Risk Sharing among Large and Small Countries

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December 12, 2017

Question and Motivation

We study perfect risk sharing among asymmetric countries.

Why?

- Countries differ in business cycle and structural characteristics.
- Policy relevance:
  - Debate on policies to enhance risk sharing; (Fatas 1998; Farhi and Werning 2017; Beblavy et al 2015)
  - Measurement of degree of international risk sharing; (Asdrubali et al 1996)
- Literature did not focus on implications of asymmetries for risk sharing.

Our Approach and Main Result

Our framework: a two-period model of 2 countries that:

- have identical initial endowment,
- differ in size and stochastic properties of output.

Perfect risk sharing among asymmetric countries results in an efficient redistribution of consumption:

- Asymmetries imply that the shadow price of domestic state-contingent output differs across borders.
- An average level of consumption under perfect risk sharing may fall (or rise) relative to its level under financial autarky.

Relevance

- Derivation of perfect risk sharing allocation in large models with asymmetric countries is hard!
- Many studies impose symmetry of equilibrium which implies equalization of consumption levels (”income pooling”).
  - Differences in the price of risk due to asymmetries among countries are then forced not to matter in determining per-capita consumption.
  - This practice may lead to spurious **“welfare reversals”**: welfare is higher under financial autarky than under complete markets.

Further Implications

Empirics and Political Economy

- Consumption growth rates may not be equalized at the time of a risk-sharing enhancing reform.
  - Empirical studies that measure degree of risk sharing by investigating correlation of consumption growth rates among countries should take this into account.
- Efficient redistribution due to risk sharing relevant to political equilibria on institutional and market reforms.
  (Persson and Tabellini 1996).
  - Pursuing risk sharing via income pooling implements transfers to offset the income effects of adjustment in the equilibrium price of risk.

Related Literature

- Early theoretical discussion of the issue in quantitative studies:

  (Cole and Obstfeld 1991; Chari et al 2002; Devereux and Engel 2003; Monacelli and Faia 2004; Tille and Pesenti 2004)

- Spurious welfare reversals and approximation errors:

  (Kim and Kim 2003).

- Optimal Portfolio choice literature:

  (Devereux and Sutherland 2007).

Outline

- Model (one factor).
- Complete Markets, financial autarky, and perfect pooling.
- Edgeworth box analysis with country size.
- Welfare and reversals.
- Generalization (two factors).
- Spurious Welfare Reversals in a DSGE model.

Analytical Framework

Two-Period Endowment Model

- First example: asymmetric volatility—a single global shock with different factor loadings for each country.
  - In second example we show that results generalize to country-specific shocks.
- We contrast three different arrangements:
  1. complete markets (CM),
  2. financial autarky (FA),
  3. income pooling (IP).
- Welfare analysis under the second order approximation, $U(C_i) = log(C_i)$ where $i = H, F$.

Model Specification

- Size: H has population n; F has (1-n).
- Endowment is deterministic in period 1:

$$Y_{H,1} = Y_{F,1} = 1.$$

- Endowment is stochastic in period 2:

$$Y_{H,2} = 1 - \gamma_H \epsilon, \; Y_{F,2} = 1 - \gamma_F \epsilon.$$

where $E(\epsilon) = 0$, $E(\epsilon^2) = \sigma_\epsilon^2$ and γ_H, γ_F are factor loadings.

- Aggregate output:

$$Y_{W,1} = 1.$$

$$Y_{W,2} = nY_{H,2} + (1-n)Y_{F,2} = 1 - \gamma_W \epsilon.$$

where $\gamma_W = n\gamma_H + (1-n)\gamma_F$.

Complete Markets Allocation

Nonlinear Solution

- Consumption **growth rates** in each country are equalised across all states of nature.
- In **levels**, consumption in each country is a fraction of the world endowment (Obstfeld and Rogoff 1996):

$$C_{i,t} = \mu_i Y_{W,t}.$$

where $\mu_i = \frac{\frac{Y_{i,1}}{Y_{W,1}} + \beta E\left(\frac{Y_{i,2}}{Y_{W,2}}\right)}{1+\beta}$ and $i = H, F$.

Complete Markets Allocation

First and Second Order Approximation

$$C_{i,t} = \mu_i Y_{W,t}.$$

where $\mu_i = \frac{\frac{Y_{i,1}}{Y_{W,1}} + \beta E\left(\frac{Y_{i,2}}{Y_{W,2}}\right)}{1+\beta}$ and $i = H, F$.

- **Up to first order approx:** $E\left(\frac{Y_{H,2}}{Y_{W,2}}\right) \approx 1$ implying that $C_{H,t} = C_{F,t}$, in both periods $t = 1, 2$.
- **Up to second order approx:** $E\left(\frac{Y_{H,2}}{Y_{W,2}}\right) \approx 1 + (\gamma_W - \gamma_H)\gamma_W \sigma_\epsilon^2$.

$$\mu_H = 1 + \frac{\beta\gamma_W(\gamma_W - \gamma_H)}{1+\beta}\sigma_\epsilon^2, \mu_F = 1 + \frac{\beta\gamma_W(\gamma_W - \gamma_F)}{1+\beta}\sigma_\epsilon^2.$$

Efficient Consumption Share

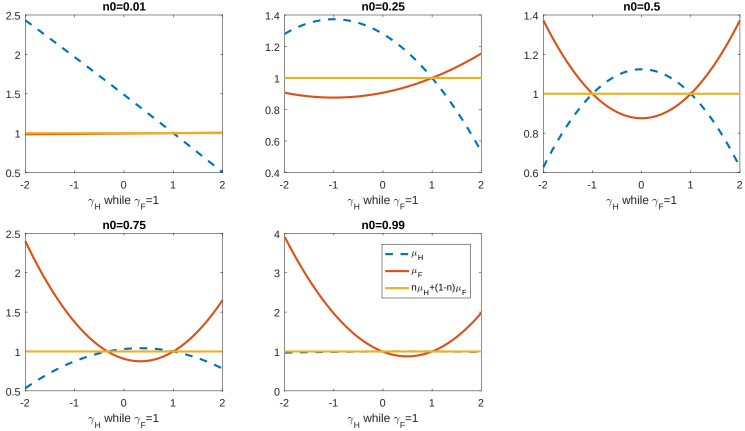
Analytical Representation

$$\mu_i = 1 + \frac{\beta\gamma_W(\gamma_W - \gamma_i)}{1+\beta}\sigma_\epsilon^2$$

where $i = H, F$ and $\gamma_W = n\gamma_H + (1-n)\gamma_F$.

- Up to second order, μ_i can be higher or lower than 1:
  - $\mu_i = 1$ if $\gamma_W = 0$ or $\gamma_W = \gamma_H$.
  - Home and Foreign consumption share increase in the difference $(\gamma_W - \gamma_i)$.
  - γ_i can be negative which can result in negative γ_W.
  - γ_W is a function of the size of the country: the larger the country the larger its influence on the world output and γ_W.

Efficient Consumption Share



Different Risk-Sharing Arrangements

Levels vs Growth Rates of Consumption

- Consumption **LEVELS are equalized** under IP:

  $C_{H,t} = C_{F,t}$ where $t = 1, 2$.

  ...as well as (by construction) under FA in period 1:

  $C_{H,1} = 1 = C_{F,1}$
  (recall: $C_{H,2} = 1 - \gamma_H \epsilon \neq C_{F,2}$ yet $\overline{C}_H = \overline{C}_F = 1$.)

- **may not be equalized** under CM, because of the endogenous adjustment of income to the price of risk:

  $C_{H,1} = \mu_H, C_{H,2} = \mu_H Y_{H,2}$ with $\overline{C}_H = \mu_H$.

- Consumption **GROWTH RATES** instead are **equalized under both CM and IP**, not under FA.

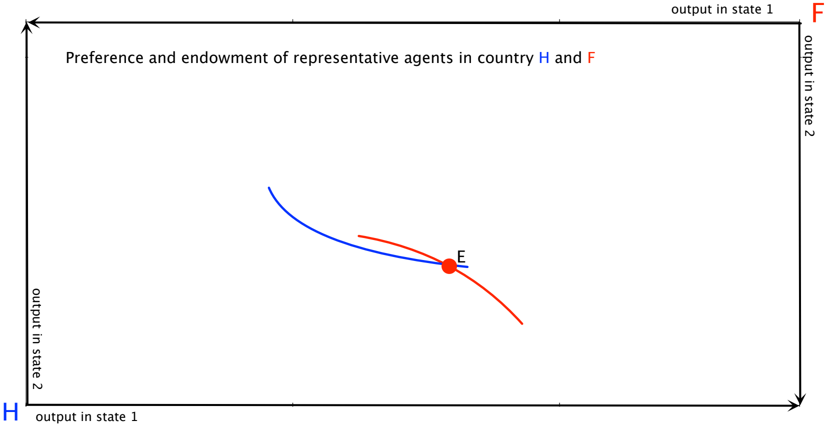
Edgeworth Box Analysis

Unconventional use of the box:

- Individual preferences and endowment of representative agents of two countries H and F.
- When we allow the number of agents in H to be infinitesimal: World price of risk dictated by autarky price in F.
  - In the graph to follow, we set $\gamma_H > \gamma_F$ and $\gamma_F = 1$. Recall that $Y_{i,2}$ subject to a mean zero shock.

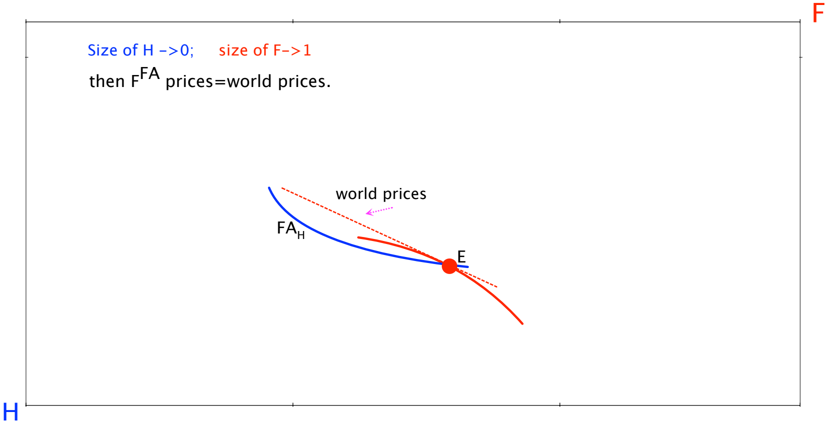
Edgeworth Box

Financial Autarky



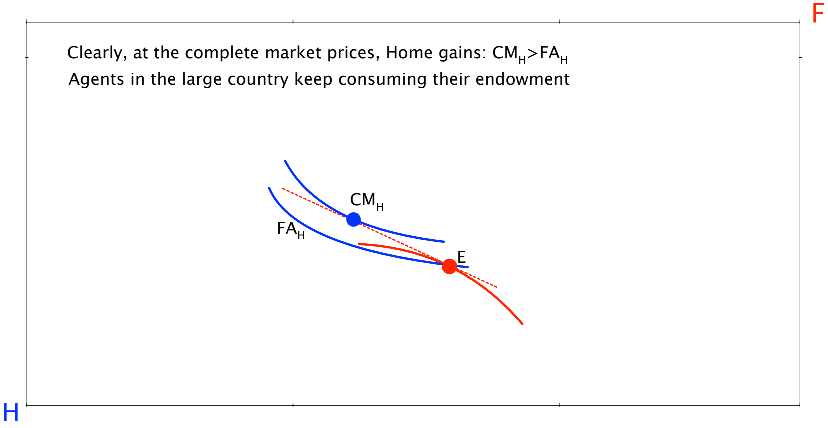
Edgeworth Box

Budget Constraint at World Prices that coincide with F Autarky Prices



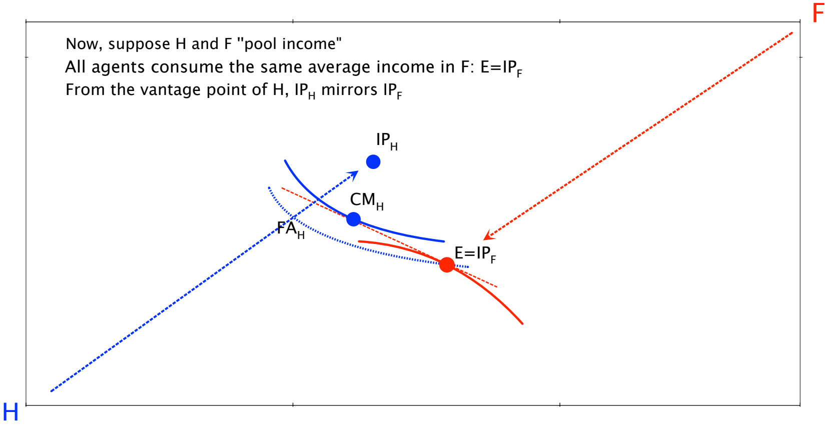
Edgeworth Box

Complete Markets when H and F differ in size



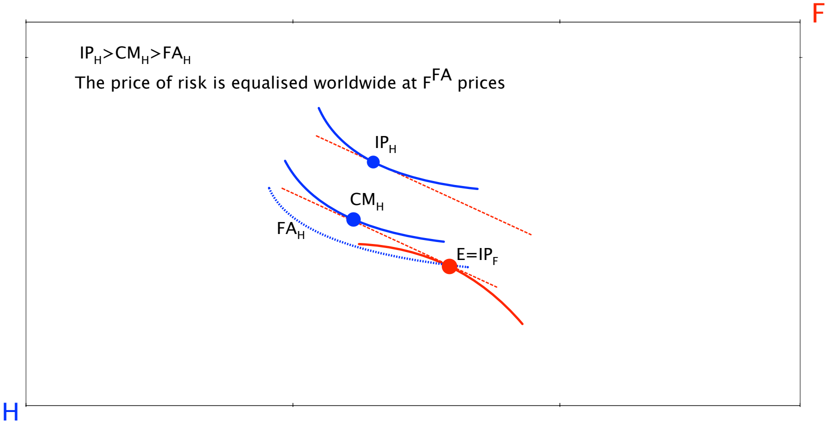
Edgeworth Box

Consumption under Income Pooling (H and F differing in size)



Complete Markets vs Income Pooling

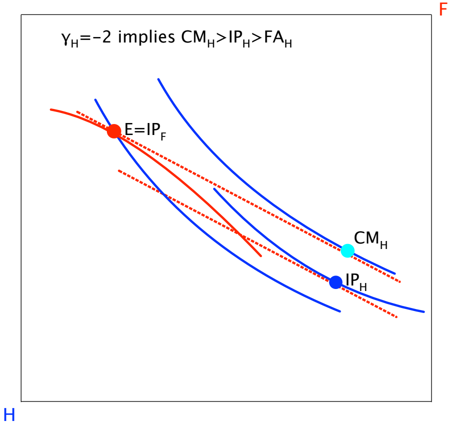
Home has high volatility of output, with $\gamma_H = 2$
Income Pooling is a good deal for Home



Income Pooling vs Complete Markets

IP may however be a bad deal for Home

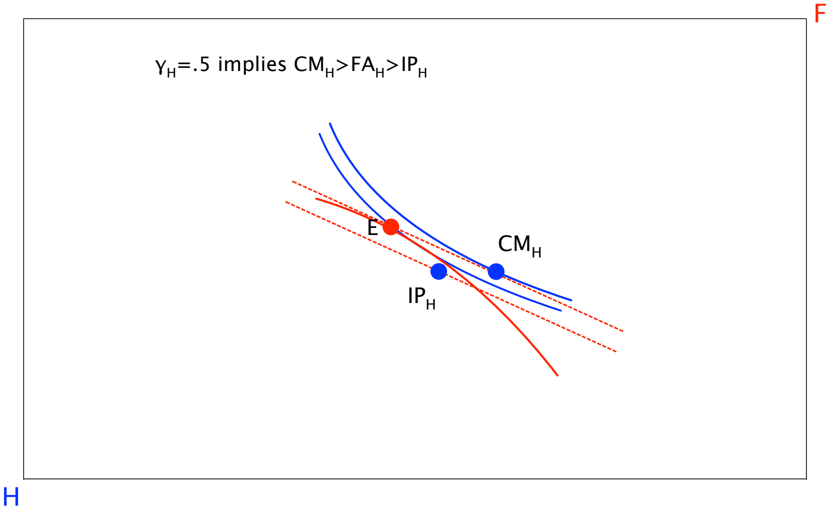
Home output variance high but negative covariance with F $\gamma_H = -2$, $(\gamma_F = 1)$



Financial Autarky vs Income Pooling

Home may even be better off in FA

volatility of output not too high, e.g. $\gamma_H = 0.5$, $(\gamma_F = 1)$



Welfare Analysis in the General Case

Second Order Approximation

We now reconsider welfare ranking using:

$$E(log(C_H)) = E(log(\overline{C}_H)) + \frac{1}{\overline{C}_H} E(C_H - \overline{C}_H) - \frac{1}{\overline{C}_H^2} E(C_H - \overline{C}_H)^2$$

Home loss under different risk sharing arrangements:

- FA: $L_H^{FA} = \frac{\beta}{2}\gamma_H^2\sigma_\epsilon^2$.
- CM: $L_H^{CM} = -\beta(\gamma_W - \gamma_H)\gamma_W\sigma_\epsilon^2 + \frac{\beta}{2}\gamma_W^2\sigma_\epsilon^2$.
- IP: $L_H^{IP} = \frac{\beta}{2}\gamma_W^2\sigma_\epsilon^2$.

Ranking Welfare

as a Function of Size and Volatility

Table 1: **Ranking of CM, FA, IP for Home and Foreign country depending on home size n and factor loading γ_H (setting $\gamma_F = 1$):**

n	γ_H				
0.01	< -199	$(-199, -99)$	$(-99, -0.98)$	$(-0.98, 1)$	> 1
0.25	< -7	$(-7, -3)$	$(-3, -0.6)$	$(-0.6, 1)$	> 1
0.5	< -3	$(-3, -1)$	$(-1. - 0.33)$	$(-0.33, 1)$	> 1
0.75	< -1.67	$(-1.67, -0.33)$	$(-0.33, -0.14)$	$(-0.14, 1)$	> 1
0.99	< -1.02	$(-1.02, -0.01)$	$(-0.01, -0.005)$	$(-0.005, 1)$	> 1
home	$IP \succ CM \succ FA$	$IP \succ CM \succ FA$	$CM \succ IP \succ FA$	$CM \succ FA \succ IP$	$IP \succ CM \succ FA$
foreign	$CM \succ FA \succ IP$	$CM \succ IP \succ FA$	$IP \succ CM \succ FA$	$IP \succ CM \succ FA$	$CM \succ IP \succ FA$

- For $-99 < \gamma_H < -.01$ (given $\gamma_F = 1$), implying negative covariance of output, the Home country prefers:
  - complete markets if its size is **small** (n=0.01).
  - income pooling if its size is **large** (n=.99).

Generalization: Two Factor Model

Model Description

- Deterministic endowment in period 1:

$$Y_{H,1} = Y_{F,1} = 1.$$

- Stochastic endowment in period 2:

$$Y_{H,2} = 1 - \epsilon_H, \, Y_{F,2} = 1 - \epsilon_F.$$

$$Y_{W,2} = nY_{H,2} + (1-n)Y_{F,2} = 1 - n\epsilon_H - (1-n)\epsilon_F.$$

- $E(\epsilon_H) = E(\epsilon_F) = 0$, $E(\epsilon_H^2) = \sigma^2_{\epsilon_H}$, $E(\epsilon_F^2) = \sigma^2_{\epsilon_F}$, $E(\epsilon_H \epsilon_F) = \sigma_{\epsilon_H \epsilon_F}$.
- $U(C_i) = log(C_i)$ where $i = H, F$.

Complete Market Allocation

in the Two-Factor Model

britishsss

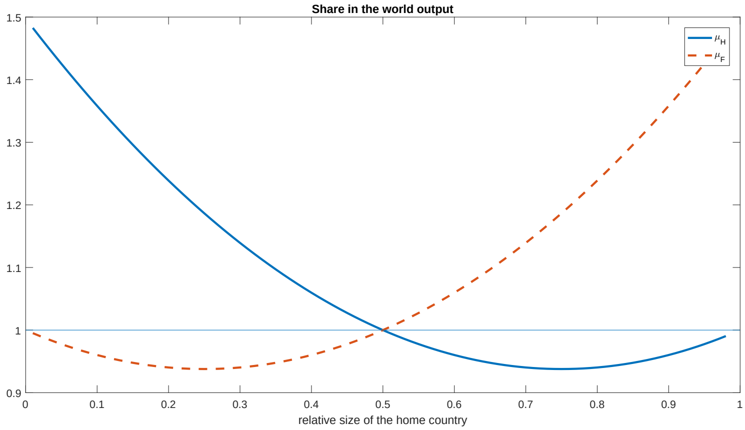
- Consumption in each country is a fraction of the world endowment with $\mu_i = \frac{\frac{Y_{i,1}}{Y_{W,1}} + \beta E\left(\frac{Y_{i,2}}{Y_{W,2}}\right)}{1+\beta}$.
- $E\left(\frac{Y_{H,2}}{Y_{W,2}}\right) \approx 1 + n(n-1)\sigma^2_{\epsilon_H} + (1-n)^2\sigma^2_{\epsilon_F} + 2n(1-n)\sigma_{\epsilon_H\epsilon_F}$.
- Home consumption share:

$$\mu_H = 1 + \frac{\beta(1-n)(n(\sigma_{\epsilon_H\epsilon_F} - \sigma^2_{\epsilon_H}) - (1-n)(\sigma_{\epsilon_H\epsilon_F} - \sigma^2_{\epsilon_F}))}{1+\beta}.$$

- if $n \to 0$ then $\mu_H = 1 + \frac{\beta(\sigma^2_{\epsilon_F} - \sigma_{\epsilon_H\epsilon_F})}{1+\beta}$.
- In general, $\frac{\partial \mu_H}{\partial \sigma^2_{\epsilon_H}} = -\frac{\beta(1-n)n}{1+\beta}$ and $\frac{\partial \mu_H}{\partial \sigma_{\epsilon_H\epsilon_F}} = \frac{\beta(1-n)(2n-1)}{1+\beta}$.

Optimal Consumption Shares

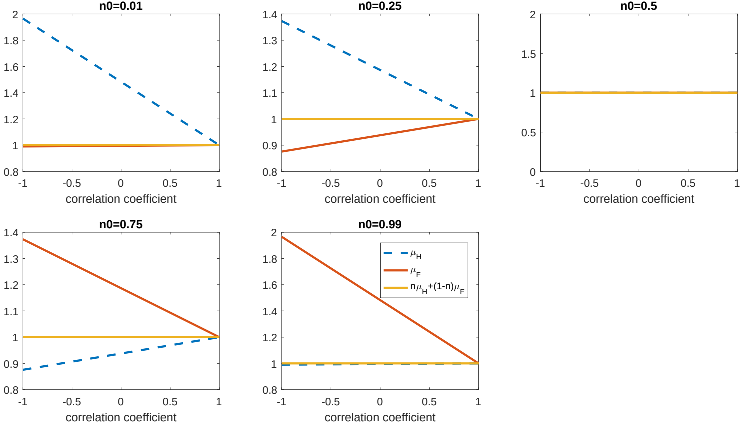
$\sigma_{\epsilon_H} = \sigma_{\epsilon_F} = 1,\ \sigma_{\epsilon_H \epsilon_F} = 0$



$$\mu_H = 1 + \frac{\beta \sigma_\epsilon^2 (n-1)(2n-1)}{1+\beta},\ \mu_F = 1 + \frac{\beta \sigma_\epsilon^2 n(2n-1)}{1+\beta}.$$

Optimal Consumption Shares

$\sigma_{\epsilon_H} = \sigma_{\epsilon_F} = 1,\ \sigma_{\epsilon_H \epsilon_F} \in [-1, 1]$



What Do We Learn?

- Countries that are ex ante identical but for size, may end up with a different level of (consumption) demand with full risk sharing.
- Smaller countries gain. Intuitively, the equilibrium price of risk is close to the FA prices of the larger country.
- The gains are larger, the more negative the covariance of output is.

Consumption Level

Two-Factor Model

1. By construction, we have an example in which consumption levels in period 1 are equalized under financial autarky.

   $$C_{H,1} = 1, C_{H,2} = 1 - \epsilon_H, \overline{C}_H = 1.$$

2. as well as with income pooling:

   $C_{H,t} = C_{F,t}$ where $t = 1, 2.$:

   $$C_{H,1} = 1, C_{H,2} = (1 - n\epsilon_H - (1-n)\epsilon_F), \overline{C}_H = 1.$$

3. Under complete markets, however, because of the endogenous adjustment of income to the price of risk, Consumption is not equalized in period 1.

   $$C_{H,1} = \mu_H, C_{H,2} = \mu_H(1 - n\epsilon_H - (1-n)\epsilon_F), \overline{C}_H = \mu_H.$$

Welfare

Two-Factor Model

Home loss under different risk sharing arrangements:

- FA: $L_H^{FA} = \frac{\beta}{2}\sigma^2_{\epsilon_H}$.
- CM: $L_H^{CM} = -\beta(1-n)(n(\sigma_{\epsilon_H\epsilon_F} - \sigma^2_{\epsilon_H}) - (1-n)(\sigma_{\epsilon_H\epsilon_F} - \sigma^2_{\epsilon_F})) + \frac{\beta}{2}(n^2\sigma^2_{\epsilon_H} + (1-n)^2\sigma^2_{\epsilon_F} + 2n(1-n)\sigma_{\epsilon_H\epsilon_F})$.
- IP: $L_H^{IP} = \frac{\beta}{2}(n^2\sigma^2_{\epsilon_H} + (1-n)^2\sigma^2_{\epsilon_F} + 2n(1-n)\sigma_{\epsilon_H\epsilon_F})$.

Welfare Ranking

as a Function of Size and Volatility

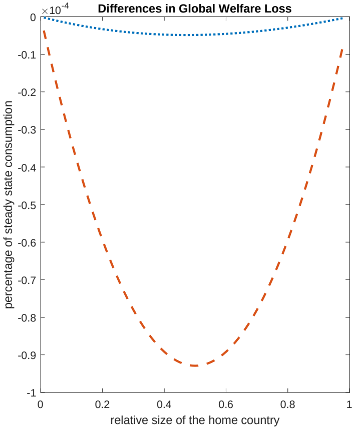
Table 2: **Ranking of CM, IP, FA for Home and Foreign country depending on home size n and $\sigma^2_{\epsilon_H}$ ($\sigma_{\epsilon_F} = 1$ and $\sigma_{\epsilon_H \epsilon_F} = 0$)**

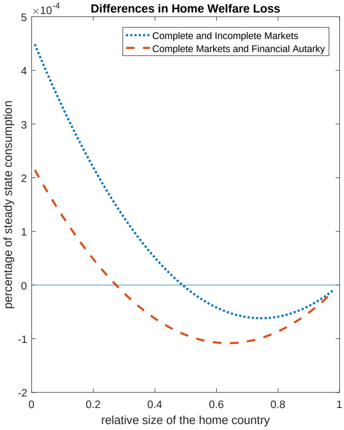
n	$\sigma^2_{\epsilon_H}$			
0.01	< 0.98	$(0.98, 99)$	$(99, 199)$	> 199
0.25	< 0.6	$(0.6, 3)$	$(3, 7)$	> 7
0.5	< 0.33	$(0.33, 1)$	$(1, 3)$	> 3
0.75	< 0.14	$(0.14, 0.33)$	$(0.33, 1.67)$	> 1.67
0.99	< 0.005	$(0.005, 0.01)$	$(0.01, 1.02)$	> 1.02
home	$CM \succ FA \succ IP$	$CM \succ IP \succ FA$	$IP \succ CM \succ FA$	$IP \succ CM \succ FA$
foreign	$IP \succ CM \succ FA$	$IP \succ CM \succ FA$	$CM \succ IP \succ FA$	$CM \succ FA \succ IP$

- For $0.01 < \sigma^2_{\epsilon_H} < 99$ (given $\sigma_{\epsilon_F} = 1$) Home country prefers:
  - complete markets if its size is **small** (n=0.01).
  - income pooling if its size is **large** (n=.99).

Reversals in Quantitative Models

Spurious Results





Conclusions

- We reconsider the implications of risk sharing arrangements among countries that differ in size and business cycle.
- As the shadow price of future contingent output differs across borders, reforms that enhance risk-sharing lead to a change in relative consumption vis-à-vis the status quo.
- Quantitative analysis: asymmetries among countries generally incompatible with explicit/implicit equalization of consumption levels or income pooling.

Research Directions

- Interaction of financial frictions with other distortions.
  - In models with additional distortions, pecuniary externalities may lower welfare—see e.g. Auray and Equyem (2014), who allow for nominal rigidities but constrain monetary policy.
- Political economy of institutional reform and capital market integration.
  - large countries would prefer arrangement implementing income pooling.
  - small countries may prefer complete markets, unless plagued by cycles of large amplitude.
- Design of empirical assessment of risk sharing around times of capital market/institutional reforms.

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Complete Markets Derivation (1)

$$\max U(C_H) = u(C_{H,1}) + \sum_{s=1}^{S} \beta \pi(s) u(C_{H,2}(s))$$

$$s.t. C_{H,1} + \sum_{s=1}^{S} \frac{p(s)}{1+r} C_{H,2}(s) = Y_{H,1} + \sum_{s=1}^{S} \frac{p(s)}{1+r} Y_{H,2}(s)$$

Home and Foreign Euler equations:

$$C_{H,2}(s) = \left[\pi(s) \beta \frac{(1+r)}{p(s)} \right]^{\frac{1}{\rho}} C_{H,1}; \; C_{F,2}(s) = \left[\pi(s) \beta \frac{(1+r)}{p(s)} \right]^{\frac{1}{\rho}} C_{F,1}$$

Complete Markets Derivation (2)

Resource constraint:

$$nC_{H,1} + (1-n)C_{F,1} = nY_{H,1} + (1-n)Y_{F,1} = Y_{W,1}$$
$$nC_{H,2}(s) + (1-n)C_{F,2}(s) = nY_{H,2}(s) + (1-n)Y_{F,2}(s) = Y_{W,2}(s)$$

Combining resource constraint and Euler equations:

$$Y_{W,2}(s) = \left[\pi(s)\beta\frac{(1+r)}{p(s)}\right]^{\frac{1}{\rho}} Y_{W,1}.$$

$$\frac{C_{H,1}}{Y_{W,1}} = \frac{C_{H,2}(s)}{Y_{W,2}(s)} = \frac{C_{H,2}(s')}{Y_{W,2}(s')} = \mu_H.$$

where $\mu_i = \frac{\frac{Y_{i,1}}{Y_{W,1}^{\rho}} + \beta E\left(\frac{Y_{i,2}}{Y_{W,2}^{\rho}}\right)}{{Y_{i,1}}^{1-\rho} + \beta E\left(Y_{W,2}^{1-\rho}\right)}$ and $i = H, F$.