Collateral Constraints and Macroeconomic Asymmetries*

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Abstract

Using Bayesian methods, we estimate a nonlinear general equilibrium model where occasionally binding collateral constraints on housing wealth drive an asymmetry in the link between housing prices and economic activity. The estimated model shows that, as collateral constraints became slack during the housing boom of 2001-2006, expanding housing wealth made a small contribution to consumption growth. By contrast, the housing collapse that followed tightened the constraints and sharply exacerbated the recession of 2007-2009. The empirical relevance of this asymmetry is corroborated by evidence from state- and MSA-level data.

`KEYWORDS`: Housing; Collateral Constraints; Occasionally Binding Constraints; Nonlinear Estimation of DSGE Models; Great Recession.

`JEL CLASSIFICATION`: E32, E44, E47, R21, R31

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1 Introduction

Collateral constraints drive an asymmetry in the relationship between house prices and economic activity and are a central mechanism to explain the collapse of the Great Recession. When housing wealth is high, collateral constraints are slack, and the sensitivity of borrowing and spending to changes in house prices is small. Conversely, when housing wealth is low, collateral constraints are tight, and borrowing and spending move with house prices in a more pronounced fashion. We develop this argument in two steps. First, we construct a nonlinear general equilibrium model and estimate it with Bayesian likelihood methods. The estimated model implies that, as collateral constraints became slack during the housing boom of 2001-2006, expanding housing wealth made a small contribution to consumption growth. By contrast, the subsequent housing collapse tightened the constraints and sharply exacerbated the recession of 2007-2009. Second, we present evidence from state- and MSA-level data that corroborates the asymmetry inferred from the estimated model.

Figure 1 offers a first look at the data that motivates our analysis and elucidates the basic asymmetry captured by our model. The top panel superimposes the time series of U.S. house prices and of U.S. aggregate consumption expenditures over the 1976–2011 period. The correlation coefficient is 0.55, a substantial but not extreme value. The bottom panel is a scatterplot of changes in consumption and changes in house prices, together with the predicted values of a regression of consumption growth on a third-order polynomial in house price growth. The scatterplot highlights that most of the positive correlation seems to be driven by periods when house prices are low, during both the 1990-1991 and the 2007-2009 recessions. However, excluding periods with declines in house prices would result in almost no correlation between consumption and house prices.

At the core of our analysis is a standard monetary DSGE model augmented to include a housing collateral constraint along the lines of Kiyotaki and Moore (1997), Iacoviello (2005), and Liu, Wang, and Zha (2013). As in these papers, we allow for the dual role of housing as a durable good and as collateral for "impatient" households. To this framework, we add two empirically realistic elements that generate important nonlinearities. First, in line with recent U.S. experience, monetary policy is constrained by the zero lower bound (ZLB). Second, the housing collateral constraint binds only occasionally, rather than at all times. We use Bayesian estimation methods to validate the nonlinear dynamics of the model against quarterly U.S.

data. The estimation involves inferring when the collateral constraint is binding, and when it is not, through observations that do not include the Lagrange multiplier for the constraint. We assume that the total supply of housing is fixed, although housing reallocation takes place across households in response to an array of shocks which also influence the price of housing. This assumption has the advantage that house price movements do not matter when the borrowing constraint is slack. By contrast, when the constraint is binding, the interaction of house prices with borrowing and spending decisions has a first-order effect on the macroeconomy, especially when monetary policy is unable to adjust the interest rate.

The nonlinear solution of the model allows us to capture the state-dependent effects of shocks based on whether housing wealth is high or low. We quantify the contribution of collateral constraints to business cycles in two ways. First, we show that during the 1990-1991 and the 2007-2009 recessions, as collateral constraints became binding, they were a key force in exacerbating the consumption decline. The amplification due to collateral constraints is so large in the 2007-2009 period that, in their absence, the zero lower bound would not have been reached. Second, we show that an estimated model that excludes collateral constraints is forced to rely on consumption preference shocks to explain the severe drop in consumption of the Great Recession, and that a posterior odds ratio greatly favors the model with collateral constraints.

Our model estimated on national data motivates additional empirical analysis that we conduct using data from U.S. states and Metropolitan Statistical Areas (MSAs). Regional data have the advantage is that variation in house prices and economic activity is greater than at the aggregate level. We choose measures of regional activity to match our model counterparts for consumption, employment and credit. Part of our empirical analysis looks for instruments for house price changes as a way to isolate housing preference shocks from other shocks that are more likely to jointly move both housing and other endogenous variables, as done by Mian and Sufi (2011).[1] In all cases, we verify that the asymmetries uncovered using the model estimated on national data are just as pronounced when using regional data. In particular, we find statistically significant differences in the reaction of the activity measure of interest to changes in housing prices depending on whether housing prices are high or low.[2]

[1] We are keenly aware that house prices are endogenous both in theory and in the data. Our modeling strategy attributes most of the variation in house prices to shocks to housing preferences, as in recent work by Liu, Wang, and Zha (2013).

[2] We classify house prices as high in a particular state when they are above a state-specific linear trend. We have experimented with alternative definitions with little change in the asymmetries uncovered.

Our analysis is related to two distinct bodies of work. Our model belongs to a literature that has looked at the role of financial frictions in business fluctuations, including during the Great Recession. Our regional empirical analysis builds on an expanding literature that has linked changes in measures of economic activity, such as consumption and employment, to changes in house prices.

A spate of recent papers has quantified the importance of financial shocks in exacerbating the Great Recession using a general equilibrium framework. Recent notable examples include Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), Jermann and Quadrini (2012), Christiano, Motto, and Rostagno (2013). The common thread among these papers is that financial shocks are key drivers of the Great Recession. The occasionally binding nature of the constraints and the estimation approach applied to a nonlinear DSGE model are the two elements that set our work apart. In our model, financial constraints endogenously become slack or binding, so that financial shocks are not required to effectively counteract or enhance an otherwise constant set of financial constraints. In this respect, our work extends the basic mechanisms in Mendoza (2010) who also considers occasionally binding financial constraints in a calibrated small open economy setting with an exogenous interest rate. Our extensions make it possible to construct quantitative counterfactual exercises and to consider policy alternatives in an empirically validated model for the United States.[3] For instance, we use the estimated model to gauge the effects of policies aimed at the housing market in the context of a deep recession.

As for the regional empirical analysis, our paper relates to a growing literature pointing to a prominent role for housing collateral in influencing consumption and employment. Recent contributions include Case, Quigley, and Shiller (2005), Campbell and Cocco (2007), Mian and Sufi (2011), Mian, Rao, and Sufi (2012), and Abdallah and Lastrapes (2012). Despite the emphasis on collateral constraints, this literature has failed to recognize that such a channel implies asymmetric relationships for house price increases and declines with other measures of aggregate activity and has not embedded this channel in a model for policy analysis, as we do.[4]

Section 2 presents a basic, partial-equilibrium model that illustrates how collateral constraints may imply an asymmetry in the relationship between house prices and consumption. Section 3 presents the general equilibrium extension of the basic partial equilibrium model.

[3] Gust, Lopez-Salido, and Smith (2012) also estimate a nonlinear DSGE model that takes into account the zero lower bound on nominal interest rates, but do not consider financial frictions.

[4] Our paper is also related to the work of Lustig and van Nieuwerburgh (2010), who find that in times when US housing collateral is scarce nationally, regional consumption is about twice as sensitive to income shocks. However, the channel they emphasize – time variation in risk-sharing among regions – is different from ours.

Sections 4 and 5 describe the estimation method and results for the general equilibrium model. Section 6 presents additional evidence on asymmetries in the relationship between house prices and other measures of economic activity based on state and MSA-level data. Section 7 considers an experiment which highlights how the same policy – a transfer to indebted borrowers – can have different effects depending on whether house prices are high or low. Section 8 concludes.

2 The Basic Model: Collateral Constraints and Asymmetries

To fix ideas regarding the fundamental asymmetry introduced by collateral constraints, it is useful to work through a basic model and analyze its implications for how consumption responds to changes in house prices. Throughout this section, we sidestep general equilibrium considerations and assume that the price of housing is exogenous. We relax all these assumptions in the full DSGE model of the next section.

Consider the problem of a household that has to choose profiles for goods consumption c_t, housing h_t, and borrowing b_t. The household's problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t + \mathsf{j} \log h_t \right), \tag{1}$$

where E_0 is the conditional expectation operator. The household is subject to the following constraints:

$$\begin{aligned} c_t + q_t h_t &= y + b_t - R b_{t-1} + q_t \left(1 - \delta_h\right) h_{t-1}; && (2) \\ b_t &\leq \mathsf{m} q_t h_t; && (3) \\ \log q_t &= \rho_q \log q_{t-1} + \varepsilon_{q,t}. && (4) \end{aligned}$$

The first constraint is the budget constraint. Income y is fixed and normalized to one. The term b_t denotes one-period debt. The gross one-period interest rate is R. Housing, which depreciates at rate δ_h, has a price q_t in units of consumption. The second constraint is a borrowing constraint that limits borrowing to a maximum fraction m of housing wealth. The third equation describes the price of housing, q_t, which follows an AR(1) stochastic process, where $\varepsilon_{q,t}$ is a zero-mean, i.i.d. process with variance σ_q^2.

Denoting with λ_t the Lagrange multiplier on the borrowing constraint, the Euler equation for consumption is given by:

$$\frac{1}{c_t} = \beta R E_t\left(\frac{1}{c_{t+1}}\right) + \lambda_t. \tag{5}$$

To develop the intuition for our result, it is useful to consider a log-linear approximation of equation (5) in a steady state without shocks. Under the assumption that $\beta R < 1$, the borrowing constraint binds, and leverage (the ratio of debt to housing wealth) is at its upper bound given by the maximum loan-to-value ratio (LTV) m. In that steady state, $\overline{\lambda} > 0$, and $\overline{c} = \overline{y} - ((R-1)\,\mathsf{m} - \delta_h)\,\overline{q}\overline{h}$. Solving this equation forward and linearizing, one obtains the following expression for consumption in percent deviation from steady state, $\widehat{c}_t$:

$$\widehat{c}_t = -\frac{1-\beta R}{\overline{\lambda}} E_t\left(\lambda_t - \overline{\lambda} + \beta R\left(\lambda_{t+1} - \overline{\lambda}\right) + \beta^2 R^2\left(\lambda_{t+2} - \overline{\lambda}\right) + \ldots\right). \tag{6}$$

Expressing the Euler equation as above shows that consumption depends negatively on current and future expected borrowing constraints. As shown by equation (3), increases in q_t will loosen the borrowing constraint. So long as they keep λ_t positive, increases and decreases in q_t will have roughly symmetric effects on c_t. However, large enough increases in q_t lead to a fundamental asymmetry. The multiplier λ_t cannot fall below zero. Consequently, large increases in q_t can bring λ_t to its lower bound and will have proportionally smaller effects on c_t than decreases in q_t. Intuitively, an impatient borrower prefers a consumption profile that is declining over time. A temporary jump in house prices will enable such a profile (high c today, low c tomorrow) without borrowing all the way up to the limit.

More formally, the household's state at time t is its housing h_{t-1}, debt b_{t-1} and the current realization of the house price q_t, and the optimal decision are given by the consumption choice $c_t = \mathcal{C}(q_t, h_{t-1}, b_{t-1})$, the housing choice $h_t = \mathcal{H}(q_t, h_{t-1}, b_{t-1})$ and the debt choice $b_t = \mathcal{B}(q_t, h_{t-1}, b_{t-1})$ that maximize expected utility subject to (2) and (3), given the house price process. Figure 2 shows the optimal leverage and the consumption function obtained from the model outlined above.[5] As the figure illustrates, high house prices are associated with slack borrowing constraints, and with a lower sensitivity of consumption to changes in house prices. Instead, when household borrowing is constrained – an outcome that is more likely when house prices are low and the initial stock of debt is high – the sensitivity of consumption

[5] The policy functions in Figure 2 are obtained via value and policy function iteration. The calibrated parameters are $\beta = 0.99$, $\mathsf{j}= 0.12$, $\mathsf{m}= 0.9$, $R = 1.005$, $\delta = 0.01$. The resulting steady-state ratio of housing wealth to annual income ratio is 1.5. For the house price process we set $\rho_q = 0.96$ and $\sigma_q = 0.0175$, in order to match a standard deviation of the quarterly growth rate of house prices equal to 1.77 percent, as in the data.

to changes in house prices becomes large. This idea is developed further both in the full model and in the empirical analysis to follow.

3 The Full Model: Demand Effects in General Equilibrium

To quantify the importance of the asymmetric relationship between house prices and consumption, we embed the basic mechanisms described in Section 2 in an estimated general equilibrium model. Our starting point is a standard monetary DSGE model along the lines of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). The model features nominal wage and price rigidities, a monetary authority using a Taylor rule, habit formation in consumption and investment adjustment costs.[6] To this framework we add three main elements. First, housing has a dual role: it is a durable good (which enter the utility function separately from consumption and labor), and it serves as collateral for "impatient" households. The supply of housing is fixed (its price varies endogenously), but housing reallocation takes place across "patient" and "impatient" households in response to an array of shocks. Second, we allow for the collateral constraint on borrowing to bind only occasionally. The estimation exercise allows us to infer when the constraint binds using observations that do not include the hidden Lagrange multiplier on the constraint. Third, in line with the U.S. experience since 2008, monetary policy is potentially constrained by the zero lower bound.

Our assumption that housing is in fixed supply and plays no role in production (unlike in the work of Liu, Wang, and Zha 2013 and Iacoviello and Neri 2010) has the important advantage that the model behaves essentially like the one in Christiano, Eichenbaum, and Evans (2005) when the borrowing constraint is found to be slack. With a slack borrowing constraint, housing prices only passively respond to movements in the macroeconomy, but play no feedback effect on other macro variables.

Below, we describe the key model features. Appendix A provides additional details as well as a list of all necessary conditions for an equilibrium.

[6] The benchmark model abstracts from trends, excludes TFP shocks, and assumes fixed capacity utilization. All these assumptions are relaxed as part of sensitivity analysis presented in Section 5.4.

3.1 Households

Within each group of patient and impatient households, a representative household maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathsf{z}_t \left(\Gamma \log \left(c_t - \varepsilon c_{t-1} \right) + \mathsf{j}_t \log h_t - \frac{1}{1+\eta} n_t^{1+\eta} \right); \tag{7}$$

$$E_0 \sum_{t=0}^{\infty} \left(\beta'\right)^t \mathsf{z}_t \left(\Gamma' \log \left(c_t' - \varepsilon c_{t-1}' \right) + \mathsf{j}_t \log h_t' - \frac{1}{1+\eta} n_t'^{1+\eta} \right) \tag{8}$$

Variables accompanied by the prime symbol refer to impatient households. The terms c_t, h_t, n_t are consumption, housing, and hours. The discount factors are β and β', with $\beta' < \beta$. The term j_t captures shocks to housing preferences. An increase in j_t shifts preferences away from consumption and leisure and towards housing, thus resulting in an increase in housing demand and, ultimately, housing prices. The term z_t captures a shock to intertemporal preferences. A rise in z_t increases households' willingness to spend today, acting as a consumption demand shock. The shock processes follow:

$$\log \mathsf{j}_t = (1-\rho_J) \log \bar{\mathsf{j}} + \rho_J \log \mathsf{j}_{t-1} + u_{j,t}, \tag{9}$$

$$\log \mathsf{z}_t = \rho_Z \log \mathsf{z}_{t-1} + u_{z,t}. \tag{10}$$

where $u_{j,t}$ and $u_{z,t}$ are n.i.i.d. processes with variance σ_J^2 and σ_Z^2. Above, ε measures habits in consumption. The scaling factors $\Gamma = (1-\varepsilon) / (1-\beta\varepsilon)$ and $\Gamma' = (1-\varepsilon) / (1-\beta'\varepsilon)$ ensure that the marginal utilities of consumption are $1/\bar{c}$ and $1/\bar{c}'$ in the non-stochastic steady state.

Patient households maximize utility subject to a budget constraint that in real terms reads:

$$c_t + q_t h_t + b_t + i_t = \frac{w_t n_t}{x_{w,t}} + q_t h_{t-1} + \frac{R_{t-1} b_{t-1}}{\pi_t} + r_{k,t} k_{t-1} + div_t, \tag{11}$$

where investment and capital are linked by:

$$k_t = \mathsf{a}_t \left(i_t - \phi \frac{\left(i_t - i_{t-1}\right)^2}{\bar{i}} \right) + (1-\delta_k) k_{t-1}, \tag{12}$$

and a_t follows:

$$\log \mathsf{a}_t = \rho_K \log \mathsf{a}_{t-1} + u_{k,t},$$

where $u_{K,t}$ is a n.i.i.d. process with variance σ_K^2. Patient agents choose consumption c_t, investment i_t, capital k_t (which depreciates at the rate δ_k), housing h_t (priced, in units of consumption, at q_t), hours n_t and loans to impatient households b_t to maximize utility subject to (11) and to (12). The term a_t is an investment shock affecting the technology transforming investment goods into capital goods. This type of shock has recently been identified as an important source of aggregate fluctuations (Justiniano, Primiceri, and Tambalotti 2011). Loans are set in nominal terms and yield a riskless nominal return of R_t. The real wage is w_t, the real rental rate of capital is $R_{k,t}$. The term $x_{w,t}$ denotes the markup (due to monopolistic competition in the labor market) between the wage paid by the wholesale firm and the wage paid to the households, which accrues to the labor unions. Finally, $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, div_t are lump-sum profits from final good firms and from labor unions.[7] The formulation in (12) allows for convex investment adjustment costs, parameterized by ϕ.

Impatient households do not accumulate capital and do not own final good firms. Their budget constraint is given by:

$$c'_t + q_t h'_t + \frac{R_{t-1} b_{t-1}}{\pi_t} = \frac{w'_t n'_t}{x'_{w,t}} + q_t h'_{t-1} + b_t + div'_t; \tag{13}$$

Impatient households face a borrowing constraint that limits the amount they can borrow, b_t, to a fraction m of the house value. We start from the constraint of the basic model of Section 2 and extend it with an eye to empirical realism. Specifically, we allow for – but do not impose – the possibility that borrowing constraints adjust to reflect the market value of the housing stock only sluggishly. Accordingly, the constraint takes the form:

$$b_t \leq \gamma \frac{b_{t-1}}{\pi_t} + (1-\gamma)\, \mathsf{m} q_t h'_t \tag{14}$$

where $\gamma > 0$ measures the degree of inertia in the borrowing limit, and m is the steady-state loan-to-value ratio.[8] Such specification mimics the common practice that borrowing constraints are fully reset only for those who move or refinance their mortgage. While our model lacks the heterogeneity at the microeconomic level that could capture this phenomenon, this specification

[7] We consider a cashless economy as in Woodford (2003).

[8] An interpretation of this borrowing constraint is that, with multi-period debt contracts, the borrowing constraint on housing is reset only for households that acquire new housing goods or choose to refinance. Of course, in the face of home equity line of credits, adjustments of the borrowing constraint may also reflect lenders' perceived changes in the collateral value. Justiniano, Primiceri, and Tambalotti (2013), who study the determinants of household leveraging and deleveraging in a calibrated dynamic general equilibrium model, adopt an analogous specification.

captures the empirical finding that measures of aggregate debt tend to lag house prices movements. For instance, a regression of household mortgage debt on its lag and on housing wealth yields coefficients of 0.89 on lagged debt, and of 0.10 on housing wealth. Both coefficients are statistically significant ($t - statistics$ of 45 and 7 respectively), and the R^2 is 0.97.[9]

3.2 Wholesale Firms

To allow for nominal price rigidities, we differentiate between competitive flexible price/wholesale firms that produce wholesale goods, and retail/final good firms that operate in the final good sector under monopolistic competition subject to implicit costs to adjusting nominal prices following Calvo-style contracts. Wholesale firms hire capital and labor supplied by the two types of households to produce wholesale goods y_t. They solve:

$$\max \frac{y_t}{x_{p,t}} - w_t n_t - w_t' n_t' - r_{k,t} k_{t-1}. \tag{15}$$

Above, $x_{p,t} = P_t / P_t^w$ is the price markup of final over wholesale goods, where P_t^w is the nominal price of wholesale goods. The production technology is:

$$y_t = n_t^{(1-\sigma)(1-\alpha)} n_t'^{\sigma(1-\alpha)} k_{t-1}^{\alpha}. \tag{16}$$

In (16), the non-housing sector produces output with labor and capital. The parameter σ measures the labor income share that accrues in the economy to impatient households. When σ approaches zero, the model boils down to a model without collateral effects.

3.3 Final Goods Firms, Nominal Rigidities and Monetary Policy

There are Calvo-style price rigidities and wage rigidities in the final good sector. As in Bernanke, Gertler, and Gilchrist (1999), final good firms (owned by patient households) buy wholesale goods y_t from wholesale firms in a competitive market, differentiate the goods at no cost, and

[9] Mortgage debt and housing wealth are from Table B.100 of the Financial Accounts. We divide both series by the GDP deflator and HP-filter them (with $\lambda = 10,000$). The regression – over the 1980Q1–2011Q4 period – might not capture adequately the specification in the borrowing constraint for three reasons. First, the Financial Accounts data are on aggregate housing wealth – housing wealth held both by borrowers and savers. Second, the data on debt include gross mortgage debt – debt held both by borrowers without any other financial assets, and by savers who hold other financial assets alongside mortgage debt. Last, the constraint above may not bind in periods of high housing prices, thus weakening the link between housing wealth and mortgage debt.

sell them at a markup $x_{p,t}$ over the marginal cost. The CES aggregates of these goods are converted back into homogeneous consumption and investment goods by households. Each period, a fraction $1-\theta_\pi$ of retailers set prices optimally, while a fraction θ_π cannot do so, and index prices to the steady state inflation $\overline{\pi}$. The resulting consumption-sector Phillips curve is:

$$\log\left(\pi_t/\overline{\pi}\right) = \beta E_t \log\left(\pi_{t+1}/\overline{\pi}\right) - \varepsilon_\pi \log\left(x_{p,t}/\overline{x}_p\right) + \mathtt{u}_{p,t}, \tag{17}$$

where $\varepsilon_\pi = \left(1-\theta_\pi\right)\left(1-\beta\theta_\pi\right)/\theta_\pi$ measures the sensitivity of inflation to changes in the markup, $x_{p,t}$, relative to its steady-state value, $\overline{x}_p$, whereas the term $\mathtt{u}_{p,t}$ denotes an i.i.d. price markup shock, $\mathtt{u}_{p,t} \sim N\left(0, \sigma_P^2\right)$.

Wage setting is modeled in an analogous way. Households supply homogeneous labor services to unions. The unions differentiate labor services as in Smets and Wouters (2007), set wages subject to a Calvo scheme and offer labor services to labor packers who reassemble these services into the homogeneous labor composites n_c and n'_c. Wholesale firms hire labor from these packers. The pricing rules set by the union imply the following wage Phillips curves:

$$\begin{aligned}
\log\left(\omega_t/\overline{\pi}\right) &= \beta E_t \log\left(\omega_{t+1}\overline{\pi}\right) - \varepsilon_w \log\left(x_{w,t}/\overline{x}_w\right) + \mathtt{u}_{w,t}, & (18)\\
\log\left(\omega'_t/\overline{\pi}\right) &= \beta' E_t \log\left(\omega'_{t+1}/\overline{\pi}\right) - \varepsilon'_w \log\left(x'_{w,t}/\overline{x}'_w\right) + \mathtt{u}_{w,t} & (19)
\end{aligned}$$

where $\omega_t = \frac{w_t \pi_t}{w_{t-1}}$ and $\omega'_t = \frac{w'_t \pi_t}{w_{t-1}}$ denote wage inflation for each household type, and $\mathtt{u}_{w,t} \sim N\left(0, \sigma_W^2\right)$ denotes an i.i.d. wage markup shock.[10] As in Justiniano, Primiceri, and Tambalotti (2011) and Smets and Wouters (2007), price and wage markup shocks act as supply-side disturbances, both moving output and inflation in opposite directions.

Monetary policy follows a modified Taylor rule that allows for interest rate smoothing and responds to year-on-year inflation and GDP[11] in deviation from steady state, subject to the zero lower bound:

$$R_t = \max\left[1, R_{t-1}^{r_R}\left(\frac{\pi_t^A}{\overline{\pi^A}}\right)^{(1-r_R)r_\pi}\left(\frac{y_t}{\overline{y}}\right)^{(1-r_R)r_Y}\overline{R}^{1-r_R}\mathsf{e}_t\right]. \tag{20}$$

[10] We assume that there are two unions, one for each household type. While the unions choose slightly different wage rates, reflecting the different desired consumption profiles of the two household types, we assume that the Calvo probability of changing wages is the same.

[11] Wholesale goods y_t are different from the CES aggregates of these goods that comprise total GDP. We make use of the result that the two are equal within a local region of the steady state. See e.g. Iacoviello (2005).

The term $\overline{R}$ is the steady-state nominal real interest rate in gross terms, and $\log \mathsf{e}_t = \rho_R \log \mathsf{e}_{t-1} + \mathsf{u}_{r,t}$ (with $\mathsf{u}_{r,t} \sim N\left(0, \sigma_R^2\right)$) denotes an autoregressive monetary policy shock.[12] As in Christiano, Eichenbaum, and Rebelo (2011), the presence of the ZLB creates an additional, important nonlinearity: shocks that move output and prices in the same direction can be amplified by the inability of central bank to adjust short-term interest rates.

4 Estimation of the Full Model

We use Bayesian estimation methods to size the deep structural parameters of the model including the share of impatient households. A subset of the model parameters are calibrated based on information complementary to the estimation sample.

4.1 Calibration and Priors

The calibrated parameters are reported in Table 1. We set $\beta = 0.995$, implying a steady-state 2% annual real interest rate. The capital share $\alpha = 0.3$ and the depreciation rate $\delta_k = 0.025$ imply a steady-state capital to annual output ratio of 2.1, and an investment to output ratio of 0.21. The weight on housing in the utility function $\overline{j}$ is set at 0.04, implying a steady-state ratio of housing wealth to annual output of 1.5. The maximum loan-to-value ratio m is set at 0.9. The labor disutility parameter η is set at 1, implying a unitary Frisch labor supply elasticity. The steady-state gross price and wage markups $\overline{x}_p$ and $\overline{x}_w$ are both set at 1.2. Finally, we set $\overline{\pi} = 1.005$ implying a 2% annual rate of inflation in steady state.

All other parameters are estimated using Bayesian methods. To this end, we select priors commonly used in the literature. The prior distributions are reported in Table 1. Our choices hew closely to those of Smets and Wouters (2007), apart from parameters that were not present in their model. In particular, we assume a rather diffuse prior for the wage share of impatient households σ (centered at 0.5) and for the inertial coefficient in the borrowing constraint γ (also centered at 0.5). A key parameter in determining the asymmetries is the discount factor of the impatient agents, β'. Values of this parameter that fall below a certain threshold imply that impatient agents never escape the borrowing constraint. In that case, the model has no asymmetries (except for the presence of the ZLB), regardless of shocks size, and produces a large correlation between housing price growth and consumption growth, since the borrowing

[12] Year-on-year inflation (expressed in quarterly units, like the interest rate) is defined as $\pi_t^A \equiv \left(P_t/P_{t-4}\right)^{0.25}$.

constraint holds all the time with equality. Conversely, when β' takes on higher values, closer to discount factor of patient agents, modest increases in house prices suffice to make the borrowing constraint slack (even though the constraint is expected to bind in the long run). We set the prior mean for β' at 0.99 – corresponding to an annualized discount rate of 4 percent – with a standard deviation of 0.0015.

4.2 Data

The estimation of the model is based on observations for six series: total real household consumption, price (GDP deflator) inflation, wage inflation, real investment, real housing prices, and the Federal Funds Rate. The observations span the period from 1985Q1 to 2011Q4 (Appendix B describes the data in detail). Our model features six observables and six shocks – investment-specific shocks, wage markup, price markup, monetary policy, intertemporal preferences, and housing preferences.

Prior to estimation, we use a one-sided HP filter (with a value of λ =100,000) in order to remove the low-frequency components of consumption, investment and housing prices. The one-sided HP filter has two advantages. First, it yields plausible estimates of the trend and the cycle for these variables. For instance, as shown by the solid lines of Figure 4, consumption and house prices were respectively 8 and 30 percent below trend at the trough of the Great Recession. Second, as argued for instance by Stock and Watson (1999), the one-sided filter is not affected by the correlation of current observations with subsequent observations. Section 5.4 documents that our results are robust to the inclusion and joint estimation of linear deterministic trends.

4.3 Solution and Likelihood

The proliferation of state variables renders standard global solution algorithms inoperable. Moreover, the occasionally binding constraint on borrowing and the non-negativity constraint on the policy interest rate make first-order perturbation methods inapplicable. We solve the model using the piecewise linear method sketched in Appendix C and described more fully in Guerrieri and Iacoviello (2014). Essentially, depending on whether the zero lower bound binds or not, and depending on whether the collateral constraint on housing binds or not, we identify four regimes. The piecewise linear solution method involves linking the first-order approximation of the model around the same point under each regime. Importantly, the solution that the algorithm produces is not just linear, with different sets of coefficients depending on each of the four regimes, but rather, it can be highly nonlinear. The dynamics in each regime may crucially depend on how long one expects to be in that regime. In turn, how long one expects to be in that regime depends on the state vector.

The model solution takes the form:

$$X_t = \mathbf{P}(X_{t-1}, \epsilon_t)X_{t-1} + \mathbf{D}(X_{t-1}, \epsilon_t) + \mathbf{Q}(X_{t-1}, \epsilon_t)\epsilon_t. \tag{21}$$

The vector X_t collects all the variables in the model, except the innovations to the shock processes, which are gathered in the vector ϵ_t. The matrix of reduced-form coefficients $\mathbf{P}$ is state-dependent, as are the vector $\mathbf{D}$ and the matrix $\mathbf{Q}$. These matrices and vector are functions of the lagged state vector and of the current innovations. However, while the current innovations can trigger a change in the reduced-form coefficients, X_t is still locally linear in ϵ_t.

We represent the solution in Equation (21) below in terms of observed series by premultiplying the state vector X_t by the matrix $\mathbf{H}_t$, which selects the observed variables. Accordingly, the vector of observed series Y_t is simply $Y_t = \mathbf{H}_t X_t$.[13]

Because the reduced-form coefficients in Equation (21) depend on ϵ_t, we cannot use the Kalman filter to retrieve the estimates of the innovations in ϵ_t. Instead, following Fair and Taylor (1983) we recursively solve for ϵ_t, given X_{t-1} and the current realization of Y_t, the following system of non-linear equations:[14]

$$Y_t = \mathbf{H}_t\mathbf{P}(X_{t-1}, \epsilon_t)X_{t-1} + \mathbf{H}_t\mathbf{D}(X_{t-1}, \epsilon_t) + \mathbf{H}_t\mathbf{Q}(X_{t-1}, \epsilon_t)\epsilon_t. \tag{22}$$

The vector X_t contains unobserved components, so the filtering scheme requires an initialization. We assume that the initial X_0 coincides with the model's steady state and train the filter using the first 20 observations.

Given that ϵ_t is assumed to be drawn from a multivariate Normal distribution with covari-

[13] The matrix that selects the observed variables is time-varying because we drop the interest rate from the observed vector at the zero lower bound. In that case, we also assume that monetary policy shock is zero, unless the notional rate implied by the model is positive when the observed rate is still zero. In that case, we select the notional rate as observed and reinstate the monetary policy shock.

[14] There is in principle the possibility of multiple solutions for ϵ_t to the extent that Equation (22) is highly nonlinear in ϵ_t. In our specific application we have not found evidence of this multiplicity. In theory, however, our approach to constructing the likelihood does not depend crucially on a one to one mapping between Y_t and ϵ_t. Standard results could be invoked to allow for a general correspondence between Y_t and ϵ_t when constructing the likelihood function.

ance matrix $\mathbf{\Sigma}$, a change in variables argument implies that the logarithmic transformation of the likelihood f for the observed data Y^T can be written as:

$$log(f(Y^T)) = -\frac{T}{2}\log(\det(\mathbf{\Sigma})) - \frac{1}{2}\sum_{t=1}^{T}\epsilon_t'\left(\mathbf{\Sigma}^{-1}\right)\epsilon_t + \sum_{t=1}^{T}\log(|\det\frac{\partial\epsilon_t}{\partial Y_t}|). \tag{23}$$

Notice that the inverse transformation from the shocks to the observations needed to form the Jacobian matrix $\frac{\partial\epsilon_t}{\partial Y_t}$ is only given implicitly by $(\mathbf{H}_t\mathbf{Q}(X_{t-1},\epsilon_t))\epsilon_t - (Y_t - \mathbf{H}_t\mathbf{P}(X_{t-1},\epsilon_t)X_{t-1} - \mathbf{H}_t\mathbf{D}_t) = 0$. To proceed by implicit differentiation, we verify that the determinant of $\mathbf{H}_t\mathbf{Q}(X_{t-1},\epsilon_t)$ is nonzero. Accordingly, the implicit transformation is locally invertible and the Jacobian of the inverse transformation is given by:

$$\frac{\partial\epsilon_t}{\partial Y_t} = (\mathbf{H}_t\mathbf{Q}(X_{t-1},\epsilon_t))^{-1}.$$

Derivation of this Jacobian relies on local linearity in ϵ_t of the model's solution, a property that is further discussed in Appendix B. Using this result and recognizing that $|\det((\mathbf{H}_t\mathbf{Q}(X_{t-1},\epsilon_t))^{-1})| = 1/|\det(\mathbf{H}_t\mathbf{Q}(X_{t-1},\epsilon_t))|$, the logarithmic transformation of the likelihood in Equation (23) can be expressed as:

$$log(f(Y^T)) = -\frac{T}{2}\log(\det(\mathbf{\Sigma})) - \frac{1}{2}\sum_{t=1}^{T}\epsilon_t'\left(\mathbf{\Sigma}^{-1}\right)\epsilon_t - \sum_{t=1}^{T}\log(|\det\mathbf{H}_t\mathbf{Q}(X_{t-1},\epsilon_t)|). \tag{24}$$

In our case, the Jacobian of the inverse transformation for the change in variables is known from the model's solution and does not require any additional calculations. This property of the solution allows to efficiently evaluate the likelihood in a matter of seconds, affording us crucial time savings, relative to the general approach in Fair and Taylor (1983), that make estimation of our model possible.

Our approach to solving and estimating the model provides critical time savings relative to the approach in Fair and Taylor (1983). On the solution side, they introduced a multi-step iterative shooting algorithm that collapses a dynamic problem into a series of static problems. Within each iteration, a quasi-Newton method is used to solve a system of nonlinear equations for the current endogenous variables, given predetermined conditions and given an initial guess for the future path of endogenous variables. At each iteration, the previous solution is used as a new guess for the entire path. After this step is completed, a second step is used to verify that the choice of end-point does not affect the initial dynamics of the variables. Accordingly, operationalizing their approach requires an initial guess for the full path of all endogenous variables up to what is likely to be a distant end-point. While no general result guarantees the convergence of Newton methods, an unsuitable initial guess often prevents convergence. Our solution approach is also iterative, but it is carried out in one step and only requires an initial guess relative to whether each constraint is binding or not over each sample period. On the estimation side, further critical time savings are reaped as our algorithm produces the Jacobian matrix for the inverse transformation from the shocks to the observations as a byproduct of the solution, instead of requiring the computation of numerical derivatives. A one-sided finite difference derivative, for instance, would augment computation time by a factor of n^2, where n is the number of shocks in our application. We surmise that these complications explain why Fair and Taylor's original approach has not been applied to estimate DSGE models.

5 Model Estimation Results

After a discussion of key parameter estimates, we highlight the non-linear nature of the reaction to positive and negative shocks. Counterfactual experiments show that collateral constraints on housing wealth played a key role in exacerbating the economic collapse of the Great Recession. A smaller, but important contribution to the economic collapse stemmed from the zero lower bound on nominal interest rates. Finally, we show that an estimated model that excludes collateral constraints on housing wealth is forced to rely on consumption preference shocks to explain the severe drop in consumption of the Great Recession. A posterior odds ratio greatly favors the model with collateral constraints.

5.1 Estimated Parameters

We combine the evaluation of the likelihood with prior information about the parameters in order to construct and maximize the posterior as a function of the model's parameters, given the data. We then construct the posterior density of the model's parameters using a standard random walk Metropolis-Hastings algorithm (with a chain of 50,000 draws).

The posterior modes of the estimated parameters and other statistics are reported in Table 1. Crucially, we find a sizeable fraction of impatient agents, governed by σ. Our choice of prior, a diffuse beta distribution, simply guarantees that this fraction remains bounded between 0 and 1. The posterior mode is estimated to be 0.42 and the 90% confidence interval ranges from 0.33 to 0.53. Accordingly, $\sigma = 0$, which would imply the exclusion of collateral constraints from the model, is highly unlikely. Moving to the parameters that govern nominal rigidities and monetary policy, the posterior modes for the Calvo parameters governing the frequency of price and wage adjustment are both equal to 0.92. This high degree of price and wage rigidity likely compensates the absence of real rigidities, such as variable capacity utilization, partial indexation of prices and wages, or firm-specific capital. The estimated interest rate reaction function gives less weight to output and more weight to inflation than our prior, which was centered around Taylor's canonical values of 0.5 (for output, measured at an annual rate) and 1.5 (for inflation). Finally, we found evidence of some inertia in the borrowing constraint, as shown by the estimated value of γ which equals 0.45. A positive value of γ slows down the extent to which deleveraging takes place in periods of falling housing prices, thus creating inertia in consumption.

Given the parameter estimates, key empirical properties of the model line up well with the data in several respects. First, in response to small shocks that do not make the borrowing constraint slack or the ZLB bind, the model's impulse responses, for instance those to monetary shocks, are in line with the findings of existing studies, such as Christiano, Trabandt, and Walentin (2010).[15] Second, key moments in the data line up well with those of our estimated model.[16] In the model like in the data, the correlation between the house prices and consumption – 0.69 in the data, 0.40 in the model – is higher than the correlation between house prices and investment – 0.30 in the data, 0.08 in the model. The standard deviation of consumption is 2.2 percent in the model, compared to 2.9 percent in the data. The model also captures the high volatility of house prices – their standard deviation is 13.9 percent in the model, 12.5 percent in the data.

Finally, in variance decomposition exercises, we find that about three quarters of the house price volatility is driven by the housing preference shock (as in recent work by Liu, Wang, and Zha 2013). We elaborate on this point with two experiments described below. We will focus first on a comparison of positive and negative housing shocks, and will move on to present a decomposition that highlights the role of housing shocks in the collapse of the Great Recession.

[15] Appendix B shows the impulse responses both for the estimated model and for an estimated version of the model without financial frictions.

[16] Our nonlinear model does not admit a closed form for the moments of the variables. We thus compute the model statistics on simulated series (using a long simulation with $T = 5,000$).

5.2 Responses to Positive and Negative Shocks to Housing Prices

Figure 3 illustrates the fundamental asymmetry in the estimated model and confirms key insights from the basic model discussed in Section 2. The figure considers the effects of shocks to housing preferences, governed by the process j_t in Equation (9). Two series of innovations to j_t occur between periods 1 and 8. One of the two series of innovation lowers house prices by 25 percent. The other series raises house prices by 25%. From period 9 onwards, there are no more innovations and the shock j_t follows its autoregressive process. All parameters are set to their estimated posterior mode.

The dashed lines denote the effects of the decline in house prices. This decline reduces the collateral capacity of constrained households, who borrow less and are forced to curtail their non-housing consumption even further. At its trough, consumption is 3.5 percent lower relative to its steady state. The nominal and real rigidities imply that the decline in aggregate consumption translates into lower demand for labor from firms. As a consequence, hours worked fall 2.5 percent below the baseline.

The solid lines plot the responses to a shock of same magnitude and profile but with opposite sign. In this case, house prices increase 25 percent. As in the partial equilibrium model described in Section 2, a protracted increase in house prices can make the borrowing constraint slack. The Lagrange multiplier for the borrowing constraint bottoms out at zero and remains at zero for some time, before rising as house prices revert to baseline. When the constraint is slack, the borrowing constraint channel remains operative only in expectation. Thus, impatient households discount that channel more heavily the longer the constraint is expected to remain slack. As a consequence, the response of consumption to the house price increases considered in the figure is not as dramatic as the reaction to the equally-size house price declines. At its peak, consumption rises 1.2 percent above its baseline, a magnitude one third as big as for the house price decline.[17] In turn, the increase in hours is muted, peaking at 0.5 percent above the baseline.

In experiments not reported here, we have found modest asymmetries for other shocks that affect house prices and consumption in our general equilibrium model. These shocks are likely to generate significant asymmetries only insofar as they affect house prices or collateral capacity.

[17] The shocks that lower house prices in Figure 3 are not large enough to push the model to the zero lower bound on interest rates. Accordingly, the asymmetry shown in the responses is only driven by the occasionally binding collateral constraint.

However, the asymmetry that we uncover is independent of the particular stochastic structure of the model, and needs not rely on housing demand shocks only. Potentially, in any housing model with occasionally binding constraints, one can find substantial asymmetries as long as the model can match the observed swings in house prices.

5.3 The Asymmetric Contribution of Housing to Business Cycles

In order to highlight the role of collateral constraints on housing wealth during the Great Recession, we consider two experiments. The first experiment feeds the estimated sequence of shocks for the benchmark model with collateral constraints into a model that does not encompass those constraints, but that is otherwise identical to the benchmark model (and that is not re-estimated). Furthermore, we use a similar device to illustrate the role of the zero lower bound. The second experiment compares the shocks needed to fit the same observed series for the benchmark model and for a model that is re-estimated after the exclusion of the collateral constraints.

By construction, when we feed the estimated sequence of shocks back into the benchmark model, we can get an exact match for the observed (detrended) data. Moreover, we can recover the path of all unobserved variables, including the Lagrange multiplier on the borrowing constraint. Figure 4 compares the observed data against the outcomes of two counterfactual experiments.

In the first experiment ("No Collateral Constraint"), we set σ, the wage share of impatient households, equal to 0, so that collateral constraints are ruled out. Housing prices are still matched, since housing services are essentially priced by the patient households. However, consumption diverges markedly from the observed data. When the Lagrange multiplier is estimated to be binding, as in the 1990-1991 and the 2007-2009 recessions, a large gap opens up between the observed and counterfactual consumption levels. During the Great Recession, the model without collateral constraints predicts a decline in consumption of 1.3 percent, whereas the observed decline was 5.5 percent, implying that collateral effects account for three quarters of the observed decline in (detrended) consumption. Remarkably, without collateral constraints the recession would have been curbed to such an extent that the Federal Funds rate would not have reached zero.[18] By contrast, when the Lagrange multiplier on the collateral constraint is estimated to be slack, there is little difference between the counterfactual consumption without collateral constraints and the observed consumption.

The second experiment ("No ZLB") gauges how much of the decline in consumption was due to the zero lower bound constraint. As shown in the Figure, absent the zero lower bound, nominal interest rates would have fallen to minus 2 percent, and the trough of consumption would have been one percentage point lower than the trough observed.

Figure 5 provides an additional angle to compare our model against a model without collateral constraints. Once more, to exclude the collateral constraints, we impose $\sigma = 0$, but this time we re-estimate the restricted model (posterior modes for all the parameters of the restricted model are reported in column 2 of Table 2). Figure 5 highlights the effects of different patterns of shocks needed to match the data by the benchmark model with occasionally binding collateral constraints and by the restricted model. The top panels in the figure compare the evolution of consumption and housing prices when only housing preference shocks are turned on. For both models, the evolution of housing prices are in line with observed housing prices. However, the two models differ drastically in their implications for consumption. Whereas the benchmark model closely matches the evolution of both housing prices and consumption with just the housing shocks, housing shocks have no bearing on consumption for the model without the collateral constraints. The bottom panels of Figure 5 compare the evolution of consumption and housing prices from the two models when only consumption preference shocks are turned on. These panels highlight that the restricted model is completely dependent on a sequence of consumption shocks to match the consumption data. Accordingly, the proliferation of shocks that are needed for the restricted model to fit the observed data results in a posterior odds ratio exceeding 90 to 1, overwhelmingly in favor of the model with collateral constraints.

In sum, we find it compelling to argue that lower house prices coupled with weaker households balance sheets were the main culprits for the consumption collapse during the 2007-2009 recession. By contrast, our results show that a model that excludes collateral constraints has to rely on a contagious attack of patience to explain the depth of the Great Recession.

[18] In our sample, the interest rate is at zero from 2009Q1 until the end of sample (2011Q4). According to the estimated model, the interest rate prescribed by the Taylor rule would have remained at its lower bound until 2011Q1. The estimates call for expansionary monetary shocks in 2011Q2, Q3 and Q4 to align the model with the data. The expansionary shocks occur around the time when the Federal Open Market Committee became increasingly vocal about its intentions to keep the federal funds rate at zero for an extended period.

5.4 Specification Checks and Sensitivity Analysis

We perform a series of specification checks and sensitivity analysis in order to gauge the robustness of our estimation results. First, we check that our findings are insensitive to the assumption that the initial vector of endogenous variables, X_0, is equal to its steady-state value. Second, we show that our algorithm can accurately recover the "true" structural shocks when the structural parameters are known. Third, we show that when our estimation strategy is applied to data generated from the posterior mode of the model, the estimated parameters are close to their true values. As for sensitivity analysis, we consider an alternative detrending strategy, a different shock structure, and introduce variable capacity utilization.

Initialization Scheme. Our estimation procedure makes use of the assumption that all variables are known and equal to their nonstochastic steady state in the first period. The first 20 observations are used to train the filter. As a robustness exercise, we have estimated our model under different assumptions about the values of the initial state vector X_0.[19] We confirmed the initial conditions were essentially irrelevant by period 20 and that our estimated parameters were minimally affected by the initial condition. Table 2 compares the benchmark results with the estimation results assuming a different known initial condition (see column 3), randomly sampled from the distribution of the model state variables based on the model's estimated mode.

Filtering. Our estimation procedure relies on using a nonlinear equation solver in order to filter in each period t, given X_{t-1}, the sequence of shocks ϵ_t that reproduces the observations in the vector Y_t. It is possible that small numerical errors in retrieving ϵ_t at each point in time may propagate over time and lead to inaccuracies in computing the filtered shocks. To explore the practical relevance of this possibility, we generate an artificially long sample of observables from our model. Drawing from the posterior mode of the model, we generate a time series of artificial observations of length $T = 500$. We then use our procedure to filter these shocks back and compare the filtered shocks to the "true" ones used to generate our artificial data set. The

[19] By treating the initial distribution of X_0 as known, we eliminate the conditionality of the likelihood function for the observed data Y^T on both X_0 and Y_0. Without this assumption, one needs to integrate the likelihood for Y^T over the distribution for X_0 implied by the specification of the model and the observed data, as discussed for instance by DeJong (2007), and simulation-based methods (such as the particle filter or the unscented Kalman filter) become necessary.

correlation between the "true" shocks and the filtered ones is, for all shocks, extremely high, ranging from 0.997 for the monetary shock to 0.99999987 for the inflation shock.

Identifiability. Following Schmitt-Grohé and Uribe (2012), with estimated parameters set to their posterior mode, we generate a sample of 120 observations (comparable in size to our actual dataset), and then estimate the model parameters using the same methods and procedures applied to the observed data, both with uninformative priors – pure maximum likelihood estimation (MLE) – and with our Bayesian approach. At no point does our estimation procedure make use of knowledge of the true parameter values. In this case, too, our estimation strategy comes close to uncovering the true shocks and the true values of the parameters in question. For instance, the estimated wage share of impatient households at the mode is 0.37 in the MLE case, and 0.49 in the Bayesian approach. All the other estimated coefficients are reported in columns 4 and 5 of Table 2.

Detrending Method. We use a one-sided HP filter to construct the data analogues to our model variables prior to estimation. As an alternative, we have incorporated linear deterministic trends in the model and estimated the parameters governing the trends jointly with the other parameters. Specifically, we have assumed three separate deterministic trends for TFP, investment goods technology, and housing supply technology. Given our assumptions about preferences and technology, these three separate trends yield a balanced growth path in which real consumption (together with real wages), real investment, and real house prices grow at different rates (even if the nominal shares of consumption, investment and housing expenditures remain constant). The model with deterministic trends implies slightly more persistent and more volatile shocks, presumably in order to account for the larger and more persistent deviations of the observations around their constant trends. The additional estimation results are reported in column 6 of Table 2.

Allowing for TFP Shocks and Variable Capacity Utilization. In our benchmark specification we include six observed series (inflation, wages, house prices, consumption, investment and the interest rate) and six shocks (investment-specific shocks, wage markup, price markup, monetary policy, intertemporal preferences, and preferences for housing). As a robustness exercise, we have included utilization-adjusted TFP (constructed by Fernald 2012) among the observed series and allowed for a seventh shock, a TFP shock, in a model with variable capacity utilization.[20] The estimated model with TFP shocks and variable capacity implies a slightly higher fraction of impatient households (0.52 instead of 0.42), and minor changes in the rest of the estimated parameters. As a result, the housing collapse plays a slightly larger role than in our benchmark model in accounting for the consumption decline in the Great Recession. The additional estimation results are reported in column 7 of Table 2.

6 Regional Evidence on Asymmetries

Our model estimated on national-level data motivates additional empirical analysis that we conduct using a panel of data from U.S. states and Metropolitan Statistical Areas (MSA). The advantage of these data is that variation in house prices and economic activity is greater at the regional than at the aggregate level, as documented for instance by Del Negro and Otrok (2007), who find large heterogeneity across states in regard to the relative importance of the national factors.[21] Note that, in any event, the state-level series aggregated back to the national level track their National Income and Product Accounts (NIPA) counterparts rather well.[22]

To set the stage, Figure 6 shows at the regional level house prices and several measures of activity, namely employment in the service sector, auto sales, electricity consumption, and mortgage originations. The figure focuses on two points in time, 2005 and 2008 for all the 50 U.S. states and the District of Columbia. For each state, each panel presents two dots: the green dot (concentrated in the north–east region of the graph) shows the lagged percent change in house prices and the percent change in the indicator of economic activity in 2005, at the height of the housing boom.[23] The red dot represents analogous observations for the 2008 period, in the midst of the housing crash. Fitting a piecewise linear regression to these data yields a correlation between house prices and activity that is smaller when house prices are high. This evidence on asymmetry is bolstered by the large cross-sectional variation in house prices across states over the period in question.

[20] We model variable capacity utilization in a manner similar to Smets and Wouters (2007).

[21] In the sample we analyze, the first principal component for annual house price growth accounts for 64 percent of the variance of house prices across the 50 U.S. states and the District of Columbia. The corresponding numbers for employment in the service sector, auto sales, electricity consumption, and mortgage originations are respectively 73, 90, 44, and 89 percent.

[22] For instance, over the sample period, the correlation between NIPA motor vehicle consumption growth (about 1/3 of durable expenditure) and retail auto sales growth is 0.89; and the correlation between services consumption growth and electricity usage growth is 0.54.

[23] An analogous relationship is more tenuous for house prices and employment in the manufacturing goods sector. Most goods are traded and are less sensitive to local house prices than services.

6.1 State-Level Evidence

We use annual data from the early 1990s to 2011 on house prices and measures of economic activity for the 50 U.S. states and the District of Columbia. We choose measures of economic activity that best match our model counterparts for consumption, employment and credit.[24]

Our main specification takes the following form:

$$\Delta \log y_{i,t} = \alpha_i + \gamma_t + \beta_{POS}\mathcal{I}_{i,t}\Delta \log hp_{i,t-1} + \beta_{NEG}\left(1 - \mathcal{I}_{i,t}\right)\Delta \log hp_{i,t-1} + \delta X_{i,t-1} + \varepsilon_{i,t} \quad (25)$$

where $y_{i,t}$ is an index of economic activity and $hp_{i,t}$ is the inflation-adjusted house price index in state i in period t; α_i and γ_t represent state and year fixed effects; and $X_{i,t}$ is a vector of additional controls. We interact changes in house prices with a state-specific indicator variable $\mathcal{I}_{i,t}$ that, in line with the model predictions, takes value 1 when house prices are high, and value 0 when house prices are low. We classify house prices as high in a state when they are above a state-specific linear trend separately estimated for the 1976-2011 period, a classification that lines up with the findings of the estimated model in Figure 4. Using this approach, the fraction of states with high house prices is about 20 percent in the 1990s, rising gradually to peak at 100 percent in 2005 and 2006, and dropping to 27 percent at the end of the sample. Our results were similar using two alternative definitions of $\mathcal{I}_{i,t}$. Under the first alternative definition, $\mathcal{I}_{i,t}$ equals 1 when real house price inflation is positive. Under the second definition, $\mathcal{I}_{i,t}$ equals 1 when the ratio of house prices to income is high relative to its trend (in log). In our benchmark specification, we use one-year lags of house prices and other controls to control for obvious endogeneity concerns. Our results were also little changed when instrumenting current or lagged house prices with one or more lags.

Tables 3 to 5 present our estimates when the indicators of economic activity $y_{i,t}$ are employment in the service sector, auto sales, and electricity usage respectively.

Table 3 presents the results when the measure of regional activity is employment in the non-tradeable service sector. We choose this measure (rather than total employment) since U.S. states (and MSAs) trade heavily with each other, so that employment in sectors that mainly

[24] None of the currently available measures of regional consumption aligns with national consumption data in a fully satisfactory way. See for instance the discussion in Awuku-Budu, Guci, Lucas, and Robbins (2013). We proxy total consumption with electricity consumption and with automobile sales.

cater to the local economy better isolates the local effects of movements in local house prices.[25] The first two specifications do not control for time effects. They show that the asymmetry is strong and economically relevant, and that house prices matter, at statistically conventional levels, both when high and when low. After controlling for time effects in the third specification, the coefficient on high house prices is little changed, but the coefficient on low house prices is lower. A large portion of the declines in house prices in our sample took place against the background of the zero lower bound on policy interest rates. As discussed in the model results, the zero lower bound is a distinct source of asymmetry for the effect of change in house prices. Time fixed effects allow us to parse out the effects of the national monetary policy reaching the zero lower bound and, in line with our theory, compress the elasticity of employment to low house prices. In the last two specifications, after controlling for income and lagged employment, the only significant coefficient is the one on low house prices. In column five, the coefficient on high house prices is positive, although it is low and not significantly different from zero. The coefficient on low house prices, instead, is positive and significantly different from zero, thus implying that house prices matter more for economic activity when they are low. In addition, the test for the difference between the coefficient on low house prices and the coefficient on high house prices confirms that the difference is significantly larger than zero.

Table 4 reports our results when the measure of activity is retail automobile sales. Auto sales are an excellent indicator of local demand, since autos are almost always sold to state residents, and since durable goods are notoriously sensitive to business cycles. After adding lagged car sales and personal income as controls, the coefficients on low and high house prices are both positive, but the coefficient on low house prices (estimated at 0.2) is nearly three times as large.

Table 5 reports our results using residential electricity usage as a proxy for consumption. Even though electricity usage only accounts for 3 percent of total consumption, we take electricity usage to be a useful proxy for nondurable consumption.[26] Most activities involve the use

[25] The BLS collects state-level employment data by sectors broken down according to NAICS (National Industry Classification System) starting from 1990. According to this classification (available at http://www.bls.gov/ces/cessuper.htm), the goods-producing sector includes Natural Resources and mining, construction and manufacturing. The service-producing sector includes wholesale trade, retail trade, transportation, information, finance and insurance, professional and business services, education and health services, leisure and hospitality and other services. A residual category includes unclassified sectors and public administration. We exclude from the service sector wholesale trade (which on average accounts for about 6 percent of total service sector employment) since wholesale trade does not necessarily cater to the local economy.

[26] Da and Yun (2010) show that using electricity to proxy for consumption produces asset pricing implications that are consistent with consumption-based capital asset pricing models.

of electricity, and electricity cannot be easily stored. Accordingly, the flow usage of electricity may even provide a better measure of the utility flow derived from a good than the actual purchase of the good. Even in cases when annual changes in weather conditions may affect year-on-year consumption growth, their effect can be filtered out using state-level observations on heating and cooling degree days, which are conventional measures of weather-driven electricity demand. We use these weather measures as controls in all specifications reported. As the table shows, in all regressions low house prices affect consumption growth more than high house prices. After time effects, lagged income growth and lagged consumption growth are controlled for (last column), the coefficient on high house prices is 0.12, the coefficient on low house prices is nearly twice as large at 0.19, and their difference is statistically larger than 0 at the 10 percent significance level.

Because the effects of low and high house prices on consumption work in our model through tightening or relaxing borrowing constraints, it is important to check whether measures of leverage also depend asymmetrically on house prices. We perform these checks and report the results in Appendix D, which confirms that mortgage originations depend asymmetrically on house prices too.

6.2 MSA-Level Evidence

Tables 6 and 7 present the results of evidence based on MSAs data, focusing on employment and auto sales. MSAs account for about 80 percent of the population and of employment in the entire United States. In Table 6, the results from the MSA-level regressions reinforce those obtained at the state level. After controlling for income, lagged employment and time effects, the elasticities of employment to house prices are 0.05 and 0.09 when house prices are high and low, respectively. These elasticities are larger than those found at the state level.

A legitimate concern with the panel and time-series regressions discussed so far is that the correlation between house prices and activity could be due to some omitted factor that simultaneously drives both house prices and activity. Even if this were the case, our regressions would still be of independent interest, since – even in absence of a causal relationship – they would indicate that comovement between house prices and activity is larger when house prices are low, as predicted by the model.

To support claims of causality, one needs to isolate exogenous from endogenous movements in house prices. In Table 7, we follow the methodology and insight of Mian and Sufi (2011) and use data from Saiz (2010) in an attempt to distinguish an independent driver of housing demand that better aligns with its model counterpart. The insight is to use the differential elasticity of housing supply at the MSA level as an instrument for house prices, so as to disentangle movements in housing prices due to general changes in economic conditions from movements in the housing market that are directly driven by shifts in housing demand in a particular area. Because such elasticity is constant over time, we cannot exploit the panel dimension of our dataset, and instead use the elasticity in two separate periods by running two distinct regressions of car sales on house prices. The first regression is for the 2002-2006 housing boom period, the second for the 2006-2010 housing bust period. In practice, we rely on the following differenced instrumental variable specifications:

$$\log hp_t - \log hp_s = b_0 + b_1 \text{ Elasticity} + \varepsilon_b \tag{26}$$

$$\log car_t - \log car_s = c_0 + c_1(\widehat{\log hp_t - \log hp_s}) + \varepsilon_c \tag{27}$$

where $s = 2002$ and $t = 2006$ in the first set of regressions, and $s = 2006$ and $t = 2010$ in the second set.

The first stage, OLS regressions show that the elasticity measure is a powerful instrument in driving house prices, with an R^2 from the first stage regression around 0.20 in both subperiods.[27] The second stage regressions show how car sales respond to house prices dramatically more in the second period, in line with the predictions of the model and with the results of the panel regressions. In the $2002 - 2006$ period, the elasticity of car sales to house prices is 0.24. In the $2006 - 2010$ period, in contrast, this elasticity doubles to 0.49.

Using a higher level of data disaggregation (ZIP-code level data instead of MSAs) and a sample that runs from 2007 to 2009, Mian, Rao, and Sufi (2012) find a large elasticity (equal to 0.74) of auto sales to housing wealth during the housing bust, in line with our findings. Importantly, they also find that this elasticity is smaller in zip codes with a high fraction of non-housing wealth to total wealth. One interpretation of their result – in line with our model – is that households in zip codes with high non-housing wealth might be, all else equal, less likely to face binding borrowing constraints during periods of housing price declines, because they can use other forms of wealth to smooth consumption.

[27] The F statistics on the first stage regressions are 69.1 and 67.2 for the first and the second period, well above the conventional threshold of 10 for evaluating weak instruments.

7 Debt Relief and Borrowing Constraints

So far, our theoretical and empirical results show that movements in house prices can produce asymmetries that are economically and statistically significant. We now consider whether these asymmetries are also important for gauging the effects of policies aimed at the housing market in the context of a deep recession. To illustrate our ideas, we choose a simple example of one such policy, a lump-sum transfer from patient (saver) households to impatient (borrower) households. This policy could mimic voluntary debt relief from the creditors, or a scheme where interest income is taxed and interest payments are subsidized in lump-sum fashion, so that the end result is a transfer of resources from the savers to the borrowers.

We consider this experiment against two different baselines. In one case, house prices are assumed to be below steady state, and the collateral constraint binds; in the other case, housing prices are assumed to be above steady state, and the constraint is slack. The baseline housing price changes are brought about by a sequences of unexpected shocks to housing preference.

Figure 7 shows the combined response of house prices to the baseline housing preference shocks and to the two transfer shocks. Both transfer shocks are unforeseen and are sized at 1 percent of steady-state aggregate consumption. Each transfer is governed by an AR(1) process with coefficient equal to 0.5. The first transfer starts in period 6. A series of unforeseen innovations to the shock process phases in the transfer, until it reaches a peak of 1 percent of steady-state consumption. Then, the auto-regressive component of the shock reduces the transfer back to 0. The first transfer happens against a background of housing price declines and tight borrowing constraints. The second transfer, starting in period 51, mimics the first but happens against a baseline with housing price increases and slack borrowing constraints.

The top left panel of Figure 7 shows house prices in deviation from steady state. The transfer shocks have a negligible effect on house prices, but their timing coincides with the series of housing preference shocks that change house prices. Accordingly, the marginal effect of the transfer shocks differs strikingly depending on the baseline variation in house prices, as shown in the remaining panels in Figure 7. The consumption response of borrower households is dramatically different depending on the baseline variation in house prices. When house prices are low, the borrowing constraint is tight and the marginal propensity to consume of borrower households is elevated. When house prices are high, the borrowing constraint becomes slack and the marginal propensity to consume of borrower households drops down closer to that for saver households. In reaction to the transfer, consumption of the savers declines less, and less persistently, against a baseline of housing price declines. In that case, there are expansionary spillovers from the increased consumption of borrowers to aggregate hours worked and output. Taking together the responses of savers and borrowers, the effects of the transfer on aggregate consumption are sizable when house prices are low, and small when house prices are elevated. As a consequence, actions such as mortgage relief can almost pay for themselves through their expansionary effects on economic activity in a scenario of binding borrowing constraints.

8 Conclusions

Our results show that housing prices matter more during severe recessions than during booms through their effects on collateral constraints. We document that these constraints were a key catalyst for the economic collapse of the Financial Crisis.

Our estimated model allows the assessment of costs and benefits of alternative policies aimed at restoring the efficient functioning of the housing market. For instance, policies such as debt relief can produce outsize spillovers to aggregate consumption in periods when collateral constraints are tight. Our estimates of these spillover effects are larger than estimates based on samples dominated by house price increases, as inference based on these periods can severely underpredict the sensitivity of consumption to movements in housing wealth.

Throughout the paper, we have emphasized the role of housing as collateral for households, and on the effects of changes in housing wealth on consumption. However, the mechanism at the heart of our argument has even broader applicability. For instance, to the extent that fixed assets are used for collateral by entrepreneurs, local governments, or exporters, the asymmetries highlighted here for consumption could also be relevant for fixed investment, government spending, or the trade balance.[28]

[28] See for instance Adelino, Schoar, and Severino (2013), Chaney, Sraer, and Thesmar (2012) for investment; Barboza (2011) for government spending; Klapper, Laeven, and Rajan (2012) for trade credit.

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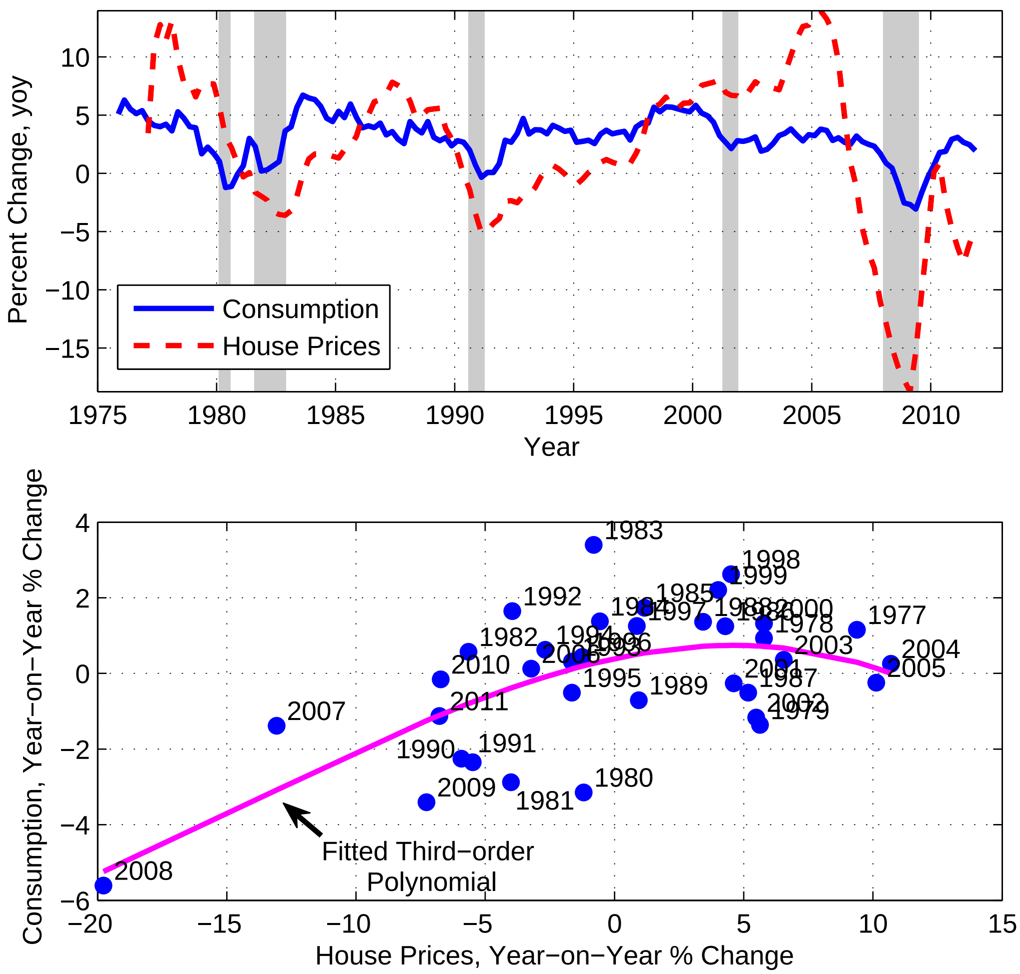
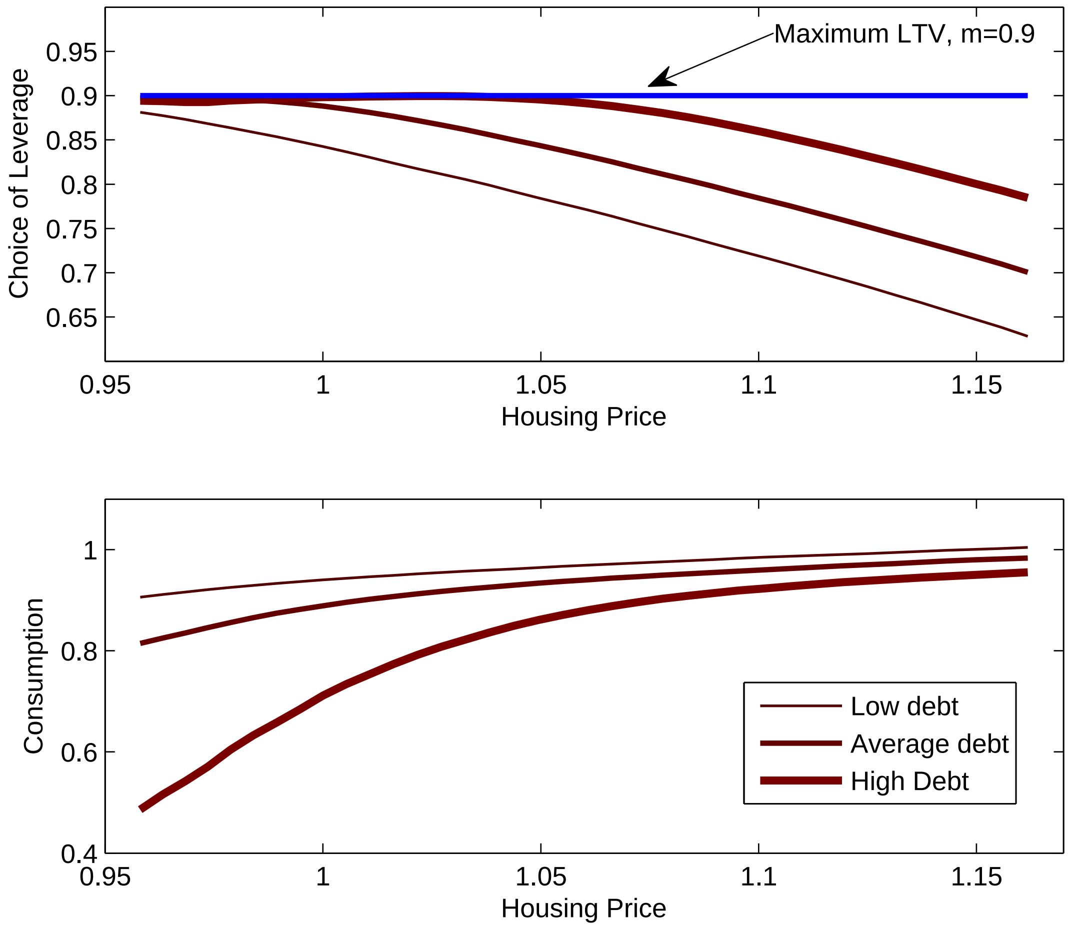
Figure 1: House Prices and Consumption in U.S. National Data



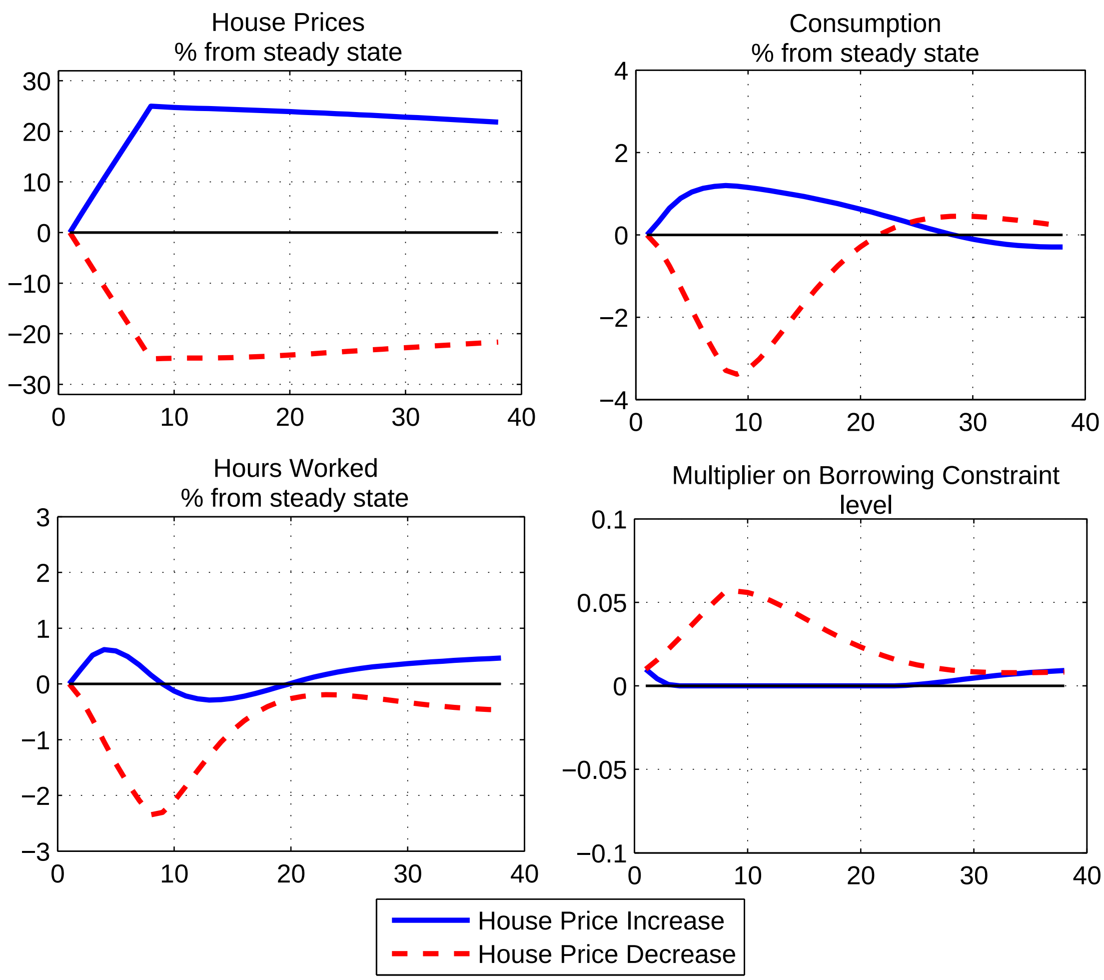
Note: Data sources are as follows. House Prices: CoreLogic National House Price Index, seasonally adjusted (Haver mnemonics: USLPHPIS@USECON), divided by the GDP deflator (DGDP@USECON). Consumption: Real Personal Consumption Expenditures, Department of Commerce–Bureau of Economic Analysis (CH@USECON). In the top panel, shaded areas indicate NBER recessions. In the bottom panel, consumption growth and house price growth are expressed in deviation from their sample mean. The data sample is from 1976Q1 to 2011Q4.

Figure 2: House Prices and Consumption in the Basic Model



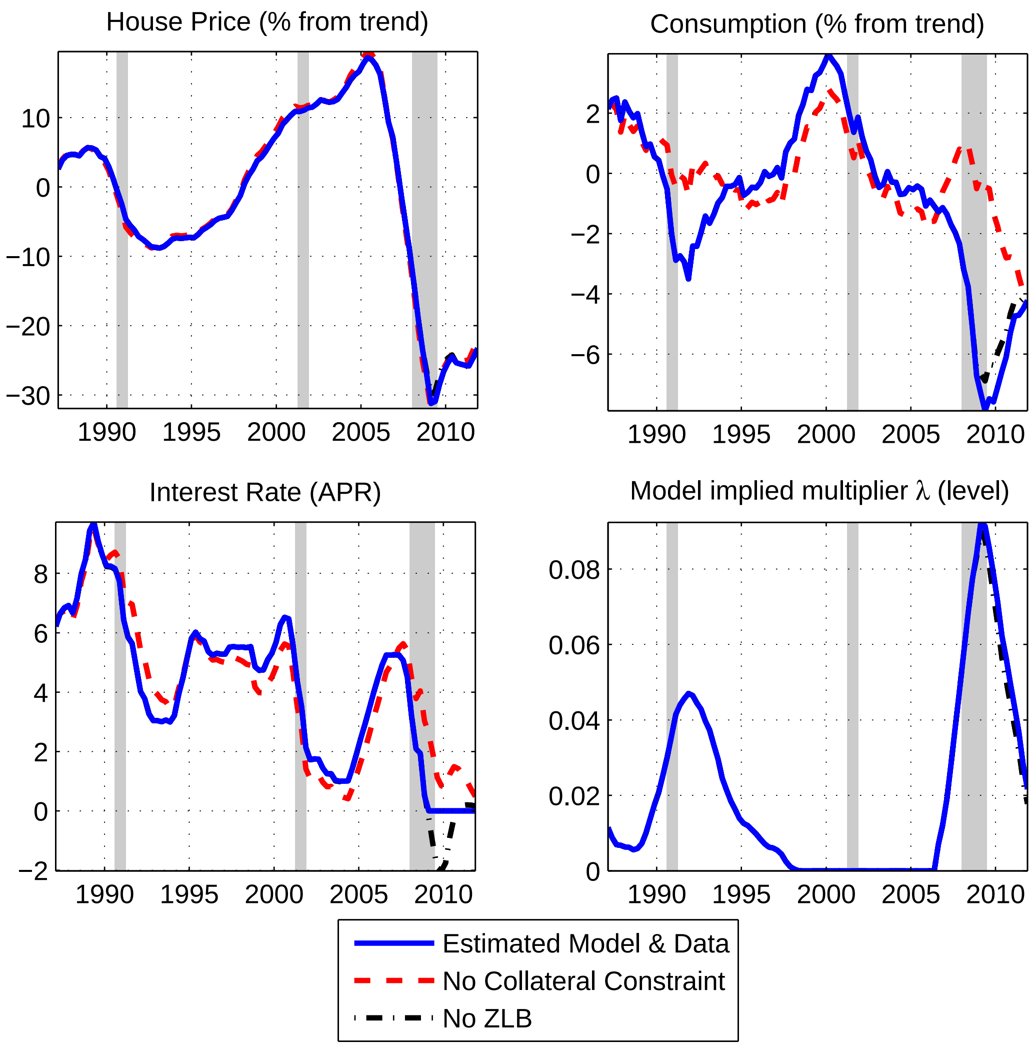
Note: Optimal leverage choice and optimal consumption as a function of the housing price for three different levels of debt, low, normal and high, when housing is at its nonstochastic steady-state value. In the top panel, low house prices move the household closer to the maximum borrowing limit given by m$=0.9$. This is more likely to happen at high levels of debt (thick line). In the bottom panel, the higher house prices are, the more likely is the household not to be credit constrained, and the consumption function becomes flatter. At high levels of debt, the household is constrained for a larger range of realizations of house prices, and the consumption function is steeper when house prices are low.

Figure 3: Impulse Responses to Positive and Negative Housing Demand Shocks in the Full DSGE Model



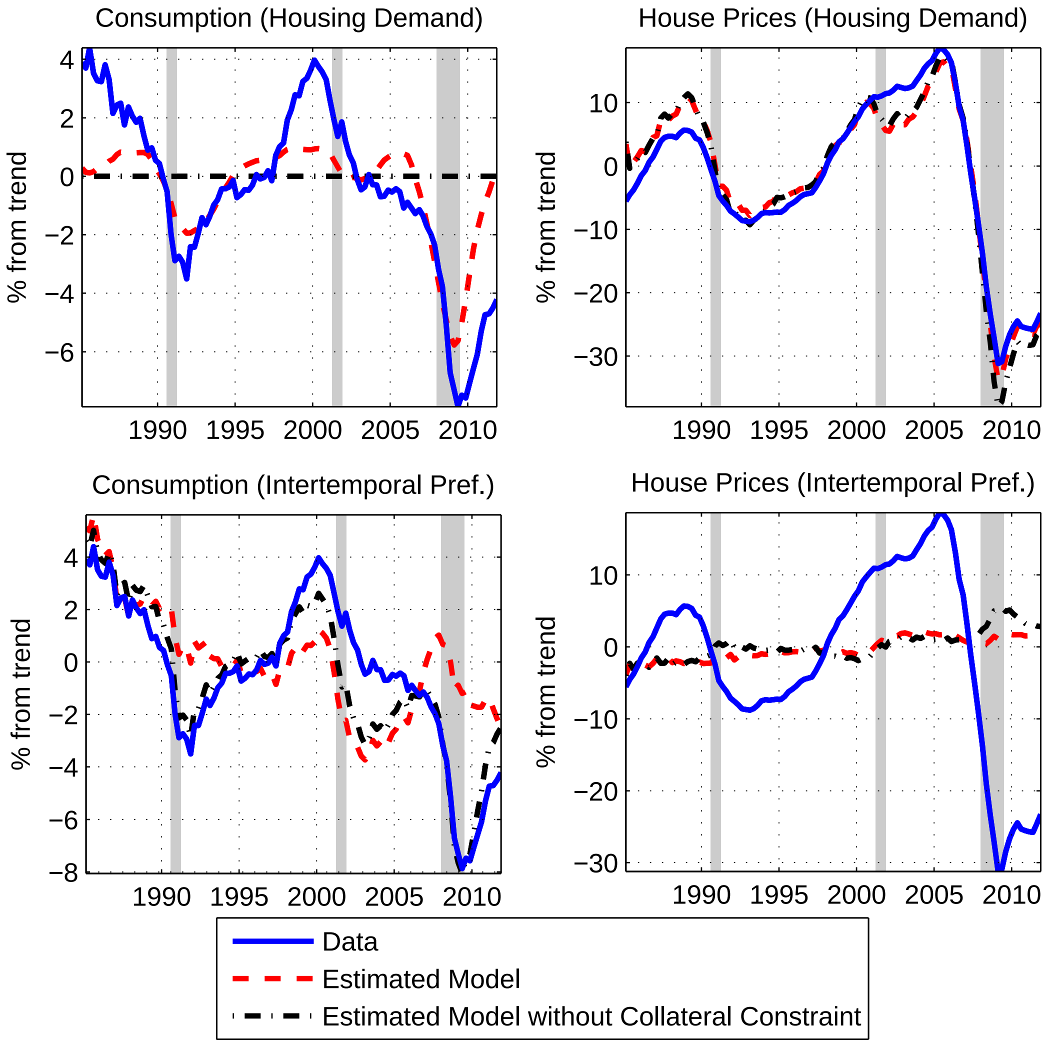
Note: Horizontal axis: horizon in quarters. The simulation shows the dynamic responses to sequence of housing demand shocks of equal size but opposite sign that move house prices up (solid lines) and down (dashed lines) by 25 percent relative to the steady state.

Figure 4: Historical Simulation of the Estimated Model



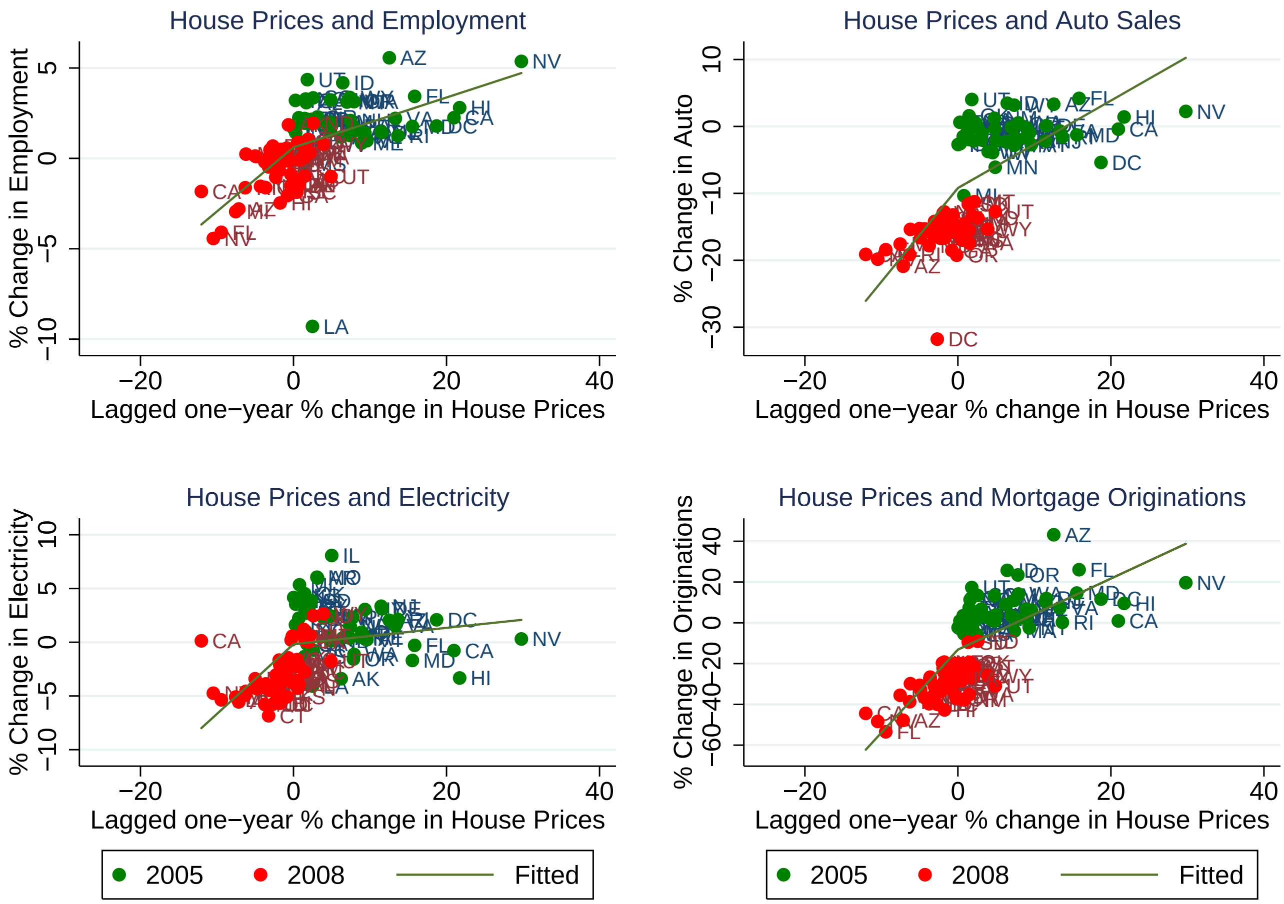
Note: The simulation shows the filtered series for house prices, consumption, interest rates and the Lagrange multiplier in the estimated model. The dashed lines show their paths feeding in the same shocks but in absence of constrained households (setting σ=0). The dash-dotted lines show the paths feeding in the same shocks but in absence of the zero lower bound on interest rates.

Figure 5: Counterfactual Consumption Paths in the Estimated Model



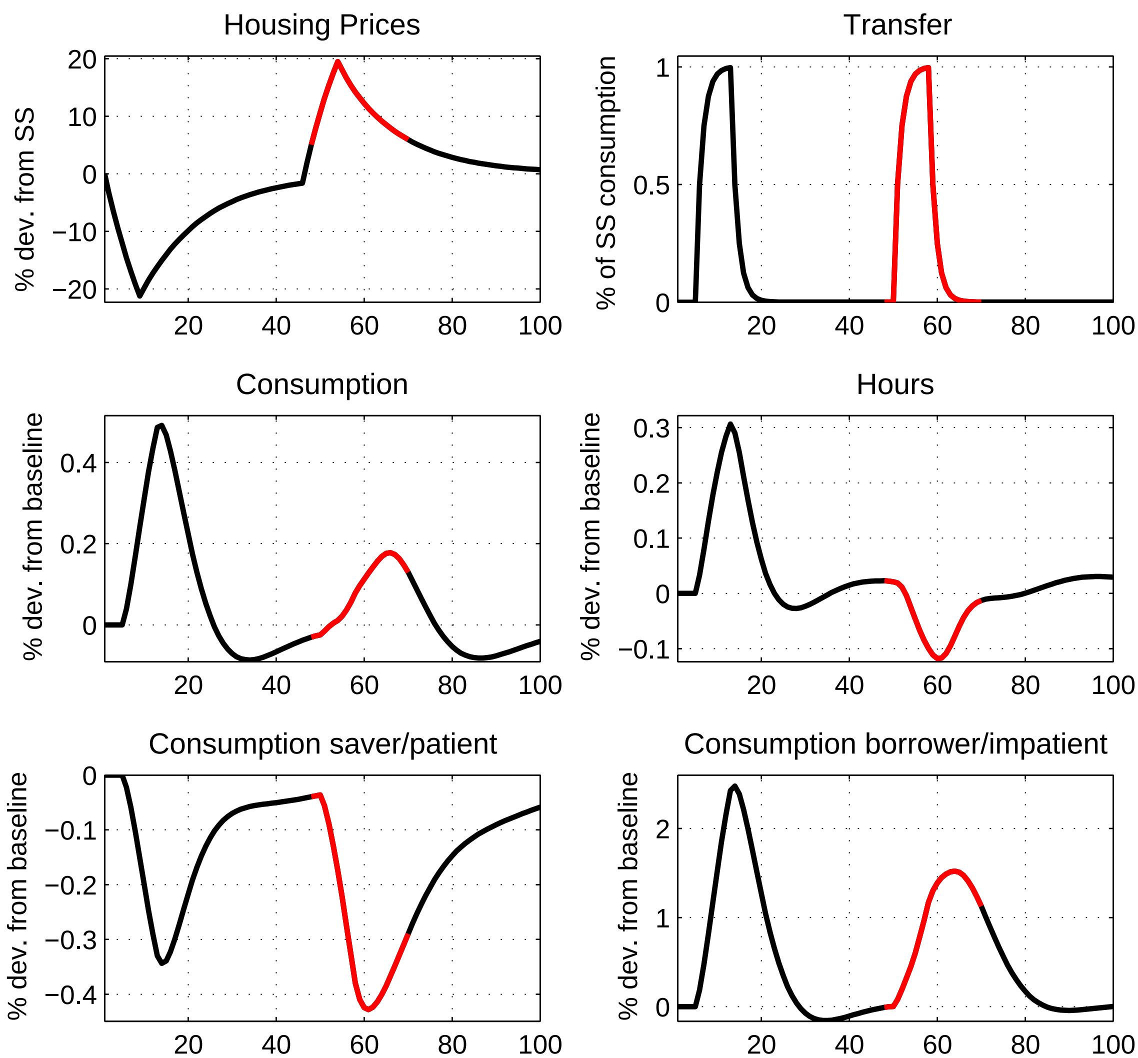
Note: The top panels compare consumption and house prices in the data (dashed line) with their model counterparts when consumption and house prices are driven by the housing preference shock. The dash-dotted line shows the counterfactual model paths for the estimated model with the restriction that $\sigma = 0$. The bottom panels redo the exercise for intertemporal preference shocks.

Figure 6: House Prices and Economic Activity by State



Note: Each panel shows house price growth and activity growth across US states in 2005 and 2008. The "fitted" line shows the fitted values of a regression of activity growth on house prices growth broken down into positive and negative changes.

Figure 7: Transfer from Lenders to Borrowers with Low and High House Prices



Note: Two unexpected lump-sum transfers from savers to borrowers sized at 1 percent of steady-state consumption. The first transfer (periods 6-13) happens against a baseline of low house prices and tight collateral constraints. The second transfer (periods 51-58) happens against a baseline of high house prices and slack collateral constraints. Housing price changes in the baseline stem from a housing preference shock. The responses of consumption, hours, savers' and borrowers' consumption are shown in deviation from baseline to isolate the partial effect of the transfer shocks. Variables are plotted in red when the constraint is slack.

Table 1: Calibrated and Estimated Parameter Values

	Calibrated Parameters	Value
m	Maximum LTV	0.9
η	labor disutility	1
β	discount factor, patient agents	0.995
$\overline{\pi}$	steady-state gross inflation rate	1.005
α	capital share in production	0.3
δ_k	capital depreciation rate	0.025
$\overline{j}$	housing weight in utility	0.04
$\overline{x}_p$	steady-state price markup	1.2
$\overline{x}_w$	steady-state wage markup	1.2

	Estimated Parameters	Prior type [mean, std]	Posterior			
			Mode	5%	Median	95%
β'	discount factor, impatients	normal [0.99, .0015]	**0.9895**	0.9875	0.9901	0.9927
ε	habit in consumption	beta [0.5, 0.1]	**0.6399**	0.5481	0.6467	0.7312
ϕ	investment adjustment cost	gamma [5, 2]	**5.0307**	3.0168	5.1009	8.3434
σ	wage share, impatients	beta [0.5, 0.20]	**0.4151**	0.3256	0.4239	0.5296
r_π	inflation resp. Taylor rule	normal, 1.5, 0.25]	**1.7385**	1.5248	1.8074	2.0945
r_R	inertia Taylor rule	beta [0.75, 0.1]	**0.5200**	0.4328	0.5447	0.6331
r_Y	output response Taylor rule	beta [0.125, 0.025]	**0.0796**	0.0581	0.0892	0.1150
θ_π	Calvo parameter, prices	beta [0.5, 0.075]	**0.9190**	0.8943	0.9146	0.9343
θ_w	Calvo parameter, wages	beta [0.5, 0.075]	**0.9170**	0.8939	0.9144	0.9325
γ	inertia borrowing constraint	beta [0.5, 0.1]	**0.4547**	0.2851	0.4322	0.5851
ρ_J	AR(1) housing shock	beta [0.75, 0.1]	**0.9934**	0.9674	0.9838	0.9941
ρ_K	AR(1) investment shock	beta [0.75, 0.1]	**0.7651**	0.7146	0.7715	0.8213
ρ_R	AR(1) monetary shock	beta [0.5, 0.1]	**0.6382**	0.5021	0.6134	0.7136
ρ_Z	AR(1) intertemporal shock	beta [0.75, 0.1]	**0.7793**	0.6896	0.7849	0.8532
σ_J	std. housing demand shock	invgamma [0.01, 1]	**0.0441**	0.0422	0.0825	0.1492
σ_K	std. investment shock	invgamma [0.01, 1]	**0.0418**	0.0288	0.0436	0.0658
σ_P	std. price markup shock	invgamma [0.01, 1]	**0.0030**	0.0027	0.0031	0.0035
σ_R	std. interest rate shock	invgamma [0.01, 1]	**0.0013**	0.0012	0.0014	0.0016
σ_W	std. wage markup shock	invgamma [0.01, 1]	**0.0100**	0.0089	0.0101	0.0116
σ_Z	std. intertemporal shock	invgamma [0.01, 1]	**0.0145**	0.0124	0.0157	0.0198

Note: Calibrated and Estimated Parameters for the Full Model. The posterior statistics are based on 50,000 draws from the posterior distribution.

Table 2: Estimation Results: Robustness Analysis

Param.	1.Benchm.	2. No constraint	3. Different initialization	4. MLE artif. data	5. Bayes artif. data	6. Lin. trend	7. TFP
			Specification Checks			Sensitivity	
β'	0.9895		0.9901	0.9896	0.9891	0.9873	0.9903
ε	0.6399	0.7514	0.6070	0.5972	0.5737	0.6386	0.6470
ϕ	5.0307	6.1677	6.9583	8.0458	4.6468	4.9900	6.1166
σ	0.4151		0.4592	0.4894	0.3704	0.4412	0.5255
r_π	1.7385	1.6830	1.8711	1.9386	1.8374	1.7642	1.6753
r_R	0.5200	0.5413	0.5932	0.4701	0.5523	0.5907	0.5438
r_Y	0.0796	0.0699	0.0792	0.0923	0.1036	0.0622	0.0917
θ_π	0.9190	0.9134	0.9200	0.9058	0.8593	0.9496	0.9104
θ_w	0.9170	0.8944	0.9164	0.9234	0.8841	0.9273	0.9167
γ	0.4547		0.4795	0.5769	0.4644	0.3993	0.5541
ρ_J	0.9934	0.9934	0.9935	0.9986	0.9832	0.9935	0.9968
ρ_K	0.7651	0.7813	0.7448	0.8103	0.7790	0.8073	0.7466
ρ_R	0.6382	0.6207	0.6102	0.6546	0.5591	0.7331	0.6252
ρ_Z	0.7793	0.7379	0.8387	0.8061	0.7584	0.8540	0.7998
σ_J	0.0441	0.0473	0.0454	0.0257	0.0819	0.0527	0.0316
σ_K	0.0418	0.0451	0.0564	0.0584	0.0413	0.0383	0.0533
σ_P	0.0030	0.0029	0.0029	0.0031	0.0033	0.0030	0.0030
σ_R	0.0013	0.0014	0.0013	0.0013	0.0016	0.0014	0.0013
σ_W	0.0100	0.0102	0.0101	0.0098	0.0103	0.0097	0.0100
σ_Z	0.0145	0.0226	0.0147	0.0144	0.0130	0.0163	0.0156
τ_C						0.0073	
τ_K						0.0107	
τ_q						0.0043	
ζ_K							0.3960
ρ_A							0.8578
σ_A							0.0065

Note: Column (1) reports the mode of the parameters for the benchmark model. Column (2) reports the mode imposing no credit constraints. Column (3) reports the mode for the benchmark model using a different initial condition. Columns (4) and (5) report maximum likelihood estimates (MLE) or posterior mode (Bayes) using artificial data generated by the model with parameters set at the values in (1). Column (6) reports the mode for the model with linear deterministic trends. The parameters τ_C, τ_K, τ_q are the implied growth rates for real consumption, real investment, real house prices. Column (7) reports the mode of the model with TFP shocks and variable utilization, where ρ_A and σ_A are AR(1) coefficient and standard deviation of the TFP shock. The parameter ζ_K measures the curvature (between 0 and 1) of the utilization cost function, where 0 (1) indicates that utilization can be changed at an arbitrarily small (large) cost.

Table 3: State-Level Regressions: Employment in Services and House Prices

	% Change in Employment (Δemp_t)				
Δhp_{t-1}	**0.14***** (0.01)				
Δhp_high_{t-1}		**0.07***** (0.01)	**0.08***** (0.01)	**0.03*** (0.02)	**0.02** (0.01)
Δhp_low_{t-1}		**0.24***** (0.02)	**0.12***** (0.02)	**0.08***** (0.02)	**0.07***** (0.02)
Δemp_{t-1}				**0.26***** (0.08)	**0.23***** (0.09)
$\Delta income_{t-1}$					**0.07**** (0.03)
pval difference		**0.000**	**0.100**	**0.013**	**0.017**
Time effects	**no**	**no**	**yes**	**yes**	**yes**
Observations	1071	1071	1071	1020	1020
States	51	51	51	51	51
R-squared	0.12	0.16	0.66	0.72	0.73

Note: Regressions using annual observations from 1991 to 2011 on 50 States and the District of Columbia. Robust standard errors in parenthesis. ***,**,*: Coefficients statistically different from zero at 1, 5 and 10% confidence level. pval is the p-value of the test for difference between low-house price and high-house prices coefficient.

Data Sources and Definitions: Δhp is the inflation–adjusted (using the GDP deflator) percent change in the FHFA All Transactions House Price Index (available both at State- and MSA-level). Δemp is the percent change in employment in the Non-Tradable Service Sector which includes: Retail Trade, Transportation and Utilities, Information, Financial Activities, Professional and Business Services, Education and Health Services, Leisure and Hospitality, and Other Services (source: BLS Current Employment Statistics: Employment, Hours, and Earnings - State and Metro Area). $\Delta income$ is the percent change in the inflation–adjusted state-level disposable personal income (source: Bureau of Economic Analysis).

Table 4: State-Level Regressions: Auto Sales and House Prices

	% Change in Auto Sales ($\Delta auto_t$)				
Δhp_{t-1}	**0.24*****				
	(0.03)				
Δhp_high_{t-1}		**-0.05**	**0.16*****	**0.11*****	**0.07****
		(0.04)	(0.04)	(0.03)	(0.03)
Δhp_low_{t-1}		**0.62*****	**0.33*****	**0.27****	**0.20****
		(0.05)	(0.06)	(0.11)	(0.09)
$\Delta auto_{t-1}$				**0.23**	**0.21**
				(0.17)	(0.17)
$\Delta income_{t-1}$					**0.34*****
					(0.11)
pval difference		**0.000**	**0.040**	**0.137**	**0.155**
Time effects	**no**	**no**	**yes**	**yes**	**yes**
Observations	969	969	969	918	918
States	51	51	51	51	51
R-squared	0.02	0.06	0.86	0.87	0.88

Note: State–level Regressions using annual observations from 1992 to 2011 on 50 States and the District of Columbia. Robust standard errors in parenthesis. ***,**,*: Coefficients statistically different from zero at 1, 5 and 10% confidence level. pval is the p-value of the test for difference in the coefficients for low-house prices and high-house prices.

Data Sources and Definitions: $\Delta auto$ is the percent change in inflation–adjusted auto sales, "Retail Sales: Motor vehicle and parts dealers" from Moody's Analytics Database. Auto sales data are constructed with underlying data from the US Census Bureau and employment statistics from the BLS. The two Census Bureau surveys are the quinquennial Census of Retail Trade, a subset of the Economic Census, and the monthly Advance Retail Trade and Food Services Survey. See Table 3 for other variable definitions.

Table 5: State-Level Regressions: Electricity Consumption and House Prices

	% Change in Electricity Consumption ($\Delta elec_t$)				
Δhp_{t-1}	**0.11*** **				
	(0.02)				
Δhp_high_{t-1}		**0.03**	**0.09*** **	**0.14*** **	**0.12*** **
		(0.02)	(0.02)	(0.03)	(0.03)
Δhp_low_{t-1}		**0.24*** **	**0.16*** **	**0.22*** **	**0.19*** **
		(0.03)	(0.03)	(0.04)	(0.04)
$\Delta elec_{t-1}$				**-0.41*** **	**-0.41*** **
				(0.02)	(0.02)
$\Delta income_{t-1}$					**0.15*** **
					(0.05)
pval difference		**0.000**	**0.105**	**0.058**	**0.090**
Time effects	**no**	**no**	**yes**	**yes**	**yes**
Weather Controls*	**yes**	**yes**	**yes**	**yes**	**yes**
Observations	1071	1071	1071	1020	1020
States	51	51	51	51	51
R-squared	0.04	0.04	0.08	0.12	0.12

Note: State–level Regressions using annual observations from 1990 to 2011 on 50 States and the District of Columbia. Robust standard errors in parenthesis. ***,**,*: Coefficients statistically different from zero at 1, 5 and 10% confidence level. pval is the p-value of the test for difference in the coefficients for low-house prices and high-house prices.

Data Sources and Definitions: $\Delta elec$ is the percent change in Residential Electricity Consumption (source: the U.S. Energy Information Administration's Electric Power Monthly publication. Electricity Power Annual: Retail Sales - Total Electric Industry - Residential Sales, NSA, Megawatt-hours). See Table 3 for other variable definitions. All regressions in the Table control separately for number of heating degree days and number of cooling degree days in each state (source: U.S. National Oceanic and Atmospheric Administration's National Climatic Data Center).

Table 6: MSA Level: Employment in Services and House Prices

	% Change in Employment (Δemp_t)				
Δhp_{t-1}	**0.134***** (0.006)				
Δhp_high_{t-1}		**0.104***** (0.008)	**0.058***** (0.007)	**0.049***** (0.008)	**0.048***** (0.008)
Δhp_low_{t-1}		**0.183***** (0.009)	**0.099***** (0.008)	**0.095***** (0.010)	**0.094***** (0.010)
Δemp_{t-1}				**0.033** (0.041)	**0.031** (0.041)
$\Delta income_{t-1}$					**0.021*** (0.011)
pval difference		**0.0000**	**0.0003**	**0.0001**	**0.0000**
Time effects	no	no	yes	yes	yes
Observations	5390	5390	5390	5147	5147
MSA	262	262	262	262	262
R-squared	0.09	0.10	0.37	0.39	0.39

Note: MSA–level Regressions using annual observations from 1992 to 2011 on 262 MSAs (102 MSAs were dropped since they had incomplete or missing data on employment by sector). Robust standard errors in parenthesis. ***,**,*: Coefficients statistically different from zero at 1, 5 and 10% confidence level. pval is the p-value of the test for difference in the coefficients for low-house prices and high-house prices.

Data Sources and Definitions: $\Delta income$ is the percent change in MSA-level inflation-adjusted personal income (source: BEA, Local and Metro Area Personal Income Release). For employment (Δemp) and house prices (Δhp), see Table 3.

Table 7: MSA Level: Auto Registrations and House Prices

	Cross-sectional Regressions			
	Sample 2002-2006 (Housing Boom)		Sample 2006-2010 (Housing Bust)	
	Δhp	Δcar	Δhp	Δcar
Elasticity	**-7.26***** (0.87)		**4.69***** (0.57)	
Δhp		**0.24***** (0.06)		**0.49***** (0.08)
Method	OLS	IV	OLS	IV
Observations	254	254	254	254
R-squared	0.22	0.35	0.21	0.48

Note: Regressions using housing supply elasticity at the MSA level as an instrument for house prices in a regression of MSA car registrations on MSA house prices. ***,**,*: Coefficients statistically different from zero at 1, 5 and 10% confidence level. The housing supply elasticity is taken from Saiz (2010) and measures limits on real-estate development due to geographic factors that affect the amount of developable land, as well as factors like zoning restrictions. The elasticity data are available for 269 cities: we dropped 15 areas because they were covering primary metropolitan statistical areas (PMSA), which are portions of metropolitan areas, rather than complete MSAs.

Data Sources: Car Registrations are retail (total less rental, commercial and government) auto registrations from Polk Automotive Data. Δcar is the percent change in car registrations. See Table 3 for other data sources.

Appendix for "Collateral Constraints and Macroeconomic Asymmetries"

Appendix A Equilibrium Conditions of the Full Model

We summarize here the equations describing the equilibrium of the full model. Let $u_{c,t}$ (and $u_{c',t}$), $u_{h,t}$ (and $u_{h',t}$), $u_{n,t}$ (and $u_{n',t}$) denote the time-t marginal utility of consumption, marginal utility of housing and marginal disutility of labor (inclusive of the shock terms: that is, $u_t = \mathsf{z}_t \left(\Gamma \log\left(c_t - \varepsilon c_{t-1}\right) + \mathsf{j}_t \log h_t - \frac{1}{1+\eta} n_t^{1+\eta}\right)$, and $u_{c,t}$ is the derivative of u_t with respect to c_t). Let Δ be the first difference operator, and let overbars denote steady states. The set of necessary conditions for an equilibrium is given by:

- Budget constraint for patient households:

$$c_t + q_t \Delta h_t + i_t - \frac{R_{t-1} b_{t-1}}{\pi_t} = \frac{w_t n_t}{x_{w,t}} + r_{k,t} k_{t-1} - b_t + div_t, \tag{A.1}$$

  where lump-sum dividends from ownership of final goods firms and from labor unions are given by $div_t = \frac{x_{p,t}-1}{x_{p,t}} y_t + \frac{x_{w,t}-1}{x_{w,t}} w_t n_t$.

- Capital accumulation equations for patient households:

$$u_{c,t} q_{k,t} \left(1 - \phi \Delta i_t\right) = u_{ct} - \beta u_{c,t+1} q_{k,t+1} \phi \Delta i_{t+1}, \tag{A.2}$$

$$u_{c,t} q_{k,t} = \beta u_{c,t+1} \left(r_{k,t+1} + q_{k,t+1} \left(1 - \delta_k\right)\right), \tag{A.3}$$

$$k_t = \mathsf{a}_t \left(i_t - \phi \frac{\left(i_t - i_{t-1}\right)^2}{\bar{i}}\right) + \left(1 - \delta_k\right) k_{t-1}, \tag{A.4}$$

  where $q_{k,t}$ is the Lagrange multiplier on the capital accumulation constraint.

- Other optimality conditions for patient households:

$$u_{c,t} = \beta u_{c,t+1} \left(R_t / \pi_{t+1}\right), \tag{A.5}$$

$$\frac{w_t}{x_{w,t}} u_{ct} = u_{nt}, \tag{A.6}$$

$$q_t u_{c,t} = u_{h,t} + \beta E_t q_{t+1} u_{c,t+1}. \tag{A.7}$$

- Budget and borrowing constraint and optimization conditions for impatient households:

$$c'_t + q_t \Delta h'_t + \frac{R_{t-1}}{\pi_t} b_{t-1} = \frac{w'_t}{x'_{w,t}} n'_t + b_t + div'_t, \tag{A.8}$$

$$b_t \leq \gamma \frac{b_{t-1}}{\pi_t} + \left(1 - \gamma\right) \mathsf{m} q_t h'_t, \tag{A.9}$$

$$(1-\lambda_t)\,u_{c,t} = \beta' E_t\left(\frac{R_t - \gamma\lambda_{t+1}}{\pi_{t+1}} u_{c,t+1}\right), \tag{A.10}$$

$$\frac{w'_t}{x'_{w,t}} u_{c',t} = u_{n',t}\ , \tag{A.11}$$

$$q_t u_{c',t} = u_{h',t} + \beta' q_{t+1} u_{c',t+1} + u_{c',t}\lambda_t\,(1-\gamma)\,\mathsf{m} q_t, \tag{A.12}$$

where lump-sum dividends from labor unions are given by $div'_t = \frac{x_{w',t}-1}{x_{w',t}} w'_t n'_t$.

- Firm problem, aggregate production, and Phillips curves:

$$\begin{aligned}
y_t &= n_t^{(1-\sigma)(1-\alpha)} n_t^{\prime\sigma(1-\alpha)} k_{t-1}^{\alpha}, && \text{(A.13)}\\
(1-\alpha)(1-\sigma)\,y_t &= x_{p,t} w_t n_t, && \text{(A.14)}\\
(1-\alpha)\,\sigma y_t &= x_{p,t} w'_t n'_t, && \text{(A.15)}\\
\alpha y_t &= x_{p,t} r_{k,t} k_{t-1}, && \text{(A.16)}\\
\log(\pi_t/\overline{\pi}) &= \beta E_t \log(\pi_{t+1}/\overline{\pi}) - \varepsilon_\pi \log(x_{p,t}/\overline{x}_p) + \mathsf{u}_{p,t}, && \text{(A.17)}\\
\log(\omega_t/\overline{\pi}) &= \beta E_t \log(\omega_{t+1}\overline{\pi}) - \varepsilon_w \log(x_{w,t}/\overline{x}_w) + \mathsf{u}_{w,t}, && \text{(A.18)}\\
\log(\omega'_t/\overline{\pi}) &= \beta' E_t \log\left(\omega'_{t+1}/\overline{\pi}\right) - \varepsilon'_w \log\left(x'_{w,t}/\overline{x}'_w\right) + \mathsf{u}_{w,t}. && \text{(A.19)}
\end{aligned}$$

Above, $\omega_t = \frac{w_t \pi_t}{w_{t-1}}$ and $\omega'_t = \frac{w'_t \pi_t}{w'_{t-1}}$denote wage inflation for each household type, and $\varepsilon_\pi = \frac{(1-\theta_\pi)(1-\beta\theta_\pi)}{\theta_\pi}$, $\varepsilon_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w}$, $\varepsilon'_w = \frac{(1-\theta_w)(1-\beta'\theta_w)}{\theta_w}$.

- Monetary policy:

$$R_t = \max\left(1, R_{t-1}^{r_R}\left(\frac{\pi_t^A}{\overline{\pi^A}}\right)^{(1-r_R)r_\pi}\left(\frac{y_t}{\overline{y}}\right)^{(1-r_R)r_Y}\overline{R}^{1-r_R}\mathsf{e}_{r,t}\right), \tag{A.20}$$

where π_t^A is year-on-year inflation (expressed in quarterly units) and is defined as $\pi_t^A \equiv (P_t/P_{t-4})^{0.25}$.

- Market clearing:

$$h_t + h'_t = 1. \tag{A.21}$$

By Walras' law, the good's market clears, so that $y_t = c_t + c'_t + k_t - (1-\delta_k)\,k_{t-1}$.

Equations A.1 to A.21, together with the laws of motion for the exogenous shocks described in Section 3, define a system of 21 equations in the following variables: c, c', h, h', i, k, y, b, n, n', w, w', π, q, R, λ, x_p, x_w, x'_w, r_k, q_k.

We use the methods described in Appendix C and more fully developed in Guerrieri and Iacoviello (2014) to solve the model subject to the two occasionally binding constraints given by equations A.9 and A.20.

Appendix B Additional Details on Estimation

Data. Data sources for the estimation are as follows.

1. Consumption.

   Model Variable: $\widetilde{C}_t = \log \frac{c_t + c'_t}{\bar{c} + \bar{c}'}$.

   Data: Real Personal Consumption Expenditures, from Bureau of Economic Analysis – BEA – (Haver Analytics code: CH@USECON), log transformed and detrended with one-sided HP filter (with $\lambda = 100,000$).

2. Price Inflation.

   Model Variable: $\widetilde{\pi}_t = \log \frac{\pi_t}{\bar{\pi}}$.

   Data: quarterly change in GDP Implicit Price Deflator, from BEA (DGDP@USECON), minus 0.5 percent.

3. Wage Inflation.

   Model Variable: $\widetilde{\omega}_t = \log \frac{\sigma\omega_t + (1-\sigma)\omega'_t}{\bar{\pi}}$.

   Data: Real Compensation per Hour in Nonfarm Business Sector (LXNFR@USECON), log transformed, detrended with one-sided HP filter (with $\lambda = 100,000$), first differenced, and expressed in nominal terms adding back price inflation $\widetilde{\pi}_t$.

4. Investment.

   Model Variable: $\widetilde{i}_t = \log \frac{i_t}{\bar{i}}$.

   Data: Real Private Nonresidential Fixed Investment, from BEA (FNH@USECON), log transformed and detrended with one-sided HP filter (with $\lambda = 100,000$).

5. House Prices.

   Model Variable: $\widetilde{q}_t = \log \frac{q_t}{\bar{q}}$.

   Data: Corelogic House Price Index (USLPHPIS@USECON) divided by the GDP Implicit Price Deflator, log transformed and detrended with one-sided HP filter ($\lambda = 100,000$).

6. Interest Rate.

   Model Variable: $\widetilde{r}_t = R_t - 1$.

   Data: Effective Federal Funds Rate, annualized percent (FEDFUNDS@USECON), divided by 400 to express in quarterly units.

Local linearity of the Policy Functions. The solution of the model can be described by a policy function of the form:

$$X_t = \mathbf{P}(X_{t-1}, \epsilon_t)X_{t-1} + \mathbf{D}(X_{t-1}, \epsilon_t) + \mathbf{Q}(X_{t-1}, \epsilon_t)\epsilon_t. \tag{28}$$

The vector X_t collects all the variables in the model, except the innovations to the shock processes, which are gathered in the vector ϵ_t. The matrix of reduced-form coefficients $\mathbf{P}$ is state-dependent, as are the vector $\mathbf{D}$ and the matrix $\mathbf{Q}$. These matrices and vector are functions of the lagged state vector and of the current innovations. However, while the current innovations can trigger a change in the reduced-form coefficients, X_t is still locally linear in ϵ_t. To illustrate this point, Figure A.1 shows how the policy function for impatient agents' consumption c'_t – one of the elements of X_t – depends on the realization of the housing preference shock $u_{j,t}$ – one of the elements of ϵ_t – when all the other elements of X_{t-1} are held at their steady–state value. The top panel shows the consumption function: this function is piecewise linear, with each of the rays corresponding to a given number of periods in which the borrowing constraint is expected to be slack. The bottom panel shows the derivative of the consumption function with respect to $u_{j,t}$. As the consumption function is piecewise linear, the derivative is not defined at the threshold values of the shock $u_{j,t}$ that change the expected duration of the regime. However, each of these threshold points for different shocks is a set of measure zero.[29]

Realizations of the shock $u_{j,t}$ above a threshold will imply that the borrowing constraint is temporarily slack. When the constraint is slack, the constraint will be expected to be slack for a number of periods which increases with the size of the shock. Accordingly, consumption will respond proportionally less, and the $\mathbf{Q}_{c',u_j}$ element of the matrix $\mathbf{Q}$ that defines the impact sensitivity of c' to u_j will be smaller.

Appendix C Solution Method for the Full Model and Accuracy Checks

Solution Method. We use a piecewise–linear solution approach to find the equilibrium allocations of the model in Section 3. This method resolves the problem of computing decision rules that approximate the equilibrium well both when the borrowing constraint binds, and when it does not (similar reasoning applies to the nonnegativity constraint on the interest rate, as described at the end of this Section).

The economy features two regimes: a regime when collateral constraints bind; and a regime in which they do not, but are expected to bind in the future.[30] With binding collateral constraints, the linearized system of necessary conditions for an equilibrium can be expressed as

$$\mathcal{A}_1 E_t X_{t+1} + \mathcal{A}_0 X_t + \mathcal{A}_{-1} X_{t-1} + \mathcal{B} u_t = 0, \tag{B.1}$$

where $\mathcal{A}_1$, $\mathcal{A}_0$, and $\mathcal{A}_{-1}$ are matrices of coefficients conformable with the vector X collecting the model variables in deviation from the steady state for the regime with binding constraints; and where u is the vector collecting all shock innovations (and $\mathcal{B}$ is the corresponding conformable matrix). Similarly, when the constraint is not binding, the linearized system can be expressed as

$$\mathcal{A}_1^* E_t X_{t+1} + \mathcal{A}_0^* X_t + \mathcal{A}_{-1}^* X_{t-1} + \mathcal{B}^* u_t + \mathcal{C}^* = 0, \tag{B.2}$$

[29] It is straightforward to prove that the points where the derivative of the decision rule is not defined are of measure zero given a choice of process for the stochastic innovations. By construction, there are only countably many of these points. If there were uncountably many, a shock could lead to a permanent switch in regimes, which is ruled out by the solution method.

[30] If one assumes that the constraints are not expected to bind in the future, the regime with slack borrowing constraints becomes unstable, since borrowers' consumption falls over time and their debt rises over time until it reaches the debt limit, which contradicts the initial assumption.

where $\mathcal{C}^*$ is a vector of constants. When the constraint binds, we use standard linear solution methods to express the decision rule for the model as

$$X_t = \mathcal{P} X_{t-1} + \mathcal{Q} u_t. \tag{B.3}$$

When the collateral constraints do not bind, we use a guess-and-verify approach. We shoot back towards the initial conditions, from the first period when the constraints are guessed to bind again. For example, if the constraints do not bind in t but are expected to bind the next period, the decision rule for period t can be expressed, starting from B.2 and using the result that $E_t X_{t+1} = \mathcal{P} X_t$, as:

$$X_t = -\left(\mathcal{A}_1^* \mathcal{P} + \mathcal{A}_0^*\right)^{-1} \left(\mathcal{A}_{-1}^* X_{t-1} + \mathcal{B}^* u_t + \mathcal{C}^*\right). \tag{B.4}$$

We proceed in a similar fashion to compute the allocations for the case when collateral constraints are guessed not to bind for multiple periods or when they are foreseen to be slack starting in periods beyond t. As shown by equation B.4, the model dynamics when constraints are not binding depend both on the current regime (through the matrices $\mathcal{A}_1^*, \mathcal{A}_0^*$ and $\mathcal{A}_{-1}^*$) and on the expectations of future regimes when constraints will bind again (through the matrix $\mathcal{P}$, which is a nonlinear function of the matrices $\mathcal{A}_1$, $\mathcal{A}_0$ and $\mathcal{A}_{-1}$).

It is straightforward to generalize the solution method described above for multiple occasionally binding constraints.[31] The extension is needed to account for the zero lower bound (ZLB) on policy interest rates as well as the possibility of slack collateral constraints. In that case, there are four possible regimes: 1) collateral constraints bind and policy interest rates are above zero, 2) collateral constraints bind and policy interest rates are at zero, 3) collateral constraints do no bind and policy interest rates are above zero, 4) collateral constraints do not bind and policy interest rates are at zero. Apart from the proliferation of cases, the main ideas outlined above still apply.

Checks on Solution Accuracy. In the absence of an analytical solution for the models considered in this paper, we assess the solution algorithm used to solve the full general equilibrium model by comparing its performance against standard solution methods. As is well understood, standard global methods are subject to the curse of dimensionality, which renders such methods inoperable for our application. However, the partial equilibrium model of Section 2 of the paper can be solved with both our piecewise-linear algorithm, and with standard global solution methods. We use this smaller model to showcase the performance of our solution algorithm.

Among standard global methods, we focus on value function iteration since it is reliable,

[31] For an array of models, Guerrieri and Iacoviello (2014) compare the performance of the piecewise perturbation solution described above against a dynamic programming solution obtained by discretizing the state space over a fine grid. Their results show that this solution method efficiently and quickly computes a solution that closely mimics the nonlinear solution.

accurate, and well understood.[32] Overall, we find that key aspects of the global solution obtained through value function iteration are matched by the solution from the piecewise-linear algorithm. A key advantage of our algorithm is that it can handle the solution of models for which the curse of dimensionality renders standard global solution methods infeasible.

In Figures A.3 and A.4 we compare the simulated paths for house prices, consumption, leverage and debt using alternative solution methods. In Figure A.3, we consider impulse responses to negative and positive house price shocks. In Figure A.4, we generate a realization of house prices drawing shock innovations for 50 periods from the stochastic AR(1) process described in equation 4.

The "piecewise-linear" lines are computed using our method. The "nonlinear stochastic" lines refer to the nonlinear model solution obtained using global methods (value function iteration) under the assumption that the agents know and act upon the future distribution of the random shocks. The "nonlinear deterministic" lines refer to the perfect foresight case, solved using global methods under the assumption that agents ignore the future variance of shocks (that is, each period they expect that future shock innovations will equal zero with probability one, only for these expectations to be dashed when new shocks are realized). Finally, the "linear" lines refer to the model solved using brute force linearization under the – counterfactual – assumption that the borrowing constraint is always binding.

As can be seen from the figures, the nonlinear (value function iteration) and the piecewise linear method deliver very similar dynamics for the variables of interest. The similarity of the simulation paths causes the business cycle statistics (reported in Table A.1) to be in broad agreement for those two methods. As expected, leverage and debt are on average lowest in the full stochastic case, since buffer stock motives – ignored by construction or by design in the other cases – cause agents to save more and reduce indebtedness. However, our method comes remarkably close to matching the dynamics of the full nonlinear method under perfect foresight. As first-order perturbation solutions ignore the possibility of future shocks, it is not surprising that our piecewise-linear method would not be able to capture precautionary motives present in the full stochastic non-linear solution. By contrast, the linearized solution that assumes that the constraint is always binding cannot capture the asymmetry of consumption and grossly overestimates its volatility.

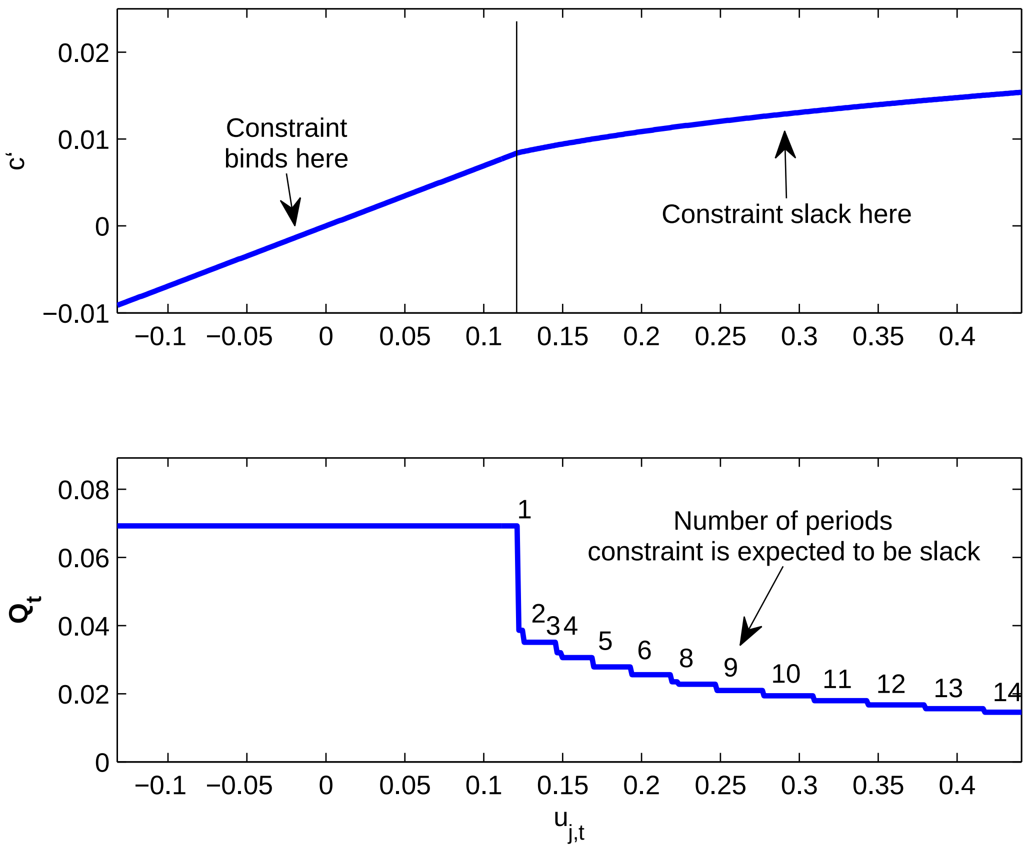
As a further metric to judge to accuracy of our solution method, the last column of Table A.1 reports the welfare cost for a household of using the approximated policy functions instead of the nearly-exact one (which we take to be the solution obtained via value function iteration) in order to solve the problem. The welfare cost of using the piecewise linear policy function is small (about 0.01% of lifetime consumption), and is one order of magnitude smaller than the cost of using the linearized policy function.

[32] Our state variables are the level of debt, the housing stock and the house price process. We discretize the AR(1) house price process with using Tauchen's method (Tauchen 1986) with 101 grid points. We pick a solution range for housing and debt between −60 and +60 percent of their steady state values, discretized over 100 points for debt and 110 points for housing. In between iterations, we use Howard's improvement step. We verified that increasing the number of grid points did not materially change any of the results.

Appendix D State-Level Evidence on House Prices and Mortgage Originations

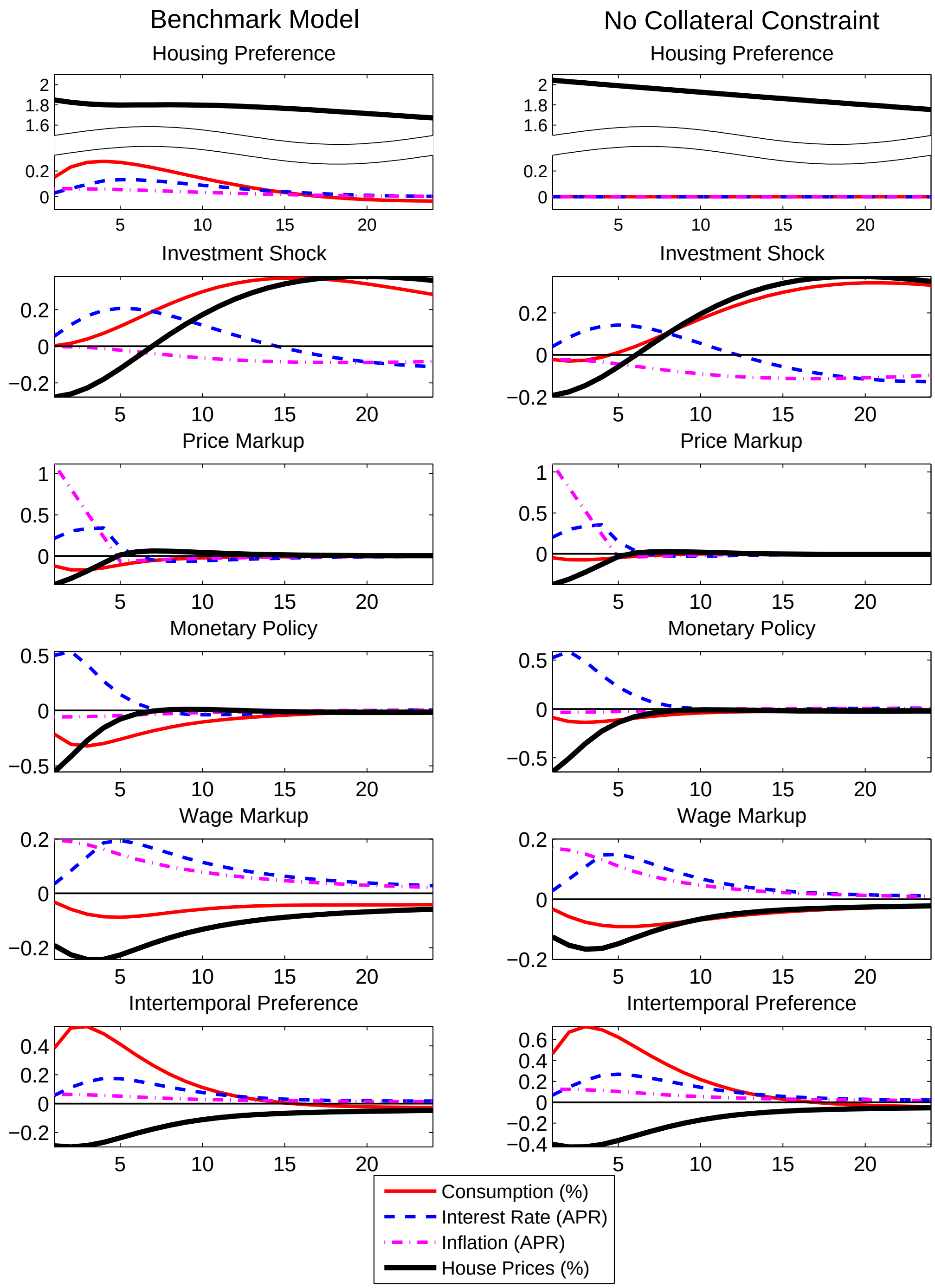
Because the effects of low and high house prices on consumption work in our model through tightening or relaxing borrowing constraints, it is important to check whether measures of credit also depend asymmetrically on house prices. Table A.2 shows how mortgage originations at the state level respond to changes in house prices. We choose mortgage originations because they are available for a long time period, and because they are a better measure of the flow of new credit to households than the stock of existing debt. In all of the specifications in Table A.2, mortgage originations depend asymmetrically on house prices, too.

Figure A.1: Local Linearity of the Policy Functions



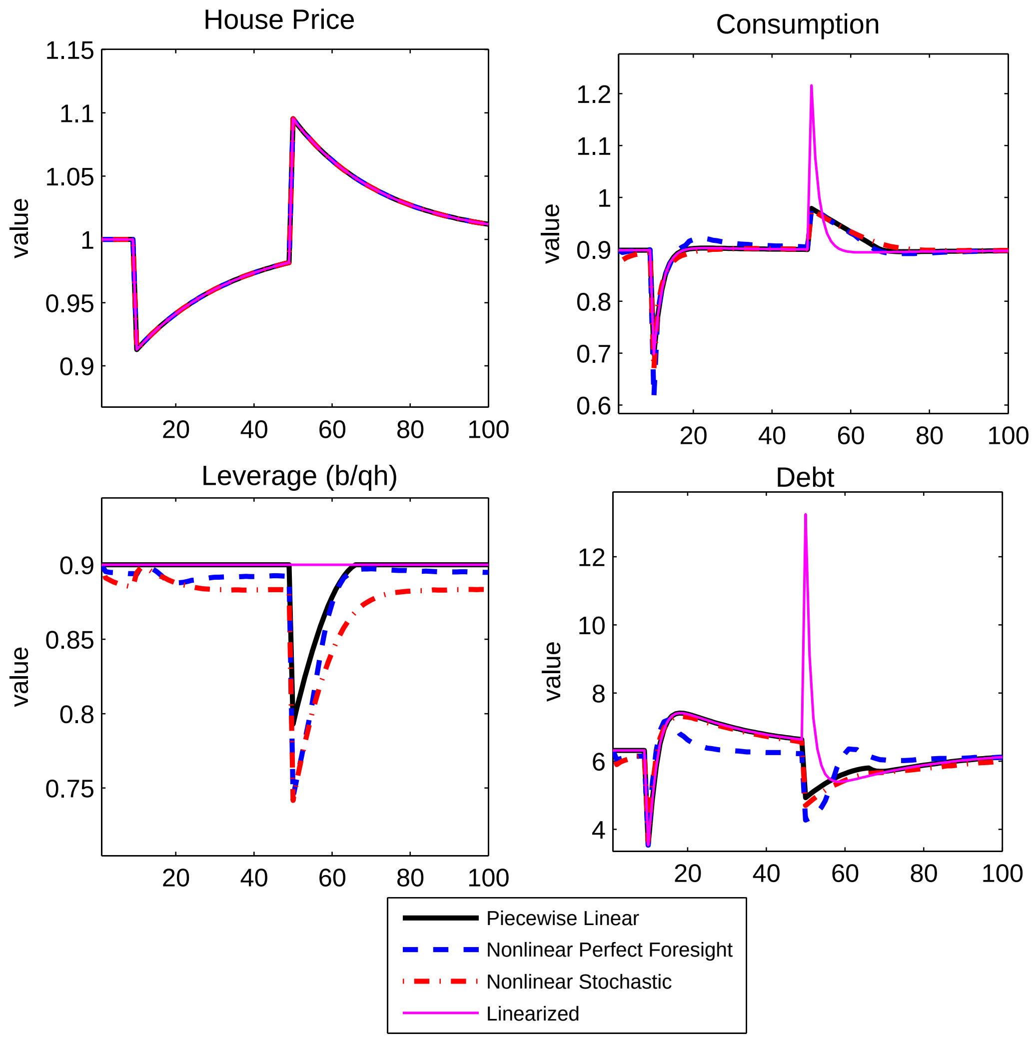
Note: The top panel plots consumption of the impatient agent (in deviation from the steady state) as a function of various realizations of the housing preference shock. The bottom panel plots the slope of the consumption function. The consumption function has a kink when the borrowing constraint becomes binding, and becomes flatter the larger the realization of the housing preference shock.

Figure A.2: Impulse Responses to All Shocks for the Estimated Model



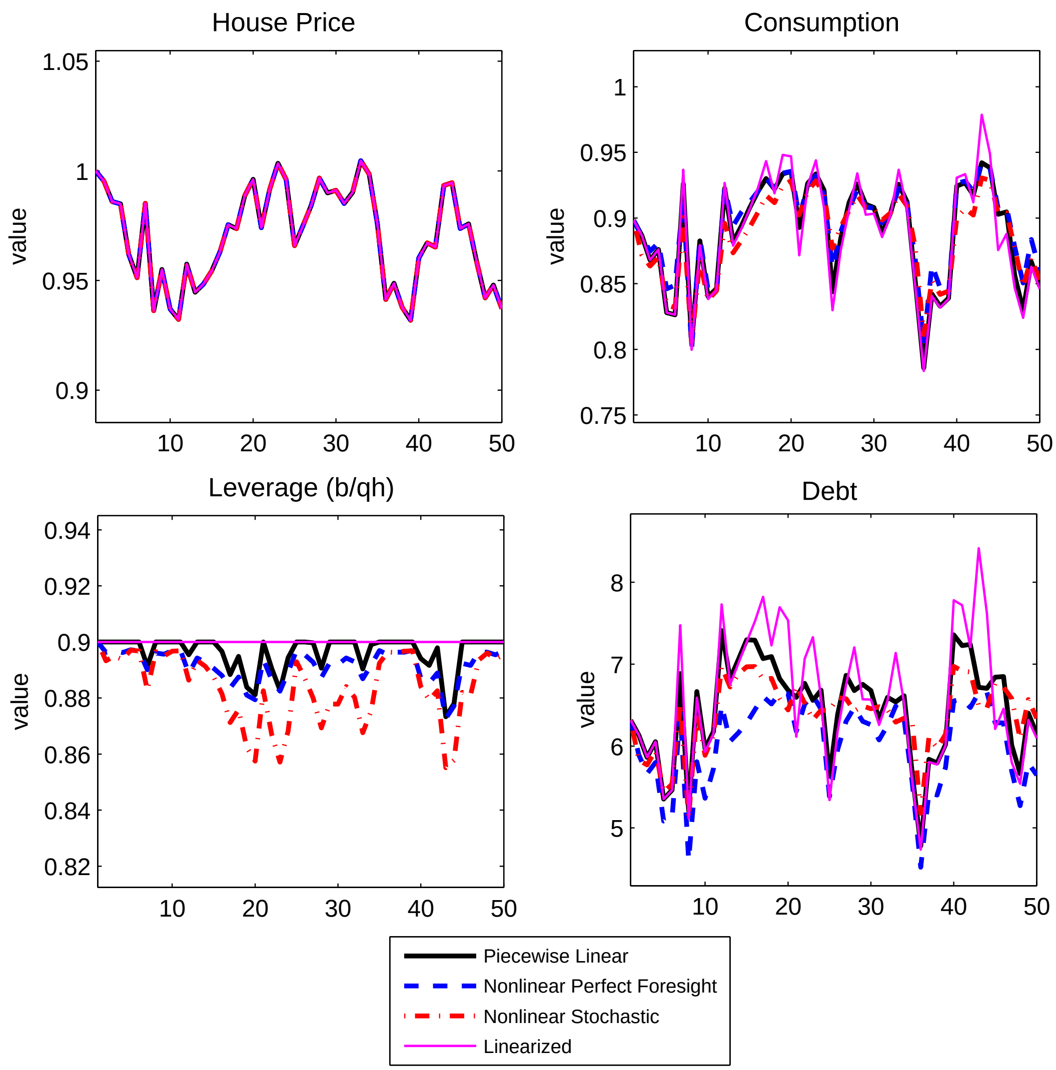
Note: Horizontal axes: horizon in quarters. The panels to the left show the impulse responses of house prices, consumption, interest rate and inflation to an estimated one standard deviation shock in the benchmark model. The panels to the right repeat the exercise for the estimated model without collateral constraints.

Figure A.3: Accuracy of Solution Method: Impulse Responses



Note: Horizontal axes: horizon in quarters. Impulse Responses of the basic model to a negative house price shock in period 10 and a positive house price shock in period 50.

Figure A.4: Accuracy of Solution Method: Simulated Time Series for the Basic Model



Note: Simulation of the basic model for 50 periods using identical realizations for the exogenous random shock to house prices.

Table A.1: Accuracy of the Solution Method

Solution Method	Log Consumption		Correlations		$\frac{b}{qh}$	Δ Welfare
	st.dev	skewness	$\log q, \log c$	$\log q, \frac{b}{qh}$	mean	
Linear	5.97%	-0.04	0.47	0.00	0.900	0.160%
Piecewise Linear	4.70%	-1.01	0.59	-0.63	0.884	0.011%
Nonlinear Perfect Foresight	4.32%	-0.94	0.55	-0.63	0.880	0.010%
Nonlinear Stochastic	3.89%	-0.99	0.67	-0.72	0.867	-

Note: Selected properties of the basic model using different solution algorithms. These properties are based on the outcomes of a simulation of 5,000 observations using identical realizations for the exogenous random shocks.

The column labeled "Δ Welfare" indicates the annuity value of the transfer τ (as a percent of current consumption) that would make an agent using the solution method in the first column indifferent between using that method and using the Nonlinear Stochastic solution. Letting (c_t^*, h_t^*) denote the consumption and housing policy in the nonlinear stochastic case, and $\left(\widetilde{c}_t, \widetilde{h}_t\right)$ the consumption policy in the linear case, the two associated value functions are $W_t^* = u\left(c_t^*, h_t^*\right) + \beta E_t W_{t+1}^*$ and $\widetilde{W}_t = u\left(\widetilde{c}_t, \widetilde{h}_t\right) + \beta E_t \widetilde{W}_{t+1}$. The transfer τ is the solution to the following equation: $u\left(\widetilde{c}_t\left(1+\tau\right), \widetilde{h}\right) + \beta E_t \widetilde{W}_{t+1} = W_t^*$. By design, the nonlinear stochastic solution attains the highest level of welfare. Note that the linear and piecewise linear solution method could lead to spurious welfare reversals since they linearize the constraints of the original nonlinear problem thus transforming the original problem. To avoid this problem, we use these methods only to compute the borrowing and housing policy, and then obtain the consumption policy c nonlinearly from the budget constraint.

Table A.2: State-Level Regressions: Mortgage Originations and House Prices

	% Change in Mortgage Originations ($\Delta mori_t$)				
Δhp_{t-1}	**1.10*** **				
	(0.18)				
Δhp_high_{t-1}		**-0.41***	**1.08*** **	**1.46*** **	**1.54*** **
		(0.24)	(0.16)	(0.21)	(0.33)
Δhp_low_{t-1}		**3.13*** **	**1.85*** **	**2.53*** **	**2.67** **
		(0.59)	(0.68)	(0.90)	(1.11)
$\Delta mori_{t-1}$				**-0.20*** **	**-0.20*** **
				(0.02)	(0.02)
$\Delta income_{t-1}$					**-0.63**
					(1.04)
pval difference		**0.000**	**0.211**	**0.160**	**0.181**
Time effects	**no**	**no**	**yes**	**yes**	**yes**
Observations	1020	1020	1020	969	969
States	51	51	51	51	51
R-squared	0.01	0.03	0.58	0.53	0.53

Note: State–level Regressions using annual observations from 1992 to 2011 on 50 States and the District of Columbia. Robust standard errors in parenthesis. ***,**,*: Coefficients statistically different from zero at 1, 5 and 10% confidence level. pval is the p-value of the test for difference in the coefficients for low-house prices and high-house prices.

Data Sources and Definitions: $\Delta mori$ is the percent change in "Mortgage originations and purchases: Value" from the U.S. Federal Financial Institutions Examination Council: Home Mortgage Disclosure Act. See Table 3 for other variable definitions.