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Sentiment and the U.S. business cycle

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Psychological factors are commonly believed to play a role on cyclical economic fluctuations, but they are typically omitted from state-of-the-art macroeconomic models.

This paper introduces “sentiment” in a medium-scale DSGE model of the U.S. economy and tests the empirical contribution of sentiment shocks to business cycle fluctuations.

The assumption of rational expectations is relaxed. The paper exploits, instead, observed data on expectations in the estimation. The observed expectations are assumed to be formed from a *near*-rational learning model. Agents are endowed with a perceived law of motion that resembles the model solution under rational expectations, but they lack knowledge about the solution’s reduced-form coefficients. They attempt to learn those coefficients over time using available time series at each point in the sample and updating their beliefs through constant-gain learning. In each period, however, they may form expectations that fall above or below those implied by the learning model. These deviations capture excesses of optimism and pessimism, which can be quite persistent and which are defined as sentiment in the model. Different sentiment shocks are identified in the empirical analysis: waves of undue optimism and pessimism may refer to expected future consumption, future investment, or future inflationary pressures.

The results show that exogenous variations in sentiment are responsible for a sizable (above forty percent) portion of historical U.S. business cycle fluctuations. Sentiment shocks related to investment decisions, which evoke Keynes’ animal spirits, play the largest role. When the model is estimated imposing the rational expectations hypothesis, instead, the role of structural investment-specific and neutral technology shocks significantly expands to capture the omitted contribution of sentiment.

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1. Introduction

Economists have always recognized the importance of expectations for aggregate economic behavior. Some of the most influential economic thinkers of the past century attributed explicitly to the volatility of expectations a prime role in explaining the existence and depth of business cycles.

Keynes emphasized in the *General Theory* the importance of changes in expectations that are not necessarily driven by rational probabilistic calculations, but which are rather motivated by what he famously labeled “animal spirits”. In particular, entrepreneurs’ animal spirits related to their investment decisions were theorized of being a major determinant of economic fluctuations. Pigou (1927) also thought of business cycles as being largely driven by expectations and he stressed entrepreneurs’ errors of optimism and pessimism as key drivers of fluctuations in real activity.

Expectations maintain a central, although different, role in modern state-of-the-art general equilibrium models. Expectations are almost universally modeled as formed according to the rational expectations hypothesis. As a result, at least in models with a determinate equilibrium, expectational errors can be solved out as a function of fundamental shocks and they disappear as autonomous sources of dynamics. Hence, there is typically no scope for fluctuations in expectations in the spirit of those emphasized by Keynes, and which are driven by animal spirits, market psychology, sentiment, or by any expectational shift that cannot be reconnected to primitive structural disturbances.[2]

In state-of-the-art DSGE models, the main sources of fluctuations are typically shocks to demand, such as exogenous shifts in preferences, risk-premia, and monetary and fiscal policies, shocks related to technology, such as Hicks-neutral or investment specific technology shocks, or to market power, such as price and wage markup shocks. While the empirical DSGE macro literature disagrees on the relative contributions of each shock, most of it implicitly agrees on assigning a nil role to explanations based on non-fundamental expectational shifts, such as swings in sentiment that are not necessarily motivated by fundamentals.

This paper offers an alternative approach. It revisits a benchmark DSGE model that is often used to characterize the dynamics of the U.S. economy at business cycle frequencies. But the model is extended to incorporate “sentiment”, which represents waves of optimism and pessimism that are exogenous to the state of the economy.

The stringent informational requirements of rational expectations are relaxed. In their place, I will exploit observed data on expectations, obtained from the Survey of Professional Forecasters, in the estimation.

The observed expectations are assumed to be, on average, the outcome of a near-rational expectation formation process, which allows for learning by economic agents. Agents form expectations based on a linear model that has the same structural form of the system solution under rational expectations (i.e., the model used by agents is correctly-specified, since it contains the same regressors). The paper, however, relaxes the assumption that economic agents in the model have an informational advantage over the econometrician estimating the model. Here, at each point in the sample, economic agents can observe only historical data up to that point and they form beliefs about the reduced-form model coefficients by estimating simple regressions. The framework allows for deviations from rational expectations, but the deviation is meant to be small: agents still use a correctly-specified model. Such small deviations set the learning literature apart from starker alternatives that abandon rational expectations to assume, for example, simple heuristic rules (e.g., Grauwe, 2012).

Although expectations are formed, on average, from the learning model, economic agents can, in every period, form expectations that deviate from the point forecasts that their learning model yields. These deviations of actual expectations from their levels that can be explained by a near-rational model with learning are interpreted as denoting exogenous waves of undue optimism or pessimism, and define the “sentiment” terms in the model. Sentiment shocks are, therefore, identified from the dynamic interactions between observed expectations and realized macroeconomic time series.

The DSGE model is estimated using Bayesian techniques and adding data on observed expectations about consumption, investment, and inflation to the set of observables to match. The main scope in the empirical analysis lies in studying the empirical contribution of these newly-defined sentiment shocks to macroeconomic fluctuations.

Main Results. The empirical results show that sentiment shocks explain a sizable portion of U.S. business cycle fluctuations. Sentiment explains more than forty percent of the variability of output and consumption at business cycle horizons, and around sixty percent of the variability of investment and inflation. The most important component of sentiment consists of sentiment related to future investment expectations, which is found to be the single main driver of business cycle movements. Inflation is driven by structural price markup shocks in the short-run; their transmission is, however, very quick, and market participants’ sentiment about inflationary pressures becomes predominant at frequencies above one year.

If learning and sentiment are shut down and the conventional assumption of rational expectations is re-imposed, technology shocks become the dominant source of aggregate fluctuations (in both forms of investment-specific and neutral technology shocks), as theorized by the RBC literature. The contribution of technological changes for booms and busts significantly rises to close the gap induced by the omitted role of households and firms’ sentiment.

[2] Animal spirits may, instead, be reintroduced under rational expectations by assuming equilibrium indeterminacy: the expectational errors in that case are not only a function of structural disturbances, but also of exogenous sunspot variables. There is a conspicuous literature, surveyed in Benhabib and Farmer (1999), which is focused on studying indeterminacy and sunspots in macroeconomic models. The work in this area has, however, been more often theoretical than empirical (Lubik and Schorfheide, 2004, and Farmer et al., 2015, are two exceptions, which provide techniques to perform econometric analyses of sunspots in general equilibrium models). This paper’s approach differs from the indeterminacy literature as it can imply self-fulfilling fluctuations even when the equilibrium is unique.

When the model is estimated under observed expectations, allowing for learning and sentiment, the degrees of some real and nominal frictions necessary to fit the data is diminished. In particular, moving away from rational expectations attenuates the degree of habit formation in consumption and sharply reduces the magnitude of adjustment costs in investment. The estimated autocorrelation of several structural disturbances is largely reduced and the responses of macroeconomic variables to some structural shocks become faster than usually estimated. Sentiment, on the other hand, is responsible for more sluggish adjustment in the economy. The model with near-rational expectations outperforms the version under rational expectations in terms of its ability to fit the set of realized and expected macroeconomic time series.

Related Literature. In previous related work (Milani, 2011), I modeled expectation shocks in a near-rational expectations environment and showed that they potentially play a large role as drivers of business cycles. That paper used a stylized three-equation New Keynesian model. The current paper extends the analysis to a more comprehensive and empirically relevant model of the U.S. economy. While the previous paper abstracted from capital accumulation, this work includes capital and investment, allowing for adjustment costs and variable capital utilization, in addition to features such as monopolistic competition in the labor market, wage stickiness, habit formation in consumption, and it exploits expectations about future consumption and investment in the estimation. In this way, the paper can disentangle the role of sentiment related to consumers' expenditure decisions and to firm's investment choices, which was the channel emphasized in Keynes' theories.

The paper adds to the expanding literature on bounded rationality and learning in macroeconomics (e.g., Evans and Honkapohja, 2001; Sargent, 1993). It exploits direct data on expectations to inform the estimation of the best-fitting learning process over the sample. Moreover, it shows that, in addition to the role of learning, a different component of expectations, which the paper defines as sentiment, is key to understand business cycles. In this way, the paper adds to the previous studies that document the empirical importance of learning in macroeconomic models, such as Milani (2007) and Slobodyan and Wouters (2012). A particularly related paper in the learning literature is Bullard et al. (2008), who introduce judgment in economic agents' learning model, showing that it can lead to near-rational exuberance equilibria. Eusepi and Preston (2011) show that learning can amplify business cycle fluctuations in a baseline RBC model driven by technology shocks.

On the methodological side, the DSGE literature has only recently started to include data on expectations in the estimation of DSGE models. Del Negro and Eusepi (2011) use data on inflation expectations to test whether rational expectations DSGE models can successfully explain the expectation series; Ormeño and Molnár (2015) add information from observed inflation expectations to discipline the estimation of models with learning. Milani (2011) introduces data on expectations regarding output, inflation, and interest rates. Hirose and Kurozumi (2012) and Milani and Rajbhandari (2012) exploit a large set of expectations series at different horizons to enhance the identification of news shocks.

The paper can be connected to the literature on multiple equilibria, sunspots, and animal spirits, although most of the modeling choices differ. The paper can be seen as an econometric evaluation of the importance of animal spirits, here defined somewhat differently, since they arise in a model in which rational expectations have been relaxed. An advantage of the approach suggested in this paper is that the existence of self-fulfilling fluctuations in expectations are not conditional on indeterminacy of the equilibrium, which in a model as the one considered in this paper, would be mostly due to a failure by monetary policy to satisfy the Taylor principle. Self-fulfilling fluctuations and expectations-driven business cycles may arise here in a model in which monetary policy still responds aggressively toward inflation and a unique determinate equilibrium exists.

The paper has also points of contact with the literature on news and anticipated shocks (e.g., Beaudry and Portier, 2006). That literature has mostly emphasized news about future technology as sources of fluctuations, although recently other types of news have been considered (e.g., Fujiwara et al., 2011; Khan and Tsoukalas, 2012; Schmitt-Grohé and Uribe, 2012 and Milani and Treadwell, 2012). The sentiment variables identified here do not represent news about future improvements in technology or future monetary or fiscal policies, but they are intended to capture unjustified and possibly persistent waves of optimism and pessimism that are orthogonal to the observed state of the economy. The paper shows that the identified sentiment in the model is indeed correlated to innovations obtained from available survey data on sentiment, supporting the interpretation proposed in this paper.

The paper is also part of a recent strand of literature that assigns a prominent role to real-time data. Orphanides (2001) provides an early contribution. More recently, Lubik and Matthes (2016) estimate a model with real-time data and least-squares learning to investigate the implications of data misperceptions for monetary policy. They show that policymakers' misperceptions in real-time may give rise to indeterminacy. Casares and Vázquez (2016) examine the implications of real-time data and data revisions on the coefficients of estimated DSGE models.

In the aftermath of the Financial Crisis, a growing number of papers have started to add 'sentiment', variously defined, or more broadly 'behavioral' elements to macroeconomic frameworks. Angeletos et al. (2016) enlarge the state space to include an exogenous random variable, which aims to capture deviations of higher-order beliefs from first-order beliefs. The extra term is interpreted as a "confidence shock". Benhabib et al. (2015) use a simple model in which firms' production decisions and households' saving and labor supply decisions need to take place before the realization of demand. Under imperfect information, households form expectations about future income partly based on their 'sentiments', while firms receive only noisy signals about demand. Their signal extraction problem can generate equilibria where fluctuations in output are driven by time-varying sentiment. These works retain the assumption of rational expectations, while my paper introduces sentiment in an environment with bounded rationality and learning. Gabaix (2016) introduces an 'inattention', or 'myopia', parameter to a stylized New Keynesian model: agents pay less attention to events or disturbances farther into the future (behavioral expectations are obtained by pre-multiplying rational expectations by a cognitive discount factor, which decreases with the forecast horizon). Fuster et al. (2010) propose 'natural expectations' as a middle way between rational and naïve expectations: agents with natural expectations form beliefs characterized by extrapolation in the short-run, but they fail to capture the long-run mean reversion of macroeconomic time series (they systematically overestimate the persistence of disturbances). My approach is more firmly rooted in the adaptive learning literature, which has a long tradition in macroeconomics, as a rigorous way to model deviations from rational expecctations.

Finally, other recent studies have, instead, focused on second-moment, rather than first-moment shocks, by investigating the potential for uncertainty shocks as a source of economic fluctuations (e.g., Bloom, 2009). I abstract from those here.

2. Model

The current generation of DSGE models joins elements from the RBC tradition (explicit microfoundations, dynamic optimization, capital accumulation, and technology shocks) and elements from the New Keynesian tradition (imperfect competition, sticky prices, sticky wages, an interest rate rule for monetary policy). Economic agents are typically assumed to form model-consistent rational expectations. To be taken to the data, the model needs to incorporate a variety of real and nominal frictions, along with a combination of serially-correlated exogenous shocks.

This paper follows in this tradition by assuming that fluctuations in the U.S. economy at business cycle frequencies can be summarized by a medium-scale DSGE model, based on Christiano et al. (2005) and Smets and Wouters (2007), and which has been used as benchmark in several studies. Similar models have been developed and fitted to U.S. data by Justiniano et al. (2010) and Del Negro et al. (2007), among countless others. Since later we will relax the assumption of rational expectations, we replace the mathematical expectation operator E_t with the indicator for subjective expectations $\widehat{E}_t$. Our aim is, therefore, to assess whether sentiment has a role in a benchmark DSGE model that is known to fit U.S. data well.

We report the set of loglinearized equations here.[3]

$$y_t = c_y c_t + i_y i_t + u_y u_t + \varepsilon_t^g \tag{1}$$

$$c_t = c_1 c_{t-1} + (1 - c_1)\widehat{E}_t c_{t+1} + c_2(l_t - \widehat{E}_t l_{t+1}) - c_3(r_t - \widehat{E}_t \pi_{t+1} + \varepsilon_t^b) \tag{2}$$

$$i_t = i_1 i_{t-1} + (1 - i_1)\widehat{E}_t i_{t+1} + i_2 q_t + \varepsilon_t^i \tag{3}$$

$$q_t = q_1 \widehat{E}_t q_{t+1} + (1 - q_1)\widehat{E}_t r_{t+1}^k - (r_t - \widehat{E}_t \pi_{t+1} + \varepsilon_t^b) \tag{4}$$

$$y_t = \Phi_p(\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^a) \tag{5}$$

$$k_t^s = k_{t-1} + u_t \tag{6}$$

$$u_t = u_1 r_t^k \tag{7}$$

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i \tag{8}$$

$$\mu_t^p = \alpha(k_t^s - l_t) + \varepsilon_t^a - w_t \tag{9}$$

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 \widehat{E}_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p \tag{10}$$

$$r_t^k = -(k_t - l_t) + w_t \tag{11}$$

$$\mu_t^w = w_t - \left(\sigma_l l_t + \frac{1}{1 - h/\gamma}\left(c_t - \frac{h}{\gamma} c_{t-1}\right)\right) \tag{12}$$

[3] The reader is referred to the extensive appendix in Smets and Wouters (2007) for a step-by-step derivation of these equations.

$$w_t = w_1 w_{t-1} + (1-w_1)\widehat{E}_t(w_{t+1}+\pi_{t+1}) - w_2\pi_t + w_3\pi_{t-1} - w_4\mu_t^w + \varepsilon_t^w \tag{13}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)[\chi_\pi \pi_t + \chi_y(y_t - \Phi_p \varepsilon_t^a)] + \varepsilon_t^r \tag{14}$$

The composite coefficients are given by:

$$\begin{array}{lll}
c_y = 1 - i_y - g_y; & i_y = \delta k_y; & u_y = r_k^* k_y; \\
c_1 = h/(1+h); & c_2 = (\sigma_c - 1)(W_*^h L_*/C_*)/\sigma_c(1+h); & c_3 = (1-h)/[\sigma_c(1+h)]; \\
i_1 = 1/(1+\beta); & i_2 = 1/[(1+\beta)\varphi]; & \\
q_1 = \beta(1-\delta); & & \\
z_1 = (1-\psi)/\psi; & & \\
k_1 = 1-\delta; & k_2 = \delta(1+\beta)\varphi; & \\
\pi_1 = \iota_p/(1+\beta\iota_p); & \pi_2 = \beta/(1+\beta\iota_p); & \\
\pi_3 = [1/(1+\beta\iota_p)][(1-\beta\xi_p)(1-\xi_p)/(\xi_p(\phi_p-1)\varepsilon_p+1)]; & & \\
w_1 = 1/(1+\beta); & w_2 = (1+\beta\iota_w)/(1+\beta); & w_3 = \iota_w/(1+\beta); \\
w_4 = [1/(1+\beta)][(1-\beta\xi_w)(1-\xi_w)/(\xi_w(\phi_w-1)\varepsilon_w+1)]. & &
\end{array} \tag{15}$$

Eq. (1) is the economy's aggregate resource constraint: output is denoted by y_t, consumption by c_t, investment by i_t, and variable capacity utilization by u_t. The term ε_t^g denotes an exogenous government spending shock. The coefficients c_y, i_y, and u_y, denote the steady-state shares of consumption, investment, and resources used to vary capital utilization, expressed as a fraction of steady-state output.

Eq. (2) is the Euler equation for consumption. Consumption depends on expected future consumption, on past consumption, through the assumption of habit formation in households' preferences, on current and expected hours of work l_t, on the ex-ante real interest rate ($r_t - E_t\pi_{t+1}$, and on a risk-premium shock ε_t^b. It is perhaps more common in the macro literature to assume a preference shock with the power of shifting the Euler equation; the risk-premium shock has similar implications, but with the advantage of helping the model match the comovement between consumption and investment by entering also in (4). The main coefficients of interest in this equation are h, the degree of habit formation in consumption, and σ_c, the inverse of the intertemporal elasticity of substitution in consumption.

Eq. (3) represents the first-order condition for investment. Current investment depends on lagged and expected investment, and on the value of capital stock q_t. The term ε_t^i represents a disturbance that accounts for investment-specific technological change. The dynamics of the value of capital q_t is characterized by Eq. (4): it depends on its future expected value, on expectations about the rental rate on capital $E_t r_{t+1}^k$, and on the ex-ante real interest rate, adjusted for the risk-premium disturbance. The elasticity of investment to q_t is governed by the coefficient φ, which represents adjustment costs in investment.

Eq. (5) denotes a Cobb-Douglas aggregate production function: the technology to produce output requires capital services k_t^s and labor hours, which enter with shares α and $(1-\alpha)$. The term ε_t^a denotes the neutral technology shock, while the coefficient Φ_p accounts for the existence of fixed costs in production. The model assumes variable capital utilization. As Eq. (6) shows, capital services used to produce output are, therefore, a function of the whole capital stock in the previous period (given the assumption that capital becomes effective after a one-quarter lag) and the capital utilization rate u_t. The degree of capital utilization is varied depending on the rental rate of capital (Eq. (7)); the relation depends on the parameter $0 \le \psi \le 1$, which is a positive function of the elasticity of the capital adjustment cost function, but normalized to be between 0 and 1.

The capital accumulation Eq. (8) shows that capital, net of depreciation, changes due to new investment and to the efficiency of these new investments, captured by the investment-specific technology process ε_t^i.

Eqs. (9) and (10) summarize the equilibrium in the goods market. Inflation π_t is a function of both lagged and expected inflation, and it depends on the time-varying price mark-up μ_t^p and on the price mark-up shock ε_t^p. The price mark-up μ_t^p equals the difference between the marginal product of labor $(\alpha(k_t^s - l_t) + \varepsilon_t^a)$ and the real wage w_t.

Eq. (11) shows that the rental rate of capital is a function of the capital to labor ratio and of the real wage.

Eqs. (12) and (13) describe the labor market. The wage mark-up μ_t^w captures the difference between the real wage and the marginal rate of substitution between consumption and leisure, given by $\left(\sigma_l l_t + \frac{1}{1-h}(c_t - hc_{t-1})\right)$, where σ_l is the inverse of the Frisch elasticity of labor supply. The real wage depends on its past and expected future values, on current, past, and expected inflation, and on the wage mark-up. Wage dynamics is also affected by the wage mark-up disturbance ε_t^w. The importance of the backward-looking terms in the inflation and wage equations are driven by the indexation to past inflation coefficients ι_p and ι_w; the slopes of the curves are an inverse function of the Calvo price and wage stickiness coefficients ξ_p and ξ_w.

Finally, Eq. (14) serves as an approximation of monetary policy decisions in the economy. The monetary authority follows a Taylor rule with partial adjustment, moving the short-term nominal interest rate r_t in response to changes in inflation and the output gap. The Taylor rule is simplified with respect to the one used in Smets and Wouters (2007). Potential output is not defined here as the level of output in the same economy, but under flexible prices. Given that the estimation is complicated by the addition of sentiment shocks, learning, and so forth, I avoid augmenting the state-space with extra equations for the flexible price economy: the output gap is simply defined here as the deviation of output from a potential level of output driven exclusively by technology.

All exogenous shocks, except the monetary policy shock, which is *i.i.d.*, are assumed to evolve as AR processes as in Smets and Wouters (2007).[4] The government spending shock is allowed to respond to innovations in technology as $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \tilde{\varepsilon}_t^g + \rho_{ga}\tilde{\varepsilon}_t^a$, where $\tilde{\varepsilon}_t^g$ and $\tilde{\varepsilon}_t^a$ are spending and technology innovations and ρ_{ga} is a coefficient to be estimated.

Therefore, the model summarizes the dynamics for fourteen endogenous variables. Smets and Wouters (2007) use observables for seven of the variables. Moreover, there are seven structural disturbances that are unobserved and are obtained through filtering. To the observables and shocks in Smets and Wouters (2007), I will add available observable data on expectations about consumption, investment, and inflation, and expectational, or sentiment, shocks, which are defined in the next section.

3. Relaxing rational expectations: learning and sentiment

Economic agents in the model form expectations about future aggregate consumption, investment, hours of work, inflation, real wages, the rental rate of capital, and the value of capital. The literature typically assumes that such expectations are formed according the rational expectations hypothesis. Here, we relax the strong informational assumptions imposed by rational expectations to exploit direct, observed, data on expectations, and to investigate the role of sentiment on the economy.

Agents are assumed to form expectations using their perceived model of the economy, which is assumed to include the same endogenous variables that appear in the minimum state variable (MSV) solution of the system under rational expectations. The departure from rational expectations consists of agents' lacking knowledge about the reduced-form model coefficients (for example, they lack knowledge about Calvo coefficients and, as a result, they cannot recover the reduced-form coefficients in the model solution of the system) and, for now, of the realizations of the unobserved structural disturbances.[5] Later, as a robustness check, agents will be allowed to observe disturbances as well. Moreover, when agents solve their optimization problems, they behave in accordance with Kreps and Kamien (1998) anticipated utility approach: in each period, agents maximize their expected lifetime utility taking their beliefs and perceived model as constant, even though the beliefs will be possibly updated over the next periods; this assumption, which is standard in the adaptive learning literature, rules out active experimentation by agents. While the model departs from rational expectations, the departure is not drastic and the model can be intended as *near*-rational.

Economic agents use historical data to infer the unknown coefficients over time. They do so by estimating the following Perceived Law of Motion (PLM)

$$Y_t = a_t + b_t Y_{t-1} + \epsilon_t \tag{16}$$

where $Y_t = [c_t, i_t, q_t, l_t, k_t, r_t^k, \pi_t, w_t, r_t]'$, and a_t and b_t are vectors and matrices of coefficients. Restrictions with ones and zeros on the coefficients are used to select variables that do or do not enter the MSV solution (q_t, l_t, and r_t^k do not enter the model in lags and, hence, the corresponding coefficients on their lagged values in b_t equal 0).[6] Agents use data on the endogenous variables to form expectations, but they do not observe the structural disturbances that would also enter the PLM, but that are typically unobserved to the econometrician. Besides cognitive consistency, I regard this as the most empirically realistic description of the information available to forecasters.

In each period t, agents are assumed to observe values of the endogenous variables up to $t-1$. This assumption is mainly motivated by the attempt to be consistent with the timing in the Survey of Professional Forecasters: when survey participants are asked in period t for their forecasts for period $t+1$, they can observe historical data only up to $t-1$. The assumption is also typical in the theoretical learning literature as a means to avoid simultaneity issues in self-referential models. Therefore, agents, in each period t, form expectations using observations up to $t-1$ along with their beliefs, which they have previously updated by running regressions of $(t-1)$-dated variables on $(t-2)$-dated variables.

The beliefs are recursively updated following a constant-gain learning algorithm as

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \mathbf{\bar{g}} R_t^{-1} X_t (Y_t - \widehat{\phi}_{t-1}{}' X_t)' \tag{17}$$

$$R_t = R_{t-1} + \mathbf{\bar{g}}(X_t X_t' - R_{t-1}) \tag{18}$$

where $X_t \equiv [1, Y_{t-1}]'$, and $\widehat{\phi}_t = [a_t, b_t]'$. Eq. (17) describes the updating of beliefs regarding the model solution coefficients, while Eq. (18) describes the updating of the corresponding precision matrix R_t.

Given knowledge of the endogenous variables in $(t-1)$ and given the state of recently updated beliefs, observed expectations are assumed to be formed as follows

$$\widehat{E}_{t-1} Y_{t+1} = \left(I + \widehat{b}_{t-1}\right)\widehat{a}_{t-1} + \widehat{b}_{t-1}^2 Y_{t-1} + \widehat{d}\alpha_t, \tag{19}$$

[4] Smets and Wouters (2007) assume MA(1) components in the price and wage markup shocks. We assume that they also follow AR processes here.

[5] Chung and Xiao (2013) discuss how the assumption that agents cannot observe fundamental shocks is needed to satisfy the guiding principle of "cognitive consistency" in the adaptive learning literature, i.e., the principle that agents in the model should not be endowed with more information than econometricians have available when working with the model.

[6] No restrictions are imposed on the variance-covariance matrix of ϵ_t.

where α_t is the vector collecting the different sentiment shocks $\alpha_t = [\alpha_t^c, \alpha_t^i, \alpha_t^\pi]'$, and d is a selection matrix with elements equal to 1 for expectations for which an observable is available and 0 otherwise.

Expectations can, therefore, be decomposed in two parts. One consists of the endogenous reaction of expectations to the state of the economy, given the agents' learning beliefs: this is the forecast implied by the near-rational learning model (i.e., the right-hand side except $d\alpha_t$). The other consists of the component of expectations that cannot be rationalized as being derived as the outcome of a near-rational model. This second component accounts for exogenous movements in expectations that are unrelated to observed fundamentals.

Expectations in the model about consumption, investment, and inflation, will be matched to the corresponding observable expectation variables in the empirical analysis. Those observed expectations are assumed to be formed from the near-rational learning model specified above. But in each period, agents are allowed to deviate from the point forecasts that arise from their near-rational model: they can form forecasts that are unduly optimistic or pessimistic, given the state of the economy and their most recent updated beliefs. These deviations, which are the component of expectations that cannot be explained as the outcome of the near-rational learning model, are defined as "sentiment" in the model (with the different sentiment shocks included in the vector α_t). Sentiment, therefore, captures exogenous waves of optimism and pessimism, which cannot be explained by existing economic conditions.[7]

4. Sentiment and the business cycle: estimation approach

4.1. Observed expectations and real-time data

The model is estimated using Bayesian methods to match the following set of observables: Real GDP, Real Consumption, Real Investment, Hours worked, Real Wage, Inflation, Federal Funds rate, Expected Real Consumption, Expected Real Investment, and Expected Inflation.[8] Therefore, to extract and investigate the role of sentiment shocks, I add to the same variables that are used in Smets and Wouters (2007) information on expectations about future consumption, investment, and inflation. The structural "deep" parameters, the shock parameters, the learning, and sentiment, parameters will all be jointly estimated.

The data on expectations are obtained from the Survey of Professional Forecasters (SPF), hosted by the Federal Reserve Bank of Philadelphia. I use the mean of expectations across forecasters for levels of consumption, investment, and for the inflation rate. In each period t, agents in the model, in the same way as forecasters in the survey, form expectations about variables in $t+1$, knowing the values of endogenous variables up to $t-1$ (forecasters at each t are also asked for their estimate of variables in $t-1$ and the vast majority of them simply reports the latest BEA data release for variables in $t-1$). In the SPF, the forecasts that will be used in the empirical analysis correspond to the column *'dVariable3'*, i.e., expectations about values of the variable one-quarter-ahead.[9]

Expectations about consumption correspond to the series "*Forecasts for the quarterly and annual level of real personal consumption expenditures (RCONSUM)*". Expectations about investment are obtained by adding the series for nonresidential and residential investment: "*Forecasts for the quarterly and annual level of real nonresidential fixed investment (RNRESIN)*" and "*Forecasts for the quarterly and annual level of real residential fixed investment (RRESINV)*". These forecasts are available starting from 1981:Q3, which is, therefore, chosen as the sample starting date for the main estimation in the paper. Expectations about inflation are calculated from the price level series "*Forecasts for the quarterly and annual level of the GDP Price Index (PGDP)*"; the series is available from 1968:Q3. The base year changes over the sample as the base year for the GDP price deflator used to compute real variables changes. The series have, therefore, been transformed to maintain the same base year (chosen to be 2009) across the whole sample.[10]

Given the focus on identifying the learning process of economic agents in real-time and on disentangling the components of expectations that can be rationalized as the outcome of a learning model or attributed to exogenous sentiment, it is important that the estimation captures as closely as possible the information set available to agents at each point in the sample. For this reason, I choose to use real-time data in the estimation.[11]

[7] For variables for which a corresponding observable series is not available, the expectations in the empirical analysis will be simply equal to those implied by the learning model, with no excess optimism or pessimism.

[8] Justiniano et al. (2010) include consumer durables in their definition of investment. I use the typical aggregate series for consumption and investment here, since they are the only ones for which real-time data and expectations are available.

[9] Given the timing of the survey (mailed mid-quarter), and the delay at which macroeconomic data are released, it can be argued that these forecasts correspond to almost two-quarter-ahead expectations. In the Robustness section, the estimation is, therefore, repeated using forecasts from the column *'dVariable2'*, which correspond to the same quarter expectations (expectations for t based on $t-1$ information), and may be alternatively interpreted as one-quarter-ahead forecasts.

[10] In every quarter in which there is a change of base year, the levels of the real-time variables or their expectations are rescaled using the observed growth rates, which are available in t: for realized variables, the growth rate, corresponding to vintage t, between t and $t-1$ can be used; for expectations, the forecasters' expected growth rates at each t, between $t-1$ and t or between t and $t+1$ are always available. Other changes in the SPF over the sample relate to the definitions of the variables. The series for output referred to GNP prior to 1992, when it switched to GDP. The correlations between the two series for the common samples were, however, close to 1. The same is true for GNP and GDP deflator series, where the shift also occurred in 1992.

[11] Real-time data in the estimation of DSGE models have been considered, for example, in Casares and Vázquez (2016).

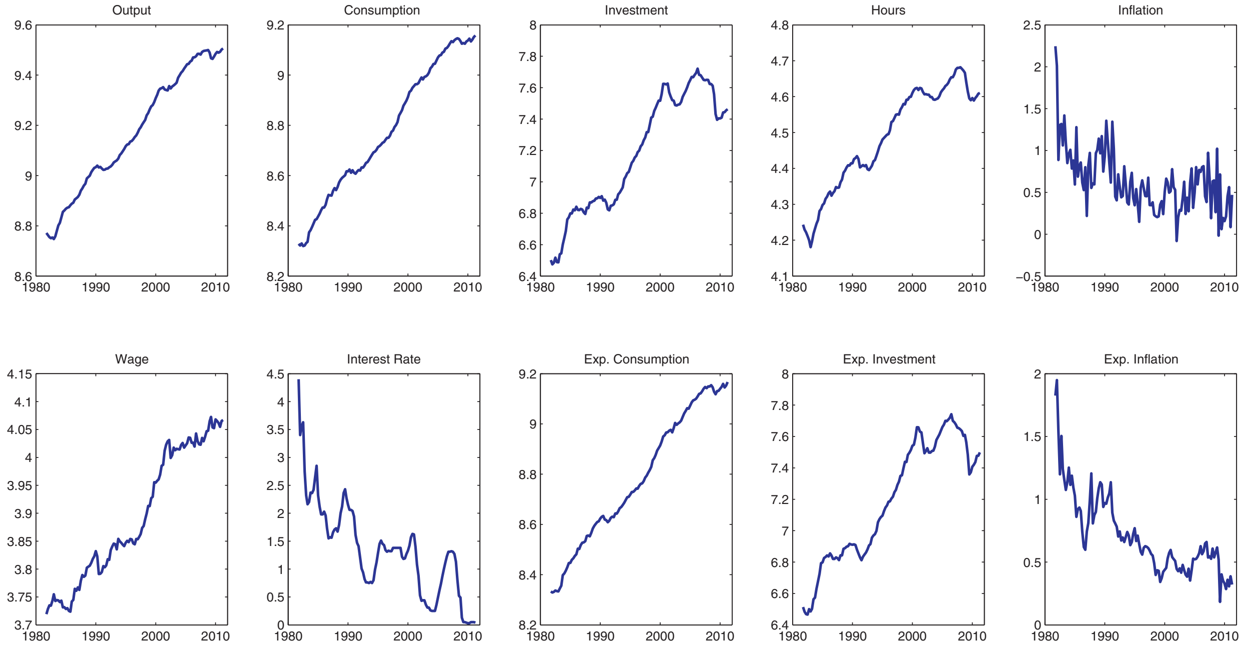

Fig. 1. Raw Data Series. *Note*: output, consumption, investment, wage, consumption expectations, investment expectations are all expressed in real terms and are reported in log levels; hours are also in log levels. The interest rate is levels and converted to quarterly, while inflation and expected inflation are obtained as the log first difference of the price level. All data correspond to real-time vintages (the series were redefined to maintain the same base year across the sample, when necessary).

For each variable being forecasted, the SPF provides a link to “Real-time data for this variable”. I use those to better approximate the information set available to forecasters in real time and as the observable series in the model. The realized data series for each variable are hence obtained from the corresponding Real Time Data Set for Macroeconomists’ website, also hosted by the Federal Reserve of Philadelphia, with the exception of the Federal Funds rate (which is not subject to revision), which is obtained from the FRED database, made available by the Federal Reserve of St. Louis.

For consumption, investment, and inflation, therefore, I use the real-time data series corresponding to the forecasts described above. In the benchmark estimation, I use the first-release version of the data. I will later assess the robustness of the results to the alternative assumptions: the use of final-vintage, revised, data, and the addition of measurement error in the estimation to account for possible inaccuracies in the measurement of real-time information sets. I use the real-time real GDP series (ROUTPUT) as measure of output. Hours are computed using the total aggregate weekly hours index (H) divided by civilian noninstitutional population (POP). I compute real wages as total wage and salary disbursements, private industries (WSD), divided by total aggregate hours and by the GDP deflator. The definition for wages is somewhat different from the one used in Smets and Wouters (2007). I choose to use a related definition, for which real-time data are available, rather than the same series they use, but for which real-time data do not exist. Finally, as measure of the short-term nominal interest rate, I use the Federal Funds rate (FEDFUNDS) from FRED. The annualized values are converted into quarterly rates for the estimation.

The estimation sample spans the years from 1981:III to 2011:I; the starting date is chosen due to the availability of expectations data (available only from 1981:III for consumption and investment expectations). All variables are at quarterly frequency. The raw variables, before any detrending, that will be used in the estimation are shown in Fig. 1.

4.2. Trends and state-space system

I present the state-space system for the model in its more general form, with the variables in levels, rather than in detrended or growth rate form.[12] The estimation on raw data is in the spirit of Canova (2014) and Canova and Ferroni (2011), and it permits to evaluate different detrending procedures.

The state space system can be written as:

$$Y_t^{OBS} = \bar{H} + H(T_t + \xi_t) \tag{20}$$

$$\xi_t = A_t + F_t \xi_{t-1} + G\omega_t \tag{21}$$

[12] Later in the paper, I will evaluate the robustness of the results to using the growth rates of the variables in the estimation.

where $\xi_t = [Y_t, \widehat{E}_t Y_{t+1}, \varepsilon_t, \alpha_t]'$, $\omega_t \sim N(0, \sigma_\omega^2)$.

Eq. (20) is the measurement equation that relates observed data series to the variables in the model and it separates between noncyclical, or trend, (T_t), and cyclical components (ξ_t). The vector $\bar{H}$ may contain steady state parameters or simply the sample mean of the variables, and the matrix H selects variables for which observables are available from the state vector.

Eq. (21), instead, represents the DSGE model for the cyclical components of the series. Under rational expectations, the equation corresponds to the rational expectation solution of the system (1)–(14), which has constant coefficients $A_t = A = 0$ and $F_t = F$. With observed expectations and learning, it is obtained by replacing rational expectations with survey expectations, and allowing survey expectations to derive from the near-rational learning model as in (19). The vectors and matrices of coefficients are possibly time-varying as a result of agent's learning process, as modeled in (17) and (18).

I consider two detrending options. The first is a linear trend, which can be expressed as $T_t = \delta_0 + \delta_1 t$. Besides its simplicity, an advantage of the linear trend is that it is probably more likely than more sophisticated alternatives to mimic the trend and cycle decomposition that forecasters had in mind when communicating their survey forecasts over the sample. An extension of the linear trend specification, intended to even better capture the trend estimation by forecasters in real time, consists of adopting a recursive linear trend. In this case, the trend coefficients δ_0 and δ_1 are estimated using only information from $t = 1$ up to $t = \tau$, at each point τ in the sample. In line with the spirit of the learning approach, when forming expectations, agents also learn about the trend, and use the trend coefficient they have estimated on time series available up to $t - 1$ to forecast variables in $t + 1$. This is the second approach that will be considered in the estimation.

I allow the trends to differ across each variable.[13] Also to minimize a priori assumptions, I allow trends to potentially matter for each observable variable, including inflation and the interest rate (which actually display declining trends over the sample, but are often treated as stationary in DSGE estimations). Expectation series are also detrended, and their trends are allowed to differ from those of the corresponding realized variables (but whether the trends differ from or match with those of the realized variables has been found to be uninfluential for the results).

4.3. Bayesian estimation and priors

The priors for the model coefficients are shown in Table 1. The majority of prior choices follow Smets and Wouters (2007). There are, however, some differences. The prior for the intertemporal elasticity of substitution is a Gamma with mean 2 and standard deviation 0.5. The degree of habit formation has prior mean 0.5, rather than the higher 0.7 used by Smets and Wouters. The priors for the Calvo coefficients here are Beta with mean 0.7, rather than 0.5, to be more consistent with the recent micro-level evidence on price stickiness that finds price durations between 3 and 4 quarters. The disturbances' autoregressive coefficients all follow Beta prior distributions with means equal to 0.5 and standard deviations 0.2. Inverse Gamma distributions are used for shocks' standard deviations: they have prior means equal to 0.3 in all cases, except for the shocks related to investment, which have a mean of 1, given the a priori expectation that exogenous shifts in investment efficiency may have higher volatility. For the main learning parameter, I assume a Beta prior with mean 0.025 and standard deviation 0.01, which spans the range of calibrated constant gain parameters used in the theoretical adaptive learning literature.

The model is estimated using full-information Bayesian techniques. Draws are generated using the Metropolis-Hastings algorithm. I run 400,000 draws, discarding the initial 40% as burn-in. The parameter posterior distributions are usually well-behaved. When bimodality exists, I will point it out in the discussion of the results.

4.4. Initialization of the Agents' learning process

Besides detrending details, another factor that may potentially affect the results is the initialization of the agents' learning process. I consider three alternatives. The preferred initialization can again be chosen on the basis of its ability to fit the data; moreover, I will show later in the paper that the empirical conclusions are robust to different choices of initial beliefs at the beginning of the sample.

To avoid imposing arbitrary assumptions about initial values on the main estimation results, I first estimate the model for a presample period. The initialization requires full-information Bayesian estimation, since also some unobservable variables, which need to be obtained by filtering, enter the MSV solution. The model is therefore estimated on the 1964:I-1981:II sample (with initial date chosen since labor hours are available from 1964). I consider the results under three main options (I also considered other closely-related alternatives, without effects on the results).

4.4.1. REE from 1964-1981 pre-sample

In the first, I estimate the model in the presample period under the assumption of rational expectations. Under this approach, when moving to the main estimation on the second sample (1981–2011), I set the initial beliefs as equal to

[13] I have also performed the estimation for the case in which the restriction that a common trend exists among real variables is imposed, to allow for a balanced growth path. I have chosen to relax this restriction here (and, therefore, I do not explicitly include growth around a balanced growth path), since the fit becomes substantially worse than the variable-specific trend assumption. The assumption of a common trend among real variables is even more strongly rejected with real-time, than revised final-vintage data. Therefore, I prefer to use a more empirically-oriented specification, which fits the data better, than a 'more rigorous' theoretical specification, which fits the data less accurately and might lead to spurious conclusions.

Table 1
Prior distributions and Posterior estimates, baseline model.

Param.	Prior distributions	Posterior distributions: Rational expectations		Sentiment Shocks (NS)		Sentiment Shocks (S)	
φ	$\Gamma(4, 1.5)$	5.96	[3.83,8.41]	2.67	[1.94,3.74]	2.69	[1.90,3.66]
σ_c	$\Gamma(2, 0.5)$	1.65	[1.26,2.10]	1.02	[0.84,1.22]	1.91	[1.47,2.38]
h	$B(0.5, 0.15)$	0.70	[0.44,0.81]	0.48	[0.31,0.66]	0.34	[0.18,0.62]
σ_l	$\Gamma(2, 0.75)$	1.27	[0.47,2.70]	1.59	[0.57,2.95]	1.87	[0.73,3.29]
ξ_w	$B(0.7, 0.1)$	0.80	[0.69,0.93]	0.71	[0.54,0.87]	0.73	[0.60,0.85]
ξ_p	$B(0.7, 0.1)$	0.88	[0.78,0.99]	0.89	[0.81,0.95]	0.89	[0.81,0.95]
ι_w	$B(0.5, 0.15)$	0.58	[0.36,0.80]	0.47	[0.24,0.72]	0.46	[0.19,0.76]
ι_p	$B(0.5, 0.15)$	0.14	[0.04,0.43]	0.23	[0.08,0.47]	0.24	[0.09,0.46]
ψ	$B(0.5, 0.15)$	0.84	[0.71,0.94]	0.84	[0.71,0.94]	0.84	[0.72,0.94]
$\Phi_p - 1$	$\Gamma(0.25, 0.12)$	0.38	[0.20,0.66]	0.55	[0.29,0.83]	0.53	[0.32,0.79]
ρ_r	$B(0.75, 0.1)$	0.83	[0.78,0.87]	0.83	[0.75,0.90]	0.83	[0.74,0.90]
χ_π	$N(1.5, 0.25)$	1.74	[1.24,2.13]	1.49	[1.09,1.95]	1.44	[1.02,1.82]
χ_y	$N(0.125, 0.05)$	0.06	[0.02,0.10]	0.06	[0.02,0.11]	0.06	[0.02,0.12]
ρ_g	$B(0.5, 0.2)$	0.95	[0.92,0.97]	0.95	[0.91,0.98]	0.95	[0.92,0.98]
ρ_b	$B(0.5, 0.2)$	0.23	[0.04,0.78]	0.10	[0.02,0.22]	0.08	[0.02,0.18]
ρ_i	$B(0.5, 0.2)$	0.68	[0.54,0.81]	0.14	[0.03,0.31]	0.09	[0.02,0.20]
ρ_a	$B(0.5, 0.2)$	0.97	[0.93,0.99]	0.95	[0.90,0.98]	0.95	[0.91,0.98]
ρ_p	$B(0.5, 0.2)$	0.73	[0.14,0.91]	0.09	[0.02,0.23]	0.08	[0.02,0.18]
ρ_w	$B(0.5, 0.2)$	0.18	[0.05,0.35]	0.13	[0.03,0.26]	0.10	[0.02,0.22]
ρ_{ga}	$N(0.5, 0.25)$	0.35	[0.17,0.52]	0.34	[0.15,0.52]	0.34	[0.18,0.52]
σ_g	$\Gamma^{-1}(0.3, 1)$	0.61	[0.53,0.68]	0.60	[0.53,0.69]	0.60	[0.53,0.68]
σ_b	$\Gamma^{-1}(0.3, 1)$	0.30	[0.14,0.38]	0.68	[0.60,0.77]	0.68	[0.60,0.76]
σ_i	$\Gamma^{-1}(1, 5)$	0.74	[0.59,0.93]	2.00	[1.77,2.30]	1.98	[1.76,2.25]
σ_a	$\Gamma^{-1}(0.3, 1)$	0.59	[0.52,0.68]	0.58	[0.51,0.66]	0.59	[0.50,0.66]
σ_p	$\Gamma^{-1}(0.3, 1)$	0.12	[0.08,0.19]	0.26	[0.23,0.30]	0.26	[0.23,0.30]
σ_w	$\Gamma^{-1}(0.3, 1)$	0.46	[0.38,0.56]	0.96	[0.84,1.11]	0.95	[0.84,1.07]
σ_ε	$\Gamma^{-1}(0.3, 1)$	0.17	[0.15,0.19]	0.20	[0.18,0.23]	0.20	[0.18,0.23]
ρ_{α_c}	$B(0.5, 0.2)$			0.70	[0.57,0.83]	0.69	[0.57,0.69]
ρ_{α_i}	$B(0.5, 0.2)$			0.85	[0.77,0.93]	0.86	[0.76,0.94]
ρ_{α_π}	$B(0.5, 0.2)$			0.74	[0.62,0.86]	0.70	[0.57,0.81]
σ_{α_c}	$\Gamma^{-1}(0.3, 1)$			0.51	[0.45,0.59]	0.50	[0.44,0.57]
σ_{α_i}	$\Gamma^{-1}(1, 5)$			1.43	[1.26,1.63]	1.43	[1.27,1.61]
σ_{α_π}	$\Gamma^{-1}(0.3, 1)$			0.12	[0.10,0.13]	0.12	[0.11,0.14]
$\bar{\mathbf{g}}$	$B(0.025, 0.01)$			0.013	[0.01,0.017]	0.012	[0.008,0.015]

Note: Γ denotes Gamma distribution, B denotes Beta distribution, N denotes Normal distribution, Γ^{-1} denotes Inverse Gamma distribution, and U denotes Uniform distribution. The prior distributions are expressed in terms of mean and standard deviation, except for the Uniform prior, for which lower and upper bounds are shown. Posterior means and 95% credible intervals have been calculated over 400,000 Metropolis-Hastings draws, discarding a burn-in of 40% draws. The table reports the posterior estimates for the model under rational expectations and under learning and sentiment (for the benchmark specification with non-separable, NS, preferences and for the separable, S, alternative). The sample is 1981:III-2011:I.

their rational expectations equilibrium obtained from the presample period, i.e. $\widehat{\phi}_{t=0} = \widehat{\phi}_{RE}$. The precision matrix is similarly initialized as $R_{t=0} = XX'_{RE}$, which is also the value obtained in the presample estimation under rational expectations. Both $\widehat{\phi}_{t=0}$ and $R_{t=0}$ are obtained as means across MH draws, after a burn-in period, in the rational expectations DSGE estimation. The interpretation of this initialization is as follows: agents living in the 1964–1981 period are assumed to have had enough time to converge to the rational expectations equilibrium. The post-1981 sample may be interpreted as a new regime: agents start from their beliefs that they have formed by living in the pre-1981 regime and gradually learn about the new structure of the economy in the second sample.[14]

4.4.2. Ending point of learning beliefs from 1964 to 1981 pre-sample

The second option is more agnostic. I estimate also the model in the presample 1964–1981 period under non-fully rational expectations and learning. The initialization in 1964 is left as uninformative as possible: all variables in the perceived law of motion are assumed to evolve as AR(1) with an autoregressive coefficient equal to 0.9. This choice assigns agents the knowledge that macroeconomic variables are persistent, but it doesn't endow them with information on more complicated dynamic interactions among variables. The learning process is, therefore, given time to update in the presample estimation, and the state of beliefs at the end of the presample (1981:III) is then set as the initial set of beliefs for the main post-1981 estimation. The 70 quarterly periods in the presample estimation provide sufficient time to remove the most severe effects of initial conditions.

[14] We do not follow the practice of starting from RE estimates, since those require estimation over the full sample, which cannot be in the agents' information set in 1964.

Table 2
Forecast error variance decomposition (model with rational expectations).

	ε_t^g	ε_t^b	ε_t^i	ε_t^a	ε_t^p	ε_t^w	ε_t^r
Business Cycle horizon (4–24)							
y_t	0.017	0.008	0.387	0.404	0.119	0.057	0.008
c_t	0.026	0.019	0.279	0.484	0.123	0.059	0.010
i_t	0.073	0.002	0.660	0.139	0.081	0.040	0.004
π_t	0.054	0.005	0.398	0.149	0.250	0.130	0.015

Note: The numbers refer to posterior means across MH draws for the forecast-error variance shares due to the seven structural shocks. The results refer to the estimation under rational expectations.

4.4.3. Fully uninformative: initial beliefs equal to zero

Finally, as a further check, I estimate the model without a presample estimation. The beliefs are simply set at zero in 1981.

The benchmark results described in the next section will refer to the estimated model version that delivers the highest marginal likelihood (i.e., the case with linear detrending and initial beliefs derived from presample estimation under learning). Posterior estimates and other results for the full set of estimated specifications will be discussed in the Robustness Section.

5. Sentiment and the business cycle: empirical results

5.1. Business cycle evidence under rational expectations

For the sake of comparison, I start by estimating the model under the conventional assumption of rational expectations. Agents have perfect knowledge regarding the model parameters, other agents' preferences and constraints, the distribution of the shocks, and so forth. Expectational errors in this scenario (given that the equilibrium exists and is unique) are simply a function of structural innovations and do not represent an autonomous source of fluctuations in the model.

The estimation under rational expectations is similar to the one in Smets and Wouters (2007), but with the difference that here I use real-time data, rather than revised data. Moreover, I consider a different detrending procedure, the sample is limited to the post-1981 period and extended to 2011, and some series definitions differ, given the need here to match the real-time series on realized variables and their forecasts (for example, the wage series is different form the one in Smets and Wouters). There are some minor differences in priors and model specifications.

The parameters estimated for the DSGE model under rational expectations are shown in Table 1. The estimation reveals significant degrees of real frictions, such as investment adjustment costs (with a posterior estimate for $\varphi = 5.96$, which updates the prior toward larger values) and habit formation in consumption ($h = 0.70$), which are necessary to fit the sluggishness of macroeconomic data. Nominal rigidities are also essential: the posterior mean estimate for the Calvo coefficient in price-setting falls on the high side at 0.88, probably as a result of a less restrictive prior (while Smets and Wouters impose a prior with mean 0.5, I allow let here the data free to move to regions with higher price stickiness), and for the Calvo wage-stickiness coefficient is equal to 0.80. Indexation to past inflation is important in wage-setting ($\iota_w = 0.58$), but less so in price-setting ($\iota_p = 0.14$).

Structural disturbances related to government spending and technology are very persistent, with autoregressive coefficients above 0.9. The investment-specific technology shock and the price markup shock are also persistent with autoregressive coefficients equal to 0.68 and 0.73. The wage markup shock has only a limited serial correlation, a result that differs from the corresponding estimate in Smets and Wouters and that is in large part due to the choice of relaxing the assumption of a common trend between the real wage and other real variables. The posterior mean for the autocorrelation of the risk-premium disturbance is quite low (0.23). The estimation, however, reveals a clear bimodality: one mode is characterized by a very large degree of habit formation in consumption, but a low serial correlation of the exogenous risk-premium shock, the other by a more moderate degree of habit formation, but by a substantially serially-correlated risk premium. Bivariate posterior scatter plots indicate a strong negative relation between the two coefficients. The high habits-low autocorrelation mode, however, achieves higher probability and is, therefore, visited much more often by the MCMC sampler.

Figs. 2 and 3 show the impulse responses of output and inflation to selected shocks. Many impulse responses show the usual hump-shaped patterns. Output responds sluggishly to investment-specific, technology, wage markup, and monetary policy shocks. The peak effect for the risk-premium shock happens two quarters after the initial impact, whereas peaks are more delayed for the previous shocks, ranging from four quarters for the investment-specific to eight/ten quarters for the technology and wage markup shocks. Inflation adjusts somewhat more quickly to the shocks.

The variance decomposition for the model with rational expectations is shown in Table 2.

The shock that is responsible for the largest portion of fluctuations is the investment-specific shock, which is the dominant shock at high frequencies, explaining 63% of output variability at horizons below one year (not shown in the Table), and it is also important at business-cycle frequencies, with a share of the forecast error variance for output of 38.7%; a predominant role for this shock has been found also in Justiniano et al. (2010). Technology shocks are the main contributors

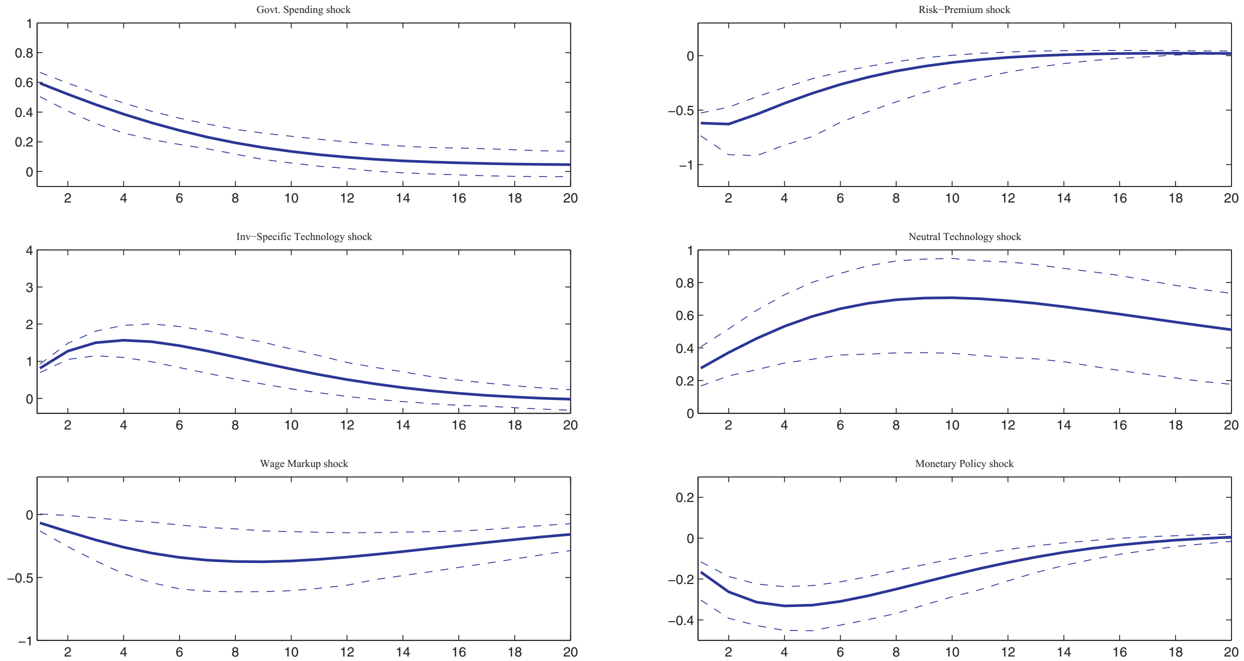

Fig. 2. Impulse response functions of output y_t to government spending, risk-premium, investment-specific technology and neutral technology shocks, wage markup shocks, and monetary policy shocks, under rational expectations. The graphs show mean impulse responses across MCMC draws, along with 5% and 95% percentile error bands.

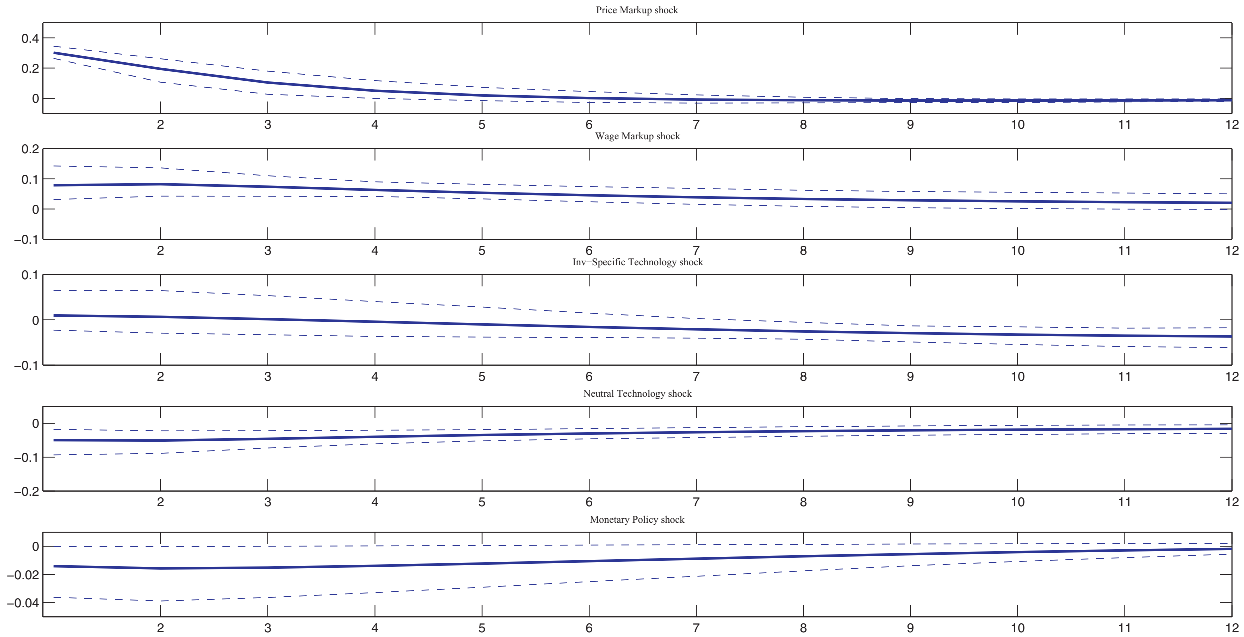

Fig. 3. Impulse response function of π_t to price markup, wage markup, investment-specific, neutral technology, and monetary policy shocks, under rational expectations. The graphs show mean impulse responses across MCMC draws, along with 5% and 95% percentile error bands.

at business-cycle horizons: in addition to the investment-specific shock, the Hicks-neutral technology shock accounts for another 40% of fluctuations.

The variance of inflation is mostly driven by the price markup shock at high frequencies, and by investment-specific shocks at lower frequencies, with technology, price, and wage markup shocks also playing a major role.

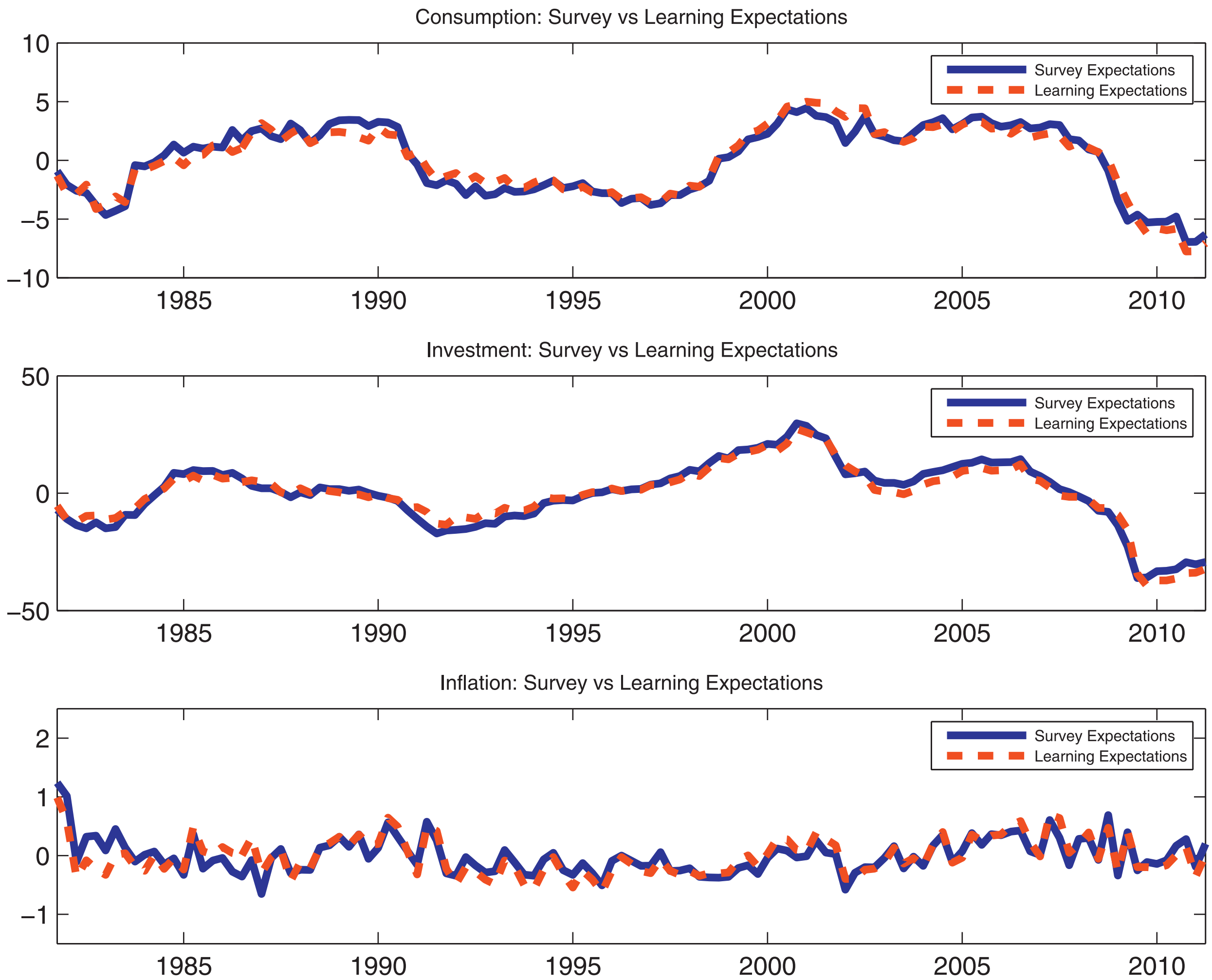

Fig. 4. Survey-based expectations versus near-rational expectations from learning model (detrended variables). The solid blue lines indicate survey expectations from the Survey of Professional Forecasters for consumption, investment, and inflation. The dashed red lines refer to the model-implied expectations, based on the near-rational learning model. They represent the endogenous component of expectations (i.e., the part that reflects an adjustment of expectations based on the state of the economy, excluding sentiment). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5.2. Learning and sentiment

I now move to estimate the version of the model that relaxes the stringent informational assumptions imposed by rational expectations. Economic agents form subjective expectations from a near-rational model and can deviate from near-rational forecasts because of exogenous changes in "sentiment". Observed expectations are used to better identify the economic agents' learning process over the sample and the expectation components that can be attributed to sentiment.

Table 1 shows the posterior estimates for the best-fitting version, which is the one with simple linear detrending and initial agents' beliefs set in 1981 to match those obtained from the presample estimation under learning. To gauge the sensitivity of results to the various assumptions, I will present the estimates for the alternative detrending and learning initialization, along with other sensitivity checks, later in Tables 5 and 6.

It can be noticed that the near-rational learning model provides a successful approximation of how survey forecasters form expectations in real-time. Fig. 4 displays the survey-based expectations, along with the implied expectations from the near-rational learning model, which represent the endogenous component of expectations, excluding sentiment, in (19). The learning expectations track observed survey forecasts relatively closely for most of the sample, with few episodes of divergence between the two (the deviations, however, can be quite persistent). Given that sentiment is identified in the estimation as the part of expectations that cannot be explained by the near-rational learning model, these results reassure us that an important role for sentiment is not likely to arise from a severe misspecification of the agents' forecasting model.[15]

[15] Branch and Evans (2006) also provide evidence that survey expectations are well explained by constant-gain learning models.

There are four areas in which the results under learning and sentiment provide insights that go beyond traditional results under rational expectations: the role of real frictions, or of the so-called “mechanical” sources of persistence, the ability to fit both macroeconomic realizations and the corresponding expectations, the response of macroeconomic variables to structural innovations, and the sources of business cycles.

5.2.1. Mechanical sources of persistence

When direct data on expectations are used to replace rational expectations, the estimation points toward smaller degrees of real frictions that are necessary to fit the persistence in the data. In particular, the posterior mean for the elasticity of the investment adjustment cost function is considerably reduced from $\varphi = 5.96$ under rational expectations to $\varphi = 2.67$ with observed expectations and learning. The lower magnitude of adjustment costs removes some of the delays and sluggishness in the responses of output and investment to shocks. The estimated degree of habit formation in consumption h also falls from 0.70 to 0.48.

The intermediate level of habit formation obtained in the estimation with subjective expectations and learning is mostly due to the estimation’s attempt to close the non-separability between consumption and leisure (by concurrently moving σ_c closer to 1) and, at the same time, to lower the sensitivity of consumption to the ex-ante real interest rate (by raising the estimated degree of habit formation). Therefore, I re-estimate the model with separable preferences between consumption and leisure to assess the role of this channel. The posterior estimates show that the degree of habit formation becomes lower (0.34 rather than 0.70 as under rational expectations).

Turning to nominal rigidities, the level of price stickiness remains similar between rational and subjective expectations estimations, whereas the estimated wage stickiness is reduced to 0.71, indicating wages that are re-optimized on average every ten months. Wage indexation to past inflation is moderately lower under subjective expectations.

The mean estimates for the elasticity of labor supply vary between rational expectations and learning, but, as indicated by the wide 95% credible sets, the uncertainty surrounding their estimation is substantial. A key parameter in models with learning is the constant gain: here the gain is estimated equal to 0.013, which is only slightly lower than the value of 0.0183 estimated in Milani (2007) without using survey data.[16]

One of the main differences in terms of estimation results concerns the properties of some of the shocks: the estimated persistence for the investment-specific shock is reduced from 0.68 to 0.14 and for the price markup shock falls from 0.73 to 0.09. The risk-premium shock is close to i.i.d., with an autoregressive coefficient equal to 0.10. Sentiment shocks are, instead, identified as quite persistent with autocorrelations in the 0.7–0.85 range.

The estimation results are suggestive that subjective expectations and learning help in capturing some of the persistence in macroeconomic data. Fig. 5 helps in summarizing the evidence. The figure shows posterior distributions for selected endogenous and exogenous sources of persistence. The first panel compares the posterior distributions for the investment adjustment cost coefficient obtained for the model under rational expectations and under learning. The second panel overlaps the posterior distributions for the autoregressive coefficient related to the investment-specific technology disturbance, both under rational expectations and learning. Given that endogenous and exogenous sources of persistence can be interchangeable for some variables, the third and fourth panels show the posterior distributions for the sum of the coefficients capturing the endogenous mechanism and the autoregressive coefficient for the exogenous shock, instead of distributions for single coefficients. The third panel refers to sources of persistence in consumption (habits plus the serial correlation of the risk-premium disturbance) and the fourth to sources of persistence in inflation (endogenous indexation to past inflation plus serial correlation in the price markup shock). The posterior distributions indicate that large degrees of structural and exogenous persistence are needed to fit the data under the assumption of rational expectations. If rational expectations are replaced by observed expectations in a model with learning, there is less need for additional sources of persistence: the relevant posterior distributions all markedly shift to the left.

5.2.2. Model fit

Given that the model with near-rational expectations and sentiment is compared with a current leading framework for empirical work in macroeconomics, known to fit U.S. time series remarkably well, it is natural to consider whether there are differences in fit with the rational expectations version. To allow such comparison, I add the three SPF expectation series to the list of observables under rational expectations, so that both versions are estimated on identical data sets. The observation equations are now extended to include *i.i.d.* measurement error (the survey expectation series are equal to the model-implied rational expectation plus a measurement error). I evaluate the model fit by computing the Marginal Data Densities (MDD), using Geweke’s Modified Harmonic Mean approach. The model with learning and sentiment significantly outperforms the alternative under rational expectations: the MDD equals 2769 under learning and 2148 under rational expectations. This paper, therefore, may provide one approach to address the difficulties that rational expectations DSGE models seem to encounter in matching simultaneously realized macroeconomic data and the corresponding expectations, as documented, for example, in Cole and Milani (2017) in their study of misspecification using a DSGE-VAR approach.

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[16] A constant gain equal to 0.013 means that agents weigh the current observation as 1, the $t-1$ observation as 0.987, the $t-2$ observation as 0.987^2, and so forth.

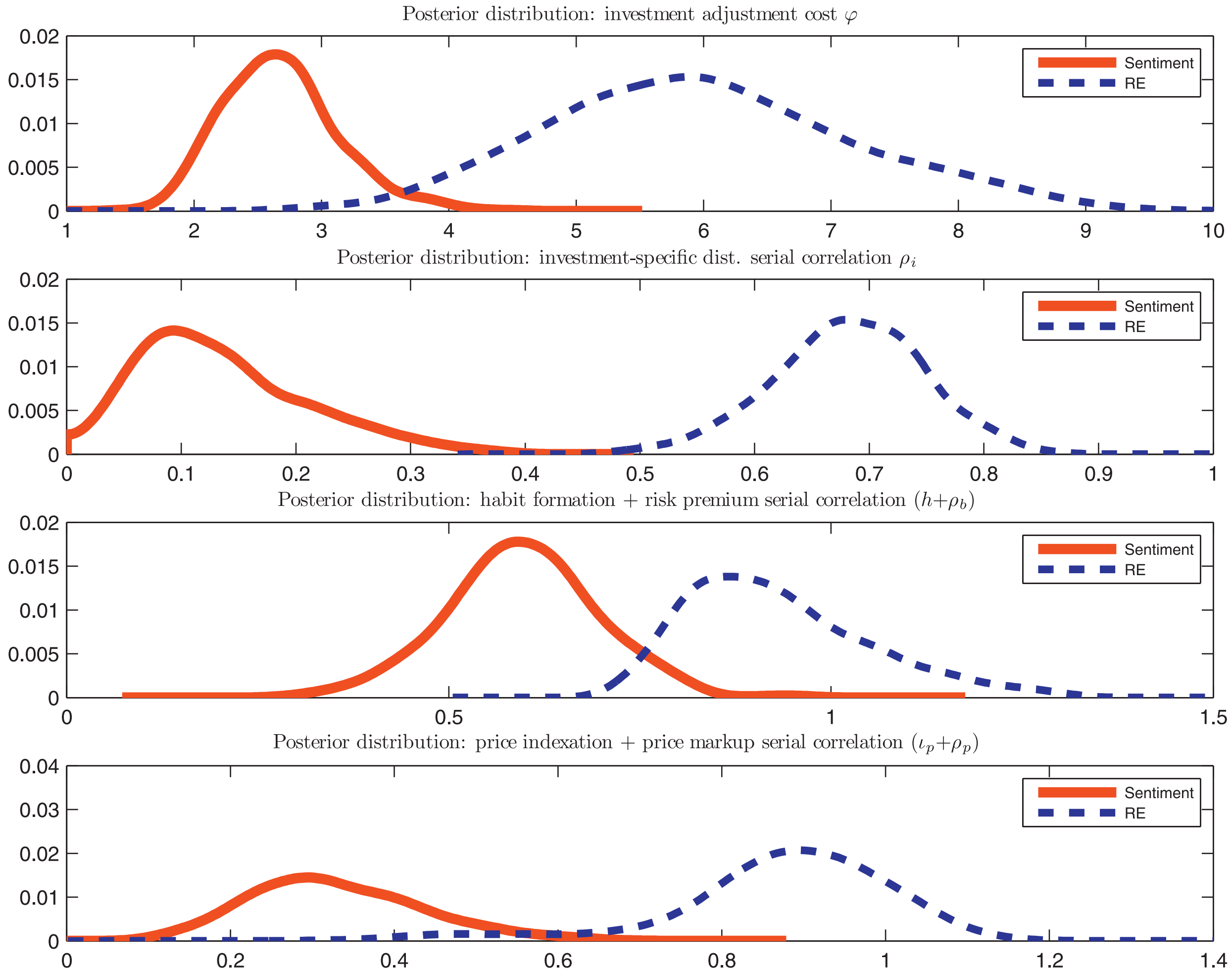

Fig. 5. Posterior distributions: endogenous and exogenous sources of persistence. Note: the top panel shows the posterior distributions for the investment-adjustment cost coefficient. The solid red line refers to the distribution obtained from the model with subjective expectations, learning, and sentiment, the dashed blue line to the distribution obtained from the rational expectations estimation. The second panel shows the posterior distribution for the autoregressive coefficient related to the investment-specific technology disturbance. The third panel shows the posterior distribution for the sum of the habit formation in consumption coefficient and the serial correlation of the risk-premium shock. The bottom panel shows the posterior distribution for the sum of the inflation indexation coefficient and the serial correlation of the price markup shock. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5.2.3. Responses to structural and sentiment shocks

Figs. 6 and 7 overlap the responses of output and inflation to some of the most influential structural and sentiment disturbances (given that impulse responses are time-varying under learning, to simplify the presentation in the graph, I report average impulse responses over the sample). The figures show the mean impulse responses across the last 50,000 MCMC draws, along with error bands corresponding to the 5th and 95th percentiles.

The first set of impulse responses shows that output responds rather quickly to structural innovations. The response to the government spending and risk premium shocks reach their peak effects on impact, while the investment-specific shock generates a peak after only one quarter. Particularly for the case of the risk-premium and investment-specific shocks, the effects are transmitted more quickly to the economy in the estimation that uses subjective rather than rational expectations.

Sentiment shocks, instead, produce longer adjustments. The sentiment shock related to investment leads to a larger and more persistent response of output compared with the corresponding investment-specific structural shock. The output response is hump-shaped with stronger effects between one and two years after the initial impact. The magnitude of the effect for the consumption sentiment shock is roughly similar to the magnitude for the risk-premium shock, except for short-horizons, where the risk-premium dominates. Both sentiment shocks lead to sluggish adjustment in output with more forceful effects that are delayed by at least one year.

Fig. 7 displays the impulse response functions for inflation. The top panel shows the responses to the cost-push (price-markup and wage-markup) and inflationary sentiment shocks. The bottom panel shows the responses to the neutral and

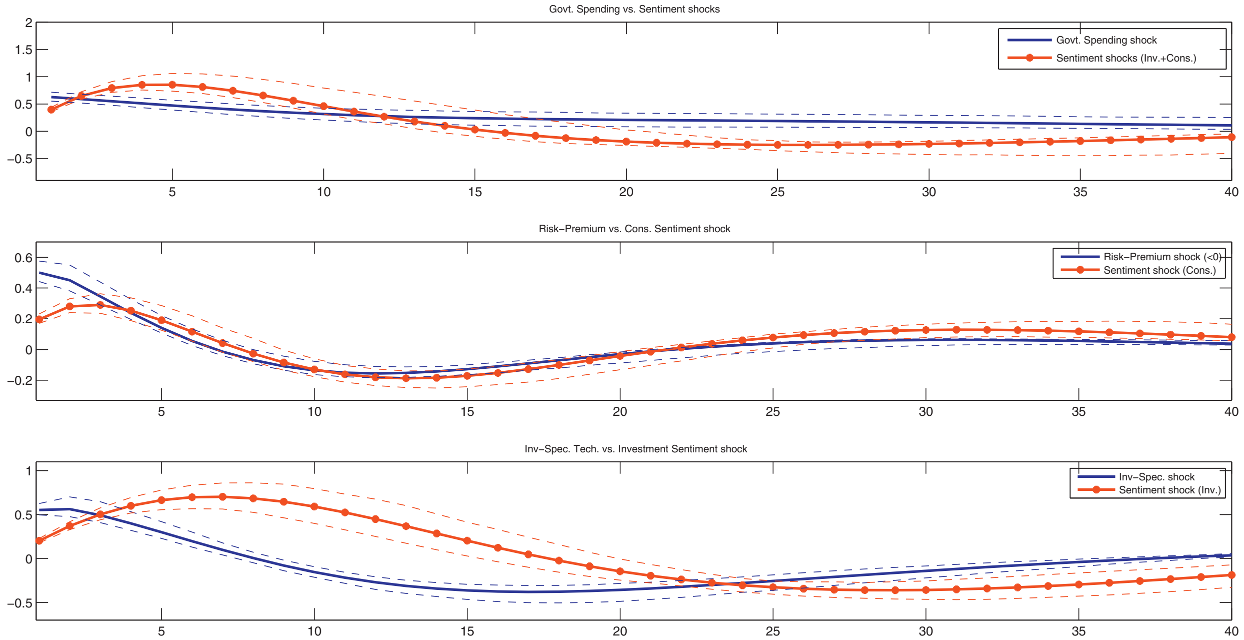

Fig. 6. Impulse response function of y_t to structural and sentiment shocks, under observed expectations. The structural shocks considered are government spending, risk-premium, and investment-specific. The sentiment shocks are those related to consumption and investment expectations.

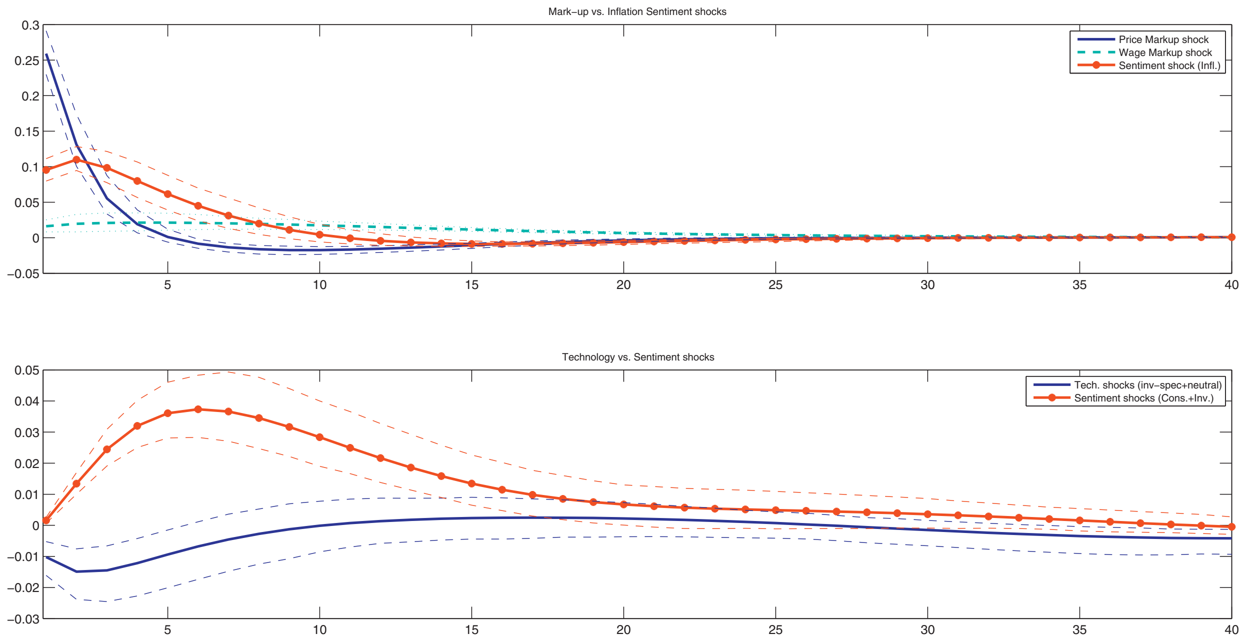

Fig. 7. Impulse response function of inflation to structural and sentiment shocks, under observed expectations. The top panel shows the responses to price markup, wage markup, and inflation sentiment shocks. The bottom panel shows the sum of the responses to the investment-specific and technology shocks, together with the sum of the responses to the consumption and investment sentiment shocks.

investment-specific technology shocks and to the two demand-related sentiment shocks. The price markup shock leads to a large immediate response in inflation, but the adjustment is very quick. Fluctuations in inflation over the medium term are mostly driven by sentiment about future inflation pressures, with the wage markup shock playing a role at longer horizons. Technology shocks lead to a negative sluggish response in inflation. Sentiment about aggregate demand, however, plays an even more important role over the business cycle by producing persistent adjustments in inflation.

Table 3
Forecast error variance decomposition (model with survey expectations, learning, and sentiment).

	Smets-Wouters shocks	Sentiment α_t^c	α_t^i	α_t^π	% Sentiment
Short-Run (0–4)					
y_t	0.786	0.051	0.157	0.007	**21.4%**
c_t	0.649	0.342	0.005	0.004	**35.1%**
i_t	0.653	0.002	0.336	0.009	**34.7%**
π_t	0.704	0.013	0.001	0.282	**29.6%**
Business Cycle (4–24)					
y_t	0.56	0.025	0.350	0.065	**44.0%**
c_t	0.599	0.127	0.166	0.108	**40.1%**
i_t	0.385	0.037	0.531	0.047	**61.5%**
π_t	0.419	0.119	0.135	0.327	**58.1%**

Note: The numbers refer to the average forecast-error variance shares across MH draws for the model estimation with observed expectations, learning and sentiment.

5.2.4. Sources of business cycles

What are the main drivers of business cycle fluctuations? The literature is divided between explanations focused on technology shocks and explanations based on demand shocks. On the other hand, shifts in expectations that are unrelated to fundamentals, psychological forces and market sentiment, waves of optimism and pessimism, typically receive a zero weight as drivers of fluctuations in state-of-the-art general equilibrium macroeconomic models.

By relaxing the assumption of rational expectations and using data on observed expectations, this paper can test the contribution of sentiment to aggregate fluctuations.

Table 3 shows the forecast error variance decomposition for short-run (here 0 to 4 quarters) and business cycle frequencies (here 4–24 quarters, but results were similar for a definition based on 6 to 32 quarters).

Sentiment shifts are indeed a major contributor of business cycle fluctuations. The ensemble of sentiment shocks explains 44% of output fluctuations. The most important driver of output fluctuations at business cycle frequencies appears to be the sentiment shock related to investment expectations, which accounts by itself for 35% of the variance. The structural investment-specific technology shock is dominant, among the remaining shocks, accounting for 19% of fluctuations.[17] The key role of sentiment linked to investment decisions is clearly reminiscent of Keynes' animal spirits, which he also discussed in relation to entrepreneurs' investment behavior. Sentiment shocks explain more than 60% of the variability of investment and 40% of the variability in consumption.

Inflation is also largely driven by sentiment shifts. Inflation sentiment is dominant over business cycle horizons, accounting for almost 60% of the inflation forecast error variance.

While sentiment shocks are particularly important at business cycle frequencies, they also play a role in creating noise at higher frequencies. Sentiment explains between 21% and 35% of short-run fluctuations in the same variables, with investors' sentiment again playing the largest role, among the sentiment shocks, for movements in output. Consumers' sentiment also matters, accounting for a third of aggregate consumption variability in the short-run. Some of the structural shocks have become less persistent in the model with observed expectations. As a result, they are mostly important at horizons below one year: the risk premium shock is the main determinant of short-run consumption movements, the investment-specific shock is the main determinant of short-run investment, and the price markup shock is the main determinant of short-run inflation. Government spending and investment-specific innovations are the main drivers of output variability at horizons below one year.

The empirical results seem to suggest that structural shocks are important, but they have a large and immediate impact on the economy, rather than a prolonged one. At business cycle frequencies, sentiment becomes a major source of fluctuations.

One of the most striking differences between the conclusions in the model with rational expectations and in the model with subjective expectations and sentiment is given by the role of technology shocks. When exogenous shifts in expectations due to sentiment are permitted in the model, sentiment accounts for a large share of cyclical fluctuations in consumption, investment, and output. The two technology shocks account for about a quarter or less of their changes. But when we follow the previous literature by shutting down sentiment and imposing rational expectations (and hence implicitly assuming that any learning that may have taken place has already converged to the rational expectations equilibrium), the contribution of technology jumps to levels around 80% of fluctuations, in line with the RBC literature view, to capture the now omitted

[17] To save space, Table 3 shows the variance shares for the seven 'fundamental' shocks lumped together. The full set of individual results is available in the working paper version (Milani, 2016).

Table 4
VAR with expectation shocks: forecast error variance decomposition.

	FEVD Share due to 'Sentiment' Shocks			
	All Exp.	$E_t c_{t+1}$ only	$E_t i_{t+1}$ only	$E_t \pi_{t+1}$ only
Output	61.07%	35.66%	38.55%	28.33%
Inflation	27.49%	15.70%	6.18%	16.45%
FFR	20.99%	16.21%	7.47%	5.06%

Note: The VAR is estimated on SPF forecasts (one-quarter-ahead) for consumption, SPF forecasts for investment, SPF forecasts for inflation, real output, inflation, and the Federal Funds Rate. The sample is 1981:III to 2011:I. The expectations variables are ordered first, and the expectational, or 'sentiment', shocks are identified through a Cholesky decomposition. The variance decomposition shares ($h = \infty$) in the first column refer to the sum for the three sentiment shocks (in the VAR, no attempt is made to identify them individually). In the following three columns, only one expectation series at a time is included in the VAR.

role of sentiment. For the case of inflation, technology and markup shocks rise to close the gap created by the omission of sentiment.

Overall, the results show that macroeconomic models may miss an important channel by removing, by assumption, sentiment, or similar psychological forces, from their analyses.

5.3. External evidence on the importance of sentiment shocks

As seen in Section 5.2.4, and documented in the literature, constant-gain learning models can provide a close approximation of survey expectations. We can be reassured, therefore, that the results do not arise from a gross misspecification of the expectation formation model. The results so far, however, have been based on a specific structural model. The DSGE environment required introducing specific theoretical assumptions about agents' learning and how to model sentiment. To make sure that sentiment is indeed a large driver of aggregate fluctuations, it is necessary to verify that a more data-driven approach can produce similar findings.

This section aims to provide some independent evidence that can serve as additional validation of the role played by sentiment shocks over the business cycle.

To this scope, I estimate a VAR model on the following variables: SPF expectations for consumption, investment, and inflation, and realized series for real output, inflation, and the Federal Funds rate. The data series are the same used in the estimation of the DSGE model and are detrended in the same way. The identification of the expectational, or sentiment, shocks exploits the timing of the Survey of Professional Forecasters: expectations are predetermined with respect to the other macroeconomic variables, as they are based on $t-1$ information (the same identifying assumption has been exploited in Leduc and Sill, 2013). Therefore, expectations are ordered first in the VAR, and the shocks are identified through a Cholesky decomposition. I do not attempt to identify the three separate sentiment shocks, as done in the DSGE model, but I can identify the overall impact of sentiment on the economy. Table 4 shows the results. The VAR provides additional evidence on the importance of sentiment: the expectational shocks explain 61% of output fluctuations, 27% of inflation fluctuations, and 21% of interest rate fluctuations.

As an additional exercise, I consider the expectations series one at a time in the VAR. The shares due to sentiment remain large and range from 28 to 38.5% of the output variance, with sentiment related to investment expectations playing the largest role.

The results on the importance of sentiment, therefore, do not hinge on specific modeling choices, but they seem a feature of the data. The model presented in the paper offers a theoretical framework to incorporate, assess, and match the empirical relevance of sentiment shocks.

5.4. But is it really sentiment?

In the estimation, we have identified sentiment as the component of expectations that cannot be rationalized as coming from a near-rational forecasting model, which allows for learning by economic agents. But sentiment in the model is obtained without using any data and information that may reflect actual sentiment, optimism or pessimism, degree of confidence, and so forth, in the economy. Can the new disturbances be really interpreted as sentiment then? In this section, we provide evidence that the identified sentiment shocks are really related to excess optimism and pessimism in expectations about the future state of the economy, even if no sentiment data were used in their calculation.

Fig. 8 shows that our sentiment shocks indeed capture exogenous shifts in aggregate optimism and pessimism. Using the available survey indicators of sentiment can be informative. The figure shows scatter plots between the consumption sentiment (top-left panel) and the investment sentiment (remaining three panels) disturbances obtained from the DSGE

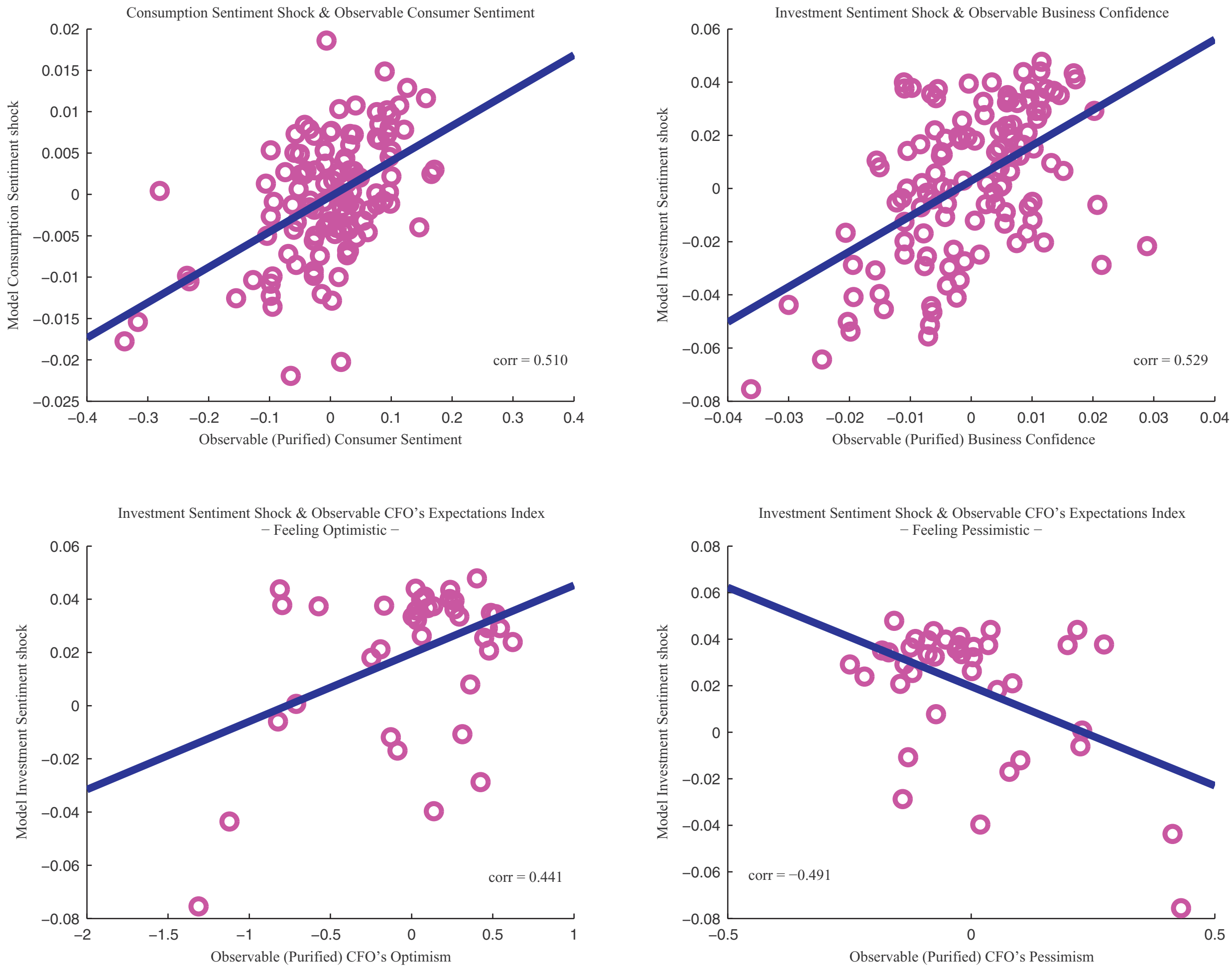

Fig. 8. Scatter plots: sentiment shocks from DSGE model estimation versus observed survey sentiment indicators (purified component). The upper left scatter plot shows the relation (with OLS regression line) between the DSGE-model-based consumption sentiment shock and the exogenous component of the University of Michigan Consumer Sentiment series (obtained by regressing the sentiment index on economic indicators, specifically output, inflation, interest rates). The upper right scatter plot shows the relation between the DSGE-model-based investment sentiment shock and the exogenous component in the Business Confidence Index (obtained by regressing the dependent variable on the same vector of economic indicators). The two bottom panels show the relation between the same model-based investment sentiment and the CFO's optimism index (business executive responders feeling optimistic about future business conditions) and the CFO's pessimism index (business executive responders feeling pessimistic about future business conditions), respectively. Both are also purified to extract the exogenous component of the indexes. The implied correlation coefficients are superimposed on every panel.

model estimation, and the purified (or exogenous) components obtained by regressing the corresponding survey sentiment indicators on a vector of endogenous variables (detrended output, inflation, interest rates). The first panel relates the model-implied consumer sentiment shock to the exogenous component of the University of Michigan Consumer Sentiment Index (using the same 1981–2011 sample). The second panel matches the model's investment sentiment series to the purified Business Confidence Indicator obtained from the OECD's Business Tendency Surveys for Manufacturing, USA. The bottom panels compare the obtained investment sentiment to the 'CFO Expectations Index: percentage of responders feeling more optimistic about the U.S. economy' and the 'CFO Expectations Index: percentage of responders feeling more pessimistic about the U.S. economy', respectively. Both series are published as part of Duke Fuqua School of Business' CFO Magazine Business Outlook Survey; the data are available starting from 2001.

The scatter plots reveal a strong positive relation between our model-based sentiment shocks and the observed sentiment indicators obtained from survey data and purified from their dependence on macroeconomic variables. The correlation coefficients are slightly above 0.5 between model and survey's consumption sentiment series and between model and survey's investment sentiment series. The DSGE investment sentiment series has a 0.44 correlation with the CFO's optimism index and -0.49 correlation with the pessimism index.

We can also rule out that the identified sentiment simply spuriously reflects other factors. The correlation between consumption and investment sentiments and the utilization-adjusted tfp measure calculated by Fernald (2014), for example,

Table 5
Posterior estimates: robustness checks (different data and trend assumptions).

Robustness cases	Posterior means								
	φ	h	ρ_b	ρ_i	ρ_p	ρ_{α_c}	ρ_{α_i}	ρ_{α_π}	Sentiment share in y_t
i) RLT	3.52	0.47	0.1	0.21	0.09	0.89	0.87	0.83	**48.6%**
ii) Growth Rates	6.97	0.18	0.80	0.26	0.09	0.74	0.74	0.78	**64.6%**
iii) Final-vintage data	3.08	0.50	0.62	0.58	0.11	0.81	0.81	0.75	**39.6%**
iv) 1q-ahead forecasts	2.22	0.50	0.10	0.16	0.08	0.71	0.87	0.65	**37.9%**
v) ZLB	2.56	0.50	0.10	0.12	0.12	0.74	0.87	0.66	**45.4%**
vi) M.E.	2.23	0.76	0.47	0.55	0.23	0.22	0.70	0.76	**50.5%**

Note: case (i) reports the results for the estimation using a recursively updated linear trend as detrending option; case (ii) for the estimation in the growth rates of the variables (except for inflation and interest rate); case (iii) for the estimation using final-vintage, revised, data (rather than real-time); case (iv) for the estimation using same-quarter (one-quarter-ahead) SPF expectations; case (v) for the estimation that excludes the ZLB part of the sample; case (vi) for the estimation in which measurement error is appended to the observation equations for real-time data series. For each robustness check, we show posterior mean estimates, 2.5 and 97.5 percentiles, for a selection of model parameters (investment-adjustment cost, habit formation, and autoregressive coefficients for the risk-premium, investment-specific, price-markup, and sentiment disturbances), and the share of forecast error variance of output that can be explained by the ensemble of sentiment shocks.

are equal to −0.016 and 0.05, respectively. The correlation of consumption and investment sentiments with credit spreads (BAA-AAA) are equal to 0.03 and −0.12 if the sample ends before the Financial Crisis, and to −0.12 and −0.30 if the sample includes it. The correlation between the investment-specific technology shock and credit spreads in Justiniano et al. (2010), for example, is much stronger at −0.70. Therefore, investment sentiment may be in part affected by financial stress, but there is a large additional component that is unaccounted for by financial conditions.

Recently, Ben Zeev and Khan (2015) have documented a large role for investment-specific news shocks for business cycles in a VAR context, identified using the maximum-forecast-error-variance approach and relative price of investment data. It is beyond the scope of the paper to investigate the relation between their news shock and the investment sentiment studied in this paper, but both point to an important expectational component, related to investment, as a major driver of business cycles. Interestingly, while in their VAR they find large effects of investment news, they notice how in a DSGE model, investment-specific news shocks explain a nil share of output variance. They call for structural mechanisms that can match the relevance of these expectational investment shocks in DSGE models. The current paper provides one such framework, although the interpretation is of sentiment, rather than based on news about the future.

In the following section, we provide further evidence on the validity of the sentiment interpretation by repeating the analysis with the inclusion of survey sentiment indicators to the set of observables that need to be matched in the estimation.

6. Robustness analysis

In this section, I investigate the sensitivity of the main results to a range of alternative assumptions. They can be grouped into two areas: one collecting the sensitivity checks related to the choice of data, detrending options, and samples, and the other referring to the modeling assumptions for expectations, learning, and sentiment.

The results for the first set of robustness checks are shown in Table 3. To make the table more easily readable, I report the results only for a selection of the most relevant parameters and for the share of output explained by sentiment; the full set of parameters are available in the associated working paper version (Milani, 2016).

First, the estimation is repeated under an alternative detrending assumption, by assuming a recursively-updated linear trend.

I also re-estimate the model using growth rates of the variables, rather than raw data in levels, and final-vintage, revised, data, rather than real-time first-vintage data (the use of revised data is more common in macroeconomic work, but likely less appropriate for the current framework). In another estimation, I consider same-quarter SPF forecasts (which it can be argued are a better representation of one-quarter-ahead forecasts), in place of the following-quarter expected series. Given the uncertainty in how to select the most appropriate vintage for real-time data, and how to deal with data revisions, I also re-estimate the model now allowing for measurement error in the real-time series that are subject to revisions (all except the Federal Funds rate). Finally, the benchmark estimation included the period 2009–2011, which was characterized by the zero-lower-bound in the nominal interest rate. To confirm that those years don't affect the results, the model is re-estimated ending the sample in 2008:IV.

Table 6 shows, instead, the results for the second set of robustness checks, which refer to the learning process and sentiment shocks.

Table 6
Posterior estimates: robustness checks (different learning and sentiment assumptions).

Robustness cases	Posterior means								
	φ	h	ρ_b	ρ_i	ρ_p	ρ_{α_c}	ρ_{α_i}	ρ_{α_π}	Sentiment share in y_t
vii) I.B. = REE	2.95	0.40	0.08	0.14	0.09	0.77	0.84	0.49	**38.9%**
viii) I.B. = 0	3.68	0.46	0.08	0.14	0.09	0.83	0.91	0.71	**64.7%**
ix) PLM = VAR(2)	3.61	0.58	0.09	0.13	0.09	0.77	0.86	0.73	**54.2%**
x) PLM = MSV	2.58	0.49	0.09	0.11	0.08	0.66	0.81	0.69	**33.4%**
xi) Corr. sent	2.65	0.49	0.08	0.11	0.09	0.63	0.88	0.66	**57.3%**
xii) i.i.d. Sent.	4.66	0.68	0.12	0.13	0.09	–	–	–	**12%**
xiii) Observed Conf.	2.76	0.48	0.09	0.12	0.09	0.69	0.85	0.71	**42.8%**

Note: case (vii) reports the results under a different initialization of the agents' learning process: agents' initial beliefs correspond to those in the rational expectations equilibrium (REE) for the previous sample (1964–1981); case (viii) shows the results when the initial beliefs are set at zero; case (ix) shows the results obtained under a different perceived law of motion (PLM): agents use a VAR(2) as their PLM, rather than the benchmark VAR(1); case (x) shows the results for the case in which agents use the MSV solution as their PLM and are able to observe all exogenous disturbances; case (xi) shows the results for the case in which sentiment disturbances are allowed to have non-zero correlation; case (xii) refers to the estimation in which sentiment is forced to be i.i.d.; case (xiii) shows the results obtained for the estimation in which data on consumer confidence and business confidence are included to the list of observables: sentiment in the model is linked to the observable series up to an i.i.d. measurement error. For each robustness check, we show posterior mean estimates, 2.5 and 97.5 percentiles, for a selection of model parameters (investment-adjustment cost, habit formation, and autoregressive coefficients for the risk-premium, investment-specific, price-markup, and sentiment disturbances), and the share of forecast error variance of output that can be explained by the ensemble of sentiment shocks.

The table first reports the results obtained under the initializations of the learning process that were not favored by the data, and which consisted in fixing the initial beliefs either at their REE value for the pre-sample, 1964–1981, period, or at zero.

In the benchmark estimation, agents are assumed to observe the values of endogenous variables, but, as econometricians, are unable to observe disturbances. This choice seems more empirically realistic and satisfies the principle of "cognitive consistency" (e.g., Chung and Xiao, 2013), the symmetry in knowledge between agents within the model and researchers working with the model. To examine the sensitivity of the results, however, I start by partially relaxing this assumption, and assuming, instead, that agents are at least able to approximate the unobserved VARMA(1,1) structure with a higher-order VAR(2), which they use as their PLM. Then, I abstract from cognitive consistency arguments and I assume that agents are now able to observe the exogenous disturbances as they would under rational expectations; therefore, they use the full MSV solution as their PLM.

Another potentially restrictive assumption has been the absence of correlation among sentiment shocks. I now allow for non-zero correlations by assuming that sentiment disturbances evolve as a VAR(1), rather than independent AR processes.[18]

As seen in Table 1, the relaxation of rational expectations leads to a fall in the estimated autocorrelation for some disturbances (mainly, the risk-premium, investment-specific, and price markup disturbances). We can examine to what extent this change is due to the assumption of serially-correlated sentiment series, by re-estimating the model forcing sentiment to be *i.i.d.*

As a final check, I repeat the estimation by adding observable survey data on sentiment and requiring the DSGE sentiment disturbances to match such observables. Measurement error is added to allow for a non-structural stochastic component. The following measurement equations related to sentiment are added to the existing set of measurement equations collected in (20):

$$Survey\ Consumer\ Sent_t = h_0^{cs} + h_1^{cs}\alpha_t^c + me_t^c \tag{22}$$

$$Survey\ Business\ Sent_t = h_0^{bs} + h_1^{bs}\alpha_t^i + me_t^i, \tag{23}$$

where I use the University of Michigan Consumer Sentiment Index and the Business Confidence Surveys: Business Confidence Index (both purified of their endogenous components as described in the previous section) as observable series, h_0^{cs}, h_1^{cs}, h_0^{bs}, h_1^{bs}, are coefficients, and where me_t^c and me_t^i denote measurement error terms, which are *i.i.d.* and with mean zero and variances $\sigma^2_{me_c}$, $\sigma^2_{me_i}$.

There are no major shifts in the estimates, with few exceptions. The estimate of the investment adjustment cost coefficient remains high if the estimation is performed on growth rates of the variables. The degree of habit formation is, instead, reduced further, but at the cost of a more serially-correlated risk premium. The other estimation that leads to some larger

[18] I've also estimated an alternative case, not shown for brevity, in which each sentiment series is allowed to respond to other sentiments' contemporaneous innovations.

differences is the one with measurement error in all real-time series. The estimate of habits is larger, and the persistence of consumption sentiment lower. The risk-premium and investment-specific technology shocks are somewhat more persistent when the estimation is done on final-vintage data.

The shares of economic fluctuations that can be attributed to sentiment shocks remain large and in line with those discussed in the paper. They range from 33% to 65% for the role of sentiment on output. The share would be lower (12%) under *i.i.d.* sentiment, but this case is strongly at odds with the data, and mainly considered to study its implications for the estimated persistence of structural disturbances.[19]

7. Conclusions

The role of psychological factors in booms and busts has been emphasized in the early stages of economic thought by prominent economists as Keynes and Pigou, and it still prominently features in discussions about business cycles by economic observers. Yet, current macroeconomic theory, and particularly empirical work in macroeconomics, have taken another route and typically abstract from psychology almost entirely.

This paper suggested an approach to reintroduce psychology at the center of macroeconomic analysis, by modeling 'sentiment' in a microfounded DSGE model of the U.S. economy. The paper's main objective was to investigate whether the typically omitted sentiment matters for aggregate fluctuations.

The empirical results indeed show that the literature should probably take sentiment and psychological elements more seriously. Sentiment shocks are found to explain more than forty percent of U.S. output fluctuations at business cycle horizons. The main contributor to fluctuations is, in particular, sentiment associated to expectations regarding future investment decisions. Sentiment also explains a large portion of the variability in inflation rates.

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[19] The shares due to sentiment also rise in different instances to 70% or more for consumption and investment (not shown in the table, but available in the working paper version).

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