Borrower heterogeneity within a risky mortgage-lending market *

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May 20, 2017

Abstract

We propose a model of a risky mortgage-lending market in which we take explicit account of heterogeneity in household borrowing conditions, by introducing two borrower types: one with a low loan-to-value (LTV) ratio, one with a high LTV ratio, calibrated to U.S. data. We use such framework to study a deleveraging shock, modeled as an increase in housing investment risk, that falls more strongly on, and produces a larger contraction in credit for high-LTV type borrowers, as in the data. We find that this deleveraging experience produces significant *aggregate* effects on output and consumption, and that the contractionary effects are orders of magnitudes higher in a model version that takes account of borrower heterogeneity, compared to a more standard model version with a representative borrower.

Keywords: Borrowing Constraints, Loan-to-Value ratio, Heterogeneity, Financial Amplification
JEL-Codes: E23, E32, E44

*The work on this paper is part of FinMaP ('Financial Distortions and Macroeconomic Performance', contract no. SSH.2013.1.3-2), funded by the EU Commission under its 7th Framework Programme for Research and Technological Development. We thank Pierre Monnin, Davide Furceri, Gregory Thwaites, our discussant Gee Hee Hong and participants at the CEP-IMF Workshop on 'Monetary Policy, Macroprudential Regulation and Inequality' for useful comments.

1 Introduction

Empirically, the distribution of loan-to-value (LTV) ratios of household borrowers in the U.S. economy, at the onset of the financial crisis, documents stark differences in household leverage, and features, among it, a small fraction of highly indebted households. We present a macroeconomic model of the household mortgage market in which we take account of this fact by extending an otherwise standard model with a low-LTV type and a high-LTV type borrower group. We show that a deleveraging process, modeled as a shock to the riskiness of housing investment of borrowers, that falls mostly on the high LTV group of borrowers produces a sizeable macroeconomic contraction, orders of magnitude larger than when the model features a representative borrowing agent. We argue that adding features of heterogeneity into core macroeconomic modeling frameworks may thus be of paramount importance, and may help to understand why a small part of the economy can have large effects on the aggregate macroeconomy, such as was the case for the subprime mortgage market at the beginning of the financial crisis.

The key contributions of our paper are thus twofold. One, we document that a contraction of household credit, brought about by an increase in household borrower riskiness, produces negative effects on *aggregate* economic variables, such as GDP and aggregate consumption. This is important against the background that most models in the literature attribute little effect of household credit on aggregate macroeconomic variables. For example, Justiniano, Primiceri, and Tambalotti (2015) show that leveraging (or deleveraging) cycles can have only a moderate impact on macroeconomic aggregates because the responses of Borrowers and Savers cancel out in the aggregate, i.e. when negative shocks hit the credit cycle, Borrowers work more and cut their consumption in both goods and housing, while Savers behave in the opposite way. This phenomena of 'washing out' is typical in this class of models. See, for example, Iacoviello and Neri (2010) and Kiyotaki, Michaelides, and Nikolov (2011). Two, the negative effects are amplified when we take explicit account of borrower heterogeneity and when the contraction of overall credit falls on highly indebted households. This regards the effect on aggregate GDP and consumption; but also the outstanding overall debt level and the house price, which, in conventional models often displays little variation in this model class. Empirically, house prices and the home mortgage loans to GDP ratio in the US have experienced large swings over the leveraging and deleveraging cycle, for the period of 1975-2012, as reported in Figure 1. While the increase in both variables was moderate in the first part of the period, a huge run-up is evident since the 2000s until the peak of the financial crisis.[1] We argue that a model with explicit heterogeneity of

[1]A large portion of debt outstanding comprised of securitized mortgages and debt held by Government-Sponsored Enterprises (GSEs). By the end of 2009, GSEs accounted for about 54% of all mortgage originations, while commercial banks, federal and related agencies and life insurance companies reached around 31%, 6% and 2%, respectively. After 2009, GSEs completely collapsed, and federal agencies have been the major source of mortgage financing. See Figure 7 in Appendix.

borrowers' LTV ratios, that produces quantitatively more pronounced swings, constitutes a mechanism by which the leveraging (or deleveraging) cycle may contribute more to the business and financial cycle.

[Fig. 1 about here.]

We develop our results in a state-of-the-art dynamic stochastic general equilibrium (DSGE) model, that comes in a baseline, and in an extended model version. Both versions feature a household sector that consists of Savers and two types of Borrowers, described in more detail below. In the baseline model, borrowing and lending directly takes place between these agents. In the extended model a role for a financial intermediary, a banking sector, is included, which Savers use for their deposits and from which Borrowers obtain loans. The rest of the model is standard; the production side features a competitive final good sector, as well as an intermediate goods sector that is subject to nominal rigidities; a monetary policy authority follows a Taylor rule.

The household sector requires a more detailed description. All household types consume goods and housing services, the Saver (patient) lends to Borrowers (impatient). Borrowers, who use their housing as collateral in a mortgage contract, come in two types: a low-LTV type Borrower and high-LTV type; the different LTV ratios arise, endogenously, from differences in the idiosyncratic housing investment risk of each borrower group, following the literature on risky mortgages, e.g., Forlati and Lambertini (2011).[2,3] We calibrate the LTV ratios of the model from the empirical LTV distribution from the Fannie Mae and Freddie Mac database, which covers around 12 million in home purchases of single-family loans issued in the US, and we simulate a drop in the LTV ratios occurred between the pre-crisis and post-crisis period. Figure 2 reports LTV distributions for the period of 2000-2006 (solid line) and 2009 (dashed line). The Figure reveals deep heterogeneity in the distribution, and a small portion of households that holds mortgages with an LTV ratio almost equal to 100% of the value of the house. Moreover, the distribution has changed since the financial crisis, accounting for lower LTV ratios. We calibrate the model to the period of 2000-2006. The low-LTV type borrower is calibrated to the lower 74-th percentile of the sample distribution, containing all LTV ratios lower than 80%, which has an average LTV ratio equal to 67%. The high-LTV type borrower is calibrated to the upper 26-th percentile of the sample, containing all LTV ratios between 80% and 100%, which displays a mean LTV ratio equal to 91%. We contrast this 'heterogeneous borrowers' model to an more conventional

[2] Quint and Rabanal (2014), Lambertini, Nuguer, and Uysal (2015) and Ferrante (2015) employ similar setups.

[3] To be precise, the low-LTV and the high-LTV type Borrower are not single (representative) agents, but are borrower *groups*, that each consists of many members. The members of a borrowing group face idiosyncratic housing investment risk, that is key for the modeling of the risky mortgage contract. However, since there is perfect risk sharing among all members of a borrower group, and they thus have the same consumption and housing demand decisions, we use the terms 'low(high)-LTV type borrower' and 'low(high)-LTV type borrower *group*' interchangeably.

case of a representative borrowing agent[4], in which case the two borrower groups' LTV ratios are identical and calibrated to the overall mean of the LTV distribution, equal to 73%. The latter is called the 'homogenous borrowers' version.

[Fig. 2 about here.]

We then use the model to conduct the following experiment. We study the deleveraging effects of an unanticipated increase in the volatility of idiosyncratic housing investment risk that mimics the drop in LTV ratios observed at the onset of the crisis. The LTV distribution on loan-level data collected from the Fannie Mae and Freddie Mac database, shows that after the financial crisis, outstanding loans have been issued at a lower LTV ratio, and the deepest drop occurred in 2009.[5] See Figure 2 (dashed line). In the 'homogeneous borrowers' version, this produces a drop of the (economy-wide) LTV ratio from 73% to 69%, and is similar in spirit to the exercise in Forlati and Lambertini (2011). In our 'heterogeneous borrowers' version, instead, the fall in the economy-wide average LTV ratio is the same, but the re-evaluation of the riskiness, and thus the bulk of the contraction in credit, falls more strongly on high-LTV borrowers, whose LTV drops from 91% to 85%, as in the data. On the other hand, the LTV of low type borrowers drops only from 67% to 64%. A re-evaluation of the riskiness of high-LTV type households leads to a wave of defaults when house prices drop and this group find themselves underwater, i.e. the mortgage repayment is higher than the current value of the house which has been used to pledge against borrowing. Despite featuring the same drop in the economy-wide average LTV ratio, the 'heterogeneous borrowers' version of the model shows a substantially amplified drop in aggregate consumption and output, leading to a deep recession. It also produces more pronounced swings in asset prices and sharp reactions in the total debt level. In the extended model, the presence of a banking sector and a role for financial intermediation contributes to additional amplification in both 'homogenous borrowers' and 'heterogeneous borrowers' economy.

This paper is related to different strands of the literature. First, the paper adds to the literature on macro-financial linkages where financial frictions are incorporated in the New Keynesian DSGE models. A large body of literature, in most instances building on the seminal contributions of Kiyotaki, Moore, et al. (1997) and Bernanke, Gertler, and Gilchrist (1999), has studied the amplification mechanism of shocks through credit market imperfections, on real variables, credit variables and asset prices. In order to consider such amplification mechanism, most of the literature has introduced a representative Saver and representative Borrower in a standard DSGE model. (See Iacoviello (2005), Iacoviello and Neri (2010), Mendicino and Punzi (2014), Campbell and Hercowitz (2009), Gerali, Neri, Sessa, and Signoretti (2010), Iacoviello (2015), Justiniano, Primiceri, and Tambalotti (2014) and Justiniano, Primiceri, and Tambalotti (2015) for

[4]Again, there is not strictly speaking a 'representative borrowing agent'. Instead 'the borrower' is composed of many members i that face idiosyncratic housing investment risk, but there is perfect risk sharing among all i members.

[5]Similarly, Bokhari, Torous, and Wheaton (2013) provide evidence that before the financial crisis about 25% of Borrowers held mortgage loans with a $80\% < LTV \leq 100\%$

a non-exhaustive list of the literature on household borrowing and housing). In contrast to this literature, we depart from the representative borrower assumption. To our knowledge, only Punzi and Rabitsch (2015) have so far introduced borrower heterogeneity. Namely, in that paper, we introduce heterogeneity in investors' ability to borrow from collateral in a Kiyotaki-Moore style macro model, calibrated to the quintiles of the leverage-ratio distribution of US non-financial firms. There, we find that financial amplification intensifies, because of stronger asset price reactions among highly levered investors. This paper is closely related, but with a focus on the mortgage and housing market. Moreover, the mechanism is fundamentally different. In Punzi and Rabitsch (2015), an additional amplification on output arises from the fact that loans affect the productive capacity of the borrowing agents; this is not typically the case for household debt, and, in fact, there is little to no effect on aggregate real variables in response to sources of shocks other than the deleveraging/ riskiness shock.[6]

Second, the paper contributes to the empirical literature on wealth heterogeneity by developing a dynamic model with the aim to replicate household default and prolonged recessions. At the empirical level, Mian, Rao, and Sufi (2013) documents the importance of heterogeneity in wealth, debt and liquidity assets across U.S. households, showing that leveraged households do not have the same marginal propensity to consume. Similarly, Kaplan, Violante, and Weidner (2014) show that households with little or no liquid wealth have a higher marginal propensity to consume out of their income. Therefore, heterogeneity matters and it should be taken into account in macroeconomic models.

Third, the paper contributes to the growing literature on risk shocks and endogenous LTV ratios. Christiano, Motto, and Rostagno (2014) allow for time-varying volatility of cross-sectional idiosyncratic uncertainty in a model with a financial accelerator mechanism *à* la Bernanke-Gertler-Gilchrist (BGG). They find that risk shocks are crucial in understanding the drivers of business cycle fluctuations. As regards mortgage defaults, this paper is closely related to Forlati and Lambertini (2011), Quint and Rabanal (2014), Lambertini, Nuguer, and Uysal (2015) and Ferrante (2015). These papers all share the principal idea that idiosyncratic housing risk shocks generate an endogenous default decision which lead to underwater mortgages and house price collapse, triggering a credit crunch and deep recession.[7] We contribute to this strand of the literature

[6]In particular, in response to technology, housing preference or monetary policy shocks, the results are similar to Justiniano, Primiceri, and Tambalotti (2015), in that the responses of Borrowers and Savers (nearly) net out in aggregate. This regards at least the effects on aggregate output and consumption. The responses of debt levels become significantly amplified in the 'heterogeneous borrower' version as well, even in response to these standard shocks. This may be a notable result as well, especially for (macroprudential) regulators interested in keeping debt levels contained. Nevertheless, we see the key result of the present paper of heterogeneity in LTV ratios as lying in the additional amplification on real aggregate consumption and output, as we obtain in response to the deleveraging of mortgage risk shocks.

[7]Alternative mechanisms can be found in Lambertini, Mendicino, and Punzi (2013), who introduce news shocks to generate an excess credit boom, and Burnside, Eichenbaum, and Rebelo (2011) who introduce heterogeneous expectations to generate boom and bust in the housing market.

by allowing for Borrowers' heterogeneity and assuming that the credit contraction and default on outstanding mortgage loans falls primarily on high-LTV type Borrowers.

The rest of the paper is organized as follows. Section two presents the baseline theoretical model. Section three presents the extended model with a banking sector. Section four discusses the calibration of parameters. Section five discusses in detail the model experiment of a deleveraging experience, initiated through an unanticipated increase of housing investment riskiness and presents results of the corresponding impulse responses. We also show results in response to other, more standard shocks. Section six concludes.

2 Baseline Model

The baseline economy features (i) a household sector, consisting of a Saver and two types of Borrowers groups, (ii) a production sector that consists of a competitive final good sector and an intermediate goods sector that faces monopolistic competition and nominal rigidities, and (iii) a monetary policy authority.

2.1 Households

The economy is populated by two types of households that work, consume and buy real estate, and decide on their asset position: *patient* (denoted by s) and *impatient* (denoted by bH and bL for the high-LTV and the low-LTV type, respectively). Patient households have a higher propensity to save, therefore they have higher discount factor, i.e. $\beta_s > \beta_b$. Thus, in equilibrium patient agents save while impatient agents borrow. Housing is treated as a durable good with its demand depending on both the service flow and asset value of housing units. The model allows for constrained agents who collateralize the value of their homes.

Households supply labor, $n_{j,t}$, and derive utility from consuming goods, $c_{j,t}$, and housing services, $h_{j,t+1}$, as following:

$$\max E_0 \sum_{t=0}^{\infty} (\beta_j)^t \left[\frac{c_{j,t}^{1-\sigma_c}}{1-\sigma_c} + \varepsilon_t^h \varkappa \frac{h_{j,t+1}^{1-\sigma_H}}{1-\sigma_H} - \frac{\upsilon_j^n}{\eta} (n_{j,t})^{\eta} \right], \tag{2.1}$$

where $j = \{s, bH, bL\}$ denotes the different types of households. As common in the literature, housing services are assumed to be proportional to the stock of houses held by the household. $\varkappa$ is the weight of housing preference in the utility function and ε_t^h is a shock to the preference for housing services, υ_j^n is a weighting parameter on the disutility from labor. Households have total mass equal to one, out of which patient households represent a fraction α_s, impatient households a fraction α_b, where $\alpha_s + \alpha_b = 1$. We further denote with α_{bH} and α_{bL} the fraction (out of total borrowers) of high-LTV and a low-LTV type borrowers, respectively, where, again $\alpha_{bH} + \alpha_{bL} = 1$.

2.1.1 Patient households (Savers)

Patient households are indexed by s, and present mass α_s of households. They accumulate properties for housing purposes, $h_{s,t+1}$ and asset holdings, $d_{s,t+1}$. They also receive (real) dividends from firms, $\Delta_{s,t}$. Thus, they maximize their expected utility subject to the following budget constraint,

$$c_{s,t} + q_{h,t}(h_{s,t+1} - (1-\delta_h)h_{s,t}) + d_{s,t+1} = w_{s,t}n_{s,t} + \tfrac{R_{t-1}}{\pi_t}d_{s,t} + \Delta_{s,t}, \tag{2.2}$$

where $q_{h,t}$ is the real price of housing, $w_{s,t}$ are real wages. Real variables are expressed in units of the final good price.[8] R_{t-1} is the nominal gross interest rate on assets holdings (deposits) between $t-1$ and t. The stock of houses depreciates at rate δ_h.

Savers maximize 2.1 subject to 2.2 with respect to $c_{s,t}$, $n_{s,t}$, $h_{s,t+1}$, and $d_{s,t+1}$. The first order conditions are, respectively:

$$\begin{aligned}
\lambda_{s,t}^{BC} &= (c_{s,t})^{-\sigma_c}, & (2.3)\\
v_s^n (n_{s,t})^{\eta-1} &= w_{s,t}\lambda_{s,t}^{BC}, & (2.4)\\
q_{h,t}(c_{s,t})^{-\sigma_c} &= \beta_s E_t \left\{ (c_{s,t+1})^{-\sigma_c} [q_{h,t+1}] (1-\delta_h) \right\} + \varepsilon_t^h \varkappa h_{s,t+1}^{-\sigma_H}, & (2.5)\\
(c_{s,t})^{-\sigma_c} &= \beta_s E_t \left\{ (c_{s,t+1})^{-\sigma_c} \frac{R_t}{\pi_{t+1}} \right\}. & (2.6)
\end{aligned}$$

2.1.2 Impatient households (Borrowers)

Impatient households are indexed by bj and come in $j = H, L$ types, a high-LTV and a low-LTV borrower group; each type presents a mass $(1-\alpha_s)\alpha_{bj}$ of households. Borrower type j accumulates properties for housing purposes, $H_{bj,t}$, and receives from lenders (Savers in the baseline model) a one-period defaultable loan, $L_{bj,t}$, collateralized by the value of the house they purchase. The mortgage contract follows closely Forlati and Lambertini (2011), who introduce idiosyncratic risk and the possibility of default – in a setup similar to Bernanke, Gertler, and Gilchrist (1999) – to housing investment. Borrower group j's budget constraint, expressed in nominal terms, is given by:

$$\begin{aligned}
&P_t c_{bj,t} + Q_{h,t}(h_{bj,t+1} - (1-\delta_h)h_{bj,t}) + [1 - F_{bj,t}(\bar{\omega}_{bj,t})]R_{Zj,t}L_{bj,t} \\
&\quad = W_{bj,t}n_{bj,t} + L_{bj,t+1} - Q_{h,t}(1-\delta_h)G_{bj,t}(\bar{\omega}_{bj,t})h_{bj,t}),
\end{aligned} \tag{2.7}$$

where P_t is the final (consumption) good price, $Q_{h,t}$ the nominal house price $W_{bj,t}n_{bj,t}$ is the nominal labor income of borrower group j, and $L_{bj,t+1}$ are (nominal) loans taken out from the lender (Saver) at t to be repaid in period $t+1$. $R_{Zj,t}$ is the gross contractual

[8] We denote nominal variables with upper case letters, and real variables with lower case letters, deflated by the final consumption good price. E.g. $Q_{h,t}$ ($q_{h,t}$), $W_{s,t}$ ($w_{s,t}$), $D_{s,t}$ ($d_{s,t}$), $H_{s,t}$ ($h_{s,t}$), are the nominal (real) house price, wage rate, asset holdings or housing stock, respectively.

state-contingent loan rate paid to the lender by non-defaulting borrowers of borrower group j. It is determined at time t after the realization of shocks and in order to satisfy the participation constraint of lenders, explained below. Not all borrowers repay the contracted loans; fraction $G_{bj,t}(\bar{\omega}_{bj,t})$ of borrower group j's housing stock is seized by the lender in case of default. $[1 - F_{bj,t}(\bar{\omega}_{bj,t})]$ indicates the fraction of loans that the lender is repaid. As in Bernanke, Gertler, and Gilchrist (1999) and Forlati and Lambertini (2011), the seized housing stock is destroyed during the foreclosure process.

Each borrower group j consists of many members, indexed by i. Borrower group j decides on its total housing investment, $h_{bj,t+1}$, and the state-contingent mortgage rates to be paid next period on the contracts signed this period. Borrower group j assigns equal resources to each of its i members to purchase the housing stock $h^i_{bj,t+1}$, where $\int_i h^i_{bj,t+1} di = h_{bj,t+1}$. All i members of borrower group j are identical ex ante. After finalizing the mortgage contract, the i-th member, having in hand housing stock $h^i_{bj,t+1}$, experiences an idiosyncratic shock $\omega^i_{bj,t+1}$ such that her ex post housing value is $\omega^i_{bj,t+1} Q_{h,t+1} h^i_{bj,t+1}$. This captures the idea that housing investment is risky. The random variable $\omega^i_{bj,t+1}$is an i.i.d. idiosyncratic shock which is log-normally distributed with cumulative distribution $F_{bj,t}(\omega^i_{bj,t+1})$. We allow for idiosyncratic risk but no aggregate risk in the housing market, therefore $E_t(\omega^i_{bj,t+1}) = 1$. This implies that $\log(\omega^i_{bj,t}) \sim N(-\frac{\sigma^2_{\omega_{bj},t}}{2}, \sigma^2_{\omega_{bj},t})$, where $\sigma_{\omega_{bj},t}$ is a time-varying standard deviation for each type of borrower group, which follows an AR(1) process.

After realization of the idiosyncratic shock, member i of borrower group j decides whether to repay the mortgage or to default. Define the threshold value $\bar{\omega}_{bj,t}$ as the value of the idiosyncratic shock for which repayment of the loan at rate $R_{Zj,t}$ is incentive compatible with the member-i-borrower

$$\bar{\omega}_{bj,t+1}(1 - \delta_h) Q_{h,t+1} h_{bj,t+1} = L_{bj,t+1} R_{Zj,t+1}. \tag{2.8}$$

Loans with high realizations of the idiosyncratic random variable, $\omega^i_{bj,t+1} \in [\bar{\omega}_{bj,t+1}, \infty]$, are repaid, while loans with low realizations, $\omega^i_{bj,t+1} \in [0, \bar{\omega}_{bj,t+1}]$, are defaulted on. In case of default, the defaulting members loose their housing stock[9], which goes to lenders. However, lenders need to costly verify the default state by paying a monitoring cost to assess and seize the collateral connected to the defaulted loan, which is assumed to be a fraction μ_{bj} of the housing value, $\mu_{bj} \bar{\omega}_{bj,t+1} Q_{h,t+1} h_{bj,t+1}$.

We follow Bernanke, Gertler, and Gilchrist (1999), Forlati and Lambertini (2011), Quint and Rabanal (2014) and Lambertini, Nuguer, and Uysal (2015) in defining a one-period mortgage contract which guarantees lenders, assumed to be risk neutral, a predetermined rate of return on their total loans to borrower group j. At time t, the lender makes total loans $L_{bj,t+1}$ to borrower group j, and demands the gross rate of return R_t. Therefore the expected return from granted mortgages should guarantee

[9] We follow Forlati and Lambertini (2011) in assuming that, despite the i-th borrower's loss of her housing stocks in case of default, there is perfect consumption insurance among all members of each borrower group, so that consumption and housing investment of each group are ex post equal across members of the group.

lenders a certain funding rate equal at least to rate, R_t. This leads to the following participation constraint:

$$R_t L_{bj,t+1} = \left\{ (1-\mu) \int_0^{\bar{\omega}_{bj,t+1}} \omega^i_{bj,t+1} (1-\delta_h) Q_{h,t+1} h_{bj,t+1} f_{t+1}(\omega^i_{bj}) d\omega^i_{bj} \right\} + \left\{ \int_{\bar{\omega}_{bj,t+1}}^{\infty} R_{Zj,t+1} f_{t+1}(\omega^i_{bj}) d\omega^i_{bj} \right\}, \tag{2.9}$$

where $f(\omega^i_{bj})$ is the probability density function of ω^i_{bj}. Equation 2.9 states that the return on total loans the lender expects to obtain comes from two components: one, the housing stock, net of monitoring costs and depreciation, of the defaulting borrowers, i.e. of all i members with low realizations of the idiosyncratic shock (equal to the first term on the right hand side); and, two, the repayment by the non-defaulting borrowers, i.e. from all i members with high realizations of the idiosyncratic shock (equal to the second term on the right hand side). Once the idiosyncratic and aggregate shocks hit the economy, the threshold values $\bar{\omega}_{bj,t+1}$ and the state-contingent mortgage rate $R_{Zj,t}$ are determined, to fulfill the above participation constraint. Denote with $G_{t+1}(\bar{\omega}_{bj,t+1}) \equiv \int_0^{\bar{\omega}_{bj,t+1}} \omega^i_{bj,t+1} f_{t+1}(\omega^i_{bj}) d\omega^i_{bj}$ the expected value of the idiosyncratic shock for the case $\omega^i_{bj,t+1} \in [0, \bar{\omega}_{bj,t+1}]$ multiplied by the probability of default, while $\Gamma_{t+1}(\bar{\omega}_{bj,t+1}) \equiv \bar{\omega}_{bj,t+1} \int_{\bar{\omega}_{bj,t+1}}^{\infty} f_{t+1}(\omega^i_{bj}) \omega^i_{bj} + G_{t+1}(\bar{\omega}_{bj,t+1})$ is the expected share of housing values, gross of monitoring costs that goes to lenders in case of default. Substituting in from equation 2.8 into 2.9, and using the just defined expressions, the participation constraint can be written, in real terms, as following:

$$R_t l_{bj,t+1} = \left[\Gamma_{t+1}(\bar{\omega}_{bj,t+1}) - \mu_{bj} G_{t+1}(\bar{\omega}_{bj,t+1}) \right] (1-\delta_h) q_{h,t+1} h_{bj,t+1} \pi_{t+1}, \tag{2.10}$$

where $l_{bj,t+1}$ is the real debt position of borrower group j, i.e. $l_{bj,t+1} = L_{bj,t+1}/P_t$, $q_{h,t+1} = Q_{h,t+1}/P_{t+1}$ is the real house price and $\pi_{t+1} = P_{t+1}/P_t$ is consumer price inflation. Under 2.10, borrower group j is subject to a constraint that limits its leverage by a fraction of the expected future value of its current housing wealth at time. We can also define the debt ratio $\frac{l_{bj,t+1}}{q_{h,t+1} h_{bj,t+1}}$, which defines each group Borrowers' leverage position, and the (endogenous) loan-to-value ratio of borrower group j as:

$$\Gamma_{t+1}(\bar{\omega}_{bj,t+1}) - \mu_{bj} G_{t+1}(\bar{\omega}_{bj,t+1}).$$

The part of the housing stock that all members of borrower group j are left with, after accounting for defaulting members, is

$$\int_{\bar{\omega}_{bj,t+1}}^{\infty} (1-\delta_h) q_{h,t+1} h_{bj,t+1} f_{t+1}(\omega^i_{bj}) d\omega^i_{bj} = \left[1 - G(\bar{\omega}_{bj,t+1})\right] (1-\delta_h) q_{h,t+1} h_{bj,t+1}.$$

We can now combine the above expression with equations 2.7, 2.8 and 2.10 and rewrite the Borrowers' budget constraint, in real terms, as follows:

$$\begin{aligned} & c_{bj,t} + q_{h,t}(h_{bj,t+1} - (1-\delta_h)h_{bj,t}) + \tfrac{R_{t-1}}{\pi_t} l_{bj,t} \\ & = w_{bj,t} n_{bj,t} + l_{bj,t+1} + \left[1 - \mu_{bj} G_{bj,t}(\bar{\omega}_{bj,t})\right](1-\delta_h) q_{h,t} h_{bj,t}. \end{aligned} \tag{2.11}$$

The optimization problem of borrower group j is then given by maximizing equation 2.1 subject to equations 2.11 and 2.10. Denoting with $\lambda^{BC}_{bj,t}$ the Lagrange multiplier on the constraint 2.11, and with $\lambda^{PC}_{bj,t}$ the Lagrange multiplier on the constraint 2.10, the first order conditions with respect to $c_{bj,t}$, $n_{bj,t}$, $h_{bj,t+1}$, $l_{bj,t+1}$, and $\bar{\omega}_{bj,t+1}$ are:

$$\lambda^{BC}_{bj,t} = \left(c_{bj,t}\right)^{-\sigma_c}, \tag{2.12}$$

$$\upsilon^n_{bj}\left(n_{bj,t}\right)^{\eta-1} = w_{bj,t}\lambda^{BC}_{bj,t}, \tag{2.13}$$

$$q_{h,t}\left(c_{bj,t}\right)^{-\sigma_c} = \beta_b E_t\left\{\left(c_{bj,t+1}\right)^{-\sigma_c}\left[q_{h,t+1}\right](1-\delta_h)\left[1-\mu_{bj}G_{bj,t+1}(\bar{\omega}_{bj,t+1})\right]\right\} + \varepsilon^h_t\ \varkappa h^{-\sigma_H}_{bj,t+1}, \tag{2.14}$$

$$\left(c_{bj,t}\right)^{-\sigma_c} = \beta_b E_t\left\{\left(c_{bj,t+1}\right)^{-\sigma_c}\frac{R_t}{\pi_{t+1}} + \lambda^{PC}_{bj,t} R_t\right\}, \tag{2.15}$$

$$0 = \begin{array}{c} \lambda^{PC}_{bj,t+1}[\Gamma'_{t+1}(\bar{\omega}_{bj,t+1}) - \mu_{bj}G'_{t+1}(\bar{\omega}_{bj,t+1})](1-\delta_h)E_t q_{h,t+1} h_{bj,t}\pi_{t+1} \\ -\beta_{bj}E_t\left(c_{bj,t+1}\right)^{-\sigma_c}\mu_{bj}G'_{t+1}(\bar{\omega}_{bj,t+1})(1-\delta_h)E_t q_{h,t+1} h_{bj,t}. \end{array} \tag{2.16}$$

2.2 Production

The production side of the economy consists of a competitive final good sector, and an intermediate goods sector. The latter operates under monopolistic competition and sticky prices.

2.2.1 Final goods producers

The final good, y_t, is produced by perfectly competitive firms using $y_t(i)$ units of each type of intermediate good i and a constant elasticity of substitution technology:

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\xi-1}{\xi}} di\right]^{\frac{\xi}{\xi-1}}, \tag{2.17}$$

From standard profit maximization, input demand for the intermediate good i is obtained as:

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\xi} y_t, \tag{2.18}$$

where P_t is the CES-based final (consumption) price index given by

$$P_t = \left[\int_0^1 P_t(i)^{1-\xi} di\right]^{\frac{1}{1-\xi}}. \tag{2.19}$$

2.2.2 Intermediate goods producers

Output of producer i, denoted $y_t(i)$, is produced using the following technology:

$$y_t(i) = \varepsilon_t^z n_{s,t}(i)^{\gamma_s} \left[n_{bH,t}(i)^{\gamma_H} n_{bL,t}(i)^{\gamma_L} \right]^{1-\gamma_s}, \tag{2.20}$$

where $n_s(i)$ and $n_{bj}(i)$ is labor demanded by firm i from patient and each of the two impatient agents, respectively.[10] γ_s is the share of labor of Savers in the production function, and $(1-\gamma_s)$ the share of labor from the two borrower groups, further split up into each group $j = H, L$, where $\gamma_H + \gamma_L = 1$. ε_t^z is an aggregate productivity shock. As in Iacoviello (2005) and Iacoviello and Neri (2010), we assume that different labor types are unit-elastic. There is a continuum of monopolistically competitive firms indexed by i, with total mass one. At time t, each intermediate firm is able to revise its price with a probability $(1-\theta)$ as in Calvo (1983). Intermediate firms are owned by patient households. Producer i takes as given demand for her good i, equation 2.18, and the production function, equation 2.20, and chooses optimal inputs $n_{s,t}$, $n_{bj,t}$, for $j = H, L$ and for all periods k, and the price of good i, $P_t(i)$, such as to maximize her lifetime expected discounted profits:

$$\max E_t \sum_{k=t}^{\infty} (\beta_s\theta)^{k-t} \frac{U_{C_{st+k}}}{U_{C_{st}}} \left\{ \begin{array}{c} P_t(i)\, y_{t+k}(i) - W_{s,t+k} n_{s,t+k}(i) - \sum_{bj=1}^{H,L} W_{bj,t+k} n_{bj,t+k}(i) \\ +MC_{t+k}(i) \left[\varepsilon_{t+k}^z n_{s,t+k}(i)^{\gamma_s} \left[n_{bH,t+k}(i)^{\gamma_H} n_{bL,t+k}(i)^{\gamma_L} \right]^{1-\gamma_s} - y_{t+k}(i) \right] \end{array} \right\}$$

The first order conditions that result from this problem can be summarized, already expressed in real terms, as:

$$w_{s,t} = \gamma_s mc_t(i) \frac{y_t(i)}{n_{st}(i)}, \tag{2.21}$$

$$w_{bj,t} = (1-\gamma_s)\gamma_{bj} mc_t(i) \frac{y_t(i)}{n_{bj}(i)}, \text{ for } j = H, L, \tag{2.22}$$

$$\widetilde{p}_t(i) = \frac{\xi}{\xi-1} \frac{E_t \sum_{k=t}^{\infty} (\beta^s\theta)^{k-t} \frac{U_{C_{st+1}}}{U_{C_{st}}} \pi_{t+k}^{\xi} y_{t+k} mc_{t+k}}{E_t \sum_{k=t}^{\infty} (\beta^s\theta)^{k-t} \frac{U_{C_{st+1}}}{U_{C_{st}}} \pi_{t+k}^{\xi-1} y_{t+k}}, \tag{2.23}$$

where $\widetilde{p}_t(i) \equiv \frac{P_t(i)}{P_t}$ is the optimal relative price of firm i, and $w_{st} = \frac{W_{st}}{P_t}$ and $w_{bjt} = \frac{W_{bjt}}{P_t}$ are real wages. The last equation uses the fact that the real marginal cost $mc_t = MC_t/P_t$, is equal for all producers i, since it is a function of (aggregate) wage rates only, i.e. $mc_t(i) = mc_t = \frac{1}{\varepsilon_t^z} \frac{w_{st}^{\gamma_s} \left[w_{bH,t}(i)^{\gamma_H} w_{bL,t}(i)^{\gamma_L} \right]^{1-\gamma_s}}{[\gamma_s]^{\gamma_s} [\gamma_H(1-\gamma_s)]^{\gamma_H(1-\gamma_s)} [\gamma_L(1-\gamma_s)]^{\gamma_L(1-\gamma_s)}}$. From equation

[10] For similar specification, see Justiniano, Primiceri, and Tambalotti (2014), Mendicino and Punzi (2014) and Brzoza-Brzezina, Gelain, and Kolasa (2014).

2.19, one can derive the link of the optimal price $\widetilde{p}_t(i)$ to aggregate price behavior under the Calvo setting as

$$1 = (1-\theta)\left(\widetilde{p}_t(i)\right)^{1-\xi} + \theta\pi_t^{\xi-1}. \tag{2.24}$$

We also define firm i's period t (real) profits:

$$\Delta_t(i) = y_t(i) - w_{s,t}n_{s,t}(i) - \sum_{bj=1}^{H,L} w_{bj,t}n_{bj,t}(i) \tag{2.25}$$

2.3 Housing Producers

Housing producers combine a fraction of the final goods purchased from retailers as investment goods, $i_{h,t}$, to combine it with the existing housing stock in order to produce new units of installed houses. Housing production is subject to an adjustment cost specified as $\frac{\psi_h}{2}\left(\frac{i_{h,t}}{h_{t-1}} - \delta_h\right)^2 h_{t-1}$, where ψ_h governs the slope of the housing producers adjustment cost function. Housing producers choose the level of $i_{h,t}$ that maximizes their profits

$$\max_{i_{h,t}} q_{h,t}i_{h,t} - \left(i_{h,t} + \frac{\psi_h}{2}\left(\frac{i_{h,t}}{h_{t-1}} - \delta_h\right)^2 h_{t-1}\right).$$

From profit maximization, it is possible to derive the supply of housing

$$q_{h,t} = \left[1 + \psi_h\left(\frac{i_{h,t}}{h_{t-1}} - \delta_h\right)\right], \tag{2.26}$$

where q_t^h is the relative price of capital. In the absence of investment adjustment costs, q_t^h, is constant and equal to one. The usual housing accumulation equation defines aggregate housing investment:

$$i_{h,t} = h_t - (1-\delta_h)\,h_{t-1}. \tag{2.27}$$

2.4 Monetary Policy

The Central Bank follows a standard Taylor-type rule which responds to changes in inflation and output:

$$\frac{R_t}{\bar{R}} = (R_{t-1})^{\phi_R}\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi(1-\phi_R)}\left(\frac{y_t}{y}\right)^{\phi_Y(1-\phi_R)}\varepsilon_t^r \tag{2.28}$$

where ϕ_π is the coefficient on inflation in the feedback rule, ϕ_Y is the coefficient on output, and ϕ_R determines the degree of interest rate smoothing. ε_t^r is an i.i.d. monetary policy shock.

2.5 Market Clearing

Define aggregate consumption and the aggregate housing stock as

$$c_t = \alpha_s c_{s,t} + \alpha_b \sum_{bj=1}^{H,L} \alpha_{bj} c_{bj,t},$$

$$h_t = \alpha_s h_{s,t} + \alpha_b \sum_{bj=1}^{H,L} \alpha_{bj} h_{bj,t}.$$

To close the model, we need to specify the aggregate market clearing conditions. Market clearing in the assets market implies:

$$\alpha_s d_{s,t} = \alpha_b \left[\alpha_{bH} l_{bH} + \alpha_{bL} l_{bL}\right]. \tag{2.29}$$

Labor market clearing requires:

$$\int_0^1 n_{s,t}(i)di = \alpha_s n_{s,t}, \int_0^1 n_{bj,t}(i)di = \alpha_b \alpha_{bj} n_{bj,t}, \text{ for } j = H, L. \tag{2.30}$$

Good market clearing, obtained by combining aggregate versions of equations 2.2, 2.11, 2.29, 2.30 and 2.25, reads[11]:

$$y_t = c_t + q_{h,t} i_{h,t}. \tag{2.31}$$

2.6 Exogenous Factors

Shocks to productivity, ε_t^z, house preference, ε_t^h and monetary policy, ε_t^r, follow an autoregressive process of order one:

$$\ln \upsilon_t = \rho_\upsilon \ln \upsilon_{t-1} + \varepsilon_{\upsilon,t},$$

where $\upsilon = \{z, h, r\}$, ρ_υ is the persistence parameter and $\varepsilon_{\upsilon,t}$ is a i.i.d. white noise process with mean zero and variance σ_υ^2.

Similar to Forlati and Lambertini (2011), the idiosyncratic housing investment shocks, $\omega_{bj,t}$, for $j = H, L$, follow a log-normal distribution, i.e. $log(\omega_{bj,t}) \sim N(-\frac{\sigma^2_{\omega_{bj},t}}{2}, \sigma^2_{\omega_{bj},t})$. The standard deviations, $\sigma^2_{\omega_{bj},t}$, are time-varying and follow an AR(1) process, such that:

$$\ln \frac{\sigma_{\omega_{bj},t}}{\sigma_{\omega_{bj}}} = \rho_\omega \ln \frac{\sigma_{\omega_{bj},t-1}}{\sigma_{\omega_{bj}}} + \varepsilon^{bj}_{\omega,t}, , \text{ for } j = H, L,$$

where $\varepsilon^{bj}_{\omega,t}$ is an i.i.d. shock.

[11]Strictly speaking, the market clearing condition, reads $\frac{y_t}{s_t} = c_t + q_{h,t} i_{h,t}$, where variables s_t denotes price dispersion among Calvo price setters. However, since we consider a first order approximation of the model to solve for the model dynamics, we can safely ignore this term.

3 Extended Model

In the extended model a role for a financial intermediary, a banking sector, is included, which Savers use for their deposits and from which Borrowers obtain loans. The financial intermediary is part of the household sector, and has mass α_{fi}; the total household mass remains equal to one, so that now $\alpha_s + \alpha_b + \alpha_{fi} = 1$.

3.1 Banking Sector

The banking sector follows a similar setup as in Kollmann, Enders, and Müller (2011) and Kollmann (2013). We assume there is a bank which, at time t, receives deposits from the Savers, denoted $d_{fi,t}$, and make loans to each type of Borrowers, $l^{bj}_{fi,t}$, for $bj = H, L$, where subscript fi stands for holdings of these types of assets by the financial intermediary. The financial intermediary faces a bank capital constraint which requires the capital ($\sum_{bj=1}^{H,L} l^{bj}_{fi,t} - d_{fi,t}$) not be smaller than a fraction ϕ of the bank's assets $\sum_{bj=1}^{H,L} l^{bj}_{fi,t}$. The banking sector maximizes

$$\max E_0 \sum_{t=0}^{\infty} \beta^t_{fi} \ln(c_{fi,t}),$$

subject to the flow of funds

$$c_{fi,t} + \frac{R^d_{t-1}}{\pi_t} d_{fi,t} + \sum_{bj=1}^{H,L} l^{bj}_{fi,t+1} + \Gamma_c \left(d_{fi,t+1}, \sum_{bj=1}^{H,L} l^{bj}_{fi,t+1} \right) + \Gamma_x (x_t) = d_{fi,t+1} + \frac{R^l_{t-1}}{\pi_t} \sum_{bj=1}^{H,L} l^{bj}_{fi,t}$$

and capital constraint

$$d_{fi,t+1} \leq (1-\phi) \sum_{bj=1}^{H,L} l^{bj}_{fi,t+1},$$

where $c_{fi,t}$ denotes the financial intermediary's consumption (dividends) and β_{fi} is its discount factor; $\Gamma_c > 0$ denotes real marginal operating cost of collecting deposits and extending loans, assumed to be linear, i.e. $\Gamma_c(d_{fi,t+1}, \sum_{bj=1}^{H,L} l^{bj}_{fi,t+1}) = \Gamma_s d_{fi,t+1} + \sum_{bj=1}^{H,L} \Gamma_{bj} l^{bj}_{fi,t+1}$. We follow Kollmann, Enders, and Müller (2011) in assuming that the bank can hold less capital than the required level, but that this is costly (e.g., because the bank then has to engage in creative accounting). Such cost is a convex function of the excess capital of the bank, x_t, and follow the properties that $\Gamma_x(x_t) > 0$ for $x_t < 0$, $\Gamma''_x \geq 0$, and $\Gamma_x(0) = 0$. Therefore the bank pays a positive cost if $x_t < 0$. Excess bank capital is given by

$$x_t = (1-\phi) \sum_{bj=1}^{H,L} l^{bj}_{fi,t+1} - d_{fi,t+1}$$

The first order conditions with respect to $c_{fi,t}$, $d_{fi,t+1}$, and $l^{bj}_{fi,t+1,}$, for $j = H, L$, are:

$$\lambda^{BC}_{fi,t} = \frac{1}{c_{fi,t}}, \tag{3.1}$$

$$\frac{1}{c_{fi,t}}[1 - \Gamma_s + \Gamma'_x] = \beta_{fi} E_t(\frac{1}{c_{fi,t+1}} \frac{R^d_t}{\pi_{t+1}}), \tag{3.2}$$

$$\frac{1}{c_{fi,t}}[1 + \Gamma_B + \Gamma'_x] = \beta_b E_t(\frac{R^l_t}{c_{fi,t+1}\pi_{t+1}}). \tag{3.3}$$

3.2 Market Clearing

Asset market clearing now implies:

$$\alpha_{fi} d_{fi,t} = \alpha_s d_{s,t},$$
$$\alpha_{fi}\left[l^{bH}_{fi,t} + l^{bL}_{fi,t}\right] = \alpha_b\left[\alpha_{bH} l_{bH} + \alpha_{bL} l_{bL}\right].$$

The definition of aggregate consumption becomes:

$$c_t = \alpha_s c_{s,t} + \alpha_b \sum_{bj=1}^{H,L} \alpha_{bj} c_{bj,t} + \alpha_{fi} c_{fi,t}.$$

4 Parameterization

The model is parameterized at a quarterly frequency, aimed at capturing key features for the US over the period 1975-2006.[12] Table 1 reports these ratios under the homogenous scenario, i.e. all Borrowers are treated equally, and under the heterogenous scenario, i.e. we specify a high-LTV type and low-LTV type Borrower group. The parameter values are summarized in Table 2.

[Table 1 about here.]

[Table 2 about here.]

The discount factor of the Savers, β_s, is set equal to 0.99 to match the annualized steady state interest rate of 4%, while the Borrowers' discount factor, β_b, is assumed to be lower than the Saver's discount factor and equal to 0.98.[13] The inverse of the

[12] Data source: U.S. Bureau of Economic Analysis, the Federal Reserve System, the Mortgage Bankers Association - National Delinquency Survey, Fannie Mae and Freddie Mac. See Appendix 1.

[13] Similarly, Justiniano, Primiceri, and Tambalotti (2014) and Justiniano, Primiceri, and Tambalotti (2015) choose 0.998 for the lenders discount factor and 0.99 for the borrowers discount factor in order to distinguish the relative impatience of the two groups. Iacoviello (2015) has a similar value for the Savers, but sets a much lower discount factor for the Borrowers, equal to 0.94.

Frisch elasticity of labor supply, η, is set equal to 2 and the labor disutility parameter, v_j^n, is equal to 1, implying log preferences. The coefficients on relative risk aversion for consumption goods, σ_c, and housing services, σ_H, are both set to 1. Housing stocks depreciate at a rate of 0.0089 and the weight of housing in utility function, $\varkappa$, is set to 0.075. We follow Justiniano, Primiceri, and Tambalotti (2014) and Justiniano, Primiceri, and Tambalotti (2015) in taking into account evidence on the ratios from the Survey of Consumer Finances (SCF) when setting the parameters between patient and impatient households in the production function. According to the SCF in 1992, 1995, 1998, 2001 and 2007, the average share of borrowers is about 60%; therefore the share of Savers' labor income in the goods sector, γ_s, is set at 0.64, with a complimentary share of $(1-\gamma_s)$ for borrowers, whereof each borrower group's share, γ_H and γ_L, in Borrowers' Cobb-Douglas labor supply are set to 0.5.[14] Those values also help to match the ratio of hours worked between Borrowers and Savers of about 1.1. We set ξ equal to 11, which implies a steady-state markup of 10% in the goods sector.

The housing investment adjustment cost parameter, ψ_h, is set to equal 14 as in Iacoviello and Neri (2010).[15] For the monetary policy rule we chose a coefficient for the interest rate inertia, ρ_R, equal to 0.8, a moderate reaction to the output gap, $\rho_Y = 0.125$, and a reaction to inflation of $\rho_\pi = 1.5$. The Calvo probability of changing price is set to 0.75, implying that prices are fixed for a year on average, a fairly standard value in the literature. Similar to Iacoviello and Neri (2010), the technology shock and housing demand shock follow an autoregressive process of order one, with persistence of 0.95 and 0.96, respectively. The standard deviation is set to 0.01 for technology shock and 0.04 for housing demand shock. Monetary policy shocks are i.i.d. with a variance equal to 0.23%.

Turning to the parameters related to the mortgage contract – monitoring cost, μ_j, the autoregressive coefficient and standard deviation of the idiosyncratic housing risk shock, $\rho_{\omega bj}$ and $\sigma_{\omega bj}$ –, we focus our attention on targeting the values from a detailed empirical LTV ratio distribution. This approach thus differs from Forlati and Lambertini (2011), who aim to match the US delinquency rate, and who obtain an average LTV ratio which is lower than standard values find in the literature (see Iacoviello (2005), Iacoviello and Neri (2010), Iacoviello (2015), Justiniano, Primiceri, and Tambalotti (2014), Justiniano, Primiceri, and Tambalotti (2015), Kiyotaki, Moore, et al. (1997), Brzoza-Brzezina, Gelain, and Kolasa (2014) and Mendicino and Punzi (2014)). Instead we calibrate the LTV ratios for the economy-wide average (for the 'homogenous

[14] Iacoviello (2005) use a value of 0.36 and Iacoviello and Neri (2010) estimate a value of 0.21. Justiniano et al. (2014) calibrate the average share of borrowers equal to 0.61 to match the relative labor income of 0.64 in the SCF. Kaplan, Violante, and Weidner (2014) call *hand-to-mouth* (HtM) households such Borrowers who spend all of their available income every period, and estimate the fraction of them equal to 0.31 using data from the Survey of Consumer Finance (SCF) during the period 1989-2010.

[15] The housing investment adjustment cost parameter is responsible mostly in determining how fast the housing stock is (allowed to) rebound after a shock to housing investment risk that destroyed part of the housing stock. All the other variables are largely unaffected by alternative calibration of the housing adjustment cost.

borrowers' version) and for the two different groups of Borrowers (for the 'heterogeneous borrowers' version). In particular, we choose values to match the mean values of the loan-to-value ratios distribution from the Fannie Mae and Freddie Mac database, covering around 12 million in home purchases of single-family loans issued during the period of 2000-2006. Figure 2 (solid line) reports the cumulative distribution of LTV ratios, which reveals clear heterogeneity: loans that fall into category $0 < LTV \leq 80\%$, that is, all holdings of mortgage loans issued with a LTV ratio less or equal to 80%, constitute 74 percent of total loans; this lower 74-th percentile of the sample distribution has an average LTV ratio equal to 67%. The upper 26-th percentile of the sample, containing all holdings of mortgage loans issued with a LTV ratio between 80% and 100% ($80\% < LTV \leq 100\%$), displays a mean LTV ratio equal to 91%.[16] In contrast, the homogenous borrower model with a representative borrowing agent shows an overall mean of the LTV distribution of 73%. See Table 3, Panel (a).

To obtain the economy-wide LTV ratio of 73% in the homogenous model version, the standard deviation of the idiosyncratic housing risk shock is set equal to σ_ω=0.1125, together with monitoring cost μ=0.12. To match the LTV ratios under the heterogeneous scenario (i.e. low-LTV type =67% and high-LTV type =91%), we choose $\sigma_{\omega L}$=0.147 and $\sigma_{\omega H}$=0.028, the monitoring cost for both groups, μ_j, remains at 0.12. Each Borrower's size, α_{bj}, is set to the share of mortgage applications under a specific LTV ratio, $\alpha_{bL} = 0.74$ and $\alpha_{bL} = 0.26$.

Figure 2 (dashed line) documents the changes in the distribution of LTV ratios in 2009: the economy-wide average LTV ratio drops from 73% to 69%, while the mean of the LTV ratio belonging to the lower 76-th percentile of the sample distribution drops from 67% to 64%, and the mean LTV ratio of the upper 24-th percentile of the sample drops from 91% to 85%. See Table 3, Panel (c). To capture this deleveraging experience in our model, in which (the change in) LTV ratios endogenously arise(s) from the (change in) importance of housing investment risk, we proceed as follows: we ask what the levels of $\sigma_{\omega L}$ and $\sigma_{\omega H}$ (or σ_ω) would be which would be needed to replicate the lower LTV ratios in 2009, of 64% and 85% (73%) for the low-LTV and the high-LTV type (for the representative Borrower in the homogenous borrower scenario) respectively, if this deleveraging experience were permanent. In order to replicate these stylized facts in our model, the standard deviation of idiosyncratic housing investment risk needs to increase by 20% in the homogenous borrower model, i.e. σ_ω increases from 0.1125 to 0.1350; in the heterogeneous borrower scenario, the riskiness of the high-LTV type Borrowers, $\sigma_{\omega H}$, is required to increase by 91%, from $\sigma_{\omega H}$=0.028 to $\sigma_{\omega H}$=0.053, to generate the drop from an LTV ratio of 91% to 85%, while simultaneously an increase in the riskiness of the low-LTV type Borrowers, $\sigma_{\omega L}$, of 13%, from $\sigma_{\omega L}$=0.147 to $\sigma_{\omega L}$=0.166, generates the drop from an LTV ratio of 67% to 64%. We set the persistence of the shocks to housing investment risk very high and equal to 0.99, capturing the idea that the

[16] Similarly, Bokhari, Torous, and Wheaton (2013) analyze single-family home mortgages originated in the US over the period 1986 to 2010 and find that about 76% of the loans contain an average LTV ratio up to 80%, 13% contain an average LTV ratio between 80% and 90%, while 11% contain an average LTV ratio between 90% and 100%.

deleveraging shock at the onset of the crisis is perceived an an (almost) permanent re-evaluation of the riskiness of investment risk in the mortgage market.[17] Table 3, Panel (b)-(c), shows that, over the average period of 2008-2010, the main drop in the LTV distribution occurred in 2009.

[Table 3 about here.]

In the extended model version, we also need to specify the parameters attributed to the financial intermediary, the bank. We calibrate the required bank capital ratio equal to 0.08.[18] This value reflects the rules defined under Basel III, which requires that the total risk-weighted capital requirements, which is defined as total (Tier 1 and Tier 2) capital divided by total risk-weighted assets, be at least 8%. The discount factor is set equal to the Savers' discount factor. The bank operating cost coefficient is set equal to 0.0018, while the cost on banks' excess capital is set to 0.1264, similar to Kollmann, Enders, and Müller (2011).

5 Simulation Results: Impulse Responses

5.1 Baseline Model: Idiosyncratic Housing Investment Risk Shocks

Figure 3 plots the dynamic responses to a deleveraging experience, resulting from an unanticipated increase in the volatility of idiosyncratic housing investment risk, $\sigma_{\omega bj,t}$. The Figure compares two scenarios: the 'homogeneous borrowers' scenario (solid line), where all borrowers face the same LTV ratio, and where the economy-wide average LTV ratio drops from 73% to 69%; and the 'heterogeneous borrowers' scenario (dashed line), where there exists two types of Borrowers, low-LTV type Borrower ($0 < LTV \leq 80\%$) versus high-LTV type Borrower ($80\% < LTV \leq 100\%$), who experience a drop in their respective groups' LTV ratios from 67% to 64% and 91% to 85%, as described in section 4.

The responses in the 'homogenous borrower' and the 'heterogeneous borrower' scenarios are qualitatively similar. The increase in the standard deviations of idiosyncratic housing investment risk increases default rates, monitoring costs and the external finance premium. As a result of the mortgage risk shock, Borrowers' financial conditions worsen, more members of each borrower group default on their loans and loose their housing stock in the process, while non-defaulting members pay a higher mortgage

[17] In particular, we do not believe that this is necessarily the 'correct' persistence if we were to parameterize the model for typical variations in investment riskiness (or LTV ratios) in 'normal times', over the business cycle. Instead, when capturing the idea that, particularly the riskiness of the small market segment of subprime mortgages was evaluated too optimistically, the adjustments in evaluating riskiness of this market were of more permanent nature. There is no comparable value in the literature; Forlati and Lambertini (2011) choose a lower persistence, equal to 0.9 in a similar exercise. While this leads to a somewhat smaller drop in LTV ratios in our experiment, it leaves our results largely unchanged.

[18] Kollmann, Enders, and Müller (2011) show that the capital ratios of US commercial banks have been in the range of 7-8%.

interest rate. The tightening of credit conditions that occurs with the drop in loan-to-value ratios reduces the ability to borrow from the housing stock, forcing borrowing agents to reduce their consumption, housing stock, and increase hours worked. At the same time, Savers increase their consumption and cut hours worked, and reduce their lending in response to falling interest rates.

The heterogeneous model version in which the bulk of the credit crunch and the adjustments in LTV ratios falls on the high-LTV type Borrower group displays a substantially amplified drop in real and financial variables, relative to the homogenous model version, despite being subject to the same drop in the economy-wide average LTV ratio. In particular, the heterogeneous borrower model version leads output and aggregate consumption to decrease three times as much as in the homogenous model version (panels 'Output' and 'Total Consumption'). This highlights the differences in household net worth and wealth effects across types of Borrowers. The disruption of the highly leveraged Borrowers' balance sheet decreases their ability to take new mortgage loans to finance their spending, and, because of a higher marginal propensity for both consumption and housing goods, results in a more severe drop in housing demand and consumption. The fall in housing demand experienced by all Borrowers in the heterogeneous borrowers version (panel 'House Demand Borrowers') is twice the drop in the homogenous borrower version, and driven by the strong cutback in high-LTV type borrowing (panel 'Borrowing Type H'). The strong tightening of collateral constraint on high-LTV type Borrowers also leads them to cut back severely also on consumption goods and demand for new mortgage loans, therefore total lending decreases an additional 8% over and above the drop of 12% in the homogeneous scenario (panel 'Total Lending'). The heterogeneous model version leads to a fall in the rate of return on outstanding loans approximately twice as large as that found in the homogeneous scenario, leading Savers to reduce their lending more. Moreover, while output and total consumption display a rapid drop and rebound, total lending falls substantially and persistently. Savers, in order to smooth consumption and avoid a drop in their utility, increase the demand of goods and housing services. The increase in Savers' housing demand, and the presence of housing adjustment costs, leads to an increase in housing investment after the initial drop. Both model versions display similar impact behavior of residential investment and house prices, but the heterogeneous borrower version produces more pronounced swings, shedding light on the importance of borrower heterogeneity in magnifying asset prices fluctuations.

Significant Borrowers' heterogeneity arises from different Borrowers' quality, i.e. Borrowers' balance sheet channel, combines with an housing net worth channel. This can be showed by re-arranging the optimal demand of houses (Eq. 2.14) and the first order condition with respect to $\bar{\omega}_{bj,t+1}$ (Eq. 2.16). It is possible to re-write Eq. 2.16 as follows:

$$\lambda^{PC}_{bj,t+1} = \frac{\beta_{bj}}{\pi_{t+1}} \frac{\mu_{bj} G^{'}_{t+1}(\bar{\omega}_{bj,t+1})}{[\Gamma^{'}_{t+1}(\bar{\omega}_{bj,t+1}) - \mu_{bj} G^{'}_{t+1}(\bar{\omega}_{bj,t+1})]} \lambda^{BC}_{bj,t+1} \tag{5.1}$$

thus, the lagrangian multipliers of the borrowing constraints are a negative function of the loan-to-value (LTV) ratio for each group of Borrowers:

$$\lambda^{PC}_{bj,t+1} = \frac{\beta_{bj}}{\pi_{t+1}} \frac{\mu_{bj} G^{'}_{t+1}(\bar{\omega}_{bj,t+1})}{[LTV_{bj,t}]} \lambda^{BC}_{bj,t+1} \tag{5.2}$$

During crisis, high-LTV type Borrowers experience larger drop in their loan-to-value ratios, $LTV_{bH,t}$, therefore, *ceteris paribus*, their borrowing constraint in Eq. 2.10 binds the strongest LTV ratio falls. Through the Borrowers' balance sheet channel, collateral constraints become tighter, and an additional unit of borrowing results to be more expansive for the high-LTV type Borrowers who would need to provide more collateral for the same amount of mortgage loans. As a result, high-LTV type Borrowers reduce their demand for mortgage loans and houses much more than the low-LTV type. The highest tightening of high-LTV type Borrowers constraints also implies a stronger impact of net worth channel. Indeed, because of a binging borrowing constraints, consumption responses are more sensitive to fluctuations in the credit markets, thus Borrowers' net worth falls and the high-LTV type Borrowers is much more affected as their marginal utility of borrowing is higher. Thus, the drop in consumption for the high-LTV type Borrowers is more severe.

Heterogeneity across Borrowers arises also from different marginal rates of substitution between houses and consumption. This can also be showed by re-arranging Eq. 2.14, as follows:

$$\varkappa h^{-\sigma_H}_{bj,t+1} = \left(\frac{1}{\varepsilon^h_t}\right) \left\{ q_{h,t} \left(c_{bj,t}\right)^{-\sigma_c} - \beta_b E_t \left\{ \left(c_{bj,t+1}\right)^{-\sigma_c} q_{h,t+1} \left(1-\delta_h\right) \left[1 - \mu_{bj} G_{bj,t+1}(\bar{\omega}_{bj,t+1})\right] \right\} \right\} \tag{5.3}$$

then,

$$\frac{\varkappa h^{-\sigma_H}_{bj,t+1}}{c^{-\sigma_c}_{bj,t}} = \left(\frac{1}{\varepsilon^h_t}\right) \left\{ q_{h,t} - \beta_b E_t \left\{ \left(\frac{c_{bj,t+1}}{c_{bj,t}}\right)^{-\sigma_c} q_{h,t+1} \left(1-\delta_h\right) \left[1 - \mu_{bj} G_{bj,t+1}(\bar{\omega}_{bj,t+1})\right] \right\} \right\} \tag{5.4}$$

where $\frac{\varkappa h^{-\sigma_H}_{bj,t+1}}{c^{-\sigma_c}_{bj,t}} = MRS^{H,C}_{bj}$ is the current marginal utility of dwellings expressed in units of marginal utility of consumption. Eq. 5.4 shows that the marginal rate of substitution between houses and consumption is given by the current house prices and the discounted expected value of future house prices net of seized housing stock. Clearly, $MRS^{H,C}_{bL} \neq MRS^{H,C}_{bH}$ because higher leveraged Borrowers will tend to default more when a negative idiosyncratic risk shock hits the economy and more of their housing stock is seized, implying heterogeneous marginal rate of substitution. The Borrowers' net worth channel, which leads Borrowers with higher leverage to experience larger marginal propensity to consume relative to housing wealth, reinforces the Borrowers' balance sheet channel where the strongest deleveraging process brings high-LTV type Borrowers to default more and loose the house.[19]

[19]Figure 8 in the Appendix shows the heterogeneity between low-LTV and high-LTV type Borrowers relative to the lagrangian multiplier of the borrowing constraint and the marginal rate of substitution between houses and consumption.

To sum up, the collapse in the endogenous LTV ratios and particularly strong deleveraging process for the high-LTV type Borrower, generates a stronger amplification of the financial accelerator mechanism, despite the same reduction in loan-to-value ratios on average. The increased riskiness of high-LTV type households leads to a wave of defaults when the quality of this type's borrowers' housing stock drops and they find themselves underwater, i.e. the mortgage repayment is higher than the current value of the house which has been used to pledge against borrowing. The reallocation in terms of consumption, housing and borrowing of the high leverage group is so strong as to drag down the entire economy. Mian, Rao, and Sufi (2013) emphasize, in empirical work, the role of heterogeneity in the marginal propensity to consume in affecting the aggregate consumption through the distribution of wealth losses.[20] The results of our theoretical model are in line with these findings. In fact, higher leverage increases the marginal propensity to consume for higher indebted households relative to less indebted ones and the real impact at a given aggregate loss in wealth is amplified.[21]

[Fig. 3 about here.]

5.2 Extended Model: Idiosyncratic Housing Investment Risk Shocks

This section discusses the transmission of housing investment risk shocks and the resulting deleveraging episode in the extended model version, that includes a role of the banking system. The financial intermediary, the bank, collects deposits from saving households and makes loans to (both types of) borrowing household groups. In this setting the bank has to finance a fraction of loans using its own equity. Figure 4 documents that the presence of the banking sector further amplifies the model dynamics in response to idiosyncratic housing risk shocks. We continue to report impulse responses for the homogeneous borrowers model version (solid line) and the heterogeneous borrowers model version (dotted line).

As in the baseline model, the mortgage risk shock leads to worsening Borrowers' financial conditions, and to more members of each borrower group defaulting on their loans. Therefore, banks suffer losses because of foregone mortgage payments, higher internal costs and mortgage defaults. These loan losses translate into a decrease in banks' assets; the bank thus deleverages, in order to avoid facing negative excess bank capital and additional costs to engage in creative accounting, which leads to a further contraction of credit from the real economy. The higher risk premium charged by banks discourages new demand from loans, depressing house prices even more.

[20] King (1994) emphasizes that the marginal propensity to consume out of wealth is much higher for credit-constrained households.

[21] Mian, Rao, and Sufi (2013) provide evidence that households with a loan-to-value (LTV) ratio of 90% show to have a marginal propensity to consume out of housing wealth three times as large as that found in households facing a LTV ratio equal to 30%.

Thus, through the bank balance sheet channel, the deleveraging process of banks amplifies and propagates default shocks to the real economy. Because of the more pronounced loan losses in the heterogeneous model version, the heterogeneous borrower model with a banking sector produces a larger amplification to the real economy: we observe a drop in total consumption and output of around 3.5% deviations from steady state, relative to a drop of only 2% in the homogenous borrower model. It should be noted that this mechanism, i.e. the presence of the banking sector, leads to a further amplification in these two variables in both the homogenous and the heterogeneous borrower version. The presence of banks leads house prices to decrease more relative to the baseline model. This occurs because, given a lower interest rate income, Savers increase their demand for houses by less, relative to the baseline model. Through the bank balance sheet channel, banks increase their leverage ratio and decrease deposit rates in order to smooth out their losses. Therefore, the aggregate housing stock decreases more, contributing also to a more pronounced house price drop, relative to a model that abstracts from a banking system. Residential investments follow a similar pattern. With lower house prices, the Borrowers' net worth channel is reinforced, as more dramatic drop in house prices imply lower Borrowers' consumption, which reduce aggregate consumption by an additional 1.8% over the drop of 1.6% generated in a model without banking sector.

Total lending decreases as well, and the heterogeneous model version also generates an extra amplification. However, the drop is less pronounced relative to a model that abstracts from the banking sector. This occurs because, on impact, low-LTV type Borrowers endogenously experience easier credit standards, allowing them to increase their mortgage demand, at a given lower house price. Despite the fact that the fraction of this group is 74% of the full sample, total lending still decreases but with smaller magnitude compared to the baseline model. The model shows a switching of funding allocation in the banking strategy. Since the collateral value shrinks in a declining asset market, banks will be less willing to lend out to high-LTV type because those borrowers may not be able to repay their debts through asset sales. Consequently, banks increase the supply of loans to the low-LTV type. Even if, some Borrowers have access to better credit standards, the negative wealth effect and larger financial accelerator mechanism put in place by the high-LTV type Borrowers, generate a more pronounced drop in their consumption, dragging down the aggregate consumption level. The interaction between the bank's 'financial acceleration' and the wealth of high-LTV type Borrowers thus generate a much deeper recession.

[Fig. 4 about here.]

5.3 Extended Model: Other Shocks

In this section we analyze the effects of TFP, housing demand and monetary policy shocks. Figures 5 and 6 plot impulse responses to a 1% productivity increase (first row), to a 1% increase in housing preference (second row) and to a 1% increase in the policy rate. The case of the 'homogeneous scenario' is depicted with a solid line, the 'heterogeneous scenario' with a dotted line. We report only basic variables for these shocks.

An increase in productivity leads to an increase in output and total consumption. The increased TFP leads to a housing boom, which generates an increase in house prices due to Borrowers' increased housing demand. Demand for credit increases as well, at aggregate and group levels. Borrowers with a high LTV ratio show a pronounced wealth effect and a substantial increase in demand for credit. In comparison to the homogenous setting, the model version with heterogeneous borrowers generates a higher response on total lending. Total consumption also shows moderate amplification. The remaining real variables are invariant to the use of a homogeneous scenario versus a heterogenous one. The response of asset prices is also similar, as the additional increase in the demand of goods and housing from Borrowers with higher LTV ratio in the heterogeneous borrower case nets out the less pronounced increase from Borrowers with a lower LTV ratio.

The second row of Figures 5 and 6 plots impulse responses to an increase in housing preferences, or similarly an increase in the demand for housing, which generates a prolonged increase in house prices. This shock increases Borrowers' collateral capacity, allowing them to borrow and consume more. Because of a higher marginal propensity to consume, Borrowers increase their consumption of goods and housing, which, despite Savers reducing them, strengthens the increase in total consumption. The impact on total consumption is quantitatively small, however the heterogeneous case generates an additional amplification effect to some degree; on impact, consumption rises 0.37% instead of only 0.21% in the homogeneous case. The response to total lending is, again, more strongly amplified, rising 0.93% in the case of heterogeneous borrowers compared to only 0.79% in the homogeneous case.

Third row of Figures 5 and 6 plot impulse responses to an contractionary monetary policy. Consumption, output and asset prices all decrease. Again, the heterogeneous case generates greater amplification in total lending, leading to an decrease of 0.9% over 0.7% in the homogenous case, while the output and total consumption responses are virtually identical.

[Fig. 5 about here.]

[Fig. 6 about here.]

In summary, comparing the heterogeneous model version with the homogenous model version, standard shocks (productivity, housing preference, monetary shocks) generate an additional amplification in the level of household indebtedness, coming from the heterogeneity in wealth and marginal propensities to consume of low-LTV and high-LTV type borrowers, yet there is only modest to no influence on other macroeconomic variables, such as output or aggregate consumption. We conclude that it is thus primarily risk shocks, as documented in previous sections, for which a model with heterogeneous borrowers is able to generate a more pronounced amplification in real and financial variables in the transmission mechanism.

6 Conclusion

This paper sheds light on the importance of borrower heterogeneity for the quantitative consequences of real and financial variables in response to a deleveraging episode, that results from an increase in housing investment risk in a risky mortgage market. We contrast two model versions: a model version with explicit consideration of a low-LTV type and a high-LTV type borrower group, that accounts for the empirical stylized fact that households' loan-to-value ratios vary significantly over its distribution; and, a model version where a representative borrower faces a loan-to-value ratio equal to the mean value of the loans distribution.

The contractionary effects of a credit disruption that falls more heavily on high-LTV type borrowers, as in the data, are orders of magnitude more severe compared to a standard model with a representative borrowing agent, despite the same economy-wide drop in LTV across the two settings. Output and aggregate consumption drop three times as much, and an additional amplification is obtained also for the responses of total household debt level, housing investment and asset prices.

In an extended model version we add a banking sector, which offers an additional channel of amplification, because banks are themselves leveraged agents. Since the collateral value shrinks in a declining asset market, banks will be less willing to lend out to high-LTV type Borrowers because of their inability to repay debts through asset sales. The interaction between banks' financial friction and the wealth effects of high-LTV type Borrowers generates an even deeper recession.

Other than the housing investment risk shock, we consider also more standard sources of shocks (productivity, housing preferences, monetary policy) and find that the model version with heterogeneous borrowers leads mostly to an additional amplification in total lending, and less so for other aggregate variables.

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Borrower heterogeneity within a risky mortgage-lending market

Technical Appendix

A Data and Sources

Aggregate Consumption. Real Personal Consumption Expenditure (seasonally adjusted, billions of chained 2005 dollars), divided by the Civilian Noninstitutional Population (Source: Bureau of Labor Statistics). Source: Bureau of Economic Analysis (BEA).

Gross Domestic Product. Real Gross Domestic Product (seasonally adjusted, billions of chained 2005 dollars), divided by CNP16OV. Source: BEA.

Residential Investment. Real Private Residential Fixed Investment (seasonally adjusted, billions of chained 2005 dollars), divided by CNP16OV. Source: BEA.

Inflation. Quarter on quarter log differences in the implicit price deflator for the nonfarm business sector, demeaned. Source: Bureau of Labor Statistics (BLS).

Nominal Short-term Interest Rate. 3-month Treasury Bill Rate (Secondary Market Rate), expressed in quarterly units. Source: Board of Governors of the Federal Reserve System.

Real House Prices. Census Bureau House Price Index (new one-family houses sold including value of lot) deflated with the implicit price deflator for the nonfarm business sector. Source: Census Bureau.

Hours in Consumption Sector. Total Nonfarm Payrolls (Source: Saint Louis Fed Fred2) less all employees in the construction sector (Source: Saint Louis Fed Fred2), times Average Weekly Hours of Production Workers, divided by CNP160V. Source: BLS.

Real Wage in Consumption-good Sector. Average Hourly Earnings of Production/ Nonsupervisory Workers on Private Nonfarm Payrolls, Total Private, divided by the price index for Personal Consumption Expenditure (source: BEA). Source: BLS.

Households and nonprofit organizations home mortgages liability (seasonally adjusted, millions of current dollars), divided by the implicit price deflator and divided by the Civilian Noninstitutional Population. Source: The Federal Reserve Board.

Seriously delinquent mortgages, not seasonally adjusted, percentage of total mortgages. Source: Mortgage Bankers Association, National Delinquency Survey.

Loan-to-value ratios: Fannie Mae and Freddie Mac database, covering around 12 million in home purchases of single-family loans issued during the period of 2000-2006.

[Fig. 7 about here.]

[Fig. 8 about here.]

Table 1: Steady-state Ratios

Variable	Homogeneous scenario	Heterogeneous scenario
Annual short-term interest rate	4.04	4.04
Consumption, Savers	67.96	68.08
Consumption, Borrowers	32.04	31.92
Consumption, Borrower low-type	16.02	16.07
Consumption, Borrower high-type	16.02	15.84
Housing Demand, Savers	71.03	70.41
Housing Demand, Borrowers	28.97	29.59
Housing Demand, Borrower low-type	14.48	13.97
Housing Demand, Borrower high-type	14.48	15.62
Hours worked, Savers	74.71	74.69
Hours worked, Borrowers	89.01	89.15
Hours worked, Borrower low-type	44.50	65.72
Hours worked, Borrower high-type	44.50	23.42
Loans	0.56	0.61
Loans low-type	0.56	0.32
Loans high-type	0.56	1.41
Loan-to-Value Ratio , avg	73.00	73.40
Loan-to-Value Ratio low-type	73.00	67.09
Loan-to-Value Ratio high-type	73.00	91.38
Default Rate on Mortgages low-type (annual)	1.24	1.67
Default Rate on Mortgages high-type(annual)	1.24	0.27
External Finance Premium low-type (annual)	0.19	0.28
External Finance Premium high-type(annual)	0.19	0.04
Mortgage Interest Rate low-type (annual)	4.30	4.38
Mortgage Interest Rate high-type (annual)	4.30	4.14

Table 2: Parameters' Values

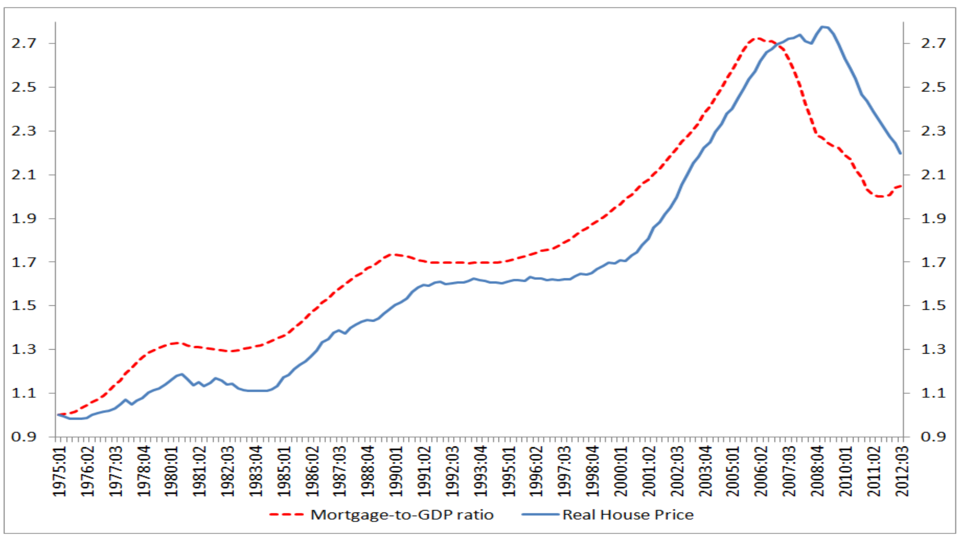
		Homogeneous scenario	Heterogeneous scenario
β_s	Discount factor Savers	0.99	0.99
β_b	Discount factor Borrowers	0.98	0.98
α_s	Fraction of Savers	0.50	0.50
α_b	Fraction of Borrowers	0.50	0.50
σ_c	Relative risk aversion on consumption	1	1
σ_H	Relative risk aversion on housing services	1	1
γ_s	Saver labor share in production	0.64	0.64
γ_L, γ_H	Borrower j type share Borrowers' labor	0.5	0.5
ν_j^n	Labor disutility parameter	1	1
η	Labor supply aversion	2	2
$\varkappa$	Housing preference parameter	0.075	0.075
X_{ss}	Marg. cost of production	11	11
ψ_k	Adj cost housing	14	14
δ_h	Housing depreciation parameter	0.0089	0.0089
θ	Calvo parameter	0.75	0.75
ϕ_π	Taylor-rule parameter, inflation	1.5	1.5
ϕ_r	Taylor-rule parameter, int. rate smoothing	0.8	0.8
ϕ_y	Taylor-rule parameter, output	0.125	0.125
ρ_z	AR(1) coefficient on TFP shocks	0.95	0.95
ρ_h	AR(1) coefficient on housing demand shocks	0.96	0.96
ρ_r	AR(1) coefficient on monetary policy shocks	0	0
σ_z	Standard deviation on TFP shocks	0.01	0.01
σ_h	Standard deviation on housing demand shocks	0.04	0.04
σ_r	Standard deviation on monetary policy shocks	0.0023	0.0023
μ_L, μ_H	Monitoring Cost	0.12	0.12
$\rho_{\omega,L}$	AR(1) coefficient on riskiness shock	0.99	0.99
$\rho_{\omega,H}$	AR(1) coefficient on riskiness shock	0.99	0.99
$\sigma_{\omega,L}$	Standard deviation on riskiness shock	0.1125	0.147
$\sigma_{\omega,H}$	Standard deviation on riskiness shock	0.1125	0.028
$\epsilon_{\sigma_{\omega,L}}$	Standard deviation on variance of riskiness shock	0.2	0.1278
$\epsilon_{\sigma_{\omega,H}}$	Standard deviation on variance of riskiness shock	0.2	0.91
n_{b_L}	Size of low-LTV Group	0.5	0.26
n_{b_H}	Size of high-LTV Group	0.5	0.74
β_{fi}	Discount factor banks	0.99	0.99
ϕ	Bank capital ratio	0.08	0.08
Γ_c	Banks' operating costs	0.0018	0.0018
Γ_c	Banks' excess capital	0.1264	0.1264
α_{fi}	Fraction of banks	0.05	0.05

Table 3: Share of Borrowers from LTV Distribution

	LTV Distribution	LTV Value	Share of Borrowers
	Panel (a) Period: 2000-2006		
Homogeneous Scenario	$0 < LTV <= 100$	0.73	100%
Heterogeneous Scenario	$0 < LTV <= 80$	0.67	74%
	$80 < LTV <= 100$	0.91	26%
	Panel (b) Period: 2008-2010		
Homogeneous Scenario	$0 < LTV <= 100$	0.71	100%
Heterogeneous Scenario		0.66	74%
		0.87	26%
	Panel (c) Period: 2009		
Homogeneous Scenario	$0 < LTV <= 100$	0.69	100%
Heterogeneous Scenario		0.64	74%
		0.85	26%

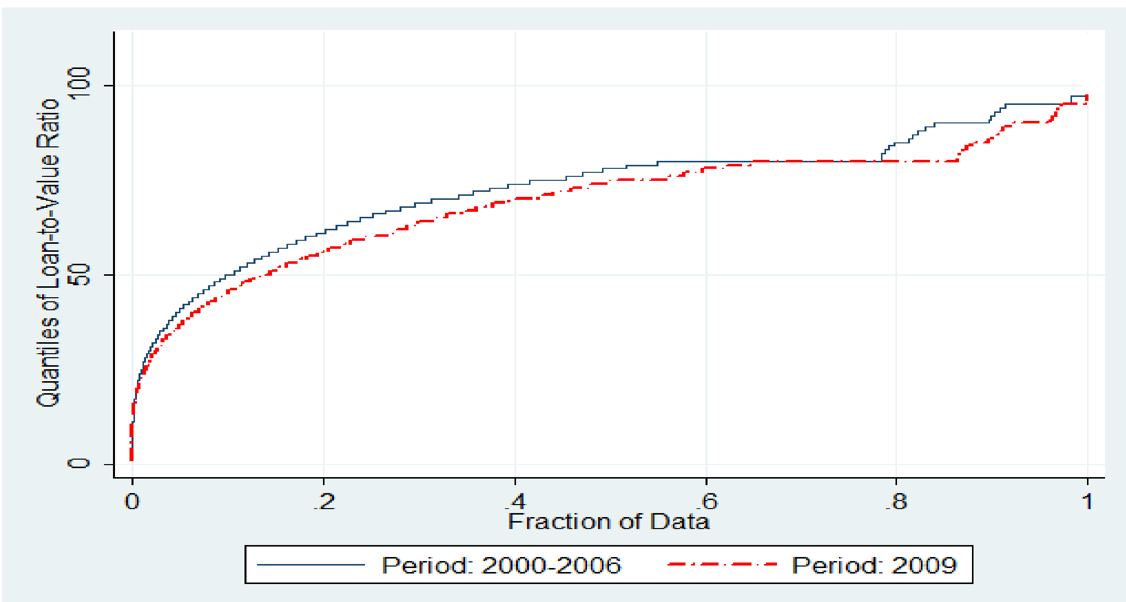
Data Sources: Fannie Mae and Freddie Mac database (holdings of 12 million in home purchases of single-family loans).

Fig. 1: Real house price (left side) and mortgage-to-real estate ratio (right side) for the U.S



Data Sources: Home mortgages of U.S. households and nonprofit organizations (Flow of Funds). Real new one-family houses sold including value of lot deflated with the implicit price deflator for the nonfarm business sector (Census Bureau).

Fig. 2: Loan-to-Value Distribution



Data Sources: Fannie Mae and Freddie Mac database (holdings of 12 million in home purchases of single-family loans).

Fig. 3: Idiosyncratic Housing Investment Risk Shock in the Baseline Model: 'homogeneous borrowers' version (solid black line) versus 'heterogeneous borrowers' version (dotted blue line). All impulse responses are expressed in % deviations from steady states, except the LTV ratios which are expressed in levels.

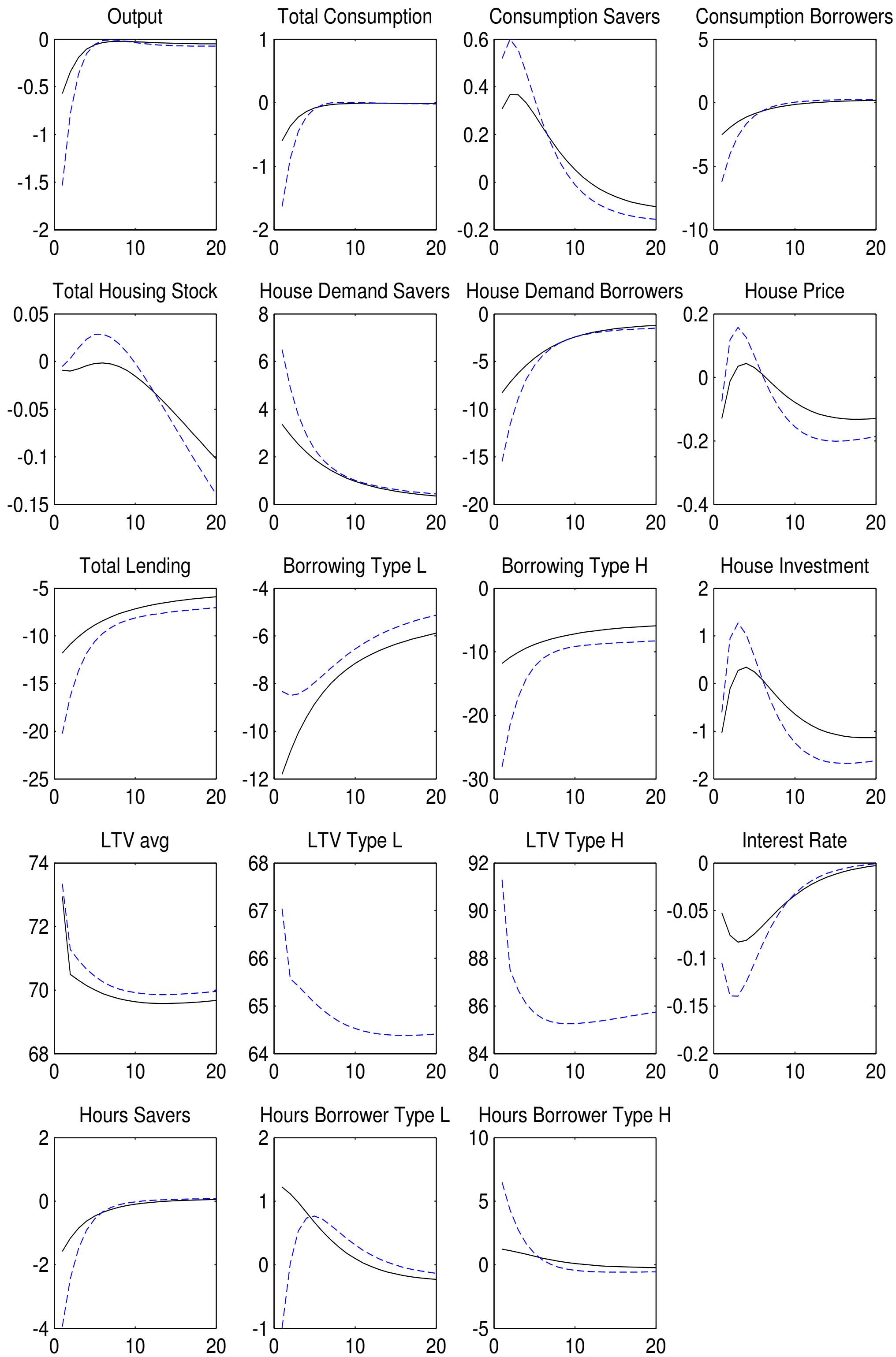
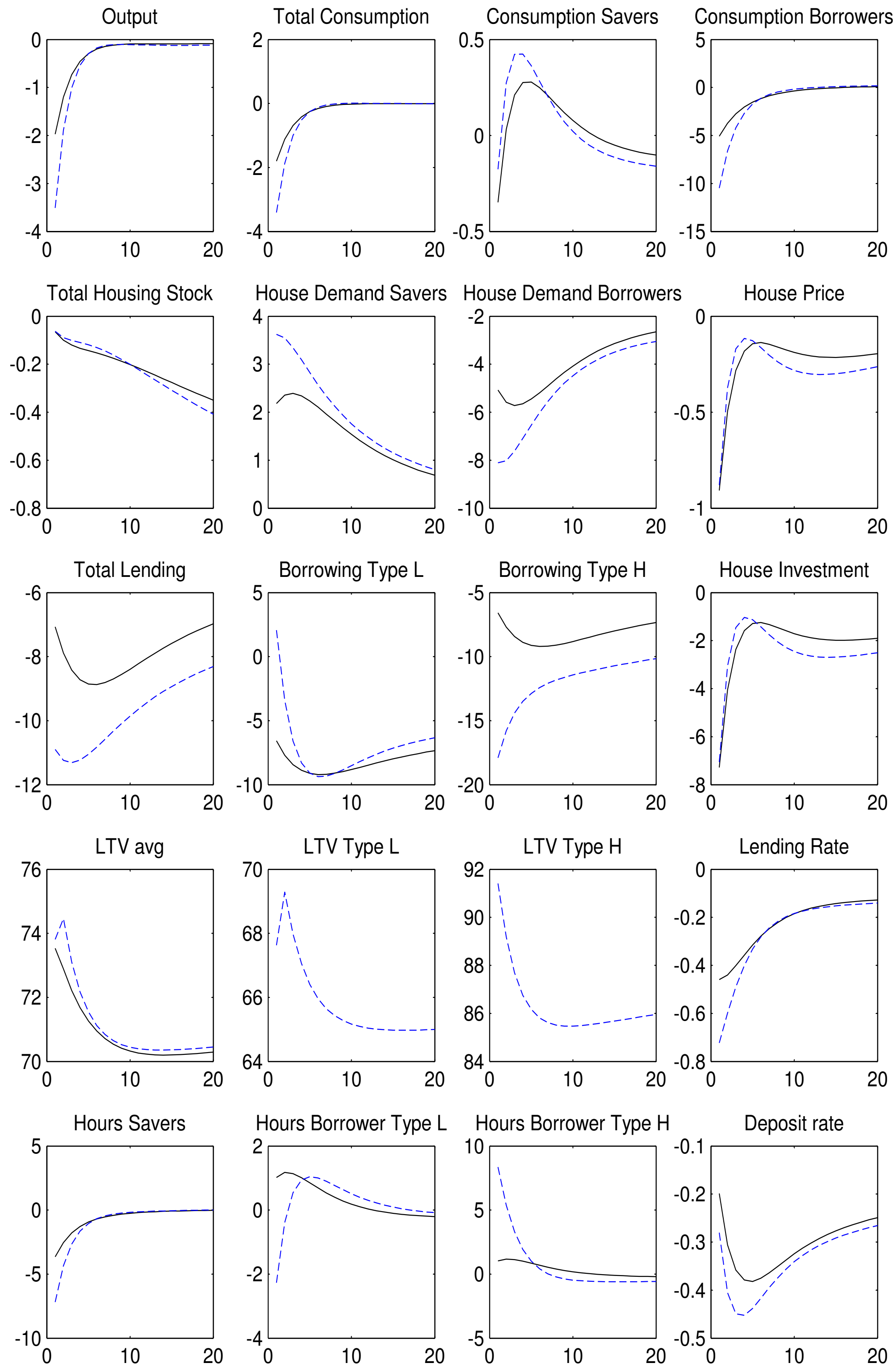


Fig. 4: Idiosyncratic Housing Investment Risk Shock in the Extended Model: 'homogeneous borrowers' version (solid black line) versus 'heterogeneous borrowers' version (dotted blue line). All impulse responses are expressed in % deviations from steady states, except the LTV ratios which are expressed in levels.



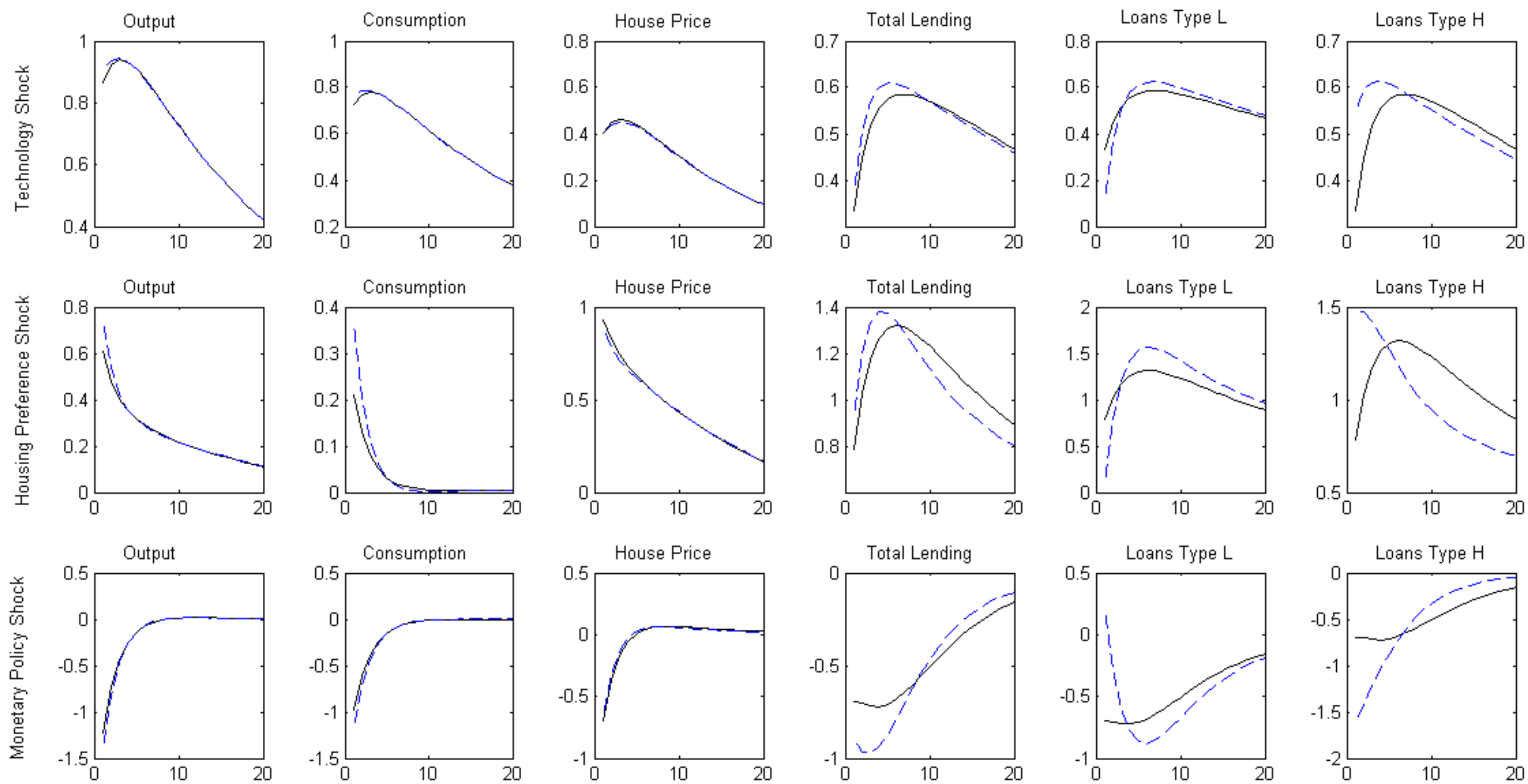


Fig. 5: Different Sources of Shock in the Extended Model. Homogeneous model version (solid line) versus heterogeneous model version (dashed line). All impulse responses are expressed in % deviations from steady state.

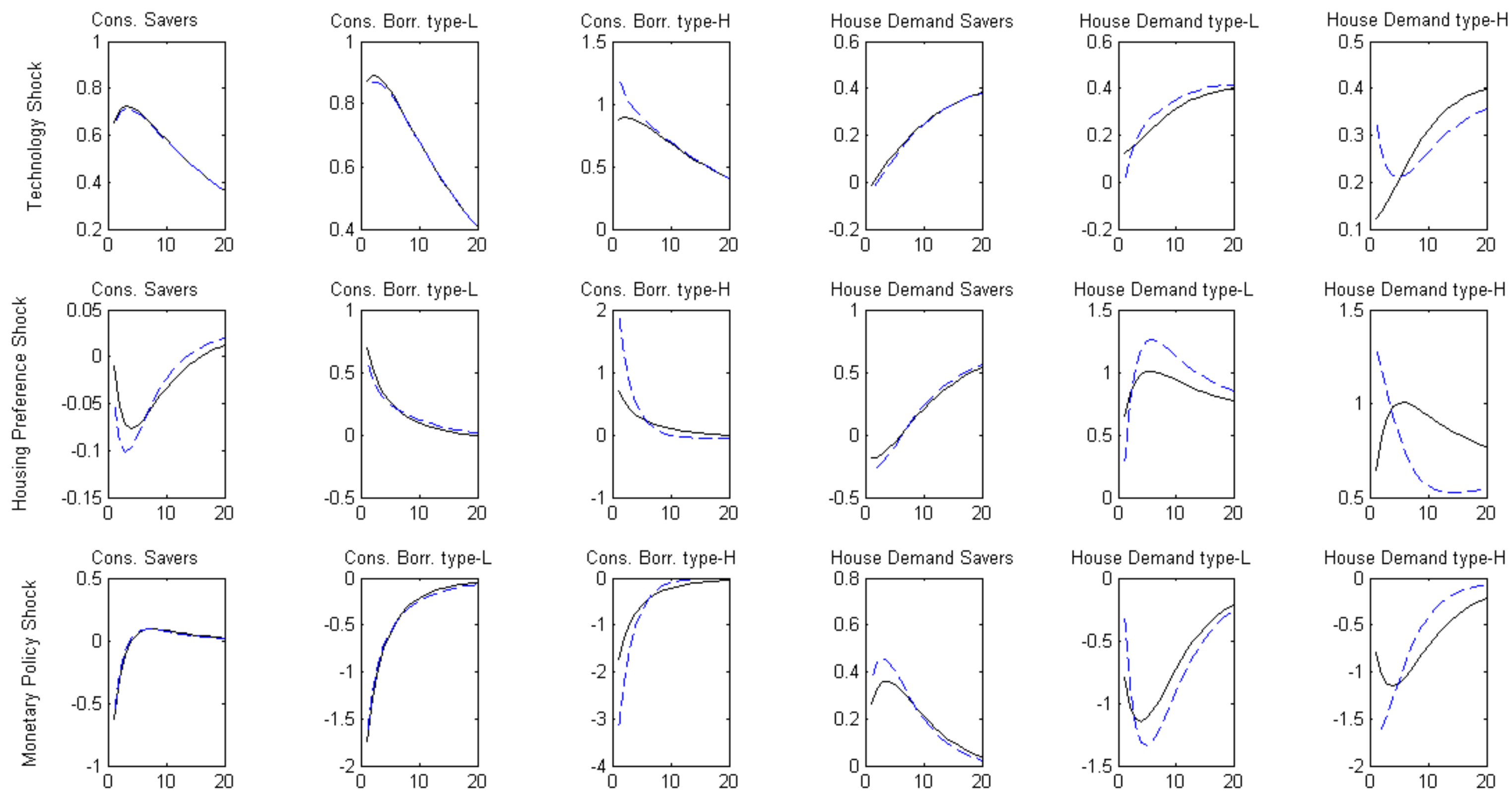
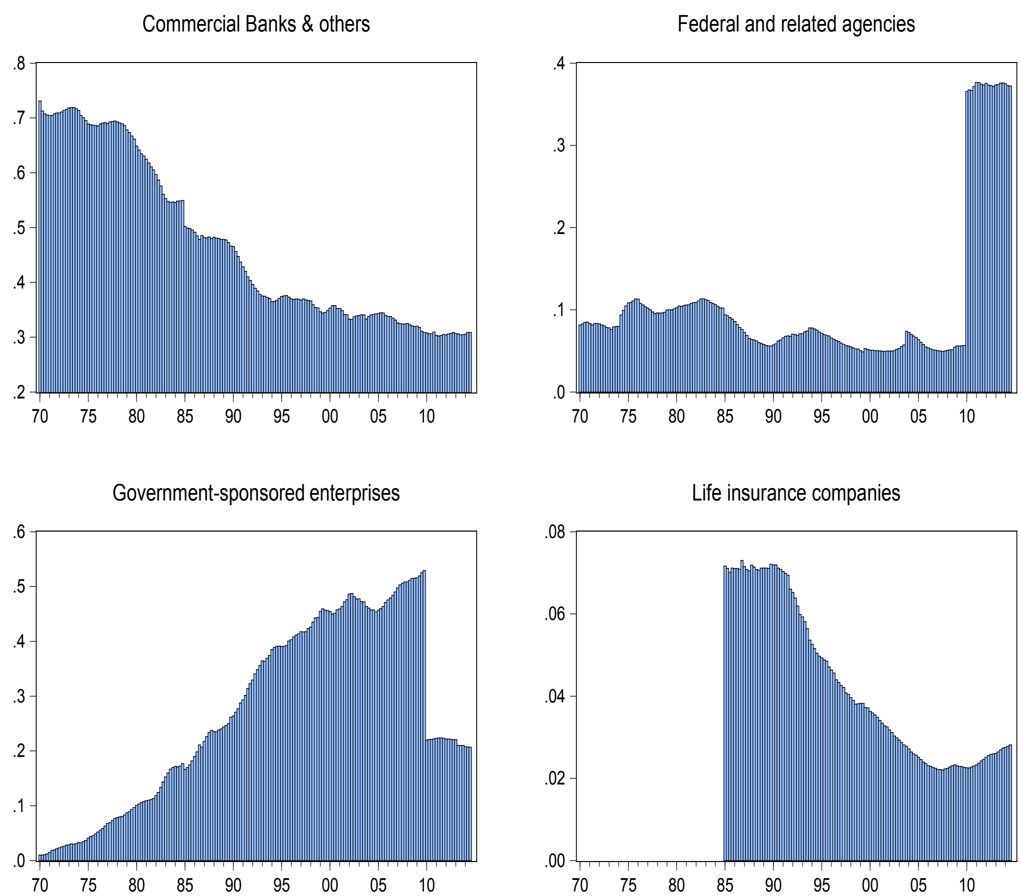


Fig. 6: Different Sources of Shock in the Extended Model. Homogeneous model version (solid line) versus heterogeneous model version (dashed line). All impulse responses are expressed in % deviations from steady state.

Fig. 7: Funding for Mortgages. In percent, by source. Data 1970q1-2014q1.



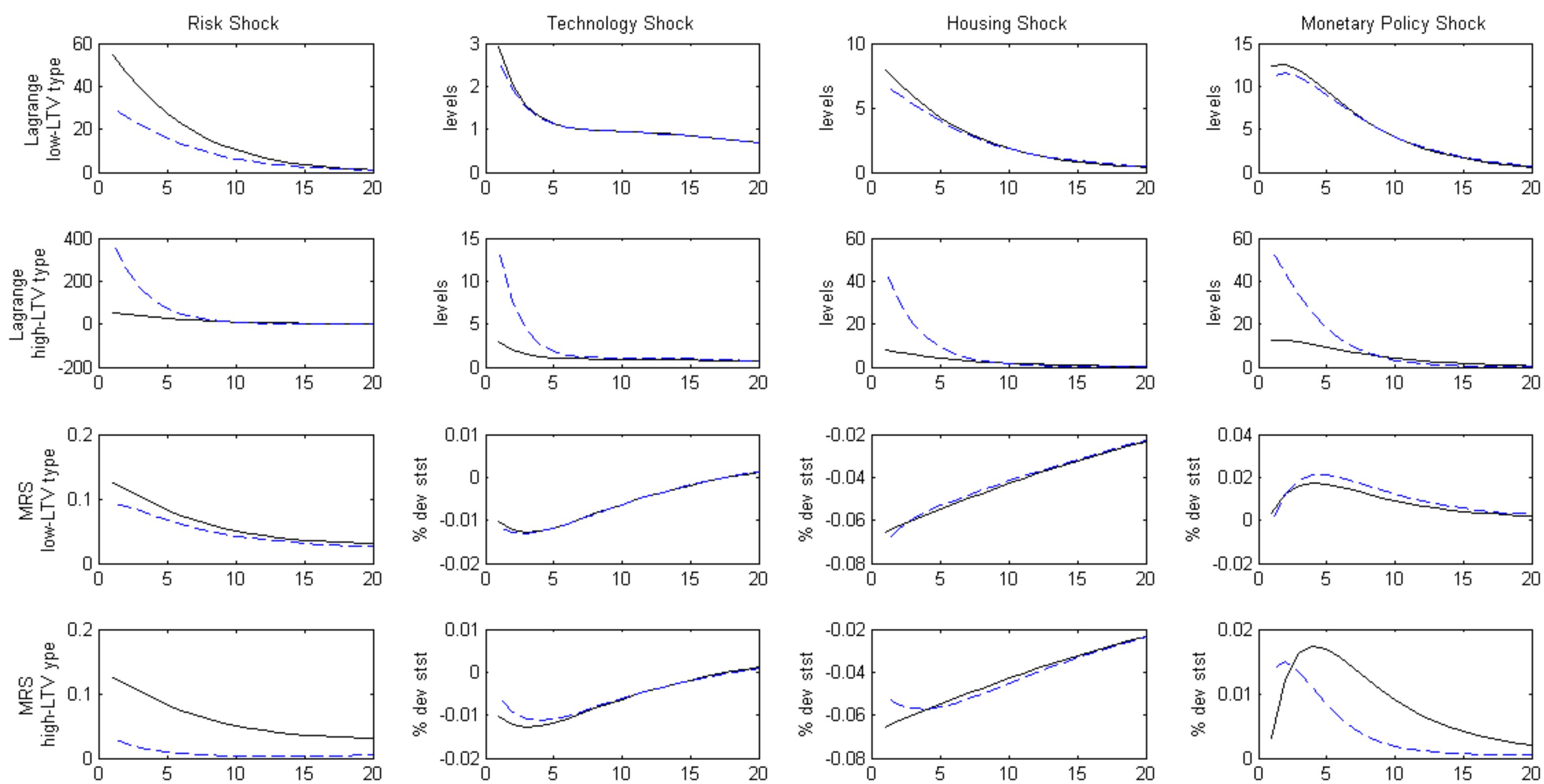


Fig. 8: Different Sources of Shock in the Benchmark Model. Homogeneous model version (solid line) versus heterogeneous model version (dashed line). Lagrange multipliers on borrowing constraint are expressed in level. MRS=marginal rate of substitution between houses and consumption. MRS are expressed in % deviations from steady state.