The Role of Firm-Level Productivity Growth for the Optimal Rate of Inflation

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– PRELIMINARY AND INCOMPLETE, DO NOT QUOTE –

Abstract

Empirical data suggest that firms tend to move systematically through the productivity distribution over time. A sticky-price model with firm-level productivity growth fits this data and predicts that the optimal long-run inflation rate is positive and between 0.5% and 1.5% per year. In contrast, the standard sticky-price model cannot fit this data and predicts optimal long-run inflation near zero. The positive long-run inflation rate is geared toward the fast-growing new firms and helps them to align their real price with their firm-level productivity growth. In a two-sector extension of the model with firm-level productivity growth, the optimal long-run inflation rate weights the sector with the stickier prices more heavily.

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1 Motivation

Empirical data suggest that productivity growth rates in new firms tend to exceed productivity growth rates in incumbent firms. One prominent explanation for this is learning by doing, i.e., the idea that firms accumulate knowledge as by-product of producing goods and services.[1] Since firms' learning curves flatten out in the course of time, new firms see their productivity grow faster than incumbent firms (Bahk and Gort (1993)). Another prominent explanation for new firms expanding at a faster rate than incumbent firms is embodied productivity growth, i.e., the idea that best-practice technology is embodied in new firms and expands at a faster rate than the technology installed in incumbent firms (Jensen, McGuckin, and Stiroh (2001)).

In economies in which firms set their price based on marginal costs, firm-level productivity growth affects a firm's price setting. Ideally, a profit-maximizing firm sets its nominal price in a way that guarantees that its real price exceeds its real marginal costs by some markup. In this ideal case, the real price of new firms with relatively fast firm-level productivity growth falls over time, because firm-level productivity growth reduces firm-level marginal costs. However, when firms adjust their nominal price only infrequently, as empirical data suggest, new firms will find it difficult to reduce their real price in line with their firm-level productivity growth.

A positive long-run inflation rate erodes real prices and, therefore, can help new firms to reduce their real price over time. In contrast to new firms, however, a positive long-run inflation rate can affect incumbent firms adversely. In the case in which incumbent firms expand their productivity less than new firms, they prefer to reduce their real price less than new firms, or to even increase it. The purpose of this paper is to quantify the long-run inflation rate that optimally resolves the tradeoff between new and incumbent firms, and to identify the factors that shift the optimal long-run inflation rate in favor of either new or incumbent firms.

Using a basic New Keynesian model that incorporates firm-level productivity growth

[1] Empirical work refers to plants, products, product lines, or establishments, among other things, and I use the term "firm" to collectively refer to these units.

and new firms that expand at a faster rate than incumbent firms, I find that the optimal long-run inflation rate is between 0.5% and 1.5% per year. The model is calibrated to the US economy, and the calibration relies on firm-level data. The positive optimal inflation rate arises from the learning-by-doing dynamics in new firms. In a version of the model without learning by doing, the optimal long-run inflation rate is between -1.2% and -0.6% per year. This negative optimal inflation rate arises from embodied productivity growth, which makes the technology of incumbent firms to become obsolete at a faster rate. To preserve their profit-maximizing markup, incumbent firms therefore prefer to increasing their real price, and this requires a negative inflation rate.

Apparently, learning by doing dominates the role of embodied productivity growth for the optimal long-run inflation rate and makes this rate positive. Learning by doing triggers large and rapid changes in firm-level productivity, whereas embodied productivity growth triggers only small and gradual changes in firm-level productivity. Consequently, the price distortions in new firms from learning by doing are larger than the price distortions in incumbent firms from embodied productivity growth, and the optimal long-run inflation rate is geared towards new firms.

Another factor that emphasizes the role of new firms for the optimal long-run inflation rate and, therefore, makes this rate more positive, is a large market share of new firms. New firms have a large market share, when households find it difficult to substitute away from the relatively expensive products of new firms, and when the portion of new firms in the market is high. Interestingly, a factor that does not influence the aggregate inflation rate much is the degree of firms' price stickiness.

Firm-level productivity growth can differ among new and incumbent firms, but it can also differ among economic sectors. For example, while embodied productivity growth accounts for about two thirds of total productivity growth in Manufacturing (Sakellaris and Wilson (2004)), embodied productivity growth accounts for basically all productivity growth in Services (Foster, Haltiwanger, and Krizan (2006)). Another important difference across sectors is the degree of price stickiness. Therefore, in order to obtain a reliable estimate of the optimal long-run inflation rate, I also consider a two-sector model, which varies firm-level productivity growth and the degree of price stickiness across economic sectors.

In the two-sector model, each sector has its own optimal long-run inflation rate, and this creates a tradeoff for the government when it selects the aggregate long-run inflation rate. This tradeoff between sectors arises in addition to the tradeoff within each sector between new and incumbent firms. To resolve the tradeoff between sectors, the government tilts the optimal aggregate long-run inflation rate towards the optimal long-run inflation rate in the sector with the more sticky prices because thereby it shifts the price adjustment to the sector with the more flexible prices, where it is least distortive.

The literature on the optimal long-run inflation rate has not yet examined thoroughly the role of firm-level productivity growth, and this paper contributes to closing this gap in the literature.[2] Three related papers examine the role of firm-level and sectoral factors, but their mechanisms and results differ from the ones in this paper. Wolman (2011) examines the role of sectoral productivity growth and finds that the government weighs the sector with stickier prices more heavily and that mild deflation is socially optimal. Schmitt-Groh and Uribe (2012) examine quality bias in the officially measured inflation rate in one and two-sector models and find that price stability is optimal if non-quality adjusted prices are sticky. Finally, Janiak and Monteiro (2011) examine entry and exit of heterogenous firms in a flexible-price model with a cash-in-advance constraint and find that the long-run inflation rate affects the level of aggregate productivity.

A main finding in this paper is that firm-level productivity growth can justify a positive optimal long-run inflation rate. Recently, Billi (2011) and Coibion, Gorodnichenko, and Wieland (forthcoming), among others, show that the zero lower bound on nominal interest rates can also justify a positive optimal long-run inflation rate. Incorporating firm-level productivity growth into the analysis suggests that the welfare costs of pursuing a positive

[2]However, the literature on the optimal long-run inflation rate, which is reviewed in Schmitt-Grohe and Uribe (2010), has examined a long list of factors, and I leave many of them out of my analysis in order to focus it on a lean model. Among these factors are monetary and transaction frictions (e.g., Friedman (1969), Aruoba and Schorfheide (2011), Berentsen, Menzio, and Wright (2011)), downwardly rigid nominal wages (e.g., Kim and Ruge-Murcia (2009)), or a positive trend growth rate in aggregate productivity (e.g., Amano, Moran, Murchison, and Rennison (2009)).

long-run inflation rate are smaller than what the zero-lower-bound literature estimates.

This paper is also related to the literature on the role of firm entry and exit for optimal monetary policy. Bergin and Corsetti (2008), Bilbiie, Ghironi, and Melitz (2008), Faia (2009), and Bilbiie, Fujiwara, and Ghironi (2011) analyze optimal monetary policy in models with firm entry and exit and sticky prices. However, while these authors use models with aggregate productivity growth and homogenous firms, I use a model with firm-level productivity growth and heterogenous firms.

This paper continues as follows. Section 2 describes the one-sector model. Section 3 contains the calibration, and Section 4 derives the optimal long-run inflation rate. Section 5 extends the model to two sectors and incorporates sectoral asymmetries, and Section 6 derives the optimal long-run inflation rate in the two-sector model.

2 Model

This section describes a monetary model with firm-level productivity growth and with exogenous firm entry and exit. The model features sticky nominal prices and represents a cashless economy without aggregate uncertainty.

2.1 Firms

In order to set up the model, I index firms by $j \in [0, 1]$ and let each firm produce a single product variety. The technology of firm j needs labor ℓ_{jt} as the sole input to produce output y_{jt}:

$$y_{jt} = \hat{q}^{t-s_{jt}} \left(\frac{\hat{a}^t \hat{g}^{s_{jt}}}{1 + \overline{\lambda} \lambda^{s_{jt}}} \right) \ell_{jt} \ . \tag{1}$$

The integer variable $s_{jt} = 0, 1, 2, \ldots$ indicates the firm's age at time t, and parameters $\hat{a}, \hat{q}, \hat{g}, \overline{\lambda}$, and λ determine how firm-level productivity evolves over the lifetime of the firm. Specifically, $\overline{\lambda} \geq 0$ and $\lambda \in [0, 1]$ capture scope and speed of learning by doing, respectively. When $\overline{\lambda}$ is large, the firm begins production with a low level of productivity and experiences large productivity gains over time. Furthermore, when λ is close to zero, the firm learns quickly and, therefore, productivity gains realize fast. Learning by doing yields large increments to learning when the firm is young and diminishing increments to learning when the firm ages. Reasons for (post-entry) learning are managers' accumulating experiences, workers' learning by doing, or economies of scale. Hornstein and Krusell (1996) use a similar model of learning by doing.

Parameter $\hat{a} \geq 1$ denotes the growth rate in the firm's productivity component that is common to other firms. The term $\hat{q}^{t-s_{jt}}$ indexes the initial level of productivity in a new firm to the firm's date of market entry. Thus, $\hat{q} > 0$ is the growth rate in technological change embodied in new firms. Only new firms use the new technology, i.e., incumbent firms cannot retool. Finally, $\hat{g} > 0$ determines the growth rate in the productivity component that is specific to incumbent firms once the increments to learning approach zero.

I rearrange the technology of firm j according to

$$y_{jt} \quad = \quad \left(\frac{a^t g^{s_{jt}}}{1 + \overline{\lambda}\lambda^{s_{jt}}} \right) \ell_{jt} \ ,$$

with $a = \hat{a}\hat{q}$ and $g = \hat{g}/\hat{q}$. Here, a denotes aggregate productivity growth, which arises from both embodied and common productivity growth, and g denotes the rate at which incumbent firms become obsolete relative to new firms. In the special case when g equals unity and $\overline{\lambda}$ equals zero, all firms are equally productive, as in the basic New Keynesian model derived in, e.g., Woodford (2003).

Panel A of Figure 1 illustrates the role of learning by doing and incumbent and embodied productivity growth for (detrended) firm-level productivity, $(\hat{g}/\hat{q})^{s_{jt}}/(1+\overline{\lambda}\lambda^{s_{jt}})$. When the firm is young, its productivity grows fast as a result of learning by doing. When the firm ages, learning by doing fades away, and the firm's productivity starts to decline relative to the productivity in new firms as a result of embodied productivity growth. Panel B also illustrates the role of embodied productivity growth for the level of productivity in new firms.

Firms enter and exit the economy continuously. At the beginning of a period, $\delta \in [0, 1)$ new firms enter the economy, while at the end of a period, δ firms exit the economy.[3] The

[3] If I were to consider endogenous firm entry, as in, e.g., Bilbiie, Fujiwara, and Ghironi

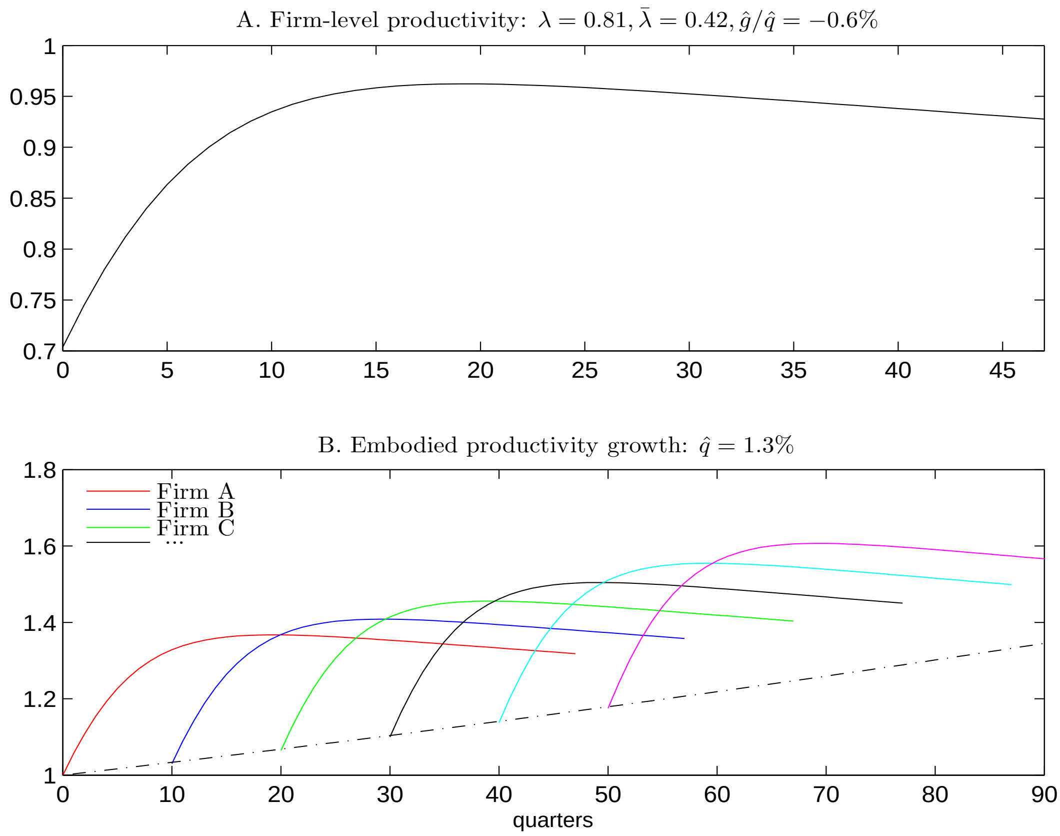

Figure 1: Panel A shows (detrended) firm-level productivity, $(\hat{g}/\hat{q})^{s_{jt}}/(1+\overline{\lambda}\lambda^{s_{jt}})$, over the lifetime of firm j. Panel B shows the role of embodied productivity growth (dashed line) for the level of productivity in new firms. The calibration, which underlies both panels, is described in Section 3.

exit of firms occurs randomly and, therefore, firms with various levels of productivity are equally exposed to exit. In reality, a firm with high productivity may exit because a major shift in consumer taste occurs, a new regulation is passed, or product liability legislation is changed; because a new firm crowds the established firm out of the market by supplying a close substitute; or because the established firm starts exporting and stops selling at home.

When a new firm enters the economy, it sets a price for its product. In subsequent periods, the firm resets its price with probability $(1-\alpha)$, $\alpha \in [0, 1)$, each period until

(2011), the number of firms evolves according to $N_t = (1-\delta)[N_{t-1} + N_{Et-1}]$. In this case, the steady-state fraction of new over all firms, N_E/N, also depends on only the exit rate, $N_E/N = \delta/(1-\delta)$. This suggests that endogenous firm entry adds little to my results on the optimal long-run inflation rate derived for exogenous firm entry.

exit. Firm j sets its nominal price P_{jt} to solve

$$\max_{P_{jt}} \sum_{i=0}^{\infty} \kappa^i \; \Omega_{t,t+i} \left[P_{jt} - W_{t+i} \left(\frac{1 + \overline{\lambda} \lambda^{s_{jt+i}}}{a^{t+i} g^{s_{jt+i}}} \right) \right] y_{jt+i} \qquad \text{s.t.} \qquad y_{jt+i} = \left(\frac{P_{jt}}{P_{t+i}} \right)^{-\theta} y_{t+i} \; . \tag{2}$$

$\Omega_{t,t+i}$ discounts nominal payoffs, and $\kappa = \alpha(1-\delta)$ is the probability to produce at current prices in the next period. When the firm sets its price, it anticipates the evolution of its productivity. The constraint in the firm's problem is the household demand for product j, derived below, and P_t, W_t, and y_t denote the aggregate price level, the nominal wage, and the aggregate output, respectively. Wages are identical across firms because firms hire labor in a perfectly competitive labor market, as in, e.g., Melitz (2003).

The optimal price of firm j equates the discounted sum of marginal revenues to the discounted sum of marginal costs. I rearrange this condition to obtain:

$$\begin{aligned}
\left(\frac{P_{jt}^{\star}}{P_t} \right) g^{s_{jt}} &= \frac{\theta}{\theta - 1} \left(\frac{N_t + \overline{\lambda} \lambda^{s_{jt}} N_{\lambda t}}{D_t} \right) \; , \\
N_t &= w_t / a^t + (\kappa/g) \beta \pi_{t+1}^{\theta} N_{t+1} \; , \\
N_{\lambda t} &= w_t / a^t + (\kappa \lambda/g) \beta \pi_{t+1}^{\theta} N_{\lambda t+1} \; , \\
D_t &= 1 + \kappa \beta \pi_{t+1}^{\theta - 1} D_{t+1} \; .
\end{aligned} \tag{3}$$

The (gross) inflation rate π_t is the change in the aggregate price level, and N_t, $N_{\lambda t}$, and D_t denote auxiliary variables. The optimal price of firm j, $P_{jt}^{\star}$, depends on firm-level productivity, which depends on the firm's age, s_{jt}. Consequently, because firms of different age adjust their price in a given period, adjusting firms set various optimal prices.

2.2 Household

The representative household maximizes discounted lifetime utility:

$$\max_{\{\ell_t, c_{jt}, Q_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(\ell_t) \right] \; , \quad 0 < \beta < 1 \; , \tag{4}$$

where c_t is aggregate consumption, and ℓ_t is aggregate labor. The function u is twice continuously differentiable, increasing, and concave. The function h is twice continuously differentiable, increasing, and convex. The household is subject to the budget constraint

$$\Omega_{t,t+1} Q_{t+1} + \int_0^1 P_{jt} c_{jt} \, dj \quad \leq Q_t + (1 - \tau_L) W_t \ell_t + D_t + T_t \ . \tag{5}$$

It selects a financial portfolio of nominal claims with payoff Q_{t+1}. The price of this portfolio at date t is $\Omega_{t,t+1} Q_{t+1}$, where $\Omega_{t,t+1}$ is the unique discount factor, to be determined by complete financial markets. The household consumes and it receives $(1 - \tau_L) W_t \ell_t$ as labor income net of taxes. While the labor income tax τ_L is not essential for the main results, it will facilitate characterizing them analytically. The household also receives profits D_t from the ownership of firms and a lump-sum transfer T_t from the government. Terminal conditions (not shown) require household solvency. The household's preference for intermediate products is $c_t = (\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} \, dj)^{\frac{\theta}{\theta-1}}$, with $\theta > 1$. The household's optimization yields the product demand, $c_{jt}/c_t = (P_{jt}/P_t)^{-\theta}$, the cost-minimal price of aggregate consumption, $P_t = (\int_0^1 P_{jt}^{1-\theta} \, dj)^{\frac{1}{1-\theta}}$, and $P_t c_t = \int_0^1 P_{jt} c_{jt} \, dj$.

2.3 Equilibrium and the balanced growth path

In the decentralized economy, firms set prices according to equation (2); the household maximizes lifetime utility (4) subject to the budget constraint (5) and the definition of aggregate consumption c_t; product markets clear at $y_{jt} = c_{jt}$; the labor market clears at $\ell_t = \int_0^1 \ell_{jt} \, dj$; financial markets clear at $Q_t = 0$; the resource constraint $y_t = c_t$ holds; and the government sets τ_L, ensures $T_t = \tau_L W_t \ell_t$, and controls the nominal short-term interest rate i_t, which is the payoff to a one-period nominal bond, $(1 + i_t)^{-1} = \beta \Omega_{t,t+1}$.

In the model, aggregate variables will grow at constant rates, because there are no aggregate shocks to perturb the balanced growth path of the economy. Furthermore, I assume that the population does not growth at the balanced growth path, such that ℓ_t is constant, and that the government maintains a constant long-run inflation rate.

2.4 Aggregation

Firms differ from one another in two dimensions, namely, in the level of their productivity and in the length of their price spell. Differences in the first dimension arise from firm entry and from assuming that firm-level productivity grows over the lifetime of a firm, whereas differences in the second dimension arise from staggered pricing of firms.

To aggregate firms' prices to the aggregate price level, I replace firm index j by two new indices, n and k, each representing one dimension of heterogeneity, and denote the current price of firm j as[4]

$$P_{jt} = P^{\star}_{t-(n+k),t-k} \ , \quad n = 0, 1, 2, \ldots , \ k = 0, 1, 2, \ldots \ .$$

The first subscript, $t - (n + k)$, indicates the date of market entry. The second subscript, $t - k$, indicates the date of the last price change. Thus, index k denotes the length of the price spell, and index n denotes the time between market entry and last price change.

The price level P_t comprises the prices of all cohorts of firms. For the moment, I consider the cohort that entered $s \geq 0$ periods ago, at date $t - s$, and normalize its mass to unity. At date t, the weighted average price of this cohort, $\Lambda_t(s)$, is

$$\Lambda_t(s) = (1-\alpha) \sum_{k=0}^{s-1} \alpha^k (P^{\star}_{t-s,t-k})^{1-\theta} + \alpha^s (P^{\star}_{t-s,t-s})^{1-\theta} \ , \tag{6}$$

if $s \geq 1$, and $\Lambda_t(s) = (P^{\star}_{t,t})^{1-\theta}$ if $s = 0$. Upon entry ($s = 0$), all firms in a cohort s set the same optimal price. At subsequent dates ($s \geq 1$), some firms change their prices, while others keep their price, and therefore the price distribution of the cohort s fans out.

At date t, the mass of cohort s is equal to $(1-\delta)^s\delta$ because firm exit diminishes the cohort's mass over time. Summing over all cohorts s yields the unit mass of firms that underlies the price level, $1 = \sum_{s=0}^{\infty}(1-\delta)^s\delta$. Thus, the price level $P_t^{1-\theta} = \int_0^1 P_{jt}^{1-\theta} \, dj$ is equal to the sum over cohort prices $\Lambda_t(s)$, each weighted by the mass $(1-\delta)^s\delta$ of its

[4]This approach is related to Dotsey, King, and Wolman (1999). Unlike in my approach, however, they consider a finite-dimensional state vector of prices and firms with homogenous productivity.

cohort:

$$P_t^{1-\theta} = \sum_{s=0}^{\infty}(1-\delta)^s \delta \Lambda_t(s) \ . \tag{7}$$

In order to rearrange this equation, I consider the optimal prices of two firms denoted j and j'. Both firms adjust their price at the same date $t-k$. However, while firm j is a new firm with age $s_{jt-k} = 0$, firm j' is a firm with age $s_{j't-k} = n$. Relating the pricing equations (3) for both firms to one another yields:

$$P^{\star}_{t-k,t-k} \quad = \quad g^n \left(\frac{1+\overline{\lambda}(N_\lambda/N)}{1+\lambda^n \overline{\lambda}(N_\lambda/N)} \right) P^{\star}_{t-(n+k),t-k} \ , \tag{8}$$

where N_t and $N_{\lambda t}$ are constant at the balanced growth path. The equation states that optimal prices of firms with different age are proportional to one another. Proportionality corresponds to the differential, in terms of expected discounted marginal costs, between incumbent and new firm. Marginal costs differ across firms with different age because these firms maintain different levels of productivity.

Combining equations (6), (7), and (8), and defining the real price of a new firm as $p^\star = P^\star_{t,t}/P_t$ yields the aggregate inflation rate as function of $p^\star$:

$$1 \quad = \quad \{\delta + (1-\alpha)\overline{m}\} \left(p^\star\right)^{1-\theta} + \kappa \pi^{\theta-1} \ .$$

The term in curly brackets indicates that the δ firms that are new at date t maintain the optimal real price $p^\star$. Furthermore, there is a fraction $(1-\alpha)$ of incumbent firms that reset prices optimally, and optimal prices of incumbent firms are equal to the optimal price of new firms after accounting for the differential in marginal costs between incumbent and new firms. Parameter $\overline{m}$, which is defined as

$$\overline{m} \quad = \quad \delta \sum_{n=0}^{\infty} (1-\delta)^{n+1} \ g^{(n+1)(\theta-1)} \left(\frac{1+\overline{\lambda}(N_\lambda/N)}{1+\lambda^{n+1}\overline{\lambda}(N_\lambda/N)} \right)^{\theta-1} \ ,$$

accounts for these differentials, as it is a weighted sum of differentials in marginal costs between incumbent firms of all ages and new firms.

Aggregation also involves combining the technology of firms to the aggregate technology. To this end, I combine the technology of firms, labor-market clearing, and product demand, and this yields:

$$y \;=\; \ell/\Delta \;, \tag{9}$$

where aggregate output in the steady state is aggregate output at the balanced growth path divided by aggregate productivity growth, $y = y_t/a^t$. Thus, economic growth is exogenous and arises from common and embodied productivity growth, $\hat{a}$ and $\hat{q}$, respectively.[5] Furthermore, $1/\Delta$ is the endogenous steady-state level of productivity, with

$$\Delta = \int_0^1 \left(\frac{1+\overline{\lambda}\lambda^{s_{jt}}}{g^{s_{jt}}}\right)\left[\frac{P_{jt}}{P_t}\right]^{-\theta} dj \;,$$

which is constant. The term in round brackets, which is absent in the basic New Keynesian model, shows that both, the ratio of incumbent to embodied productivity growth, $\hat{g}/\hat{q}$, and learning by doing affect the steady-state level of productivity. For example, a large scope for learning by doing ($\overline{\lambda}$ large) depresses the initial level of productivity in new firms and, thereby, $1/\Delta$. The term in square brackets, which also occurs in the basic model, shows the effect of cross-sectional price dispersion.[6] Price dispersion implies that the household consumes an uneven distribution of products, substituting expensive for less expensive products, and this reduces $1/\Delta$.

2.5 Decentralized relative to planned economy

I represent the decentralized economy relative to the planned economy. This representation illustrates the nature of distortions in the decentralized economy and whether or not the

[5]While $\hat{g}$ affects firm-level productivity growth, it does not affect aggregate productivity growth as a result of non-selective firm exit.

[6]With firm-level productivity growth, price dispersion arises not only from staggered pricing, but also from firm-level productivity. Therefore, prices will differ from one another even if they are fully flexible; this is distinct from the price dispersion in Yun (2005), which arises exclusively from staggered pricing. This consequence of firm-level productivity growth helps to improve the model's fit to the large amount of price dispersion observed in micro data.

government faces a policy tradeoff. The decentralized economy consists of the aggregate technology (9) and the household's optimality condition $h_\ell(\ell_t)/u_c(y_t) = (1-\tau_L)w_t$, and rearranging these equations yields:

$$y = R(\pi)\frac{\ell}{\Delta_e} \ , \qquad \frac{h_\ell(\ell)}{u_c(y)}\left(\frac{\mu(\pi)}{1-\tau_L}\right) = \frac{1}{\Delta_e} \ . \tag{10}$$

Here, I define the relative price distortion, $R(\pi) = \Delta_e/\Delta$, the average markup, $\mu(\pi) = 1/(w\Delta_e)$, and the markup distortion, $\mu(\pi)/(1-\tau_L)$. While the relative price distortion arises only from staggered pricing of firms, the markup distortion arises also from monopolistic competition among firms.

Parameter Δ_e derives from the solution of the planned economy that consists of two equations, which are similar to the decentralized economy:

$$y^e = \frac{\ell^e}{\Delta_e} \ , \qquad \frac{h_\ell(\ell^e)}{u_c(y^e)} = \frac{1}{\Delta_e} \ . \tag{11}$$

The planner employs the aggregate technology and equates the marginal rate of substituting labor for consumption to the marginal rate of transformation. Furthermore, the planner resolves an important tradeoff at the firm level: while some firms can produce a given amount of a product with less labor than other firms, the household prefers to consume an even distribution of all products. Parameter $1/\Delta^e = (\int_0^1 [g^{s_{jt}}/(1+\overline{\lambda}\lambda^{s_{jt}})]^{(\theta-1)}dj)^{1/(\theta-1)}$ arises from resolving this tradeoff optimally and indicates the efficient amount of output dispersion. Finally, comparing decentralized and planned economy to one another shows that when relative price distortion and average markup distortion are equal to unity, and thus eliminated, then decentralized and planned economy coincide.

3 Calibration

The parameters that are likely to matter for the long-run inflation rate are those pertaining to firm-level productivity growth. Jensen, McGuckin, and Stiroh (2001) estimate both embodied and incumbent productivity growth in labor productivity of U.S. manufacturing plants, controlling for aggregate productivity growth by time effects. They estimate that

Table 1: Calibration

Parameter	Value	Description
a	2.0% per year	Aggregate productivity growth
$\hat{a}$	0.7% per year	Common productivity growth
$\hat{g}$	0.7% per year	Incumbent productivity growth
$\hat{q}$	1.3% per year	Embodied productivity growth
λ	0.81	Speed of learning
$\overline{\lambda}$	0.42	Scope of learning
δ	8.0% per year	Firm turnover rate
$(1-\alpha(1-\delta))^{-1}$	2 quarters	Median price duration
θ	$4\frac{1}{3}$	30% steady-state markup
ν	0.25	Labor supply elasticity
β	0.995	4% annual real interest rate

Notes: See main text for explanation.

the initial productivity level of plants that enter in 1992 is 46.8% higher than the initial productivity level of plants that enter in 1963 (see β_{92} in their Table 2). This estimate implies a rate $\hat{q}$ of embodied productivity growth equal to 1.3% per year. To estimate the rate of incumbent productivity growth, the authors use a sample in which a cohort contains only plants that survive the entire sample period. Based on this sample, they estimate that the 1967 cohort shows a 18.7 % productivity gain in 1992 relative to 1967 (see λ_5 in their Table 3). This estimate implies a rate $\hat{g}$ of incumbent productivity growth equal to 0.7% per year.

The rate a of aggregate productivity growth is set to 2% per year, based on estimates in Basu, Fernald, and Kimball (2006) for non-durable manufacturing goods. This rate implies that the fraction of aggregate growth that is due to embodied growth, $\log \hat{q}/\log a$, is equal to two-thirds, which is consistent with results in, e.g., Sakellaris and Wilson (2004). Also, using $a = \hat{a}\hat{q}$, the rate $\hat{a}$ of common productivity growth is equal to 0.7% per year.

To calibrate speed and scope of learning by doing, which also pertain to firm-level productivity, I exploit the level effects of learning by doing. One such level effect of learning by doing is the size of new firms relative to incumbent firms, and I calibrate the scope of learning, $\overline{\lambda}$, to match a relative size of new firms equal to 60%, measuring size as hours worked per plant. This calibration is consistent with evidence provided in Jensen, McGuckin, and Stiroh (2001) that the size of the average new plant relative to the average plant in the industry is close to 60% (see their Table 1). Furthermore, this calibration yields a progress ratio $1 + \bar{\lambda}$, i.e., the total increase in a firm's productivity from learning by doing, equal to 1.42, which falls into the range of industry estimates in Jovanovic Nyarko (1995), but is below the calibrated range from 1.5 to 2.5 used in Klenow (1998).

To calibrate the speed of learning λ, I assume it takes two years for a firm to close 75% of its learning gap, $1 - 1/(1 + \overline{\lambda})$, i.e., the firm's productivity level after learning is completed minus its initial productivity level. This speed of learning is smaller than the three years used in Yorukoglu (1998), but is larger than the (slightly more than) one year used in Hornstein and Krusell (1996). This speed of learning may also be compared to case studies, which provide compelling evidence for learning by doing.[7] However, case studies usually examine well-defined learning tasks (assembly of a car, say), whereas examining the introduction of a new product involves taking a broader perspective on learning (one that also involves organisational learning, say).

I calibrate the rate δ of firm turnover to 8% per year, which implies that in the model, the number of new firms over all firms is about one-third.[8] This fits with the evidence provided in Jensen, McGuckin, and Stiroh (2001), that on average, the number of new plants over all plants in an industry is about one-third (see their Table 1). This calibration of δ is also broadly consistent with the 10% annual production depreciation rate, used in Bilbiie, Ghironi, and Melitz (2012).

The remaining parameters are calibrated as follows. I set the probability α for a firm not to adjust its price equal to 0.5105, which yields a median price duration in the truncated price distribution (i.e., the one incorporating firm exit) equal to 2 quarters, as in Wolman (2011). Furthermore, I set the static markup to 30%, which yields that θ is equal to $4\frac{1}{3}$. Finally, I set the discount factor β to 0.995, which implies a 4% annual real interest rate after accounting for aggregate productivity growth, and parameterize

[7]E.g., Levitt, List, and Syverson (2012) examine learning by doing in an automobile assembly plant, and refer to numerous further case studies.

[8]When I convert annual into quarterly rates, I solve $0.08 = \delta \sum_{s=0}^{3}(1 - \delta)^s$ to account for firm exit throughout the year.

the period utility function as $u(c) = \log(c)$ and $h(\ell) = \eta_L \ell^{1+\nu}/(1+\nu)$, with labor-supply elasticity ν equal to 0.25 and η_L equal to 3. Table 1 summarizes the calibration.

4 The optimal long-run inflation rate

The policy problem that I solve to derive the optimal long-run inflation rate consists of an optimizing government that uses a restricted set of policy instruments, i.e., the long-run inflation rate and the labor income tax, to maximize steady-state welfare.

4.1 The model without learning by doing

I begin to compute the optimal long-run inflation rate in the model without learning by doing. This model corresponds to an economy with only firms that realize no further increments in learning, and allows me to derive the following proposition analytically.

Proposition 1: In the model without learning by doing, in which $\overline{\lambda} = 0$, the optimal long-run inflation rate that maximizes steady-state welfare is:

$$\pi^\star = \hat{g}/\hat{q} \ ,$$

and the optimal labor income tax is equal to $\tau_L^\star = -1/(\theta - 1)$. In this case, the decentralized economy (10) coincides with the planned economy (11) and, therefore, is first best.

My calibration implies that in the model without learning by doing, $\pi^\star$ is negative and equal to -0.63% per year. The key equation to understand this first main result is the firms' pricing equation (3).[9] For a new firm without learning by doing and denoting $w = w_t/a^t$, this equation can be rearranged as

$$0 = \sum_{s=0}^{\infty} (\kappa\beta\pi^\theta)^s \left[\frac{p^\star}{\pi^s} - \frac{\theta}{\theta - 1} \frac{w}{(\hat{g}/\hat{q})^s} \right] \ . \tag{12}$$

[9] Proposition 1 does not hinge on assuming time-dependent pricing, since other pricing assumptions also will not interfere with the government's ability to reconcile decentralized and planned economy.

Square brackets contain the difference between the (constrained) optimal real price, $p^\star/\pi^s$, and the desired real price, $\frac{\theta}{\theta-1}\frac{w}{(\hat{g}/\hat{q})^s}$, which is the static markup times firm-level marginal costs. Firm-level marginal costs increase over time because the firm's technology becomes obsolete at rate $\hat{g}/\hat{q}$ relative to the technology embodied in the new firms to come. Accordingly, when the firm can adjust its nominal price only infrequently, it will also find it difficult to adjust its real price in line with its real marginal costs.

The negative optimal long-run inflation rate, $\pi^\star = \hat{g}/\hat{q}$, helps the firm to increase its real price over time and, thereby, to continuously align it with its increasing real marginal costs.[10] Consequently, the firm has no reason to actually change its nominal price, and this prevents distorted relative prices. Furthermore, the firm continuously maintains the static markup and, therefore, the optimal labor income tax remedies the average markup distortion. This is shown in Figure 2, which contains the distortions $\mu(\pi)/(1-\tau_L^\star)$ and $R(\pi)$ and ticks $\pi^\star$ (bold lines). Evidently, the optimizing government faces no policy tradeoff, because it can eliminate both distortions and, therefore, reconcile decentralized and planned economy.

Panel A in Figure 2 also shows that for π either sufficiently above or below $\hat{g}/\hat{q}$, the average markup $\mu(\pi)$ exceeds the static markup, which is equal to $(1-\tau_L^\star)$. When π is sufficiently above $\hat{g}/\hat{q}$, adjusting firms set a higher nominal price than otherwise, because they anticipate π to excessively erode their real price over time and, thereby, to compress their markup to below the static markup. The higher prices of adjusting firms elevate $\mu(\pi)$. Further, when π is below $\hat{g}/\hat{q}$, firms that cannot adjust their price see their marginal costs decline at a faster rate than their real price, and this also elevates $\mu(\pi)$.[11]

Similarly, Panel B shows that relative prices are distorted when π deviates from $\hat{g}/\hat{q}$. In this case, firms do not manage to continuously realize the static markup and, therefore, adjust their price whenever they can. This disperses relative prices because only a subset of firms adjust their price in each period. Both Panel A and B also contain the distortions

[10] The negative optimal long-run inflation rate arises from new firms. In the model without learning by doing, they set their nominal price to below the average price because their productivity exceeds the average level of productivity. In contrast, incumbent firms do not create any inflation because they keep their nominal price constant.

[11] King and Wolman (1999) describe similar effects in the basic New Keynesian model.

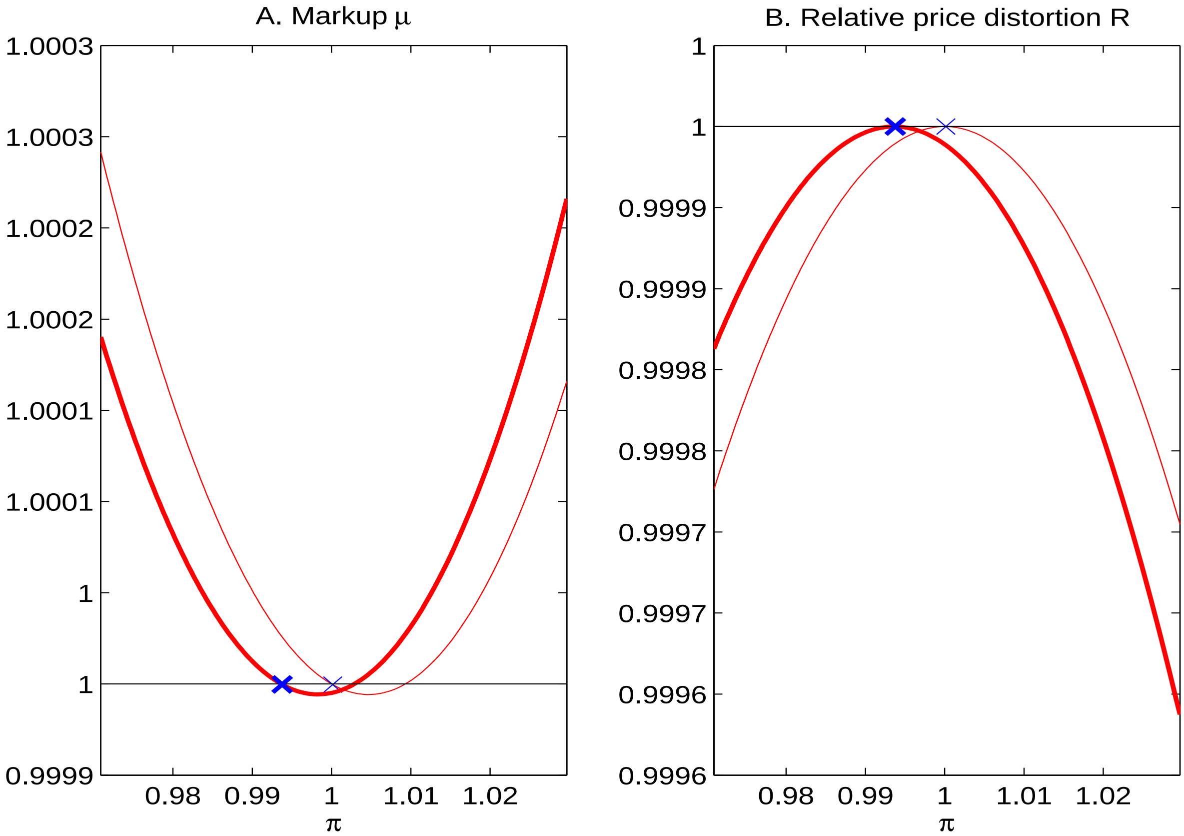

Figure 2: Distortions in the model without learning by doing, but with incumbent and embodied productivity growth (bold lines). Panel A shows the markup distortion $\mu(\pi)/(1-\tau_L^\star)$, and Panel B shows the relative price distortion $R(\pi)$. In both panels, π is the annualized gross inflation rate. Thin lines indicate the distortions in the basic New Keynesian model.

in the basic New Keynesian model, with $\overline{\lambda} = 0$ and $\hat{g}/\hat{q} = 1$ (thin lines). They show that firm-level productivity growth merely re-centers the distortions.

4.2 The model with learning by doing

In the model with learning by doing, the optimal long-run inflation rate is positive and equal to 1.04% per year, which is a second main result in this paper. Thus, incorporating learning by doing into the model increases the optimal long-run inflation rate by 1.67 percentage points, i.e., from -0.63% to 1.04%. With learning by doing, however, $\pi^\star$ can no longer be computed analytically. Therefore, I compute it numerically by maximizing steady-state welfare keeping $\tau_L^\star = -1/(\theta - 1)$ (which I vary below).

To explain the substantial increase in the optimal long-run inflation rate, Figure 3 plots

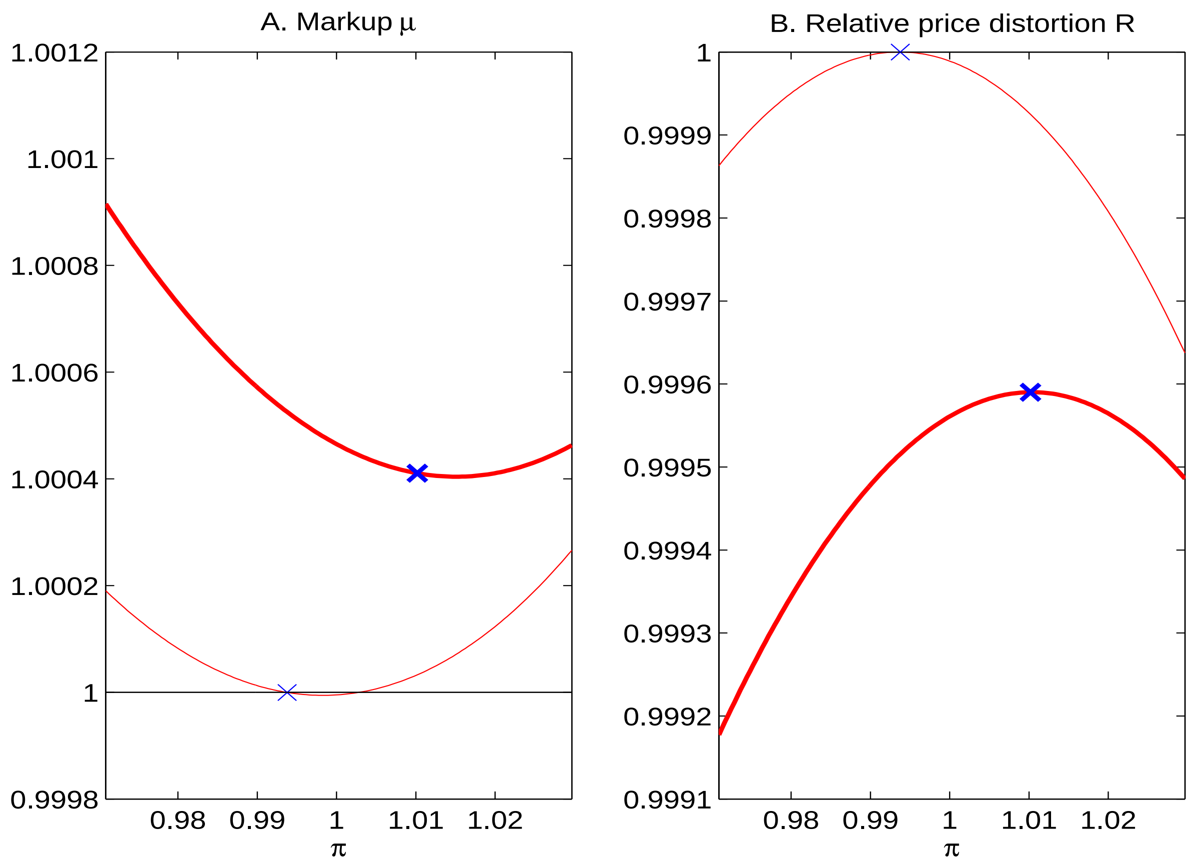

Figure 3: Distortions in the model with learning by doing and incumbent and embodied productivity growth (bold lines). Panel A shows the markup distortion $\mu(\pi)/(1-\tau_L^\star)$, and Panel B shows the relative price distortion $R(\pi)$. In both panels, π is the annualized gross inflation rate. Thin lines indicate the distortions in the model without learning by doing.

the distortions $\mu(\pi)/(1-\tau_L^\star)$ and $R(\pi)$ in the models with learning by doing (bold lines) and without learning by doing (thin lines). In contrast to the model without learning by doing, no long-run inflation rate can eliminate the distortions in the model with learning by doing. In this model, thus, the government faces a policy tradeoff, and the figure shows that this tradeoff is resolved optimally by a positive, instead of a negative, optimal long-run inflation rate (ticked).

The policy tradeoff arises because new firms, which experience learning-by-doing dynamics, prefer a positive long-run inflation rate. However, new firms coexist with incumbent firms, which no longer learn and thus prefer a negative long-run inflation rate, as shown in Proposition 1. Thus, the government can use the long-run inflation rate to help either new or incumbent firms, but it cannot help both of them at the same time.

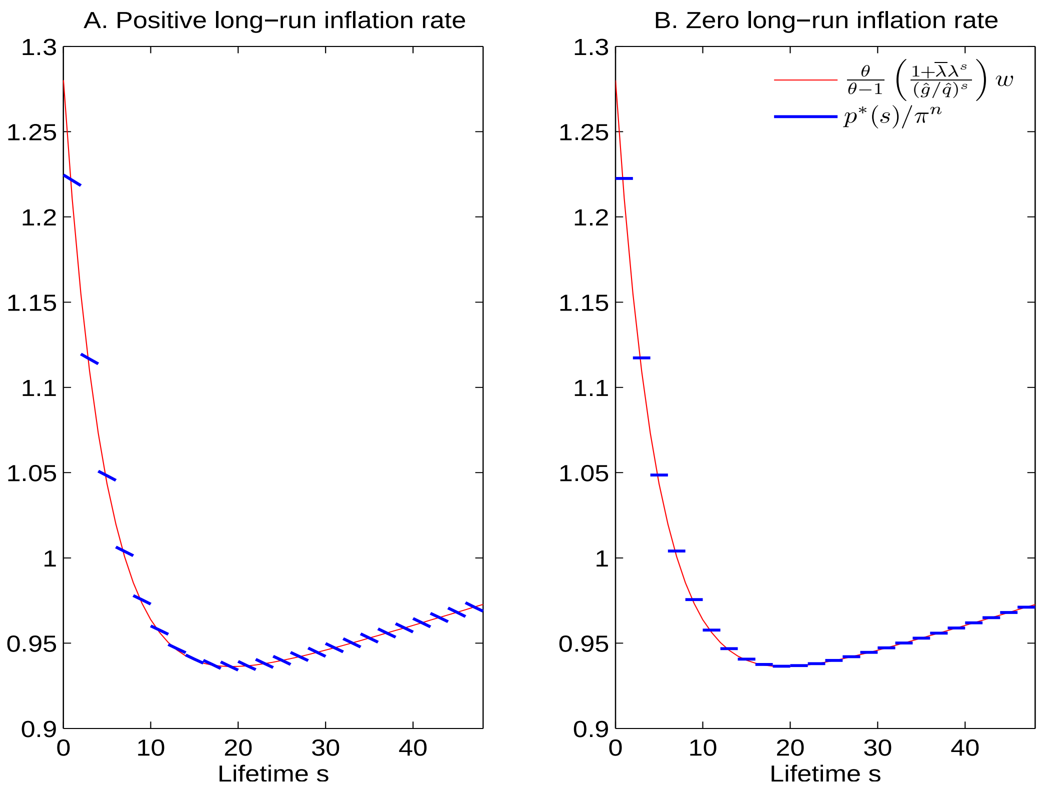

Figure 4: Firm-level marginal costs (solid line) and a sample path of a firm's real price (bars) over the lifetime of the firm. s denotes the age of the firm. Panel A shows the case when the long-run inflation rate is 1.04% per year. Panel B shows the case when the long-run inflation rate is zero. The firm's price is assumed to change every two quarters.

To illustrate this, I revisit pricing equation (3) of a new firm with learning by doing:

$$0 = \sum_{s=0}^{\infty}(\kappa\beta\pi^{\theta})^s \left[\frac{p^\star}{\pi^s} - \frac{\theta}{\theta-1}\left(\frac{1+\bar{\lambda}\lambda^s}{(\hat{g}/\hat{q})^s}\right)w\right] . \tag{13}$$

The firm sees its real marginal costs decline as a result of learning by doing, which is governed by $\bar{\lambda}$ and λ^s. When the firm can adjust its nominal price only infrequently, it will find it difficult to reduce its real price, $p^\star$, in order to align it with its declining real marginal costs. The positive optimal long-run inflation rate helps the firm to reduce its real price over time. However, when learning-by-doing dynamics fade out as the firm ages (s increases), the fact that the firm's technology becomes obsolete at rate $\hat{g}/\hat{q}$ relative to the technology of newer firms begins to dominate the firm's pricing. In this situation, the firm prefers a negative long-run inflation rate, as shown in Proposition 1.

The role of the positive optimal long-run inflation rate for the firm's pricing is shown in Panel A in Figure 4, which contains a sample path of a firm's real price (bars) and the evolution of firm-level marginal costs (solid line) over the lifetime of the firm. Panel B shows the corresponding plot for the case without inflation. When the firm is young, a positive long-run inflation rate reduces the gap between the firm's real price and its marginal costs. However, when the firm ages, the positive inflation rate widens this gap.

But why does the optimizing government weigh new firms more than incumbent firms and, therefore, selects a positive long-run inflation rate? Figure 4 shows that learning by doing triggers rapid changes in marginal costs of new firms, whereas embodied productivity growth triggers only gradual changes in marginal costs of incumbent firms. Accordingly, nominally sticky prices, which prevent a firm from aligning its real price with its marginal costs, constrain new firms more than incumbent firms. Therefore, the optimal long-run inflation rate is geared towards helping new firms, and this holds true despite the fact that learning by doing dominates the dynamics of firm-level marginal costs for only a relatively short period, about one third, of the lifetime of the average firm.

4.3 Robustness

Calibration of model parameters is crucial to quantify the optimal long-run inflation rate, and I explore in Figure 5, how robust the optimal long-run inflation rate is with respect to changing $\hat{g}/\hat{q}$, λ, $\overline{\lambda}$, α, δ, and θ. Each panel in Figure 5 shows the optimal long-run inflation rate when one parameter is varied keeping other parameters fix at their benchmark value, except for λ and $\overline{\lambda}$, which are set to keep the relative size of new firms at 60% and the speed of learning at two years, as in Section 3. I find that parameters governing learning by doing and the market share of new firms are important to determine $\pi^{\star}$, whereas other parameters, including the one for price stickiness, are less important.

Panel (a) shows that $\pi^{\star}$ is fairly insensitive to the rate $\hat{g}/\hat{q}$, at which incumbent firms become obsolete relative to new firms. Reducing this rate to below its benchmark value (ticked) also reduces $\pi^{\star}$, akin to Proposition 1, because incumbent firms now prefer to increase their real price at a faster rate. However, the moderate quantitative effects of varying $\hat{g}/\hat{q}$ suggest that largely independent of $\hat{g}/\hat{q}$, nominally sticky prices constrain

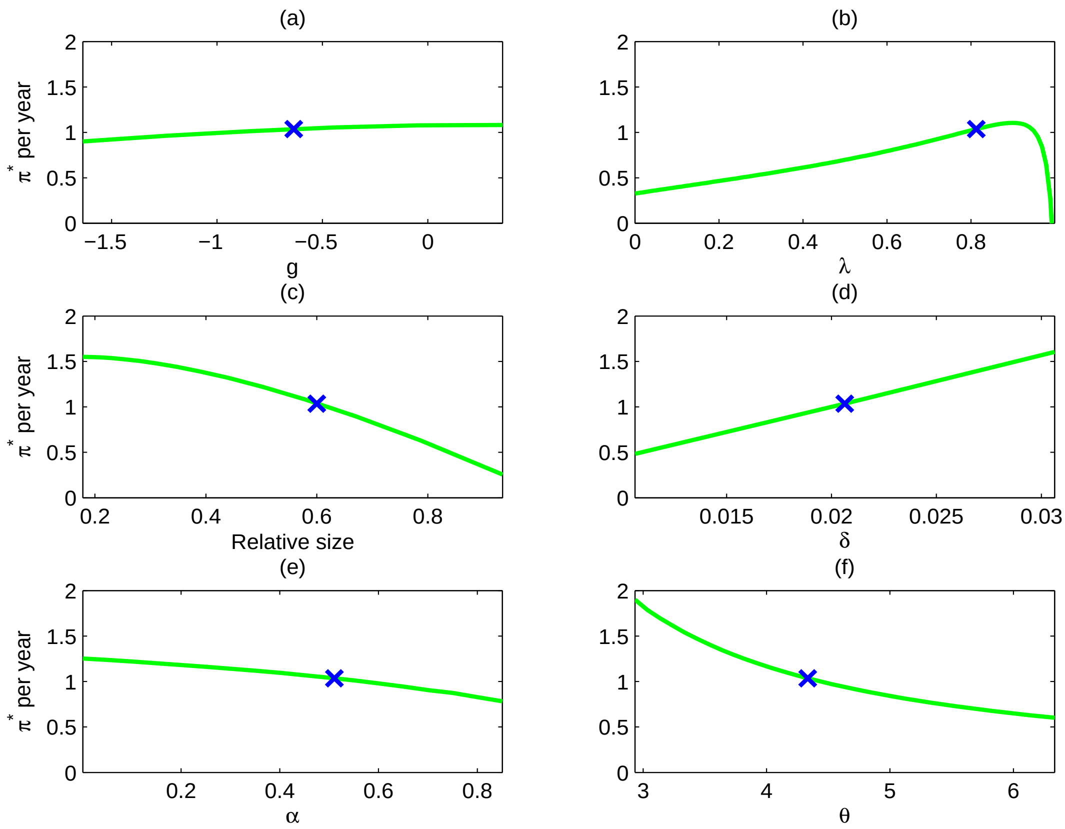

Figure 5: Robustness of the optimal long-run inflation rate with respect to various parameters. In computing $\pi^\star$ in Panel (b), $\overline{\lambda}$ is kept fix at 0.42.

new firms considerably more than incumbent firms.

However, $\pi^\star$ does depend on the amount of learning by doing. Panel (b) shows that reducing the speed of learning from high to low, which corresponds to increasing λ from zero to unity, first increases $\pi^\star$ and then decreases it. While $\pi^\star$ is already positive for instantaneous learning, $\lambda = 0$, reducing the speed of learning makes a firm learn for a longer period of time and, therefore, increases $\pi^\star$. Further reducing the speed of learning dampens the learning dynamics and, therefore, reduces $\pi^\star$. In the boundary case $\lambda = 1$, $\pi^\star$ converges to $\hat{g}/\hat{q}$. In Panel (c), a small size of new firms relative to incumbent firms, which in the model requires a large scope of learning $\overline{\lambda}$ and thus, creates large learning dynamics in new firms, yields a large $\pi^\star$. Increasing the relative size of new firms corresponds to lowering the scope of learning and, therefore, reduces $\pi^\star$.

Panel (e) shows that increasing the amount of price stickiness reduces $\pi^\star$ only moderately. Prices of new firms are more flexible than prices of incumbent firms, because a firm sets its price in its first period unconstrained, whereas this firm is subject to a sticky price with likelihood α in each subsequent period. When prices are flexible (α small), this asymmetry between new and incumbent firms is quantitatively small. However, when prices are sticky, this asymmetry matters and tends to increase the amount of price stickiness in incumbent firms relative to new firms. This shifts $\pi^\star$ in favor of incumbent firms and, therefore, reduces it. The finding that $\pi^\star$ is nevertheless fairly insensitive to the amount of price stickiness differs from, e.g., Khan, King, and Wolman (2003), in which the amount of price stickiness is a core determinant of the optimal long-run inflation rate.

Finally, Panels (d) and (f) yield that $\pi^\star$ increases in the market share of new firms. Increasing the rate of firm turnover in Panel (d) increases the portion of new firms in the market and thereby the market share of new firms. Furthermore, Panel (f) shows that a price-inelastic product demand (θ small) yields a higher $\pi^\star$ than otherwise. Inelastic product demand implies that households find it difficult to substitute away from the relatively expensive products of new firms, and this also preserves the market share of new firms.

5 The role of sectoral asymmetries

Differences in firm-level productivity growth between new and incumbent firms coexist with differences in magnitude and composition of productivity growth between economic sectors. For example, productivity growth in manufacturing (Goods) is about 2% per year, whereas productivity growth in retail trade (Services) is about 1% per year (Foster, Haltiwanger, and Krizan (2006)). As another example, embodied productivity growth accounts for about two-thirds of sectoral productivity growth in Goods, whereas it accounts for basically all sectoral productivity growth in Services (Foster, Haltiwanger, and Krizan (2006)). A further difference between economic sectors is the amount of price stickiness.

I incorporate these sectoral asymmetries into my analysis by extending it to a two-sector model. Sectors differ from one another in terms of magnitude and composition of productivity growth, and firms in one sector differ from the firms in the other sector in terms of their firm-level productivity growth, their degree of price stickiness, and their likelihood to survive. Such asymmetries are not only a realistic feature, which allows me to refine my estimate of the optimal long-run inflation rate, but the literature has also shown that they can imply important policy tradeoffs.

5.1 Firms and household

As stated above the model now has two sectors, $z = 1, 2$, and each sector contains many firms that produce intermediate products. Firms in a sector z enter and exit continuously at the rate $\delta_z \in [0, 1)$, and exiting firms are drawn randomly. Firm $j \in [0, 1]$ in a sector z uses the technology $y_{zjt} = (a_z^t g_z^{s_{zjt}} \ell_{zjt})/(1 + \overline{\lambda}_z \lambda_z^{s_{zjt}})$, with $a_z = \hat{a}_z \hat{q}_z$ and $g_z = \hat{g}_z/\hat{q}_z$. Here, a_z denotes sectoral productivity growth, $\hat{a}_z$, $\hat{q}_z$, and $\hat{g}_z$ denote common, embodied, and incumbent productivity growth in sector z, respectively, and $\overline{\lambda}_z$ and λ_z denote scope and speed of learning in sector z, respectively. Firm j's pricing problem is analogous to the one in equation (2), after incorporating the sectoral asymmetries, one of which is the probability to produce tomorrow at current prices, $\kappa_z = \alpha_z(1 - \delta_z)$. Further, firm j hires labor ℓ_{zjt} in an economy-wide, competitive labor market.

The household uses the preference $c_t = c_{1t}^{\psi} \; c_{2t}^{1-\psi}$, with $\psi \in (0, 1)$, to combine consumption in a sector z, c_{zt}, to aggregate consumption c_t. The corresponding price level equals $P_t = (P_{1t}/\psi)^{\psi} \, (P_{2t}/(1-\psi))^{1-\psi}$. Further, the household uses the preference $c_{zt} = (\int_0^1 c_{zjt}^{\frac{\theta-1}{\theta}} \; dj)^{\frac{\theta}{\theta-1}}$, with $\theta > 1$, to combine the intermediate products to the consumption in a sector z. The corresponding price level in a sector z equals $P_{zt} = (\int_0^1 P_{zjt}^{1-\theta} \; dj)^{\frac{1}{1-\theta}}$. The household also solves an intertemporal problem, which is similar to the one described in Section 2.2.

5.2 Decentralized relative to planned economy

Along the lines of the one-sector model, I represent the decentralized economy with two sectors relative to the planned economy with two sectors. To this end, I let $p_z^{\star} = P_{z,t,t}^{\star}/(\eta_z^t P_t)$ denote the relative price of a new firm in a sector z, $p_z = P_{zt}/(\eta_z^t P_t)$ the relative price in a sector z, and $\pi_z = P_{zt}/P_{zt-1}$ the inflation rate in this sector. These variables are constant in steady state. Parameters $\eta_1 = (a_2/a_1)^{1-\psi}$ and $\eta_2 = (a_1/a_2)^{\psi}$ represent trends in P_{zt}/P_t in a sector z, which depend on the differential in sectoral productivity growth. I also let $\pi = P_t/P_{t-1}$ denote the long-run inflation rate, which I assume is constant in steady state.

The decentralized economy with two sectors consists of the aggregate technology, the intratemporal household optimality condition, and two aggregate distortions that are indexed by the long-run inflation rate:

$$y = R(\pi)\frac{\ell}{\Delta_e} \, , \qquad \frac{h_\ell(\ell)}{u_c(y)}\left(\frac{\mu(\pi)}{1-\tau_L}\right) = \frac{1}{\Delta_e} \, . \tag{14}$$

$R(\pi)$ denotes the aggregate relative price distortion, $\mu(\pi)$ denotes the aggregate markup, and $\mu(\pi)/(1-\tau_L)$ denotes the aggregate markup distortion. When both aggregate distortions are equal to unity, then the decentralized and the planned economy coincide with one another, as in the one-sector model.

These aggregate distortions are functions of the sectoral relative price distortion $\rho_z(\pi)$ and the sectoral markup $\mu_z(\pi)$, with $z = 1, 2$:

$$R(\pi) = \left[\psi\left(\frac{\mu_2(\pi)}{\mu_1(\pi)}\right)^{1-\psi}\rho_1(\pi)^{-1} + (1-\psi)\left(\frac{\mu_1(\pi)}{\mu_2(\pi)}\right)^{\psi}\rho_2(\pi)^{-1}\right]^{-1} \, , \tag{15}$$

$$\mu(\pi) = \mu_1(\pi)^{\psi}\mu_2(\pi)^{1-\psi} \, . \tag{16}$$

The aggregate markup is a weighted geometric mean of the sectoral markup defined as $\mu_z(\pi) = p_z/(w\Delta_z^e)$, indicating with $1/\Delta_z^e$ the efficient amount of output dispersion in a sector z. Furthermore, the aggregate relative price distortion is a weighted mean of the sectoral relative price distortion defined as $\rho_z(\pi) = \Delta_z^e/\Delta_z$. The weights depend on ψ and $1-\psi$ and on the ratio of sectoral average markups, which is proportional to the relative price p_2/p_1. Thus, uneven sectoral average markups distort the relative price of sectoral consumption and, therefore, the allocation of the household's expenditure across sectors. This source of the aggregate relative price distortion is absent in the one-sector model. $\mu_z(\pi)$ and $\rho_z(\pi)$ are functions of π and depend on model parameters.

Table 2: Calibration of sectoral asymmetries

Parameter	Value	Description
a_1	1.08% per year	Aggregate productivity growth
$\hat{a}_1$	0.0% per year	Common productivity growth
$\hat{g}_1$	0.0% per year	Incumbent productivity growth
$\hat{q}_1$	1.08% per year	Embodied productivity growth
δ_1	11% per year	Firm turnover rate
$(1-\alpha_1(1-\delta_1))^{-1}$	3 quarters	Median price duration
ψ	60%	Relative size of Services sector

Notes: See main text for explanation.

5.3 Calibration of sectoral asymmetries

Productivity growth in the Goods sector 2 is calibrated as in the one-sector model in Table 1. To calibrate productivity growth in the Services sector 1, I use estimates obtained by Foster, Haltiwanger, and Krizan (2006) for firm-level data on the U.S. retail trade industry. They estimate that labor productivity in this industry grows by 11.43% between 1987 and 1997, and that this increase arises exclusively from productivity growth embodied in new firms. These estimates imply that both the rate a_1 of sectoral productivity growth and the rate $\hat{q}_1$ of embodied productivity growth are equal to 1.08% per year. Furthermore, I use $a_1 = \hat{a}_1\hat{q}_1$ to obtain a rate $\hat{a}_1$ of common productivity growth equal to zero percent, and also set the rate $\hat{g}_1$ of incumbent productivity growth equal to zero percent.

Speed and scope of learning by doing in Services are the same parameters as in Goods. I calibrate the rate δ_1 of firm turnover to 11% per year, which implies that in Services, the number of new firms over all firms is about 0.44 and, thus, higher than in Goods. I set the relative size ψ of Services to 60%, as in Wolman (2011). Further, I set the probability α_1 for a firm not to adjust its price equal to 0.6864, which yields a median price duration in the truncated price distribution equal to 3 quarters, also as in Wolman (2011). Thus, Services prices are stickier than Goods prices. Table 2 summarizes this calibration, and remaining parameters are from Table 1.

6 The optimal long-run inflation rate with sectoral asymmetries

6.1 The two-sector model without learning by doing

I first derive the optimal long-run inflation rate that maximizes steady-state welfare, in the special case in which there is no learning by doing and the discount factor β approaches unity. Maximizing steady-state welfare in this case is equivalent to minimizing only one of the two aggregate distortions in the decentralized equilibrium (14) because these distortions are equal to one another. Minimizing only one of the two aggregate distortion instead of maximizing steady-state welfare simplifies deriving analytical results.

To show that the aggregate distortions are the same in the special case, I use the aggregate markup (16) to rewrite the aggregate relative price distortion (15) as

$$R(\pi) = \left(\frac{\psi}{\rho_1(\pi)\mu_1(\pi)} + \frac{1-\psi}{\rho_2(\pi)\mu_2(\pi)} \right)^{-1} \mu(\pi)^{-1} \ . \tag{17}$$

Equations (??)–(??) imply that $\rho_z(\pi)\mu_z(\pi) = \theta/(\theta-1)$, where $z = 1, 2$, in the special case with $\overline{\lambda}_z = 0$ and $\beta \to 1$. Thus, equation (17) collapses to

$$R(\pi) = \left(\frac{\theta/(\theta-1)}{\mu(\pi)} \right) \ . \tag{18}$$

Without loss of generality, I assume that the labor income tax perfectly offsets the static markup, i.e., $1-\tau_L = \theta/(\theta-1)$.[12] Equation (18) thus states that the aggregate distortions are inversely equal to one another. The third main result in this paper follows from minimizing $\mu(\pi)$ (or maximizing $R(\pi)$) and shows how the optimizing government resolves a policy tradeoff that arises between asymmetric sectors.

Proposition 2: In the two-sector model without learning by doing, $\overline{\lambda}_z = 0$, and in which

[12]In the limit $\beta \to 1$, I obtain the same optimal π if τ_L is selected optimally.

$\beta \to 1$, the optimal long-run inflation rate that maximizes steady-state welfare solves:

$$0 \;=\; \omega(\pi^\star)\left(\frac{\pi^\star - (g_1/\eta_1)}{(g_1/\eta_1)}\right) + [1-\omega(\pi^\star)]\left(\frac{\pi^\star - (g_2/\eta_2)}{(g_2/\eta_2)}\right) \;, \tag{19}$$

with $\eta_1 = (a_2/a_1)^{(1-\psi)}$ and $\eta_2 = (a_1/a_2)^{\psi}$. The weight fulfills the condition that $\omega(\pi) \in [0,1]$ and depends on the long-run inflation rate:

$$\omega(\pi) \;=\; \left[1 + \left(\frac{1-\psi}{\psi}\right)\left(\frac{\kappa_2}{\kappa_1}\right)\left(\frac{a_1}{a_2}\right)^{\theta-1}\left(\frac{1-\kappa_1(\eta_1\pi)^{\theta}/g_1}{1-\kappa_2(\eta_2\pi)^{\theta}/g_2}\right)\left(\frac{1-\kappa_1(\eta_1\pi)^{\theta-1}}{1-\kappa_2(\eta_2\pi)^{\theta-1}}\right)\right]^{-1} \;,$$

with $\kappa_z = \alpha_z(1-\delta_z)$ and $z = 1, 2$.

Equation (19) shows that in the two-sector model without learning by doing, the government faces a policy tradeoff between a long-run inflation rate equal to either g_1/η_1 or g_2/η_2, and it resolves this tradeoff optimally using $\omega(\pi)$.[13] In contrast to Proposition 1, thus, the optimal long-run inflation rate in Proposition 2 generally does not recover the first-best allocation in the two-sector model without learning by doing.[14] The policy tradeoff arises from a lack of policy instruments that work at the sectoral level. Namely, while the government can use the long-run inflation rate to fully offset the distortions in either sector 1 or sector 2, this instrument is not able to fully offset the distortions in both sectors at the same time.

The weight $\omega(\pi)$ in Proposition 2 depends on various sectoral asymmetries, but the predominant asymmetry is the amount of price stickiness in a sector z, which is shown in Figure 6. For a particular value of α_1, reducing the value of α_2 increases the weight on sector 1. The optimizing government weights the sector with the stickier prices more

[13]Alternatively, using equation (19), $\pi^\star$ corresponds to a weighted harmonic mean:

$$\pi^\star = \left(\frac{\omega(\pi^\star)}{g_1/\eta_1} + \frac{1-\omega(\pi^\star)}{g_2/\eta_2}\right)^{-1} \;.$$

[14]One special case in which the policy tradeoff disappears and the optimized decentralized economy is first best arises when $g_1/\eta_1 = g_2/\eta_2$ or, equivalently, $a_1 g_1 = a_2 g_2$. In this case, Proposition 2 yields $\pi^\star = a_1 g_1$. Accordingly, this case generalizes Proposition 1, for the limit $\beta \to 1$, to a two-sector model with asymmetric price stickiness and sectoral productivity growth. Another special case in which the policy tradeoff disappears arises when firms in sector 2, say, have flexible prices. In this case, Proposition 2 yields $\pi^\star = g_1/\eta_1$.

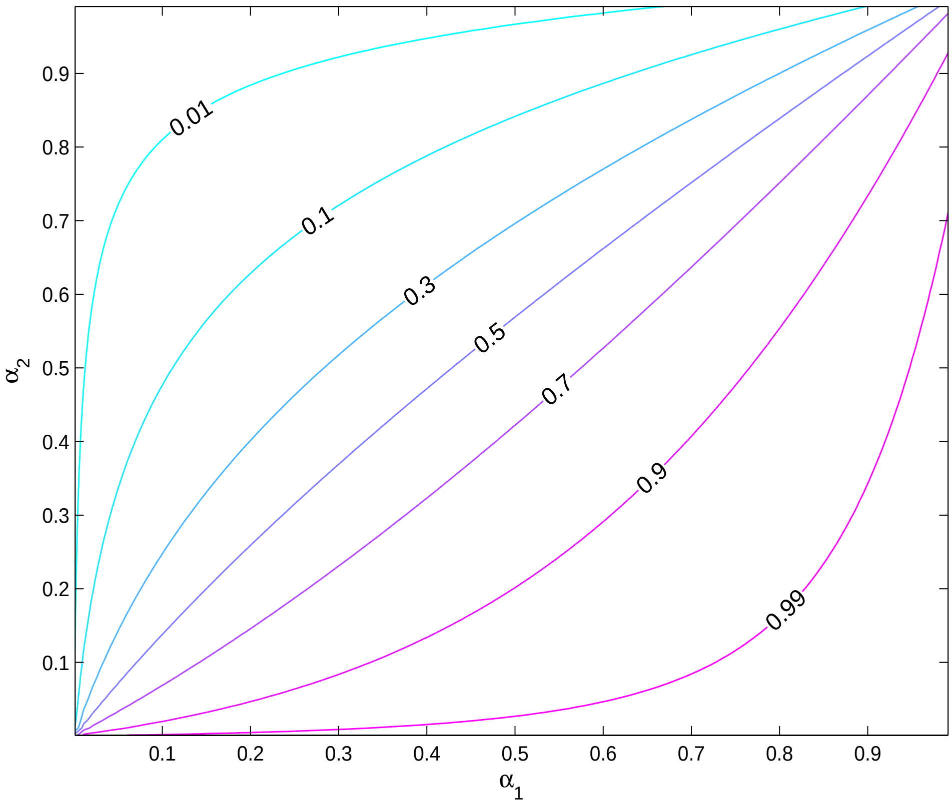

Figure 6: Weight $\omega(\pi)$ as a function of the degree of price stickiness α_1 and α_2 in the two sectors. Lines indicate the combinations of α_1 and α_2 that yield a particular value of $\omega(\pi)$. The long-run inflation rate is the one in Proposition 2, and the calibration is the one in Section 5.3.

heavily, because thereby it shifts the price adjustment to the sector with the more flexible prices, where it is least distortive. This phenomenon is known as the "stickiness principle" in the literature on the optimal inflation *stabilization* policy (e.g., Aoki (2001), Mankiw and Reis (2003), Benigno (2004), and Eusepi, Hobijn, and Tambalotti (2011)). An important consequence of Proposition 2 is that in a model with firm-level productivity growth, this principle applies equally to the choice of the optimal *long-run* inflation rate.

The policy tradeoff in equation (19), which the optimizing government resolves using the stickiness principle, has two sources. One is the rate g_z at which incumbent firms become obsolete relative to new firms in a sector z. To illustrate this, I consider the case $a_1 = a_2$, in which equation (19) reduces to

$$0 \quad = \quad \omega(\pi^\star)(\pi^\star - g_1)/g_1 + [1 - \omega(\pi^\star)](\pi^\star - g_2)/g_2 \; .$$

A natural interpretation of g_z is that it represents the long-run inflation rate that eliminates all distortions in a sector z. This is similar to Proposition 1, in which $\pi^\star = g$ eliminates all distortions in the one-sector model. In the two-sector model, however, no long-run inflation rate is optimal in both sectors at the same time, as long as $g_1 \neq g_2$, and this creates a policy tradeoff. In the case in which this is the only tradeoff, the optimal long-run inflation rate is negative and equal to -0.99% per year. This inflation rate is closer to g_1, which is equal to -1.07% per year, than to g_2, which is equal to -0.64% per year, because Services prices are stickier than Goods prices.

The second source of the policy tradeoff is the differential a_1/a_2 in sectoral productivity growth, incorporated into η_z. To illustrate this, I consider the case $g_1 = g_2 = 1$, in which equation (19) reduces to

$$0 \;=\; \omega(\pi^\star)(\eta_1\pi^\star - 1) + [1 - \omega(\pi^\star)](\eta_2\pi^\star - 1) \;,$$

where $\eta_z\pi$ is equal to the long-run inflation rate π_z in a sector z. Thus, in both sectors, the optimizing government targets π_z equal to zero percent per year. This is optimal because without (effective) firm-level productivity growth, firm-level marginal costs are constant, as in the basic New Keynesian model. In the two-sector model with sectoral productivity growth, however, the relative price P_{1t}/P_{2t} is trending at rate a_2/a_1, because relative productivity gains in one sector reduce the relative price in this sector, and this triggers the inflation differential $\pi_1/\pi_2 = a_2/a_1$. Therefore, the optimizing government, which only controls π, cannot achieve $\pi_z = 1$ in both sectors at the same time, and this creates a policy tradeoff.

In the case in which this is the only tradeoff, the optimal long-run inflation rate is equal to -0.20% per year. This inflation rate implies that π_1, which is equal to 0.17% per year, is closer to zero than π_2, which is equal to -0.74% per year, because Services prices are stickier than Goods prices. A trending relative price is also the policy tradeoff analyzed in Wolman (2011). Wolman uses more general models of how firms set prices than the one used here, but he also finds that mild deflation is the optimal policy.

I now return to the case in Proposition 2, which combines both sources of the policy

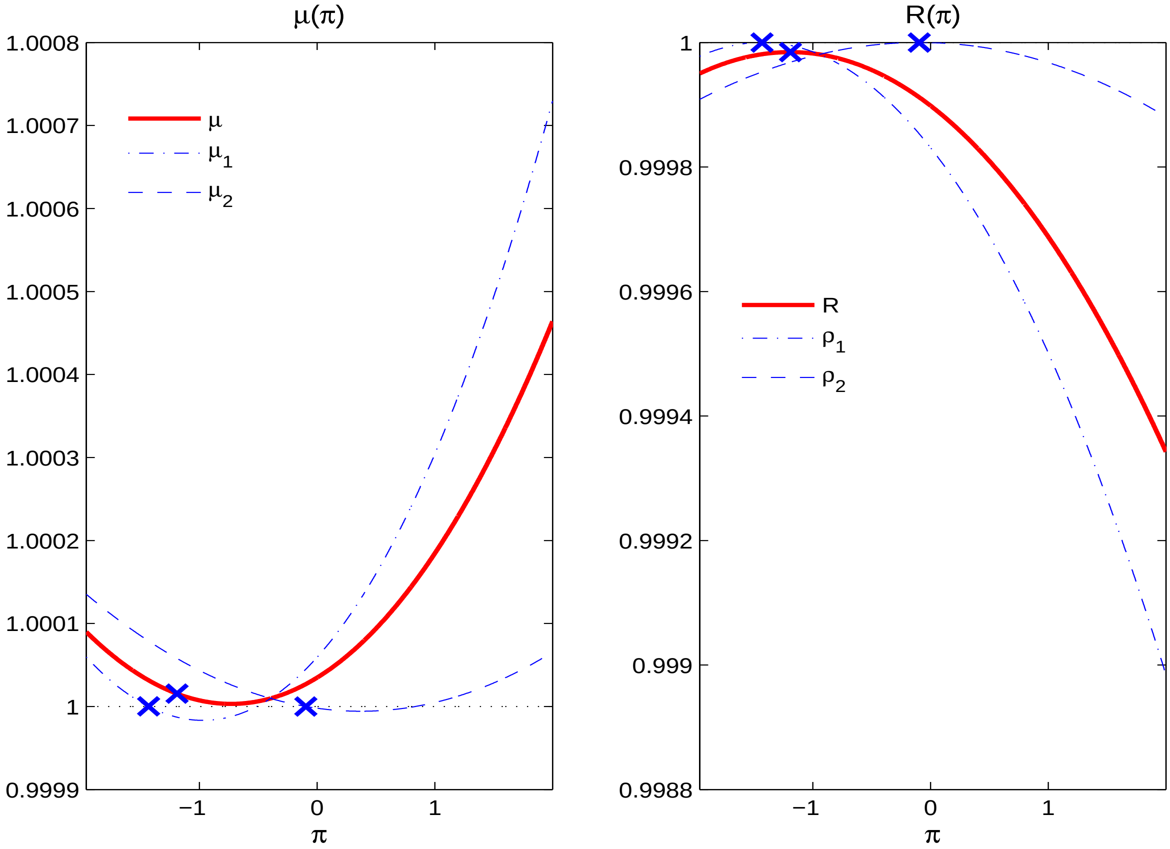

Figure 7: Aggregate distortions (bold lines) and sectoral distortions (thin lines) in the two-sector model without learning by doing and $\beta = 0.995$. Panel A shows the markup distortion $\mu(\pi)/(1 - \tau_L^{\star})$, and Panel B shows the relative price distortion $R(\pi)$. In both panels, π is the annualized net inflation rate.

tradeoff. Figure 7 shows both, aggregate distortions (bold lines) and sectoral distortions (thin lines) in this case. The optimal long-run inflation rate (ticked) is equal to -1.19% per year and indicates that both sources of the policy tradeoff work into the same direction. Furthermore, $\pi^{\star}$ is closer to -1.43%, which would minimize the distortions in sector 1, than to -0.10%, which would minimize the distortions in sector 2, because Services prices are stickier than Goods prices. Figure 7 also shows that the tradeoff between sectors is smaller than the tradeoff between new and incumbent firms within a sector, which is shown in Figure 3.

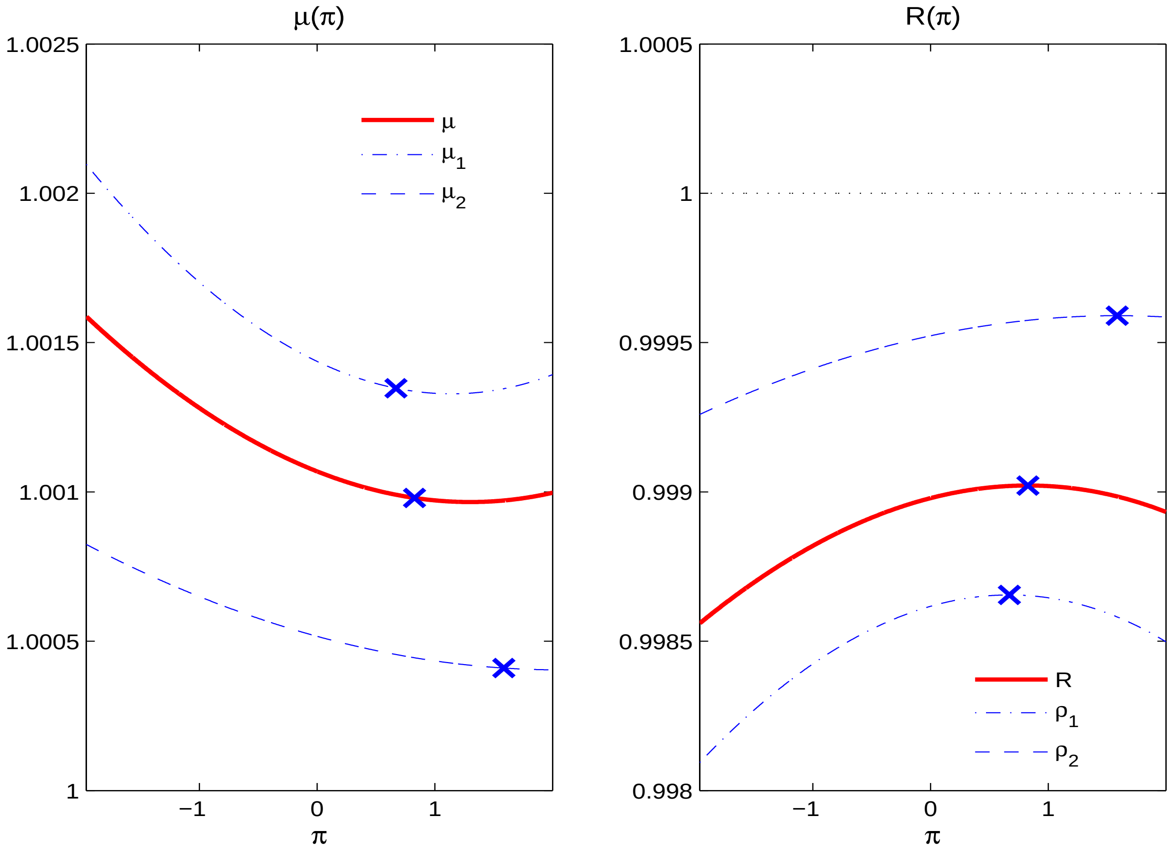

Figure 8: Aggregate distortions (bold lines) and sectoral distortions (thin lines) in the two-sector model with learning by doing and $\beta = 0.995$. Panel A shows the markup distortion $\mu(\pi)/(1-\tau_L^\star)$, and Panel B shows the relative price distortion $R(\pi)$. In both panels, π is the annualized net inflation rate.

6.2 The two-sector model with learning by doing

In the model with learning by doing, the tradeoff between sectors coexists with the tradeoff between new and incumbent firms within a sector. In this model, the optimal long-run inflation rate is positive and equal to 0.82% per year. Thus, incorporating learning by doing into the two-sector model increases the optimal long-run inflation rate by about two percentage points, i.e., from −1.19% to 0.82%. I compute the optimal long-run inflation rate numerically by maximizing steady-state welfare keeping $\tau_L^\star = -1/(\theta - 1)$ (which I vary below).

Figure 8 shows both, aggregate distortions (bold lines) and sectoral distortions (thin lines) in the model with learning by doing. Aggregate distortions imply that the government's overall policy tradeoff is best resolved by a positive long-run inflation rate (ticked).

Sectoral distortions imply that in Services, which is the sector with the relatively stickier prices, learning by doing creates larger distortions than in Goods, which is the sector with the relatively flexible prices. Thus, more sticky prices exacerbate the government's policy tradeoff.

Figure 8 also shows that the stickiness principle continues to resolve the tradeoff between sectors in the model with learning by doing, because $\pi^\star$ is closer to 0.67%, which would minimize the distortions in Services, than to 1.59%, which would minimize the distortions in Goods. Furthermore, the figure shows that the significance of new firms continues to resolve the tradeoff between new and incumbent firms in the two-sector model with learning by doing, because the optimizing government still prefers to help new firms rather than incumbent firms by using a positive long-run inflation rate.

Overall, the result in the one-sector model that the long-run inflation rate is positive after incorporating firm-level productivity growth, is robust to incorporating sectoral asymmetries in the magnitude and composition of productivity growth and the amount of price stickiness in a sector.

7 Conclusion

To be added.

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