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Should pensions be progressive?

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ABSTRACT

The present paper quantitatively characterizes the consequences of rising pension progressivity in an overlapping generations model with idiosyncratic income, disability and longevity risk as well as endogenous labor supply at the intensive and extensive margin. Focusing on the German pension system which is purely earnings related, we increase the degree of progressivity and compute the optimal mix between flat and earnings-related pensions.

We find that a flat-rate pension share of roughly 30% maximizes aggregate economic efficiency, since improved insurance provision dominates higher labor supply distortions. Disability risk significantly increases the optimal progressivity level, while endogenous retirement has important macroeconomic implications. Since our results are robust for a wide range of parameter specifications, they indicate that at least in Germany a move towards more redistribution within the pension system is efficient.

1. Introduction

During the last decade, many OECD countries have legislated pension reforms in order to reduce the labor supply distortions induced by the pension system. In order to strengthen the tax-benefit linkage, countries have reduced incentives for early retirement, extended the calculation periods for calculating pension benefits or even introduced so-called notional defined contribution (NDC) systems, in which each contribution accrued during working life will be converted into an annuity at the date of retirement. As a result of these reform efforts, the pension progressivity index, which measures the linkage between earnings when working and pension benefits during retirement, declined significantly in most OECD countries in recent years, confer OECD (2005) and OECD (2011).

The observed decline in pension progressivity does however not come without cost. Since negative income shocks during the employment phase are now more directly transmitted to the retirement phase, old-age poverty risk for low income households will increase. From an ex ante point of view it is not clear, whether the benefits from reduced labor supply distortions really dominate the cost of reduced insurance provision against labor market risk. Obviously, the latter and the former effects work in opposite directions in terms of efficiency. Consequently, the present paper asks whether the focus on

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reducing labor market distortions in the pension system is really efficient. Or could it rather be optimal to make the pension system more progressive in order to reduce the risk of old-age poverty? What is the optimal pension progressivity that balances benefits and costs perfectly and what are the dominating forces that determine this optimal level?

In order to quantitatively clarify these questions, we develop an eighty-period overlapping generations model à la Auerbach and Kotlikoff (1987) with endogenous labor supply and idiosyncratic wage risk. Labor supply is endogenous along two margins, the number of hours an individual works in the market (intensive) and the date at which he retires (extensive). Wage risk is due to permanent and transitory components, whereas the former identify household's skill level. In addition, we allow for skill specific disability risk, since currently about 20% of new entries into the German pension system are due to disability (DRV, 2010). In this setup we study the macroeconomic, welfare and efficiency consequences of different degrees of progressivity in the pension system. We therefore proceed as follows: we first compute an initial equilibrium which is calibrated to the German economy and features the statutory pension system with a perfect tax-benefit linkage. We then increase the share of income-independent benefits and compute a full transition path of the economy up to a new long-run equilibrium and report the resulting macroeconomic effects and welfare consequences. Next, we simulate the considered policy reform once again with compensating lump-sum transfers in order to quantify the respective aggregate efficiency consequences. Finally, the optimal flat and earnings-related benefit combination that maximizes aggregate efficiency is determined.

The overlapping generations framework has already been used by Huggett and Ventura (1999) as well as Nishiyama and Smetters (2008) to quantify the long-run macroeconomic and welfare effects of different pension designs. Fehr and Habermann (2008) compute transition paths between long-run equilibria and therefore are able to quantify the efficiency effects of different degrees of progressivity in the German pension formula. The German pension system features a nearly perfect linkage between contributions and pension benefits, so that labor supply distortions as well as the insurance provision are currently both fairly low. The study finds that a reduction of the tight tax-benefit linkage would increase economic efficiency, since the better insurance provision dominates higher labor supply distortions. However, Fehr and Habermann (2008) only apply a very stylized model of the household sector and the respective income dynamics. In contrast, our model is much more detailed in that it is computed on a yearly basis and features both disability risk and labor supply decisions at the extensive margin. We would expect rising insurance benefits and an increase in optimal progressivity due to disability risk. Of course, increased pension progressivity will distort labor supply at the intensive margin, but it is not clear at all whether it induces people to retire earlier or later since income effects for different skill classes go in opposite directions.

Our simulations show that higher pension progressivity reduces factor inputs, output as well as pension benefits and increases future tax and contribution rates. Lower benefits and higher taxes reduce welfare of existing pensioners, while low-skilled workers may benefit due to the reform. Future cohorts most likely gain from higher pension progressivity, since they experience positive insurance effects. Overall, a flat-rate pension share of 30–40% maximizes aggregate efficiency gains. As expected, disability risk raises the optimal progressivity significantly. This result is very robust with regard to alternative wage processes and mortality assumptions. Of course, lower risk aversion and higher labor supply elasticities (at the intensive margin) reduce optimal progressivity. Overall, this indicates that at least in Germany a move towards more redistribution within the pension system is efficient and that some of the recent reforms in Western countries discussed above might have gone too far.

The remainder of the paper is arranged as follows: the next section describes the base version of the general equilibrium model we use in our quantitative analysis. Section 3 discusses the calibration of the initial equilibrium and Section 4 reports our simulation results. Section 5 concludes. Additional material on the model structure, computation and initial calibration is provided in the Appendices.

2. The model economy

This section provides a description of the base model version we use to quantify our results. A more formal equation-based description of our fully specified model and a definition of the respective equilibrium path can be found in Appendix Appendix A.

2.1. Demographics

Our model is populated by J overlapping generations. At any discrete point t in time, a new generation is born with the same mass as the previous one. At the beginning of life, individuals are (exogenously) assigned a skill level $s \in \mathcal{S}$. During their life-cycle, they only survive from period to period with skill- and age dependent survival probabilities $\psi_{j,s}$, where $\psi_{J,s} = 0$. Since our model abstracts from annuity markets, individuals that die before the maximum age of J may leave accidental bequests that will be distributed in a lump-sum fashion across all working individuals. Agents retire at age j_r and start to receive pension benefits which are financed by proportional payroll taxes payed up to the double of average labor income. In the following, we will, for the sake of simplicity, omit the indices t and s wherever possible.

2.2. Endowments and preferences

When individuals enter the labor market, they are assigned a skill level s according to the probability distribution ϖ_s. Since they remain in it forever, skill level may be interpreted as a permanent shock. Assets are initially zero ($a_1 = 0$) and restricted throughout the whole life cycle to be greater or equal to zero, i.e. agents might be liquidity constrained. During their working phase, they may work up to the maximum time endowment of 1 in each period. Time that is not devoted to working is consumed as leisure ℓ_j. When working in the market, agents will accumulate pension claims $ep_j \in \mathcal{P}$ that define their pension payments when retired. Labor productivity e_j depends on individual's skill level s as well as idiosyncratic shocks $\eta_j \in \mathcal{E}$. We assume productivity shocks to be independent across individuals and to be identically distributed across individuals of a specific skill level. Consequently, individuals' state is characterized by

$$z_j = (s, a_j, ep_j, \eta_j) \in \mathcal{Z}.$$

The budget constraint is

$$a_{j+1} = (1+r)a_j + y_j(1-\tau) + b_j + p_j - T(y_j, p_j, ra_j) - (1+\tau_c)c_j,$$

where future assets a_{j+1} are derived from current assets (including interest), gross income from labor $y_j = we_j\eta_j(1-\ell_j)$ (which is due to the wage rate for effective labor, individual productivity $e_j\eta_j$, and hours worked), accidental bequests b_j and pensions p_j, net of payroll taxes τy_j, income taxes $T(\cdot)$ and consumption expenditures c_j (including consumption taxes).

Accumulated pension claims consist of both a flat and a perfectly earnings related part. Specifically we let

$$ep_{j+1} = ep_j + \left[\lambda + (1-\lambda)\frac{y_j}{\overline{y}}\right],$$

where $\overline{y}$ indicates the average labor income of the economy.[1] When $\lambda = 0$, agents face a perfectly earnings related system, whereas $\lambda = 1$ means that the pension system is completely flat. Note that $ep_{j+1} = ep_j$ holds after retirement.

Our model assumes a preference structure that is represented by a time-separable, nested utility function. In order to vary risk aversion and intertemporal substitution independently, we follow the approach of Epstein and Zin (1991) and formulate the maximization problem of a representative consumer at age j and state z_j recursively as

$$V(z_j) = \max_{c_j, \ell_j} \{u(c_j, \ell_j)^{1-1/\gamma} + \psi_{j+1}\beta E_j[V(z_{j+1})^{1-\mu}]^{(1-1/\gamma)/1-\mu}\}^{1/(1-1/\gamma)}$$

where the parameters γ and μ define the intertemporal elasticity of substitution between consumption and leisure in different years and the degree of (relative) risk aversion, respectively. Note for the special case of $\mu = 1/\gamma$ we are back at the traditional expected utility specification, see Epstein and Zin (1991, p. 266).

2.3. Technology

We let the production technology in our model be represented by a Cobb–Douglas production function $Y = \theta K^{\epsilon} L^{1-\epsilon}$, where K measures aggregate capital and L aggregate labor in efficiency units. The parameter ϵ denotes the share of capital in production and θ is a technology parameter. We assume capital to depreciate at the constant rate δ_k and firms to pay corporate taxes on output net of labor and depreciation costs at rate τ_k. Since a continuum of firms produces with the same technology under perfect competition, net marginal products of capital and labor equal the interest rate for capital r and the wage rate for effective labor w. Finally, in order to account for technological progress, we follow Kotlikoff et al. (2007) and assume time augmenting technological change.[2] Consequently, individual time endowment increases by κ for any individual from period to period.

2.4. Government activity

The government in our model splits into a tax and a pension system. The budgets of both systems are closed separately. While the consumption tax rate is used to balance the tax system, pension contributions are chosen in a way that pension contributions equal pension payments.

The tax system. In each period t, the government issues new debt and collects taxes on consumption τ_c as well as labor and capital income. The income tax code is oriented towards the German tax system where labor is taxed progressively and capital at a constant rate (after a basic allowance). Pension contributions are completely exempt from income taxation while pension benefits are fully taxed.

The pension system. The pension system is run on a pay-as-you-go basis. It collects contributions at a rate τ from all working households and pays pension benefits after retirement. The latter are determined by the sum of earning points

[1] This implies that the average earning point accumulated per year is equal to 1. Note that due to the contribution ceiling in the German pension system and in our model the amount of earning points that can be accumulated per year is limited to 2.

[2] Note that, due to the utility function not being of Cobb–Douglas type, we cannot assume labor augmenting technological change, since this would not be consistent with a balanced growth path.

accumulated during the working periods and the actual pension amount (APA) which reflects the monetary value of one earning point, i.e.

$$p_j = ep_{j_r} \times APA$$

where ep_{j_r} denotes accumulated earning points at retirement and APA the actual pension amount. Over time, APA grows with gross labor earnings.

2.5. Equilibrium conditions

Given a specific fiscal policy, an equilibrium path of the economy has to solve the household decision problem, reflect competitive factor prices, and balance aggregate inheritances with unintended bequests. Furthermore aggregation must hold, and consumption tax and pension contribution rate have to balance the tax and pension system's budgets. Since we assume a closed economy setting, output has to be completely utilized for private consumption C_t, public consumption G_t and investment purposes, i.e.

$$Y_t = C_t + G_t + (1+\kappa)K_{t+1} - (1-\delta_k)K_t,$$

aggregate savings have to balance capital demands of firms and the government and aggregate labor supply has to be employed by firms.

3. Calibration and initial equilibrium

In our model, one period covers one year. We assume agents to start their economically relevant life at the age of 20 and to live up to a maximum of 100 years. In order to get a reasonable classification of skills, we use the International Standard Classification of Education (ISCED) of the UNESCO. We thereby merge levels 0–2 (primary and lower secondary education), levels 3 and 4 (higher secondary education) and levels 5 and 6 (tertiary education) in order to receive 3 skill levels, i.e. $\mathcal{S} = \{1, 2, 3\}$. The initial probability distribution ϖ_s is calculated using data from the German Socio-Economic Panel (SOEP), a description of which can be found in Wagner et al. (2007). In this representative data set, low-, medium- and high-skilled individuals represent 26%, 55% and 19% of the population, respectively. Survival probabilities for the medium skill class $\psi_{j,2}$ are taken from the 2008 Life Tables for Germany reported in HMD (2013). von Gaudecker and Scholz (2007) document a positive correlation between lifetime earnings and life expectancy at age 65 which differs up to 6 years between the lowest and the highest earnings group considered in their study. Since our skill-levels are less differentiated, we compute probabilities $\psi_{j,s}$ for the low- and the high-skilled individuals, so that life expectancy between those two differs by about 5 years, i.e. it increases from 75.8 to 78.1 and 80.4 for the low-, medium- and high-skilled class, respectively. Therefore, the models average life expectancy almost exactly matches the respective one from the 2007/2009 German life tables.

3.1. Labor productivity, idiosyncratic risk and the distribution of income

We use data of the SOEP to quantify wage risk. Specifically, we calculate deflated hourly wages $w_{i,j,t,s}$ of individuals i at age j and time t who belong to cohort c and classify them according to the three education levels s mentioned above. Interns and civil servants are excluded from our data set, since they tend to have quite different wage dynamics. In addition, we exclude the upper and the lower percentile of the hourly wage distribution in order to rule out errors in the data. We end up with a total of 12,8581 observations with 20,271, 87,481, and 20,829 in the three different classes, respectively.

With this data, we estimate a version of the model in Storesletten et al. (2004). Specifically, we assume that

$$\log w_{i,j,t,s} = \phi_{0,s} + \phi_{1,j,s} + \phi_{2,t,s} + \phi_{3,c,s} + \log \eta_{i,j,t,s} + \nu_{i,j,t,s} \tag{1}$$

with the autoregressive risk term

$$\log \eta_{i,j,t,s} = \varrho_s \cdot \log \eta_{i,j-1,t-1,s} + \epsilon_{i,j,t,s}.$$

Thereby, $\phi_{0,s}$ is the intercept, $\phi_{1,j,s}$ is an age fixed effect, $\phi_{2,t,s}$ a time fixed effect, $\phi_{3,c,s}$ a cohort fixed effect, the innovation $\epsilon_{i,j,t,s}$ is normally distributed with mean 0 and variance $\sigma^2_{\epsilon,s}$, and the iid measurement errors $\nu_{i,j,t,s}$ are normally distributed with mean 0 and variance $\sigma^2_{\nu,s}$. Owing to the perfect collinearity between age, time and cohort, we obviously cannot estimate all the three fixed effects simultaneously. As Heathcote et al. (2005) or Huggett et al. (2011) point out, the choice of fixed effect may have an impact on the resulting age profile of wages. While the former strongly argue in favor of using time effects, the latter take a more agnostic view. In order to deal with this issue, we follow Huggett et al. (2011) and estimate two separate models of income inequality, one with a time effect and one with a cohort effect. Our baseline calibration will feature the parameters of the time effect model. Results from the cohort effect model will be presented in a sensitivity analysis. We determine our parameters in two steps: we first calculate fixed effects via OLS. From the residuals we then estimate the life cycle risk process via a restricted Maximum-Likelihood Method for each schooling level s, thereby exploiting the panel structure of our data. Our estimation results concerning productivity risk are shown in Table 1.

First, we find strong autocorrelation coefficients of roughly 0.95 for all three skill classes, which is typical for this specification of a risk process, see e.g. Storesletten et al. (2004) or Guvenen (2009). Second, we find the variance of the

Table 1
Parameter estimates for individual productivity risk (time effects).

	Low-skilled	Middle-skilled	High-skilled
AR(1) correlation ϱ	0.95666	0.95687	0.95828
	(0.00232)	(0.00099)	(0.00191)
Transitory variance σ_ϵ^2	0.02321	0.02812	0.03538
	(0.00609)	(0.00362)	(0.00991)
Total variance σ_η^2	0.27367	0.33326	0.43299
	(0.00600)	(0.00360)	(0.00978)
Error variance σ_ν^2	0.09212	0.08423	0.09565
	(0.00285)	(0.00132)	(0.00315)

Table 2
Measures of income inequality.

Indicator		Percentage share of		Gini index
		Lowest 10%	Highest 10%	
Net Income	Model	3.5	21.6	0.286
	Data[a]	3.6	24.0	0.290
Assets	Model	0.0	37.5	0.588
	Data[a]	−1.1	61.0	0.799

[a] *Source*: SVR (2009).

overall wage risk process σ_η^2 to increase with the education level s. Fossen and Glocker (2011) draw a similar picture using German data. However, they argue that unemployment is significantly lower among the well educated and therefore it is not clear from an ex ante perspective, whether labor market risk increases or decreases with education.

From the intercept and age fixed effects, we construct productivity profiles according to $e_j = \exp(\phi_{0,s} + \phi_{1,j,s})$. In our simulation model, we use these profiles in between the ages 20 and 63. Note that with rising age, the estimates become more and more biased upwards, since individuals with low productivity e.g. due to bad health, might already have become retired. Hence, from the age of 63 onwards, we assume in line with Eisensee (2005) that productivity depreciates quadratically until it finally reaches zero at age 80. The productivity profiles with time effects can be seen in the left part of Fig. 8 in Section 4.6.1. We thereby normalized wages for the highest educational group at age 20 to 1.

In terms of income uncertainty, we discretize the estimated AR(1) process, using a Rouwenhorst method as described in Kopecky and Suen (2010) with 3 approximation points, since this algorithm already delivers reliable results with a small number of nodes. Table 2 compares some model outcomes with income and asset inequality measures in Germany for 2007 taken from SVR (2009).

We find quite a good match of both the lowest and highest percentile of the income distribution as well as the Gini index. Of course, since individuals are not allowed to run into debt and we do not explicitly account for very high income earners, the asset distribution is more equal in our model than in reality.

3.2. Preferences and technology

In order to calibrate the parameters of the utility function we first set the intertemporal elasticity of substitution (IES) γ at 0.5, which is in the range of commonly used parameters in these types of models, see İmrohoroğlu and Kitao (2009) or Conesa et al. (2009, p. 33). The coefficient of relative risk aversion μ is set to 4.0 in the benchmark calibration. Using preferences of the Epstein–Zin (1991) type makes the paper different from most of the macro and public-finance literature. However, it allows us to explore the impact of risk aversion and the IES in isolation in the sensitivity analysis. Values of μ up to 5 (Cecchetti et al., 2000, p. 792) or even 8 (Meyer and Meyer, 2005, p. 260) are perceived as reasonable in the literature.

We let individual preferences over consumption and leisure be represented by the instantaneous CRRA utility function

$$u(c,\ell) = \{c^{1-1/\rho} + \alpha \ell^{1-1/\rho}\}^{1/(1-1/\rho)},$$

where ρ denotes the intra-temporal elasticity between consumption and leisure and α is a taste parameter for leisure consumption. We then chose $\rho = 0.6$ to obtain realistic labor supply elasticities. In order to set average hours worked in the economy at 0.4, which implies a 40 h work week length, we let $\alpha = 1.95$. With this preference specification, the Frisch (constant marginal utility of wealth) labor supply elasticity[3]

$$\eta^{\text{Frisch}} = \frac{\ell}{1-\ell}[\xi\gamma + (1-\xi)\rho] \qquad \text{with } \xi = \frac{\alpha^\rho w^{1-\rho}}{1+\alpha^\rho w^{1-\rho}}$$

is 0.87. This is in the range of values reported in Conesa et al. (2009, p. 43) or Ludwig et al. (2012, p. 102). Finally, we chose a value of 0.992 for β to obtain a capital to output ratio of 3.5, which is observed in Germany in year 2008 (IdW, 2009).

IdW (2009) reports a wage share in production of 65.2% for Germany in 2008. Since the wage share in our model is given by $1-\epsilon$, we set ϵ at 0.35. In addition, we choose a value of 1.3 for the technology parameter θ in order to normalize the wage rate for effective labor to 1. We let the depreciation rate δ_k on capital be 4.2%. This guarantees investment to amount to 19.4% of GDP, which is slightly higher than the value of 19.3 reported in IdW (2009) for Germany. Finally, we assume a growth rate κ of 1.3% for individual time endowment, which is in line with the long run average growth rate for Germany reported in Erber and Fritsche (2009).

3.3. Government policy

Government tax policy in our model reflects quite well the German tax system. Specifically, we set the debt to output ratio at 60% and fix the consumption tax rate at 17%, which guarantees a consumption tax revenue to output share of 10.1%. This share is slightly lower than the value of 10.7% reported in IdW (2009). We apply the German income tax code of the year 2005 to labor and pension income, i.e. the marginal tax rate schedule rises after a basic allowance from 15.8% to 44.3%. We assume, in line with German law, that pension contributions are deductible from tax while pension income is fully taxable and apply the German income splitting method. The resulting tax schedule is shown in Fig. 1.

In addition we tax returns from savings above a threshold of 4200€ linearly at the rate 26.4%. This reflects the recent reform of capital income taxation in Germany. Finally, we set the corporate tax rate τ_k at 15% which yields a revenue to output ratio of 3% that is slightly higher than the value of 2.1% reported in IdW (2009). Overall, tax revenues amount to 23.4% of GDP which is in line with the respective figure of 23.8% for Germany in 2008 reported in IdW (2009).

With respect to the pension system we set the retirement age j_r at age 63 and specify an APA value (i.e. the replacement rate) which yields the current contribution rate τ of 19.9% and a benefit level of 12.3% of GDP which is only slightly higher than the figure for Germany in 2008, see IdW (2009).

Fig. 2 reports the cross-sectional profiles for gross labor income, consumption and assets in the initial long-run equilibrium. All profiles are normalized to average gross labor income. Since we do not explicitly consider human capital formation, gross labor incomes increase faster and decline earlier than in reality. The right side shows the resulting profiles of consumption and assets over the life cycle. The shape and level of the consumption profile is quite realistic, see Rick (2010, p. x, xi). Coppola (2011, p. 47) reports a similar peak of the asset profile at age 50–55 and a maximum level of roughly 180, 000€ for Germany.

Table 3 again summarizes our chosen parameters and the respective calibration targets. Appendix B discusses the numerical solution algorithm which is applied in this study.

4. Simulation results

The remainder of this paper will mainly focus on the macroeconomic, welfare and efficiency consequences of progressive pension arrangements and the optimal combination of flat and earnings-related benefits. In order to quantify the various effects of pension progressivity, we always proceed in the same fashion. We start from an initial equilibrium with the parameterization that we described in the previous section ($t=2008$), specifically with $\lambda=0$. Then, we change the parameter λ once and for all in period $t=2009$ of the transition and compute a full transition path up to a new long run equilibrium $t=\infty$. Note that the change in λ will affect only the accumulation of new earnings point ep_{j+1} along the transition. Those points that were earned in the initial equilibrium will be unaffected by the reform. This especially applies to households that were already retired before the reform took place. In the first subsection we explain how welfare and efficiency effects are computed given this path. The next subsection considers a rather extreme scenario, namely the change from a fully earnings-related to a fully flat pension system in the base version of the model. Precisely because this reform is so drastic, it is the best one to discuss the economic consequences of pension progressivity. The following two subsections extend the base version of the model by successively introducing disability risk and respective pension benefits as well as endogenous retirement. Subsection five determines the optimal degree of pension progressivity for the different model versions while the final subsection presents sensitivity calculations for the fully specified model.

[3] The derivation of the Frisch elasticity for our specific preference structure is available upon request.

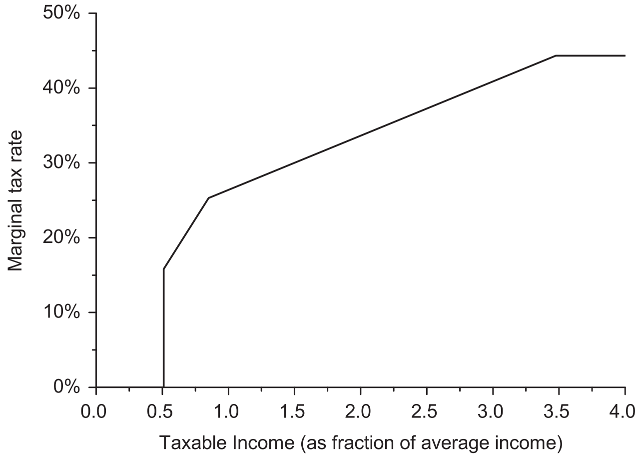

Fig. 1. Marginal tax rates for labor and pension income.

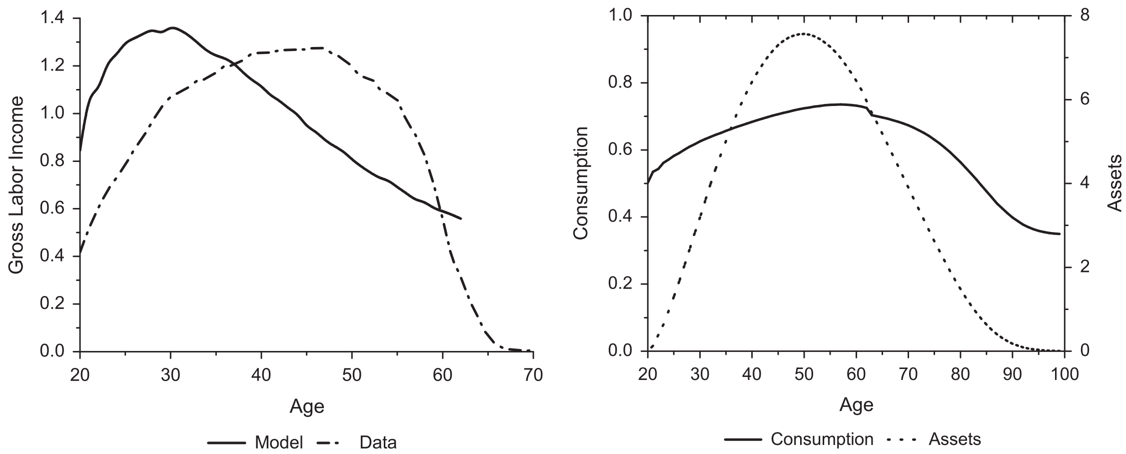

Fig. 2. Cross-sectional profiles for gross income, consumption and assets.

Table 3
Parameter values and calibration targets.

Parameter		Value	Target
Demographics and labor productivity			
Survival probabilities	$\psi_{j,2}$		HMD (2013)
	$\psi_{j,1}, \psi_{j,3}$		von Gaudecker and Scholz (2007)
Skill distribution	ϖ_s	0.26, 0.55, 0.19	Estimated from SOEP data
Retirement age	j_r	63	Average for old-age pension
Labor productivity	e_j, η_j		Estimated from SOEP data
Preference and technology parameters			
(relative) Risk aversion	μ	4.0	Cecchetti et al. (2000, p. 792)
Inter-temporal elasticity of substitution	γ	0.50	İmrohoroğlu and Kitao (2009)
Intra-temporal elasticity of substitution	ρ	0.60	Labor supply elasticities
Leisure preference	α	1.95	Average hours worked 0.4
Time discount factor	β	0.992	Capital output ratio 3.5
Capital share in production	ϵ	0.35	IdW (2009)
Technology parameter	θ		Wage rate for effective labor of 1
Depreciation of capital	δ_k	0.042	Investment to GDP ratio 19.4%
Technological progress	κ	0.013	Erber and Fritsche (2009)
Government policy			
Debt to GDP ratio		0.60	Target value issued by government
Consumption tax rate	τ_c	0.17	Revenue to output share 10.0%
Income tax code	$T(\cdot)$		German tax law
Corporate tax rate	τ_k	0.15	Revenue to output share 3%
Pension system	*APA*		Contribution rate τ is 19.9%
	λ	0.0	German pension law

4.1. Computation of welfare and efficiency effects

The concept we apply to quantify welfare effects is compensating variation à la Hicks. Due to the homogeneity of our utility function,

$$u[(1+\phi)c_j,(1+\phi)\ell_j]=(1+\phi)u[c_j,\ell_j]$$

holds for any c_j, ℓ_j and ϕ. In consequence, since utility is additively separable with respect to time, if consumption and leisure were simultaneously increased by the factor $1+\phi$ at any age, life-time utility would increase by the same factor. With this considerations let us again turn to our simulation model. Assume an individual at state z_j had utility $V_{2008}(z_j)$ in the initial long-run equilibrium path and $V_t(z_j),\ t>2008$ after the policy reform. The compensating variation between the baseline and the reform scenario for the individual characterized by z_j is then given as

$$\phi=\frac{V_t(z_j)}{V_{2008}(z_j)}-1.$$

ϕ then indicates the percentage change in both consumption and leisure individual z_j would require in the initial equilibrium in order to be as well of as after the policy reform. The other way round we may say that an individual is ϕ better (or worse) off in terms of resources after the reform. If $\phi>0$, the reform is therefore welfare improving for this individual and vice versa.

A special rule applies to individual not having entered their economically relevant phase of life in the year before we conduct our pension reforms (the so-called future generations). We evaluate their utility behind the Rawlsian veil of ignorance, i.e. from an ex ante perspective where neither their skill level nor any labor market shock has been revealed. The concept of compensating variation thereby applies likewise.

The solid line in Fig. 3 shows the possible individual welfare consequences resulting from a generic reform experiment. For the sake of simplicity, we only consider a representative individual for each cohort.

The numbers on the abscissa denote birth years of different cohorts. Since households become economically active at age 20, the last cohort that was already participating in markets in year 2008 was born in 1988. This point is indicated by the intersection of the two axes. Consequently, when talking about future generations in the following, we mean all cohorts born after 1989.

The solid line in Fig. 3 indicates cohort-specific welfare consequences. As can be seen, the considered reform redistributes from currently living to future cohorts. In order to isolate the pure efficiency effects of the reform, we apply the hypothetical concept of a Lump-Sum Redistribution Authority (LSRA) used by Auerbach and Kotlikoff (1987) in a separate simulation. The LSRA thereby proceeds as follows: to all generations already being economically active in 2008 it pays lump-sum transfers or levies lump-sum taxes in order to make them as well off after the reform as in the initial equilibrium. Consequently their compensating variation amounts to zero. Having done that, the LSRA might have run into debt or build up some assets. It now redistributes this debt or assets across all future generations in a way that they all face the same compensating variation, confer the dashed line in Fig. 3. This variation can be interpreted as a measure of efficiency. Consequently, if the variation is greater than zero, the reform is Pareto improving after compensation and vice versa. With this concepts in hand, we can now proceed to our simulation results and the question of optimal progressivity of the pension system.

4.2. Flat pensions in the base version of the model

In this subsection, we will present some detailed simulation results from the transition towards a fully flat pension system, i.e. we set $\lambda=1$ from year 2009 onwards. The introduction of flat pensions comes with two reverse effects. On the

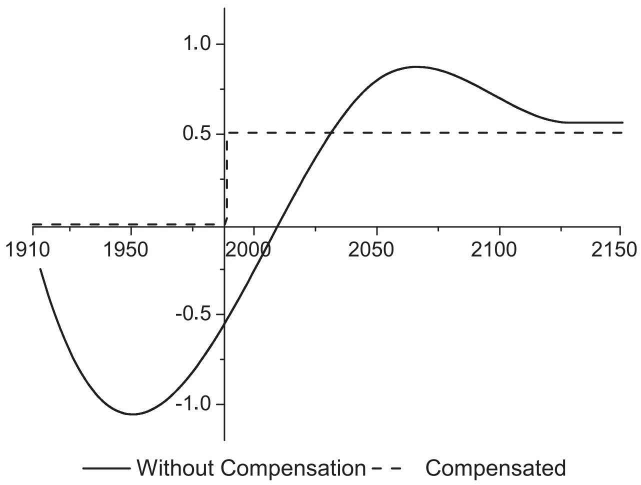

Fig. 3. Generic welfare consequences of a reform.

Table 4
Macroeconomic effects of flat pensions (base model version).

Year	2009	2020	2030	2040	2050	2060	∞
Macroeconomic aggregates							
Labor input	−5.6	−4.9	−4.8	−4.8	−4.7	−4.7	−4.7
Capital	0.0	−2.2	−2.8	−3.0	−3.0	−2.9	−3.0
Prices							
Wage	2.1	0.9	0.5	0.5	0.4	0.4	0.4
Interest rate	−0.3	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1
Consumption tax rate	1.6	2.0	2.2	2.3	2.4	2.4	2.4
Pension system							
Expenditure (in % of GDP)	−0.1	0.1	0.4	0.5	0.5	0.4	0.5
Contribution rate	0.5	0.8	1.2	1.4	1.3	1.4	1.4

one hand, whereas under the earnings related system a part of pension contributions was recognized as implicit savings, with flat pensions the complete contribution works as implicit tax.[4] Hence, the change to a flat pension benefit system increases labor supply distortions and decreases welfare even if contributions would remain constant. On the other hand, a non-earnings-related pension also provides insurance against labor market risk which tends to increase welfare.

Macroeconomic implications. First we consider the macroeconomic implications of our pension reform. Remember that the reform is introduced in year 2009. To clarify the impact of the reform on factor markets, Table 4 reports in the upper part the changes of employment and capital in and after year 2009. Since λ changes from 0 to a value of 1 immediately in year 2009, any worker can suddenly accumulate one earning point per year and in consequence the whole contribution to the pension system is perceived as a tax. This severely distorts labor supply so that labor input falls by 5.6% immediately. After the reform year labor increases slightly again since future cohorts experience a fall in incomes. Since the capital stock needs a longer time to adjust, the decline in household labor supply comes along with a sharp increase in wages and a drop in interest rates. During the transition the drop in labor input reduces individual savings and capital accumulation. With the decline in capital, factor markets again have to adjust and therefore wages decline and interest rates increase slightly again during the transition.

Due to the sharp drop in labor, output, household income and therefore aggregate consumption fall immediately. Afterwards this decline continues due to the fall in capital stock and wages. As government expenditure is held fix per capita and income tax revenues decline, the consumption tax rate has to increase in order to balance the government budget. In the pension system, the actual pension amount (APA), i.e. the monetary value of one earning point, is coupled with the evolution of lagged aggregate household income. Consequently, reductions in labor income also reduce pension benefits in our model. However, the fall in output and labor dominate, so that the expenditure share in GDP as well as the contribution rate steadily increase during the transition to the new long-run equilibrium.

Welfare and efficiency. With the above discussion in mind, we can now turn to the welfare effects of our reform. Table 5 summarizes welfare consequences measured in compensating variation for different cohorts. For agents already taking economic decisions in the reform year, we disentangle welfare effects in several ways. The left part reports for cohorts existing in the reform year average welfare changes grouped by their skill level. For future generations, we apply the concept of ex ante welfare and therefore only report one aggregate number per cohort. The first two columns indicate birth year of the respective cohorts and their age in the reform year 2009.

Retirees face tremendous welfare losses from the reform. These are due to the strong increase in consumption taxes and the downward adjustment of the actual pension amount and benefit level from year 2010 onwards. Obviously, welfare losses for retirees decrease with rising skill level since the fraction of pension benefits in total income declines. This also explains why welfare losses slightly decline for younger retirees. For the working cohorts in the reform year, welfare effects are not so clear-cut. Here the intra-generational redistribution from rich towards poor households induced by the progressive pension formula becomes most obvious. As a consequence, low-skilled workers realize significant welfare gains of about 1.0% of remaining resources while high-skilled workers lose. The implied redistribution effect becomes even stronger when we disaggregate cohorts according to their productivity level in the right part of Table 5. We thereby merge the lower one sixth and the higher one sixth of the overall productivity distribution in the states “low” and “high”. The remaining part of the distribution is captured in state “middle”.[5] As reported in Table 5, welfare of individuals in the bottom one sixth of the income distribution rises by up to 2.5%, while average welfare especially of the highest one sixth of the productivity distribution decreases by more than 1%. Noticeably, welfare gains are higher for older workers since they are closer to retirement.

Future generations gain from the introduction of flat pensions. This is mainly due to the sharp increase in wages and the insurance provision through the pension system outweighing the losses from labor market distortions. However, through time welfare gains decreases, since individual assets and therefore accidental bequests decline, which redistributes towards the earlier cohorts.

[4] For a detailed discussion of the implicit tax and savings rate during the life cycle see Fehr and Kindermann (2010, p. 423).

[5] We chose this partition of the work force due to computational reasons.

Table 5
Welfare effects of flat pensions (base model version).[a]

Birth year	Age in 2009	Without LSRA						With LSRA
		By skill level			By productivity			
Retirees		*Low*	*Middle*	*High*				
1920	89	−2.44	−2.32	−2.08				0.00
1940	69	−2.22	−2.09	−1.87				0.00
Workers		*Low*	*Middle*	*High*	*Low*	*Middle*	*High*	
1960	49	0.93	0.23	−0.63	2.50	−0.15	−1.18	0.00
1980	29	1.03	0.50	−0.58	2.07	0.21	−0.77	0.00
Future generations								
2000	9		0.35					−0.46
2020	–		0.18					−0.46
2060	–		0.22					−0.46
∞	–		0.20					−0.46

[a] In percent of initial resources.

Table 6
Macroeconomic effects of flat pensions (with disability).

Year	2009	2020	2030	2040	2050	2060	∞
Macroeconomic aggregates							
Labor input	−6.2	−5.2	−5.2	−5.2	−5.2	−5.1	−5.2
Capital	0.0	−2.6	−3.1	−3.0	−2.9	−2.9	−3.0
Prices							
Wage	2.3	0.9	0.8	0.8	0.8	0.8	0.8
Interest rate	−0.3	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1
Consumption tax rate	1.7	2.2	2.4	2.5	2.6	2.6	2.6
Pension system							
Expenditure (in % of GDP)	−0.1	0.1	0.3	0.4	0.3	0.3	0.4
Contribution rate	0.5	0.7	1.0	1.2	1.1	1.1	1.2

Finally, lets turn to welfare effects after LSRA compensation payments. As mentioned above, the LSRA makes all existing cohorts as well off as in the benchmark simulation and redistributes resources across future generations to make them all face the same welfare changes. The evolution of welfare after compensation is depicted in the right part of Table 5. We find that the reform induces losses for any future generation of 0.46% of initial resources. The introduction of flat pensions comes along with two major efficiency consequences: on the one hand, insurance provision against labor market risk causes efficiency to rise while, on the other hand, increasing labor market distortions reduce it. We find in this reform scenario that the latter outweighs the former and therefore the introduction of completely flat pension benefits is Pareto inferior. Before we alter the flat-benefit fraction λ we check how these results depend on the specific institutional structure of the base model version.

4.3. Disability risk and disability pensions

Beneath labor market shocks, individuals are exposed to disability risk. As already mentioned in the introduction, about 20% of new entries into the German pension system were due to disability. Consequently, we follow DíazGiménez and DíazSaavedra (2009) and assume that individuals face exogenous disability shocks that force them to retire early and receive disability pensions. The individuals' state now changes to

$$z_j = (s, a_j, ep_j, \eta_j, d_j, o_j) \in \mathcal{Z},$$

where both the disability state d_j and the retirement state o_j change from 0 to 1 in the moment the agent receives the disability shock.[6] If disabled persons retire before age 60 the calculation of earning points assumes that they had worked up to the age of 60 with their average productivity. Hence, their pension gets subsidized in order to correct for the missing years of work. If they retire before age 63 – the so-called normal retirement age for disability pensions – the sum of earning points will be reduced by 3.6% for each year of early retirement. This adjustment is restricted to three years, so that a disabled worker who has to retire before age 60, will face an adjustment factor of $(63-60) \cdot 3.6 = 10.8\%$. In order to account

[6] For simplicity we introduce the retirement state already in this subsection, although we have to distinguish between disability and old-age retirement only with an active retirement choice in the next subsection.

for utility costs of this bad health shock, we restrict individual leisure consumption to a value of $h=0.85$, which reflects the time cost of health care, e.g. visiting a doctor, or the utility costs of a reduced quality of life. This generates a reasonable welfare loss from disability, see below.

Starting from age 30, we assume the probabilities of becoming disabled to be positive. Hagen et al. (2010) report disability risk for workers with different skill levels between the ages 30 and 59. They find that the probability to become disabled increases exponentially with age. Consequently, we also assume exponential growth of disability risk throughout the working life and extrapolate the probabilities mentioned above from age 59 up to age 70. Fig. 4 compares the evolvement of disability risk in Hagen et al. (2010) on the left hand side with the probabilities used in our model on the right hand side. Interestingly, disability risk seems to decrease from age 58 to 59. According to Hagen et al. (2010), this however is due to the fact that agents facing disability shocks so late in life might also be eligible to a regular old-age pension. Since becoming eligible for disability pensions is quite some effort – one e.g. needs detailed medical examinations and to fill out many forms – individuals may choose the easier way of becoming regular old-age pensioners. Hence, the probability estimates tend to be biased downwards towards age 59.

The left part of Fig. 5 shows the loss in income for individuals becoming disabled at different ages in the initial equilibrium measured as compensating variation à la Hicks. Interestingly, the higher an agent's skill level, the higher his losses from disability. This is not intuitive in a perfectly earnings related pension system. However, it becomes clear from the fact that initial income is very similar in the three skill classes, but the higher skilled face the steeper increase in labor productivity throughout the life cycle, see Fig. 8. Consequently, their forgone earnings from disability are especially large at the beginning of the life cycle, while at older ages this difference shrinks noticeably. Furthermore, utility costs are lowest around the age of 45. This is due to the fact that labor income peaks at this point in the life cycle. Since earning points of a disability pensioner are calculated by assuming he had worked the remaining years until age 60 earning his average income, benefits from this disability subsidy reach their maximum at age 45 and decline afterwards. According to Torrance et al. (1982) direct utility losses from disabilities that make individuals unable to work may amount from 30% to 70%. Consequently, we feel that a maximum average welfare loss of 22% is a quite conservative estimate.

On the aggregate level the introduction of disability pensions only slightly changes the initial equilibrium reported above. We adjust the discount rate in order to keep the capital output ratio at 3.5 and the initial APA value (i.e. the pension replacement rate) in order to keep the contribution rate at 19.9%. As in reality, the shares of disability and old-age pensions of aggregate pension benefits are 12% and 88%, respectively. Since we keep the initial consumption tax rate at 17%, the public good share in output slightly adjusts.[7]

With disability pensions the implicit tax rate decreases especially for low-skilled households. Consequently, when we implement the flat pension reform, there is a stronger reduction in labor supply in the short run. In 2009 labor input is reduced by 6.2% instead of 5.6% in Table 4. In the medium and long run labor input rises again slightly but the relative reduction is always stronger than before. Since the capital stock falls similarly as in the base model, wages now increase stronger. Lower income tax revenues induce a higher increase in consumption taxes compared to Table 4. The resulting welfare consequences for different cohorts of Table 7 can be directly compared to Table 5.

Most retirees now experience stronger welfare losses since benefits are reduced more and consumption taxes are increased more than before. Although disability pensioners consume less leisure than regular old-age pensioners, their pension benefits are significantly lower, so that goods consumption has less weight in their utility. This explains why they are hurt less by the consumption tax increase in the right part of Table 7. The increase in the consumption tax rate during the transition also explains why disabled retirees of age 49 in the reform year are worse off than those who are 20 years older. Since the implicit tax rate jumps up much stronger after the reform than before, low-skilled older workers experience much lower welfare gains than in the simulation without disability risk. Younger workers are better off than before since disability risk is now better insured. This also explains why especially future cohorts are much better off. On first sight surprisingly, aggregate efficiency in the right column further decreases compared to Table 5. However, the right part of Fig. 5 documents that now many low-skilled workers are better off when they get disabled. Consequently, in the present calibration, the flat benefit system induces an “overinsurance” of disability risk. The insurance benefits are therefore maximized at a lower progressivity level.

4.4. Endogenous retirement

The next extension of the model allows for an explicit choice of retirement similar as in SánchezMartín (2010). Starting at age 60, agents may choose to exit the labor force and retire. Retirement is driven by the change in welfare from exiting the labor force, i.e. an agent chooses to retire if

$$V(z^r_{j+1}) \geq V(z^w_{j+1}),$$

where z^r_{j+1} and z^w_{j+1} indicate the pensioner and the worker state, respectively. If an individual retires, he/she can spend more time on leisure consumption, but has to accept lower income from pension benefits. Besides the already mentioned normal retirement age of 63 for disability pensions, the German pension system has a (regular) normal retirement age for old-age

[7] Appendix C provides some key figures of the initial equilibrium of the different model versions.

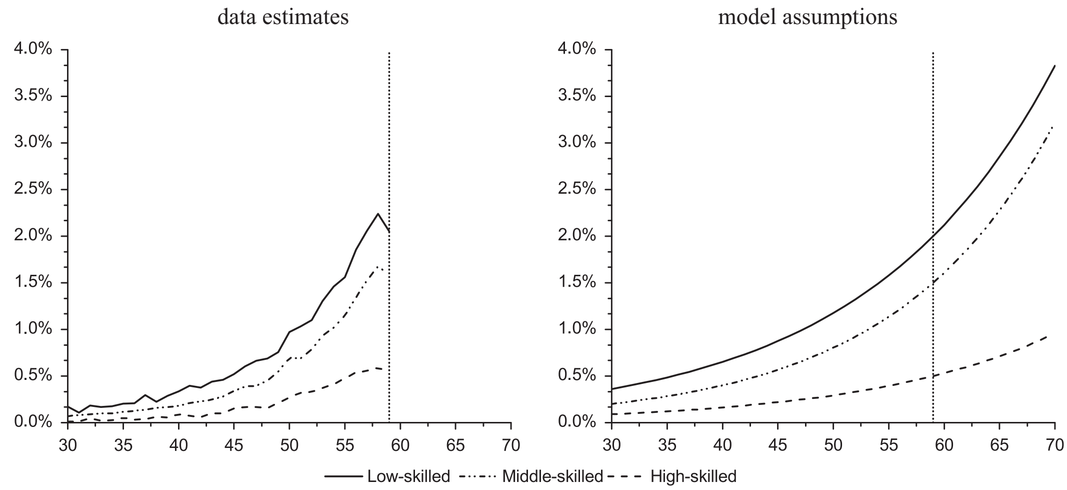

Fig. 4. Disability risk for different skill levels.

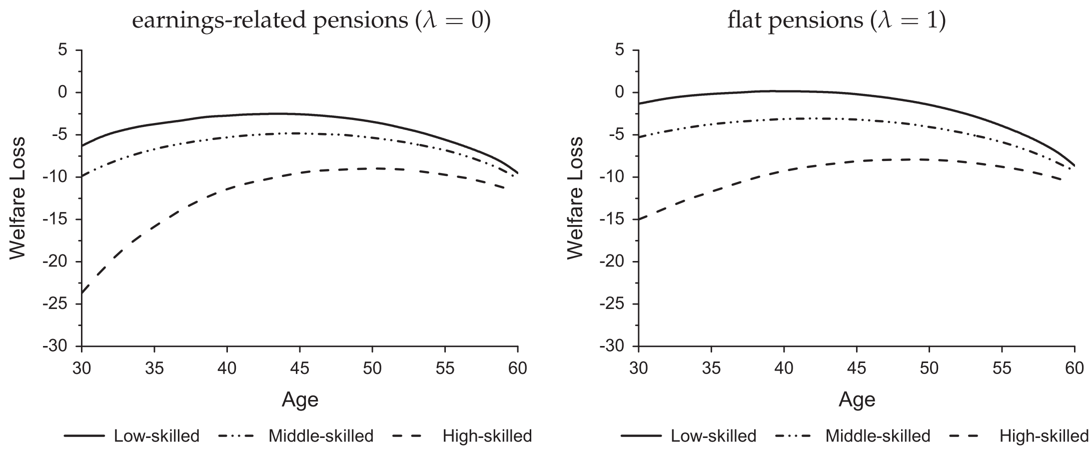

Fig. 5. Welfare loss from disability by age (without and with flat pensions).

Table 7
Welfare effects of flat pensions (with disability).[a]

Birth year	Age in 2009	Without LSRA							With LSRA
		By skill level			By productivity/status				
Retirees		*Low*	*Middle*	*High*		Old-age		Disab.	
1920	89	−2.29	−2.56	−2.31		−2.80		−2.50	0.00
1940	69	−2.32	−2.19	−1.99		−2.61		−2.23	0.00
Workers		*Low*	*Middle*	*High*	*Low*	*Middle*	*High*		
1960	49	0.19	−0.21	−0.85	2.15	−0.44	−1.33	−2.58	0.00
1980	29	1.34	0.75	−0.44	3.65	0.08	−1.11	–	0.00
Future generations									
2000	9		0.54						−0.60
2020	–		0.39						−0.60
2060	–		0.40						−0.60
∞	–		0.38						−0.60

[a] In percent of initial resources.

pensioners, which is currently age 65. For every year a regular worker retires before this age, he again has to incur a reduction in pension benefits of 3.6%. Fig. 6 summarizes the factors by which pensions will be adjusted for different types of retirees.

When we allow people to retire based on these economic considerations, we get a retirement pattern which is shown in Fig. 7. The model does a moderate job in matching the actual retirement behavior. While we almost perfectly fit the number of retirees at ages 60 and 63, we are lacking the huge peak at age 65. There maybe several reasons for this. On the one hand,

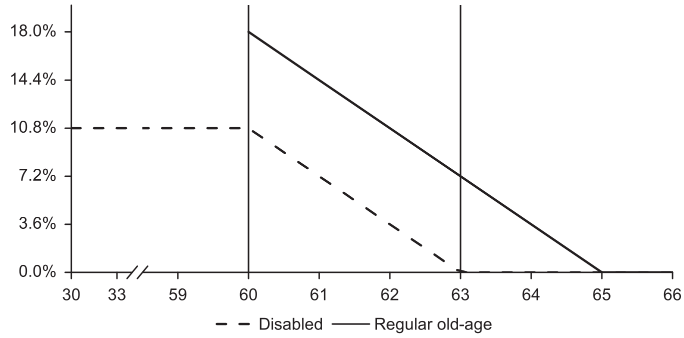

Fig. 6. Adjustment factors by groups of pensioners.

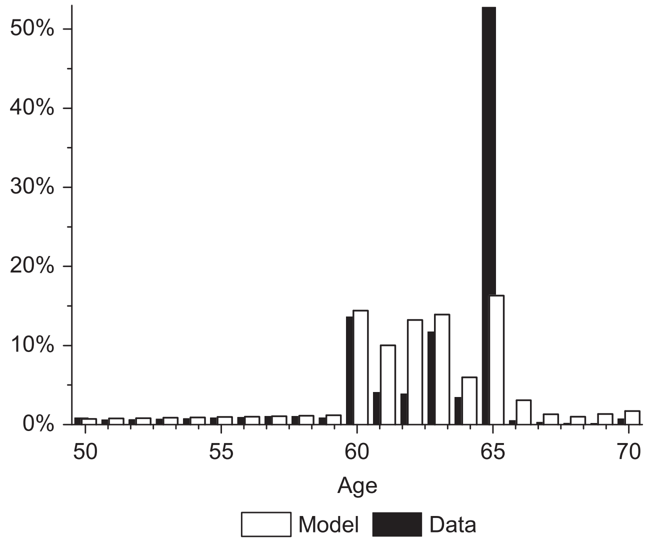

Fig. 7. Retirement pattern in model and data.
**Source*: DRV (2010). Own calculations for year 2009.

there are many more institutional restrictions on when people are actually eligible for retirement that our model does not capture. For example, people can retire early due to health problems or unemployment, there are special rules for long term insured, etc. On the other hand, besides pure economic considerations there are numerous other aspects which affect the decision to retire in practice. For example, people might also have other benefits (such as social acceptance) besides the pure labor income from staying on the job.[8]

Again, when we compute the initial equilibrium, we adjust the discount rate, the replacement rate and the public goods provision in order to generate the same capital–output ratio and identical tax and contribution rates as in the base version of the model.

Table 8 reports the macroeconomic consequences of the flat benefit reform with disability risk and endogenous retirement in detail. Compared to Table 6 the initial drop in labor input is stronger (from −6.2% to −6.7%) but during the transition the fall in labor is almost the same as in Table 6. On the other hand, the capital stock is now crowded out much stronger and falls in the long run by 3.6% (instead of 3.0% in Table 6). As a result wages are significantly lower during the transition. When introducing flat pensions, individual pension benefits cannot be increased anymore by working longer hours within a year, but only by working more years. As a consequence, the average retirement age rises sharply by more than one year initially for all skill classes. During the transition, the retirement age for lower skilled falls again slightly while it increases further for high-skilled households. Of course, this reflects the income redistribution during the transition. The resulting welfare consequences are reported in Table 9.

Again, due to the initial fall in labor input, the welfare of all existing retirees decreases further compared to Table 7. At the same time, older and middle-aged workers benefit since they can now decide to retire later. The same applies to future cohorts who were born before the reform. However, cohorts born after the reform lose compared to Table 7 due to the lower

[8] In principle, such non-monetary rewards could be captured indirectly in order to better match the actual retirement behavior. However, the focus here is on the change in retirement behavior due to the reform.

Table 8
Macroeconomic effects of flat pensions (with endogenous retirement).

Year	2009	2020	2030	2040	2050	2060	∞
Macroeconomic aggregates							
Labor input	−6.7	−4.9	−5.0	−5.1	−5.1	−5.1	−5.1
Capital	0.0	−2.6	−3.1	−3.3	−3.3	−3.4	−3.6
Prices							
Wage	2.4	0.9	0.7	0.7	0.6	0.6	0.5
Interest rate	−0.4	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1
Consumption tax rate	1.8	2.0	2.3	2.4	2.5	2.6	2.6
Pension system							
Expenditure (in % of GDP)	−0.1	−0.2	0.2	0.3	0.3	0.4	0.4
Contribution rate	0.5	0.2	0.8	1.1	1.1	1.1	1.2
RA ($s=1$) (in month)	0	13	11	10	9	7	8
RA ($s=2$) (in month)	0	13	13	13	13	12	12
RA ($s=3$) (in month)	0	14	15	17	18	15	15

Table 9
Welfare effects of flat pensions (with endogenous retirement).[a]

Birth year	Age in 2009	Without LSRA							With LSRA
		By skill level			By productivity/status				
Retirees		*Low*	*Middle*	*High*		Old-age		Disab.	
1920	89	−2.82	−2.65	−2.36		−3.01		−2.62	0.00
1940	69	−2.39	−2.25	−2.06		−2.76		−2.35	0.00
Workers		*Low*	*Middle*	*High*	*Low*	*Middle*	*High*		
1960	49	0.22	−0.13	−0.65	2.45	−0.34	−1.37	−2.70	0.00
1980	29	1.40	0.81	−0.29	3.69	0.19	−1.06	–	0.00
Future generations									
2000	9		0.58						−0.59
2020	–		0.29						−0.59
2060	–		0.25						−0.59
∞	–		0.22						−0.59

[a] In percent of initial resources.

wage growth. While endogenous retirement significantly affects the intergenerational income redistribution, the last column in Table 9 documents that it does not change much aggregate efficiency.

4.5. Optimal progressivity of the pension system

While it turned out in the last subsection that a completely flat benefit system would reduce aggregate efficiency, it is not clear yet, whether a partially progressive pension system will also lead to efficiency losses. Having in mind that labor market distortions rise quadratically in tax rates, there may be a combination of flat and earnings-related pensions that improves efficiency. This is clarified in Table 10 which shows aggregate efficiency gains and losses for different choices of λ for our different model versions. Note that the aggregate efficiency value for $\lambda=0.0$ is zero since nothing has changed and the value for $\lambda=1.0$ in the right column was already reported in the respective tables above.

Table 10 reveals an interesting insight into the opponent efficiency effects of pension progressivity. With an increasing flat pillar in the pension system, obviously both insurance provision and labor market distortions rise. However, at lower values of λ, the insurance effect clearly dominates labor supply distortions. Consequently, in the base version of the model aggregate efficiency gains are maximized at a flat benefit fraction of 20%. With higher values of λ, aggregate efficiency starts to decline again as labor supply distortions outweigh the insurance effects. However, the flat benefit fraction of pensions has to increase to 50% until pension progressivity finally generates efficiency losses.

When disability risk is added to the model in the second line of Table 10 the improved insurance provision induces a positive efficiency effect for lower values of λ. Consequently, aggregate efficiency gains are now much higher initially. The optimal progressivity of the pension system increases to $\lambda=0.3$ compared to the base model.[9] When λ increases further labor supply distortions rise, but also insurance gains decline at some point since people receive more insurance as actually

[9] Actually, if we take a closer look at values of λ it even increases to $\lambda=0.32$!

Table 10
Aggregate efficiency effects of alternative progressivity levels.[a]

Model version	λ									
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Base	0.054	**0.080**	0.063	0.037	−0.001	−0.065	−0.139	−0.224	−0.331	−0.463
+ disability	0.182	0.309	**0.355**	0.325	0.218	0.054	−0.125	−0.298	−0.453	−0.596
+ retirement	0.175	0.307	**0.373**	0.343	0.232	0.075	−0.105	−0.271	−0.430	−0.585

[a] In percent of initial resources.

demanded. Consequently, efficiency effects turn negative at a flat pension fraction of 70% and efficiency losses are even higher than in the base case for more progressive pension systems.

The next line shows that endogenous retirement has a very modest impact on aggregate efficiency. Especially for higher flat benefit fractions, aggregate efficiency improves slightly compared to the previous model version. We have already reported above that the increase in progressivity induces households to substitute work at younger ages for later retirement. As it seems the additional extensive margin reduces efficiency losses from labor supply distortions slightly. However, the optimal mix of flat and earnings-related pensions remains stable.

4.6. Sensitivity analysis

In this section we present some sensitivity analysis with respect to central assumptions of the model. Since the insurance benefits heavily depend on the estimated income process, we first present alternative estimation strategies and discuss their consequences for optimal progressivity. Then we report the sensitivity of our results with respect to alternative parameter specifications of the model. Note that we only focus on the derived optimal values for λ. More detailed information on macroeconomic and welfare effects for suboptimal levels of λ is available upon request.

4.6.1. Earnings profiles and earnings risk

As already mentioned in the calibration section, there is a discussion in the literature whether to use cohort or time effects in the estimation of earnings profiles and risk. In order to take this discussion into account, we performed two additional estimations. In the first, we estimated Eq. (1) with cohort instead of time fixed effects. Owing to the perfect collinearity of year, age and cohort, this only affects the mean wage profiles of the three different education levels. The risk process, however, stays the same. The profiles with year and cohort fixed effects are shown in the right part of Fig. 8. We find a similar pattern as Heathcote et al. (2005) or Huggett et al. (2011), namely that both the mean and the variance of wages are higher at older ages under cohort effects. In our case, the change in variance has to be solely attributed to a higher wage rate for high-skilled workers.[10]

For our second sensitivity estimation, we restrict our data to cohorts born in 1960 and later and to the years 2000–2008.[11] By doing so, we would be able to identify whether there were significant changes in the means or risk properties of wages in recent years or for younger cohorts. We again use Eq. (1) with time effects. The mean wage profiles for the three different schooling levels did not change significantly.[12] Yet, there are slight differences in the risk properties of wages. Table 11 summarizes our estimation results. While the autocorrelation coefficients of wage risk are not very much affected by this sample selection, the overall process variance for the lower education group has increased a bit.

Table 12 shows the optimal pension progressivity in our two sensitivity cases. In the row cohort effects, we used the wage profiles shown in the right part of Fig. 8 and the same wage risk process as shown in the calibration section. Since the variance in wages between the different skill groups increases substantially in this case, the optimal progressivity of the pension system increases to a value of 0.42. With ex ante insurance effects being larger, the efficiency gain in this experiment amounts to 0.611 instead of 0.373. If, on the other hand, we keep the wage profiles from the time effects estimation and employ the wage risk process determined from our restricted sample, there is not much of a change in optimal progressivity. This is not very surprising, since the estimated risk processes do not differ so much from the benchmark case. As the variance of wages rises a little bit for the lower educated, the optimal progressivity of the pension system increases to 0.34 and the efficiency gain is slightly higher than in the benchmark scenario.

[10] The major difference between our estimation framework and that of Huggett et al. (2011) is that the latter only come up with an estimate of the variance of earnings over the life cycle, but are not concerned about the autocorrelation of wage shocks. In that sense, their analysis could also be done with repeated cross-sections. We, on the other hand, estimate autocorrelated wage shocks over the life cycle and want to exploit the panel structure of our data to do so.

[11] The original sample included cohorts born in 1925 or later and the years 1984–2008.

[12] Note that we can obviously only estimate wage profiles up to the age of 2008 − 1960 = 48 in this case. Since the mean profiles did not change, we use the profiles estimated in the calibration section.

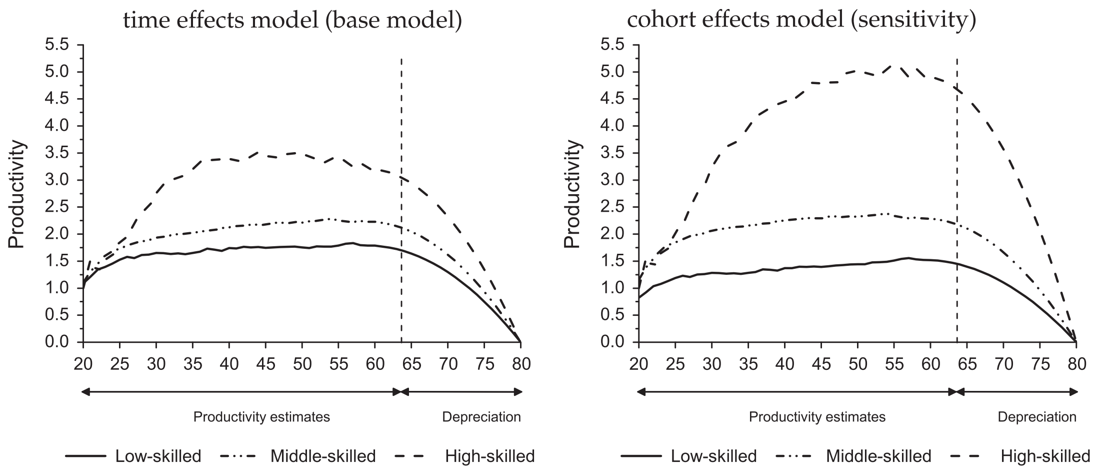

Fig. 8. Productivity throughout the life cycle: time vs. cohort fixed effects.

Table 11
Parameter estimates for individual productivity risk (young sample).

	Low-skilled	Middle-skilled	High-skilled
AR(1) correlation ϱ	0.93421	0.95262	0.94754
	(0.00944)	(0.00225)	(0.00450)
Transitory variance σ_ε^2	0.04063	0.03210	0.04249
	(0.01721)	(0.00637)	(0.01549)
Total variance σ_η^2	0.31927	0.34695	0.41584
	(0.01622)	(0.00627)	(0.01504)
Error variance σ_ν^2	0.09392	0.08555	0.09282
	(0.00536)	(0.00175)	(0.00379)

Table 12
Sensitivity analysis (1).

Earnings process	λ_{opt}	Efficiency[a]
Cohort effects	0.42	0.661
Young sample	0.34	0.381

[a] In percent of initial resources.

Table 13
Sensitivity analysis (2).

Parameter	λ_{opt}	Efficiency[a]
$\mu=2$	0.04	0.014
$\mu=0$	0.00	0.000
$\rho=1$	0.19	0.074
$\gamma=0.8$	0.38	0.359
$\tau=0.13$	0.37	0.350
Uniform mortality	0.32	0.351

[a] In percent of initial resources.

4.6.2. *Preference, mortality and budget parameters*

Of course, the optimal progressivity of the pension system depends also on the specific parameter values of the model. Insurance benefits mainly depend on risk aversion while labor supply distortions depend on labor supply elasticities. Consequently, the first two rows of Table 13 report the optimal progressivity when we reduce relative risk aversion. If we simulate the traditional CRRA utility specification (i.e. when relative risk aversion is reduced from 4.0 to 2.0), benefits from insurance provision decrease sharply so that efficiency gains from higher progressivity are very small and the optimal progressivity is already reached at $\lambda=0.04$. Of course, when households are risk neutral (i.e. when μ is reduced to 0), then insurance benefits ceased to exist so that it is optimal to have a perfectly earnings-related pension system as in the benchmark. Positive λ values only generate labor supply distortions and aggregate efficiency losses.

Of course, efficiency gains as well as the optimal progressivity would rise, if we increased risk aversion in the model.[13] In the third line of Table 13 we simulate the model with a Cobb–Douglas sub-utility function $u(c,\ell)=c^{\alpha}\ell^{1-\alpha}$ with $\alpha=0.4$. In this case the Frisch elasticity can be computed from $\eta^{Frisch}=(\ell/(1-\ell))[\gamma(1-\alpha)-\alpha]$. Now the Frisch elasticity increases from 0.87 to 1.05. Consequently, the optimal progressivity decreases in the third line of Table 13 to 0.19. The next line shows the consequences of a higher intertemporal elasticity of substitution γ. In this case the intertemporal substitution of labor supply towards the end of the employment phase will rise. We have argued already above that flexibility at the extensive margin reduces labor supply distortions at the intensive margin. Consequently, efficiency gains from higher progressivity increase and the optimal progressivity level rises to $\lambda=0.38$.[14] In the next line we analyze the sensitivity of optimal progressivity with respect to the generosity of the pension system. In the initial equilibrium we reduce the replacement rate so that the initial contribution rate declines from 19.9% to 13%. The lower contribution rate implies lower labor supply distortions so that the optimal fraction of flat pensions increases to 37%. Finally, in the last line of Table 13 we report the impact when we get rid of the skill-dependent life expectancy. In Germany there is currently a discussion to increase pension progressivity in order to balance the redistribution from poor towards rich pensioners arising from differences in life expectancy, see Breyer and Hupfeld (2009). Our results indicate that skill-specific mortality has a negligible impact on economic efficiency and the optimal progressivity level.[15]

We have also simulated our fully extended model with constant factor prices (i.e. a small open economy), but the consequences for economic efficiency were not significant. The latter is not surprising since factor price changes mainly affect the intergenerational redistribution but not economic efficiency.

5. Discussion

This paper aims at analyzing the economic effects of pension progressivity and the optimal progressivity of the pension system. In order to clarify this question, we construct a model of overlapping generations which features realistic labor income, longevity and disability risk, endogenous labor supply at both the intensive and the extensive margin. We calibrate our base year to the German economy and study the transition from the fully earnings related pension regime towards systems with a flat pillar. We find that higher pension progressivity reduces employment, capital accumulation and output growth and increases future tax and contribution rates. Nevertheless a pension system consisting of 30% flat and 70% earning related benefits maximizes aggregate efficiency, since positive insurance effects dominate the rising labor supply distortions. Our results indicate that this optimal progressivity level is quite robust with respect to the wage process as well as preference and policy parameters. Consequently, Germany should implement policy reforms which increase redistribution within the pension system. Furthermore, recent reforms in Western countries which reduced pension progressivity might have gone too far.

Nevertheless our approach also has some shortcomings. First, we neglect the information problem to identify the true disability level which is very important in practice and discussed in the theoretical and empirical literature, see e.g. Diamond and Sheshinski (1995) or Duggan et al. (2007). In our setup, there is only one disability shock which makes the person automatically eligible to disability insurance benefits. This simplification could be justified, since this type of uncertainty is not the major source of income risk. Considering various disability levels, including information constraints, computational time would dramatically increase however it would unlikely change the central economic argument. A second shortcoming of the model relates to the fact that we completely neglected means-tested minimum income benefits. At the moment, only about 2.5% of retirees in Germany receives such benefits. However, since old-age poverty is expected to rise substantially in the future, this fraction would increase significantly. On the one hand, a means-tested minimum income guarantee also provides insurance against labor market risk and old-age poverty. Hence, it might be that the efficiency gains from insurance are dampened when such benefits are incorporated. On the other hand, means-tested minimum income guarantees also severely distorts old-age savings. Consequently, it is not clear how this feature affects the optimal progressivity which indicates that means-tested income guarantees should be left to future research. Finally, we have not considered the interaction between the progressivity of the income tax system and the optimal progressivity of the pension system. Already Fehr and Habermann (2008) have shown that optimal pension progressivity will be higher (lower) when the progressivity of the income tax system is reduced (increased). The question is then which is the optimal combination of income tax and pension progressivity. Since the answer is complicated by the fact that the former redistributes annual income while the latter redistributes lifetime income, it has to be left to future research.

Conflict of interest

This article reflects the personal views of the authors and not those of the German Council of Economic Experts.

[13] As discussed in Cecchetti et al. (2000) higher values for risk aversion are not unrealistic.
[14] We have also increased γ in the model without endogenous retirement. In this case there is no effect on optimal progressivity.
[15] This should hardly be surprising, since the claim of Breyer and Hupfeld (2009) is purely based on equity and not related to efficiency arguments.

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Appendix A. Technical appendix

In this appendix we want to give a more equation based definition of our model and define what an equilibrium path is.

A.1. Detailed formulation of the household problem

In order to state the household optimization problem in a recursive fashion, we first want to define in detail what the household state vector is.

Definition 1 (*Household state vector*). Let

$$z = (j, s, a, ep, \eta, d, o) \in \mathcal{Z} = \mathcal{J} \times \mathcal{S} \times \mathcal{A} \times \mathcal{P} \times \mathcal{E} \times \mathcal{D} \times \mathcal{R}$$

where $\mathcal{J} = \{1, 2, \ldots, J\}$, $\mathcal{S} = \{1, 2, \ldots, S\}$, $\mathcal{A} = [0, \infty]$, $\mathcal{P} = [0, \infty]$, $\mathcal{E} = [0, \infty]$, $\mathcal{D} = \{0, 1\}$, and $\mathcal{R} = \{0, 1\}$. Then z completely describes the individual state of a household. In the following we will use the abbreviation:

$$z_j = (s, a, ep, \eta, d, o)$$

for the sake of simplicity.

With this definition of the household state, we can now turn to the description of the household optimization problem. We define this problem separately for working and retired agents. The decision problem of an individual with state $z_j = (s, a_j, ep_j, \eta_j, 0, 0)$ at age j and time t, i.e. an individual that is still participating in the labor market, is given by

$$V_t\left(z_j\right) = \max_{c,\ell}\Bigg\{u(c,\ell)^{1-1/\gamma} + \beta\psi_{j+1,s}\Big[(1-\pi^d_{j+1,s}) \int_{\mathcal{E}}(1-o_{j+1})V_{t+1}(z^w_{j+1})^{1-\eta} + o_{j+1}V_{t+1}(z^r_{j+1})^{1-\eta})\pi(\eta'|\eta,s)\, d\eta' + \pi^d_{j+1,s}V_{t+1}(z^d_{j+1})^{1-\eta}\Big]^{(1-1/\gamma)/1-\eta}\Bigg\}^{1/(1-1/\gamma)},$$

with the terminal condition $V_t(z_{J+1}) = 0,\ \forall t$. The three different combinations for z_{j+1} define the states in which the agent is still working in the next period, in which he chooses to retire and in which he receives a disability shock, i.e.

$$z^w_{j+1} = (s, a_{j+1}, ep_{j+1}, \eta_{j+1}, 0, 0), \quad z^r_{j+1} = (s, a_{j+1}, ep_{j+1}, 0, 0, 1) \quad \text{and} \quad z^d_{j+1} = (s, a_{j+1}, ep_{j+1}, 0, 1, 1).$$

The continuous state variables a, ep and η are subject to the following laws of motions: assets evolve according to

$$a_{j+1} = (1 + r_t)a_j + y + b - \tau_t y - T_t(y, 0, r_t a_j) - (1 + \tau_{c,t})c,$$

with labor income $y = w_t e_j \eta_j (1-\ell)$,[16] accidental bequests b, the pension contribution rate τ_t, the tax schedule $T_t(\cdot)$ and the consumption tax rate $\tau_{c,t}$. As already mentioned above, individuals accumulate earning points according to the formula

$$ep_{j+1} = ep_j + \left[\lambda_t + (1-\lambda_t)\frac{y}{\overline{y}_t}\right]$$

with the economy wide average labor income $\overline{y}_t$ at time t. Finally, log labor productivity shock evolves according to the AR (1) process

$$\log \eta_{j+1} = \varrho \log \eta_j + \varepsilon \quad \text{with} \quad \varepsilon \sim N(0, \sigma^2_\varepsilon).$$

According to this process, we can calculate the distribution function $\pi(\cdot|\cdot,\cdot)$ of η_{j+1} conditional on the realization η_j and the individual skill level s. Note that the labor productivity shock does not play a role for retired agents. Hence, we set η_j to zero when an agent chooses to retire.

An important factor for calculating expected future utility is the retirement decision in the next period o_{j+1}. Note that the retirement decision is made after all current shocks are revealed, i.e. when disability status and labor productivity are known. The individual then makes a retirement decision via comparing utilities from working for another period $o_{j+1} = 0$

[16] Thereby, $e_j = e(z_j)$ denotes labor productivity that depends on age and skill level.

and switching to the state of a pensioner $o_{j+1}=1$, i.e.

$$o_{j+1}=\begin{cases}1 & \text{if } V(z^r_{j+1})>V(z^w_{j+1}) \text{ and}\\ 0 & \text{otherwise.}\end{cases}$$

For the already retired, the optimization problem is much simpler. Since they do not work anymore, their leisure consumption is already determined and they therefore only have to decide about regular consumption via

$$V_t(z_j)=\max_c\{u(c,\ell)+\beta\psi_{j+1,s}V_{t+1}(z_{j+1})\}$$

where $\ell=1$ for old-age and $\ell=h$ for disability pensioners. Next periods state vector is then given by

$$z_{j+1}=(s,a_{j+1},ep_j,0,d_j,1),$$

where assets evolve according to

$$a_{j+1}=(1+r_t)a_j+p-T_t(0,p,r_ta_j)-(1+\tau_{c,t})c.$$

Thereby

$$p(z_j)=\begin{cases}ep_j\times APA_t & \text{if } o_j=1 \text{ and}\\ 0 & \text{otherwise}\end{cases}$$

defines pension payments that depend on the amount of earning points accumulated during the working periods. Note that individuals above age 60 will not receive any accidental bequests in our model.

Having specified the household decision problem in detail, we finally want to define household choices and the measure of households. The former thereby defines the decisions of households at a specific state, the latter at which states households are located.

Definition 2 (*Optimal household decisions and measure of households*). For any $t\in\mathcal{T}=\{1,2,\ldots,\infty\}$ let

$$\mathbb{H}_t=\{c_t(\cdot),\ell_t(\cdot),o_t(\cdot),a_t{}'(\cdot),ep_t{}'(\cdot)\}$$

with functions

$$c_t(\cdot):\mathcal{Z}\rightarrow[0,\infty],\quad \ell_t(\cdot):\mathcal{Z}\rightarrow[0,1],\quad o_t(\cdot):\mathcal{Z}\rightarrow\{0,1\},$$
$$a_t{}'(\cdot):\mathcal{Z}\rightarrow[0,\infty] \quad\text{and}\quad ep_t{}'(\cdot):\mathcal{Z}\rightarrow[0,\infty].$$ [17]

Furthermore let for any $t\in\mathcal{T}$

$$\xi_t(\cdot):\mathcal{Z}\rightarrow[0,\infty].$$

be a measure on the measurable space

$$(\mathcal{Z},P(\mathcal{J})\times P(\mathcal{S})\times B(\mathcal{A})\times B(\mathcal{P})\times B(\mathcal{E})\times P(\mathcal{D})\times P(\mathcal{R})),$$

where B is the Borel σ−algebra of a continuous set and P the power set of a discrete set. We say that $\mathbb{H}_t$ is a set of optimal household decisions, if – given prices and public policy – $\mathbb{H}_t$ satisfies the above optimization problem. Furthermore, we denote by ξ_t the measure of households at time t.

A.2. Firms behavior

Next, we want to define firms behavior.

Definition 3 (*Production plan and price set*). For any $t\in\mathcal{T}$ let $\mathbb{Q}_t=\{K_t,L_t\}$ be the production plan for the firms and denote by $\mathbb{P}_t=\{r_t,w_t\}$ the set of prices for capital and labor.

The factor prices are set in a competitive way, given the production plan $\mathbb{Q}_t$.

Definition 4 (*Competitive factor prices*). We say that factor prices are competitive, if

$$r_t=(1-\tau_k)\left(\frac{\partial Y_t}{\partial K_t}-\delta_k\right)^{18}\quad\text{and}\quad w_t=\frac{\partial Y_t}{\partial L_t}.$$

A.3. Governmental activity

The last sector to specify is the government sector.

[17] $a_t{}'$ and $ep_t{}'$ thereby define future assets a_{j+1} and earning points ep_{j+1} depending on the current state and consumption decisions.
[18] Note that, due to the perfect competition assumption, corporate taxes in our model works as additional taxes to capital income of households.

Definition 5 (*Tax and pension policy*). Let $\mathbb{G}_t = \{G_t, \tau_k, T_t(\cdot,\cdot,\cdot), B_t, \tau_{c,t}, \lambda_t, APA_t, \tau_t\}$ define governmental policy at time t, where G_t is public expenditure, τ_k the corporate tax rate, T_t the income tax schedule, B_t government debt holdings and $\tau_{c,t}$ the consumption tax rate at time t. APA_t and τ_t define the actual pension amount per earnings points and τ_t the contribution rate to the pension system.

The government adjusts the consumption tax rate in order to balance the tax system.

Definition 6 (*Budget balance of the tax system*). Given a set of household decisions $\mathbb{H}_t$, a measure of households ξ_t and a set of production plans $\mathbb{Q}_t$ and prices $\mathbb{P}_t$, we say that the tax system is balanced at time $t \in \mathcal{T}$, if

$$G_t + (1+r_t)B_t = \tau_c \int_{\mathcal{Z}} c_t(z) d\xi_t + T_y + \frac{\tau_k}{1-\tau_k} r_t K_t + (1+\kappa)B_{t+1}$$

with

$$T_y = \int_{\mathcal{Z}} T_t(w_t e(z)\eta(1-\ell_t(z)), p(z), r_t a_t(z))\, d\xi_t$$

defines revenue from income taxation.

On the other hand, the pension contribution rate is adjusted to balance the pension budget.

Definition 7 (*Budget balance of the pension system*). Given a set of household decisions $\mathbb{H}_t$, a measure of households ξ_t and a set of prices $\mathbb{P}_t$, we say that the tax system is balanced at time $t \in \mathcal{T}$, if

$$\tau_t \int_{\mathcal{Z}} w_t e(z)\eta(1-\ell_t(z)) d\xi_t = \int_{\mathcal{Z}} p(z) d\xi_t.$$

A.4. Equilibrium definition

Having specified the behavior of households, firms and the government, we can now turn to the definition of a competitive equilibrium in our model. Before we do that, we want to specify the economic environment households, firms and the government face.

Definition 8 (*Economic environment*). The economic environment in our model is defined as the set

$$\mathbb{E} = (\{\psi_{j,s}\}_{j\in\mathcal{J},s\in\mathcal{S}}, \{\pi^d_{j,s}\}_{j\in\mathcal{J},s\in\mathcal{S}}, \pi(\cdot|\cdot,\cdot), \Gamma(\cdot), \theta, \epsilon, \delta_k, \kappa)$$

of individual survival probabilities ψ, disability risk probabilities π^d, the conditional distribution function π for the evolvement of η, a distribution function for bequests $\Gamma : \mathcal{Z} \to [0,1]$ that satisfies

$$\int_{\mathcal{Z}} \Gamma(z) d\xi_t = 1.$$

and production technology θ, ϵ, δ_k and κ.

Obviously, when we denote individual bequests as $b(z) = \Gamma(z)Q_t$, where

$$Q_t = \frac{1}{1+\kappa} \int_{\mathcal{Z}} (1+r_t)a_{t-1}{}'(z)\big(1-\psi_{t-1,j,s}\big)\, d\xi_t,$$

we assure that no resources are wasted. Finally, we define an equilibrium path.

Definition 9 (*Competitive equilibrium*). Given an economic environment $\mathbb{E}$ and initial conditions K_1 and ξ_1, a competitive equilibrium is a set of optimal household decisions $\{\mathbb{H}_t\}_{t\in\mathcal{T}}$, production plans $\{\mathbb{Q}_t\}_{t\in\mathcal{T}}$, competitive factor prices $\{\mathbb{P}_t\}_{t\in\mathcal{T}}$ and budget balancing tax and pension policies $\{\mathbb{G}_t\}_{t\in\mathcal{T}}$ that satisfy the following conditions:

1. Market clearance:

$$K_t = \int_{\mathcal{Z}} a_t(z)\, d\xi_t - B_t, \quad \text{(Capital market)}$$

$$L_t = \int_{\mathcal{Z}} (1-\ell_t(z))e(z)\eta\, d\xi_t, \quad \text{(Labor market)}$$

$$\theta K_t^{\epsilon} L_t^{1-\epsilon} = \int_{\mathcal{Z}} c_t(z)\, d\xi_t + G_t + (1+\kappa)K_{t+1} - (1-\delta_k)K_t \quad \text{(Goods market).}$$

2. Law of motion: for any subset $C \subseteq \{2,\ldots,J\} \times \mathcal{S} \times \mathcal{A} \times \mathcal{P} \times \mathcal{E} \times \mathcal{D} \times \mathcal{R}$,

$$\xi_{t+1}(C) = \int_{\mathcal{Z}} \psi_{j+1,s} P_t(z,C)\, d\xi_t,$$

where

$$P_t(z,C)=\begin{cases}(1-\pi^d_{j+1,s})\pi(\eta'|\eta,s) & \text{if } (j+1,s,a'_t(z),ep'_t(z),\eta',0,0)\in C \text{ and } z=(j,s,a,ep,\eta,0,0),\\ 1-\pi^d_{j+1,s} & \text{if } (j+1,s,a'_t(z),ep'_t(z),0,0,1)\in C \text{ and } z=(j,s,a,ep,\eta,0,0),\\ \pi^d_{j+1,s} & \text{if } (j+1,s,a'_t(z),ep'_t(z),0,1,1)\in C \text{ and } z=(j,s,a,ep,\eta,0,0),\\ 1 & \text{if } (j+1,s,a'_t(z),ep'_t(z),0,d,1)\in C \text{ and } z=(j,s,a,ep,\eta,d,1),\\ 0 & \text{otherwise.}\end{cases}$$

For any $C=C_J\times C_S\times C_A\times C_P\times C_E\times C_D\times C_R\subseteq\{1\}\times\mathcal{S}\times\mathcal{A}\times\mathcal{P}\times\mathcal{E}\times\mathcal{D}\times\mathcal{R}$,

$$\xi_{t+1}(C)=\begin{cases}\sum_{s\in C_s}\varpi_s\left[\int_{C_E}\varphi_{0,\sigma^2_\varepsilon}(\eta)\,d\eta\right] & \text{if } 0\in C_A,\ 0\in C_P,\ 0\in C_D \text{ and } 0\in C_R,\\ 0 & \text{otherwise.}\end{cases}$$

Hereby, $\varphi_{0,\sigma^2_\varepsilon}$ denotes the probability density function of the normal distribution with mean 0 and variance σ^2_ε.

Definition 10 (*Stationary equilibrium*). A stationary equilibrium is a competitive equilibrium in which per capital variables and functions, as well as prices are constant over times and aggregate variables grow at the constant rate κ.

Appendix B. Computational appendix

This section gives an overview over the solution methods used to solve our model numerically. We distinguish between a micro- and a macroeconomic solution method. The former is used to solve the household problem, while the letter serves to compute equilibrium prices and quantities.

B.1. Solving the household problem

In order to compute a solution of the complex household problem, we discretize the continuous elements of our state space $\mathcal{Z}$. We therefore choose $\hat{\mathcal{A}}=\{a^1,\dots,a^{n_A}\}$, $\hat{\mathcal{P}}=\{ep^1,\dots,ep^{n_P}\}$, and $\hat{\mathcal{E}}=\{\eta^1,\dots,\eta^{n_E}\}$. We use the algorithm described in Kopecky and Suen (2010) to obtain an approximation to the distribution of η with our set $\mathcal{E}$ and a suitable probability function $\hat{\pi}(\cdot|\cdot,\cdot)$. For all the resulting discrete values of z_j we compute the optimal decision of households from the household optimization problem described above. Since $V_{t+1}(\cdot)$ consequently is also only known in a discrete set of points z_{j+1}, this maximization problem cannot be solved analytically. Therefore we have to use the following numerical maximization and interpolation algorithms to compute households optimal decision:

1. Compute household decisions at the last possible age J for all possible z_J. Note that the terminal condition $V_t(z_{J+1})=0$, households are not allowed to work anymore and they die for sure in the next period. Hence, they consume all their resources.
2. Find the solution to the household optimization problem for all possible z_j recursively using Powell's algorithm, see Press et al. (2001, 406 ff.). Since this algorithm requires a continuous function, we have to interpolate $V_{t+1}(z_{j+1})$. Having computed the data $V_{t+1}(z_{j+1})$ for all $z_{j+1}\in\mathcal{S}\times\mathcal{A}\times\mathcal{P}\times\mathcal{E}\times\mathcal{D}\times\mathcal{R}$ in the last step, we can now find a piecewise polynomial function $sp_{t+1,j+1}$ that satisfies the interpolation conditions

$$sp_{t+1,j+1}(\mathrm{a}^k,ep^l)=EV(z_{j+1}) \tag{2}$$

for all $k=1,\dots,n_A$, $l=1,\dots,n_P$. In this paper we use multidimensional spline interpolation, see Habermann and Kindermann (2007).

B.2. The macroeconomic computational algorithm

The computation method for the macroeconomic model follows the Gauss–Seidel procedure of Auerbach and Kotlikoff (1987). We start with a guess for quantities and government policy. Then we compute prices, optimal household decisions, and value functions. This involves a discretization of the state space which is explained in the previous section. Next we obtain the measure of households and new macroeconomic quantities as well as the social security tax rate and the consumption tax rate that balances government's budgets. This information allows us to update the initial guesses. The procedure is repeated until the initial guesses and the resulting values for quantities, prices and public policy have sufficiently converged.

Appendix C. Initial equilibria

See Table 14.

Table 14
Initial equilibria of different model variants.

Variant	(1)	(2)	(3)
Productivity risk	yes	yes	yes
Disability risk	no	yes	yes
Endogenous retirement	no	no	yes
Time discount factor δ	0.992	0.989	0.989
Goods market (in % of GDP)			
Private consumption	59.4	59.8	59.8
Public consumption	21.2	20.8	20.7
Investments	19.4	19.4	19.4
Exports–imports	0.0	0.0	0.0
Capital market (in %)			
Capital/GDP	353.2	353.3	353.2
Interest rate	4.9	4.9	4.9
Tax System (in % of GDP)			
Tax revenues	23.4	23.0	22.9
Consumption tax	10.1	10.2	10.2
Income tax	10.3	9.8	9.7
Pension system			
Pension benefits/GDP (in %)	12.3	12.3	12.4
Disability/total PB (in %)	–	12.3	11.6
Replacement rate (in %)	53.4	44.6	46.0
Average retirement age	63.0	59.5	59.5
Old-age	63.0	63.0	63.1
Disability	–	51.6	50.9

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