us concentrate on terms $\frac{f_g'(x_g)}{q_g'(x_g)}$ and$\frac{dN}{df_g}$ now. For the assumed specification of M_g:

$$\frac{f_g' \left(x_g\right)}{q_g' \left(x_g\right)} = -\frac{1}{x_g^{\alpha_g - 1}}.$$

From the formula derived in the proof of Proposition 1:

$$N \left(f_g \left(x_g\right)\right) = \left(1 - \left(\frac{\kappa_l \cdot f_g \left(x_g\right)}{f_g \left(x_g\right) - \frac{w}{p}}\right)^{\alpha_l}\right)^{\frac{1}{\alpha_l}}$$

which implies:

$$\frac{dN}{df_g} = \left(\left(\frac{f_g \left(x_g\right) - \frac{w}{p}}{\kappa_l \cdot f_g \left(x_g\right)}\right)^{\alpha_l} - 1\right)^{\frac{1-\alpha_l}{\alpha_l}} \cdot \frac{\frac{w}{p}\kappa_l}{\left(f_g \left(x_g\right) - \frac{w}{p}\right)^2}.$$

Observe that $\frac{dN}{df_g} \to +\infty$ as $x_g \to \underline{x_g}$ and that $\frac{dN}{df_g}$ is decreasing in f_g (and hence it is decreasing in x_g). This all means that the inequality can be written in a tractable form:

$$\beta p\left(-W''\left(m'\left(x_g;m,z\right)\right)\right)\cdot wz\cdot\frac{dN}{df_g}\cdot\frac{1}{x_g^{\alpha_g-1}}>\frac{\kappa_g}{\left(q_g\left(x_g\right)\right)^2}$$

It is easy to see that the RHS is an increasing function of x_g which takes the value of $\frac{\kappa_g}{\left(q_g\left(\underline{x_g}\right)\right)^2}>0$ at $\underline{x_g}$ and converges monotonically to $+\infty$ for $x_g\to+\infty$. The LHS is a decreasing function of x_g. This follows from our analysis of $\frac{dN}{df_g}$, the fact that $\frac{1}{x_g^{\alpha_g-1}}$ is decreasing in x_g (recall that $\alpha_g>1$) and because the policy m' is increasing in disposable income which increases with x_g which coupled with the assumption that $W'''>0$ implies that $-W''$ (which is positive since $W''<0$) decreases in x_g. The LHS is continuous on $\left(\underline{x_g},+\infty\right)$ and its limit is $+\infty$ for $\underline{x_g}$ and 0 for $+\infty$. Thus, curves represented by the LHS and RHS intersect and this intersection is unique (we denote it by $\tilde{x}_g$) which completes the proof. □