The Role of Firm-Level Productivity Growth for the Optimal Rate of Inflation

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Motivation

Central banks all over the world pursue inflation targets.

A core concern of central banks is to understand how the levels of their inflation target affect the economy.

Existing work studies the role of, e.g., transaction frictions, downwardly rigid wages, sticky prices, and the ZLB for the optimal inflation target, i.e., long-run inflation rate.

This paper studies the role of “firm-level productivity growth” and sticky prices for the optimal choice of the long-run inflation rate.

Motivation

Empirical work shows that productivity growth rates of *new firms* and *incumbent firms* systematically differ from one another.

Learning By Doing:

- New firms with little experience see their productivity grow faster than incumbent firms with a lot of experience (e.g., Bahk Gort 1993, Cooper Johri 2005).

Embodied Productivity Growth:

- Best-practice technology is embodied in new firms and improves at a faster rate than technology installed in incumbents (e.g., Jensen, McGuckin, Stiroh 2001).

Plan of talk

Analysis:

Examine New Keynesian model with firm-level productivity growth & sticky prices.

Quantify optimal long-run inflation rate using a calibration based on firm-level data.

Result:

The optimal long-run inflation rate is positive and between 0.5% and 1.5% per year, despite sticky prices.

CB faces a tradeoff absent in other models, namely to minimize the distortions in either new or incumbent firms.

CB resolves this tradeoff depending on the relative importance of each set of firms.

Main mechanism

In basic NK model without FLPG, a positive (long-run) inflation rate distorts real prices of *all* firms and, thus, depresses output.

With FLPG, a positive inflation rate still distorts real prices of *incumbent* firms.

At the same time, however, it reduces distortions in real prices of *new* firms.

Productivity of new firms grows fast relative to productivity of incumbent firms.

Thus, new firms see their marginal costs decline, and want to reduce their price.

A positive inflation rate erodes real prices provided *nominal* prices are sticky.

Price erosion helps new firms to align their real price with marginal costs.

Related work

Related work assumes that firms have common productivity (Kahn et al. 2003, Kim Ruge-Murcia 2009, Schmitt-Grohe Uribe 2010, Billi 2011, Coibion et al. 2012).

In the data, however, firms (products, plants) differ substantially in their productivity.

Assuming firm-level productivity *shocks* greatly helps to reconcile macro models with key facts about prices (e.g., Nakamura Steinsson (2008)).

In these macro models, a productivity increase or decrease is equally likely.

On average, thus, a firm maintains its rank in the productivity distribution.

Related work

In the data, however, firms tend to systematically change their rank in the productivity distribution over their lifetime as a result of:

- Learning by doing (e.g., Bahk Gort 1993, Cooper Johri 2005).
- Embodied productivity growth. (e.g., Jensen, McGuckin, Stiroh 2001).
- Incumbent productivity growth (e.g., Foster, Haltiwanger, Krizan 2006).
- Various productivity growth rates in economic sectors (e.g., Wolman 2011).

I explore the role of these factors for the optimal long-run inflation rate.

Model overview

* New Keynesian model with nominally sticky prices.
* A firm's productivity grows deterministically and exogenously.
* Firms (products, plants) enter and exit the market at exogenous rate δ.
* Exiting firms are randomly selected.
* A new firm sets its price optimally.
* Incumbent firms set prices with probability $(1 - \alpha)$ in each period.
* There are no firm-level productivity shocks.

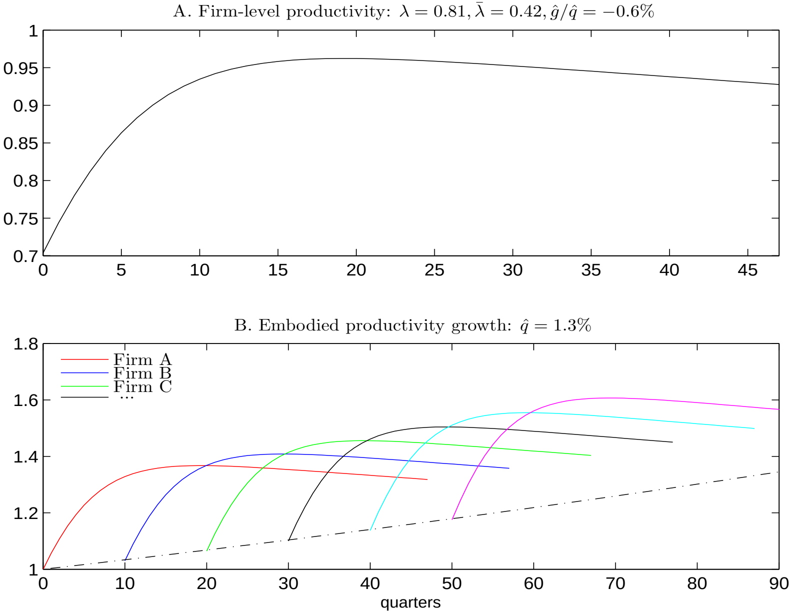
Technology of firm j

$$y_{jt} = \hat{q}^{t-s_{jt}} \left(\frac{\hat{a}^t \hat{g}^{s_{jt}}}{1 + \overline{\lambda} \lambda^{s_{jt}}} \right) \ell_{jt}$$

* $s_{jt} = 0, 1, 2, \ldots$ is firm's age.
* $\hat{q}$ is *embodied* productivity growth.
* $\hat{g}$ is *incumbent* productivity growth.
* $\overline{\lambda} \geq 0$ and $0 \leq \lambda < 1$ are scope and speed of learning by doing (LBD).
* $\hat{a}$ is *common* productivity growth.

Rewrite technology as:

$$y_{jt} = \left(\frac{a^t g^{s_{jt}}}{1 + \overline{\lambda} \lambda^{s_{jt}}} \right) \ell_{jt} , \quad \text{with} \quad a = \hat{a}\hat{q}, \quad g = \hat{g} / \hat{q}.$$


A. Firm-level productivity: $\lambda = 0.81, \bar{\lambda} = 0.42, \hat{g}/\hat{q} = -0.6\%$
1
0.95
0.9
0.85
0.8
0.75
0.7
0
5
10
15
20
25
30
35
40
45
B. Embodied productivity growth: $\hat{q} = 1.3\%$
Firm A
Firm B
Firm C
...
1.8
1.6
1.4
1.2
1
0
10
20
30
40
50
60
70
80
90
quarters

Firm j's pricing decision

Optimal price of firm $j \in [0, 1]$ maximizes expected discounted profits ($\kappa = \alpha(1 - \delta)$):

$$\max_{P_{jt}} \sum_{i=0}^{\infty} \kappa^i \, \Omega_{t,t+i} \left[P_{jt} y_{jt+i} - W_{t+i} y_{jt+i} \left(\frac{1 + \bar{\lambda} \lambda^{s_{jt+i}}}{a^{t+i} g^{s_{jt+i}}} \right) \right]$$

$$s.t. \quad y_{jt+i} = \left(\frac{P_{jt}}{P_{t+i}} \right)^{-\theta} y_{t+i}$$

When the firm sets its price, it anticipates evolution of its productivity.

Optimizing firms differ in their productivity and, thus, set various optimal prices.

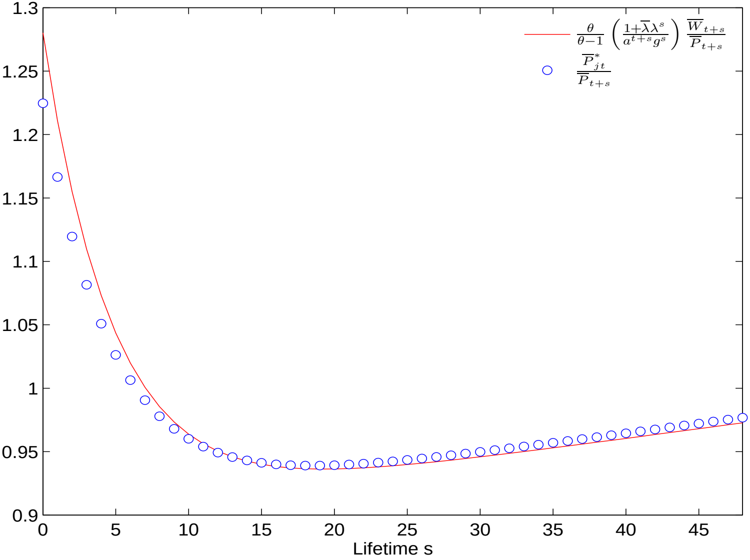
Firm j's optimal price

In the steady state, the pricing equation of a new firm is:

$$0 = \sum_{s=0}^{\infty} (\kappa\beta\pi^{\theta})^{s} \left[\frac{\overline{P}_{jt}^{\star}}{\overline{P}_{t+s}} - \frac{\theta}{\theta - 1} \left(\frac{1 + \overline{\lambda}\lambda^{s}}{a^{t+s} g^{s}} \right) \frac{\overline{W}_{t+s}}{\overline{P}_{t+s}} \right].$$

New firm anticipates the decline in its future marginal costs.

Therefore, it sets a lower optimal price, $\overline{P}_{jt}^{\star}$, than without declining marginal costs.



$\frac{\theta}{\theta-1}\left(\frac{1+\overline{\lambda}\lambda^s}{a^{t+s}g^s}\right)\frac{\overline{W}_{t+s}}{\overline{P}_{t+s}}$
$\frac{\overline{P}^*_{jt}}{\overline{P}_{t+s}}$
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0
5
10
15
20
25
30
35
40
45
Lifetime s

Representative household

$$\max_{\{B_t,\ell_t,c_{jt}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(\ell_t)\right] \quad s.t. \quad c_t = (\int_0^1 c_{jt}^{1-1/\theta}\, dj)^{\frac{\theta}{\theta-1}}$$

$$B_t + \int_0^1 P_{jt} c_{jt}\, dj \leq (1 + i_{t-1})B_{t-1} + (1-\tau_L)W_t \ell_t + T_t + D_t \ .$$

Price level $P_t = \int_0^1 (c_{jt}/c_t)\, P_{jt}\, dj$, and $\pi_t = P_t/P_{t-1}$ denotes the inflation rate.

Steady state price level

Firm-level growth and staggered pricing yield 2-dimensional firm heterogeneity.

Thus, denote price of firm j as $\quad P_{jt} = P^{\star}_{t-(n+k),t-k}$ with $n, k = 0, 1, 2, \dots$

Weighted average price of the cohort with age $s \geq 0$ is:

$$\Lambda_t(s) = \begin{cases} (1-\alpha)\sum_{k=0}^{s-1}\alpha^k \left(P^{\star}_{t-s,t-k}\right)^{1-\theta} + \alpha^s \left(P^{\star}_{t-s,t-s}\right)^{1-\theta} & \text{if} \quad s \geq 1\ , \\ \left(P^{\star}_{t,t}\right)^{1-\theta} & \text{if} \quad s = 0\ . \end{cases}$$

Unit mass of operating firms is composed of many entry cohorts:

$$1 = \delta \sum_{s=0}^{\infty} (1-\delta)^s\ .$$

Steady state price level

Akin to Dotsey, King, Wolman 1999, aggregate price level is:

$$P_t^{1-\theta} = \delta \sum_{s=0}^{\infty} (1-\delta)^s \Lambda_t(s) \ ,$$

Pricing equation implies that optimal prices of firms with different levels of productivity map into one another (N is a function of parameters):

$$\overline{P}^{\star}_{t-k,t-k} = g^n \left(\frac{1+\overline{\lambda} N}{1+\lambda^n \overline{\lambda} N} \right) \overline{P}^{\star}_{t-(n+k),t-k} \ .$$

E.g., with $k=0$, optimal price of a new firm $\propto$ optimal prices of incumbent firms.

Steady state price level

Combining these equations yields, using $p^\star = \overline{P}^\star_{t,t}/\overline{P}_t$:

$$
\begin{aligned}
1 &= \left\{\delta + (1-\alpha)\overline{m}\right\}(p^\star)^{1-\theta} + \kappa\pi^{\theta-1} \\
\overline{m} &= \delta\sum_{n=0}^{\infty}(1-\delta)^{n+1} g^{(n+1)(\theta-1)}\left(\frac{1+\overline{\lambda}N}{1+\lambda^{n+1}\overline{\lambda}N}\right)^{\theta-1} .
\end{aligned}
$$

$\overline{m}$ contains marginal costs of incumbent firms that reset prices at date t, relative to marginal costs of new firm.

Steady state aggregate technology

Aggregate output $\overline{y}_t$ grows at rate $a = \hat{a}\hat{q}$ along the balanced growth path.

Denote steady state output as $y = \overline{y}_t / a^t$, and combine firms' technology to:

$$
\begin{aligned}
y &= \ell / \Delta \\
\Delta &= \int_0^1 \left(\frac{1 + \overline{\lambda} \lambda^{s_{jt}}}{g^{s_{jt}}} \right) \left(\frac{\overline{P}_{jt}}{\overline{P}_t} \right)^{-\theta} dj \, .
\end{aligned}
$$

Firm-level growth alters *level* of aggregate productivity, $1/\Delta$, and thus output y.

Price dispersion makes household substitute expensive for cheap products. Uneven distribution of output across firms reduces output y.

Aggregate steady state

In aggregate steady state, decentralized economy can be summarized as

$$\frac{h_\ell(\ell)}{u_c(y)}\left(\frac{\mu(\pi)}{1-\tau_L}\right)=\frac{1}{\Delta_e} \quad , \qquad y=R(\pi)\frac{\ell}{\Delta_e} \; .$$

$1/\Delta_e$ is *efficient amount of output dispersion in the planned economy*.

$R(\pi)=\Delta_e/\Delta$ is effect of relative price dispersion on output.

$\mu(\pi)=1/(w\Delta_e)$ is average markup.

$R(\pi)$ and $\mu(\pi)/(1-\tau_L)$ represent all distortions in the decentralized economy.

Calibration for US economy

A time period t corresponds to one quarter.

Preferences are log-utility of consumption and $h(\ell) = \eta_L \ell^{1+\nu}/(1+\nu)$.

Parameter	Value	Description
a	2% p.a.	Aggregate growth $a = \hat{a}\hat{q}$ (BFK 2006)
$\hat{a}$	0.7% p.a.	Common growth (derived)
$\hat{g}$	0.7% p.a.	Incumbent growth (JMS 2001)
$\hat{q}$	1.3% p.a.	Embodied growth (JMS 2001)
$\bar{\lambda}$	0.42	Scope for learning
λ	0.81	Speed of learning
δ	8% p.a.	12 years expected lifetime (JMS 2001)
$(1-\alpha(1-\delta))^{-1}$	2 qrts	Median price duration (BK 2004)
θ	4.33	30% steady-state markup
ν	0.25	Labor supply elasticity
β	0.995	4% p.a. real interest rate
τ_L	-0.30	labor income tax offseting $\theta/(\theta-1)$

Calibration for US economy

$\overline{\lambda}$ and λ determined jointly by relative size of new firms and speed of learning:

- Set relative size (hours worked per firm) to 60% (JMS 2001).
- Assume it takes 2 years for firms to close 75% of learning gap, $1 - 1/(1 + \overline{\lambda})$ (Hornstein Krusell 1996 use 1 year, Yorukoglu 1998 uses 3 years).

Calibration implies:

- Productivity of firms with age ≤ 5 years relative to average productivity is 99%.
- Price of new firms with age ≤ 5 years relative to average price level is 101%.

The optimal inflation rate in the model without LBD

Case without LBD, $\overline{\lambda} = 0$ and $\lambda = 1$, allows to derive analytical results.

Interpret as economy with only incumbent firms with no further increments in learning.

Proposition 1:

Without LBD and using $g = \hat{g}/\hat{q}$, the optimal long-run inflation rate that maximizes steady state welfare is:

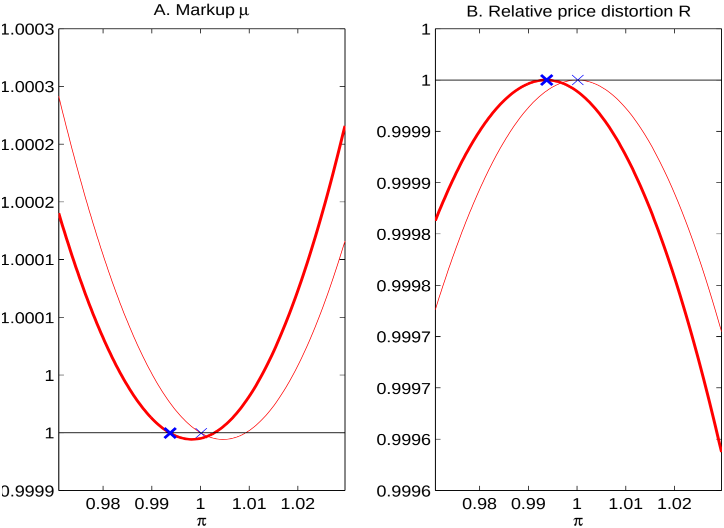
$$\begin{aligned} \pi^{\star} &= \hat{g}/\hat{q} \\ &= -\mathbf{0.63}\% \text{ per year.} \end{aligned}$$

The optimal inflation rate in the model without LBD

A new firm sets its price according to (using $w = \overline{w}_t / (\hat{a}\hat{q})^t$ and $p^\star = \bar{P}^\star_{t,t} / \bar{P}_t$)

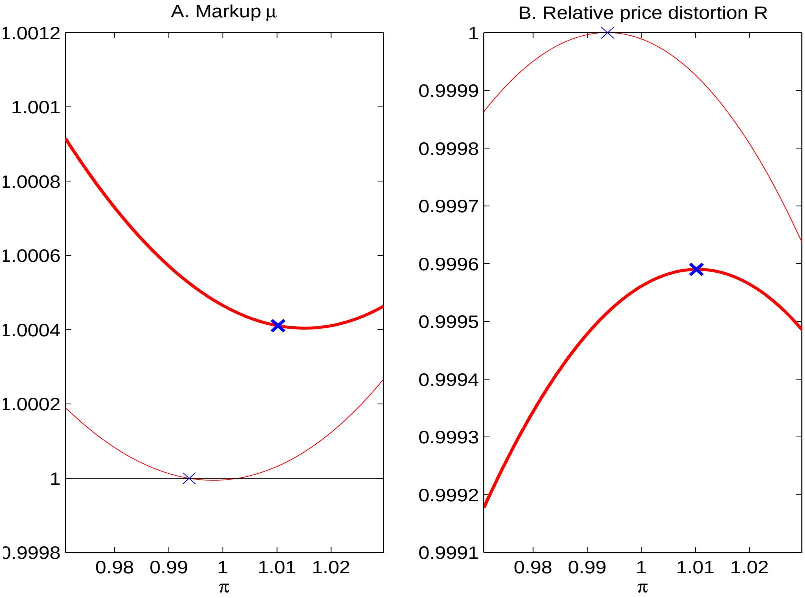
$$0 = \sum_{s=0}^{\infty} (\kappa \beta \pi^{\theta})^s \left[\frac{p^\star}{\pi^s} - \frac{\theta}{\theta - 1} \frac{w}{(\hat{g}/\hat{q})^s} \right] .$$

- $\hat{g} > 1$ indicates positive productivity growth of incumbent firms.
- Relative to newest firms, however, incumbents become obsolete at rate $\hat{g}/\hat{q} < 1$.
- Thus, real marginal costs of incumbent firms increase.
- Thus, firm's desired real price (markup over real marginal costs) also increases.
- $\pi < 1$ maps sticky nominal prices, which cannot increase, into increasing real prices.



$$\frac{h_\ell(\ell)}{u_c(y)}\left(\frac{\mu(\pi)}{1-\tau_L}\right)=\frac{1}{\Delta_e} \qquad y = R(\pi)\frac{\ell}{\Delta_e}$$

The optimal inflation rate in the model **with LBD is $+1.04$% per year**



The optimal inflation rate in the model with LBD

With LBD, CB faces a **tradeoff** and, thus, no longer achieves efficient allocation.

With LBD, a new firm sets its price as

$$0 = \sum_{s=0}^{\infty} (\kappa\beta\pi^{\theta})^s \left[\frac{p^\star}{\pi^s} - \frac{\theta}{\theta - 1} \left(\frac{1 + \overline{\lambda}\lambda^s}{(\hat{g}/\hat{q})^s} \right) w \right] .$$

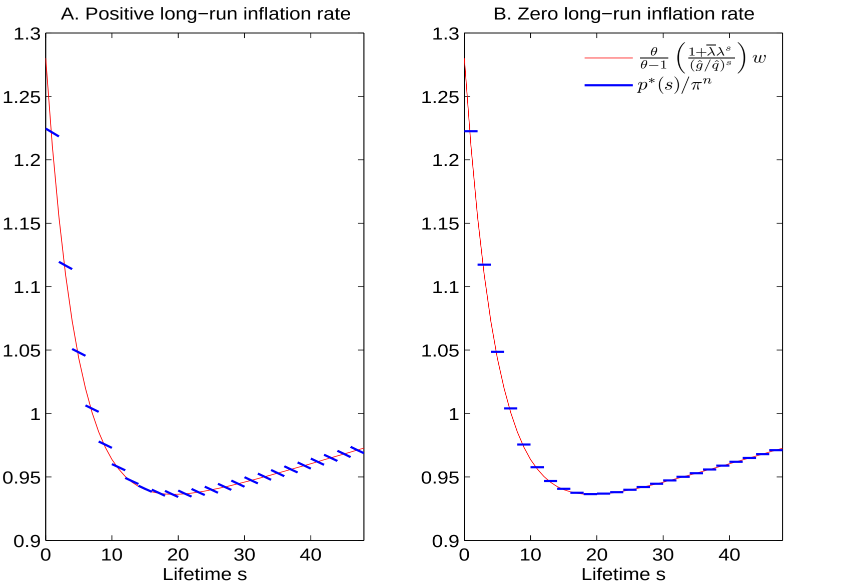
The inflation rate that is optimal for the new firm moves 1:1 with firm's productivity:

$$\pi_t^\star \propto \left(\frac{(\hat{g}/\hat{q})^t}{1 + \overline{\lambda}\lambda^t} \right) .$$

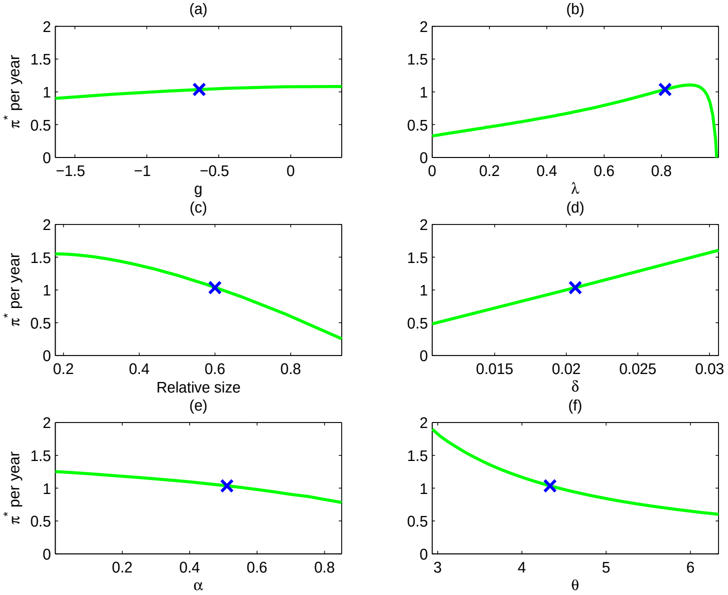
But this rate is suboptimal for old firms, say, which prefer $\pi^\star = \hat{g}/\hat{q}$.

Thus, CB can minimize distortion in **either** new **or** incumbent firms, but not both.

The role of the inflation rate in the model with LBD



Robustness of the optimal inflation rate to various calibrations



How should CB trade off distortions in new versus incumbent firms?

CB weighs new firms heavily, if their market share is large & LBD is important, i.e.:

- If it is difficult to substitute away from relatively expensive goods (θ small).
- If the number of new firms in each period is large (δ large).
- If the scope for learning is reasonably large ($\overline{\lambda}$ large).
- If the speed of learning is reasonably slow (λ large).

Price stickiness is **not** important (as long as it exists), because it affects new and incumbent firms similarly.

Role of sectoral asymmetries

Magnitude and composition of productivity growth differs across economic sectors:

- While Goods grow at 2%, Services grow at only 1% per year.
- While embodied growth accounts for 66% of total growth in Goods, it accounts for 100% in Services (Foster, Haltiwanger, Krizan 2006).

Also, prices in Services change less frequently than prices in Goods.

How do sectoral asymmetries in magnitude and composition of productivity growth and price stickiness affect the optimal inflation rate?

Model extension with sectoral asymmetries

- Two sectors, Services and Goods, denoted by $z = 1, 2$.
- Aggregate output: $y_t = y_{1t}^{\psi} \, y_{2t}^{1-\psi}$.
- Asymmetry in magnitude and composition of prod. growth: a_z, $\hat{a}_z$, $\hat{q}_z$, $\hat{g}_z$.
- Asymmetry in frequency of price changes, α_z, and survival probability, δ_z.
- (Asymmetry in LBD, $\overline{\lambda}_z$, λ_z.)

Steady state of two-sector model

In aggregate steady state, decentralized two-sector economy can be summarized as

$$\frac{h_\ell(\ell)}{u_c(y)}\left(\frac{\mu(\pi)}{1-\tau_L}\right)=\frac{1}{\Delta_e}\quad , \qquad y=R(\pi)\frac{\ell}{\Delta_e}\ ,$$

with

$$\mu(\pi) \quad = \quad \mu_1^{\psi}\mu_2^{1-\psi}$$

$$R(\pi) \quad = \quad \left[\psi\left(\frac{\mu_2}{\mu_1}\right)^{1-\psi}\rho_1^{-1}+(1-\psi)\left(\frac{\mu_1}{\mu_2}\right)^{\psi}\rho_2^{-1}\right]^{-1} .$$

$\mu_z(\pi)$ is average markup in sector z.

$\rho_z(\pi)$ is effect of RPD on output in sector z relative to planned economy.

Calibration for US economy

Sector 1 represents Services, Sector 2 represents Goods.

Parameter	Value	Description
a_1	1.08% p.a.	Sectoral growth $a_1 = \hat{a}_1\hat{q}_1$ (FHK 2006)
$\hat{a}_1$	0% p.a.	Common growth (derived)
$\hat{g}_1$	0% p.a.	Incumbent growth (FHK 2006)
$\hat{q}_1$	1.08% p.a.	Embodied growth (FHK 2006)
δ_1	11% p.a.	9 years expected lifetime
$(1-\alpha_1(1-\delta_1))^{-1}$	3 qrts	Median price duration (BK 2004)
a_2	2% p.a.	Aggregate growth $a = \hat{a}\hat{q}$ (BFK 2006)
$\hat{a}_2$	0.7% p.a.	Common growth (derived)
$\hat{g}_2$	0.7% p.a.	Incumbent growth (JMS 2001)
$\hat{q}_2$	1.3% p.a.	Embodied growth (JMS 2001)
δ_2	8% p.a.	12 years expected lifetime (JMS 2001)
$(1-\alpha_2(1-\delta_2))^{-1}$	2 qrts	Median price duration (BK 2004)
ψ	0.6	Relative size of Services (W 2011)
$\overline{\lambda}$	0.42	Scope for learning
λ	0.81	Speed of learning

The optimal inflation rate in the two-sector model without LBD

Akin to Proposition 1, the special case without LBD yields analytical results.

Proposition 2:

In the limit $\beta \to 1$, the optimal *aggregate* long-run inflation rate π solves

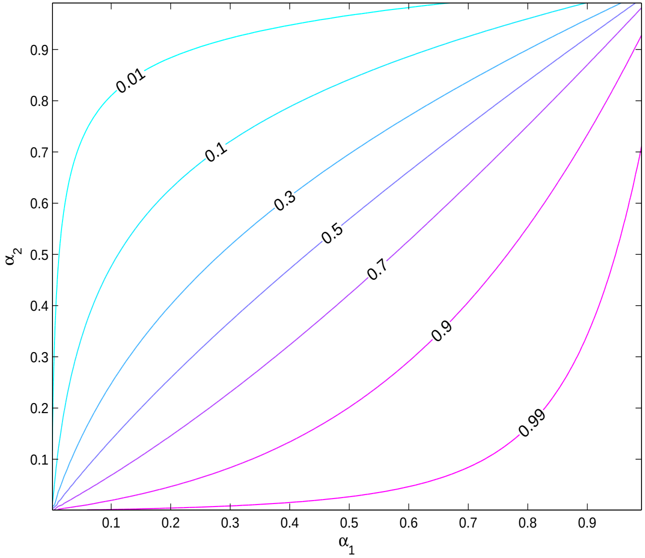
$$0 = \omega(\pi^\star)\left(\frac{\pi^\star - (g_1/\eta_1)}{(g_1/\eta_1)}\right) + (1-\omega(\pi^\star))\left(\frac{\pi^\star - (g_2/\eta_2)}{(g_2/\eta_2)}\right),$$

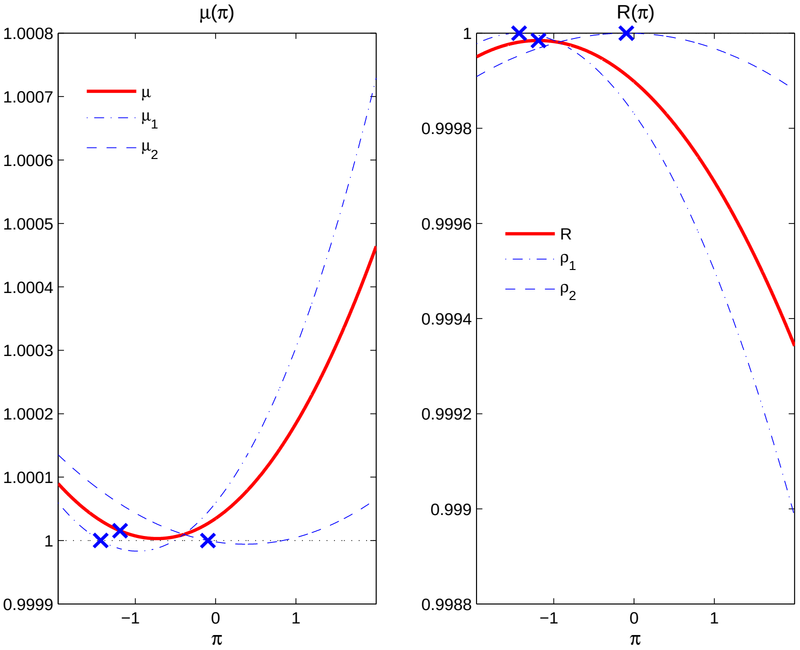
with $\eta_1 = (a_2/a_1)^{(1-\psi)}$ and $\eta_2 = (a_1/a_2)^{\psi}$.

I.e., CB faces the tradeoff $\pi^\star = g_1/\eta_1$ versus $\pi^\star = g_2/\eta_2$, and resolves it using ω.

Calibrated model yields $\pi^\star = -\mathbf{1.19}\%$ per year.

Weight $\omega(\pi^\star)$ implies that $\pi^\star$ weighs sector with stickier prices more heavily




μ(π)
1.0008
1.0007
1.0006
1.0005
1.0004
1.0003
1.0002
1.0001
1
0.9999
−1
0
1
π
μ
$μ_1$
$μ_2$
R(π)
1
0.9998
0.9996
0.9994
0.9992
0.999
0.9988
R
$ρ_1$
$ρ_2$

Special case: No sectoral productivity growth differential

With $a_1 = a_2$, optimal aggregate inflation rate solves:

$$0 = \omega(\pi^\star)\left(\frac{\pi^\star - g_1}{g_1}\right) + (1 - \omega(\pi^\star))\left(\frac{\pi^\star - g_2}{g_2}\right).$$

- Recall that $\pi^\star = g$ in one-sector model without LBD.
- With two sectors, g_z represents the inflation rate that is optimal in sector z.
- However, with $g_2 > g_1$, no π is optimal in both sectors at the same time.
- Weight ω makes CB pull $\pi^\star$ toward the Services sector with more sticky prices.

Calibrated model yields $g_1 = -1.07\%$, $g_2 = -0.64\%$, and $\pi^\star = -\mathbf{0.99}\%$ per year.

Special case: No firm-level productivity growth (Wolman 2011)

With $g_z = 1$, optimal aggregate inflation rate solves:

$$0 = \omega(\pi^\star)\left(\eta_1 \pi^\star - 1\right) + \left(1 - \omega(\pi^\star)\right)\left(\eta_2 \pi^\star - 1\right) .$$

- Since $\eta_z \pi = \pi_z$, CB strives for zero *sectoral* inflation rates, $\pi_z^\star = 1$.
- Since $a_1 \neq a_2$, relative price, $\bar{P}_{1t} / \bar{P}_{2t}$, is trending, so that $\pi_1 / \pi_2 = a_2 / a_1$.
- Therefore, using ω, CB trades off $\pi_1^\star = 1$ against $\pi_2^\star = 1$.

Calibrated model yields $\pi^\star = -\mathbf{0.20}\%$ per year, with $\pi_1^\star = .17\%$ and $\pi_2^\star = -.74\%$.

Calibration implies, survival growth (g_z) matters more than sectoral growth (a_z) for $\pi^\star$.

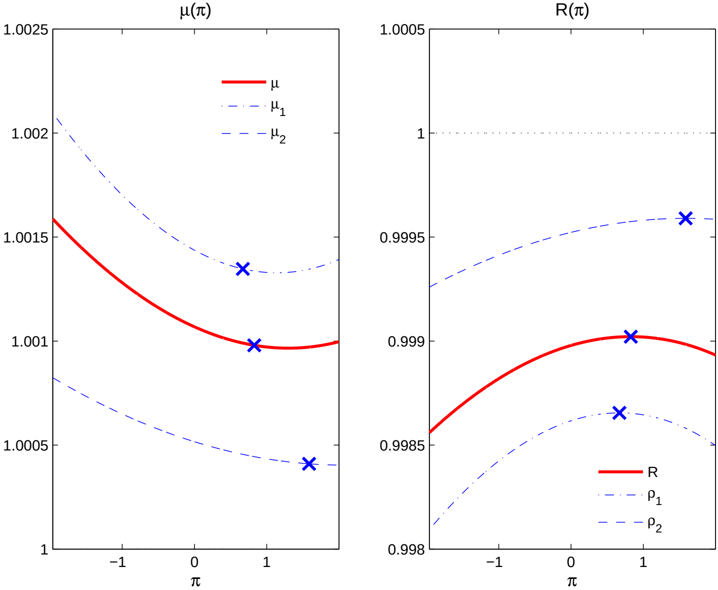
Related work

$\omega(\pi)$ incorporates "stickiness principle", i.e., optimal monetary policy weighs sector, or country, with the stickier prices more heavily (Goodfriend King 1997).

Aioki 2001, Mankiw Reis 2003, Benigno 2004: Stickiness principle applies to *temporary* changes in relative price across sectors.

Proposition 2, Wolman 2011: Stickiness principle also applies to optimal *long-run* inflation rate.

The optimal inflation rate **with two sectors and LBD is** $+0.82\%$ **per year**



The optimal inflation rate with two sectors and LBD

With two sectors and LBD, policy tradeoff btw sectors coexists with policy tradeoff btw new and incumbent firms within a sector.

The optimal price of a new firm in sector z, $p^\star_z = \bar{P}^\star_{z,t,t}/(\eta^t_z \bar{P}_t)$, fulfills:

$$0 = \sum_{i=0}^{\infty} (\kappa_z \pi_z^\theta \beta)^i \left[\frac{p^\star_z}{(\eta_z \pi)^i} - \frac{\theta}{\theta - 1} \left(\frac{1 + \bar{\lambda}_z \lambda_z^i}{g_z^i} \right) w \right] .$$

- Learning-by-doing dynamics, $\bar{\lambda}_z$, and λ_z,
- survival and vintage growth, $g_z = \hat{g}_z / \hat{q}_z$, and
- sectoral productivity growth, a_z (via η_z),

interact with price stickiness, α_z, and relative sector size, ψ, to determine the optimal inflation rate.

Summary

Incorporating firm-level productivity growth into a NK model tends to increase the optimal long-run inflation rate from zero to between 0.5 and 1.5% per year:

One-sector model with vintage & survival growth, and no LBD: $\pi^\star = -0.63\%$.

One-sector model with vintage & survival growth, and LBD: $\pi^\star = 1.04\%$.

Two-sector model with vintage & survival growth, and LBD: $\pi^\star = 0.82\%$.

Optimizing central bank trades off the distortions in new versus incumbent firms.

The more important are new firms and LBD, the higher is the optimal inflation rate.