Speculative Bubbles, Heterogeneous Beliefs, and Learning

******

Jan Werner

University of Minnesota

..

Asset Price Bubbles

- Recent Price Bubbles:

  Japanese Bubble of 1980s',

  Dot.com Bubble,

  US Housing Bubble,

  Chinese Warrant Bubble

  Bitcoin Bubble,

  Dow Jones at 26,000 (?).

- Understanding Price Bubbles.

  What should policy makers do about price bubbles?

Understanding Asset Price Bubbles

- price = fundamental value + bubble.
- But what is the “fundamental value”?
  - I. Discounted present value of future dividends.
    Rational Price Bubble
  - II. Agents’ marginal valuation of future dividends, that is, willingness to pay if obliged to hold the asset forever.
    Speculative Bubble.

Rational Price Bubble

- The present value at date t of an asset with dividend stream $\{x_t\}$ is

$$PV_t(x) = \sum_{\tau=t+1}^{\infty} \frac{1}{R_t^{\tau}} E^*[x_\tau]$$

Discounted present value of future dividends.

  - $E^*(\cdot)$ is expectation under equivalent martingale measure, or risk-neutral pricing measure.
  - R_t^{τ} is date-t risk-free return for maturity τ.

Speculative Bubble

- Fundamental value: marginal value of buying an additional share of the asset at date t and holding it forever

$$V_t^i(x) = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} E^i[\frac{u'(c^i_{\tau+1})}{u'(c^i_\tau)}\ x_\tau]$$

$\frac{u'(c^i_{\tau+1})}{u'(c^i_\tau)}$ is the marginal rate of substitution between consumption in $\tau+1$ and τ.

Speculative Bubble

- There is speculative bubble at date t if

$$p_t > \max_i V_t^i(x).$$

  That is, asset price exceeds all agents' fundamental valuations.

- Agent who buys the asset pays more than his willingness to pay if obliged to hold it forever. Hence, speculative trade: Buy in order to sell at a later date.

Heterogeneous Beliefs

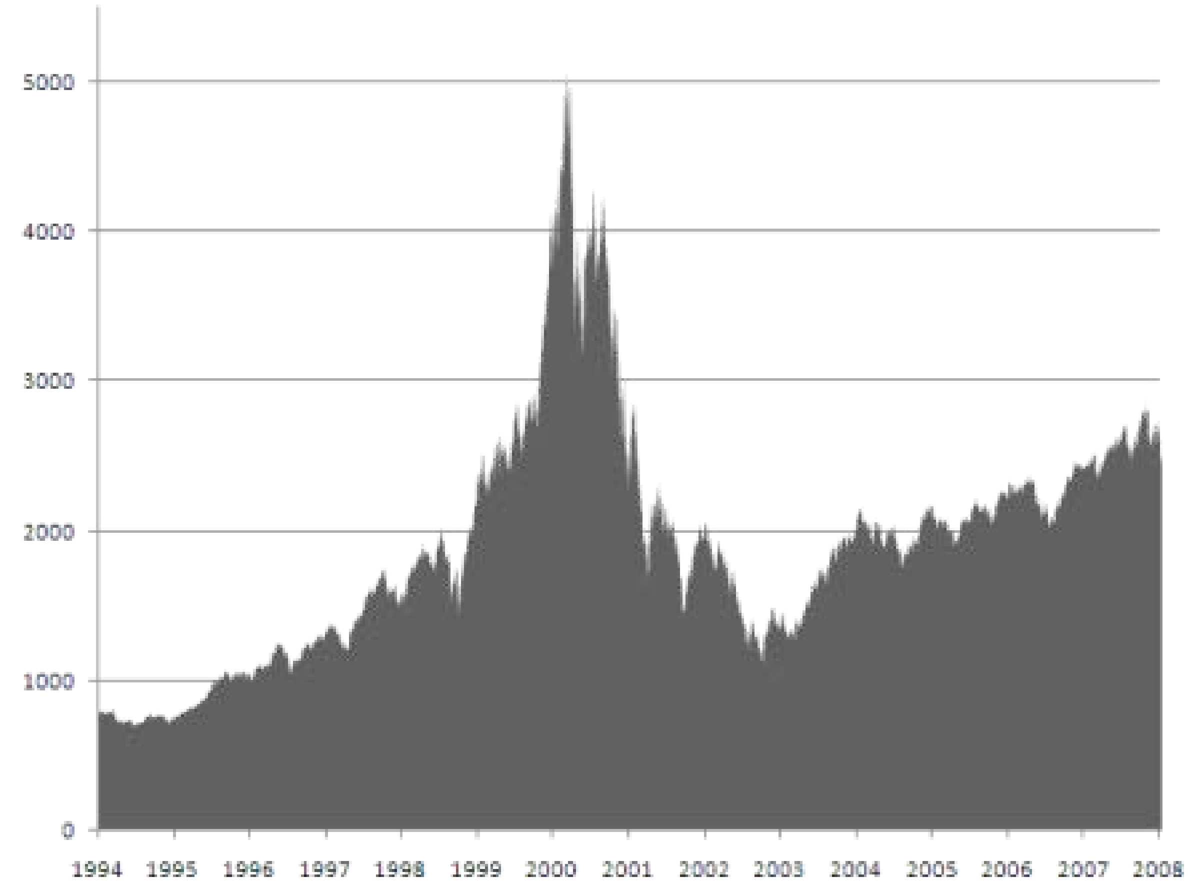
- For risk-neutral traders with different probability beliefs, V_t^i is the discounted expected value of dividends under i's belief:

$$V_t^i(x) = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} E^i[x_\tau],$$

- Speculative bubbles can occur in markets with heterogeneus beliefs and short-sales constraints (**Harrison and Kreps (1978)**):
  - Optimists hold the asset; pessimists ‘sit on the sidelines’
  - Switching of optimism.
  - Price is strictly higher than the most optimistic belief.

Dot.com Bubble

- Dot.com Bubble 1999-2001.
  - Nasdaq Index peaked on March 10, 2000, at 5048.
  - 1000 $\%$ return on internet stocks between February 1998 and early 2000.
  - Volume of trade and share turnover 3-4 times higher than for non-internet stocks.
  - Stringent short sales restrictions (lockups) and significant heterogeneity of investors' beliefs.
    E. Ofek and M. Richardson (2003) and Hong, Scheinkman, and Xiong (2006).


5000
4000
3000
2000
1000
0
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008

Chinese Warrants Bubble

- 18 put warrants issued by Chinese companies between 2005-2008.
  - Boom in the Chinese stock market in 2005-2007.
- Warrants expected to expire out of the money. Should have had nearly zero prices.
- Speculative trade in warrants:
  - 328 % daily turnover on average.
  - Average daily trading volume 1.29 billion yuan.
- Fig. 1 – WuLiang stock prices and warrant prices. **Xiong and Yu (2011).**

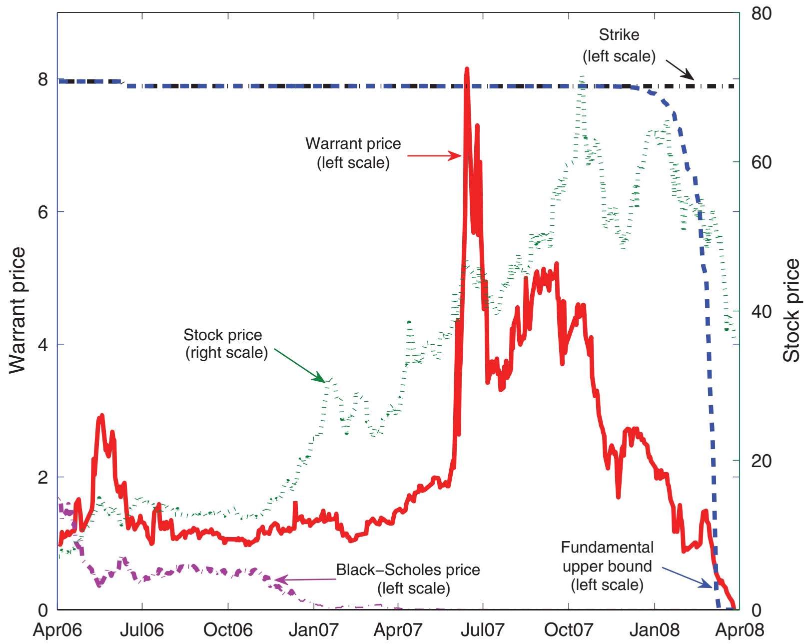


Figure 1. Prices of WuLiang Put Warrant

Notes: This figure shows the daily closing prices of WuLiang stock and its put warrant, along with WuLiang war-rant strike price, upper bound of its fundamental value assuming WuLiang stock price drops 10 percent every day before expiration (maximum allowed per day in China's stock market), and its Black-Scholes price using WuLiang stock's previous one-year rolling daily return volatility.

Heterogeneous Beliefs?

- "Dogmatic" beliefs with permanent switching of optimism: *Harrison and Kreps (1978)*.
- Overconfidence, or bias in belief updating: *Scheinkman and Xiong (2003, 2006)*.
- Heterogeneous priors and learning. Bayes Learning leads to switching of optimism: *Morris (1996)*.
- Ambiguous but common beliefs: *Werner (2015)*.

Questions

- Toward general theory of speculative bubbles:
  - When do heterogeneous beliefs give rise to speculative trade and bubbles?
  - Bayesian learning and speculative bubbles. When do heterogeneous priors give rise to speculative trade and bubbles?
  - Bayesian learning and the dynamics of price bubbles.

Literature

- Key references:
  - Harrison and Kreps (1978), Morris (1996).
- Also Miller (1977), Scheinkman and Xiong (2003, 2006), Harris and Raviv (1993), Slawski (2009).
- Werner (2015)

Outline

- Outline:
  - 1. Valuation Switching and Speculative Bubbles.
  - 2. Speculative Trade with Learning.
  - 3. Example.
  - 4. Merging of Beliefs and Disapperance of Bubbles.

Asset Market with no Short-Selling.

- Set S of states of at each date. The product set S^∞ represents all sequences of states.
  Notation: s_t is date-t state; s^t is t-history $(s_0, \dots, s_t)$.
- Single asset with dividend process $\{x_t\}$ on (S^∞, Σ), measurable w.r. to $\mathcal{F}_t$.
- I agents; each agent i is risk-neutral; the same discount factor β.
  Endowments are e_t^i, positive, and bounded. Initial asset holdings are $\bar{h}_0^i$. Asset supply is $\bar{h}_0 = \sum_i \hat{h}_0^i > 0$.
- No short sales.

Heterogeneous Beliefs

- Agents have different probability measures on (S^∞, Σ). Probability measure of agent i is P_i.

  Notation: E^i is the expectation under probability measure P_i. E^i_t is conditional expectation at t.

- Agent's i utility function of $\{c_t\}$ is

$$\sum_{t=0}^{\infty} \beta^t E^i[c_t].$$

- Budget constraints are

$$c_t + p_t h_t = e^i_t + [p_t + x_t] h_{t-1}, \quad \text{with } h_t \geq 0.$$

Equilibrium

- Equilibrium: Prices p and allocation $\{c^i, h^i\}$ such that each (c^i, h^i) is optimal for i, and markets clear.
  - Market clearing: $\sum_i c_t^i = \bar{e}_t + \bar{h}_0 x_t, \quad \sum_i h_t^i = \bar{h}_0.$

- Equilibrium price p_t must satisfy

$$p_t = \max_i \beta E_t^i [p_{t+1} + x_{t+1}]. \tag{1}$$

Agent(s) whose expectation is the maximizing one in (1) holds the asset. He is the optimist about the short-term gain at s^t.

Asset Prices

- Market belief $\hat{P}_t$ is the maximizing probability in (1). $\hat{P}$ derived from $\hat{P}_t$ is the risk-neutral pricing measure.

- It holds

$$p_t = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} E_t^{\hat{P}}[x_\tau],$$

price $=$ discounted sum of expected dividends under $\hat{P}$.

(no rational bubble).

- Fundamental value of the asset under ith belief is

$$V_t^i = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} E_t^i[x_\tau],$$

Speculative Bubbles

- It follows from (1) that

$$p_t \geq V_t^i \quad \text{for every i, every t}$$

- Speculative bubble in event s^t if

$$p_t(s^t) > max_i\, V_t^i(s_t). \tag{2}$$

If (2) holds, then the agent who buys the asset in s^t pays the price exceeding her valuation of the asset if she were to hold the asset forever.

- Question: Under what conditions on heterogeneous beliefs is there speculative bubble?

Valuation Dominance and Switching

- Agent i is valuation dominant in s^t if

$$V^i_\tau(s^\tau) = \max_j \; V^j_\tau(s^\tau)$$

  for every successor event s^τ of s^t, for all $\tau > t$.

- If there is no valuation dominant agent at s^t, then beliefs exhibit valuation switching at s^t.
  If so for all s^t, then perpetual valuation switching.

- Theorem 1: *If beliefs exhibit valuation switching in s^t, then there is speculative bubble in s^t.*

Theorem 1

- Proof of Theorem 1:
  - Suppose by contradiction that $p_t(s^t) = V_t^i(s^t)$ for some i.
  - Then $p_\tau(s^\tau) = V_\tau^i(s^\tau)$ for every successor s^τ.
  - Agent i is not valuation dominant, hence

$$V_\tau^j(s^\tau) > V_\tau^i(s^\tau) = p_\tau(s^\tau),$$

    for some j and some successor s^τ.

    But this is a contradiction.
- Sufficient but not necessary. HK (1978) example.

Heterogeneous Priors and Learning

- There is a family of probability measures P_θ on (S^∞, Σ) parametrized by θ in Θ.
- Agent i has prior belief μ^i on $(\Theta, \mathcal{G})$. Prior μ^i induces joint distribution Π_{μ^i}

$$\Pi_{\mu^i}(A \times B) = \int_A P_\theta(B)\mu^i(d\theta).$$

$\mu^i(\cdot|s^t)$ is agent's i posterior on Θ, $P_{\mu^i}(\cdot|s^t)$ is conditional probability of the future given the past (on Σ).

Notation: E_t^i is the conditional expectation under $P_{\mu^i}(\cdot|s^t)$.

Valuation Dominance and MLR Order

- Question: What conditions on priors give valuation switching?
- Suppose that prior μ^i has density f_i on $\Theta \subset R$. Prior μ_i dominates μ^j in the Maximium Likelihood Ratio order if

$$\frac{f_i(\theta')}{f_i(\theta)} \geq \frac{f_j(\theta')}{f_j(\theta)} \qquad \text{for every } \theta' \geq \theta.$$

- Proposition 2: *If μ^i dominates μ^j in MLR order for every $j \neq i$ and if $V_\theta(s^t)$ is non-decreasing in θ for every s^t, then agent i is valuation dominant.*

  Proof: MLR-dominance implies first-order stochastic dominance, and is preserved by posteriors.

Learning with i.i.d. Dividends

- $\{x_t\}$ is an i.i.d. process. Then

$$V_t^i = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} E_t^i[x_{t+1}] = E_t^i[x_{t+1}] \frac{\beta}{1-\beta}$$

Here, valuation dominance is equivalent to dominance of posterior mean dividends.

- MLR-dominance among priors implies valuation dominance if $E_\theta[x]$ is increasing in θ.

Example

- $0-1$ dividends, Morris (1996):

  x_t can take values 0 or 1; $\theta \in [0,1]$ is probability of high dividend.

  Posterior mean $E_t^i[x_{t+1}]$ equals the posterior probability of high dividend.

- For beta prior with density $f(\theta) \sim \theta^{\alpha-1}(1-\theta)^{\beta-1}$, where $\alpha > 0$ and $\beta > 0$, the posterior probability of high dividend given k "successes" in t periods is

$$\nu(t,k) = \frac{(k+\alpha)}{(t+\alpha+\beta)}.$$

Example

- Take the uniform prior μ^u with $f_u \equiv 1$ and posterior

$$\nu^u(t,k) = \frac{(k+1)}{(t+2)},$$

and Jeffreys prior μ^J with $f_J(\theta) \sim \frac{1}{\sqrt{\theta(1-\theta)}}$ and

$$\nu^J(t,k) = \frac{(k+1/2)}{(t+1)}.$$

- There is perpetual valuation switching for μ^u and μ^J.
- By Theorem 1, there is speculative bubble in equilibrium with uniform and Jeffrey's priors.

Example

- More generally for beta priors, agent i is valuation dominant if and only if $\alpha_i \geq \alpha_j$ and $\beta_i \leq \beta_j$ for every j.

- Valuation dominance is equivalent to the Monotone Likelihood Ratio order for beta priors.

Merging of Beliefs and Bubbles

- Let P^0 be the true probability measure on (S^∞, Σ).

  P^0 is absolutely continuous with respect to P^i if $P^0(A) > 0$ implies $P^i(A) > 0$.

- Blackwell and Dubins (1962) merging of opinions:
  If the true probability P^0 is absolutely continuous with respect to agent's belief P^i, then conditional beliefs for the future given the past $P^i(\cdot|s^t)$ merge with true conditionals $P^0(\cdot|s^t)$ P^0-a.e.

Merging of Beliefs and Bubbles

- Theorem 2: *Suppose that P^0 is absolutely continuous with respect to P^i for every i. Then*

$$\lim_t [V_t^i - V_t^0] = 0, \quad P^0 - a.e.$$

Further, P^0 is absolutely continuous with respect to the market belief $\hat{P}$ and

$$\lim_t [p_t - V_t^0] = 0, \quad P^0 - a.e.$$

so that speculative bubble vanishes in the limit P^0-a.e.

Proof: Use Kabanov, Liptser and Shiryaev (1985) to show that $\hat{P}$ is absolutely continuous.

Bayes Consistency and Bubbles

- θ_0 is the true parameter.
  Absolute continuity of P^0 w.r.t. to P^i requires $\mu^i(\theta_0) > 0$ – may not hold for infinite parameter sets.

- Prior μ^i is consistent at θ_0 if posterior μ^i_t converges weak-star to Dirac point-mass at $\theta_0, \;\; P^0 - a.e.$

- Proposition 3: *For i.i.d. dividends, if $E_\theta[x]$ is continuous in θ, all agents' priors are consistent at θ_0 and absolutely continuous with respect to each other, then the hypotheses of Theorem 2 hold.*
  In particular, speculative bubble vanishes in the limit P^0-a.e.

  In the $0-1$ i.i.d. example, μ^u and μ^J are consistent at arbitrary θ_0. Fundamental valuations and the price converge to $\frac{\beta}{(1-\beta)}\theta_0$.

Bayes Consistency and Bubbles

- Consistency of priors:
  - Holds for every θ_0 in the support of prior μ^i if Θ is finite-dimensional. **Freedman (1963) and Schwartz (1965).**
  - Can be problematic if Θ is infinite-dimensional; Bayesian non-parametrics.
  - Misspecified priors.

    Persistent speculative bubble with misspecified prior – **Slawski (2009).**

Conclusions

- Speculative trade and bubbles are likely with heterogeneous priors and Bayes learning.
- Bubbles may vanish in the long run, or be permanent.