Skew-Normal Shocks in the Linear State Space Form DSGE Model

Grzegorz Grabek[†], Bohdan Kłos[‡], Grzegorz Koloch[ħ]

[†] Bureau of Applied Research, National Bank of Poland

[‡] Bureau of Applied Research, National Bank of Poland

[ħ] Bureau of Applied Research, National Bank of Poland; Institute of Econometrics, Warsaw School of Economics

First version — December 2010 / This version — January 2011

Abstract

Observed macroeconomic data – notably GDP, inflation and interest rates – can be, and usually are skewed. Economists attempt to fit models to data by matching first and second moments or co-moments, but skewness is usually neglected. It is so probably because skewness cannot appear in linear (or linearized) models with Gaussian shocks, and shocks are usually assumed to be Gaussian. Skewness requires non-linearities or non-Gaussian shocks. In this paper we introduce skewness into the DSGE framework assuming skewed normal distribution for shocks while keeping the model linear (or linearized). We argue that such a skewness can be perceived as structural, since it concerns the nature of structural shocks. Importantly, the skewed normal distribution nests the normal one, so that skewness is not assumed, but only allowed for. We derive elementary facts about skewness propagation in the state space model and, using the well-known Lubik-Schorfheide model, we run simulations to investigate how skewness propagates from shocks to observables in a standard DSGE model. We also assess properties of an *ad hoc* quasi-maximum likelihood estimator of models' parameters, shocks' skewness parameters among them.

Contents

1 Introduction

Skewness is a statistical feature of observed economic data. For an arbitrary random variable, like output, inflation or an interest rate, skewness is typically manifested by the lack of symmetry in the probability distribution function which governs this variable[1]. If a random variable follows a skewed distribution, then values of this variable above the expected value are either more or less probable then the values below it, i.e. either positive or negative deviations from the mean value are more probable than the opposite ones[2]. Neglecting this feature of economic data distorts the balance of macroeconomic risks which policy makers face, which limits their ability to achieve assumed objectives. Additionally, unnoticed or neglected features of economic phenomena tend to limit the insight into them, especially if data features in question seem to be structural. In this paper we argue that skewness observed in macroeconomic data constitutes such a structural feature. The argument is put forward in the domain of state space models which represent a first order approximation, i.e. linearization, of a DSGE model. We depart from the standard state space model with normally distributed innovations and extend it by allowing shocks to state variables (i.e. shocks in the transition equation) to follow a more general, skewed distribution. This can be thought of as introduction of the structural skewness into the DSGE framework.

In the DSGE framework, skewness in observed variables can appear as a manifestation of three major factors. Firstly, skewness can appear as a result of non-linearities. A trivial example is when a normally distributed variable, e.g. a shock, is squared so that it obtains a χ^2_1 distribution with skewness coefficient equal to $2\sqrt{2}$. This shock, influencing states can propagate skewness to observables. Other examples include shocks cross-products, which are typical for higher order perturbations, as well as downward nominal or real rigidities, see (Fahr and Smets, 2008; Kim and Ruge-Murcia, 2009). In such cases, shocks hitting the economy can well be symmetric. Although nonlinear DSGE models have advantages, first order approximations are used more often, especially when the model is estimated, see (Amisano and Tristani, 2007). Secondly, skewness can be a result of models' internal mechanisms, e.g. asymmetric preferences, see Christodoulakis and Peel (2009). In such a case, skewness constitutes a structural feature of the model. Such mechanisms result nevertheless in non-representative agent specifications. Also, skewness vanishes if only the first order perturbation is used. Finally even in a fully linear (or linearized) model, skewness in observables can appear as a result of the fact that shocks hitting the economy follow a skewed distribution[3]. In such a case, skewness also constitutes a structural feature of the model. This is because structural shocks constitute the only possible source of skewness in states and observables.

In this paper we exploit the latter approach, i.e. we take a linear state space model which represents a first order perturbation of a DSGE model and assume that martingale difference shocks in the transition equation have a skewed distribution. Alternatively, we could assume that measurement errors are skewed. Both approaches result in skewed observables, but the latter one lacks structural motivation. The first one, in turn, has a sound economic interpretation.

What is important for our motivation, is that in the class of linear state space models skewness in observed variables must be a reflection of skewness in stochastic disturbances. There is no other way for skewness to enter the model. Additionally, skewness in structural shocks has a sound interpretation.

Introduction of skewness gives rise to the question which family of probability distributions for shocks is appropriate for this purpose. Such a family, firstly, should nest a normal distribution, so that the typical specification of DSGE shocks is allowed for and shocks skewness can be rejected if it does not find enough

[1]Yet, it can be the case that probability distribution function of a non-skewed random variable is not symmetric.
[2]This gives rise to the positive and negative skewness respectively.
[3]Outside the framework of DSGE models Ball and Mankiw (1995) showed how combination of non-linearity (firms adjust prices to shocks that are sufficiently large to justify paying menu costs) and skewed shocks (with zero mean) to desired price levels leads to skewed observed price changes.

support in the data. Secondly, employed distribution should have properties that allow us to use the state space setting. Desired properties involve closure under linear transformations, addition of independent variables, taking joint and marginal distributions and under conditioning. Most of these features, but not all of them, are offered by the closed skewed normal distribution.

In the paper we do three things. First, we deliver elementary facts about propagation of skewness in linear state space models. Second, we conduct simulation experiments designed to capture propagation of skewness from shocks to observed variables in a quite mainstream Lubik and Schorfheide (2007) DSGE model. Finally, we develop a simple, yet useful, quasi-maximum likelihood estimation procedure, which is capable of handling skewness, but avoids numerical obstacles faced in case of maximum likelihood estimation.

2 Skewness in macroeconomic data

Distributions of many macroeconomic time series exhibit skewness or asymmetry[4]. Table 1 reports skewness coefficients for five macro aggregates in Australia, Canada, New Zealand and United Kingdom calculated over ca. 15–25 years on the basis of quarterly data. This data sample was used by Lubik and Schorfheide (2007), whose model is used in our simulation exercises. In all of the countries inflation is positively skewed, which means that we should expect more episodes of inflation above the mean, than episodes below the mean. If the mean is in line with the central banks inflation target, this means that positive deviations from the target are more probable than the negative ones implying asymmetric inflationary risks. It is thus of no of surprise, that nominal interest rates also reveal positive skewness. Real output growth rate, in turn, tends to be more often below the average. Absolute changes in exchange rates are positively skewed in the sample, whereas absolute changes of terms of trade do not seem to exhibit a consistent pattern.

Table 1. Skewness in the data

Variable	Australia 1974:1 – 2001:4	Canada 1981:2 – 2002:3	New Zealand 1988:1 – 2001:4	United Kingdom 1975:1 – 2002:3
GDP growth	−0.41	−0.55	−0.91	−0.15
Inflation	0.76	0.93	1.47	1.55
Nominal interest rate	0.59	0.90	0.61	0.30
Exchange rate	0.84	0.19	0.26	0.53
Terms of trade	−1.78	−0.13	0.51	0.23

Source: F. Schorfheide's webpage (http://www.econ.upenn.edu/~schorf/)

It is clear from this exemplary exposition that, at least for investigated samples, important macroeconomic time series reveal skewness. This stands in contrast with the fact that current workhorse of macro analysis, i.e. the new-keynesian DSGE model[5], totally abstracts from skewness of observed data, assuming that both structural innovations and measurement errors are normally distributed, hence symmetric.

Some authors test for skewness in macroeconomic time series while working with them. For example, Bai and Ng (2005) failed to find evidence of skewness in (log-differences of) output, industrial production and unemployment in US, but rejected symmetry for inflation — measured both by CPI and by GDP deflator, stock returns, manufacturing employment, consumption of durable goods and in the USD/JPY exchange rate. Fagiolo et al. (2008) find only conservative support for skewness in distribution of growth rate of output in OECD countries. Having applied tests for skewness developed by Bai and Ng (2005) on time series presented in Table 1, we have to reject symmetry for certain series, e.g. for inflation Australia and United Kingdom or GDP growth rate in Canada.

[4]We measure skewness using a skewness coefficient defined as the third central moment standardized by the second central moment to the power of 1.5, see eq. (3.5). For a review of other skewness measures see e.g. MacGillivray (1986).

[5]We focus only on the first order perturbation in this paper.

3 Skewness in linear models

This section presents the closed skewed normal distribution (section 3.1) and provides elementary facts on skewness propagation in linear state space models (section 3.2).

3.1 The closed skewed normal distribution

Let us denote a density function of a p-dimensional normal distribution with mean[6] μ and positive-definite covariance matrix Σ by $\phi_p(z;\mu,\Sigma)$. Let us also denote a cumulative distribution function of a q-dimensional normal distribution with mean μ and nonnegative-definite covariance matrix Σ by $\Phi_q(z;\mu,\Sigma)$. For $q>1$ function Φ_q does not have a closed form. We will define the closed skewed normal, possibly singular, distribution by means of the moment generating function. Then, under nonsingularity conditions, probability density function will be provided.

Definition 3.1. (*the closed skewed normal distribution — moment generating function*)
Let $\tilde{\mu}\in \mathrm{R}^p$ and $\vartheta \in \mathrm{R}^q$ be arbitrary vectors, let $\tilde{\Sigma}\in \mathrm{M(R)}_{p\times p}$ and $\Delta \in \mathrm{M(R)}_{q\times q}$ be nonnegative-definite (so possibly singular) matrices and let $D\in \mathrm{M(R)}_{q\times p}$ be an arbitrary matrix. We say that random variable z has a (p,q) dimensional closed skewed normal distribution with parameters $\tilde{\mu}$, $\tilde{\Sigma}$, D, ϑ and Δ if moment generating function of z, denoted by $M_z(t)$, is given by:

$$M_z(t)=\frac{\Phi_q(D\tilde{\Sigma}t;\vartheta,\Delta+D\Sigma D^T)}{\Phi_q(0;\vartheta,\Delta+D\Sigma D^T)}\mathrm{e}^{t^T\tilde{\mu}+\frac{1}{2}t^T\tilde{\Sigma}t}$$

which henceforth is denoted by:

$$z\sim csn_{p,q}(\tilde{\mu},\tilde{\Sigma},D,\vartheta,\Delta)$$

Note that matrices $\tilde{\Sigma}$ and Δ are allowed to be singular. If $\tilde{\Sigma}$ is not positive definite, hence $|\tilde{\Sigma}|=0$, resulting distribution is called singular. If $\tilde{\Sigma}$ is positive definite, hence $|\tilde{\Sigma}|\neq 0$, then the distribution is called nonsingular. The *csn* distribution is "closed" in the sense, that it is closed under full rank linear transformations. Full column, but deficient row rank linear transformations (dimension expansion) transform nonsingular skewed normal variables into singular ones. Singular variables remain singular under full rank transformations. Both singular and nonsingular variables can be transformed into a not-*csn* distribution under a rank deficient transformation. The skewed normal distribution — consisting of both singular and nonsingular variables — is therefore not closed under arbitrary linear transformations, which has negative consequences for maximum likelihood estimation of state space models with *csn* shocks. If $\tilde{\Sigma}$ is nonsingular, than random variable y has a probability density function.

Definition 3.2. (*the closed skewed normal distribution — probability distribution function*)
If a random variable z follows a (p,q)-dimensional, $p,q\geq 1$, nonsingular or closed skewed normal distribution with parameters $\tilde{\mu}$, $\tilde{\Sigma}$, D, ϑ and Δ, where $\tilde{\mu}\in \mathrm{R}^p$ and $\vartheta\in \mathrm{R}^q$ are arbitrary vectors, $\tilde{\Sigma}\in \mathrm{M(R)}_{p\times p}$ is positive definite, $\Delta\in \mathrm{M(R)}_{p\times p}$ is nonnegative definite and $D\in \mathrm{M(R)}_{q\times p}$ is an arbitrary matrix, than probability density function of y is given by:

$$p(z)=\phi_p(z;\tilde{\mu},\tilde{\Sigma})\frac{\Phi_q(D(z-\tilde{\mu});\vartheta,\Delta)}{\Phi_q(0;\vartheta,\Delta+D\Sigma D^T)} \tag{3.1}$$

[6] All vectors are column vectors throughout the paper.

Density function (3.1) defines a (p,q)-dimensional nonsingular closed skewed normal distribution in the sense that a random variable has (p,q)-dimensional nonsingular closed skewed normal distribution with parameters $\tilde{\mu}$, $\tilde{\Sigma}$, D, ϑ and Δ, where $|\tilde{\Sigma}| \neq 0$ if and only if its density function is given by (3.1). The probability density function (3.1) involves a probability distribution function of a q-dimensional normal distribution for, in principle, arbitrarily large q, which entails computational difficulties when working with likelihood functions based on (3.1).

Parameters $\tilde{\mu}$, $\tilde{\Sigma}$ and D have interpretation of location, scale and skewness parameters respectively. Parameters ϑ and Δ are artificial, but inclusion of these additional dimensions allows for closure of the *csn* distribution under conditioning and marginalization respectively. The q-dimension is also artificial, but it allows for closure for sums and the joint distribution of independent (not necessarily *iid*) variables. When $\tilde{\Sigma}$ and Δ are scalars, they will be denoted respectively by $\tilde{\sigma}$ and δ.

We will make use of the following:

Remark 3.3. For $p = q = 1$, $\vartheta = 0$ and $\Delta = 1$ the *csn* distribution reduces to the Azzalini skewed normal distribution, see Azzalini and Valle (1996); Azzalini and Capitanio (1999).

Such a case will be denoted by surpassing the fixed parameters ϑ and Δ, i.e by writing:

$$z \sim csn(\tilde{\mu}_u, \tilde{\sigma}_u, d_u) \tag{3.2}$$

In the next sections we will find useful the following:

Corollary 3.4. *Let $z \sim csn_{1,1}(\tilde{\mu}, \tilde{\sigma}, D, \vartheta, \delta)$ for parameters as in definition (3.1), then:*

$$\begin{aligned}
\mathrm{E}(z) &= \tilde{\mu} + \sqrt{\frac{2}{\pi}} \frac{d\tilde{\sigma}}{\sqrt{\delta + d^2\tilde{\sigma}}} \\
var(z) &= \tilde{\sigma} - \frac{2}{\pi} \frac{d^2\sigma^2}{\delta + d^2\tilde{\sigma}} \\
\mathrm{E}(z - E(z))^3 &= \left(\sqrt{\frac{2}{\pi}}\right)^3 (2 - \frac{\pi}{4})(d\tilde{\sigma})^3
\end{aligned} \tag{3.3}$$

It follows that:

Remark 3.5. $\mathrm{E}(z) = 0$ iff $\tilde{\mu} = -\sqrt{\frac{2}{\pi}} \frac{d\tilde{\sigma}}{\sqrt{\delta + d^2\tilde{\sigma}}}$.

We also need the following:

Corollary 3.6. *Let $z \sim csn_{p,q}(\tilde{\mu}, \tilde{\sigma}, D, \vartheta, \delta)$ for $p, q \geq 1$ and for parameters as in definition (3.1). Elements of z are independent iff matrices $\tilde{\Sigma}$ and D are diagonal.*

It implies that it is impossible to have $q = 1$ while keeping elements of z independent for $p > 1$.

Corollary 3.7. *Let $z \sim csn_{p,q}(\tilde{\mu}, \tilde{\Sigma}, D, \vartheta, \Delta)$ for $p, q \geq 1$ and for parameters as in definition (3.1). Let also $x \sim N(\mu_x, \Sigma_x)$, for $\Sigma_x > 0$, be independent of z. Then, $z + x \sim csn_{p,q}(\tilde{\mu} + \mu_x, \tilde{\Sigma} + \Sigma_x, D\tilde{\Sigma}(\tilde{\Sigma} + \Sigma_x)^{-1}, \vartheta, \Delta + (D(I - \tilde{\Sigma}(\tilde{\Sigma} + \Sigma_x)^{-1}))\tilde{\Sigma}D^T)$.*

Corollary 3.8. *Let $z \sim csn_{1,q}(\tilde{\mu}, \tilde{\sigma}, d, \vartheta, \delta)$ for $q \geq 1$ and for parameters as in definition (3.1). Let $\rho \neq 0$. Then, $\rho z \sim csn_{1,q}(\rho\tilde{\mu}, \rho^2\tilde{\sigma}, \frac{1}{\rho}d, \vartheta, \delta)$*

Corollary 3.9. *Let $z \sim csn_{p,q}(\tilde{\mu}, \tilde{\Sigma}, D, \vartheta, \Delta)$ for $p, q \geq 1$ and for parameters as in definition (3.1). Let A be square and have full rank. Then, $Az \sim csn_{p,q}(A\tilde{\mu}, A\tilde{\Sigma}A^T, D\tilde{\Sigma}A^T(A\tilde{\Sigma}A^T)^{-1}, \vartheta, \Delta + D\tilde{\Sigma}D^T - D\tilde{\Sigma}A^T(A\tilde{\Sigma}A^T)^{-1}A\tilde{\Sigma}D^T)$*

Corollary 3.10. *Let* $z_i \sim csn_{p,q_i}(\tilde{\mu}_i, \tilde{\Sigma}_i, D_i, \vartheta_i, \Delta_i)$ *for* $p, q_i \geq 1$, $i = 1, 2, ..., n$ *and for parameters as in definition (3.1). Then,* $\sum_{i=1}^{n} y_i \sim csn_{p,\sum_{i=1}^{n} q_i}(\tilde{\mu}^\star, \tilde{\Sigma}^\star, D^\star, \vartheta^\star, \Delta^\star)$ *where* $\tilde{\mu}^\star = \sum_{i=1}^{n} \tilde{\mu}_i$, $\Sigma^\star = \sum_{i=1}^{n} \tilde{\Sigma}_i$, $D^\star = (\Sigma_1 D_1^T, ..., \Sigma_n D_n^T)^T (\Sigma^\star)^{-1}$, $\vartheta^\star = (\vartheta_1^T, \vartheta_2^T, ..., \vartheta_n^T)^T$ *and* $\Delta^\star = \Delta^\oplus + D^\oplus \tilde{\Sigma}^\oplus D^\oplus - [\bigoplus_{i=1}^{n} D_i \tilde{\Sigma}_i](\tilde{\Sigma}^\star)^{-1}[\bigoplus_{i=1}^{n} D_i \tilde{\Sigma}_i]^{-1}$ *for* $\Delta^\oplus = \bigoplus_{i=1}^{n} \Delta_i$, $D^\oplus = \bigoplus_{i=1}^{n} D_i$ *and* $\Sigma^\oplus = \bigoplus \tilde{\Sigma}_i$*. Operator* $\oplus$ *is defined for arbitrary matrices A and B in the following way:*

$$A \oplus B = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$

3.2 Elementary facts on skewness propagation

In this section we put forward elementary facts about skewness propagation in (linear) state space models of the form:

$$\begin{aligned} y_t &= F\xi_t + He_t \\ \xi_t &= A\xi_{t-1} + u_t \end{aligned} \tag{3.4}$$

for $t = 1, 2, ..., T$, where $\xi_t \in \mathrm{R}^p$ denotes states, $y \in \mathrm{R}^m$ denotes observables, $p, m \geq 1$, $m \leq p$, measurement errors e_t and structural shocks u_t are iid martingale differences, e_t is normally distributed and u_t follows a closed skewed normal distribution as defined in section (3.1). Section (3.2.1) deals with state variables, whereas section (3.2.2) deals with observables.

As a measure of skewness we employ the skewness coefficient, which, for a random variable y, is defined as:

$$\gamma(z) = \frac{\mathrm{E}(z - \mathrm{E}(z))^3}{(\mathrm{E}(z - \mathrm{E}(z))^2)^{\frac{3}{2}}} \tag{3.5}$$

provided that the second and the third central moment of z exist[7]. We will make use of the following:

Remark 3.11. For a random variable z with a n-times differentiable moment generating function $M_z(x)$ we have:

$$\mathrm{E}(z - \mathrm{E}(z))^n = \kappa_n(z) = \frac{\partial^n \ln M_z(x)}{\partial x^n} \Big|_{x=0}$$

where $\kappa_n(z)$ denotes the n-th cumulant of z.

Remark 3.12. For a random variable y, for which $\mathrm{E}(y - \mathrm{E}(y))^n$ exists, $\gamma(y) = \frac{\kappa_{3t}(y)}{(\kappa_{2t}(y))^{\frac{2}{3}}}$

Remark 3.13. If random variables z_i, $i = 1, 2, ..., m$, are independent, than $\kappa_n\left(\sum_{i=1}^{m} \alpha_i z_i\right) = \sum_{i=1}^{m} \alpha_i^n \kappa_n(z_i)$.

3.2.1 State variables

We will start with a one-dimensional model and then extend the results to the multidimensional case. Consider the following autoregressive model:

$$\begin{aligned} \xi_t &= \rho\xi_{t-1} + u_t \\ u_t &\sim csn(\tilde{\mu}_u, \tilde{\sigma}_u, d_u) \\ \xi_0 &\sim N(\mu_{\xi_0}, \sigma_0) \end{aligned} \tag{3.6}$$

for $t = 1, 2, ..., T$, where $\xi_t, \xi_0, u_t \in \mathrm{R}$, $\rho \neq 0$, $\tilde{\sigma}_u > 0$, $d_u \in \mathrm{R}$, $\mu_{\xi_0} \in \mathrm{R}$, $\sigma_0 \geq 0$ and $\tilde{\mu}_u$ is set to a value which is consistent with[8] $\mathrm{E}(u_t) = 0$.

[7] Which is true in all cases considered in this paper.

[8] Parameter $\tilde{\mu}_u$ is therefore not free, but equals $-\sqrt{\frac{2}{\pi}} \frac{d_u \tilde{\sigma}_u}{\sqrt{1 + d_u^2 \tilde{\sigma}_u}}$ which implies that $\mathrm{E}(u_t) = 0$.

First we will investigate the effect on ξ_t exerted an innovation u_t at time $t = 1$ keeping $u_t = 0$ for $t > 1$. This is what is called an impulse response analysis. Since ξ_1 is a sum of a normally distributed variable $\rho\xi_0$ and a $csn_{1,1}$-distributed variable u_1, it is, according to corollary (3.7), a csn random variable with parameters $\tilde{\mu}_{\xi,1} = \tilde{\mu}_u$, $\tilde{\sigma}_{\xi,1} = \rho^2\sigma_0 + \tilde{\sigma}_u$, $d_{\xi,1} = d_u\frac{\tilde{\sigma}_u}{\tilde{\sigma}_{\xi,1}}$, $\vartheta_{\xi,1} = \vartheta_u$ and $\delta_{\xi,1} = \delta_u + \tilde{\sigma}_u d_u^2(1 - \frac{\tilde{\sigma}_u}{\tilde{\sigma}_{\xi,1}})$. To see how effects of u_1 propagate through ξ_t, let us notice that $\xi_t = \rho\xi_{t-1} = \rho^{t-1}\xi_1$ for $t > 1$, hence, according to corollary (3.8), variable ξ_t has a $csn_{1,1}$ distribution with parameters:

$$\begin{gathered}\tilde{\mu}_{\xi,t} = \rho\tilde{\mu}_{\xi,t-1} = \rho^{t-1}\tilde{\mu}_{\xi,1}, \quad \tilde{\sigma}_{\xi,t} = \rho^2\tilde{\sigma}_{\xi,t-1} = \rho^{2(t-1)}\tilde{\sigma}_{\xi,1}, \\ d_{\xi,t} = \frac{1}{\rho}d_{\xi,t-1} = \frac{1}{\rho^{t-1}}d_{\xi,1}, \quad \vartheta_{\xi,t} = \vartheta_{\xi,t-1} = \vartheta_{\xi,1}, \quad \delta_{\xi,t} = \delta_{\xi,t-1} = \delta_{\xi,1}\end{gathered} \tag{3.7}$$

We see that the skewness parameter of ξ_t equals $d_{\xi,t} = \frac{1}{\rho^{t-1}}d_{\xi,1} = \frac{1}{\rho^{t-1}}\tilde{d}_u\frac{\tilde{\sigma}_u}{\tilde{\sigma}_{\xi,1}}$. If $|\rho| < 1$, then $|d_{\xi,t}|$ increases with t without bound[9] regardless of the shocks' skewness parameter $\tilde{d}_u \neq 0$. If $|\rho = 1|$, then $|d_{\xi,t}|$ equals $d_{\xi,1} = \tilde{d}_u\frac{\tilde{\sigma}_u}{\tilde{\sigma}_{\xi,1}}$ for all t and if $|\rho| > 1$, then $|d_{\xi,t}|$ decreases with t reaching zero in the limit. If $\rho < 0$, then sign of $d_{\xi,t}$ additionally oscillates.

This basic fact can easily be misunderstood, because, since the magnitude of $d_{\xi,t}$, as measured for example by $|d_{\xi,t}|$, implies in some sense the absolute (i.e. left or right) strength of skewness, one could conclude that absolute skewness intensifies with time in stationary models, is time invariant in case of random walk models and evaporates with time under explosive specifications. This is, however not the case, because the variance of ξ_t also changes with t. As a consequence, skewness of ξ_t in model (3.6) with $u_t = 0$ for $t > 1$, as measured by the skewness coefficient $\gamma(\xi_t)$, see eq. (3.5), is constant over time for $\rho > 0$ and oscilates around zero with a constant amplitude for $\rho < 0$. To see this let us notice that $\xi_t = \rho^{t-1}\xi_1$, therefore remarks (3.12–3.13) imply that $\gamma(\xi_t) = sgn(\rho)^{3(t-1)}\gamma(\xi_1)$. More explicitly, employing corollary (3.4) for $\rho > 0$, we see that:

$$\gamma(\xi_t) \propto \frac{\left(\frac{\sigma_t d_t}{(1+\sigma_t d_t^2)^{\frac{1}{2}}}\right)^3}{\left(\sigma_t - \frac{2}{\pi}\frac{\sigma_t^2 d_t^2}{1+\sigma_t d_t^2}\right)^{\frac{3}{2}}} = \frac{\left(\frac{\sigma_1 d_1}{(1+\sigma_1 d_1^2)^{\frac{1}{2}}}\right)^3}{\left(\sigma_1 - \frac{2}{\pi}\frac{\sigma_1^2 d_1^2}{1+\sigma_1 d_1^2}\right)^{\frac{3}{2}}} \propto \gamma(\xi_1) \tag{3.8}$$

and taking omitted constants of proportionality (which do not depend on ρ) into account we arrive at the conclusion that $\gamma(\xi_t) = \gamma(\xi_1)$ for $t > 1$ and $\rho > 0$.

Univariate autoregressive models with iid csn-distributed shocks preserve therefore skewness (as measured by the skewness coefficient) which originates from a one-time shock occurrence regardless of the value of the autoregressive coefficient $\rho > 0$ and preserve absolute skewness for regardless of $\rho < 0$. Time independency of impulse response skewness in model (3.6) is a consequence of applying for $z_t = \xi_t$ the following more general:

Proposition 3.1. *Let $z_t \in \mathrm{R}$, for $t = 1, 2, ..., T$, be distributed according to a $csn_{1,1}$ distribution with parameters $\tilde{\mu}_{z,t}$, $\tilde{\sigma}_{z,t} > 0$, $d_{z,t}$, $\vartheta_{z,t}$ and $\delta_{z,t} > 0$. Assume that $\vartheta_{z,t} = 0$ and that $\delta_{z,t} = const$ for all t. If $\tilde{\sigma}_{z,t}d_{z,t} = const$ and $\tilde{\sigma}_{z,t}d_{z,t}^2 = const$ for all t, then absolute of the skewness coefficient of z_t, i.e. $|\gamma(z_t)|$, is constant over time, and $sgn(\gamma(z_t))$ equals $sgn(d_t)$.*

Now, still being in the univariate case, we consider propagation of skewness through ξ_t, but we drop the assumption that $u_t = 0$ for $t > 1$, i.e. we consider the full model as given by (3.6). We make use of the fact that ξ_t can be expressed as a weighted sum of innovations u_{t-k} for $k = 0, 1, ..., t-2$, and of ξ_1, i.e. we employ the moving average representation of ξ_t:

[9] I.e. $\lim_{t\to\infty} |d_{\xi,t}| = \infty$.

$$\xi_t = \rho^{t-1}\xi_1 + \sum_{k=0}^{t-2}\rho^k u_{t-k} \tag{3.9}$$

Distribution of ξ_1 is a $csn_{1,1}$ distribution with parameters[10]: $\tilde{\mu}_{\xi,1}$, $\tilde{\sigma}_{\xi,1}$, $d_{\xi,1}$, $\vartheta_{\xi,1}$ and $\delta_{\xi,1}$. Also, using corollary (3.8), random variables $v_k = \rho^k u_{t-k}$, for $k = 0, 1, ..., t-2$, have $csn_{1,1}$ distributions, but with parameters: $\tilde{\mu}_{v,k} = \rho^k \tilde{\mu}_u$, $\tilde{\sigma}_{v,k} = \rho^{2k}\tilde{\sigma}_u$, $d_{v,k} = \frac{1}{\rho^k} d_u$, $\vartheta_{v,k} = \vartheta_u$ and $\delta_{v,k} = \delta_u$. From corollary (3.10), ξ_t has therefore a $csn_{1,t}$ distribution with parameters:

$$\begin{gathered}\tilde{\mu}_{\xi,t} = \frac{1-\rho^{t-1}}{1-\rho}\tilde{\mu}_u + \rho^{t-1}\tilde{\mu}_{\xi,1}, \quad \tilde{\sigma}_{\xi,t} = \frac{1-\rho^{2(t-1)}}{1-\rho^2}\tilde{\sigma}_u + \rho^{2(t-1)}\tilde{\sigma}_{\xi,1} \\ D_{\xi,t} = r_t^T \frac{d_u \tilde{\sigma}_u}{\sigma_{\xi,t}}, \quad \vartheta_{\xi,t} = 1 \otimes \vartheta_{\xi,1} \quad \delta_{\xi,t} = \delta_{\xi,1}\end{gathered} \tag{3.10}$$

where $r_t = (\rho^{t-1}, \rho^{t-2}, ..., \rho, 1)^T$ and $\otimes$ denotes the tensor (Kronecker) product. Above formulae are valid for $|\rho| \neq 1$. To derive them we only need to notice that that $d_{v,k}\sigma_{v,k} = \rho^k d_u \tilde{\sigma}_u$ and that $d_{\xi,1}\tilde{\sigma}_{\xi,1} = d_u \tilde{\sigma}_u$. For $\rho = 1$ the difference is only that $\tilde{\mu}_{\xi,t} = (t-1)\tilde{\mu}_u + \tilde{\mu}_{\xi,1}$, $\tilde{\sigma}_{\xi,t} = (t-1)\tilde{\sigma}_u + \tilde{\sigma}_{\xi,1}$ and $r_t = (1_t)^T$.

As previously, we will show how $\gamma(\xi_t)$ depends on ρ and t[11]. To make exposition simpler, let us assume that $\mu_{\xi_0} = 0$ and $\sigma_0 = 0$, so that $\xi_1 = u_1$ and:

$$\xi_t = \sum_{k=0}^{t-1}\rho^k u_{t-k} \tag{3.11}$$

Employing remark (3.13) to representation (3.11), we see that[12]:

$$\kappa_2(\xi_t) = var(\xi_t) = \begin{cases} \frac{1-\rho^{2t}}{1-\rho^2}\kappa_2(u) & \text{if } |\rho| \neq 1, \\ t\kappa_2(u) & \text{if } |\rho| = 1. \end{cases}, \quad \kappa_3(\xi_t) = \begin{cases} \frac{1-\rho^{3t}}{1-\rho^3}\kappa_3(u) & \text{if } |\rho| \neq 1, \\ t\kappa_3(u) & \text{if } \rho = 1. \end{cases} \tag{3.12}$$

For $\rho = 1$ it is therefore the case that $\gamma(\xi_t) \propto \frac{1}{\sqrt{t}}$ which is a known property of the skewness of sums of iid variables. For $|\rho| \neq 1$ skewness coefficient of ξ_t is proportional to:

$$\gamma(\xi_t) \propto \frac{\frac{1-\rho^{3t}}{1-\rho^3}}{\left(\frac{1-\rho^{2t}}{1-\rho^2}\right)^{\frac{3}{2}}} \tag{3.13}$$

which, this time, depends on ρ. We can form the following:

Proposition 3.2. *Assume model (3.6) for ξ_t. Assume that $\mu_{\xi_0} = 0$ and $\sigma_0 = 0$ (so that $\xi_1 = u_1$). Let t be fixed. Then, $\gamma(\xi_t) = \gamma(u)$ if $\rho = 0$, whereas $\gamma(\xi_t)$ decreases when $\rho \in (0,1)$ increases as well as when $\rho \in (0,-1)$ decreases. Also, $\gamma(\xi_t)$ increases when $\rho > 1$ increases.*

This means, that the skewness coefficient is constant and maximal for $\rho = 0$ and it decreases as ρ departs from zero both to the left or to the right until it reaches 1 or -1. Proposition (3.2) states[13] how $\gamma(\xi_t)$ behaves as a function of ρ. Behavior of $\gamma(\xi_t)$ as a function of t for fixed ρ is stated in the following:

[10] Their values have already been provided in this section.
[11] Assuming $u_t = 0$ fot $t > 1$ we arrived at the conclusion that $|\gamma(\xi_t)|$ does not depend neither on ρ nor on t.
[12] We drop time indexes for u_t since shocks u_t are assumed to be iid.
[13] Only for $\rho = 0$ we conclude that $\gamma(\xi_t)$ is constant over time.

Proposition 3.3. *Let assumptions be as in Proposition (3.2), but ρ be fixed instead of t. Then $\gamma(\xi_t)$ is constant over time if $\rho = 0$ and decreases with time if $\rho > 0$ reaching the limit of $0 < \frac{(1-\rho^2)^{\frac{3}{2}}}{1-\rho^3}\gamma(u) < \gamma(u)$ for $\rho \in (0,1)$ and reaching zero for $\rho = 1$.*

This means, that (for $\rho > 0$) skewness of ξ_t evaporates with time, but it does not vanish totally in stationary specifications, reaching in the limit some fraction of $\gamma(u)$. The limiting fraction is a decreasing function of $\rho \in (0,1)$ in case of random walks, skewness evaporates totally with the decay rate of $\frac{1}{\sqrt{t}}$.

Now we will discuss the multivariate case. More specifically, we will show how skewness propagates in a model of the form:

$$\begin{aligned} \xi_t &= A\xi_{t-1} + u_t \\ u_t &\sim CSN(\tilde{\mu}_u, \tilde{\Sigma}_u, \tilde{D}_u, \vartheta_u, \Delta_u) \\ \xi_0 &\sim N(\mu_{\xi_0}, \Sigma_0) \end{aligned} \tag{3.14}$$

for $t = 1, 2, ..., T$, where $A \neq 0$, $|\Sigma_0| \geq 0$, $|\tilde{\Sigma}_u| > 0$, $|\Delta_u| > 0$, $\mathrm{E}(u_t) = 0$ and eigenvalues of A are less then one in modulus, so that model (3.14) is non-explosive[14]. In what follows, we assume[15] that $\vartheta_u = 0$ and that univariate elements of u_t are independent, see remark (3.6). The difference between models (3.6) and (3.14) for $\xi_t \in \mathrm{R}^p$ is that in the latter case it is allowed that $p > 1$. In what follows we assume for simplicity that $\mu_{\xi_0} = 0$ and $\Sigma_0 = 0$, so that $\xi_1 = u_1$.

The multivariate case differs from the univariate one in a fundamental way. The univariate model (3.6) assures that the state variable ξ_t is distributed according to a $csn_{1,t}$ distribution for all t, i.e. that the csn distribution is closed under transformations which model (3.6) applies to ξ_t. In the multivariate case this does not have to be the case, as stated in the following:

Proposition 3.4. *Let ξ_{t-1} be distributed according to a $csn_{p,q}$ for some $p, q \geq 1$ with parameters $\tilde{\mu}_{\xi,t-1}$, $\tilde{\Sigma}_{\xi,t-1} \geq 0$, $D_{\xi,t-1}$, $\vartheta_{\xi,t-1}$ and $\Delta_{\xi,t} > 0$. Let us consider a random variable $y = A\xi_{t-1}$ for a square matrix A. Then, random variable y has a csn, possibly singular, distribution if and only if $r(A^T) = r([A^T|w_i])$ for all $i = 1, 2, ..., q$ where $r(A)$ denotes rank of A and w_i denotes the i-th row of $D_{\xi,t-1}$.*

In particular y has a csn distribution if matrix A has a full rank, regardless of $D_{\xi,t}$. Proposition (3.4) states, that for a csn variable ξ_{t-1}, variable $y = A\xi_{t-1}$ has a csn distribution if and only if rows of $D_{\xi,t-1}$ are linear combinations of rows of A. For a rank deficient matrix A this is a very demanding condition, since matrix $D_{\xi,t-1}$ can, in principle, be arbitrary. Although proposition (3.4) constitutes a negative result for ξ_t as a p-dimensional variable, it has to be stressed that model (3.14) assures that elements of ξ_t are csn-distributed. Anyway, the fact that ξ_t generally does not obtain a csn distribution does not preclude us from investigating its skewness.

To determine effects of innovations u_t exerted on states ξ_t we resort to the moving average representation of ξ_t:

$$\xi_t = A^{t-1}\xi_1 + \sum_{k=0}^{t-2} A^k u_{t-k} = \sum_{k=0}^{t-1} A^k u_{t-k} \tag{3.15}$$

where the last equality follows from the simplifying assumption about ξ_1.

It has to be made explicit that we are interested in skewness coefficients of elements of ξ_t, i.e. of one-dimensional variables $\xi_{t,i}$ for $i = 1, 2, ..., p$, and not in synthetic multivariate skewness measures of ξ_t regarded as p-dimensional variables. Respective skewness coefficients will be denoted by $\gamma(\xi_{t,i}) = \frac{\kappa_3(\xi_{t,i})}{(\kappa_2(\xi_{t,i}))^{\frac{3}{2}}}$.

[14] This is true in case of DSGE models.
[15] This assumption simplifies the considerations.

Since variables u_t are independent for $t = 1, 2, ..., T$, from remark (3.13), we can see that[16]:

$$\kappa_n(\xi_t) = \kappa_n\left(\sum_{k=0}^{t-1} A^k u_{t-k}\right) = \sum_{k=0}^{t-1}\left(A^k\right)^{\circ(n)}\kappa_n(u) \tag{3.16}$$

where $A^{\circ(n)}$ denotes the n-th Hadamard (or Schur) power of matrix A, i.e. $A^{\circ(n)} = \underbrace{A \circ A \circ ... \circ A}_{n}$, for $\circ$ denoting the Hadamard (or Schur) product, i.e. elementwise multiplication. From (3.16) we see, more explicitly, that:

$$\kappa_2(\xi_{t,i}) = \sum_{k=0}^{t-1}\sum_{j=1}^{p}(a_{ij}^k)^2\kappa_2(u_{.,j}), \quad \kappa_3(\xi_{t,i}) = \sum_{k=0}^{t-1}\sum_{j=1}^{p}(a_{ij}^k)^3\kappa_3(u_{.,j}) \tag{3.17}$$

where a_{ij}^k denotes the ij-th entry of A^k and time indexes for shocks u were suppressed. Less explicit, but considerably more parsimonious expressions for $\kappa_n(\xi_{t,i})$ for all n, t and i simultaneously can be obtained using notations of tensor calculus, in particular the so called Einstein notation. The latter approach would also be advisable in case of nondegenerate dependency structure among entries of u_t. Since we do not pursuit higher-order cumulants than the third one and shocks are independent, we stay with the explicit notation (3.17).

As previously, first we try to determine skewness of $\gamma(\xi_{t,i})$ and assuming that $u_t = 0$ for $t > 1$. In this case, see eq. (3.17), $\kappa_n(\xi_t) = (A^k)^{\circ(n)}\kappa_n(u)$, which converges with t to a zero vector as long as A is nonexplosive. Unfortunately, not much can be said about skewness coefficients $\gamma(\xi_{t,i})$:

Proposition 3.5. *Assume model (3.14) for ξ_t. Assume also that $u_t = 0$ for $t > 1$ and that $u_{1,r} \neq 0$ whereas $u_{1,j} = 0$ for all $j \neq r$. Then:*

$$\gamma(\xi_{t,i}) \propto \begin{cases} 1 & \text{if } a_{i,r}^{t-1} > 0, \\ -1 & \text{if } a_{i,r}^{t-1} < 0. \end{cases} \tag{3.18}$$

where $a_{i,j}^k$ denotes the (i, j)-th element of A^k. It follows, that the series of skewness coefficients $\gamma(\xi_{t,i})$, converges with t iff there exists $t' \geq 0$, such that $a_{i,r}^k > 0$ for all $t \geq t'$ or $a_{i,r}^k < 0$ for all $t \geq t'$. Moreover, if such t' exists, then $\gamma(\xi_{t,i})$ is constant for $t > t'$.

Proposition (3.5) states, that skewness coefficient $\gamma(\xi_{k,i})$ can converge with t or oscillate around 0 with a constant amplitude, and, if it converges, than in equals its limit starting from some t. This is analogical to the univariate case when skewness coefficient was constant for $\rho > 0$ and oscillated for $\rho < 0$. Convergence of all skewness coefficients $\gamma(\xi_{t,i})$, $i = 1, 2, ..., p$, in response to all shocks $u_{1,j}$, $j = 1, 2, ..., p_u$, takes place iff there exists $K \geq 0$, such that entries of A^k have constant signs for all $k \geq K$. Note, that this is true when matrix A has only positive elements, which is rarely true in case of DSGE models.

3.2.2 Observed variables

As stated in proposition (3.4), state variables ξ_t can fall out of the *csn* distribution family starting from some $t > 1$ if the autoregressive matrix A in model (3.14) is rank deficient. In case of DSGE models, especially larger ones, this is usually the case. In what follows, we notice that even if A is rank deficient, observed variables still follow a *csn* distribution for all t. The reason for this is that H has a full row rank. To see this notice that:

[16] We drop time indices for u_t since variables u_t are assumed to be iid. Also, for ξ_t being a p-dimensional variable, $\kappa_n(\xi_t)$ denotes a vector cumulant with entries $\kappa_n(\xi_{t,i})$ for $i = 1, 2, ..., p$.

$$y_t = F\xi_t + He_t = FA^{t-1}\xi_1 + F\sum_{k=0}^{t-2} A^k Bu_{t-k} + He_t = F\sum_{k=0}^{t-1} A^k Bu_{t-k} + He_t = A_t\vartheta_t \tag{3.19}$$

where $A_t = \left[FA^{t-1}B|FA^{t-2}B|...|FAB|FB|H\right]$ and $\vartheta_t = \left[u_1, u_2, ..., u_t, e_t\right]^T$ and we assumed for simplicity that $\xi_1 = Bu_1$. Matrix A_t has a full row rank since H does and ϑ_t has a nonsingular *csn* distribution, hence y_t follows a nonsingular *csn* distribution, which may come as a surprise since ξ_t not only can have a singular *csn* distribution, but can have some other, i.e. not *csn*, distribution.

4 DSGE model with structural skewness

We employ the small open economy model of Lubik and Schorfheide (2007) (LS) which is a simplified version of Gali and Monacelli (2005) and extend it by allowing (some of the) structural shocks to follow a closed skewed normal distribution. LS model can be seen as a minimum set of equations for an open economy framework and its small size is an advantage because it reduces computational burden of simulations which we conduct. Below we present model's equation in already log–linearised form (denoted by hats over variables), more details can be found in Lubik and Schorfheide (2007) or Negro and Schorfheide (2008). The model is also implemented in YADA package (Warne, 2010).

4.1 The model

There are nine state variables in the model: a growth rate of a non–stationary world technology $\left(z_t \equiv \frac{A_t}{A_{t-1}}\right)$ where $\left(A_t\right)$ denotes the non–stationary technology, foreign output $\left(\widehat{y}_t^\star\right)$ and inflation $\left(\widehat{\pi}_t^\star\right)$, terms of trade growth rate $\left(\Delta\widehat{q}_t\right)$, domestic output $\left(\widehat{y}_t\right)$ and inflation $\left(\widehat{\pi}_t\right)$, exchange rate growth rate $\left(\Delta\widehat{e}_t\right)$, the nominal interest rate $\left(\widehat{R}_t\right)$ and the domestic potential output $\widehat{\overline{y}}_t$. The non–stationary technology process is assumed to be present in all real variables, therefore, to ensure stationarity, all real variables are expressed as deviations from A_t.

First four state variables are approximated by autoregressions with normally distributed shocks:

$$\begin{aligned} \widehat{z}_t &= \rho_z \widehat{z}_{t-1} + \widehat{\epsilon}_t^z & \widehat{\pi}_t^\star &= \rho_{\pi^\star}\widehat{\pi}_{t-1}^\star + \widehat{\epsilon}_t^{\pi^\star} \\ \widehat{y}_t^\star &= \rho_{y^\star}\widehat{y}_{t-1}^\star + \widehat{\epsilon}_t^{y^\star} & \Delta\widehat{q}_t &= \rho_q \Delta\widehat{q}_{t-1} + \widehat{\epsilon}_t^q \end{aligned} \tag{4.1}$$

Random processes $\widehat{\epsilon}_t^z$, $\widehat{\epsilon}_t^{\pi^\star}$, $\widehat{\epsilon}_t^{y^\star}$, and $\widehat{\epsilon}_t^q$ as well as the monetary policy shock $\widehat{\epsilon}_t^R$ represent structural shocks or innovations. In the original formulation of Lubik and Schorfheide (2007) they are all normally, hence symmetrically distributed. In our approach each structural shock follows a closed skewed normal $csn_{1,1}$ distribution:

$$\begin{aligned} \widehat{\epsilon}_t^z &\sim csn_{1,1}\left(\tilde{\mu}_{\hat{z}}, \tilde{\sigma}_{\hat{z}}, d_{\hat{z}}, \vartheta_{\hat{z}}, \delta_{\hat{z}}\right) & \widehat{\epsilon}_t^\pi &\sim csn_{1,1}\left(\tilde{\mu}_{\hat{\pi}^\star}, \tilde{\sigma}_{\hat{\pi}^\star}, d_{\hat{\pi}^\star}, \vartheta_{\hat{\pi}^\star}, \delta_{\hat{\pi}^\star}\right) \\ \widehat{\epsilon}_t^y &\sim csn_{1,1}\left(\tilde{\mu}_{\hat{y}^\star}, \tilde{\sigma}_{\hat{y}^\star}, d_{\hat{y}^\star}, \vartheta_{\hat{y}^\star}, \delta_{\hat{z}}\right) & \widehat{\epsilon}_t^q &\sim csn_{1,1}\left(\tilde{\mu}_{\hat{\Delta q}}, \tilde{\sigma}_{\hat{\Delta q}}, d_{\hat{\Delta q}}, \vartheta_{\hat{\Delta q}}, \delta_{\hat{\Delta q}}\right) \end{aligned} \tag{4.2}$$

which means that they can be normaly distributed, but do not have to be. We demand that parametrization of shocks makes them martingale difference sequences.

Euler equation combined with perfect risk sharing and the market–clearing condition for the foreign good gives rise to an open economy dynamic IS curve:

$$\begin{aligned}\widehat{y}_t =& \mathop{\mathbb{E}}_t \widehat{y}_{t+1} - [\tau + \alpha(2-\alpha)(1-\tau)] \left(\widehat{R}_t - \mathop{\mathbb{E}}_t \widehat{\pi}_{t+1}\right) - \rho_z \widehat{z}_t \\ & - \alpha[\tau + \alpha(2-\alpha)(1-\tau)] \mathop{\mathbb{E}}_t \Delta\widehat{q}_{t+1} + \alpha(2-\alpha)\frac{1-\tau}{\tau} \mathop{\mathbb{E}}_t \Delta\widehat{y}^{\star}_{t+1},\end{aligned} \tag{4.3}$$

Parameters τ and α denote the intertemporal substitution elasticity and the import share (hence $0 < \alpha < 1$ and for $\alpha = 0$ equation reduces to closed economy variant).

Optimal price setting by domestic firms leads to the neokeynesian Phillips curve:

$$\widehat{\pi}_t = \beta \mathop{\mathbb{E}}_t \widehat{\pi}_{t+1} + \alpha\beta \mathop{\mathbb{E}}_t \Delta\widehat{q}_{t+1} - \alpha\Delta\widehat{q}_t + \frac{\kappa}{\tau + \alpha(2-\alpha)(1-\tau)}\left(\widehat{y}_t - \widehat{\overline{y}}_t\right), \tag{4.4}$$

where $\widehat{\overline{y}}_t = -\alpha(2-\alpha)(1-\tau)/\tau \widehat{y}^{\star}_t$ is the potential output in the absence of nominal rigidities. The parameter β is the discount factor. The parameter κ is a function of underlying structural parameters (elasticities of labour supply and demand, price stickiness), and it is treated itself as structural.

Definition of consumer prices under the assumption of relative PPP allows to determine change in nominal exchange rate as:

$$\Delta\widehat{e}_t = \widehat{\pi}_t - \widehat{\pi}^{\star}_t - (1-\alpha)\Delta\widehat{q}_t. \tag{4.5}$$

The nominal interest rate is assumed to follow a policy rule:

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1-\rho_R)\left[\Psi_\pi \widehat{\pi}_t + \Psi_y \widehat{y}_t + \Psi_e \Delta\widehat{e}_t\right] + \widehat{\epsilon}^R_t, \tag{4.6}$$

where ρ_R is a smoothing parameter.

Following Lubik and Schorfheide, we use five observable variables to link the model with the data: real GDP growth, annualised inflation rate, annual nominal interest rate, change in exchange rate, and change in terms of trade. The measurement equations take the form:

$$\begin{aligned}\underline{\pi_t} &= 4\left(\frac{\pi_A}{400} + 1\right)\widehat{\pi}_t + \pi_A \\ \underline{y_t} &= \Delta\widehat{y}_t + \widehat{z}_t + \gamma_Q \\ \underline{\Delta e_t} &= \Delta\widehat{e}_t \\ \underline{\Delta q_t} &= \Delta\widehat{q}_t \\ \underline{R_t} &= 4\left(\frac{\pi_A + r_A + 4\gamma_Q}{400} + 1\right)\widehat{R}_t + \pi_A + r_A + 4\gamma_Q\end{aligned} \tag{4.7}$$

where underlined variables denote observable variables. Parameter π_A is annual rate of inflation, γ_Q is quarterly growth rate of non–stationary technology process (z_t in steady state), and r_A is an element of real interest rate $r = r_A + 4\gamma_Q$.

4.2 Simulation exercises on skewness propagation

In order to numerically assess intuition about how structural skewness propagates through the LS model, we simulated 10.000 samples of observables, each consisting of 600 observations. Two cases were considered. In the first case shocks were assumed to be normal, whereas in the second one structural skewness was introduced by assuming that exactly one shock follows a closed skew normal distribution. Parameters of *csn* distributions were chosen in such a way, that the skewness coefficient of each shock was equal to 0.50, so that structural skewness is always positive, which means shocks draws from above the mean value are more probable than those from below the mean. Behavioral parameters and standard deviations of shocks as well as autocorrelation coefficients of states were motivated by LS's central values of priors (see Lubik and Schorfheide, 2007, p. 1077). Standard deviations of measurement errors are approximately 10% of observed variables' standard deviations. Table 2 shows the basic set of parameters of LS model.

In each of the considered cases skewness of states and observables was calculated. Results are reported in Table 3, which columns contain skewness coefficients under normality (column 2), when exactly one shock is skewed (columns 3-7), and when all shocks are skewed (column 8), both in the block of state variables and observable ones. Let us first notice that the skewness of autoregressive variables[17] $\widehat{z}_t$, $\Delta\widehat{q}_t$, $\widehat{y}^\star_t$, $\widehat{\pi}^\star_t$, and $\widehat{\epsilon}^R_t$ depends on their autoregressive coefficients (which are reported in Table 2) – the higher the autoregressive coefficient, the smaller the skewness. Under reported parametrization skewness of states and observable variables when all shocks are assumed to be skewed is roughly equal to sum of skewnesses implied by each of the shocks, but in general this does not have to be the case. Furthermore, inflation does not have its own shock in the model, but factors which induce positive skewness of CPI inflation — foreign demand and foreign price dynamics, generate positive skewness of the nominal interest rate, which reveals the pattern of propagation through the monetary policy rule. And the other way round — positively skewed monetary policy shock is reflected by positive skewness of the interest rate and a negative contribution to skewness of inflation. Finally, skewness of output is driven mainly by skewness of growth rates of technology. Positive skewness of foreign inflation is also the main cause of skewness of changes in exchange rate.

Table 2. The basic set parameters of Lubik-Schorfheide DSGE model.

Behavioral			Disturbances			Measurement errors		
Parameter	Value	Remarks	Parameter	Value	Remarks	Parameter	Value	Remarks
ψ_π	1.500		$\rho_{\hat{z}}$	0.200		$\sigma_{\underline{y}}$	0.01	kept fixed
ψ_y	0.250		$\rho_{\Delta\hat{q}}$	0.400		$\sigma_{\underline{\pi}}$	0.09	kept fixed
$\psi_{\Delta e}$	0.100		$\rho_{\hat{y}^\star}$	0.900		$\sigma_{\underline{R}}$	0.09	kept fixed
ρ_R	0.600		$\rho_{\hat{\pi}^\star}$	0.800		$\sigma_{\Delta\underline{e}}$	0.16	kept fixed
α	0.150		$\rho_{\hat{\epsilon}^R}$	0.000	kept fixed	$\sigma_{\Delta\underline{q}}$	0.04	kept fixed
κ	0.500		$\sigma_{\hat{z}}$	1.000				
τ	0.500		$\sigma_{\Delta\hat{q}}$	1.900				
r_A	0.750		$\sigma_{\hat{y}^\star}$	1.890				
π_A	2.000	kept fixed	$\sigma_{\hat{\pi}^\star}$	3.000				
γ_Q	0.800		$\sigma_{\hat{\epsilon}^R}$	0.400				

σ_u — standard deviation of u; ρ_u —- autocorrelation coefficient of u
Source: Prepared by the authors

The results raise the question whether it is possible to replicate with the model, the pattern of skewness exhibited by the data presented in section 2 — positive skewness for inflation, nominal interest rate and depreciation rate, and negative skewness of output growth rates. Although such combination exists it would require highly positively skewed foreign output shock which is not reasonable.

5 Estimation of models' parameters

In order to work with (first order perturbations of) DSGE models with structural skewness we have to develop a parameter estimation technique for a (linear) state space model with skewed shocks. A state space model which represents a reduced form of a DSGE model under normal shocks is usually estimated by the Kalman

[17]The monetary policy shock $\widehat{\epsilon}^R_t$ can also be perceived as an autoregressive process with autoregression coefficient equal to zero.

Table 3. Skewness in simulated data

Variable	Normal distribution	Skew normal distribution					
		$\widehat{\epsilon}_t^{\widehat{z}}$	$\widehat{\epsilon}_t^{\Delta\widehat{q}}$	$\widehat{\epsilon}_t^{\widehat{y}^\star}$	$\widehat{\epsilon}_t^{\widehat{\pi}^\star}$	$\widehat{\epsilon}_t^{R}$	all
State variables							
$\widehat{y}$	0.00	0.00	0.00	−0.12	0.00	−0.01	−0.12
$\widehat{\pi}$	0.00	0.00	0.00	0.03	0.09	−0.03	0.08
$\widehat{r}$	0.00	0.00	−0.01	0.03	0.03	0.05	0.09
$\Delta\widehat{e}$	0.00	0.00	−0.02	0.00	−0.16	0.00	−0.19
$\widehat{z}_t$	0.00	0.47	0.00	0.00	0.00	0.00	0.47
$\Delta\widehat{q}_t$	0.00	0.00	0.41	0.00	0.00	0.00	0.41
$\widehat{y}_t^\star$	0.00	0.00	0.00	0.14	0.00	0.00	0.14
$\widehat{\pi}_t^\star$	0.00	0.00	0.00	0.00	0.21	0.00	0.22
$\widehat{\epsilon}_t^R$	0.00	0.00	0.00	0.00	0.00	0.50	0.50
Observable variables							
GDP	0.00	0.25	0.00	−0.03	0.00	0.00	0.21
Inflation	0.00	0.00	0.00	0.03	0.08	−0.03	0.08
Interest rate	0.00	0.00	−0.01	0.03	0.03	0.04	0.09
Exchange rate	0.00	0.00	−0.02	0.00	−0.16	0.00	−0.19
Terms of trade	0.00	0.00	0.40	0.00	0.00	0.00	0.40

filter (KF) maximum likelihood (ML) estimator (see e.g. Hamilton, 1994; Meinhold and Singpurwalla, 1983). Calculation of likelihood function value via KF typically constitutes also a step of Bayesian estimation, see Fernández-Villaverde (2009). Popularity of KF estimation is motivated by the fact that for normally distributed shocks and measurement errors KF produces analytical filtration, i.e. it yields exact likelihood value — not an approximation, it is fast and easy to implement. Robustness of KF for non-Gaussian shocks in the transition equation is sometimes negated, e.g. (Meinhold and Singpurwalla, 1989). Nonetheless, we keep in mind that KF is an optimal[18] linear filter for arbitrary, hence also for closed skewed normal shocks[19].

Ideally, we would like to extend the KF formulation to the case of *csn* shocks, which would allow us to perform analytical filtration and obtain exact likelihood function in each step of the ML routine for inference of parameters. It is possible, but under assumptions which are not met in case of most DSGE models. The main problem is a reduced rank of the autoregressive matrix in the transition equation and the fact that calculation of likelihood function value, which has to be a fast task for KF-type filters, requires calculation of a cumulative distribution function value of a highly dimensional normal distribution with arbitrary dependency structure. In practice, the latter task can be done only by Monte Carlo techniques. Also in comparison with numerical burden of methods like particle filtering (see for example Fernández-Villaverde and Rubio-Ramirez (2007); An (2005)) or fully Bayesian parameter estimation which simultaneously involves numerical optimization, posterior sampling e.g. via Metropolis-Hastings type of methods and highly dimensional numerical integration via e.g. MCMC-type of methods, a simple three-step KF-based (limited information) approach can be desirable:

1. Kalman filter (quasi) maximum likelihood (Q-ML) estimation[20] of models' parameters,

2. filtration of shocks conditional upon parameter estimates from step 1, testing for skewness of shocks,

3. method of moments estimation of parameters of shocks' *csn* probability distribution functions (let us call them *shocks' parameters*), conditional upon filtration from step 2.

Estimates of all the parameters obtained in step 1,but for shocks' parameters, become final estimates. Final estimates of shocks' parameters are in turn obtained in step 3, in which it is assumed that shocks have a *csn* distribution.

[18]Optimal in the sense that it produces minimal trace of one-step ahead prediction errors covariance matrix.
[19]This means that better filters are only nonlinear ones.
[20]We use the therm *quasi maximum likelihood estimator* in a very broad sense, as a case when maximum likelihood principle is applied to a misspecified (statistical) model, see White (1982). We do not relay on properties of Q-ML estimators given by (e.g.) Wedderburn (1974), Gourieroux et al. (1984), Nadler and Lee (1992).

Above procedure omits direct optimization-based estimation of shocks' parameters. It may be an advantage, because, the log-likelihood function for the *csn* distribution exhibits anomalies, e.g. improper shape, inflection points in profile likelihood (singularity of Fisher information matrix) at points where skewness vanishes, divergence of parameters of the distribution, see Azzalini and Capitanio (1999), Azzalini (2004)), see also Azzalini and Genton (2008). Only some of these anomalies may be removed via proper parametrization.[21].

5.1 Quasi-maximum likelihood estimation

A first order perturbation of a DSGE model with structural skewness obtains the following state space form:

$$
\begin{aligned}
\xi_t &= A\xi_{t-1} + Bu_t \\
y_t &= F\xi_t + e_t \\
u_t &\sim \mathrm{CSN}\left(\tilde{\mu}_u, \tilde{\Sigma}_u, \tilde{D}_u, \nu_u, \Delta_u\right) \\
\mathrm{E}(u_t) &= 0 \\
e_t &\sim \mathrm{N}\left(0, \Sigma_e\right) \\
\xi_0 &\sim \mathrm{N}\left(\mu_{\xi_0}, \Sigma_{\xi_0}\right)
\end{aligned}
\tag{5.2}
$$

for $t = 1, 2, ..., T$, where ξ_t denote states, y_t observables, e_t and u_t denote martingale difference measurement errors and structural shocks respectively, and Σ_{ξ_0} can be zero, i.e. ξ_0 can be non-stochastic. Matrices A, B and F are functions of models' deep parameters $\theta_{\mathcal{M}}$ and A can be singular (and generally is), but not explosive. Let $\mu_u = \mathrm{E}(u_t)$ and $\Sigma_u = \mathrm{D}(u_t)$. If $D_u = 0$, i.e. shocks' skewness parameter vanishes, then $\tilde{\mu}_u = \mu_u$, $\tilde{\Sigma}_u = \Sigma_u$ and shocks u_t are normally distributed. Structural shocks u_t are, by definition, independent, therefore matrices $\tilde{\Sigma}_u$, D_u and Σ_u are diagonal, see corollary (3.6), with: $\tilde{\Sigma}_u = diag(\tilde{\sigma}_{u_i}, i = 1, 2, ..., p_u)$, $D_u = diag(d_{u_i}, i = 1, 2, ..., p_u)$ and $\Sigma_u = diag(\sigma_{u_i}, i = 1, 2, ..., p_u)$. This, by remark (3.3), means, that each $u_{t,i}$, i.e. each component of u_t, has an Azzalini-type skewed normal distribution, see (Azzalini and Valle, 1996). Azzalini distribution is nested by the *csn* distribution: $u_{t,i} \sim csn_{1,1}(\tilde{\mu}_{u_i}, \tilde{\sigma}_{u_i}, d_{u_i}, 0, 1)$. Applying corollary (3.4), we see that first three central

[21] Azzalini et al. (2010) considered a more general case of estimation of a skew-symmetric distribution's parameters. A simple version of the probability density function of a scalar skew-symmetric random variable may be written, up to a scale parameter, in the form:

$$
\mathrm{f}_{SS}(z) = \mathrm{f}_0(z; \theta_a)\,\pi(z; \theta_a, \theta_b), \qquad z \in \mathcal{R} \tag{5.1}
$$

where: f_0 is a symmetric density, and π is a skewing function, such that $\pi(-z) = 1 - \pi(z) \geq 0$ for all $z \in \mathcal{R}$. The location and scale parameters are introduced via definition $Y = \nu + \omega Z$. The (closed) skew-normal distribution is a special case of skew-symmetric family, the multivariate extension is straightforward. „The class of distributions (5.1) can be obtained via a suitable censoring mechanism, regulated by $\pi(z)$, applied to samples generated by the base density f_0, [...]. Under this perspective, it is of interest to estimate the parameters of f_0 via a method which does not depend, or depends only to a limited extent, on the component $\pi(z)$, which in many cases is not known, or is not of interest to be estimated [...]", (see Azzalini et al., 2010, p. 2). A distributional invariance property of skew-symmetric distribution is a key concept in their method of estimating equations. The distributional invariance is defined in the following way: If X, Y are two random variables $X \sim \mathrm{f}_0$, $Y \sim \mathrm{f}_{SS}$, and $T(.)$ is an even function, then $T(X) \stackrel{d}{=} T(Y)$. This property ensures that for any choice of even function T_k the expected value

$$
\mathbb{E}\, T_k\left(\frac{Y - \nu}{\omega}\right) = c_k, \quad (k = 1, 2, ..)
$$

depend only on f_0, provided they exist. The authors use that feature to builds estimation equations. Ma et al. (2005) considered a semiparametric model, where the parameters of interest are mean and variance and the skewness parameter is a nuisance parameter. The authors tested properties of regular asymptotically linear (RAL) estimators of Newey (1990). Fletcher et al. (2008) tested method of moment estimators of *csn* parameters.

moments of $u_{t,i}$ are:

$$\mathrm{E}(u_{t,i}) = \kappa_1(u_{t,i}) = \tilde{\mu}_{u_i} + \sqrt{\frac{2}{\pi}} \frac{d_{u_i}\tilde{\sigma}_{u_i}}{\sqrt{\delta_{u_i} + d_{u_i}^2 \tilde{\sigma}_{u_i}}}, \quad var(u_{t,i}) = \kappa_2(u_{t,i}) = \tilde{\sigma}_{u_i} - \frac{2}{\pi} \frac{d_{u_i}^2 \tilde{\sigma}_{u_i}^2}{\delta_{u_i} + d_{u_i}^2 \tilde{\sigma}_{u_i}}$$
$$\mathrm{E}(u_{t,i} - E(u_{t,i}))^3 = \kappa_3(u_{t,i}) = \left(\sqrt{\frac{2}{\pi}}\right)^3 (2 - \frac{\pi}{4})(\frac{d_{u_i}\tilde{\sigma}_{u_i}}{\sqrt{\delta_{u_i} + d_{u_i}^2 \tilde{\sigma}_{u_i}}})^3 \tag{5.3}$$

which means, that skewness coefficients of $u_{t,i}$ are equal to:

$$\gamma(u_{t,i}) = \gamma(u_i) = \left(\frac{4-\pi}{2}\right) \frac{\left(\sqrt{\frac{2}{\pi}} \frac{d_{u_i}\tilde{\sigma}_{u_i}}{\sqrt{\delta_{u_i} + d_{u_i}^2 \tilde{\sigma}_{u_i}}}\right)^3}{\left(\tilde{\sigma}_{u_i} - \frac{2}{\pi} \frac{d_{u_i}^2 \tilde{\sigma}_{u_i}^2}{\delta_{u_i} + d_{u_i}^2 \tilde{\sigma}_{u_i}}\right)^{\frac{3}{2}}}. \tag{5.4}$$

and satisfy $|\gamma(u_{t,i})| < 0.9953$.

Note that shocks u_t, as structural ones, are required to be martingale differences for every $\tilde{\Sigma}_u$ and D_u, which is obtained by forcing $\tilde{\mu}_u$ to adjust so that $\mathrm{E}(u_{t,i}) = 0$, see first eq. in (5.3).

If $\theta_{\mathscr{M}}$ is fixed, skewness of shocks has therefore no impact on the steady state of the model. Variances of shocks are functions of $\tilde{\Sigma}_u$ and D_u, however, given $\theta_{\mathscr{M}}$, these parameters imply structure of shocks' variance and not its magnitude, see second eq. in (5.3). If shocks' variances are given (estimated), also dynamics of the model (e.g. persistence of shocks) does not depend on these parameters. This is a heuristic motivation of our estimation procedure which first step neglects skewness of the distribution, i.e. the skewing function, and approximates likelihood using the normal distribution.

Let $\theta_u = \left(\tilde{\Sigma}_u, D_u\right)$ and $\theta_e = (\Sigma_e)$. We know that observables y_t are distributed according to a $csn\left(\tilde{\mu}_{y,t}, \tilde{\Sigma}_{y,t}, D_{y,t}, \vartheta_{y,t}, \Delta_{y,t}\right)$ distribution and, given observables, likelihood function of the models' parameters $\theta = \left(\theta_{\mathscr{M}}, \theta_u, \theta_e\right)$, let it be denoted by $\mathscr{L}(\theta)$. We are interested in finding θ which maximizes $\mathscr{L}(\theta)$. Maximizer of θ, denoted by $\hat{\theta}$, will be approximated in three steps. Let[22] $\bar{\theta}_u = (\tilde{\Sigma}_u, 0) = (\Sigma_u, 0)$, $\bar{\theta} = (\theta_{\mathscr{M}}, \bar{\theta}_u, \theta_e)$ and $\bar{\mathscr{L}}(\bar{\theta}) = \mathscr{L}(\bar{\theta})$. $\bar{\mathscr{L}}(\bar{\theta})$ is the quasi-likelihood function in the sense that it represents the the original likelihood function conditioned upon $D = 0$, which means that it neglects shocks' skewness. In the first step a maximizer:

$$\bar{\theta}^\star = (\theta_{\mathscr{M}}^\star, \bar{\theta}_u^\star, \theta_e^\star) = \underset{\bar{\theta}\in\bar{\Theta}}{\arg\max} \left\{\bar{\mathscr{L}}\left(\bar{\theta}\right)\right\} \tag{5.5}$$

is found. With $D = 0$, this is a standard maximum likelihood estimation of a state space model with normally distributed shocks. Then, shocks u_t are filtered using model (5.2) with parameters $\bar{\theta}^\star$ plugged in it[23], and sample estimates of shocks' skewness coefficients γ_{u_i} for $i = 1, 2, ..., p_u$ are established. If only skewness coefficients are of interest, then the procedure ends yielding $\gamma_u = (\gamma_{u_i}, i = 1, 2, ..., p_u)$. Otherwise, original shocks parameters $\tilde{\Sigma}_u$ and D_u are recovered from γ_u and Σ_u according to equations (5.3–5.4), which results in estimates $\tilde{\Sigma}_u^\star$ and $D_u^\star$ respectively, and final estimate of θ becomes $\hat{\theta} = (\theta_{\mathscr{M}}^\star, \bar{\theta}_u^{\star\star}, \theta_e^\star)$ where $\bar{\theta}_u^{\star\star} = (\tilde{\Sigma}_u^\star, D_u^\star)$.

5.2 Simulation experiments

A single iteration of our stochastic simulation procedure looked as follows:

[22] Notice that if $D_u = 0$ then $\tilde{\Sigma}_u = \Sigma_u$

[23] In fact, filtration of shocks is a byproduct of estimation of $\bar{\theta}$ using the Kalman filter.

1. A sample of shocks and measurement errors is simulated. States and observables are computed according to (5.2).

2. Given observables from step 1, a Newton-type optimization routine is applied to find $\bar{\theta}$, i.e. the maximizer of the quasi likelihood function $\bar{\mathscr{L}}(\bar{\theta})$. If optimization fails to converge, steps 3 - 4 are skipped and estimation results are discarded[24]. In this situation a new iteration is initiated.

3. Given $\bar{\theta}$, i.e. parameters obtained in step 2, states, observables and shocks are filtered using the Kalman smoother. Shocks skewness is tested for.

4. Smoothed states are used to estimate shocks' parameters $\bar{\theta}_u^{\star\star}$ (in what follows are report results for skewness coefficients γ_u).

All parameters, except for shocks' parameters, i.e. $\theta_{\mathscr{M}}$ and θ_e, are common for all simulation trials, see Table 2. Table 4 reports shocks' skewness parameters θ_u which were used to generate three variants in this experimental setup. This variants are: normal shocks variant, which is our benchmark, moderate skewness of all shocks (CSN-1), strong skewness of all shocks (CSN-2), and the case where only one shock ($y^\star$) is moderately skewed (CSN-3)[25].

Table 4. Simulations specific skewness parameters of shocks

Simulation variant	$\tilde{d}_z$	d_z	$\tilde{d}_{\Delta q}$	$d_{\Delta q}$	$\tilde{d}_{y^\star}$	$d_{y^\star}$	$\tilde{d}_{\pi^\star}$	$d_{\pi^\star}$	$\tilde{d}_{\epsilon_R}$	d_{ϵ_R}
Normal	0.000	0.00	0.000	0.00	0.000	0.00	0.000	0.00	0.000	0.00
CSN-1 / 5x50 /	2.174	0.50	1.144	0.50	1.150	0.50	0.725	0.50	5.434	0.50
CSN-2 / 5x95 /	9.343	0.95	4.918	0.95	4.944	0.95	3.114	0.95	23.359	0.95
CSN-3 / 1x50 /	0.000	0.00	0.000	0.00	1.150	0.50	0.000	0.00	0.000	0.00

d_{u_i} — skewness coefficient of u_i; $\tilde{d}_{u_i}$ — CSN distribution hyper-parameter of shock u_i.
Source: Prepared by the authors

Random number generator for the closed skewed normal distribution follows Gupta et al. (2004, Prop. 2.5, p. 184). A straightforward adjustment was applied to guarante that shocks expected value is zero, $\mathbb{E}u = 0$ and that shocks covariance is Σ_u. The length of samples varies from 75 („small sample") up to 600 („large sample"). For each case of given length over 2000 replications were generated. Some of the parameters (e.g. standard deviations of measurement errors) were calibrated (i.e. kept fixed) over all simulations.

5.2.1 Quasi-maximum likelihood estimation of models parameters

Selected results of stochastic simulations are presented in Table 5, more details can be found in the Appendix A (Table 9–12). Table 5 reports relative percentage biases[26] and standard deviations of models parameter estimates $\theta^\star_{\mathscr{M}}$ obtained in the second step of the simulation procedure. The general point is that results obtained for the normal case (first row, the ML estimator) and for variants CSN 1-3 (rows 2-4, the Q-ML estimator) do not differ substantially, although shocks skewness is neglected during the estimation in *csn* variants. Bias of the Q-ML procedure in short sample is substantial, but this is also the case for the ML estimator. There is likely an identification problem for interest rate rule parameters (ψ_π, ψ_y $\psi_{\Delta e}$) as well as for r_A and $\sigma_{y^\star}$[27]. The magnitude of Q-ML estimators' bias and ML estimators' bias is similar. However, ML estimators are often slightly more precise (taking into account their standard deviations). The biases as well as the standard deviations

[24] The number of rejected trials varied with sample size. It was up to 40% of samples for small sample, and just a few for large samples.
[25] This shock was chosen because it posed difficulties in the estimation.
[26] The relative bias is defined as: $100\frac{\widehat{\theta}-\theta}{\theta}$.
[27] It might be seen as a support for Cochrane (2007) thesis, who noticed that parameters of the Taylor rule in a simple new-Keynesian model of economy are unidentified (the model specification issue), but if J.H. Cochrane is right, sample size should not matter. The results of our exercise indicate however, that we likely faced a data related issue.

of estimators are (approximatively) declining functions of sample size. This means that our *ad hoc* Q-ML estimators have properties of consistent estimators, at least in the problem at hand.

Table 5. Summary of Stochastic Simulation Exercises. Properties of (Q)-LM estimators of the basic set of parameters. Number of replications = 2000.

Simul. Variant	Estimator Feature	Estimator of parameter																	
		$\widehat{\psi}_{\pi}$	$\widehat{\psi}_{y}$	$\widehat{\psi}_{\Delta e}$	$\widehat{\rho}_{R}$	$\widehat{\alpha}$	$\widehat{\kappa}$	$\widehat{\tau}$	$\widehat{r}_{A}$	$\widehat{\gamma}_{Q}$	$\widehat{\rho}_{\tilde{z}}$	$\widehat{\rho}_{\tilde{d}q}$	$\widehat{\rho}_{y^{\star}}$	$\widehat{\rho}_{\pi^{\star}}$	$\widehat{\sigma}_{\tilde{z}}$	$\widehat{\sigma}_{\tilde{d}q}$	$\widehat{\sigma}_{y^{\star}}$	$\widehat{\sigma}_{\pi^{\star}}$	$\widehat{\sigma}_{\epsilon_R}$
Sample size = 75																			
Normal	Relative Bias %	32.09	87.50	56.44	2.462	1.407	6.891	-1.912	23.49	-4.276	3.530	-2.593	-2.188	-0.786	-4.970	-0.657	61.11	-1.017	3.816
Normal	Std. Deviation	1.215	0.532	0.136	0.120	0.029	0.174	0.202	0.562	0.127	0.047	0.102	0.059	0.053	0.113	0.161	3.376	0.253	0.096
CSN-1	Relative Bias %	36.14	93.96	62.88	2.797	1.708	7.383	-2.613	24.57	-4.351	3.944	-1.842	-1.965	-0.834	-5.669	-1.098	60.16	-0.418	4.659
CSN-1	Std. Deviation	1.290	0.551	0.148	0.124	0.029	0.172	0.200	0.563	0.123	0.048	0.103	0.059	0.052	0.119	0.172	3.446	0.269	0.121
CSN-2	Relative Bias %	32.33	86.84	57.45	2.567	1.577	6.856	-2.928	22.75	-4.351	3.825	-2.707	-2.031	-0.974	-5.916	-1.138	53.64	-0.949	3.536
CSR-2	Std. Deviation	1.223	0.509	0.137	0.121	0.028	0.169	0.195	0.565	0.122	0.047	0.101	0.060	0.053	0.123	0.185	3.265	0.289	0.075
CSN-3	Relative Bias %	30.38	82.57	54.77	2.556	2.028	6.054	-3.021	24.00	-4.443	3.754	-1.589	-1.935	-0.765	-4.969	-0.669	60.61	-0.499	2.871
CSN-3	Std. Deviation	1.193	0.529	0.136	0.118	0.028	0.165	0.201	0.566	0.124	0.047	0.105	0.061	0.052	0.114	0.154	3.520	0.259	0.073
Sample size = 150																			
Normal	Relative Bias %	15.72	39.72	27.51	1.463	0.769	2.522	0.443	9.338	-1.723	3.750	-0.637	-0.976	-0.580	-2.546	-0.470	40.06	-0.217	1.561
Normal	Std. Deviation	0.746	0.290	0.080	0.093	0.021	0.112	0.155	0.438	0.094	0.035	0.071	0.039	0.036	0.080	0.111	2.423	0.180	0.057
CSN-1	Relative Bias %	15.51	39.80	27.21	0.969	0.539	2.885	0.767	8.598	-1.769	3.304	-1.394	-0.865	-0.554	-3.187	-0.177	45.29	-0.466	1.224
CSN-1	Std. Deviation	0.831	0.344	0.091	0.092	0.022	0.113	0.159	0.427	0.091	0.034	0.072	0.038	0.037	0.082	0.120	2.598	0.193	0.050
CSN-2	Relative Bias %	13.72	37.51	24.14	0.346	0.007	3.752	0.907	9.116	-1.893	2.570	-1.028	-1.190	-0.549	-3.247	-0.555	44.73	-0.503	1.264
CSN-2	Std. Deviation	0.745	0.298	0.081	0.094	0.022	0.116	0.159	0.412	0.090	0.034	0.071	0.038	0.037	0.083	0.129	2.555	0.214	0.054
CSN-3	Relative Bias %	16.93	44.59	29.87	1.050	0.946	3.677	0.328	9.324	-1.688	3.081	-1.308	-1.033	-0.730	-2.601	-0.344	45.97	-0.136	2.359
CSN-3	Std. Deviation	0.855	0.362	0.094	0.095	0.022	0.120	0.162	0.427	0.091	0.035	0.073	0.039	0.037	0.080	0.114	2.742	0.175	0.093
Sample size = 600																			
Normal	Relative Bias %	2.990	7.635	5.516	0.145	-0.043	0.546	0.276	1.511	-0.190	1.305	-0.073	-0.265	-0.237	-0.675	-0.097	11.96	-0.136	0.227
Normal	Std. Deviation	0.264	0.103	0.029	0.049	0.011	0.055	0.088	0.239	0.051	0.020	0.035	0.017	0.018	0.041	0.055	0.999	0.088	0.024
CSN-1	Relative Bias %	2.859	6.533	4.820	0.056	-0.019	0.635	1.269	-0.026	0.054	1.765	-0.073	-0.196	-0.084	-0.754	-0.061	13.89	-0.157	0.003
CSN-1	Std. Deviation	0.291	0.113	0.032	0.050	0.012	0.055	0.085	0.239	0.051	0.019	0.037	0.017	0.018	0.042	0.059	0.988	0.096	0.026
CSN-2	Relative Bias %	2.680	6.525	4.537	-0.141	0.047	1.080	0.684	0.515	-0.171	1.303	-0.549	-0.254	-0.180	-0.808	-0.189	11.96	-0.127	0.332
CSN-2	Std. Deviation	0.313	0.114	0.032	0.048	0.011	0.058	0.084	0.240	0.051	0.019	0.035	0.018	0.019	0.044	0.065	0.978	0.103	0.054
CSN-3	Relative Bias %	2.705	6.400	5.109	0.075	0.091	0.601	0.170	0.766	-0.243	1.420	-0.149	-0.262	-0.221	-0.943	-0.146	10.86	-0.111	-0.019
CSN-3	Std. Deviation	0.284	0.108	0.031	0.049	0.011	0.057	0.083	0.238	0.051	0.019	0.035	0.018	0.019	0.040	0.055	0.948	0.089	0.026

Source: Prepared by the authors

Table 6. Size and Power /Rejection Ratio/ of Skewness Tests. Number of replications = 2000.

Shock Type	Var. Type	Simple 1-tailed					Simple 2-tailed					Bai-Ng 1-tailed					Bai-Ng 2-tailed					Bai-Ng χ^2				
		$\hat{z}$	$\Delta\hat{q}$	$\hat{y}^\star$	$\hat{\pi}^\star$	$\hat{\epsilon}^R$	$\hat{z}$	$\Delta\hat{q}$	$\hat{y}^\star$	$\hat{\pi}^\star$	$\hat{\epsilon}^R$	$\hat{z}$	$\Delta\hat{q}$	$\hat{y}^\star$	$\hat{\pi}^\star$	$\hat{\epsilon}^R$	$\hat{z}$	$\Delta\hat{q}$	$\hat{y}^\star$	$\hat{\pi}^\star$	$\hat{\epsilon}^R$	$\hat{z}$	$\Delta\hat{q}$	$\hat{y}^\star$	$\hat{\pi}^\star$	$\hat{\epsilon}^R$
Sample size = 75																										
Normal	sim	0.04	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.04	0.04	0.05	0.05	0.04	0.05	0.04	0.04	0.04	0.04	0.04	0.02	0.03	0.02	0.02	0.02
	est	0.04	0.04	0.05	0.04	0.04	0.04	0.04	0.05	0.04	0.04	0.05	0.05	0.05	0.04	0.05	0.04	0.04	0.04	0.03	0.04	0.02	0.02	0.02	0.02	0.02
CSN-1	est	0.51	0.51	0.48	0.50	0.51	0.38	0.39	0.37	0.37	0.39	0.46	0.46	0.46	0.46	0.46	0.27	0.28	0.26	0.28	0.26	0.17	0.17	0.17	0.17	0.16
	est	0.30	0.50	0.20	0.47	0.43	0.20	0.38	0.13	0.33	0.32	0.25	0.44	0.17	0.41	0.39	0.12	0.25	0.08	0.24	0.21	0.06	0.15	0.04	0.13	0.13
CSN-2	sim	0.96	0.96	0.96	0.96	0.96	0.91	0.91	0.91	0.91	0.91	0.94	0.93	0.93	0.93	0.93	0.80	0.79	0.79	0.80	0.79	0.83	0.81	0.80	0.81	0.78
	est	0.69	0.95	0.43	0.95	0.88	0.57	0.89	0.32	0.88	0.80	0.63	0.92	0.37	0.90	0.85	0.43	0.77	0.20	0.76	0.76	0.31	0.77	0.12	0.74	0.59
CSN-3	sim	0.04	0.04	0.49	0.04	0.04	0.04	0.06	0.37	0.04	0.04	0.05	0.05	0.46	0.04	0.05	0.03	0.05	0.26	0.03	0.04	0.02	0.03	0.17	0.03	0.03
	est	0.04	0.05	0.21	0.04	0.04	0.04	0.05	0.13	0.04	0.04	0.04	0.05	0.18	0.04	0.04	0.04	0.05	0.09	0.03	0.03	0.03	0.03	0.04	0.02	0.03
Sample size = 150																										
Normal	sim	0.04	0.05	0.04	0.04	0.04	0.04	0.05	0.04	0.04	0.05	0.05	0.04	0.05	0.05	0.05	0.03	0.03	0.03	0.04	0.04	0.03	0.02	0.02	0.03	0.03
	est	0.04	0.05	0.05	0.04	0.05	0.05	0.04	0.05	0.04	0.04	0.05	0.05	0.06	0.04	0.05	0.04	0.03	0.05	0.04	0.04	0.03	0.02	0.02	0.02	0.03
CSN-1	sim	0.77	0.77	0.78	0.78	0.77	0.66	0.66	0.66	0.67	0.67	0.76	0.76	0.76	0.75	0.75	0.57	0.59	0.57	0.58	0.58	0.44	0.43	0.43	0.44	0.44
	est	0.50	0.75	0.35	0.73	0.67	0.38	0.63	0.25	0.62	0.56	0.47	0.74	0.32	0.69	0.66	0.29	0.57	0.18	0.54	0.46	0.17	0.41	0.11	0.39	0.33
CSN-2	sim	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	0.99	0.99	0.97	0.98	0.97	0.97	0.97	1.00	1.00	1.00	0.99	1.00
	est	0.94	1.00	0.76	1.00	1.00	0.88	1.00	0.66	1.00	0.99	0.92	1.00	0.74	0.99	0.99	0.81	0.97	0.57	0.96	0.95	0.73	1.00	0.42	0.99	0.97
CSN-3	sim	0.04	0.04	0.79	0.05	0.05	0.05	0.04	0.67	0.05	0.05	0.04	0.04	0.76	0.04	0.05	0.03	0.04	0.58	0.04	0.05	0.03	0.03	0.44	0.03	0.03
	est	0.04	0.04	0.36	0.05	0.05	0.04	0.05	0.25	0.04	0.05	0.05	0.04	0.32	0.05	0.05	0.04	0.04	0.20	0.04	0.04	0.02	0.03	0.10	0.03	0.03
Sample size = 600																										
Normal	sim	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.05	0.06	0.05	0.04	0.05	0.03	0.04	0.04	0.03	0.03
	est	0.05	0.05	0.04	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.05	0.05	0.04	0.04	0.03	0.04	0.03
CSN-1	sim	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00	0.99	0.99
	est	0.97	1.00	0.87	1.00	1.00	0.93	1.00	0.79	1.00	0.99	0.96	1.00	0.85	1.00	1.00	0.93	1.00	0.75	1.00	0.99	0.85	0.99	0.60	0.99	0.98
CSN-2	sim	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	est	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CSN-3	sim	0.05	0.05	1.00	0.05	0.04	0.05	0.04	1.00	0.05	0.05	0.05	0.05	1.00	0.05	0.05	0.05	0.04	1.00	0.05	0.05	0.03	0.03	0.99	0.04	0.04
	est	0.04	0.05	0.87	0.05	0.05	0.05	0.05	0.80	0.05	0.05	0.04	0.05	0.87	0.05	0.05	0.04	0.05	0.77	0.06	0.05	0.03	0.03	0.62	0.04	0.04

The 5% critical values are 1.64 (one-tailed), 1.96 (two-tailed) and 5.99; Bai-Ng tests: Newley-West kernel, no prewithening.
Source: Prepared by the authors

5.2.2 Tests for skewness of smoothed shocks

Step 3 of the simulation procedure involves testing skewness of filtered (smoothed) shocks. We employ significance test of shocks' skewness coefficients (one and two-tailed) as well as three parametric tests developed by Bai and Ng (2005). We verify properties of these tests, since to our best knowledge their sampling distributions have not been established for the smoothed variables[28].

Skewness coefficient of a random variable X is defined as:

$$\gamma(X) = \frac{\mathrm{E}\left[(x-\mu)^3\right]}{\mathrm{E}\left[(x-\mu)^2\right]^{\frac{3}{2}}}. \tag{5.6}$$

and its sample estimate is given by:

$$\widehat{\gamma}(X) = \frac{\frac{1}{T}\sum_{t=1}^{T}\left(X_t - \overline{X}\right)^3}{\left(\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(X_t - \overline{X}\right)^2}\right)^3}. \tag{5.7}$$

where $\overline{X}$ is the sample mean. If X_t are *iid* and normally distributed then $\sqrt{T}\widehat{\gamma}(X) \xrightarrow{d} N(0,6)$.

Below we present tests for skewness proposed by Bai and Ng (2005). If X_t is weakly dependent and stationary up to sixth order, under the null hypothesis that $\gamma(X) = 0$:

$$\sqrt{T}\widehat{\gamma} = \frac{\alpha}{\widehat{\sigma}^3}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\mathbf{Z}_t + o_p(1) \tag{5.8}$$

where

$$\alpha = \begin{bmatrix} 1 & -3\sigma^2 \end{bmatrix}, \qquad \mathbf{Z}_t = \begin{bmatrix} (X_t-\mu)^3 \\ (X_t-\mu) \end{bmatrix}, \tag{5.9}$$

and:

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\mathbf{Z}_t \xrightarrow{d} N\left(\mathbf{0},\mathbf{\Gamma}\right), \qquad \mathbf{\Gamma} = \lim_{T\to\infty} TE\left(\overline{\mathbf{Z}}\overline{\mathbf{Z}}'\right) \tag{5.10}$$

with $\overline{\mathbf{Z}}$ being the sample mean of $\mathbf{Z}_t$ and Γ is the spectral density matrix at frequency 0 of $\mathbf{Z}_t$. Additionally

$$\sqrt{T}\widehat{\gamma} \xrightarrow{d} N\left(0, \frac{\alpha\mathbf{\Gamma}\alpha'}{\sigma^6}\right) \qquad \text{or} \qquad \sqrt{T}\widehat{\mu}_3 \xrightarrow{d} N\left(0,\alpha\mathbf{\Gamma}\alpha'\right) \tag{5.11}$$

Let $\widehat{\sigma}^2$ and $\widehat{\mathbf{\Gamma}}$ be consistent estimates of σ^2 and $\mathbf{\Gamma}$. Let $\widehat{\alpha} = \left[1, -3\widehat{\sigma}^2\right]$, $s\left(\widehat{\mu}_3\right) = \left(\widehat{\alpha}\widehat{\mathbf{\Gamma}}\widehat{\alpha}'\right)^{1/2}$ and $s\left(\widehat{\gamma}\right) = \left(\widehat{\alpha}\widehat{\mathbf{\Gamma}}\widehat{\alpha}'/\widehat{\sigma}^6\right)^{1/2}$. Under the null hypothesis that $\gamma = 0$

$$\widehat{\pi}_3 = \frac{\sqrt{T}\widehat{\mu}_3}{s\left(\widehat{\mu}_3\right)} = \frac{\sqrt{T}\widehat{\gamma}}{s\left(\widehat{\theta}\right)} \xrightarrow{d} N(0,1). \tag{5.12}$$

Long-run variance matrix can be obtained nonparametrically by kernel estimation, e.g. the Bartlett kernel (see Newey and West (1987)).

Possible low power of the test can be increased by applying either a one-tailed test (direction of skewness is

[28] Bai and Ng (2005) skewness tests are valid also for likely serially correlated disturbances of the linear regression model — they proved asymptotic equivalence of test based on disturbances and estimated regression residuals.

usually suspected) or a joint test of two odd moments, r_1 and r_2. Let

$$\mathbf{Y}_t = \left[\begin{array}{c} \frac{1}{\sqrt{T}} \sum_{t=1}^{T} (X_t - \mu)^{r_1} \\ \frac{1}{\sqrt{T}} \sum_{t=1}^{T} (X_t - \mu)^{r_2} \end{array} \right] \tag{5.13}$$

It can be shown that

$$\mathbf{Y}_t = \alpha \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \mathbf{Z}_t + o_p(1) \tag{5.14}$$

where

$$\alpha = \left[\begin{array}{ccc} 1 & 0 & -r_1 \mu_{r_1 - 1} \\ 0 & 1 & -r_2 \mu_{r_2 - 1} \end{array} \right], \qquad \mathbf{Z}_t = \left[\begin{array}{c} (X_t - \mu)^{r_1} \\ (X_t - \mu)^{r_2} \\ (X_t - \mu) \end{array} \right]. \tag{5.15}$$

Under the null hypothesis of symmetry, if

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \mathbf{Z}_t \xrightarrow{d} N(\mathbf{0}, \mathbf{\Gamma}), \qquad \mathbf{\Gamma} = \lim_{T \to \infty} TE\left(\overline{\mathbf{Z}}\overline{\mathbf{Z}}'\right) \tag{5.16}$$

then $\mathbf{Y}_T \xrightarrow{d} N\left(\mathbf{0}, \alpha \mathbf{\Gamma} \alpha'\right)$. Let $\widehat{\alpha}\widehat{\mathbf{\Gamma}}\widehat{\alpha}'$ be a consistent estimate of $\alpha \mathbf{\Gamma} \alpha'$. Then

$$\widehat{\mu}_{r_1, r_2} = \mathbf{Y}_T' \left(\widehat{\alpha}\widehat{\mathbf{\Gamma}}\widehat{\alpha}' \right)^{-1} \mathbf{Y}_T \xrightarrow{d} \chi^2_2 \tag{5.17}$$

We refer to the above joint test as Bai-Ng χ^2 test[29].

Table 6 reports the rejections ratios for skewness tests computed for 5% critical values. The collected data indicate, that the rejection ratio of true hypothesis (symmetric shocks) is approximately 4–5%. The distortion created by Q-LM estimator and two-sided Kalman filter (smoother) is moderate in the case of normal shocks — the size of tests is similar for simulated and estimated (smoothed) shocks. Only, the Bai-Ng joint test (χ^2 test) reject slightly less frequently. This is a feature of this test however. In the case of skew-normal shocks, the rejection ratio is sensitive to sample type. The power of test is lower for estimates shocks. The loss of test power is differentiated. There are two „difficult" shock $\hat{y}^\star$ and $\hat{z}$ where the decline is considerable and two shocks where the decline is very small. More detailed comparison of estimated and simulated shocks indicates (see Tables 13–14), that skewness of shocks is not the main reason of the „difficulties". Even normal shocks are estimated (smoothed) imprecisely. In particular the standard deviations of smoothed normal shocks are biased. Therefore it is likely a model and/or Kalman filter based smoother related issues. From the other hand, the power of test in small sample is rather low, especially when skewness is moderate (variant CSN-1 and CSN-3).

In addition, the data shown the Table suggest that Bai-Ng tests do not dominate sample skewness coefficient based tests. It is reasonable to apply one-tailed test because of its slightly higher power.

5.2.3 Estimators of shocks' skewness

Bai and Ng (2005, p. 55) noticed that skewness measured by the sample skewness coefficients is usually underestimated. That observation agrees with our findings. Taking it into account data presented in Table 7–8, one can conclude that, when shocks are skew-normal, sample skewness coefficient are biased (skewness is underestimated). The bias seems to be a declining function of sample, but the fall of bias is very slow and even in the large sample the estimates of skewness for the „difficult" shocks, $\hat{y}^\star$ and $\hat{z}$, are very imprecise[30]. The range of skewness coefficient is the second problem worth noting. The skewness coefficient of closed

[29] The Gauss code of the test can be downloaded from http://www.columbia.edu/~sn2294/

[30] We analyze smoothed shocks, but the issue exists also for simulated variables, compare Tables 13–14.

skew-normal shocks generated in this exercise is limited, it must satisfy the following condition: $|d_i| < 0.9953$. The sample skewness coefficient could be arbitrary large. Hence the sample coefficient of skewness should be simultaneously rescaled into proper range and scaled up to minimize bias, but these two transformations are contradictory. We checked several propositions of such adjustments and chose one (see Table 7–8) that keeps relative bias quite small (less then 5%), at least for moderate skewness. If the coefficient of skewness is close to the bound (0.9953), the relative bias could exceed 15% even in a large sample. Properties of two additional variants are shown in Appendix (Table 15–16).

The detailed analysis of sample moments of smoothed state variables suggest that average sample standard deviations of smoothed shocks could be different from mean of Q-ML estimates of shocks' standard deviations: $\sigma_{\hat{z}}$, $\sigma_{\Delta\hat{q}}$, $\sigma_{\hat{y}^\star}$, $\sigma_{\hat{\pi}^\star}$, and $\sigma_{\hat{\epsilon}^R}$ — see Table 13–14 in the Appendix A. The mean standard deviations of smoothed shocks and mean of Q-ML estimators are very different for „difficult" shocks: $\hat{y}^\star$ and $\hat{z}$. Therefore, the adjustments, we have tested, are a function of $\frac{\widehat{\sigma}_{qML,u_i}}{\widehat{\sigma}_{MM,u_i}}$ ratio, where $\widehat{\sigma}_{MM,u_i}$ is the sample estimate of smoothed shocks' standard deviation and $\widehat{\sigma}_{qML,u_i}$ is Q-ML estimates of shocks' standard deviation.

Taking into account equations (5.3) and (5.4) one can express skewness coefficient as a function of ratio of shape parameter to shock's variance $\frac{\tilde{\sigma}^2_{u_i}}{\sigma^2_{u_i}} \geq 1$. Assuming that this ratio is proportional to ratio of MM/Q-LM estimators, the basic form of the adjusted estimator may be written as follows:

$$\widehat{\widehat{\gamma}}_{T,u_i} = \mathrm{sig}(\widehat{\gamma}_{T,u_i})\,\mathrm{G}\left(\left(\frac{4-\pi}{2}\right)\left(\max\left(\frac{\widehat{\sigma}^2_{qML,u_i}}{\widehat{\sigma}^2_{MM,u_i}},\frac{\widehat{\sigma}^2_{MM,u_i}}{\widehat{\sigma}^2_{qML,u_i}}\right)^{\mathrm{N}(T)}\left[1+\left(\left(\frac{2}{4-\pi}\right)|\widehat{\gamma}_{T,u_i}|\right)^{\frac{2}{3}}\right]-1\right)^{\frac{3}{2}}\right), \quad (5.18)$$

where: T — is the sample size, the functions G(.), N(.) are defined as follows:

$$\begin{aligned} &g = \mathrm{G}(\widehat{\gamma}_{u_i}), \quad \widehat{\gamma}_{u_i} \in \mathscr{R}, \quad -0.9953 < g < 0.9953, \\ &n = \mathrm{N}(T), \quad T \in \mathscr{N}, \quad 0.0 < n \leq 1.0. \end{aligned} \quad (5.19)$$

The function: $0.9953\,\mathrm{erf}(.)$ is an example of G(.). The adjusted estimator of skewness coefficient is by no means a good one (see Table 7–8). It is unable to estimate skewness coefficient properly when shocks are normal, because it moves estimates out of zero. In addition, the estimates are still downward bias for large skewness — this is an unpleasant side effect of application the function $G(.)$ which enlarges the bias. Perhaps the most important disadvantage is its *ad hoc* nature. However, our approximate dominates the sample skewness coefficient in terms of (relative) bias in small and medium sample (see variants CSN-1). The biases are usually declining functions of sample size, at least in the model under consideration. Hence, sample skewness coefficient is not the only one possibility we have and the purpose of skewness estimation is affordable.

Table 7. Properties of Skewness Estimators. Part I. Number of cases=2000

Sim. Variant	Para-meter	Sample Estimator of Skewness Coefficient									Adjusted Estimator of Skewness Coefficient $N(T) = 0.5\left(1 - \frac{100}{T}\right)$								
		Bias	Bias %	Mean	Median	St. Dev	Skewness	Kurtosis	5%	95%	Bias	Bias %	Mean	Median	St. Dev	Skewness	Kurtosis	5%	95%
Sample size = 75+6																			
Normal	$d_{\tilde{z}}$	-0.008	-	-0.008	-0.005	0.261	-0.072	3.227	-0.435	0.409	-0.013	-	-0.013	-0.047	0.511	-0.013	1.832	-0.815	0.778
	$d_{\Delta\tilde{q}}$	-0.004	-	-0.004	-0.001	0.265	-0.052	3.134	-0.441	0.427	-0.003	-	-0.003	-0.007	0.323	-0.006	2.441	-0.532	0.518
	$d_{\tilde{y}^\star}$	-0.004	-	-0.004	-0.001	0.265	0.020	3.100	-0.435	0.433	-0.010	-	-0.010	-0.079	0.641	0.009	1.502	-0.929	0.921
	$d_{\tilde{\pi}^\star}$	-0.013	-	-0.013	-0.008	0.258	-0.062	2.967	-0.450	0.412	-0.014	-	-0.014	-0.017	0.325	-0.024	2.427	-0.553	0.519
	$d_{\tilde{\varepsilon}^R}$	-0.005	-	-0.005	-0.004	0.255	-0.056	3.228	-0.426	0.422	-0.006	-	-0.006	-0.012	0.335	-0.009	2.455	-0.559	0.543
CSN-1	$d_{\tilde{z}}$	-0.191	-38.248	0.309	0.302	0.288	0.377	3.695	-0.142	0.799	-0.008	-1.596	0.492	0.598	0.417	-1.195	3.865	-0.411	0.962
	d_{dq}	-0.040	-7.902	0.461	0.445	0.290	0.461	3.819	0.008	0.954	0.002	0.499	0.503	0.531	0.264	-0.546	2.980	0.015	0.884
	$d_{\tilde{y}^\star}$	-0.281	-56.220	0.219	0.215	0.278	0.072	3.349	-0.224	0.676	-0.054	-10.777	0.446	0.659	0.552	-1.106	2.960	-0.710	0.985
	$d_{\tilde{\pi}^\star}$	-0.068	-13.602	0.432	0.424	0.272	0.425	3.630	0.011	0.899	-0.010	-1.986	0.490	0.520	0.259	-0.516	2.900	0.023	0.869
	$d_{\tilde{\varepsilon}^R}$	-0.095	-18.979	0.405	0.403	0.282	0.151	3.261	-0.045	0.884	-0.027	-5.449	0.472	0.508	0.288	-0.755	3.533	-0.088	0.876
CSN-2	$d_{\tilde{z}}$	-0.356	-37.487	0.594	0.589	0.299	0.307	3.634	0.119	1.123	-0.165	-17.377	0.785	0.859	0.241	-2.629	13.063	0.363	0.991
	$d_{\Delta\tilde{q}}$	-0.087	-9.166	0.863	0.834	0.294	0.767	4.353	0.444	1.393	-0.161	-16.937	0.789	0.810	0.143	-0.935	3.848	0.517	0.976
	$d_{\tilde{y}^\star}$	-0.538	-56.643	0.412	0.399	0.296	0.419	4.144	-0.040	0.909	-0.229	-24.105	0.721	0.872	0.383	-2.295	8.199	-0.289	0.995
	$d_{\tilde{\pi}^\star}$	-0.096	-10.081	0.854	0.830	0.291	0.682	4.311	0.440	1.363	-0.156	-16.378	0.794	0.823	0.145	-0.980	4.091	0.525	0.975
	$d_{\tilde{\varepsilon}^R}$	-0.183	-19.258	0.767	0.755	0.288	0.520	4.071	0.336	1.251	-0.188	-19.820	0.762	0.797	0.168	-1.170	5.023	0.447	0.966
CSN-3	$d_{\tilde{z}}$	-0.008	-	-0.008	-0.010	0.257	-0.077	3.152	-0.448	0.406	-0.015	-	-0.015	-0.091	0.507	0.025	1.859	-0.814	0.782
	$d_{\Delta\tilde{q}}$	-0.003	-	-0.003	-0.004	0.268	-0.093	3.346	-0.445	0.438	-0.003	-	-0.003	-0.011	0.324	-0.040	2.585	-0.537	0.526
	$d_{\tilde{y}^\star}$	-0.273	-54.546	0.227	0.223	0.278	0.179	3.233	-0.212	0.694	-0.046	-9.283	0.454	0.672	0.557	-1.119	2.980	-0.701	0.987
	$d_{\tilde{\pi}^\star}$	-0.008	-	-0.008	-0.008	0.255	0.041	3.498	-0.418	0.419	-0.012	-	-0.012	-0.023	0.319	0.052	2.610	-0.530	0.522
	$d_{\tilde{\varepsilon}^R}$	-0.005	-	-0.005	-0.005	0.264	-0.105	3.048	-0.437	0.420	-0.005	-	-0.005	-0.018	0.345	-0.067	2.333	-0.568	0.538
Sample size = 150+6																			
Normal	$d_{\tilde{z}}$	-0.001	-	-0.001	-0.003	0.191	-0.015	3.186	-0.311	0.309	-0.001	-	-0.001	-0.037	0.362	0.008	2.100	-0.567	0.571
	$d_{\Delta\tilde{q}}$	0.006	-	0.006	0.006	0.192	0.023	3.441	-0.313	0.316	0.007	-	0.007	0.012	0.224	0.023	2.895	-0.366	0.372
	$d_{\tilde{y}^\star}$	0.008	-	0.008	0.010	0.198	-0.027	3.134	-0.313	0.331	0.020	-	0.020	0.116	0.476	-0.046	1.746	-0.708	0.725
	$d_{\tilde{\pi}^\star}$	-0.007	-	-0.007	-0.009	0.188	0.023	3.026	-0.313	0.302	-0.008	-	-0.008	-0.017	0.227	0.034	2.633	-0.371	0.357
	$d_{\tilde{\varepsilon}^R}$	-0.002	-	-0.002	-0.009	0.195	0.141	3.217	-0.318	0.323	-0.004	-	-0.004	-0.024	0.250	0.108	2.638	-0.405	0.410
CSN-1	$d_{\tilde{z}}$	-0.174	-34.769	0.326	0.323	0.205	0.170	3.215	-0.006	0.671	0.018	3.517	0.518	0.562	0.268	-1.081	4.501	-0.057	0.873
	$d_{\Delta\tilde{q}}$	-0.035	-6.969	0.466	0.458	0.206	0.234	3.308	0.145	0.813	-0.005	-0.919	0.496	0.509	0.185	-0.377	3.059	0.177	0.776
	$d_{\tilde{y}^\star}$	-0.248	-49.582	0.252	0.247	0.210	0.256	3.467	-0.082	0.598	0.004	0.762	0.504	0.588	0.361	-1.293	4.277	-0.344	0.917
	$d_{\tilde{\pi}^\star}$	-0.048	-9.665	0.452	0.436	0.210	0.374	3.830	0.125	0.816	-0.008	-1.607	0.492	0.500	0.190	-0.398	3.290	0.163	0.788
	$d_{\tilde{\varepsilon}^R}$	-0.073	-14.698	0.426	0.419	0.215	0.304	3.402	0.100	0.796	-0.012	-2.454	0.487	0.503	0.203	-0.481	3.381	0.148	0.797
CSN-2	$d_{\tilde{z}}$	-0.326	-34.279	0.624	0.618	0.213	0.450	3.662	0.300	0.986	-0.158	-16.597	0.792	0.818	0.137	-1.031	4.215	0.537	0.964
	$d_{\Delta\tilde{q}}$	-0.055	-5.799	0.895	0.880	0.216	0.519	3.665	0.577	1.286	-0.155	-16.289	0.795	0.808	0.103	-0.658	3.306	0.605	0.942
	$d_{\tilde{y}^\star}$	-0.474	-49.889	0.476	0.467	0.215	0.274	3.373	0.136	0.856	-0.173	-18.172	0.777	0.835	0.203	-2.288	11.548	0.409	0.979
	$d_{\tilde{\pi}^\star}$	-0.077	-8.109	0.873	0.851	0.220	0.688	4.315	0.551	1.269	-0.157	-16.527	0.793	0.803	0.105	-0.641	3.441	0.603	0.944
	$d_{\tilde{\varepsilon}^R}$	-0.131	-13.746	0.819	0.805	0.225	0.555	3.963	0.480	1.199	-0.167	-17.602	0.783	0.796	0.115	-0.790	3.985	0.573	0.942
CSN-3	$d_{\tilde{z}}$	-0.005	-	-0.005	-0.008	0.190	0.020	3.478	-0.303	0.304	-0.013	-	-0.013	-0.059	0.360	0.054	2.153	-0.579	0.557
	$d_{\Delta\tilde{q}}$	-0.009	-	-0.009	-0.007	0.192	0.033	3.181	-0.320	0.305	-0.011	-	-0.011	-0.012	0.224	0.042	2.784	-0.378	0.358
	$d_{\tilde{y}^\star}$	-0.247	-49.339	0.253	0.247	0.203	0.085	3.174	-0.075	0.605	0.013	2.570	0.513	0.592	0.356	-1.393	4.759	-0.309	0.920
	$d_{\tilde{\pi}^\star}$	0.002	-	0.002	-0.001	0.194	0.104	3.194	-0.310	0.322	0.002	-	0.002	-0.004	0.233	0.053	2.684	-0.374	0.387
	$d_{\tilde{\varepsilon}^R}$	0.007	-	0.007	0.003	0.196	-0.041	3.256	-0.319	0.328	0.009	-	0.009	0.011	0.249	-0.032	2.630	-0.408	0.409

Source: Prepared by the authors

Table 8. Properties of Skewness Estimators. Part I (cont.). Number of cases=2000

Sim. Variant	Parameter	Sample Estimator of Skewness Coefficient									Adjusted Estimator of Skewness Coefficient								
		Bias	Bias %	Mean	Median	St. Dev	Skewness	Kurtosis	5%	95%	Bias	Bias %	Mean	Median	St. Dev	Skewness	Kurtosis	5%	95%
Sample size = 600+6																			
Normal	$d_{\hat{z}}$	0.002	-	0.002	0.001	0.098	0.108	3.227	-0.155	0.167	0.003	-	0.003	0.024	0.188	0.031	2.245	-0.290	0.300
	$d_{\Delta\hat{q}}$	-0.001	-	-0.001	-0.001	0.099	-0.032	3.006	-0.166	0.165	-0.001	-	-0.001	-0.002	0.115	-0.023	2.871	-0.191	0.188
	$d_{\hat{y}^\star}$	-0.001	-	-0.001	0.002	0.097	-0.028	2.930	-0.159	0.157	-0.001	-	-0.001	0.058	0.250	-0.023	1.842	-0.377	0.373
	$d_{\hat{\pi}^\star}$	-0.001	-	-0.001	-0.004	0.097	0.099	2.914	-0.161	0.161	-0.002	-	-0.002	-0.007	0.118	0.090	2.725	-0.194	0.191
	$d_{\hat{\epsilon}^R}$	-0.000	-	-0.000	-0.002	0.099	0.029	3.337	-0.163	0.159	-0.001	-	-0.001	-0.007	0.130	0.019	2.894	-0.209	0.207
CSN-1	$d_{\hat{z}}$	-0.142	-28.436	0.358	0.358	0.109	0.069	3.075	0.181	0.543	0.026	5.279	0.527	0.535	0.121	-0.369	3.132	0.312	0.711
	$d_{\Delta\hat{q}}$	-0.014	-2.806	0.486	0.483	0.110	0.165	3.299	0.310	0.668	0.011	2.139	0.511	0.512	0.097	-0.201	3.231	0.347	0.664
	$d_{\hat{y}^\star}$	-0.221	-44.213	0.279	0.278	0.105	0.028	3.090	0.110	0.460	0.030	5.917	0.530	0.541	0.144	-0.828	5.154	0.283	0.743
	$d_{\hat{\pi}^\star}$	-0.030	-6.068	0.470	0.463	0.109	0.316	3.178	0.306	0.661	0.008	1.595	0.508	0.506	0.097	-0.030	2.961	0.351	0.670
	$d_{\hat{\epsilon}^R}$	-0.050	-9.961	0.450	0.445	0.110	0.199	3.365	0.269	0.636	0.011	2.230	0.511	0.512	0.102	-0.207	3.190	0.332	0.671
CSN-2	$d_{\hat{z}}$	-0.273	-28.689	0.677	0.673	0.112	0.234	3.399	0.502	0.868	-0.153	-16.115	0.797	0.803	0.068	-0.599	3.638	0.674	0.893
	$d_{\Delta\hat{q}}$	-0.027	-2.815	0.923	0.915	0.113	0.368	3.365	0.755	1.120	-0.142	-14.923	0.808	0.810	0.052	-0.310	3.244	0.721	0.891
	$d_{\hat{y}^\star}$	-0.416	-43.798	0.534	0.530	0.115	0.205	3.164	0.354	0.729	-0.164	-17.304	0.786	0.796	0.087	-0.763	3.598	0.620	0.910
	$d_{\hat{\pi}^\star}$	-0.051	-5.419	0.899	0.895	0.114	0.308	3.376	0.719	1.088	-0.144	-15.144	0.806	0.811	0.054	-0.381	3.171	0.710	0.888
	$d_{\hat{\epsilon}^R}$	-0.099	-10.435	0.851	0.851	0.112	0.180	3.254	0.674	1.037	-0.149	-15.705	0.801	0.806	0.056	-0.484	3.244	0.701	0.885
CSN-3	$d_{\hat{z}}$	-0.008	-	-0.008	-0.010	0.098	0.006	2.911	-0.166	0.153	-0.016	-	-0.016	-0.049	0.188	0.071	2.150	-0.303	0.281
	$d_{\Delta\hat{q}}$	-0.001	-	-0.001	-0.001	0.098	0.048	3.082	-0.158	0.160	-0.001	-	-0.001	-0.002	0.113	0.043	2.947	-0.182	0.183
	$d_{\hat{y}^\star}$	-0.219	-43.835	0.281	0.282	0.103	0.099	3.105	0.110	0.446	0.033	6.595	0.533	0.547	0.137	-0.627	4.029	0.286	0.733
	$d_{\hat{\pi}^\star}$	0.002	-	0.002	0.000	0.100	-0.040	2.937	-0.165	0.164	0.002	-	0.002	0.002	0.120	-0.038	2.726	-0.199	0.197
	$d_{\hat{\epsilon}^R}$	-0.002	-	-0.002	0.001	0.098	0.001	3.119	-0.166	0.162	-0.002	-	-0.002	0.005	0.129	0.002	2.758	-0.212	0.208

Source: Prepared by the authors

6 Concluding remarks

In the paper we stressed the fact, that skewness in observed variables can be a result of skewness in structural shocks. In fact, in a linear (or a linearized) DSGE model there is no other way to get skewed observables. Propagation of skewness in liner state-space models undergoes certain laws, e.g. skewness of states of univariate autoregressions decreases with time reaching a zero or non-zero limit for random walks and stationary specifications respectively. Simulation exercises indicate that a simple two-step quasi-maximum likelihood parameters' estimation procedure, which neglects shocks' skewness in the first step, does not distort parameter estimates of models' deterministic part, at least for the case we considered. This allows for filtration of shocks, given parameters, and then estimation of shocks' skewness parameters. Simulations also suggest that tests for skewness of filtered shocks should be taken with caution and that quality of estimates of shocks' skewness parameters in a DSGE model is shock dependant.

A Results of stochastic simulations

Table 9. Normal shocks — Kalman filter ML estimators of the basic set of parameters. Number of replications=2000

Parameter	Bias	Bias %	Mean	Mode	Median	St.Dev	Skewness	Kurtosis	5%	95%
Sample size = 75+7										
ψ_π	0.481	32.09	1.981	1.604	1.208	1.215	2.895	13.47	1.022	4.552
ψ_y	0.219	87.50	0.469	0.295	0.214	0.532	3.375	18.19	0.075	1.518
$\psi_{\Delta e}$	0.056	56.44	0.156	0.115	0.073	0.136	3.030	14.89	0.048	0.411
ρ_R	0.015	2.462	0.615	0.613	0.615	0.120	0.016	2.884	0.420	0.819
α	0.002	1.407	0.152	0.150	0.151	0.029	0.256	3.027	0.107	0.201
κ	0.034	6.891	0.534	0.511	0.432	0.174	1.202	6.239	0.299	0.842
τ	-0.010	-1.912	0.490	0.486	0.382	0.202	-0.065	2.316	0.146	0.825
r_A	0.176	23.49	0.926	0.861	0.614	0.562	0.637	3.131	0.135	1.955
γ_Q	-0.034	-4.276	0.766	0.774	0.789	0.127	-0.383	3.048	0.535	0.959
$\rho_{\hat{z}}$	0.007	3.530	0.207	0.206	0.207	0.047	0.751	7.872	0.135	0.285
$\rho_{\Delta\hat{q}}$	-0.010	-2.593	0.390	0.397	0.434	0.102	-0.358	3.284	0.214	0.552
$\rho_{\hat{y}^\star}$	-0.020	-2.188	0.880	0.888	0.926	0.059	-0.918	4.279	0.770	0.961
$\rho_{\pi^\star}$	-0.006	-0.786	0.794	0.797	0.793	0.053	-0.445	3.210	0.698	0.875
$\sigma_{\hat{z}}$	-0.050	-4.970	0.950	0.951	0.951	0.113	-0.216	3.897	0.771	1.135
$\sigma_{\Delta\hat{q}}$	-0.012	-0.657	1.888	1.886	1.815	0.161	0.126	3.074	1.623	2.149
$\sigma_{\hat{y}^\star}$	1.155	61.11	3.045	1.830	0.818	3.376	2.188	8.164	0.233	10.847
$\sigma_{\hat{\pi}^\star}$	-0.031	-1.017	2.969	2.964	2.961	0.253	0.058	3.035	2.562	3.388
$\sigma_{\hat{\epsilon}^R}$	0.015	3.816	0.415	0.405	0.390	0.096	13.21	379.4	0.315	0.549
Sample size = 150+7										
ψ_π	0.236	15.72	1.736	1.538	1.285	0.746	3.298	23.58	1.033	3.024
ψ_y	0.099	39.72	0.349	0.263	0.201	0.290	2.948	17.80	0.083	0.902
$\psi_{\Delta e}$	0.028	27.51	0.128	0.106	0.079	0.080	2.858	17.00	0.051	0.273
ρ_R	0.009	1.463	0.609	0.603	0.597	0.093	0.192	2.846	0.466	0.767
α	0.001	0.769	0.151	0.150	0.149	0.021	0.227	2.916	0.118	0.187
κ	0.013	2.522	0.513	0.503	0.438	0.112	0.594	3.666	0.349	0.713
τ	0.002	0.443	0.502	0.505	0.568	0.155	-0.086	2.777	0.243	0.763
r_A	0.070	9.338	0.820	0.777	0.699	0.438	0.425	2.720	0.159	1.608
γ_Q	-0.014	-1.723	0.786	0.792	0.819	0.094	-0.287	2.834	0.624	0.931
$\rho_{\hat{z}}$	0.008	3.750	0.208	0.205	0.187	0.035	0.428	3.301	0.155	0.270
$\rho_{\Delta\hat{q}}$	-0.003	-0.637	0.397	0.398	0.437	0.071	-0.113	3.078	0.280	0.509
$\rho_{\hat{y}^\star}$	-0.009	-0.976	0.891	0.896	0.896	0.039	-0.752	3.683	0.823	0.945
$\rho_{\hat{\pi}^\star}$	-0.005	-0.580	0.795	0.799	0.799	0.036	-0.340	3.077	0.731	0.851
$\sigma_{\hat{z}}$	-0.025	-2.546	0.975	0.977	0.950	0.080	-0.049	3.222	0.843	1.102
$\sigma_{\Delta\hat{q}}$	-0.009	-0.470	1.891	1.891	1.926	0.111	0.083	3.165	1.711	2.076
$\sigma_{\hat{y}^\star}$	0.757	40.06	2.647	1.950	1.570	2.423	2.541	11.52	0.480	7.618
$\sigma_{hat\pi^\star}$	-0.007	-0.217	2.993	2.988	2.959	0.180	0.042	2.953	2.697	3.294
$\sigma_{\hat{\epsilon}^R}$	0.006	1.561	0.406	0.400	0.398	0.057	6.028	126.6	0.335	0.498
Sample size = 600+7										
ψ_π	0.045	2.990	1.545	1.504	1.395	0.264	0.907	4.343	1.195	2.027
ψ_y	0.019	7.635	0.269	0.252	0.219	0.103	1.034	4.696	0.132	0.463
$\psi_{\Delta e}$	0.006	5.516	0.106	0.101	0.104	0.029	0.923	4.409	0.067	0.157
ρ_R	0.001	0.145	0.601	0.601	0.605	0.049	0.010	2.900	0.522	0.681
α	-0.000	-0.043	0.150	0.150	0.146	0.011	0.047	2.930	0.131	0.169
κ	0.003	0.546	0.503	0.499	0.468	0.055	0.341	3.175	0.417	0.599
τ	0.001	0.276	0.501	0.503	0.503	0.088	-0.053	3.001	0.358	0.642
r_A	0.011	1.511	0.761	0.751	0.629	0.239	0.141	2.873	0.381	1.163
γ_Q	-0.002	-0.190	0.798	0.799	0.778	0.051	-0.057	2.871	0.713	0.882
$\rho_{\hat{z}}$	0.003	1.305	0.203	0.201	0.199	0.020	0.502	3.475	0.173	0.238
$\rho_{\Delta\hat{q}}$	-0.000	-0.073	0.400	0.400	0.387	0.035	-0.042	2.836	0.342	0.457
$\rho_{\hat{y}^\star}$	-0.002	-0.265	0.898	0.899	0.898	0.017	-0.382	3.173	0.868	0.924
$\rho_{\hat{\pi}^\star}$	-0.002	-0.237	0.798	0.799	0.792	0.018	-0.323	3.095	0.766	0.827
$\sigma_{\hat{z}}$	-0.007	-0.675	0.993	0.995	0.997	0.041	-0.147	3.080	0.925	1.057
$\sigma_{\Delta\hat{q}}$	-0.002	-0.097	1.898	1.897	1.876	0.055	0.075	2.846	1.807	1.989
$\sigma_{\hat{y}^\star}$	0.226	11.96	2.116	1.915	1.894	0.999	1.577	7.571	0.906	3.931
$\sigma_{\hat{\pi}^\star}$	-0.004	-0.136	2.996	2.995	2.999	0.088	-0.021	2.987	2.849	3.141
$\sigma_{\hat{\epsilon}^R}$	0.001	0.227	0.401	0.400	0.406	0.024	0.292	3.326	0.363	0.443

Source: Prepared by the authors

Table 10. CSN-1 shocks — Kalman filter Q-ML estimator of th basic set of parameters, Number of replications=2000

Parameter	Bias	Bias %	Mean	Mode	Median	St.Dev	Skewness	Kurtosis	5%	95%
Sample size = 75+7										
ψ_π	0.542	36.14	2.042	1.608	1.212	1.290	2.634	11.44	1.032	4.713
ψ_y	0.235	93.96	0.485	0.307	0.174	0.551	3.314	17.92	0.077	1.425
$\psi_{\Delta e}$	0.063	62.88	0.163	0.114	0.074	0.148	3.036	15.54	0.048	0.452
ρ_R	0.017	2.797	0.617	0.610	0.580	0.124	0.039	2.785	0.417	0.832
α	0.003	1.708	0.153	0.151	0.147	0.029	0.309	3.073	0.107	0.203
κ	0.037	7.383	0.537	0.517	0.457	0.172	1.061	5.169	0.299	0.864
τ	-0.013	-2.613	0.487	0.486	0.497	0.200	-0.004	2.316	0.160	0.826
r_A	0.184	24.57	0.934	0.877	0.769	0.563	0.510	2.810	0.115	1.917
γ_Q	-0.035	-4.351	0.765	0.772	0.812	0.123	-0.244	2.788	0.555	0.951
$\rho_{\hat{z}}$	0.008	3.944	0.208	0.204	0.190	0.048	0.562	4.488	0.137	0.292
$\rho_{\Delta\hat{q}}$	-0.007	-1.842	0.393	0.401	0.411	0.103	-0.335	3.128	0.208	0.549
$\rho_{\hat{y}^*}$	-0.018	-1.965	0.882	0.893	0.911	0.059	-0.896	3.823	0.770	0.960
$\rho_{\hat{\pi}^*}$	-0.007	-0.834	0.793	0.796	0.789	0.052	-0.545	3.571	0.700	0.869
$\sigma_{\hat{z}}$	-0.057	-5.669	0.943	0.945	0.879	0.119	-0.134	3.378	0.749	1.136
$\sigma_{\Delta\hat{q}}$	-0.021	-1.098	1.879	1.876	1.934	0.172	0.115	3.077	1.601	2.177
$\sigma_{\hat{y}^*}$	1.137	60.16	3.027	1.783	0.322	3.446	2.217	8.208	0.255	10.86
$\sigma_{\hat{\pi}^*}$	-0.013	-0.418	2.987	2.988	2.907	0.269	0.026	3.013	2.548	3.423
$\sigma_{\hat{\epsilon}^R}$	0.019	4.659	0.419	0.406	0.400	0.121	18.16	551.8	0.312	0.551
Sample size = 150+7										
ψ_π	0.233	15.51	1.733	1.524	1.320	0.831	4.076	29.27	1.048	3.053
ψ_y	0.100	39.80	0.350	0.257	0.219	0.344	4.317	32.21	0.084	0.937
$\psi_{\Delta e}$	0.027	27.21	0.127	0.105	0.089	0.091	4.165	30.71	0.051	0.268
ρ_R	0.006	0.969	0.606	0.601	0.597	0.092	0.276	3.127	0.463	0.763
α	0.001	0.539	0.151	0.150	0.147	0.022	0.310	3.072	0.117	0.188
κ	0.014	2.885	0.514	0.502	0.495	0.113	0.711	3.727	0.356	0.723
τ	0.004	0.767	0.504	0.504	0.437	0.159	-0.048	2.705	0.243	0.767
r_A	0.064	8.598	0.814	0.786	0.946	0.427	0.362	2.727	0.158	1.568
γ_Q	-0.014	-1.769	0.786	0.791	0.817	0.091	-0.217	2.852	0.630	0.926
$\rho_{\hat{z}}$	0.007	3.304	0.207	0.205	0.194	0.034	0.274	3.038	0.152	0.268
$\rho_{\Delta\hat{q}}$	-0.006	-1.394	0.394	0.397	0.408	0.072	-0.205	3.024	0.273	0.508
$\rho_{\hat{y}^*}$	-0.008	-0.865	0.892	0.897	0.899	0.038	-0.777	3.753	0.823	0.945
$\rho_{\hat{\pi}^*}$	-0.004	-0.554	0.796	0.797	0.780	0.037	-0.293	3.071	0.732	0.852
$\sigma_{\hat{z}}$	-0.032	-3.187	0.968	0.969	0.948	0.082	-0.096	3.158	0.834	1.102
$\sigma_{\Delta\hat{q}}$	-0.003	-0.177	1.897	1.891	1.815	0.120	0.148	2.997	1.704	2.100
$\sigma_{\hat{y}^*}$	0.856	45.29	2.746	1.961	0.940	2.598	2.580	11.82	0.480	7.758
$\sigma_{\hat{\pi}^*}$	-0.014	-0.466	2.986	2.978	2.940	0.193	0.154	2.936	2.686	3.317
$\sigma_{\hat{\epsilon}^R}$	0.005	1.224	0.405	0.398	0.379	0.050	0.821	4.490	0.334	0.499
Sample size = 600+7										
ψ_π	0.043	2.859	1.543	1.502	1.432	0.291	1.661	10.72	1.169	2.070
ψ_y	0.016	6.533	0.266	0.247	0.216	0.113	1.932	12.04	0.128	0.466
$\psi_{\Delta e}$	0.005	4.820	0.105	0.100	0.092	0.032	1.706	11.04	0.064	0.162
ρ_R	0.000	0.056	0.600	0.599	0.600	0.050	0.178	3.263	0.519	0.684
α	-0.000	-0.019	0.150	0.150	0.152	0.012	-0.024	2.971	0.130	0.169
κ	0.003	0.635	0.503	0.499	0.487	0.055	0.332	3.126	0.418	0.599
τ	0.006	1.269	0.506	0.505	0.509	0.085	0.114	3.029	0.369	0.649
r_A	-0.000	-0.026	0.750	0.753	0.837	0.239	-0.001	2.920	0.354	1.150
γ_Q	0.000	0.054	0.800	0.800	0.793	0.051	0.002	2.849	0.717	0.885
$\rho_{\hat{z}}$	0.004	1.765	0.204	0.202	0.197	0.019	0.684	3.824	0.176	0.239
$\rho_{\Delta\hat{q}}$	-0.000	-0.073	0.400	0.400	0.415	0.037	-0.133	3.021	0.335	0.460
$\rho_{\hat{y}^*}$	-0.002	-0.196	0.898	0.900	0.900	0.017	-0.403	3.185	0.868	0.924
$\rho_{\hat{\pi}^*}$	-0.001	-0.084	0.799	0.800	0.802	0.018	-0.211	3.051	0.767	0.829
$\sigma_{\hat{z}}$	-0.008	-0.754	0.992	0.993	0.991	0.042	-0.125	3.055	0.923	1.059
$\sigma_{\Delta\hat{q}}$	-0.001	-0.061	1.899	1.900	1.881	0.059	-0.013	3.036	1.800	1.995
$\sigma_{\hat{y}^*}$	0.263	13.89	2.153	1.959	1.496	0.988	1.716	8.943	0.975	3.984
$\sigma_{\hat{\pi}^*}$	-0.005	-0.157	2.995	2.991	2.967	0.096	0.085	3.114	2.840	3.153
$\sigma_{\hat{\epsilon}^R}$	0.000	0.003	0.400	0.399	0.400	0.026	0.400	3.874	0.360	0.445

Source: Prepared by the authors

Table 11. CSN-2 shocks — Kalman filter Q-ML estimator of the basic set of parameters. Number of replications=2000

Parameter	Bias	Bias %	Mean	Mode	Median	St.Dev	Skewness	Kurtosis	5%	95%
Sample size = 75+7										
ψ_π	0.485	32.33	1.985	1.575	1.227	1.223	2.672	11.63	1.026	4.488
ψ_y	0.217	86.84	0.467	0.303	0.125	0.509	2.885	13.96	0.073	1.506
$\psi_{\Delta e}$	0.057	57.45	0.157	0.114	0.064	0.137	2.879	14.38	0.048	0.422
ρ_R	0.015	2.567	0.615	0.609	0.599	0.121	0.076	2.804	0.427	0.830
α	0.002	1.577	0.152	0.151	0.153	0.028	0.262	3.032	0.108	0.202
κ	0.034	6.856	0.534	0.515	0.444	0.169	1.028	5.020	0.309	0.837
τ	-0.015	-2.928	0.485	0.485	0.521	0.195	-0.031	2.365	0.156	0.812
r_A	0.171	22.75	0.921	0.872	0.448	0.565	0.596	3.038	0.121	1.942
γ_Q	-0.035	-4.351	0.765	0.770	0.769	0.122	-0.316	2.984	0.551	0.955
$\rho_{\hat{z}}$	0.008	3.825	0.208	0.205	0.203	0.047	0.449	3.526	0.137	0.290
$\rho_{\Delta\hat{q}}$	-0.011	-2.707	0.389	0.392	0.413	0.101	-0.177	3.024	0.220	0.549
$\rho_{y^\star}$	-0.018	-2.031	0.882	0.892	0.896	0.060	-1.082	4.957	0.768	0.961
$\rho_{\pi^\star}$	-0.008	-0.974	0.792	0.795	0.811	0.053	-0.308	2.944	0.699	0.874
$\sigma_{\hat{z}}$	-0.059	-5.916	0.941	0.943	0.978	0.123	-0.033	3.503	0.740	1.138
$\sigma_{\Delta\hat{q}}$	-0.022	-1.138	1.878	1.874	2.029	0.185	0.217	3.003	1.583	2.193
$\sigma_{\hat{y}^\star}$	1.014	53.64	2.904	1.791	0.630	3.265	2.368	9.373	0.253	9.801
$\sigma_{\hat{\pi}^\star}$	-0.028	-0.949	2.972	2.960	2.907	0.289	0.288	3.132	2.524	3.457
$\sigma_{\hat{\epsilon}^R}$	0.014	3.536	0.414	0.404	0.428	0.075	0.887	4.172	0.313	0.553
Sample size = 150+7										
ψ_π	0.206	13.72	1.706	1.509	1.414	0.745	3.025	17.49	1.038	3.030
ψ_y	0.094	37.51	0.344	0.261	0.147	0.298	2.840	14.27	0.084	0.890
$\psi_{\Delta e}$	0.024	24.14	0.124	0.103	0.073	0.081	2.940	17.09	0.049	0.274
ρ_R	0.002	0.346	0.602	0.597	0.563	0.094	0.245	3.173	0.458	0.767
α	0.000	0.007	0.150	0.149	0.145	0.022	0.216	3.136	0.115	0.186
κ	0.019	3.752	0.519	0.505	0.466	0.116	0.768	4.228	0.355	0.726
τ	0.005	0.907	0.505	0.503	0.505	0.159	-0.031	2.591	0.250	0.770
r_A	0.068	9.116	0.818	0.799	0.744	0.412	0.331	2.830	0.179	1.529
γ_Q	-0.015	-1.893	0.785	0.791	0.806	0.090	-0.265	2.908	0.626	0.923
$\rho_{\hat{z}}$	0.005	2.570	0.205	0.203	0.193	0.034	0.414	3.558	0.152	0.264
$\rho_{\Delta\hat{q}}$	-0.004	-1.028	0.396	0.397	0.363	0.071	-0.157	3.100	0.279	0.511
$\rho_{\hat{y}^\star}$	-0.011	-1.190	0.889	0.894	0.899	0.038	-0.786	3.987	0.819	0.944
$\rho_{\hat{\pi}^\star}$	-0.004	-0.549	0.796	0.799	0.801	0.037	-0.359	3.196	0.730	0.852
$\sigma_{\hat{z}}$	-0.032	-3.247	0.968	0.968	0.971	0.083	-0.024	3.082	0.832	1.104
$\sigma_{\Delta\hat{q}}$	-0.011	-0.555	1.889	1.887	1.837	0.129	0.234	2.973	1.686	2.112
$\sigma_{\hat{y}^\star}$	0.845	44.73	2.735	1.940	1.165	2.555	2.462	11.19	0.460	7.847
$\sigma_{\hat{\pi}^\star}$	-0.015	-0.503	2.985	2.985	2.984	0.214	0.205	3.275	2.640	3.335
$\sigma_{\hat{\epsilon}^R}$	0.005	1.264	0.405	0.396	0.381	0.054	0.791	4.076	0.331	0.502
Sample size = 606+7										
ψ_π	0.040	2.680	1.540	1.489	1.469	0.313	6.702	134.6	1.190	2.022
ψ_y	0.016	6.525	0.266	0.249	0.206	0.114	4.565	73.65	0.135	0.452
$\psi_{\Delta e}$	0.005	4.537	0.105	0.100	0.095	0.032	4.866	81.35	0.066	0.156
ρ_R	-0.001	-0.141	0.599	0.598	0.597	0.048	0.122	2.969	0.522	0.682
α	0.000	0.047	0.150	0.150	0.155	0.011	0.038	2.986	0.132	0.169
κ	0.005	1.080	0.505	0.503	0.476	0.058	1.799	27.49	0.417	0.598
τ	0.003	0.684	0.503	0.502	0.483	0.084	0.067	3.179	0.369	0.643
r_A	0.004	0.515	0.754	0.756	0.693	0.240	0.040	2.790	0.349	1.149
γ_Q	-0.001	-0.171	0.799	0.799	0.804	0.051	-0.005	2.712	0.714	0.885
$\rho_{\hat{z}}$	0.003	1.303	0.203	0.200	0.194	0.019	0.593	3.895	0.175	0.239
$\rho_{\Delta\hat{q}}$	-0.002	-0.549	0.398	0.399	0.381	0.035	-0.076	2.956	0.338	0.455
$\rho_{y^\star}$	-0.002	-0.254	0.898	0.899	0.902	0.018	-0.511	3.425	0.866	0.925
$\rho_{\pi^\star}$	-0.001	-0.180	0.799	0.799	0.797	0.019	-0.127	3.228	0.767	0.829
$\sigma_{\hat{z}}$	-0.008	-0.808	0.992	0.991	1.007	0.044	0.008	2.998	0.919	1.066
$\sigma_{\Delta\hat{q}}$	-0.004	-0.189	1.896	1.895	1.880	0.065	0.124	3.040	1.789	2.005
$\sigma_{\hat{y}^\star}$	0.226	11.96	2.116	1.913	1.590	0.978	1.891	10.32	0.951	3.873
$\sigma_{\hat{\pi}^\star}$	-0.004	-0.127	2.996	2.996	2.999	0.103	0.102	2.996	2.829	3.168
$\sigma_{\hat{\epsilon}^R}$	0.001	0.332	0.401	0.400	0.392	0.054	30.473	1198.	0.361	0.445

Source: Prepared by the authors

Table 12. CSB-3 Shocks — Kalman filter Q-ML estimator of the basic set of parameters. Number of replications=2000,

Parameter	Bias	Bias %	Mean	Mode	Median	St. Dev	Skewness	Kurtosis	5%	95%
Sample size = 75+7										
ψ_π	0.456	30.38	1.956	1.577	1.102	1.193	2.949	13.99	1.041	4.448
ψ_y	0.206	82.57	0.456	0.291	0.128	0.529	3.515	20.14	0.072	1.468
$\psi_{\Delta e}$	0.055	54.77	0.155	0.112	0.079	0.136	3.059	15.13	0.047	0.425
ρ_R	0.015	2.556	0.615	0.612	0.537	0.118	0.096	2.980	0.428	0.827
α	0.003	2.028	0.153	0.152	0.159	0.028	0.214	2.933	0.110	0.200
κ	0.030	6.054	0.530	0.506	0.459	0.165	1.004	5.453	0.305	0.829
τ	-0.015	-3.021	0.485	0.479	0.503	0.201	0.031	2.286	0.155	0.824
r_A	0.180	24.00	0.930	0.868	0.503	0.566	0.576	3.101	0.114	1.910
γ_Q	-0.036	-4.443	0.764	0.772	0.878	0.124	-0.316	2.994	0.552	0.958
$\rho_{\hat{z}}$	0.008	3.754	0.208	0.204	0.180	0.047	0.431	3.477	0.135	0.288
$\rho_{\Delta\hat{q}}$	-0.006	-1.589	0.394	0.399	0.413	0.105	-0.223	2.966	0.215	0.562
$\rho_{\hat{y}^\star}$	-0.017	-1.935	0.883	0.893	0.904	0.061	-1.047	4.513	0.769	0.963
$\rho_{\hat{\pi}^\star}$	-0.006	-0.765	0.794	0.798	0.802	0.052	-0.406	3.136	0.699	0.872
$\sigma_{\hat{z}}$	-0.050	-4.969	0.950	0.953	1.018	0.114	-0.176	3.348	0.762	1.139
$\sigma_{\Delta\hat{q}}$	-0.013	-0.669	1.887	1.889	1.820	0.154	0.078	2.987	1.637	2.146
$\sigma_{\hat{y}^\star}$	1.145	60.61	3.035	1.805	0.666	3.520	2.323	8.813	0.256	10.800
$\sigma_{\hat{\pi}^\star}$	-0.015	-0.499	2.985	2.975	3.079	0.259	0.128	2.916	2.584	3.427
$\sigma_{\hat{\epsilon}^R}$	0.011	2.871	0.411	0.402	0.405	0.073	2.334	24.70	0.318	0.538
Sample size = 150+7										
ψ_π	0.254	16.93	1.754	1.528	1.365	0.855	3.772	25.00	1.055	3.203
ψ_y	0.111	44.59	0.361	0.262	0.188	0.362	4.183	28.58	0.086	0.922
$\psi_{\Delta e}$	0.030	29.87	0.130	0.105	0.077	0.094	3.782	25.35	0.052	0.284
ρ_R	0.006	1.050	0.606	0.600	0.575	0.095	0.267	3.085	0.458	0.769
α	0.001	0.946	0.151	0.151	0.145	0.022	0.219	3.126	0.117	0.189
κ	0.018	3.677	0.518	0.508	0.552	0.120	1.043	6.782	0.351	0.731
τ	0.002	0.328	0.502	0.500	0.529	0.162	0.017	2.650	0.229	0.773
r_A	0.070	9.324	0.820	0.810	1.011	0.427	0.260	2.592	0.144	1.566
γ_Q	-0.014	-1.688	0.786	0.789	0.757	0.091	-0.161	2.785	0.631	0.933
$\rho_{\hat{z}}$	0.006	3.081	0.206	0.204	0.214	0.035	0.409	3.383	0.153	0.267
$\rho_{\Delta\hat{q}}$	-0.005	-1.308	0.395	0.397	0.376	0.073	-0.218	3.093	0.271	0.505
$\rho_{\hat{y}^\star}$	-0.009	-1.033	0.891	0.896	0.907	0.039	-0.733	3.570	0.820	0.945
$\rho_{\hat{\pi}^\star}$	-0.006	-0.730	0.794	0.796	0.804	0.037	-0.407	3.305	0.728	0.851
$\sigma_{\hat{z}}$	-0.026	-2.601	0.974	0.976	0.959	0.080	-0.156	3.014	0.840	1.100
$\sigma_{\Delta\hat{q}}$	-0.007	-0.344	1.893	1.893	1.877	0.114	0.016	3.121	1.707	2.079
$\sigma_{\hat{y}^\star}$	0.869	45.97	2.759	1.947	1.147	2.742	2.707	12.44	0.448	7.793
$\sigma_{\hat{\pi}^\star}$	-0.004	-0.136	2.996	2.994	3.055	0.175	-0.021	3.110	2.695	3.286
$\sigma_{\hat{\epsilon}^R}$	0.009	2.359	0.409	0.401	0.393	0.093	18.29	480.4	0.334	0.505
Sample size = 600+7										
ψ_π	0.041	2.705	1.541	1.500	1.400	0.284	1.968	15.065	1.168	2.053
ψ_y	0.016	6.400	0.266	0.250	0.223	0.108	2.016	13.365	0.131	0.458
$\psi_{\Delta e}$	0.005	5.109	0.105	0.100	0.094	0.031	2.046	15.703	0.065	0.158
ρ_R	0.000	0.075	0.600	0.599	0.599	0.049	0.167	3.325	0.520	0.681
α	0.000	0.091	0.150	0.150	0.152	0.011	0.066	3.126	0.132	0.169
κ	0.003	0.601	0.503	0.500	0.476	0.057	0.300	3.011	0.414	0.605
τ	0.001	0.170	0.501	0.500	0.498	0.083	-0.020	3.227	0.363	0.636
r_A	0.006	0.766	0.756	0.758	0.781	0.238	-0.011	2.911	0.362	1.151
γ_Q	-0.002	-0.243	0.798	0.799	0.803	0.051	0.017	2.836	0.716	0.880
$\rho_{\hat{z}}$	0.003	1.420	0.203	0.201	0.205	0.019	0.496	3.394	0.174	0.238
$\rho_{\Delta\hat{q}}$	-0.001	-0.149	0.399	0.399	0.392	0.035	-0.013	3.020	0.342	0.458
$\rho_{\hat{y}^\star}$	-0.002	-0.262	0.898	0.900	0.901	0.018	-0.557	3.666	0.865	0.924
$\rho_{\hat{\pi}^\star}$	-0.002	-0.221	0.798	0.799	0.799	0.019	-0.222	2.989	0.765	0.827
$\sigma_{\hat{z}}$	-0.009	-0.943	0.991	0.991	1.001	0.040	-0.037	3.264	0.924	1.057
$\sigma_{\Delta q}$	-0.003	-0.146	1.897	1.897	1.879	0.055	0.075	2.873	1.805	1.990
$\sigma_{\hat{y}^\star}$	0.205	10.862	2.095	1.929	1.748	0.948	1.849	11.305	0.952	3.820
$\sigma_{\hat{\pi}^\star}$	-0.003	-0.111	2.997	2.999	3.014	0.089	-0.011	3.101	2.852	3.147
$\sigma_{\hat{\epsilon}^R}$	-0.000	-0.019	0.400	0.399	0.383	0.026	0.535	4.694	0.361	0.443

Source: Prepared by the authors

Table 13. Simulated and estimated /smoothed/ shocks. Part Ia. Number of replications=2000

Simulation Variant	Shock Name	Simulated shocks /average in sample/					Estimated shocks /average in sample /				
		A. Mean	A. Median	A. St. Dev	A. Skewness	A. Kurtosis	A. Mean	A. Median	A. St. Dev	A. Skewness	A. Kurtosis
Sample size = 75+6											
Normal	$\hat{z}$	-0.025	-0.023	0.995	-0.009	2.988	0.001	0.004	0.859	-0.008	2.988
	$\Delta\hat{q}$	0.001	0.001	1.898	-0.001	3.007	0.002	0.002	1.877	-0.004	3.004
	$\hat{y}^\star$	0.001	0.000	1.880	-0.003	3.011	0.007	0.012	2.627	-0.004	2.985
	$\hat{\pi}^\star$	-0.015	-0.006	2.983	-0.012	3.013	-0.012	-0.004	2.929	-0.013	3.012
	$\hat{\epsilon}^R$	0.004	0.003	0.398	-0.002	2.955	-0.001	-0.001	0.405	-0.005	2.964
CSN-1	$\hat{z}$	-0.025	-0.113	0.989	0.483	3.352	0.002	-0.047	0.853	0.315	3.204
	$\Delta\hat{q}$	0.003	-0.164	1.890	0.480	3.323	0.002	-0.158	1.871	0.470	3.315
	$\hat{y}^\star$	-0.001	-0.168	1.877	0.459	3.274	-0.017	-0.128	2.608	0.223	3.101
	$\hat{\pi}^\star$	-0.013	-0.294	2.997	0.469	3.267	-0.009	-0.269	2.947	0.440	3.251
	$\hat{\epsilon}^R$	0.004	-0.032	0.399	0.475	3.304	-0.000	-0.032	0.408	0.412	3.253
CSN-2	$\hat{z}$	-0.025	-0.219	0.986	0.926	3.724	0.003	-0.101	0.850	0.605	3.423
	$\Delta\hat{q}$	0.002	-0.359	1.891	0.914	3.724	0.003	-0.340	1.870	0.879	3.689
	$\hat{y}^\star$	-0.002	-0.355	1.872	0.895	3.666	0.007	-0.210	2.503	0.420	3.262
	$\hat{\pi}^\star$	-0.014	-0.580	2.985	0.918	3.732	-0.007	-0.541	2.932	0.870	3.682
	$\hat{\epsilon}^R$	0.002	-0.074	0.399	0.905	3.685	0.000	-0.065	0.404	0.782	3.571
CSN-3	$\hat{z}$	-0.026	-0.027	0.996	-0.001	2.983	0.002	0.002	0.858	-0.008	2.985
	$\Delta\hat{q}$	-0.001	-0.001	1.899	-0.002	3.004	0.000	0.002	1.876	-0.003	3.003
	$\hat{y}^\star$	-0.006	-0.174	1.880	0.466	3.259	-0.005	-0.123	2.619	0.232	3.115
	$\hat{\pi}^\star$	0.007	0.018	2.997	-0.012	2.992	0.011	0.019	2.946	-0.009	2.995
	$\hat{\epsilon}^R$	0.004	0.004	0.399	-0.002	2.996	0.000	0.001	0.401	-0.005	3.001
Sample size = 150+6											
Normal	$\hat{z}$	-0.010	-0.012	0.999	-0.003	3.005	0.001	0.000	0.878	-0.001	2.994
	$\Delta\hat{q}$	0.004	0.004	1.898	0.007	3.000	0.005	0.008	1.880	0.006	3.000
	$\hat{y}^\star$	-0.009	-0.010	1.885	0.005	3.004	-0.014	-0.015	2.264	0.008	3.000
	$\hat{\pi}^\star$	-0.004	0.005	2.998	-0.005	2.991	-0.003	0.001	2.951	-0.007	2.992
	$\hat{\epsilon}^R$	0.002	0.002	0.399	-0.006	3.014	0.000	0.000	0.395	-0.002	3.011
CSN-1	$\hat{z}$	-0.011	-0.101	0.995	0.483	3.316	0.000	-0.052	0.871	0.330	3.197
	$\Delta\hat{q}$	0.003	-0.169	1.902	0.484	3.309	0.004	-0.166	1.886	0.470	3.297
	$\hat{y}^\star$	-0.001	-0.174	1.888	0.484	3.314	0.003	-0.099	2.349	0.255	3.145
	$\hat{\pi}^\star$	-0.007	-0.286	2.994	0.482	3.310	-0.005	-0.258	2.944	0.456	3.292
	$\hat{\epsilon}^R$	0.000	-0.036	0.399	0.484	3.321	0.000	-0.031	0.394	0.430	3.278
CSN-2	$\hat{z}$	-0.011	-0.205	0.992	0.929	3.737	0.001	-0.111	0.872	0.630	3.436
	$\Delta\hat{q}$	0.001	-0.366	1.895	0.928	3.743	0.001	-0.353	1.878	0.904	3.717
	$\hat{y}^\star$	-0.002	-0.364	1.884	0.925	3.734	0.000	-0.216	2.335	0.481	3.304
	$\hat{\pi}^\star$	0.003	-0.567	2.990	0.927	3.765	0.005	-0.528	2.942	0.881	3.717
	$\hat{\epsilon}^R$	0.001	-0.076	0.399	0.932	3.772	0.000	-0.067	0.394	0.827	3.665
CSN-3	$\hat{z}$	-0.010	-0.010	0.999	0.002	3.005	0.001	0.001	0.877	-0.005	3.001
	$\Delta\hat{q}$	0.005	0.011	1.899	-0.009	2.994	0.005	0.010	1.883	-0.009	2.999
	$\hat{y}^\star$	0.004	-0.168	1.884	0.485	3.322	0.001	-0.103	2.358	0.256	3.142
	$\hat{\pi}^\star$	-0.006	-0.006	2.998	0.003	2.993	-0.004	-0.008	2.952	0.002	3.001
	$\hat{\epsilon}^R$	0.002	0.002	0.400	0.008	3.003	-0.000	-0.001	0.398	0.007	3.005

Source: Prepared by the authors

Table 14. Simulated and estimated /smoothed/ shocks. Part Ib. Number of replications=2000

Simulation Variant	Shock Name	Simulated shocks /average in sample/					Estimated shocks /average in sample /				
		A. Mean	A. Median	A. St. Dev	A. Skewness	A. Kurtosis	A. Mean	A. Median	A. St. Dev	A. Skewness	A. Kurtosis
Sample size = 600+6											
Normal	$\hat{z}$	-0.001	-0.001	1.001	0.001	3.008	0.000	0.000	0.894	0.002	3.004
	$\Delta\hat{q}$	0.001	0.002	1.899	-0.001	3.001	0.001	0.001	1.887	-0.001	3.001
	$\hat{y}^\star$	0.001	-0.001	1.891	-0.000	3.002	-0.001	0.000	1.784	-0.001	2.998
	$\hat{\pi}^\star$	-0.003	-0.006	2.998	-0.001	2.990	-0.004	-0.006	2.953	-0.001	2.988
	$\hat{\epsilon}^R$	0.001	0.001	0.400	-0.001	2.992	0.000	0.000	0.389	-0.000	2.995
CSN-1	$\hat{z}$	0.000	-0.092	1.000	0.501	3.351	0.000	-0.058	0.893	0.359	3.230
	$\Delta\hat{q}$	0.002	-0.172	1.900	0.498	3.347	0.003	-0.166	1.888	0.488	3.337
	$\hat{y}^\star$	-0.001	-0.175	1.890	0.496	3.345	0.001	-0.088	1.814	0.280	3.166
	$\hat{\pi}^\star$	-0.003	-0.280	2.997	0.494	3.337	-0.003	-0.260	2.952	0.471	3.317
	$\hat{\epsilon}^R$	0.000	-0.037	0.400	0.496	3.338	0.000	-0.033	0.388	0.451	3.303
CSN-2	$\hat{z}$	-0.001	-0.196	0.999	0.950	3.824	-0.000	-0.119	0.893	0.679	3.533
	$\Delta\hat{q}$	0.001	-0.369	1.898	0.944	3.799	0.001	-0.359	1.886	0.926	3.779
	$\hat{y}^\star$	0.004	-0.364	1.891	0.950	3.830	0.005	-0.173	1.782	0.535	3.392
	$\hat{\pi}^\star$	-0.000	-0.584	2.999	0.942	3.786	-0.000	-0.548	2.953	0.901	3.741
	$\hat{\epsilon}^R$	-0.000	-0.078	0.400	0.943	3.787	-0.000	-0.068	0.390	0.853	3.685
CSN-3	$\hat{z}$	-0.001	-0.001	0.999	-0.001	3.007	0.000	0.002	0.891	-0.008	3.008
	$\Delta\hat{q}$	0.002	0.004	1.899	-0.001	2.994	0.002	0.004	1.886	-0.001	2.995
	$\hat{y}^\star$	-0.002	-0.177	1.889	0.494	3.337	-0.002	-0.090	1.766	0.281	3.154
	$\hat{\pi}^\star$	-0.001	-0.002	2.998	0.001	3.002	-0.001	-0.000	2.953	0.002	2.999
	$\hat{\epsilon}^R$	-0.000	-0.000	0.400	-0.001	3.002	0.000	0.000	0.388	-0.002	2.997

Source: Prepared by the authors

Table 15. Properties of skewness estimators. Part IIa. Number of cases=2000

Sim. Variant	Para-meter	Adjusted Estimator of Skewness Coefficient N(T) = 0.5									Adjusted Estimator of Skewness Coefficient N(T) = 1.0								
		Bias	Bias %	Mean	Median	St. Dev	Skewness	Kurtosis	5%	95%	Bias	Bias %	Mean	Median	St. Dev	Skewness	Kurtosis	5%	95%
Sample size = 75+6																			
Normal	$d_{\tilde{z}}$	-0.012	-	-0.012	-0.043	0.484	-0.018	1.898	-0.781	0.744	-0.011	-	-0.011	-0.024	0.375	-0.038	2.234	-0.631	0.579
	$d_{\Delta\tilde{q}}$	-0.003	-	-0.003	-0.006	0.318	-0.008	2.456	-0.530	0.512	-0.004	-	-0.004	-0.004	0.300	-0.014	2.531	-0.501	0.482
	$d_{\tilde{y}^\star}$	-0.010	-	-0.010	-0.068	0.604	0.009	1.577	-0.896	0.891	-0.008	-	-0.008	-0.029	0.438	0.017	1.987	-0.689	0.694
	$d_{\tilde{\pi}^\star}$	-0.014	-	-0.014	-0.017	0.320	-0.024	2.443	-0.541	0.511	-0.014	-	-0.014	-0.013	0.298	-0.026	2.518	-0.509	0.482
	$d_{\tilde{\epsilon}^R}$	-0.006	-	-0.006	-0.012	0.328	-0.011	2.480	-0.547	0.533	-0.006	-	-0.006	-0.008	0.300	-0.018	2.595	-0.500	0.486
CSN-1	$d_{\tilde{z}}$	-0.025	-4.977	0.475	0.569	0.402	-1.110	3.715	-0.379	0.949	-0.103	-20.623	0.397	0.450	0.335	-0.708	3.099	-0.251	0.862
	$d_{\Delta\tilde{q}}$	-0.002	-0.441	0.498	0.524	0.262	-0.531	2.956	0.014	0.878	-0.023	-4.516	0.478	0.497	0.254	-0.467	2.865	0.011	0.852
	$d_{\tilde{y}^\star}$	-0.073	-14.541	0.427	0.608	0.525	-1.047	2.930	-0.650	0.975	-0.168	-33.602	0.332	0.414	0.401	-0.718	2.798	-0.446	0.858
	$d_{\tilde{\pi}^\star}$	-0.016	-3.133	0.485	0.514	0.257	-0.496	2.869	0.022	0.861	-0.040	-8.066	0.460	0.489	0.248	-0.410	2.757	0.018	0.829
	$d_{\tilde{\epsilon}^R}$	-0.034	-6.767	0.466	0.501	0.284	-0.726	3.470	-0.084	0.867	-0.062	-12.432	0.437	0.468	0.271	-0.599	3.222	-0.068	0.830
CSN-2	$d_{\tilde{z}}$	-0.183	-19.264	0.767	0.837	0.240	-2.379	11.396	0.337	0.987	-0.275	-28.958	0.675	0.726	0.234	-1.454	6.206	0.237	0.950
	$d_{\Delta\tilde{q}}$	-0.165	-17.367	0.785	0.806	0.143	-0.917	3.812	0.513	0.974	-0.183	-19.305	0.767	0.788	0.145	-0.840	3.661	0.495	0.963
	$d_{\tilde{y}^\star}$	-0.252	-26.525	0.698	0.838	0.373	-2.123	7.521	-0.260	0.994	-0.381	-40.080	0.569	0.646	0.321	-1.287	4.697	-0.132	0.943
	$d_{\tilde{\pi}^\star}$	-0.161	-16.905	0.789	0.817	0.145	-0.954	4.013	0.519	0.972	-0.183	-19.283	0.767	0.793	0.148	-0.851	3.721	0.497	0.961
	$d_{\tilde{\epsilon}^R}$	-0.195	-20.547	0.755	0.789	0.169	-1.132	4.861	0.438	0.962	-0.226	-23.805	0.724	0.755	0.172	-0.979	4.269	0.404	0.946
CSN-3	$d_{\tilde{z}}$	-0.015	-	-0.015	-0.083	0.481	0.021	1.926	-0.787	0.746	-0.011	-	-0.011	-0.042	0.372	-0.002	2.248	-0.634	0.593
	$d_{\Delta\tilde{q}}$	-0.003	-	-0.003	-0.010	0.320	-0.040	2.598	-0.530	0.517	-0.003	-	-0.003	-0.006	0.302	-0.042	2.664	-0.503	0.491
	$d_{\tilde{y}^\star}$	-0.065	-12.925	0.435	0.627	0.530	-1.064	2.952	-0.655	0.977	-0.159	-31.859	0.341	0.430	0.402	-0.728	2.767	-0.438	0.865
	$d_{\tilde{\pi}^\star}$	-0.011	-	-0.011	-0.021	0.314	0.052	2.631	-0.521	0.515	-0.010	-	-0.010	-0.014	0.292	0.052	2.727	-0.485	0.484
	$d_{\tilde{\epsilon}^R}$	-0.005	-	-0.005	-0.017	0.338	-0.068	2.356	-0.559	0.528	-0.005	-	-0.005	-0.011	0.309	-0.071	2.462	-0.513	0.486
Sample size = 150+6																			
Normal	$d_{\tilde{z}}$	-0.001	-	-0.001	-0.046	0.402	0.010	1.987	-0.620	0.626	-0.001	-	-0.001	-0.026	0.297	0.002	2.337	-0.472	0.475
	$d_{\Delta\tilde{q}}$	0.007	-	0.007	0.014	0.227	0.024	2.876	-0.371	0.376	0.007	-	0.007	0.010	0.218	0.021	2.930	-0.357	0.362
	$d_{\tilde{y}^\star}$	0.022	-	0.022	0.145	0.541	-0.048	1.605	-0.791	0.805	0.015	-	0.015	0.067	0.364	-0.042	2.062	-0.555	0.574
	$d_{\tilde{\pi}^\star}$	-0.008	-	-0.008	-0.018	0.232	0.034	2.607	-0.376	0.365	-0.008	-	-0.008	-0.015	0.219	0.032	2.683	-0.364	0.349
	$d_{\tilde{\epsilon}^R}$	-0.005	-	-0.005	-0.026	0.258	0.105	2.592	-0.416	0.423	-0.004	-	-0.004	-0.019	0.235	0.114	2.725	-0.386	0.388
CSN-1	$d_{\tilde{z}}$	0.056	11.106	0.556	0.609	0.283	-1.293	5.113	-0.069	0.910	-0.052	-10.333	0.449	0.476	0.241	-0.728	3.651	-0.037	0.797
	$d_{\Delta\tilde{q}}$	0.000	0.025	0.501	0.514	0.186	-0.383	3.077	0.180	0.783	-0.013	-2.607	0.487	0.499	0.184	-0.367	3.027	0.173	0.767
	$d_{\tilde{y}^\star}$	0.053	10.523	0.553	0.663	0.394	-1.528	4.799	-0.405	0.955	-0.090	-17.942	0.410	0.453	0.300	-0.841	3.481	-0.228	0.815
	$d_{\tilde{\pi}^\star}$	-0.001	-0.248	0.499	0.507	0.191	-0.419	3.332	0.169	0.796	-0.020	-4.028	0.480	0.486	0.187	-0.358	3.217	0.155	0.774
	$d_{\tilde{\epsilon}^R}$	-0.001	-0.231	0.498	0.515	0.205	-0.527	3.486	0.155	0.806	-0.032	-6.405	0.468	0.481	0.199	-0.400	3.213	0.136	0.773
CSN-2	$d_{\tilde{z}}$	-0.125	-13.118	0.825	0.856	0.131	-1.224	4.828	0.578	0.978	-0.226	-23.741	0.724	0.746	0.146	-0.742	3.505	0.455	0.927
	$d_{\Delta\tilde{q}}$	-0.151	-15.853	0.799	0.812	0.103	-0.669	3.319	0.610	0.946	-0.162	-17.086	0.788	0.801	0.104	-0.638	3.286	0.597	0.938
	$d_{\tilde{y}^\star}$	-0.125	-13.156	0.825	0.890	0.201	-3.041	17.423	0.470	0.990	-0.278	-29.248	0.672	0.712	0.203	-1.306	5.869	0.303	0.928
	$d_{\tilde{\pi}^\star}$	-0.151	-15.877	0.799	0.809	0.104	-0.664	3.470	0.611	0.947	-0.168	-17.720	0.782	0.790	0.107	-0.602	3.390	0.589	0.938
	$d_{\tilde{\epsilon}^R}$	-0.157	-16.512	0.793	0.806	0.113	-0.833	4.117	0.585	0.948	-0.186	-19.617	0.764	0.778	0.119	-0.717	3.770	0.547	0.930
CSN-3	$d_{\tilde{z}}$	-0.015	-	-0.015	-0.075	0.400	0.054	2.028	-0.640	0.613	-0.010	-	-0.010	-0.036	0.295	0.053	2.420	-0.476	0.459
	$d_{\Delta\tilde{q}}$	-0.012	-	-0.012	-0.012	0.227	0.043	2.773	-0.383	0.361	-0.011	-	-0.011	-0.010	0.218	0.041	2.812	-0.366	0.348
	$d_{\tilde{y}^\star}$	0.063	12.639	0.563	0.664	0.388	-1.631	5.326	-0.374	0.956	-0.084	-16.716	0.416	0.458	0.297	-0.941	3.844	-0.205	0.819
	$d_{\tilde{\pi}^\star}$	0.002	-	0.002	-0.004	0.238	0.051	2.655	-0.380	0.393	0.002	-	0.002	-0.003	0.225	0.057	2.741	-0.359	0.371
	$d_{\tilde{\epsilon}^R}$	0.009	-	0.009	0.013	0.258	-0.032	2.580	-0.421	0.422	0.009	-	0.009	0.008	0.235	-0.032	2.727	-0.383	0.387

Source: Prepared by the authors

Table 16. Properties of skewness estimators. Part IIb. Number of cases=2000

Sim. Variant	Para-meter	Adjusted Estimator of Skewness Coefficient N(T) = 0.5									Adjusted Estimator of Skewness Coefficient N(T) = 1.0								
		Bias	Bias %	Mean	Median	St. Dev	Skewness	Kurtosis	5%	95%	Bias	Bias %	Mean	Median	St. Dev	Skewness	Kurtosis	5%	95%
Sample size = 600+6																			
Normal	$d_{\tilde{z}}$	0.003	-	0.003	0.048	0.261	0.009	1.948	-0.390	0.403	0.003	-	0.003	0.019	0.175	0.037	2.325	-0.271	0.283
	$d_{\Delta\tilde{q}}$	-0.001	-	-0.001	-0.003	0.118	-0.021	2.830	-0.194	0.194	-0.001	-	-0.001	-0.002	0.114	-0.024	2.880	-0.190	0.188
	$d_{\tilde{y}^\star}$	-0.000	-	-0.000	0.125	0.393	-0.023	1.530	-0.557	0.559	-0.001	-	-0.001	0.048	0.226	-0.023	1.927	-0.342	0.341
	$d_{\tilde{\pi}^\star}$	-0.002	-	-0.002	-0.009	0.125	0.086	2.640	-0.203	0.201	-0.002	-	-0.002	-0.007	0.117	0.091	2.742	-0.192	0.189
	$d_{\tilde{\epsilon}^R}$	-0.001	-	-0.001	-0.012	0.145	0.017	2.702	-0.229	0.228	-0.001	-	-0.001	-0.006	0.127	0.019	2.935	-0.205	0.203
CSN-1	$d_{\tilde{z}}$	0.134	26.784	0.634	0.648	0.127	-0.535	3.258	0.401	0.817	0.005	1.045	0.506	0.514	0.119	-0.340	3.114	0.296	0.689
	$d_{\Delta\tilde{q}}$	0.017	3.411	0.518	0.517	0.098	-0.208	3.229	0.352	0.671	0.009	1.888	0.510	0.511	0.097	-0.200	3.231	0.345	0.662
	$d_{\tilde{y}^\star}$	0.201	40.193	0.701	0.726	0.155	-1.820	10.893	0.427	0.895	-0.005	-0.952	0.495	0.506	0.140	-0.700	4.635	0.256	0.708
	$d_{\tilde{\pi}^\star}$	0.023	4.562	0.523	0.522	0.098	-0.052	2.974	0.365	0.686	0.005	1.011	0.505	0.503	0.097	-0.026	2.958	0.349	0.666
	$d_{\tilde{\epsilon}^R}$	0.041	8.244	0.541	0.543	0.103	-0.257	3.216	0.359	0.702	0.005	1.049	0.505	0.506	0.102	-0.197	3.185	0.326	0.665
CSN-2	$d_{\tilde{z}}$	-0.072	-7.601	0.878	0.888	0.058	-0.911	4.077	0.770	0.955	-0.172	-18.063	0.778	0.783	0.069	-0.548	3.599	0.654	0.879
	$d_{\Delta\tilde{q}}$	-0.136	-14.359	0.814	0.815	0.052	-0.322	3.231	0.727	0.896	-0.143	-15.035	0.807	0.809	0.053	-0.308	3.246	0.720	0.890
	$d_{\tilde{y}^\star}$	-0.040	-4.204	0.910	0.926	0.062	-1.516	5.676	0.784	0.979	-0.196	-20.594	0.754	0.763	0.090	-0.654	3.421	0.586	0.887
	$d_{\tilde{\pi}^\star}$	-0.132	-13.845	0.818	0.824	0.053	-0.417	3.226	0.724	0.899	-0.146	-15.405	0.804	0.809	0.055	-0.374	3.161	0.708	0.886
	$d_{\tilde{\epsilon}^R}$	-0.124	-13.064	0.826	0.832	0.053	-0.540	3.293	0.732	0.904	-0.154	-16.245	0.796	0.801	0.057	-0.473	3.238	0.695	0.881
CSN-3	$d_{\tilde{z}}$	-0.022	-	-0.022	-0.090	0.262	0.092	1.894	-0.404	0.383	-0.015	-	-0.015	-0.042	0.175	0.065	2.217	-0.284	0.265
	$d_{\Delta\tilde{q}}$	-0.001	-	-0.001	-0.002	0.116	0.042	2.909	-0.188	0.187	-0.001	-	-0.001	-0.002	0.112	0.043	2.955	-0.181	0.183
	$d_{\tilde{y}^\star}$	0.206	41.125	0.706	0.729	0.144	-1.387	7.675	0.433	0.888	-0.002	-0.319	0.498	0.510	0.134	-0.528	3.739	0.260	0.696
	$d_{\tilde{\pi}^\star}$	0.003	-	0.003	0.003	0.127	-0.038	2.637	-0.210	0.207	0.002	-	0.002	0.001	0.119	-0.038	2.744	-0.197	0.195
	$d_{\tilde{\epsilon}^R}$	-0.002	-	-0.002	0.008	0.143	0.002	2.590	-0.233	0.228	-0.002	-	-0.002	0.004	0.126	0.002	2.794	-0.208	0.204

Source: Prepared by the authors

References

AMISANO, G. AND TRISTANI, O. 2007. Euro area inflation persistence in an estimated nonlinear DSGE model. Working Paper Series 754, European Central Bank.

AN, S. 2005. Bayesian estimation of DSGE model: Lessons from second-order aproximations. Univeristy of Pennsylvania, Manuscript, mimeo.

AZZALINI, A. 2004. The skew-normal distribution and related skew normal distribution multivariate families. Paper, presented at the 20th Nordic Conference on Mathematical Statistics, Jyvaskyla, (NORDSTAT 2004).

AZZALINI, A. AND CAPITANIO, A. 1999. Applications of the multivariate skew normal distribution. *Journal of the Royal Statistical Society. Series B (Statistical Methodology)* 61:579–602.

AZZALINI, A. AND GENTON, M. G. 2008. Robust likelihood methods based on the skew-t and related distributions. *International Statistical Review* 76:106–129.

AZZALINI, A., GENTON, M. G., AND SCARPA, B. 2010. Invariance-based estimating equations for skew-symmetric distributions. Manuscript, mimeo.

AZZALINI, A. AND VALLE, A. D. 1996. The multivariate skew-normal distribution. *Biometrika* 83:715–726.

BAI, J. AND NG, S. 2005. Test for skewness, kurtosis, and normality for time series data. *Journal of Business and Economic Statstics* 23:43–60.

BALL, L. AND MANKIW, N. G. 1995. Relative-price changes as aggregate supply shocks. *The Quarterly Journal of Economics* 110:161–93.

CHRISTODOULAKIS, G. AND PEEL, D. 2009. The central bank inflation bias and the presence of asymmetric preferences and non-normal shocks. *Economic Bulletion* 29:1608–1620.

COCHRANE, J. H. 2007. Determinacy and identification with taylor rules. Working Paper 13409, NBER.

FAGIOLO, G., NAPOLETANO, M., AND ROVENTINI, A. 2008. Are output growth-rate distributions fat-tailed? Some evidence from OECD countries. *Journal of Applied Econometrics* 23:639–669.

FAHR, S. AND SMETS, F. 2008. Downward wage rigidities and optimal monetary policy in a monetary union. Manuscript.

FERNÁNDEZ-VILLAVERDE, J. 2009. The econometrics of DSGE models. Working Paper 14677, NBER.

FERNÁNDEZ-VILLAVERDE, J. AND RUBIO-RAMIREZ, J. F. 2007. Estimationg macroeconomic models: A likelihood approach. *Review of Economic Studies* pp. 1059–1087.

FLETCHER, C., NOVEAU, P., AND ALLARD, D. 2008. Estimating the closed skew-normal distributions parameters using weighted moments. Research report 40, Institue National de la Recherche Agronomique.

GALI, J. AND MONACELLI, T. 2005. Monetary policy and exchange rate volatility in small open economy. *Review of Economic Studies* 72.

GOURIEROUX, C., MONFORT, A., AND TROGNON, A. 1984. Pseudo maximim likelihood method: Theory. *Econometrica* 52:681–700.

GUPTA, A. K., GONZÁLEZ-FARÍAS, G., AND DOMÍNGUEZ-MOLINA, J. A. 2004. A multivariate skew normal distribution. *Journal of Multivariate Analysis* 89:181–190.

HAMILTON, J. D. 1994. Time Series Analysis. Princeton University Press.

KIM, J. AND RUGE-MURCIA, F. J. 2009. How much inflation is necessary to grease the wheels? *Journal of Monetary Economics* 56:365–377.

LUBIK, T. A. AND SCHORFHEIDE, F. 2007. Do central banks respond to exchange rate movements? A structural investigation. *Journal of Monetary Economics* 54:1069–1087.

MA, Y., GENTON, M., AND TSIATSIS, A. A. 2005. Locally efficient semiparametric estimators for generalized skew-elliptical distributions. *Journal of the American Statistical Association* 100:980–989.

MACGILLIVRAY, H. L. 1986. Skewness and asymmetry: measures and orderings. *The Annals of Statistics* 14:994–1011.

MEINHOLD, R. J. AND SINGPURWALLA, N. D. 1983. Understanding the Kalman filter. *The American Statistician* 37:123–127.

MEINHOLD, R. J. AND SINGPURWALLA, N. D. 1989. Robustification of Kalman filter models. *Journal of the American Statistical Association* 84:479–486.

NADLER, J. A. AND LEE, Y. 1992. Likelihood, quasi-likelihood and pseudolikelihood: Some comparisons. *Journal of Royal Statistical Society. Series B (Methodological)* 54:273–284.

NEGRO, M. D. AND SCHORFHEIDE, F. 2008. Inflation dynamics in a small open economy model under inflation targeting: Some evidence from Chile. Working Papers Central Bank of Chile 486, Central Bank of Chile.

NEWEY, W. K. 1990. Semmiparametric efficiency bounds. *Journal of Applied Econometrics* 5:99–135.

NEWEY, W. K. AND WEST, K. 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55:703–708.

WARNE, A. 2010. YADA manual — Computational details. EBC, Manuscript. URL http://www.texlips.net/yada/index.html.

WEDDERBURN, R. 1974. Quasi-likelihood functions, generalized linear models, and the Gauss-Newton method. *Biometrica* 61:439–447.

WHITE, H. 1982. Maximum likelihood estimation of misspecified models. *Econometrica* 50:1–25.