The demographic transition and monetary policy in a small open economy *

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Abstract

The existing literature shows that population aging lowers the natural (equilibrium) rate of interest (NRI) and, as a result, can impact on monetary policy. Several questions emerge. First, whether this impact is substantial or not - the existing findings are mixed. Second, how the results depend on the closed vs. open economy assumption, in particular what happens in a small open economy when the world faces a demographic change as well. Third, what happens if the central bank learns the decline of the NRI only slowly over time. Fourth, whether the declining NRI generates problems related to the zero lower bound (ZLB). We show that given the prospective demographics of our country under investigation (Poland), the decline in interest rates is substantial - almost 2 percentage points, albeit spread over a period of 40 years. The declining interest rate abroad spills over, however, to a small extent - domestic developments are key. If the central bank is slow in learning the declining NRI a long period of inflation below the target is likely. In this case the ZLB probability increases sharply, even in the near horizon.

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1 Introduction

Many economies, developed and developing alike, experience (or will soon begin to experience) a substantial demographic transition. Increasing longevity and sub-replacement fertility rates translate into ageing of societies, with the speed of this process varying over countries. Ageing affects aggregate output, pension system sustainability, structure and volume of fiscal expenditures, housing markets, and many other issues. These developments have also become of interest to central bankers and monetary economists, as changes in the demographic structure are expected to affect both the natural (equilibrium) rate of interest (NRI) and channels of monetary policy transmission.

Economic theory predicts that a decrease in the population growth rate should translate into a drop in the NRI through higher capital per worker. Moreover, an increase in longevity lengthens the planning horizon of households, inducing them to save more, and thus exerting further downward pressure on the NRI. Longer living households, dependent largely on accumulated wealth and asset income during their retirement, may prefer lower and more stable inflation rates, influencing politicians' and central bankers' preferences towards the inflation-output tradeoff (see Bullard et al. (2012) and Vlandas (2016), contrast with Juselius and Takats (2015)). Last but not least, for given inflation targets, lower NRIs translate into lower nominal interest rates, leaving less space for conventional monetary policy during slowdowns in economic activity and thus increasing the risk of hitting the zero lower bound (ZLB) constraint.

Our paper addresses several important questions related to the impact of ageing on monetary policy. We are particularly interested in the effects related to the decline in the NRI, taking into account not only its domestic determinants, but also the impact of economic openness and foreign demography. We concentrate on Poland – a small, open economy facing one of the worst demographics in the European Union, and thus a model example for our purposes. As evidenced by Figure 1, Poland is currently undergoing a rapid drop in the number of people entering the working-age period and future fertility rates are projected to remain persistently low. Moreover, mortality rates are consistently falling and the probability of reaching the retirement age is expected to increase from 75% in 2000 to almost 95% around 2100 (Figure 2). These two forces reinforce each other in leading to a rapid increase in the old-age dependency ratio (Figure 3), which is projected to reach 80% by 2060. Similar processes, though less severe in their scale, affect the entire European Union, and the euro area in particular.

We ask the following questions: (i) What is the quantitative impact of ageing on the economy, and in particular on the NRI? (ii) Should the monetary authorities be concerned about the scale and speed of the decline in the NRI? In particular, what happens if the central bank observes it only with a lag? (iii) How does population ageing affect the standard stabilization trade-offs? (iv) How do the demographic processes affect the probability of hitting the ZLB on the nominal interest rates? (v) What is the role of economic openness, and in particular the demographic transition abroad?

The impact of demographics on monetary policy has been tackled only in a limited number of papers to date. Kara and von Thadden (2016) calibrate a Blanchard-Yaari overlapping generations model to the euro area and project a decrease of the natural interest rate for the euro area from 3.9% in 2008 to 3.0% in 2030. They conclude that such adjustments are not important within the horizon that is relevant for monetary policy and do not call for an adjustment in its conduct, consistent with the outlook of some central bankers, e.g. Bean (2004). Carvalho et al. (2016) calibrate a similar model to the average of several developed countries and simulate a more significant decline of the equilibrium interest rate (1.5 percentage points between 1990 and 2014). In contrast to Kara and von Thadden (2016), they conclude that low and declining real interest rates carry important challenges for the monetary authorities.

Regarding open economy aspects, Boersch-Supan et al. (2006) underscore the importance of considering international linkages when analyzing the impact of ageing, and show that the demographic transition may induce substantial cross-country capital flows. Krueger and Ludwig (2007) consider a similar multi-country setup to document spillovers from foreign ageing and extend it with individual idiosyncratic risk to provide firmer welfare analysis. They predict large transitional changes in developed countries' current account balances and a drop in the world rate of return by 1 percentage point between 2000 and 2050. However, neither of these two papers deals explicitly with the consequences of ageing for monetary policy.

Another strand of literature signalizes that population ageing may affect the monetary policy environment and effectiveness. Gagnon et al. (2016) argue that the demography of the US is virtually the sole culprit of the recent permanent decline in real GDP growth, rate of aggregate investment and safe asset yields, suggesting that this situation is the "new normal". Wong (2016) uses household-level data to show that older people are less responsive to interest rate shocks, and that the credit channel loses importance as societies age. Imam (2015) argues that the effectiveness of monetary policy transmission weakens in older societies, with exception of the wealth and expectation channels which gain in importance. This analysis is based on a dynamic panel model and shows that indeed demographic change is associated with decreased monetary policy effectiveness.

Our paper offers a number of contributions to the current state of knowledge. First, while most of the literature models ageing by employing simplifying Blanchard-Yaari type assumptions, we opt for modelling the demographic structure in full detail. In this respect, our model draws on the full-scale life-cycle framework pioneered by Auerbach and Kotlikoff (1987). To be able to take into account demographic processes abroad, we consider an open economy setup. Since our focus is on monetary policy implications of ageing, we include the key real and nominal rigidities identified in the New Keynesian literature. This richer structure is key to deliver reliable quantitative answers, also to questions that have not been addressed in the literature before. In particular, to our knowledge, we are the first to quantitatively relate demographics to the ZLB problem and consider the impact of imperfect central bank knowledge in this context.

We offer the following answers to the questions enumerated above. First of all, we find that the impact of ageing on the economy can be substantial from the monetary policy perspective. In particular, given the currently available demographic projections, the equilibrium interest rate in Poland is projected to decline by 1.8 percenage points between 2010 and 2050. Observing the declining NRI by the central bank in real time makes the process relatively harmless for price stability. However, if the bank learns about the impact of demographic processes on the NRI only slowly over time, a prolonged period of below-target inflation follows. This deflationary bias may be sizable and amount to 0.6-1.1 percentage points during the transition process. According to our simulations, demographic developments are expected to increase the ZLB probability in Poland only moderately when the monetary authority observes the decline in the NRI. However, this risk becomes significantly higher under the learning scenario. Finally, we show that international financial flows can reduce the impact of ageing on monetary policy in a small open economy only to a limited extent, even if demographic developments abroad are less dramatic.

The paper is organized as follows. The next section exposits the model used in our analysis. Section 3 documents the construction of demographic input data and our calibration strategy. In Section 4 we discuss our simulation results, in particular pertaining to the natural interest rate, the zero lower bound, the role of central bank learning, and the extent of open-economy spillovers. Section 5 concludes.

2 Model

We construct a small open economy model with overlapping generations as well as real and nominal frictions to analyze the implications of demographic transition for monetary policy. The model economy is populated by households facing age- and time-dependent mortality risk, five types of firms, and a monetary policy authority. The model also has a closed economy variant, where we shut down international trade and cross-border financial flows.

2.1 Households

2.1.1 Optimization problem

Each household consists of a single agent who appears in our model at age 20 and is assigned age index $j = 1$. Agents can live up to 99 years ($j = J = 80$), at each year subject to age- and time-dependent mortality risk $\omega_{j,t}$. Hence, at each time period the model economy is populated by 80 cohorts of overlapping generations, with the size of cohort j denoted by $N_{j,t}$.

A j-aged household maximizes her expected remaining lifetime utility that depends on consumption $c_{j,t}$ and hours worked $h_{j,t}$ according to

$$U_{j,t} = \mathbb{E}_t \sum_{i=0}^{J-j} \beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \exp\{\varepsilon_{u,t}\} \left[\ln c_{j+i,t+i} - \phi_{j+i} \frac{h_{j+i,t+i}^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where $\varepsilon_{u,t}$ is a preference shock, β denotes the discount factor, the ratio $N_{j+i,t+i}/N_{j,t} \equiv (1-\omega_{j,t})...(1-\omega_{j+i-1,t+i-1})$ represents the probability of surviving for at least i more years, ϕ_j is the age-dependent labor disutility parameter and φ is the inverse of the Frisch elasticity of labor supply.

All households face the following budget constraint

$$P_t c_{j,t} + A_{j,t} = W_t z_j h_{j,t} + R_t^a A_{j-1,t-1} + Beq_t \tag{2}$$

where P_t denotes the aggregate price level, $A_{j,t}$ stands for the nominal stock of assets that are managed by investment funds and yield the gross nominal rate of return R_t^a, W_t is the nominal wage per effective hour, while z_j represents age-specific labor productivity. Our model features exogenous retirement upon reaching age 64 ($j = JR = 45$) and hence we set $z_j = 0$ for $j \geq JR$. Finally, since most agents die before reaching their maximum age, they leave unintentional bequests, which are redistributed equally across all living agents in form of lump-sum transfers Beq_t.

2.1.2 Demography and aggregation

Demographic processes are governed by changes in the size of initial young cohorts $N_{1,t}$ and mortality risk $\omega_{j,t}$, both of which are assumed to be exogenous. The total number of agents alive N_t and the population growth rate n_t are given by

$$N_t = \sum_{j=1}^{J} N_{j,t} \quad \text{and} \quad n_t = \frac{N_t}{N_{t-1}} - 1 \tag{3}$$

where the number of agents evolves according to

$$N_{j,t} = (1 - \omega_{j-1,t-1})N_{j-1,t-1} \tag{4}$$

To better capture the impact of expected demographic changes, we allow population growth in the steady state to differ from zero. As then the number of agents within each cohort becomes nonstationary, it is useful to define the size of cohorts relative to that of the youngest one

$$N_{j,t}^{rel} = \frac{N_{j,t}}{N_{1,t}} \tag{5}$$

and the growth rate of initial young $n_{1,t}$

$$n_{1,t} = \frac{N_{1,t}}{N_{1,t-1}} - 1 \tag{6}$$

This allows us to rewrite equations 4 and 3 in relative terms

$$N_t^{rel} = \sum_{j=1}^{J} N_{j,t}^{rel} \quad \text{and} \quad n_t = \frac{N_t^{rel}}{N_{t-1}^{rel}}(1 + n_{1,t}) - 1 \tag{7}$$

$$N_{j,t}^{rel} = \frac{(1 - \omega_{j-1,t-1})N_{j-1,t-1}^{rel}}{1 + n_{1,t}} \tag{8}$$

Then the allocations of all households can be aggregated to the following per capita variables

$$c_t = \sum_{j=1}^{J} \frac{N_{j,t}c_{j,t}}{N_t} = \sum_{j=1}^{J} \frac{N_{j,t}^{rel}c_{j,t}}{N_t^{rel}} \tag{9}$$

$$h_t = \sum_{j=1}^{J} \frac{N_{j,t}z_j h_{j,t}}{N_t} = \sum_{j=1}^{J} \frac{N_{j,t}^{rel}z_j h_{j,t}}{N_t^{rel}} \tag{10}$$

$$A_t = \sum_{j=1}^{J} \frac{N_{j,t}A_{j,t}}{N_{t+1}} = \sum_{j=1}^{J} \frac{N_{j,t}^{rel}A_{j,t}}{N_{t+1}^{rel}(1 + n_{1,t+1})} \tag{11}$$

$$Beq_t = \sum_{j=1}^{J} \frac{(N_{j-1,t-1} - N_{j,t})R_t^a A_{j-1,t-1}}{N_t} = \sum_{j=1}^{J} \frac{[N_{j-1,t-1}^{rel} - (1 + n_{1,t})N_{j,t}^{rel}]R_t^a A_{j-1,t-1}}{(1 + n_{1,t})N_t^{rel}} \tag{12}$$

2.2 Firms

There are five types of firms in the economy – investment funds, final goods producers, capital producers, intermediate goods producers and importers. The first three groups are perfectly competitive while intermediate goods producers and importers operate in a monopolistically competitive environment. All firms are risk-neutral, i.e. they maximize the expected present value of future profits, discounting them using nominal interest rates.

2.2.1 Investment funds

Investment funds use households' savings to buy and manage a portfolio of assets, transferring every period the earned gross return back to households. The portfolio consists of domestic and foreign bonds, domestic physical capital and shares of intermediate goods producers. Investment funds maximize the expected present value of gross return

$$\mathbb{E}_t \frac{1}{R_t} \left[\begin{array}{c} [R^k_{t+1} + (1-\delta)\, Q_{t+1}]k_t + R_t B_t + S_{t+1}\Gamma_t R^*_t B^*_t \\ + \int_0^1 \left[(1+n_{t+1})P^d_{t+1}(i) + F_{t+1}(i)\right] d_t(i)\, \mathrm{d}i \end{array} \right] \tag{13}$$

where R_t and R^*_t denote the gross nominal interest rates on, respectively, domestic bonds B_t and foreign bonds B^*_t, R^k_{t+1} is the nominal rental rate on capital while Q_t is the nominal price of a unit of capital k_t, which depreciates at rate δ. The nominal exchange rate is denoted by S_t and Γ_t is the risk premium term defined as follows

$$\Gamma_t = \gamma \left(\exp\left\{ \frac{S_t B^*_t}{P_{H,t}gdp_t} \right\} - 1 \right) + \exp\{\varepsilon_{\Gamma,t}\} \tag{14}$$

where gdp_t denotes real aggregate output to be defined later, $P_{H,t}$ is the producer price index for domestic production and $\varepsilon_{\Gamma,t}$ is a risk premium (exchange rate) shock. γ is a risk premium parameter and Finally, $d_t(i)$ stands for the number of shares issued by intermediate goods producing firm i that are traded at price $P^d_t(i)$ and yield dividends $F_t(i)$.[1]

The balance sheet of investment funds can be written as

$$A_t = Q_t k_t + B_t + S_t B^*_t + \int_0^1 P^d_t(i)\, d_t(i)\, \mathrm{d}i \tag{15}$$

Since we assume that all revenue from asset management is transferred back to households, the ex-post rate of return on assets is given by

$$\begin{aligned} R^a_t A_{t-1} = {} & [R^k_t + (1-\delta)\, Q_t]k_{t-1} + R_{t-1}B_{t-1} + S_t\Gamma_{t-1}R^*_{t-1}B^*_{t-1} \\ & + \int_0^1 \left[(1+n_t)P^d_t(i) + F_t(i)\right] d_{t-1}(i)\, \mathrm{d}i \end{aligned} \tag{16}$$

[1] We multiply the proceeds from selling shares by the population growth rate to ensure that the mass of firms is linked to the mass of households. This captures the fact that the number of firms in the economy depends on the size of population.

2.2.2 Final goods producers

Final goods producers serving the domestic market purchase domestic and foreign composites of goods, and produce a homogenous final good according to the following CES aggregator

$$y_t = \left[\eta^{\frac{1}{\phi}} y_{H,t}^{\frac{\phi-1}{\phi}} + (1-\eta)^{\frac{1}{\phi}} y_{F,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{17}$$

where η reflects the home bias and ϕ is the elasticity of substitution between home-made and imported composites, which are in turn made of intermediate inputs according to

$$y_{H,t} = \left[\int_0^1 y_{H,t}(i)^{\frac{1}{\mu}} \, \mathrm{d}i \right]^{\mu} \tag{18}$$

$$y_{F,t} = \left[\int_0^1 y_{F,t}(i)^{\frac{1}{\mu}} \, \mathrm{d}i \right]^{\mu} \tag{19}$$

where μ is the gross markup that depends on the elasticity of substitution between intermediate varieties and the associated price indices are

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{\frac{1}{1-\mu}} \, \mathrm{d}i \right]^{1-\mu} \tag{20}$$

$$P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{\frac{1}{1-\mu}} \, \mathrm{d}i \right]^{1-\mu} \tag{21}$$

Similarly, producers of final export goods aggregate domestically produced intermediate inputs according to

$$y_{H,t}^* = \left[\int_0^1 y_{H,t}^*(i)^{\frac{1}{\mu}} \, \mathrm{d}i \right]^{\mu} \tag{22}$$

and sell them abroad at a foreign currency denominated price $P_{H,t}^*$ defined as

$$P_{H,t}^* = \left[\int_0^1 P_{H,t}^*(i)^{\frac{1}{1-\mu}} \, \mathrm{d}i \right]^{1-\mu} \tag{23}$$

facing the following demand function

$$y_{H,t}^* = \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\phi^*} y_t^* \tag{24}$$

where y_t^* is foreign output, P_t^* denotes the foreign aggregate price level and ϕ^* is the price elasticity of export demand.

2.2.3 Capital producers

Capital producers buy undepreciated capital and combine it with investment goods i_t subject to investment adjustment costs. The thus created new capital is resold during the same period. The aggregate law of motion for capital is then

$$(1+n_{t+1})k_t = (1-\delta)\,k_{t-1} + \left[1 - S_k\left(\frac{i_t}{i_{t-1}}\right)\right] i_t \tag{25}$$

where the investment adjustment cost function is of the following form

$$S_k\left(\frac{i_t}{i_{t-1}}\right) = \frac{S_1}{2}\left[\frac{i_t}{i_{t-1}} - (1+n_t)\right]^2 \tag{26}$$

for $S_1 \geq 0$, so that the cost is zero in the steady state.

2.2.4 Importers

Importers of foreign intermediate varieties purchase each of them at price P_t^* expressed in foreign currency and then resell to domestic final goods producers at prices denominated in local currency. They face the demand for their products consistent with aggregation 19 and are subject to the Calvo friction, so that every period each importer faces a constant probability θ_F of not being able to reoptimize, in which case the price is fully indexed to domestic steady state inflation. We assume that importers are owned by foreign agents so their profits are transferred abroad.

2.2.5 Intermediate goods producers

Intermediate goods producers supply domestic and foreign markets with output produced according to the Cobb-Douglas production function

$$y_{H,t}\,(i) + y_{H,t}^*\,(i) = \varepsilon_{z,t} k_t\,(i)^{\alpha}\,h_t\,(i)^{1-\alpha} \tag{27}$$

where $\varepsilon_{z,t}$ is a productivity shock. They face the schedules implied by aggregation 18 and 22, and set their prices separately for the domestic and foreign markets in the currency of destination country. Their pricing decisions are subject to the Calvo friction, with θ_H and θ_H^* representing the probability of not receiving the reoptimization signal, in which case prices are fully indexed to local steady state inflation.

2.3 Monetary authority

The monetary authority sets the nominal interest rate according to the standard Taylor-like rule that takes into account the zero lower bound constraint

$$R_t = \max\left\{1, R_{t-1}^{\gamma_R}\left[\tilde{R}_t^e\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi}\left(\frac{\frac{gdp_t}{gdp_{t-1}}}{\frac{\tilde{gdp^e}_t}{\tilde{gdp^e}_{t-1}}}\right)^{\gamma_y}\right]^{1-\gamma_R}\exp\{\varepsilon_{R,t}\}\right\} \tag{28}$$

where gdp_t is aggregate output that we define below, $\pi_t \equiv P_t/P_{t-1}$ is the gross rate of inflation, π is the inflation target and $\varepsilon_{R,t}$ is a monetary policy shock. The coefficients γ_R, γ_π and γ_y control, respectively, the degree of interest rate smoothing, response to deviations of inflation from the target and response to deviation of output growth from its potential.

The variables $\tilde{R}_t^e$ and $\tilde{gdp}_t^e$ describe the central bank perceptions of the natural nominal interest rate $\tilde{R}_t \equiv \pi\tilde{r}_t$ and natural output $\tilde{gdp}_t$, respectively, where $\tilde{r}_t$ denotes the natural real interest rate. These natural quantities are defined as hypothetical values of the relevant variables that would be observed under fully flexible prices (i.e. $\theta_H = \theta_H^* = \theta_F = 0$) and absent stochastic shocks, but with demographic changes taken into account. Unless indicated otherwise, the perceptions of the monetary authority are assumed to be consistent with current economic developments, i.e. $\tilde{R}_t^e = \tilde{R}_t$ and $\tilde{gdp}_t^e = \tilde{gdp}_t$. Alternatively, we assume that these perceptions are linked to the actual values with a constant gain learning process as in Evans and Honkapohja (2001)

$$\tilde{R}_t^e = \tilde{R}_{t-1}^e + \lambda(\pi\tilde{r}_{t-1} - \tilde{R}_{t-1}^e) \tag{29}$$

$$\tilde{gdp}_t^e = \tilde{gdp}_{t-1}^e + \lambda(\tilde{gdp}_{t-1} - \tilde{gdp}_{t-1}^e) \tag{30}$$

so that the central bank observes the true natural interest rate and output only with a lag, and updates their current guess with a fraction λ of the previous forecast errors.[2] This way of formulating the feedback rule ensures the long-run consistency of the equilibrium with central bank targets, but also allows us to model imperfect knowledge of the emonetary authority.

2.4 Market clearing conditions

The model is closed with a standard set of market clearing conditions. Equilibrium on the final goods market implies

$$y_t = c_t + i_t \tag{31}$$

[2]The resulting estimate of the unobserved quantity is equivalent to an exponentially weighted average of its all past values.

The market clearing conditions for capital can be written as

$$\int_0^1 k_t\,(i) = k_{t-1} \tag{32}$$

while that for labor is

$$\int_0^1 h_t\,(i) = h_t \tag{33}$$

This allows us to write the aggregate production function as

$$gdp_t \equiv y_{H,t}\Delta_{H.t} + y^*_{H,t}\Delta^*_{H,t} = \varepsilon_{z,t}k^{\alpha}_{t-1}h^{1-\alpha}_t \tag{34}$$

where $\Delta_{H,t} \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{\frac{\mu}{1-\mu}} \mathrm{d}i$ and $\Delta^*_{H,t} \equiv \int_0^1 \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{\frac{\mu}{1-\mu}} \mathrm{d}i$ measure the price dispersion in domestic and export markets.

Since domestic bonds are traded only between (identical) investment funds, we have

$$B_t = 0 \tag{35}$$

Without loss of generality, the number of shares issued by each intermediate goods producing firms can be normalized to unity, which gives

$$d_t(i) = 1 \tag{36}$$

Finally, foreign bond holdings, i.e. the economy's net foreign assets position, evolves according to

$$(1 + n_{t+1})S_tB^*_t = S_t\Gamma_{t-1}R^*_{t-1}B^*_{t-1} + S_tP^*_{H,t}y^*_{H,t} - P_{F,t}y_{F,t} \tag{37}$$

2.5 Exogenous shocks

The model economy is driven by the following exogenous variables. Demographic processes are characterized by the the growth rate of initial young $n_{1,t}$ and age-specific mortality risk $\omega_{j,t}$, all of which are treated as deterministic, i.e. known to all optimizing agents. Additionally, the economy is hit by stochastic shocks to productivity $\varepsilon_{z,t}$, household preferences $\varepsilon_{u,t}$, risk in the international financial market $\varepsilon_{\Gamma,t}$, monetary policy $\varepsilon_{m,t}$, as well as foreign output y^*_t, inflation $\pi^*_t$and interest rate R^*_t. The three foreign variables may also have a deterministic component that reflects the consequences of demographic processes abroad, and which we obtain by simulating the closed economy variant of the model sketched out above. These processes are exogenous to the domestic economy due to our small open economy assumption. Stochastic fluctuations in the foreign variables are described by a VAR(1) process. Monetary policy shocks are assumed to be white noise. All other stochastic shocks are modelled as independent AR(1) processes.

3 Calibration and data sources

We calibrate the model such that the home country resembles the Polish economy and the rest of the world is represented by the euro area. The chosen values of structural parameters are reported in Table 1. They are based on the previous literature, complemented with econometric estimates performed outside the model and a moment matching exercise. Some of the parameters are calibrated such that the means of key variables implied by our demographic transition scenario during the years 2003-2012 coincide with actual Polish data covering this period. This targeted 10-year data sample can be considered as one during which the real interest rate in Poland was close to its equilibrium level.[3] Throughout, the time frequency is annual.

We set the discount factor to 0.977, which allows us to match the average real interest rate of 2.1% observed in Poland during the years 2003-2012. Our calibration of the Frisch elasticity of labor supply of 0.25 is consistent with estimates from the microeconomic literature (see e.g. Peterman, 2016). The age profiles of productivity z_j are taken from Kolasa (2016). Since these profiles are expressed in per household and not per hour terms, we adjust the age specific productivity parameters ϕ_j so that working hours in the initial steady state (i.e. before the demographic transition) are constant over the life cycle.

Physical capital is assumed to depreciate at a standard rate of 10% annually. The capital elasticity of output is calibrated at 0.25, which ensures that the investment rate is close to the average values observed in the Polish economy. This parametrization together with a markup of 2% implies that the labor income share is about 73%, which is close to the estimates in the literature . The investment adjustment cost function curvature is set to 4 and taken from an estimated DSGE model for Poland documented in Brzoza-Brzezina et al. (2016). Since our model does not feature nominal wage rigidities, we calibrate the Calvo probability faced by all intermediate goods producing firms at a somewhat high value of 0.9.

The home bias parameter in the final goods basket is set to a value that insures the share of imports in domestic demand of around 25%, which corresponds to the mean observed in the data, corrected for the import content of exports estimated for Poland by the OECD at 30%. The elasticity of substitution between home and foreign goods is is calibrated at a standard value of 1.5. The elasticity of risk premium on international financial markets is set to 0.013, which helps match the average international investment position of -55% observed

[3] Over this period inflation fluctuated around the central bank target of 2.5%, suggestng an approximately neutral stance of monetary policy. Before 2003 Poland underwent a long-lasting disinflation, and after 2012 a long period of very low (even negative) inflation.

in the Polish economy over the period 2003-2012.

The parametrization of the monetary policy feedback rule is based on econometric estimates using an extended sample of detrended quarterly data covering the period 1999-2016 (i.e. since introduction of inflation targeting in Poland), converted to annual frequency. Whenever we allow the central bank to only gradually learn about changes in the natural interest rate, we calibrate the gain parameter at 0.08. This value follows the empirical literature documenting the observed speed of learning (Branch and Evans, 2006; Malmendier and Nagel, 2016; Milani, 2011) and implies an annual learning rate of 8%.

In our simulations we treat Poland as a small open economy that is affected by demographic processes and usual business cycle fluctuations in the euro area, manifesting themselves from the Polish perspective by exogenous movements in foreign output, inflation and the nominal interest rate. The response of these three variables to a demographic transition are generated using the closed economy version of the model described in the previous section, with some of the parameters recalibrated to fit the euro area data, see the bottom panel of Table 1. This concerns the parametrization of the monetary policy rule and the Calvo probability, which are taken from Christoffel et al. (2008), as well as the discount factor, that we reset to 0.992 to match the average real interest rate of 1.2%, observed in the euro are over the period 1999-2008, i.e. since its creation until the financial crisis.[4] For lack of appropriate estimates of age-specific productivity for the euro area, we use the calculations for the US from Gourinchas and Parker (2002) as the existing evidence points at relatively small differences in the life cycle income profiles between developed economies (OECD, 1998).

Apart from the demographic transition scenario, which is purely deterministic, we also use our model in a stochastic context. For this purpose fluctuations in foreign output, inflation and the interest rate are modeled as a first-order VAR, whose parameters we estimate using detrended estimates of the euro area aggregates from the ECB Area Wide Model database and covering the period 1970-2012. Table 2 reports the properties of shocks driving the model small open economy. Four of them, namely the monetary shock as well as the three shocks describing foreign variables, are estimated outside the model as described above. The remaining three are determined in a moment matching exercise, in which we use detrended data on Polish real GDP, HICP inflation, the short-term nominal interest rate and the CPI-based real effective exchange rate over the period 1999-2016. More specifically, we minimize the distance between the model-based standard deviations, first-order autocorrelation and correlation with output and their respective data counterparts. All model-based moments are calculated using the first-order approximation of the policy functions around the point defined as the mean of the state variables in our demographic transition scenario over the

[4]As in the case of Poland, this is the longest period when the interest rate can be considered close to its equilibrium value. In this period inflation was relatively stable. After 2008 the euro area faced a prolonged crisis that pushed interest rates down for cyclical rather than strructural reasons.

period 2003-2012. We use the same weights for all matched moments, except the volatility and inertia of the nominal interest rate, for which we assign much higher weights so that the match is exact. It is important to achieve a perfect match for this variable as one of our goals is to evaluate the impact of demographic transition on the probability of hitting the ZLB. As it can be seen from Table 3, the achieved fit is very good also for other moments.

Our demographic scenarios use the past and projected estimates of the fertility and mortality rates for Poland and the euro area. We base on the historical data provided by the Eurostat and the Europop2013 projection, which encompasses years 2013-2080. Population data and age-specific death rates prior to 2015 come from the demo_pjan and demo_mlifetable series, respectively. For years where mortality rates were not documented for the oldest cohorts, we employ exponential extrapolation. Future mortality rates are taken from the main projection scenario proj_13naasmr while population projections use the no-migration variant proj_13npzms for internal consistency.

While the construction of the Polish demographic input data is straightforward, there is no dataset ready for use in the case of the euro area. We opted to approximate the demographic situation of the euro area with the demographic situation of the entire EU-28, for which the aggregate projected no-migration variant estimates are available. The projected cohort sizes were used to obtain expected aggregate mortality rates. Historical mortality rates are available for the EU-28 since 2002. For years 1986-2001 we approximate EU-28 mortality rates with French mortality rates, as EU-28 and France have had similar mortality rates in the 2000s. For the population data, we use directly the cohort sizes for years 2001-2014. For years 1995-2000 we rescale proportionately the cohort sizes reported for EU-27 (not accounting for Croatia). Population prior to 1995 is constructed using French mortality rates.

To ensure that the model predictions for the early 2000s are not affected by starting in an artificial steady state in the 1990s, we begin simulating our model in the year 1900. To that end we construct artificial population data for the years 1900-1989 in the case of Poland and years 1900-1994 for the EU-28 and backcast the sizes of historical 20-year old cohorts while holding the historical mortality rates at the earliest observed date (1990 for Poland and 1986 for France) to accurately match the existing population structure from 1990 onwards. This is important as both for the entire Europe and for Poland the consequences of the WW2 and post-war demographic boom are still visible in the age pyramids.

Finally, the sizes of 20-year olds' cohorts and mortality rates were smoothed using the Hodrick-Prescott filter (with smoothing parameters 6.25 and 10000, respectively) to avoid jumps in the demographic input data produced by data revisions and splicing historical data and projections. At the end of our projection horizon we assume that mortality rates stabilize while the rate of change of 20-year olds stays at the level projected for 2080.

4 Effects of demographic change

In this section we seek to answer several important questions about the consequences of the demographic transition for monetary policy. As it is well know from the literature, population ageing can have a sizeable impact on savings and the equilibrium interest rate. We start by documenting this effect for the euro area and Poland. Then we move to analysing various consequences for monetary policy: we compare the impact of the demographic change on inflation, under various assumptions about the speed with which the central bank notices the decline of the NRI, and we check whether the evolving demographic structure has an important impact on the trade-offs faced by the central bank. Third, we asses how much the declining NRI raises the probability of hitting the zero lower bound on the nominal interest rates. This is done both for the case when the central bank observes the declining NRI in real time and when it gradually learns about its change. Fourth, we explore the open-economy nexus. In particular we check how our main findings (in particular about the real interest rate and foreign debt) depend on the demographic transition in the euro area.

4.1 Impact of demographic transition

We begin with describing the impact that the demographic transition exerts on main macroeconomic variables. To this end we run a deterministic simulation,[5] assuming that the demographic processes in the euro area and Poland, described in Section 3, are known to all agents.

Let us begin with the euro area. The upper-left panel of Figure 6 presents the dependency ratio in the EA. The economy faces a sharp increase of this ratio, resulting from lower birth rates and higher life expectancy. Workers, facing a longer expected time in retirement, increase savings. This results in higher asset holdings and lowers the real interest rate. The decline is substantial, though spread over time. Between 2000 and 2030 the interest rate declines approximately by 1 percentage point. Other developments worth mentioning are declining labor supply, increasing real wage (due to the lower supply) and, as a consequence, a higher capital-labor ratio chosen by firms. With less labor the economy produces less goods, so that ultimately GDP per capita and capital per capita are lower after the transition is over.

Given qualitatively similar demographic developments, it should not come as a surprise that the picture for Poland is similar. Two important differences should however be mentioned. First, the demography in Poland is even worse than in the EA. As a consequence the adjustment process is more violent. Second, Poland is a small open economy. This

[5] For numerical reasons we work with two models, closed economy for the euro area and SOE for Poland. Thus, technically speaking, we run two simulations. First we simulate the euro area. Then, we take its GDP, inflation and interest rate as exogenous variables in the simulation for Poland.

means that from its perspective savings can be imported or exported abroad. This is exactly what happens. A substantial part of the savings generated by the demographic transition is invested abroad. As a result, the country's foreign debt-to-GDP ratio declines significantly, by over 20 percentage points. Nevertheless, the real interest rate decreases by more than in the euro area, from 2.3% in 2000 to below 0.5% in the 2040s. Again, this reduction is spread over time, but, as we show in the next section, can still have important implications for monetary policy.

4.2 Consequences for monetary policy

W see two potentially important consequences of the demographic transition for monetary policy. First, the central bank might notice the declining NRI only with a lag. Second, the dynamic reactions of the economy to monetary policy could change over time. In what follows we take a deeper look at both of these issues.

Learning about the NRI

In the past equilibrium interest rates have been frequently assumed constant (or at least stationary) over time. For instance, until the mid-2000s many economists and central bankers placed NRIs in the US, UK or euro area in the range of 2-3% (Laubach and Williams, 2003; Holston et al., 2016). With relatively rare exceptions, Taylor rules that were calibrated or estimated for these (and several other developed) countries assumed a constant intercept, and hence implied a constant NRI (see e.g. Taylor (1993); Smets and Wouters (2003) for standard rules and Orphanides and Williams (2002); Trehan and Wu (2007) for exceptions with time-varying NRI). Such an assumption seemed warranted by long periods of real ineterst rates that seemed stationary. Things changed somewhat after the global financial crisis erupted and NRIs apparently declined in most coutries. However, even now it is far from evident whether the declining interest rates were a symptom of long term trends, or of cyclical developments.

Given this experience coupled with the intrinsically unobservable nature of the equilibrium interest rate, it seems plausible to expect that the long-term decline of NRIs, that we expect to be happeninng due to demographic reasons, could be noticed by central banks only with a lag (see Orphanides and Williams (2002) for a discussion). This could clearly lead to overly restrictive monetary policy with all its consequences. In this section we analyze such a scenario.

As was explained in Section 2, our monetary policy rule was designed to account for a declining NRI. In the baseline calibration we assume that the central bank observes the NRI, and a closely related concept of potential output, in real time. Now we compare this result to the case when the monetary authority follows a learning proces, described by equations 29 and 30, so that every period the central bank's guesses of these two unobservable variables is being updated for a fraction λ of the last period forecast error. The resulting expected NRI and potential output are equivalent to exponentially weighted average of all their past values.

Figure 8 documents our findings by comparing the inflation rate under the baseline and learning assumptions. In the baseline scenario inflation fluctuates in the viscinity of the central banks target. The small deviations result from the bank's willingness to stabilize output next to inflation. However, if the central bank fails to timely account for the demographic trends, monetary policy becomes much more restrictive and results in a deflationary bias. Even though the NRI drops on an average rate of only 0.04 percentage point per year, the permanent bias in monetary policy has a substantial impact on inflation. In particular, inflation remains permanently below the target, the gap in the analyzed period reaches 0.7 percentage points on maximum. As the natural rate stabilizes after 2050, the deviations in its perception stop being permanently biased and inflation returns close to target.

Monetary policy trade-offs

From the central bank's point of view not only the declining NRI matters. What is also important is the trade-off faced by monetary authorities. TBA.

4.3 Probability of hitting the ZLB

During the recent decade interest rates in many countries have hit the zero lower bound (ZLB). Something that had looked like a textbook curiosity has become a part of central bank reality. While the affected banks managed to elaborate alternative tools that allowed to (at least partially) overcome the consequences posed by the constraint (e.g. quantitative easing), it seems that they still prefer to use the short-term interest rate as the main policy instrument. Keeping this in mind we decided to check whether the lower NRI raises substantially the probabiliy of hitting the ZLB. If this is the case, central banks might have to consider increasing their inflation targets to compensate for the declining NRI.

We proceed as follows. We run stochastic simulations with productivity, time preference, risk premium and foreign shocks calibrated as described in Section 3 for 100,000 periods. This exercise is repeated in the viscinity of every point on our deterministic path. The exercise is done twice. First, simulations assume that the central bank knows the NRI and potential output. Second, we check what happens if the bank learns about these unobservable variables as described in Section 4.2. Since learning can result in a long period when the economy faces persistently low inflation, we expect the probability of hitting the ZLB to be higher at some points.[6] Then, we approximate the probability of hitting the ZLB by calculating the

[6]Regarding the perceived NRI, the Bank is assumed not to learn the equilibrium rate during the stochastic

frequency of periods during which the nominal interest rate is constrained by this limit.[7]

Our findings are summarized on Figure 9. According to the baseline scenario, the probability of hitting the ZLB in the early 2000s has been relatively low (1-2%). This result is consistent with the findings of Brzoza-Brzezina et al. (2016) and is a consequence of the relatively high NRI and a high inflation target. However, as the equilibrium real rate declines, the probability increases to over 7% in 2050 and finaly stabilizes slightly below 5% in the new steady state. These values can hardly be seen as alarming, in particular when taking into account that by 2050 the ZLB problem may be nonexistent due to technological progress.

What is more interesting and possibly disturbing, are the results from the learning scenario. Not only does the probability increase to much higher levels (almost 15% in the 2040s), but the numbers for the contemporaneous times are quite high. By 2020 the probability of meeting the ZLB exceeds 6%. These results show that a slowly, but permanently declining equilibrium interest rate, if not properly accounted for, can result in a serious deterioration of monetary policy quality.[8]

This finding stipulates our main conclusion for monetary policy. If the central bank is aware of the consequences presented in this paper, the demographic transition is relatively harmless from its point of view. However, if the bank fails to timely account for the declining NRI, monetary policy may become suboptimal to a substantial degree - too contractionary and possibly leading to liquidity traps.[9]

4.4 Impact of foreign demography

Another interesting question is how demographic developments abroad modifiy the adjustment in Poland. In order to take a deeper look into the matter we run two additional experiments and comapre them to our baseline results. First, we switch off the demographic transition in the euro area. Second, we assume that the demographic prospects in the euro area are the same as in Poland. These scenarios can be seen as two extremes, with our baseline placed somewhere in between.

The findings are presented on Figure 10. A small open economy can adjust to variations in the savings-investment balance not only via the interest rate, but also by exporting/ importing capital. This effect could already be observed when analysing the path of foreign debt in Section 4.1 - foreign indebtedness declines by over 20% of GDP in the period under consideration. This means that additional savings are being partially exported (invested) abroad. Clearly, the second adjustment channel dominates the adjustment in our case.

run. We assume the perceived NRI to be constant during each stochastic simulation at the respective level from the deterministic scenario.

[7]The simulations are performed using Dynare OBC (Holden, 2016).

[8]We do not discuss in detail the adverse consequences of hitting the ZLB, this has been done in many studies, see e.g. Gust et al. (2012); Ireland (2011); Neri and Notarpietro (2014)).

[9]A formal, quantitative assesment of policy suboptimality is left for further research.

Without demographic change in the euro area savings from Poland are being exported to an even larger extent, as they flow to where the return is not undermined by demographic developments. As a result, foreign debt declines much more than in the baseline scenario, ultimately the economy even achieves an NFA surplus. If demography in the EA behaves like in Poland, the return on assets declines similarly in both economies and, as a consequence, net international capital flows are limited and Poland's foreign debt is almost constant. As expected, our baseline scenario is placed between the two extremes.

The impact of foreign developments on the interest rate is much less pronounced. As can be expected, the demographic transition in the EA adds to the declining NRI in Poland, but the difference is not large.

5 Conclusions

How does the demographic transition, resulting from lower fertility rates and higher life expectancy, affect monetary policy? While the question has already been tackled in the literature, and there is agreement that equilibrium interest rates will be affected, many issues remain unclear. First, whether the impact is large or small. Second, what happens if the central bank learns about the declining equilibrium interest rate only with a lag. Third, how does the transition affect the probability of hitting the zero lower bound on interest rates. Fourth, what role does economic openness, and in particular the demographic transition abroad, play.

Regarding the first problem, we are confident that our modeling approach, based on an OLG framework carefully calibrated to projected birth and mortality rates is able to deliver more precise simulations than (most of ???) the earlier studies. This, coupled with modeling a country that faces a particularly sharp demograpfic transition (Poland) shows that the effects can be substantial. In particular the equilibrium interest rate declines by 1.8 percenage points between 2010 and 2050.

In principle this decline should not pose a problem for monetary authorities - they should simply adjust interest rates to follow the declining natural rate. Two issues emerge, however. First, the natural rate is not directly observable, and it seems possible that monetary policy learns about its decline only with a lag. Second, a lower natural rate implies, ceteris paribus, a higher probability of hitting the zero lower bound.

We show that both problems are acute. Learning about the declining natural rate can result in a prolonged period of below-target inflation. Our simulations show a deflationary bias during the whole transition process of up to 1 percentage point. As expected, the annual probability of hitting the ZLB also increases, from a relatively benign 1-2% in the early 2000s, to almost 5% after the transition is over. Taking additionally learning into account makes the problem much more pronounced. In this case the probability of hitting the ZLB exceeds

6% already by 2020, and rises to almost 15% in the 2040s.

Last but not least we check how open economy effects play out. To this end we simulate our model under various demographic scenarios for the euro area. The main conclusion is that the role played by foreign demography translates primarily into the small economy's net foreign asset position, and much less into equilibrium interest rates. In the open economy capital flows to where the return is higher. In our baseline scenario demography in Poland worsens more than in the euro area, hence Poland's NFA position improves by over 20%. Were the euro area's demography worsen exactly as in Poland the impact on NFA would vanish as capital would find no reason to flow abroad. As a consequence the domestic equilibrium interest rate would decline even more. However, the last effect would be quantitatively small.

All in all, our main conclusion is that in spite of being spread over a long period, the demographic transition can have a significant impact on the conduct of monetary policy. In particular, if the central bank fails to timely account for the declining NRI policy may become too contractionary and possibly leading to ZLB traps. On the contrarty, if the central bank is aware of the consequences presented in this paper, the demographic transition should ramain relatively harmless from its point of view.

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Tables and figures

Table 1: Calibrated structural parameters

Parameter	Value	Description
β	0.977	Discount factor
φ^{-1}	0.25	Frisch elasticity of labor supply
δ	0.1	Capital depreciation rate
α	0.25	Capital share in output
S_1	4	Investment adjustment cost curvature
μ	1.02	Product markup
θ_H, θ_H^*, θ_F	0.9^4	Calvo probabilities
η	0.785	Home bias in final goods
ϕ	1.5	Elasticity of substitution btw. domestic goods and imports
ϕ^*	1.5	Elasticity of export demand
γ	0.013	Risk premium parameter
π_{ss}	1.025	Inflation target
γ_R	0.82^4	Interest rate smoothing
γ_π	1.67	Reaction to inflation
γ_y	0.33	Reaction to GDP growth
λ	0.08 or 1	Learning parameter
β^*	0.992	Discount factor (EA)
θ^*	0.75^4	Calvo probability (EA)
γ_R^*	0.87^4	Interest rate smoothing (EA)
γ_π^*	1.9	Reaction to inflation (EA)
γ_y^*	0.15	Reaction to GDP growth (EA)
π_{ss}^*	1.02	Inflation target (EA)

Table 2: Calibrated stochastic shocks

Parameter	Value	Description
ρ_z	0.86	Inertia of productivity shocks
ρ_u	0.41	Inertia of preference shocks
ρ_Γ	0.54	Inertia of risk premium shocks
σ_R	0.006	Standard dev. of monetary shocks
σ_{y^*}	0.011	Standard dev. of innovations to foreign output
σ_{π^*}	0.006	Standard dev. of innovations to foreign inflation
σ_{R^*}	0.012	Standard dev. of innovations to foreign interest rate
σ_z	0.008	Standard dev. of innovations to productivity shocks
σ_u	0.051	Standard dev. of innovations to preference shocks
σ_Γ	0.023	Standard dev. of innovations to risk premium shocks
$corr(y^*, \pi^*)$	0.38	Correlation of innovations to foreign output and inflation
$corr(y^*, R^*)$	0.59	Correlation of innovations to foreign output and interest rate
$corr(\pi^*, R^*)$	0.55	Correlation of innovations to foreign inflation and interest rate

Table 3: Matched data moments

Variable	Standard dev.		Autocorrelation		Corr. with GDP	
	Model	Data	Model	Data	Model	Data
GDP	1.77	1.84	0.77	0.68	1.00	1.00
Inflation	1.50	1.77	0.25	0.37	0.39	0.72
Interest rate	1.97	1.97	0.34	0.34	0.40	0.57
Real exchange rate	5.52	5.55	0.36	0.22	0.03	0.31

Figure 1: Population size of 20 year-olds, EU-28 and Poland

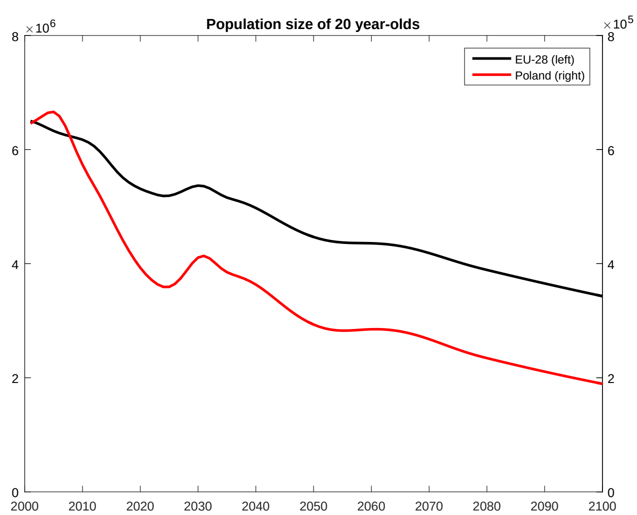


Figure 2: Probability of survival from age 20 to 65, EU-28 and Poland

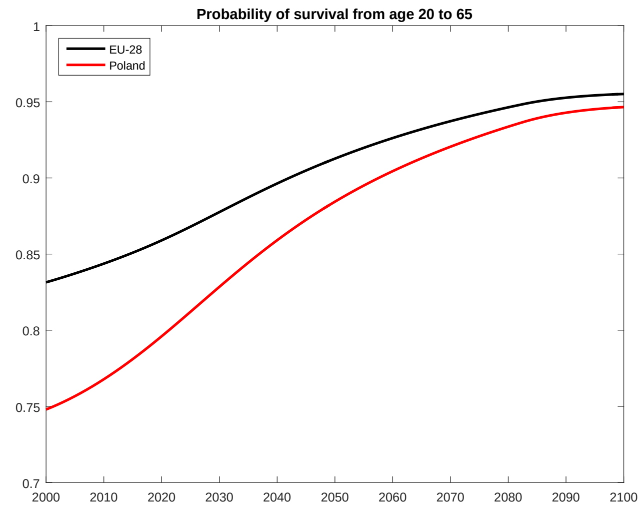


Figure 3: Old-age dependency ratio, EU-28 and Poland

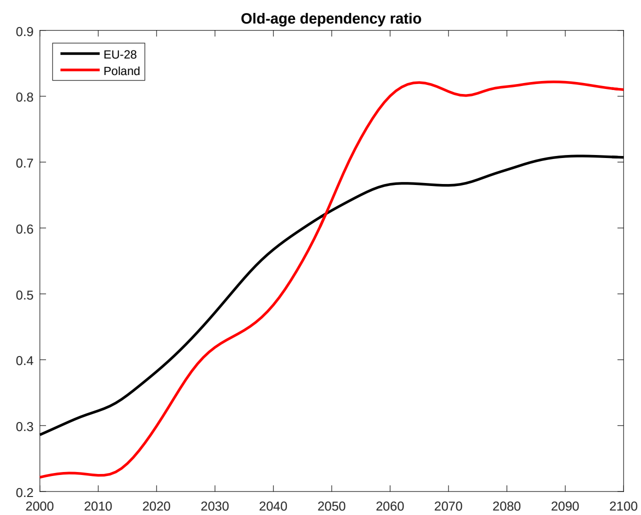


Figure 4: Age-dependent productivity profiles

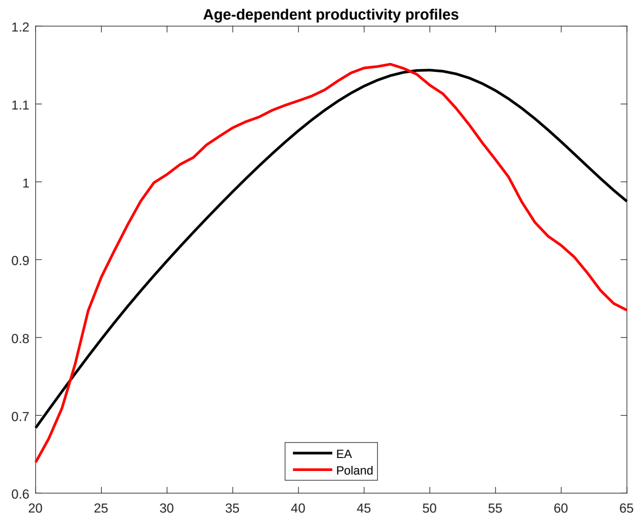


Figure 5: Age-dependent labor disutility

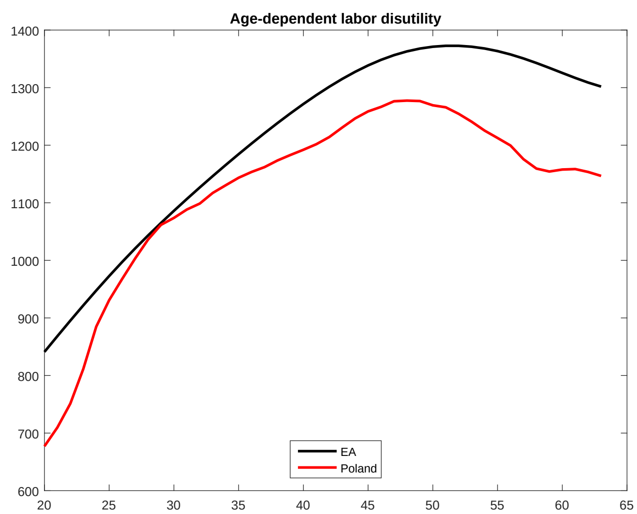


Figure 6: Impact of demographic transition on the euro area

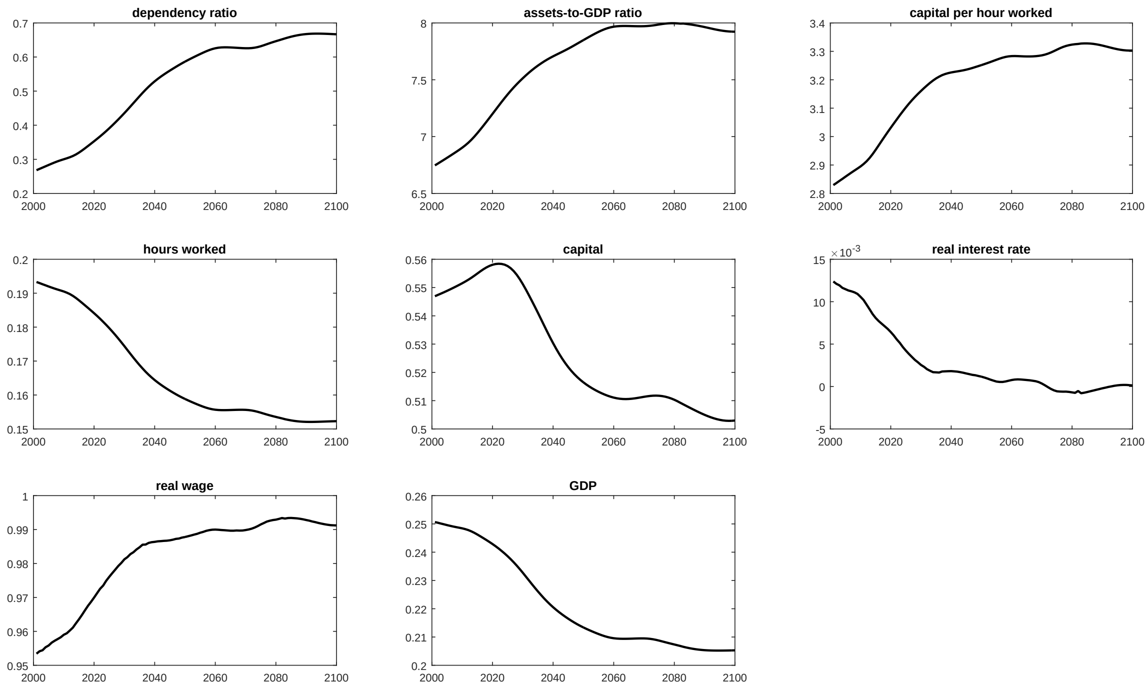


Figure 7: Impact of demographic transition on Poland

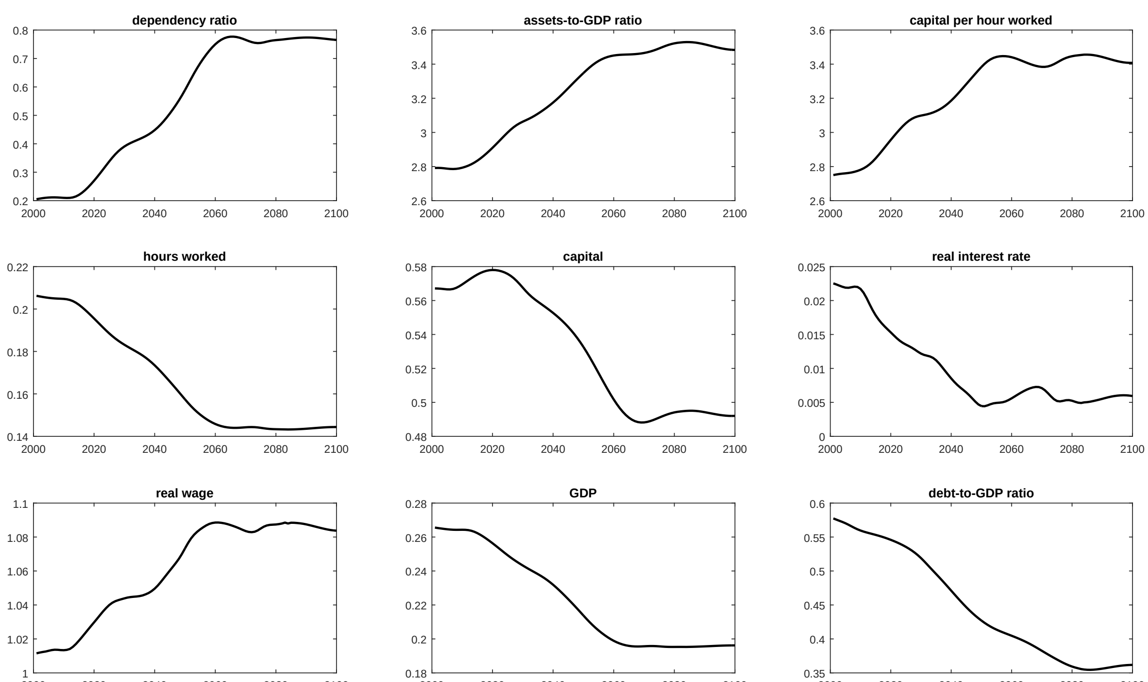
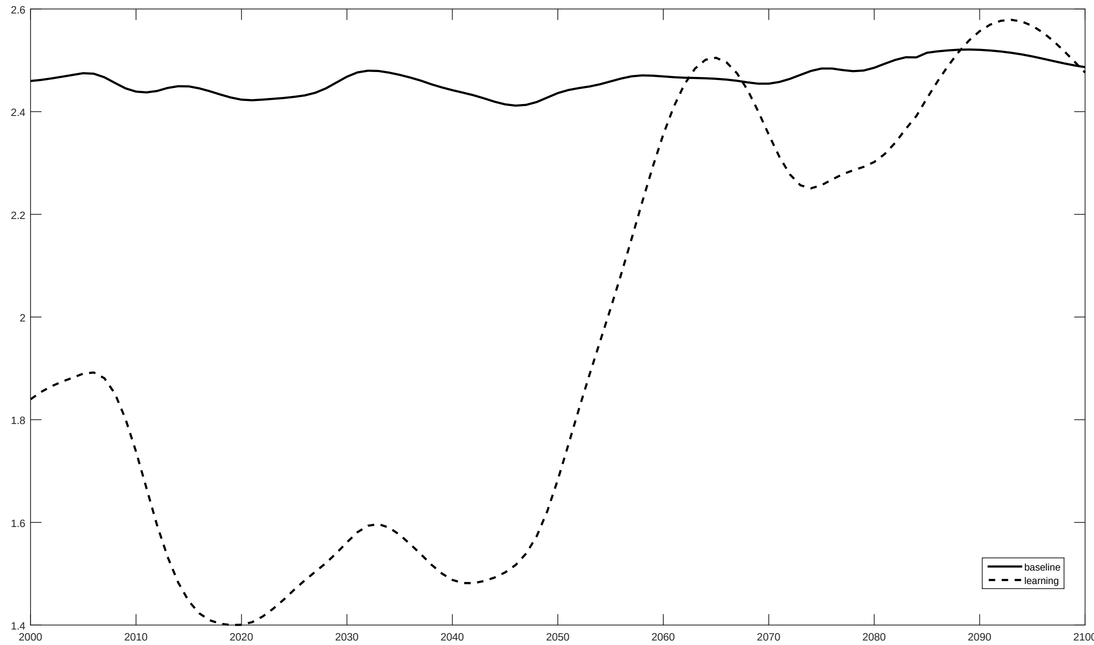
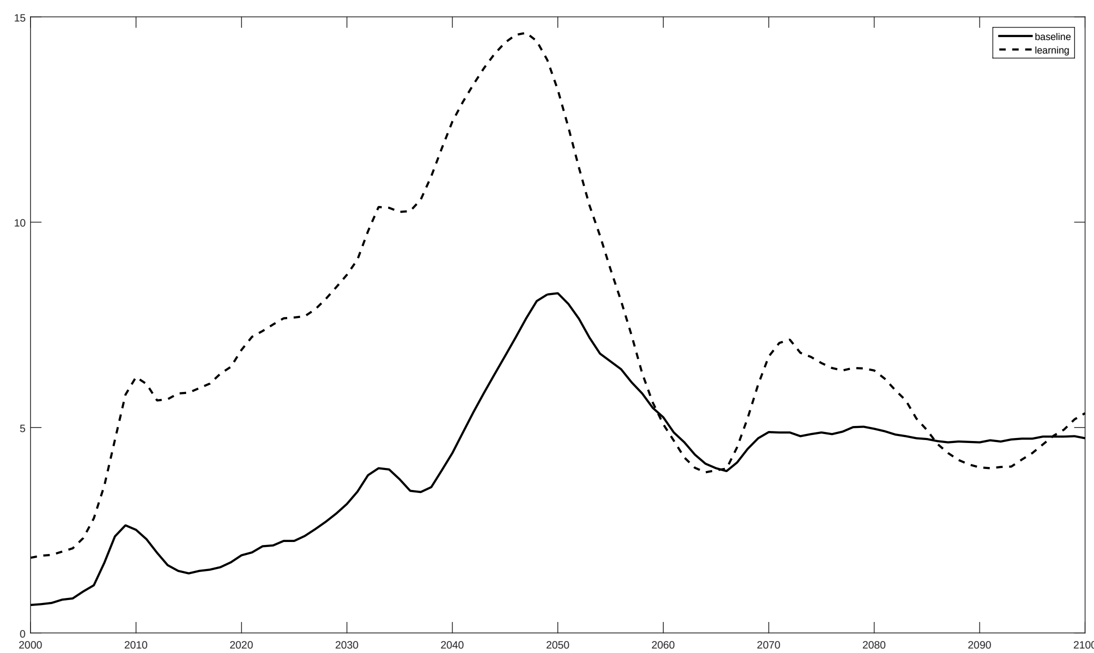


Figure 8: Inflation bias under learning about NRI



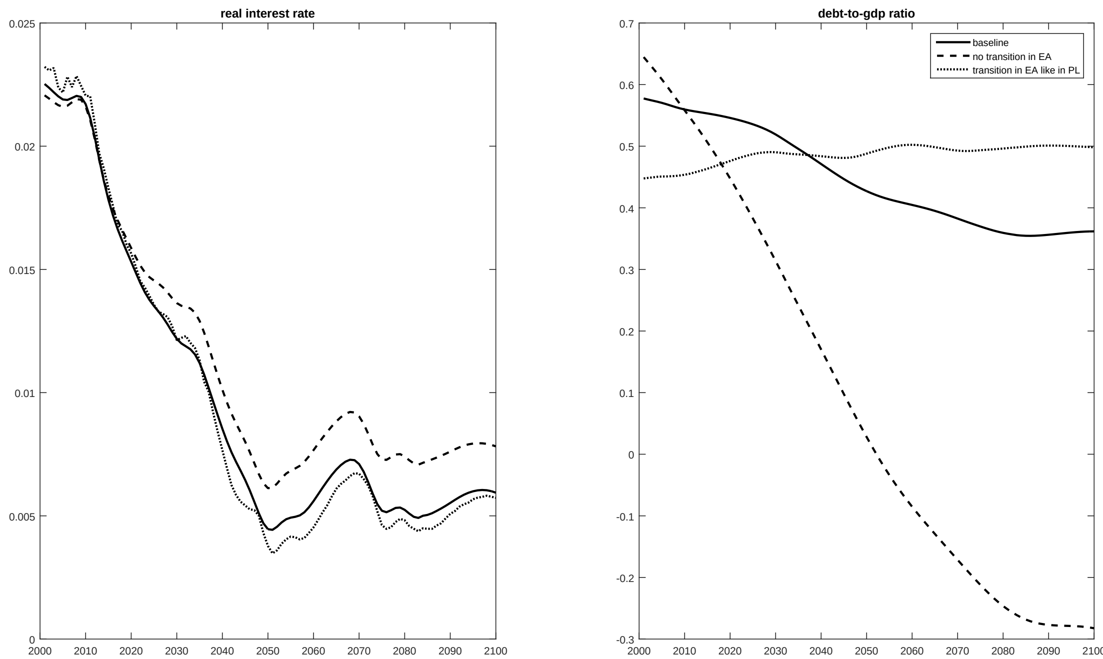
Note: solid line - baseline model; dashed line - model with NRI learning.

Figure 9: Probability of hitting the ZLB



Note: solid line - baseline scenario; dashed line - scenario with NRI learning. The missing values resulting from the failure of meeting the Blanchard-Kahn conditions at certain approximation points (eight for the baseline scenario, six for the scenario with learning) were interpolated using a cubic spline.

Figure 10: Effects of foreign demography



Note: solid line - baseline model; dashed line - model with constant demography in the EA, dotted line - model with demography in the EA same as in Poland

A Appendix

A.1 Complete list of model equations

Households

$$c_{j,t} + a_{j,t} = w_t z_j h_{j,t} + \frac{R_t^a}{\pi_t} a_{j-1,t-1} + beq_t \tag{A.1}$$

$$a_{0,t} = 0 \tag{A.2}$$

$$a_{J,t} = 0 \tag{A.3}$$

$$1 = \beta \left(1 - \omega_{j,t}\right) E_t \left[\frac{c_{j,t}}{c_{j+1,t+1}} \frac{R_{t+1}^a}{\pi_{t+1}}\right] \tag{A.4}$$

$$h_{j,t} = \left(\frac{w_t z_j}{\phi_j c_{j,t}}\right)^{1/\varphi} \qquad z_{j \geq JR} = 0 \tag{A.5}$$

Demography

$$N_{1,t}^{rel} = 1 \tag{A.6}$$

$$N_{j,t}^{rel} = \frac{\left(1 - \omega_{j-1,t-1}\right) N_{j-1,t-1}^{rel}}{1 + n_{1,t}} \tag{A.7}$$

$$N_t^{rel} = \sum_{j=1}^{J} N_{j,t}^{rel} \tag{A.8}$$

$$n_t = \frac{N_t^{rel}}{N_{t-1}^{rel}} \left(1 + n_{1,t}\right) - 1 \tag{A.9}$$

Aggregation

$$c_t = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} c_{j,t}}{N_t^{rel}} \tag{A.10}$$

$$h_t = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} z_j h_{j,t}}{N_t^{rel}} \tag{A.11}$$

$$a_t = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} a_{j,t}}{N_{t+1}^{rel} \left(1 + n_{1,t+1}\right)} \tag{A.12}$$

$$beq_t = \frac{\sum_{j=1}^{J} \left(N_{j-1,t-1}^{rel} / \left(1 + n_{1,t}\right) - N_{j,t}^{rel}\right) \left(R_t^a / \pi_t\right) a_{j-1,t-1}}{N_t^{rel}} \tag{A.13}$$

Financial intermediary

$$a_t = q_t k_t + s_t b_t^* + p_t^d \tag{A.14}$$

$$R_t q_t = E_t \left[\left(r_{t+1}^k + (1-\delta) q_{t+1} \right) \pi_{t+1} \right] \tag{A.15}$$

$$R_t s_t = E_t \left[s_{t+1} \Gamma_t R_t^* \pi_{t+1} / \pi_{t+1}^* \right] \tag{A.16}$$

$$R_t p_t^d = E_t \left[\left(p_{t+1}^d (1 + n_{t+1}) + f_{t+1} \right) \pi_{t+1} \right] \tag{A.17}$$

$$\frac{R_t^a}{\pi_t} a_{t-1} = \left(r_t^k + (1-\delta) q_t \right) k_{t-1} + s_t \Gamma_{t-1} \frac{R_{t-1}^*}{\pi_t^*} b_{t-1}^* + \left(p_t^d (1+n_t) + f_t \right) \tag{A.18}$$

Final goods producers

$$y_t = \left[\eta^{\frac{\mu-1}{\mu}} (y_{H,t})^{\frac{1}{\mu}} + (1-\eta)^{\frac{\mu-1}{\mu}} (y_{F,t})^{\frac{1}{\mu}} \right]^{\mu} \tag{A.19}$$

$$y_{H,t} = \eta (p_{H,t})^{\frac{\mu}{1-\mu}} y_t \tag{A.20}$$

$$y_{F,t} = (1-\eta) (p_{F,t})^{\frac{\mu}{1-\mu}} y_t \tag{A.21}$$

Capital goods producers

$$(1 + n_{t+1}) k_t = (1-\delta) k_{t-1} + \varepsilon_{i,t} \left[1 - \frac{S_1}{2} (1+n_t)^2 \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t \tag{A.22}$$

$$\begin{aligned} 1 = \varepsilon_{i,t} q_t & \left[1 - \frac{S_1}{2} (1+n_t)^2 \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - S_1 (1+n_t)^2 \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right] \\ & + E_t \left[\frac{\pi_{t+1}}{R_t} \varepsilon_{i,t+1} q_{t+1} S_1 (1+n_{t+1})^3 \left(\frac{i_{t+1}}{i_t} - 1 \right) \frac{i_{t+1}^2}{i_t^2} \right] \end{aligned} \tag{A.23}$$

Importers

$$\tilde{p}_{F,t} = \mu_F \frac{\Omega_{F,t}}{\Upsilon_{F,t}} \tag{A.24}$$

$$\Omega_{F,t} = (p_{F,t})^{\frac{\mu_F}{\mu_F - 1}} y_{F,t} + \theta_F \frac{\pi_{t+1}^*}{R_t^*} \left(\frac{\pi_{F,t+1}^{\zeta}}{\pi_{t+1}} \right)^{\frac{\mu_F}{1-\mu_F}} \Omega_{F,t+1} \tag{A.25}$$

$$\Upsilon_{F,t} = \varepsilon_{F,t}^{\tau} \tau_F (p_{F,t})^{\frac{\mu_F}{\mu_F - 1}} y_{F,t} + \theta_F \frac{\pi_{t+1}^*}{R_t^*} \left(\frac{\pi_{F,t+1}^{\zeta}}{\pi_{t+1}} \right)^{\frac{1}{1-\mu_F}} \Upsilon_{F,t+1} \tag{A.26}$$

Intermediate goods producers

$$\frac{r_t^k}{w_t} = \frac{\alpha}{1-\alpha}\frac{h_t}{k_{t-1}} \tag{A.27}$$

$$mc_t = \frac{1}{\varepsilon_{z,t}}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{A.28}$$

$$y_{H,t}^* = \left(p_{H,t}^*\right)^{-\phi_y} y_t^* \tag{A.29}$$

$$\tilde{p}_{H,t} = \mu_H \frac{\Omega_{H,t}}{\Upsilon_{H,t}} \tag{A.30}$$

$$\tilde{p}_{H,t}^* = \mu_H^* \frac{\Omega_{H,t}^*}{\Upsilon_{H,t}^*} \tag{A.31}$$

$$f_t = p_{H,t}y_{H,t} + s_t p_{H,t}^* y_{H,t}^* - w_t h_t - r_{k,t}k_{t-1} \tag{A.32}$$

$$\Omega_{H,t} = mc_t\left(p_{H,t}\right)^{\frac{\mu_H}{\mu_H-1}} y_{H,t} + \theta_H \frac{\pi_{t+1}}{R_t}\left(\frac{\pi_{H,t+1}^{\zeta}}{\pi_{t+1}}\right)^{\frac{\mu_H}{1-\mu_H}} \Omega_{H,t+1} \tag{A.33}$$

$$\Upsilon_{H,t} = \varepsilon_{H,t}^{\tau}\tau_H\left(p_{H,t}\right)^{\frac{\mu_H}{\mu_H-1}} y_{H,t} + \theta_H \frac{\pi_{t+1}}{R_t}\left(\frac{\pi_{H,t+1}^{\zeta}}{\pi_{t+1}}\right)^{\frac{1}{1-\mu_H}} \Upsilon_{H,t+1} \tag{A.34}$$

$$\Omega_{H,t}^* = mc_t\left(p_{H,t}^*\right)^{\frac{\mu_H^*}{\mu_H^*-1}} y_{H,t}^* + \theta_H^* \frac{\pi_{t+1}}{R_t}\left(\frac{\pi_{H,t+1}^{\zeta*}}{\pi_{t+1}^*}\right)^{\frac{\mu_H^*}{1-\mu_H^*}} \Omega_{H,t+1}^* \tag{A.35}$$

$$\Upsilon_{H,t}^* = \varepsilon_{H,t}^{\tau*}\tau_H^*\left(p_{H,t}^*\right)^{\frac{\mu_H^*}{\mu_H^*-1}} y_{H,t}^* + \theta_H^* \frac{\pi_{t+1}}{R_t}\left(\frac{\pi_{H,t+1}^{\zeta*}}{\pi_{t+1}^*}\right)^{\frac{1}{1-\mu_H^*}} \Upsilon_{H,t+1}^* \tag{A.36}$$

Inflation and price dispersion dynamics

$$\pi_{H,t}^{\zeta} = \zeta_H \pi_{t-1} + \left(1-\zeta_H\right)\pi_{ss} \tag{A.37}$$

$$\pi_{F,t}^{\zeta} = \zeta_F \pi_{t-1} + \left(1-\zeta_F\right)\pi_{ss} \tag{A.38}$$

$$\pi_{H,t}^{\zeta*} = \zeta_H^* \pi_{t-1}^* + \left(1-\zeta_H^*\right)\pi_{ss}^* \tag{A.39}$$

$$(p_{H,t})^{\frac{1}{1-\mu_H}} = \theta_H \left(\frac{p_{H,t-1}\pi_{H,t}^{\zeta}}{\pi_t}\right)^{\frac{1}{1-\mu_H}} + (1-\theta_H)(\tilde{p}_{H,t})^{\frac{1}{1-\mu_H}} \tag{A.40}$$

$$(p_{F,t})^{\frac{1}{1-\mu_F}} = \theta_F \left(\frac{p_{F,t-1}\pi_{F,t}^{\zeta}}{\pi_t}\right)^{\frac{1}{1-\mu_F}} + (1-\theta_F)(\tilde{p}_{F,t})^{\frac{1}{1-\mu_F}} \tag{A.41}$$

$$\left(p_{H,t}^*\right)^{\frac{1}{1-\mu_H^*}} = \theta_H^* \left(\frac{p_{H,t-1}^*\pi_{H,t}^{\zeta*}}{\pi_t^*}\right)^{\frac{1}{1-\mu_H^*}} + (1-\theta_H^*)\left(\tilde{p}_{H,t}^*\right)^{\frac{1}{1-\mu_H^*}} \tag{A.42}$$

$$\Delta_{H,t} = (1-\theta_H)\left(\frac{\tilde{p}_{H,t}}{p_{H,t}}\right)^{\frac{\mu_H}{1-\mu_H}} + \theta_H \Delta_{H,t-1}\left(\frac{p_{H,t}}{p_{H,t-1}}\right)^{\frac{\mu_H}{\mu_H-1}}\left(\frac{\pi_{H.t}^{\zeta}}{\pi_t}\right)^{\frac{\mu_H}{1-\mu_H}} \tag{A.43}$$

$$\Delta_{H,t}^* = (1-\theta_H^*)\left(\frac{\tilde{p}_{H,t}^*}{p_{H,t}^*}\right)^{\frac{\mu_H^*}{1-\mu_H^*}} + \theta_H^* \Delta_{H,t-1}^*\left(\frac{p_{H,t}^*}{p_{H,t-1}^*}\right)^{\frac{\mu_H^*}{\mu_H^*-1}}\left(\frac{\pi_{H.t}^{\zeta*}}{\pi_t^*}\right)^{\frac{\mu_H^*}{1-\mu_H^*}} \tag{A.44}$$

Monetary policy

$$\frac{R_t}{R_t^n} = \left(\frac{R_{t-1}}{R_t^n}\right)^{\gamma_R}\left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi}\left(\frac{gdp_t}{gdp_{t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R}\varepsilon_{R,t} \tag{A.45}$$

Alternative assumptions about R_t^n

$$R_t^n = E_t\left[\frac{R_{t+1}^a}{\pi_{t+1}}\right]\pi_{ss} \tag{A.46}$$

$$R_t^n = (1+r_t^n)\,\pi_{ss}$$

$$R_t^n = R_{t-1}^n + \lambda\left[\left(1+r_{t-1}^n\right)\pi_{ss} - R_{t-1}^n\right]$$

Market clearing

$$y_{H,t}\Delta_{H.t} + y_{H,t}^*\Delta_{H,t}^* = \varepsilon_{z,t}k_{t-1}^{\alpha}h_t^{1-\alpha} \tag{A.47}$$

$$y_t = c_t + inv_t \tag{A.48}$$

$$gdp_t = y_{H,t}\Delta_{H.t} + y_{H,t}^*\Delta_{H,t}^* \tag{A.49}$$

$$dy_t = -\frac{s_t b_t^*}{p_{H,t}gdp_t} \tag{A.50}$$

$$\Gamma_t = 1 + \gamma\left(\exp\left(dy_t\right) - 1\right) \tag{A.51}$$