

Learning Financial Structural Change and the Great Recession*

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Abstract: This paper develops a simple business-cycle model in which the financial sector originates a structural change that has large macroeconomic effects when private agents are gradually learning their economic environment. When the persistence of the unobserved process driving financial shocks to the leverage ratio changes, the responses of output and other aggregates under adaptive learning are significantly larger than under rational expectations. In our benchmark case calibrated using 2008Q4 data on leverage, debt-to-GDP and land value-to-GDP ratios, learning amplifies leverage shocks by a factor of about four, relative to rational expectations. In addition, we show that procyclical leverage reinforces the impact of learning and, accordingly, that macro-prudential policies enforcing countercyclical leverage promote stability. Finally, we illustrate both how learning with a misspecified model that ignores real/financial linkages also contributes to magnify financial shocks and how interest rate news shocks are propagated under learning with structural change.

Keywords: Borrowing Constraints, Collateral, Leverage, Learning, Financial Shocks, Recession, Structural Change

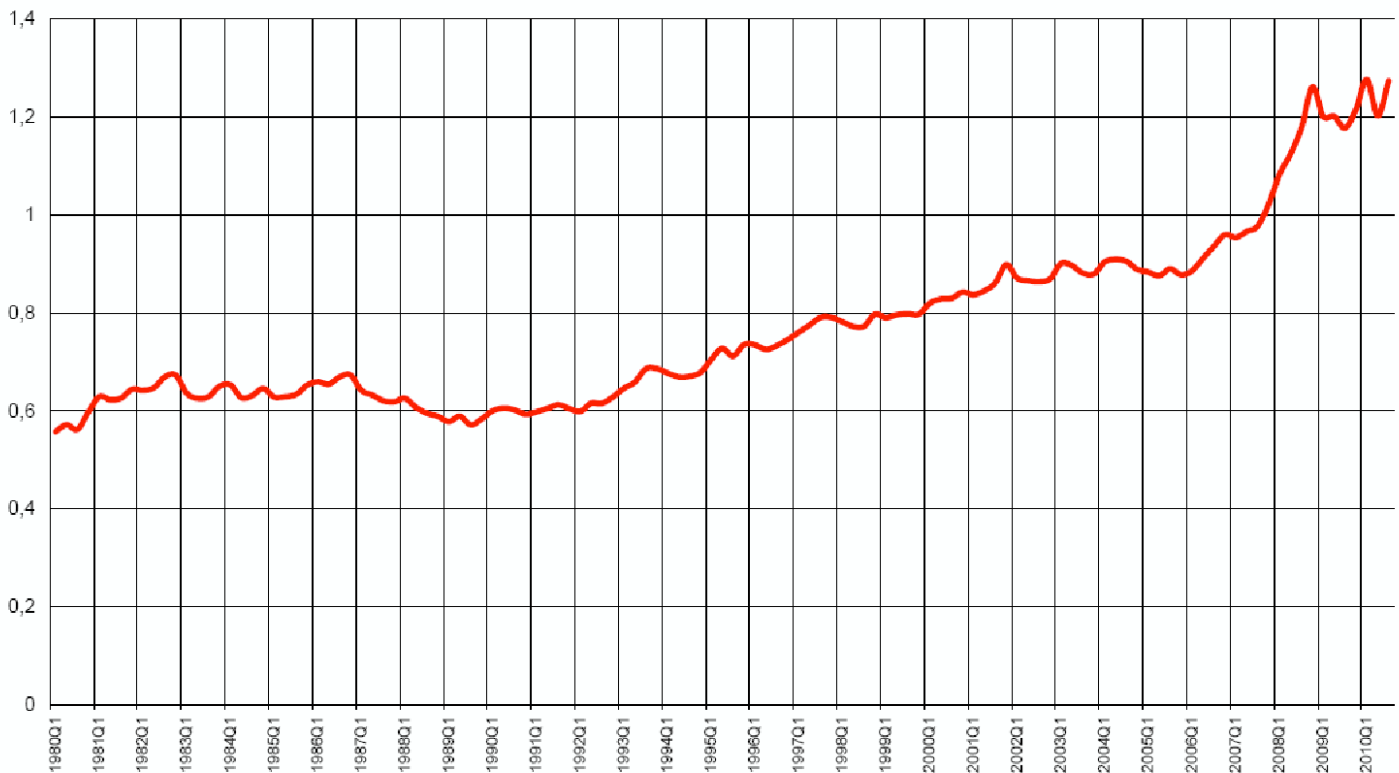
Journal of Economic Literature Classification Numbers: E32, E44, G18

1 Introduction

Both financial innovations and financial regulation (or lack of it) affect the macroeconomy and, if need be, the recent US Great Recession is a stark reminder of this fact. This paper is an attempt to capture in a simple business-cycle model the related idea that the financial sector dynamics may originate structural change that has large macroeconomic effects when private agents are gradually learning their economic environment. More specifically, we show that when the unobserved process driving financial shocks to the leverage ratio changes, the responses of output and other aggregates under adaptive learning are significantly larger than under rational expectations. In our model economy, the key random variable is the leverage ratio, which we define by how much one

can borrow out of the market value of land. Structural change takes the form of a sudden change in the persistence of shocks to the leverage ratio. More precisely, we posit that the stochastic process driving leverage may go through phases when it regains stationarity after having been nonstationary, or *vice versa*. This assumption is motivated by the data, reported in Figure 1, on US household leverage that are provided by Boz and Mendoza [4] over the period 1980Q1-2010Q3, which can roughly be split into three phases.

Figure 1: US Household Leverage Ratio 1980Q1-2010Q3. Source: Boz and Mendoza [4]



The first one is running from 1980 to the early 1990s, when leverage is flat around 60%. In the second phase, leverage trends up until the last quarter of 2008, when the financial crisis is in its most severe stage. Finally, after 2008Q4 leverage seems to become flat again. According to our definition of structural change, two episodes occur and we

focus on the last one taking place in 2008Q4, which possibly ends an era when leverage was nonstationary. More specifically, we think of leverage as following an AR(1) process, with autocorrelation going down from a value exceeding unity to a value below one in 2008Q4. In Appendix A.5, we present empirical support for this assumption.

Our main findings are derived in a model that is a simple variant of Kiyotaki and Moore [15] based on Kocherlakota [16]. We focus on financial shocks that drive up and down the leverage ratio, which according to the data in Figure 1 are very persistent. We calibrate the model using data on leverage, debt-to-GDP and land value-to-GDP ratios in 2008Q4 and we subject the economy to the large negative shock to leverage that was observed then (see Figure 1) under the assumption that the persistence of the leverage shock goes back to below unity. We compare the responses of the linearized economy under adaptive learning, following Marcet and Sargent [18] and Evans and Honkapohja [10], and under rational expectations. A major difference is of course that in the former case, agents gradually learn that the structural change took place by updating their beliefs, whereas in the latter the structural change modifies rational expectation beliefs instantaneously.

Our typical sample of results shows that learning amplifies leverage shocks by a factor of about 4 (see Figure 3). For example, our model predicts, when fed with the negative leverage shock of about -5% observed in 2008Q4, that output falls by about 1% , which is roughly by how much US GDP dropped at that time. In addition, aggregate consumption and the capital stock fall by about 1.5% and 2% , respectively. Under rational expectations, however, output drops only by a quarter of 1% while the responses of consumption and investment are divided by more than 4 at impact. Consumption and investment go down by a significantly larger margin under learning because deleveraging is more severe: land price and debt are much more depressed after the negative leverage shock hits when its persistence is overestimated by agents who are constantly learning

their environment and, because of recent past data, temporarily pessimistic. We next show that the magnitude of the consequent recession may in part be attributed to the high level of leverage (and the correspondingly high level of the debt-to-GDP ratio) observed in 2008Q4. When the same negative leverage shock occurs in the model calibrated using 1996Q1 data, when leverage was much lower, the impact on output's response is reduced by about a third and the recovery is quicker. In this sense, our model points at the obvious fact that financial shocks to leverage originate larger aggregate volatility in economies that are more levered.

In addition, we also ask whether procyclical leverage may act as an aggravating factor and our answer is positive. Under the assumption that leverage responds positively to increases in land price, the recession is further worsened by a factor of about 2.5 under learning whereas it is marginally affected under rational expectations. This experiment is motivated by the recent evidence provided by Mian and Sufi [20] and discussed in the theoretical contribution by Midrigan and Philippon [21]. In contrast, the counterfactual experiment with countercyclical leverage shows dampened effects of leverage shocks and it brings the responses of aggregate variables under learning close to their rational expectations counterpart. One possible interpretation of this finding is that macro-prudential policies enforcing countercyclical leverage have potential stabilizing effects on the economy in the face of financial shocks, at zero cost provided that non-distortionary policies are implemented (e.g. through regulation). Finally, we illustrate both how learning with a misspecified model that ignores real/financial linkages also contributes to magnify financial shocks and how interest rate news shocks are propagated under learning.

To summarize, our main finding is that leverage shocks are amplified when agents gradually learn that the structural economy reverts to a regime where the changes in the leverage level are no longer permanent. We believe it is important to acknowledge that, as Figure 1 suggests, nonstationary leverage may have played an important role

in favoring conditions that worsened the Great Recession. Looking back in time at the data in Figure 1, there is a sense in which everybody should have foreseen that leverage could not possibly increase forever. However, figuring out when leverage would stop rising was a much harder task. Our paper stresses that when such a structural change comes, its macroeconomic impact when agents adaptively learn differs much from what happens under rational expectations. Moreover, on the policy side, our analysis gives an example of a macro-prudential policy that dampens the impact of financial shocks to the macroeconomy under learning by ensuring that leverage goes down when asset prices spike up.

Related Literature: Our paper connects to several strands of the literature. The macroeconomic importance of financial shocks has recently been emphasized by Jermann and Quadrini [14], among others, and our paper contributes to this literature about credit shocks by showing how learning matters. Closest to ours are the papers by Adam, Kuang and Marcet [1], who focus on interest rate changes, and by Boz and Mendoza [4], who show how changes in the leverage ratio have large macroeconomic effects under Bayesian learning and Markov regime switching. We very much follow the approach advocated in Boz and Mendoza [4], with some differences though. Because we assume that agents are adaptively learning through VAR estimation, it is possible to enrich the model by adding capital accumulation and production. To keep the analysis as simple as possible, we solve for equilibria under learning through usual linearization techniques. In the literature, the idea that procyclical leverage has adverse consequences on the macroeconomy is forthfully developed in Geanakoplos [11] (see also Cao [6]). Although our formulation of elastic leverage is derived in an admittedly simple setup, it allows us to examine its effect in a full-fledged macroeconomic setting. Last but not least, the notion that learning is important in business-cycle models when structural change occurs has been discussed by, e.g., Bullard and Duffy [5] and Williams [24]. More recently, Eusepi and Preston [9]

have shown that learning matters in a standard RBC model when the economy is hit by shocks to productivity growth. Our paper adds to this literature by focusing on financial shocks under collateral constraints. As mentioned before, part of the paper's motivation also comes from the growing micro-evidence about the importance of households' and firms' leverage for understanding consumption and investment behaviors (e.g. Mian and Sufi [20], Chaney, Sraer and Thesmar [7]).

The paper is organized as follows. Section 2 presents the model and derives its rational expectations equilibria. Section 3 relaxes the assumption that agents form rational expectations in the short run and it studies intertemporal equilibria arising in the model under adaptive learning. Section 4 then shows how financial shocks are amplified under learning when structural change strikes and Section 5 performs a robustness analysis of this finding. Finally, Section 6 gathers concluding remarks and proofs are exposed in the Appendix.

2 The Economy with Leverage Shocks

2.1 Model

The model is essentially an extension of Kocherlakota's [16] with learning (and partial capital depreciation). A representative agent solves:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma} \quad (1)$$

where $C_t \geq 0$ is consumption and $\sigma \geq 0$ denotes relative risk aversion, subject to both the budget constraint:

$$C_t + K_{t+1} - (1 - \delta)K_t + Q_t(L_{t+1} - L_t) + (1 + R)B_t = B_{t+1} + AK_t^\alpha L_t^\gamma \quad (2)$$

and the collateral constraint:

$$\Theta_t E_t[Q_{t+1}]L_{t+1} \geq (1 + R)B_{t+1} \quad (3)$$

where K_{t+1} , L_{t+1} and B_{t+1} are, respectively, the capital stock, the land stock and the amount of new borrowing all chosen in period t , Q_t is the land price, R is the exogenous interest rate, A is total factor productivity (TFP thereafter). In our benchmark model, leverage Θ_t is exogenous and subject to random shocks whereas both the interest rate and TFP are constant over time. As we focus on financial structural change, we ignore TFP disturbances and simply notice that similar results hold when the process driving technological shocks changes as well. In addition, we report in Section 5.2 what happens under contemporaneous and news interest rate shocks. We present first the results obtained under the collateral constraint (3), which follows Kiyotaki and Moore [15]. However, quantitatively similar results hold under the margin requirement timing stressed in Aiyagari and Gertler [3] (see Section 5.1 for robustness analysis).

Denoting λ_t and ϕ_t the Lagrange multipliers of constraints (2) and (3), respectively, the borrower's first-order conditions with respect to consumption, land stock, capital stock, and loan are given, respectively, by:

$$C_t^{-\sigma} = \lambda_t \quad (4)$$

$$\lambda_t Q_t = \beta E_t[\lambda_{t+1} Q_{t+1}] + \beta \gamma E_t[\lambda_{t+1} Y_{t+1}/L_{t+1}] + \phi_t \Theta_t E_t[Q_{t+1}] \quad (5)$$

$$\lambda_t = \beta E_t[\lambda_{t+1} (\alpha Y_{t+1}/K_{t+1} + 1 - \delta)] \quad (6)$$

$$\lambda_t = \beta(1 + R)E_t[\lambda_{t+1}] + (1 + R)\phi_t \quad (7)$$

2.2 Rational Expectations Equilibria

A rational expectations competitive equilibrium is a sequence of positive prices $\{Q_t\}_{t=0}^{\infty}$ and positive allocations $\{C_t, K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty}$ such that, given the exogenous se-

quence $\{\Theta_t\}_{t=0}^\infty$ of leverage and the exogenous interest rate $R \geq 0$:

(i) $\{C_t, K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^\infty$ satisfies the first-order conditions (4)-(7), the transversality conditions, $\lim_{t \rightarrow \infty} \beta^t \lambda_t L_{t+1} = \lim_{t \rightarrow \infty} \beta^t \lambda_t K_{t+1} = 0$, and the complementarity slackness condition $\phi_t [\Theta_t Q_{t+1} L_{t+1} - (1 + R_{t+1}) B_{t+1}] = 0$ for all $t \geq 0$, given $\{Q_t\}_{t=0}^\infty$ and the initial endowments $L_0 \geq 0, B_0 \geq 0, K_0 \geq 0$;

(ii) The goods and asset markets clear for all t , that is, $C_t + K_{t+1} - (1 - \delta)K_t + (1 + R)B_t = B_{t+1} + A_t K_t^\alpha$ and $L_t = 1$, respectively.

The above definition assumes that the interest rate is exogenous. Therefore, a natural interpretation of the model is that it represents a small, open economy. Appendix A.2 presents a closed-economy variant based on Iacoviello [12], in which borrowers and lenders meet in a competitive credit market subject to collateral constraints and a constant debtor interest rate. Our findings reported below can be replicated in the closed-economy model when the economy is hit by negative financial and TFP shocks that occur simultaneously. As our focus is on how borrowers adaptively learn how the economy settles after financial structural change, we abstract both from TFP shocks and from further details regarding the lender's side, and we focus on the small-open-economy setting, as in Adam, Kuang and Marcet [1], Boz and Mendoza [4].

There is a unique (deterministic) stationary equilibrium such that the credit constraint (3) binds, provided that the interest factor $1 + R \equiv 1/\mu$ is such that $\mu \in (\beta, 1)$, that is, if lenders are more patient than borrowers. This follows from the steady-state version of (7), that is, $\phi = \lambda(\mu - \beta) > 0$. The steady state is characterized by the following great ratios, that fully determine the linearized dynamics around the steady state. From (5) and (6), it follows that the land price-to-GDP and capital-to-GDP ratios are given by $Q/Y = \gamma\beta/[1 - \beta - \Theta(\mu - \beta)]$ and $K/Y = \alpha\beta/[1 - \beta(1 - \delta)]$, respectively. Finally, (3) and (2) yield, respectively, the debt-to-GDP ratio $B/Y = \mu\Theta Q/Y$ and the consumption-to-GDP ratio $C/Y = 1 - \delta K/Y - (1/\mu - 1)(B/Y)$.

Appendix A.1 provides a linearized version, in percentage deviations from the steady state, of the set of equations (2)-(7) defining, together with the leverage law of motion $\Theta_t = \bar{\Theta}^{1-\rho_\theta} \Theta_{t-1}^{\rho_\theta} \Xi_t$, intertemporal equilibria. We assume throughout that leverage Θ is observed while the shock Ξ remains unobserved. Eliminating ϕ_t by using (7), the linearized expectational system can be written as:

$$X_t = \mathbf{A}X_{t-1} + \mathbf{B}E_{t-1}[X_t] + \mathbf{C}E_t[X_{t+1}] + \mathbf{D}\xi_t \quad (8)$$

where $X_t \equiv (c_t \ q_t \ \lambda_t \ b_t \ k_t \ \theta_t)'$ is observed whereas ξ_t is not. The derivation and the expressions of the 6-by-6 matrices \mathbf{A} , \mathbf{B} , \mathbf{C} , \mathbf{D} as functions of parameters are given in Appendix A.1.

Anticipating our results on E-stability, we now use the fact that the linearized rational expectations equilibrium around steady state can be obtained as the unique E-stable Minimal-State-Variable solution (MSV thereafter) of the form $X_t = \mathbf{M}^{\text{re}}X_{t-1}$, where \mathbf{M}^{re} solves $\mathbf{M} = [\mathbf{I}_6 - \mathbf{C}\mathbf{M}]^{-1}[\mathbf{A} + \mathbf{B}\mathbf{M}]$ and \mathbf{I}_6 is the 6-by-6 identity matrix.

To set the stage, we calibrate the model using 2006-Q4 ratios as typical of the pre-recession period.¹

Table 1. Parameter Values (2006:Q4)

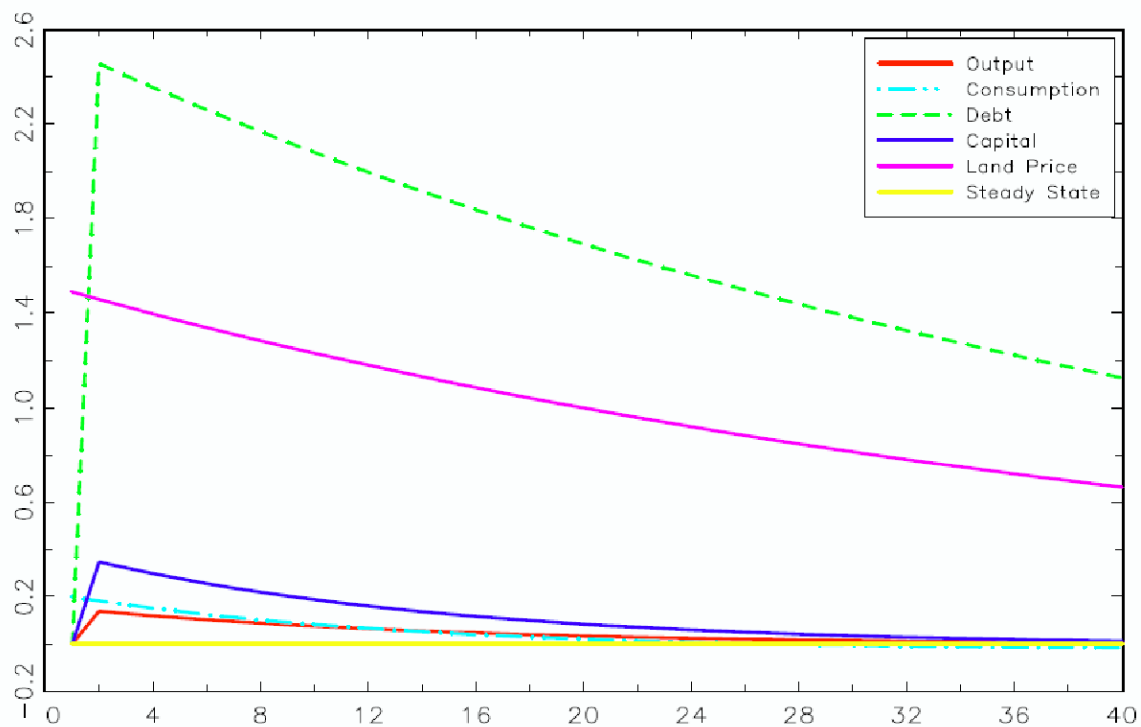
μ	β	δ	α	γ	σ	$\bar{\Theta}$	ρ_θ
0.99	0.95	0.025	0.4	0.01	1	0.95	0.98

Table 1 calibrates the model using the levels of leverage, debt-to-GDP and land value-to-GDP ratios observed in the last quarter of 2006, when $B/Y \approx 0.70$ and $QL/Y \approx 0.75$. Note that although the corresponding level of leverage $\bar{\Theta} \approx 0.95$ is already higher than past values, it continues to trend upward afterwards (see Figure 1). Under this calibration, the effect of leverage shocks on aggregate real variables is relatively small

¹In all our calibrations, the land and capital shares add up to reasonable numbers, which addresses one concern expressed in Kocherlakota [16].

under rational expectations. Figure 2 reports the rational expectations impulse response functions to a 1% leverage shock that hits the economy in period 1, which increases the land price and debt by more than one-for-one. However, its impact on output and on consumption is about one seventh and one fifth of the original shock, respectively. The purpose of the next sections is to show that shocks are further amplified under learning when structural change occurs.

Figure 2: Impulse-Response Functions Under Rational Expectations;
Parameter Values in Table 1



3 Law of Motion under Adaptive Learning

Following Marcet and Sargent [18] and Evans and Honkapohja [10], we now relax the assumption that agents form rational expectations in the short-run. The linearized

dynamic system is now:

$$X_t = \mathbf{A}X_{t-1} + \mathbf{B}E_{t-1}^*[X_t] + \mathbf{C}E_t^*[X_{t+1}] + \mathbf{D}\xi_t \quad (9)$$

where the operator E^* indicates expectations that are taken using all information available at t but that are possibly nonrational. More precisely, agents behave as econometricians by embracing the following perceived law of motion (PLM thereafter):

$$X_t = \mathbf{M}X_{t-1} + \mathbf{N} \quad (10)$$

which agents use for forecasting. In particular, (10) yields $E_t[X_{t+1}] = \mathbf{M}X_t + \mathbf{N}$ and $E_{t-1}[X_t] = \mathbf{M}X_{t-1} + \mathbf{N}$. The actual law of motion (ALM thereafter) results from combining (9) and (10) which gives:

$$[\mathbf{I}_6 - \mathbf{C}\mathbf{M}]X_t = [\mathbf{A} + \mathbf{B}\mathbf{M}]X_{t-1} + [\mathbf{B} + \mathbf{C}]\mathbf{N} + \mathbf{D}\xi_t \quad (11)$$

When \mathbf{M} coincides with $\mathbf{M}^{\mathbf{re}}$ derived in Section 2.2 and \mathbf{N} is a zeroes matrix, then agents hold rational expectations. However, beliefs captured in \mathbf{M} may differ from rational expectations and they are updated in real time using recursive learning algorithms, following Evans and Honkapohja [10]. This means that when the constant matrix \mathbf{N} is set to zero², the belief matrix \mathbf{M}_s is time-varying and its coefficients are updated using:

$$\mathbf{M}_t = \mathbf{M}_{t-1} + \nu_t \mathbf{R}_t^{-1} X_{t-1} (X_t - \mathbf{M}'_{t-1} X_{t-1}) \quad (12)$$

$$\mathbf{R}_t = \mathbf{R}_{t-1} + \nu_t (X_{t-1} X'_{t-1} - \mathbf{R}_{t-1}) \quad (13)$$

where \mathbf{R} is the estimate of the variance-covariance matrix and ν_t is the gain sequence (which equals $1/(t+1)$ under least squares and ν under constant gain, respectively LS and CG thereafter).

The mapping from the PLM (10) into the ALM (11) is given by:

$$T(\mathbf{M}, \mathbf{N}) = ([\mathbf{I}_6 - \mathbf{C}\mathbf{M}]^{-1}[\mathbf{A} + \mathbf{B}\mathbf{M}], [\mathbf{I}_6 - \mathbf{C}\mathbf{M}]^{-1}[\mathbf{B} + \mathbf{C}]\mathbf{N}) \quad (14)$$

²Allowing for a non-zero matrix \mathbf{N} as prior could possibly account for misspecification, but this turns out not to change our results much.

Adapting Proposition 10.3 from Evans and Honkapohja [10], we check that all eigenvalues of $DT_{\mathbf{M}}(\mathbf{M}, \mathbf{N})$ and of $DT_{\mathbf{N}}(\mathbf{M}, \mathbf{N})$ have real parts less than 1 when evaluated at the fixed-point solutions of the T -map (14), that is, $\mathbf{M} = \mathbf{M}^{\text{re}}$ and $\mathbf{N} = O_6$, where O_6 is the 6-by-6 zeroes matrix. Using the rules for vectorization of matrix products, we get:

$$\begin{aligned} DT_{\mathbf{M}}(\mathbf{M}^{\text{re}}, O_6) &= ([\mathbf{I}_6 - \mathbf{C}\mathbf{M}^{\text{re}}]^{-1}[\mathbf{A} + \mathbf{B}\mathbf{M}^{\text{re}}])' \otimes [\mathbf{I}_6 - \mathbf{C}\mathbf{M}^{\text{re}}]^{-1}\mathbf{C} \\ &\quad + \mathbf{I}_6 \otimes [\mathbf{I}_6 - \mathbf{C}\mathbf{M}^{\text{re}}]^{-1}\mathbf{B} \\ DT_{\mathbf{N}}(\mathbf{M}^{\text{re}}, O_6) &= [\mathbf{I}_6 - \mathbf{C}\mathbf{M}^{\text{re}}]^{-1}[\mathbf{B} + \mathbf{C}] \end{aligned}$$

All MSV solutions that we consider from now on are said to be locally E-stable when all eigenvalues of $DT_{\mathbf{M}}(\mathbf{M}^{\text{re}}, O_6)$ and $DT_{\mathbf{N}}(\mathbf{M}^{\text{re}}, O_6)$ lie within the interior of the unit circle. In practice, we numerically compute the E-stable solutions by iterating the T -map (14), as described in Evans and Honkapohja [10, p.232].

4 Learning Financial Structural Change

4.1 Time-Varying Persistence of Leverage Shocks

In this section, we show that learning amplifies leverage shocks when there is financial structural change. More precisely, by this we mean that the stochastic process driving Θ goes through a phase such that ρ_{θ} falls from above one to below one. This is meant to capture the structural break that occurs in 2008Q4 (see Figure 1), when leverage seems to become flat again. Therefore, the model is calibrated according to Table 2, so as to deliver the leverage, debt-to-GDP and land value-to-GDP ratios observed in the last quarter of 2008, that is $\bar{\Theta} \approx 1.26$, $B/Y \approx 0.69$ and $QL/Y \approx 0.55$.

Table 2. Parameter Values (2008:Q4)

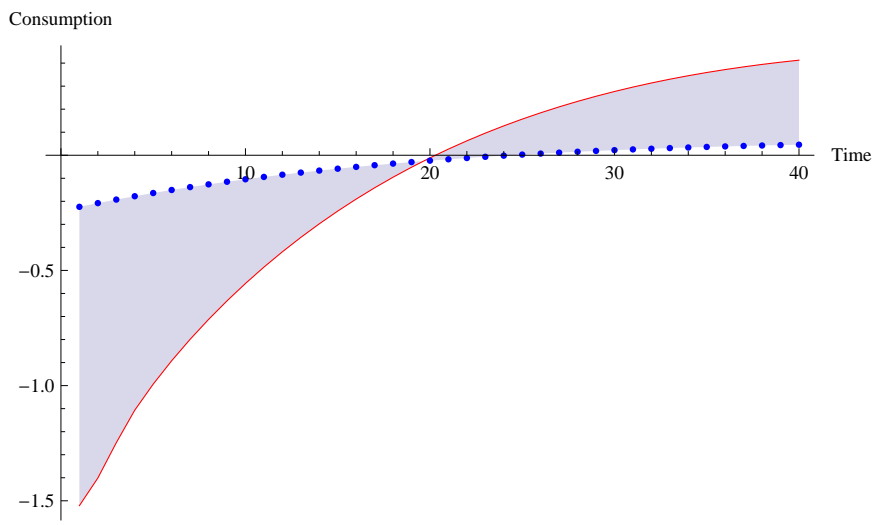
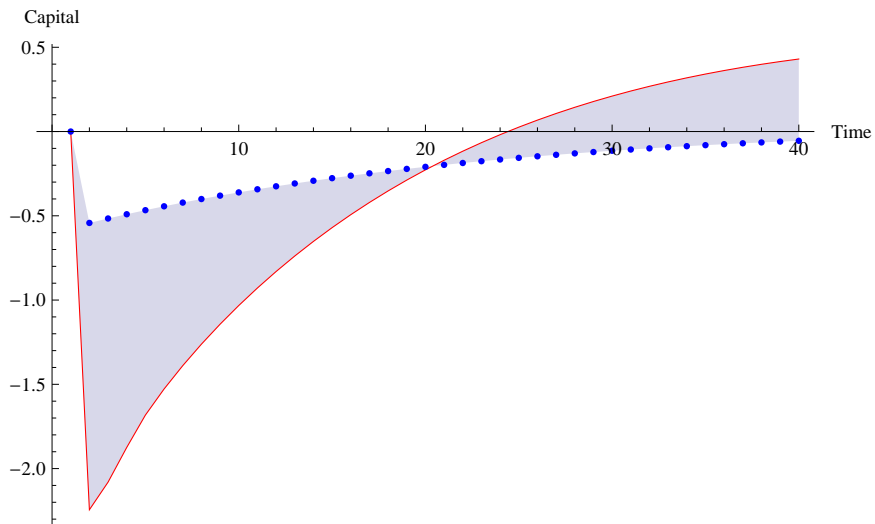
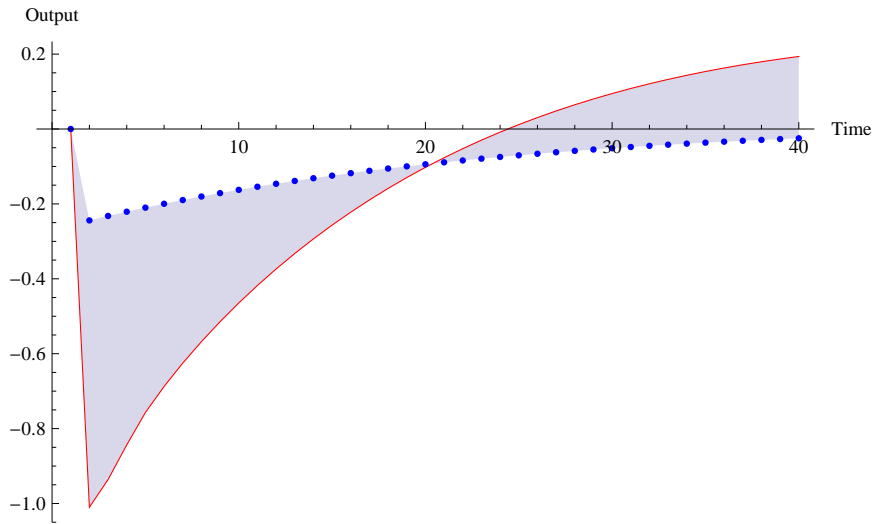
μ	β	δ	α	γ	σ	$\bar{\Theta}$
0.98	0.99	0.025	0.45	0.01	1	1.26

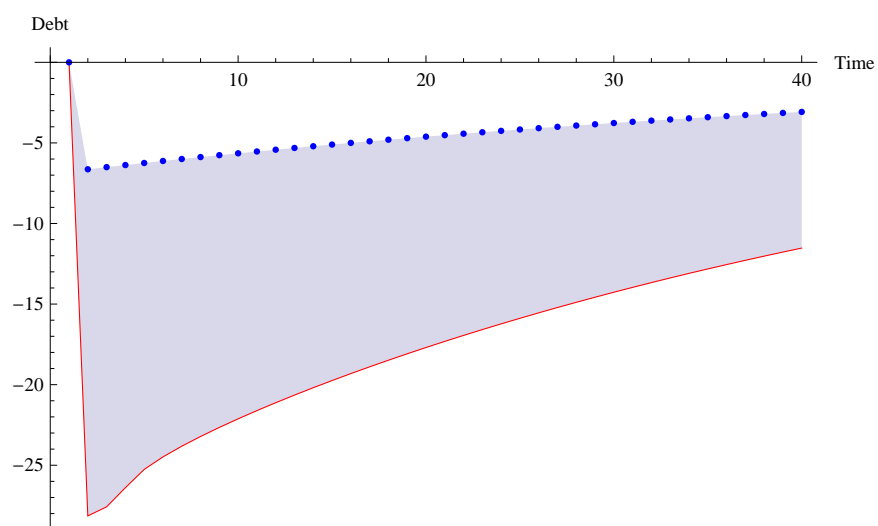
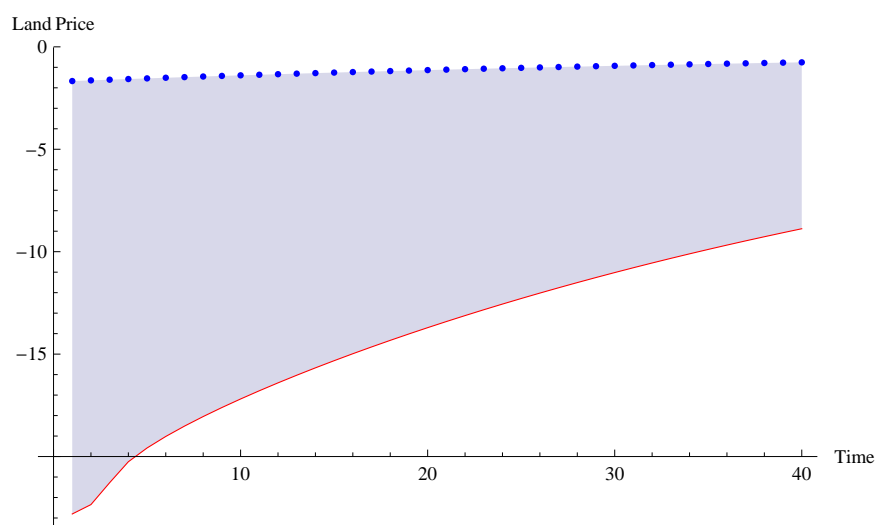
The experiment that embodies our main result is the following. We assume that in the decade preceding the financial collapse of 2008Q4, the agents in our model economy have learned that ρ_θ was larger than one, reflecting the leverage trend in Figure 1 that starts in the early 1990s. This means that agents' beliefs encapsulated in matrix \mathbf{M} of the PLM (10) reflect that $\rho_\theta > 1$. Then in 2008Q4, structural change occurs and the financial system reverts to his previous regime such that $\rho_\theta < 1$. This is also the time when a large negative shock to leverage of about -5% happens (see Figure 1). The (pseudo-)impulse functions in Figure 3 report the reaction of the economy's aggregates under two assumptions, after structural change brings ρ_θ down from 1.015 to 0.98. We choose $\rho_\theta = 1.015$ to be close to the highest possible values that guarantees E-stability. In the first case, agents know immediately that structural change has happened and their beliefs jump to the new RE equilibrium $\mathbf{M}^{\mathbf{re}}$ with $\rho_\theta = 0.98$. This is the blue dotted line in Figure 3. The second scenario captured in the solid red curve in Figure 3 is when agents gradually learn using (12)-(13), with $\rho_\theta = 0.98$. Although Figure 3 assumes CG learning (with $\nu = 0.01$), similar results occur under LS learning.

Figure 3 shows that the negative leverage shock is significantly amplified under learning. In particular, the impact on output and capital is roughly four times larger and the consumption drop is multiplied by about seven compared to the rational expectations outcome. This follows from the fact that deleveraging is much more severe under learning: the fall in land price is more than ten times larger and the debt decrease is multiplied by about seven compared to RE.³

³In Figure 3, debt falls by much more than output. This implies that the debt-to-GDP ratio - a common definition of aggregate leverage - falls by a large amount as well.

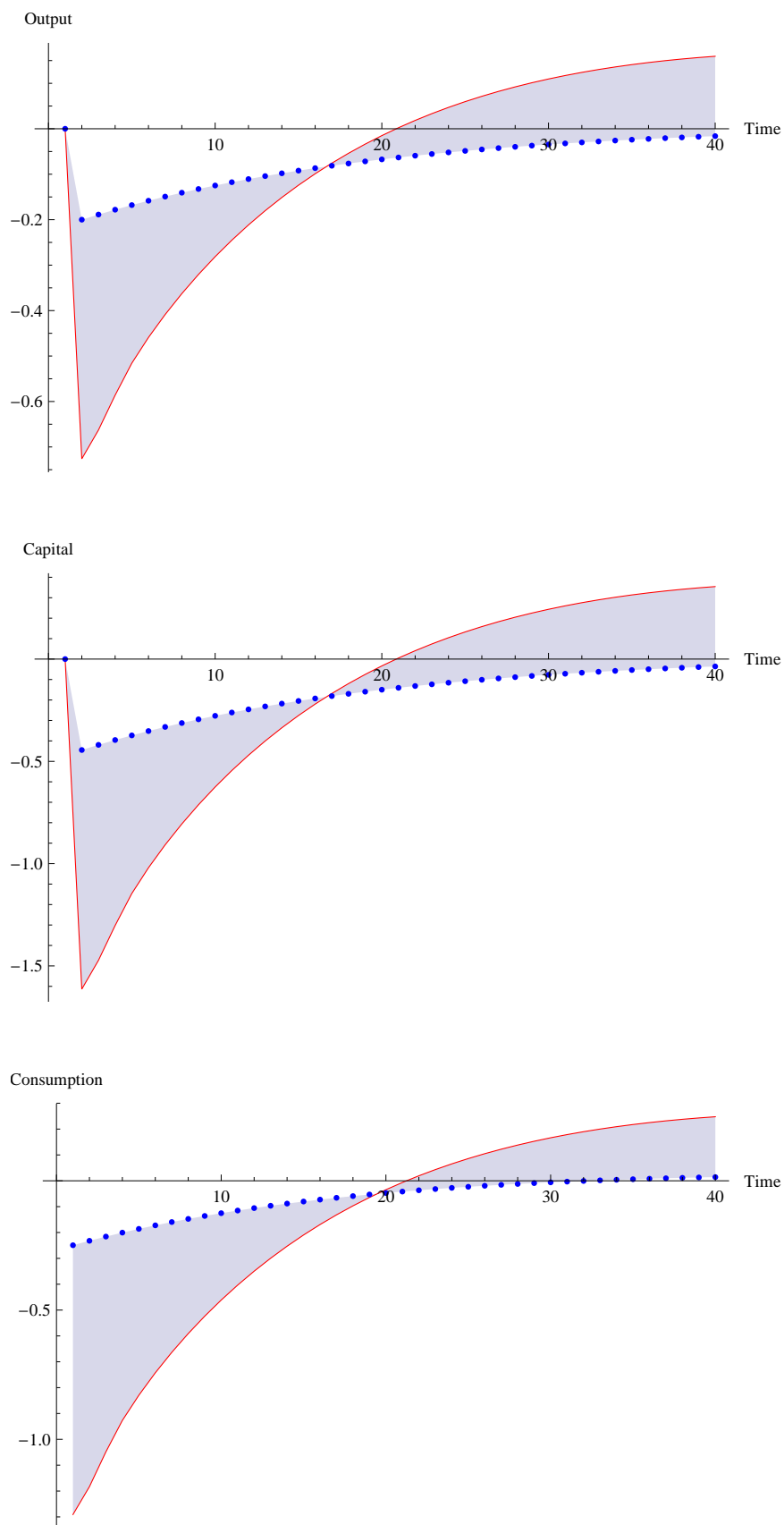
Figure 3: Responses to a -5% Leverage Shock under Learning (Red Solid Line) and Rational Expectations (Blue Dotted Line); Parameter Values in Table 2

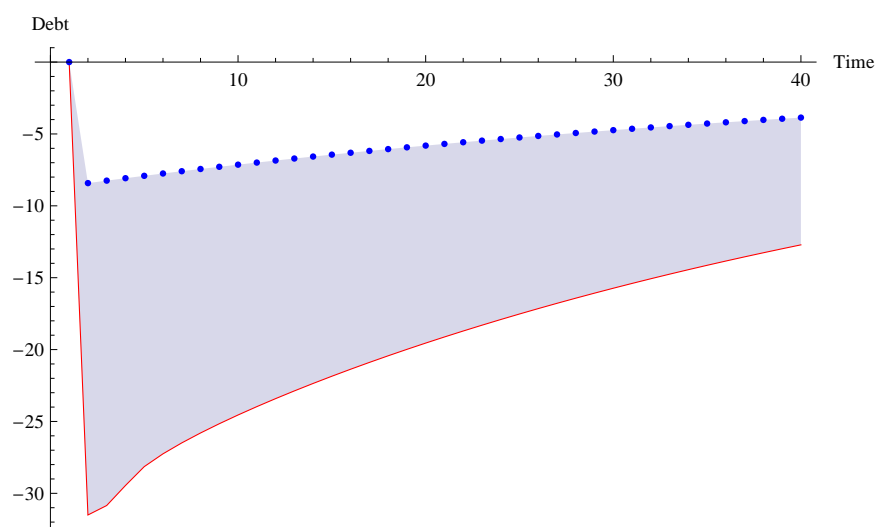
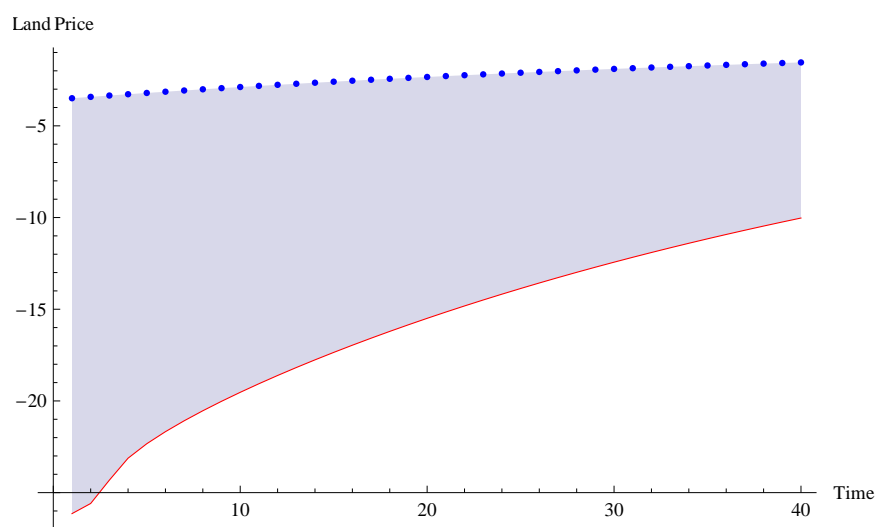




In summary, because agents incorrectly believe that the impact of the negative leverage shock will increase over time, they expect a much larger fall in land price and a much tighter borrowing constraint than under rational expectations, which in turn depresses consumption, investment and output. In this sense, agents become pessimistic when structural change triggers incorrect beliefs. More technically, setting ρ_θ slightly larger than one implies that \mathbf{M} has its highest eigenvalue close to unit root. Note that the magnitudes of output's and consumption's responses roughly match data, whereas investment is too volatile in our model economy without investment adjustment costs. Finally, Figure 3 shows that both capital and output overshoot their long-run levels, because initial deleveraging finances additional capital investment later on. This does not happen under rational expectations.

To measure how the leverage level matters for the response to a financial shock, we now calibrate the model using data from the first quarter of 1996, that is $\bar{\Theta} \approx 0.73$, $B/Y \approx 0.34$ and $QL/Y \approx 0.48$. According to most measures, this period corresponds to the starting point of the housing price “bubble”. The lower level of leverage implies that both the debt-to-GDP and the land value-to-GDP are correspondingly lower than their 2008Q4 levels. Figure 4 replicates the same experiment as above, when a -5% shock to leverage hits the economy and ρ_θ goes down from 1.015 to 0.98. Direct comparison of Figures 3 and 4 reveals that higher leverage increases the effect of the shock on aggregates by about 50% at impact under learning. In this sense, the larger the level of leverage the deeper the recession that follows after a negative financial shock. In addition, Figures 3 and 4 show that the recovery is quicker when the level of leverage is smaller.





4.2 Policy Implication: Elastic Leverage

In this section, we show that elastic leverage acts as an additional mechanism which further magnifies the impact of leverage shocks under learning. More precisely, we now incorporate into the model the feature that leverage responds to changes in the land price, which accords with the evidence documented by Mian and Sufi [20] on US micro data for the 2000s. To capture this, we now replace the borrowing constraint given by (3) with:

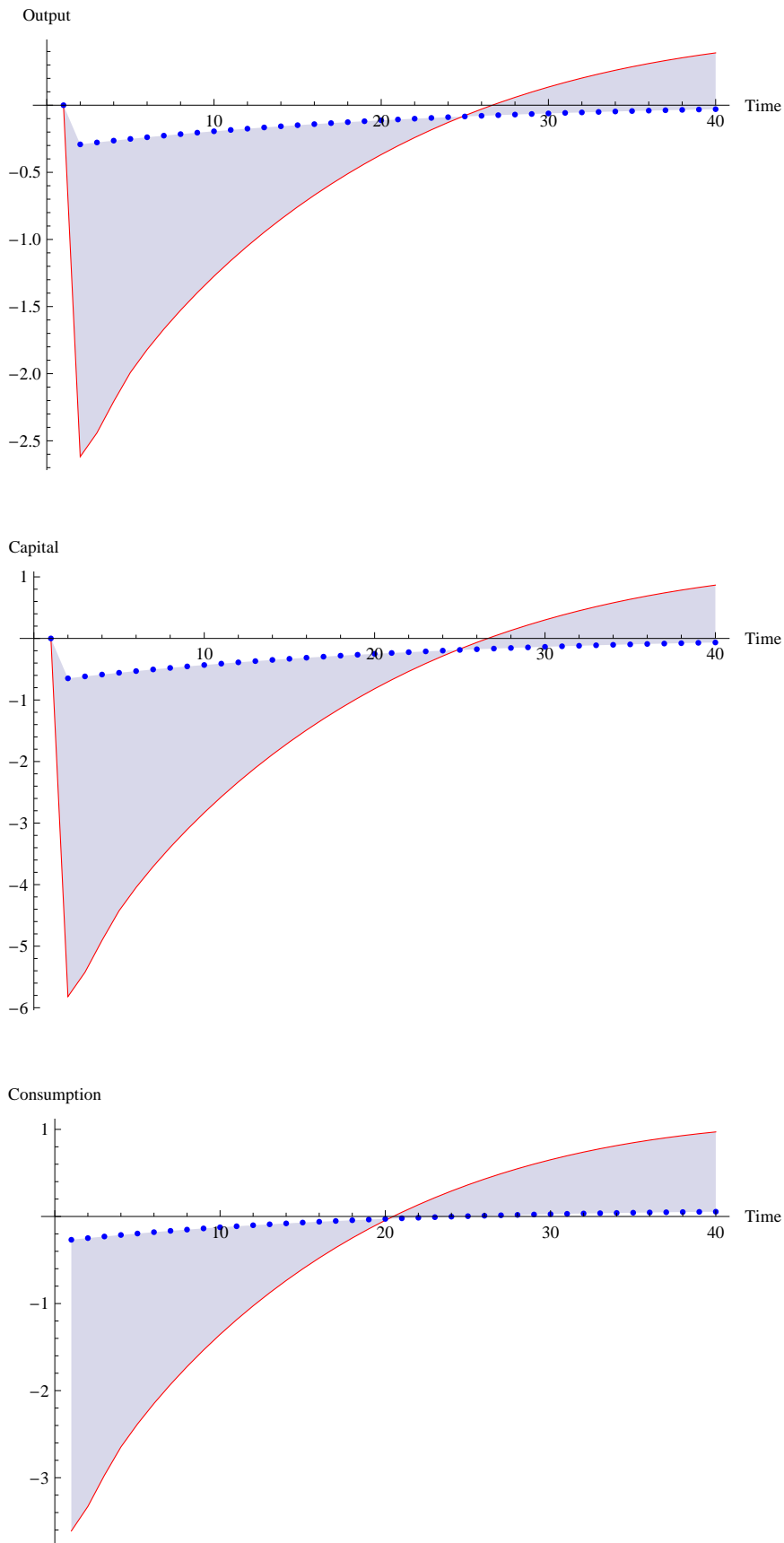
$$\tilde{\Theta}_t E_t[Q_{t+1}]L_{t+1} \geq (1 + R)B_{t+1} \quad (15)$$

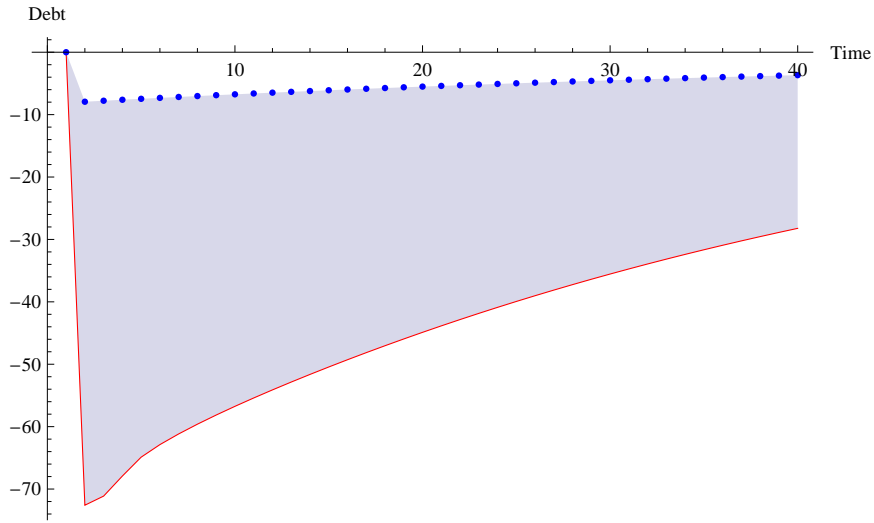
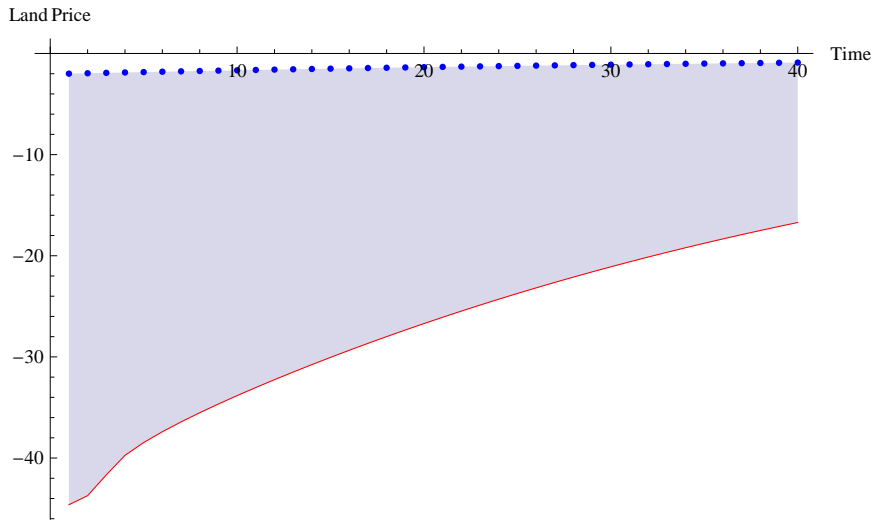
where

$$\tilde{\Theta}_t \equiv \Theta_t \left\{ \frac{Q_{t+1}}{Q} \right\}^\varepsilon \quad (16)$$

and Q is the steady-state value of land price. In Appendix A.3, we show how (16) can be derived in a simple setting with ex-post moral hazard and costly monitoring, similar to Aghion et al. [2]. In addition, the corresponding first-order conditions and linearized dynamics are derived in Appendix A.3. Following Midrigan and Philippon [21], we use $\varepsilon = 0.5$ as a benchmark value that is typical of the pre-Great Recession period. The other parameters are kept unchanged and set as in Table 2. The first observation is that, compared to the case $\varepsilon = 0$ presented in Section 4.1, the range of values for ρ_θ such that the REE is E-stable narrows. To ensure E-stability, we therefore assume that $\rho_\theta = 1.01$ prior to structural change which brings down ρ_θ to 0.98 again. Direct inspection of Figures 3 and 5 shows that the amplification of the -5% leverage shock is magnified by a factor greater than two under elastic leverage. This is because deleveraging is much more severe when the fall in housing prices contributes to decreasing further the leverage ratio, over and above the negative shock. In addition, procyclical leverage delays the recovery.

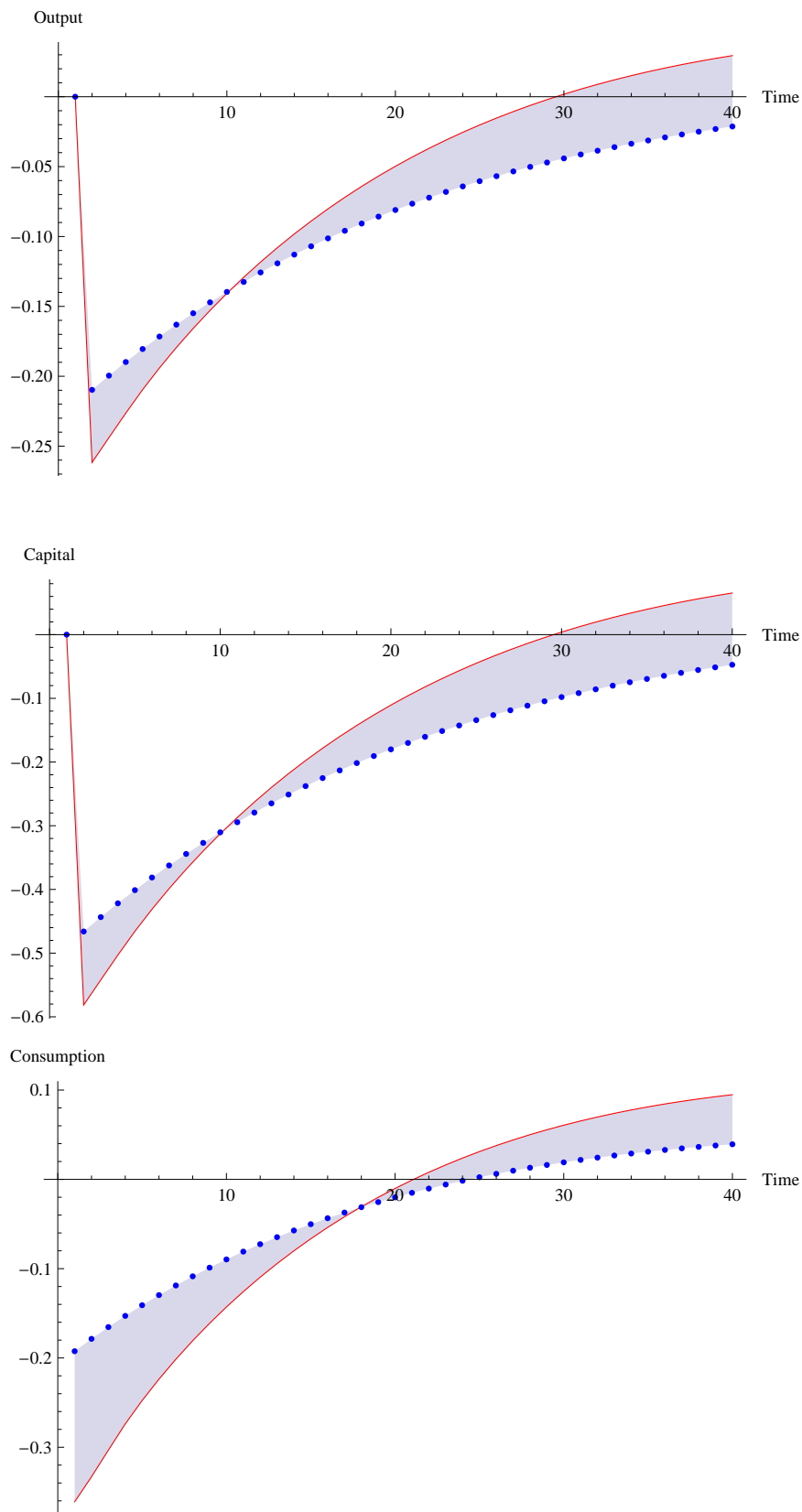
($\varepsilon = 0.5$ and other Parameter Values in Table 2; Learning: Red Solid Line; Rational Expectations: Blue Dotted Line)



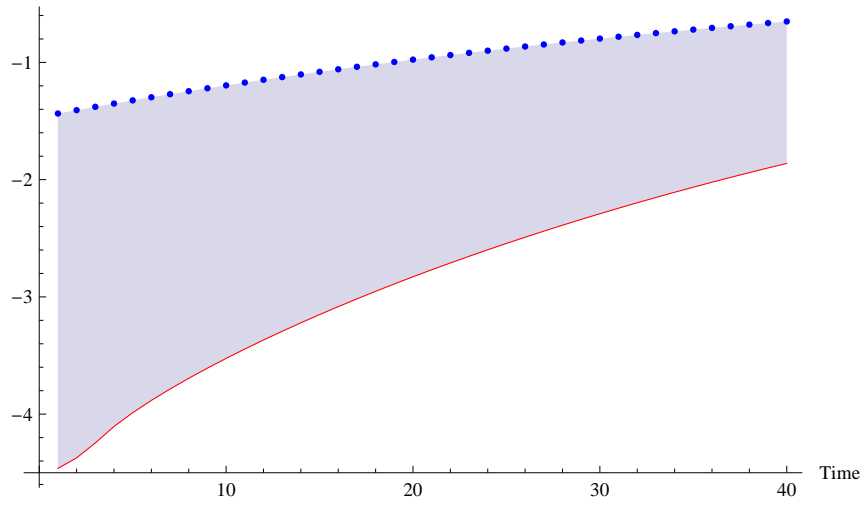


$(\varepsilon = -0.5$ and other Parameter Values in Table 2; Learning: Red Solid Line; Rational

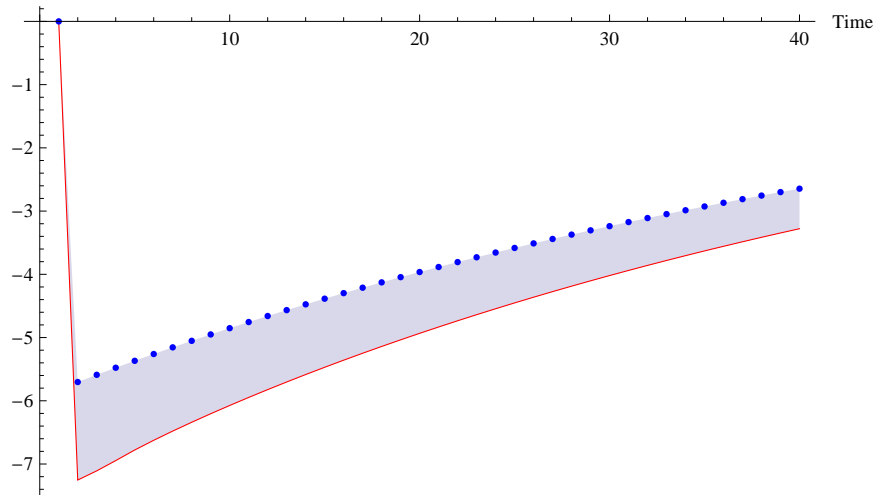
Expectations: Blue Dotted Line)



Land Price



Debt



We now ask the counterfactual question: what would be the reaction of the economy to the same shock, under the same parameter values but with the leverage being now mildly countercyclical and, in particular, $\varepsilon = -0.5$?⁴ The answer is depicted in Figure 6. Comparing Figures 5-6 shows that countercyclical leverage dampens by a significant margin the responses to financial shocks and it brings learning dynamics closer to its rational expectations counterpart. As a consequence, a much smaller recession follows a negative leverage shock: though agents anticipate a too large deleveraging effect because they overestimate the persistence of the adverse leverage shock, the land price fall now triggers an *increase* in countercyclical leverage, which dampens the impact of the negative shock.

4.3 Learning with a Misspecified Model

In this section, we explore the idea that forecasting agents may ignore important real/financial linkages. More precisely, we assume that when forming their beliefs and when estimating matrix \mathbf{M} in (10), agents set $\mathbf{M}(1,6) = \mathbf{M}(3,6) = \mathbf{M}(5,6)$. This means that they incorrectly believe that leverage shocks affect only financial variables (land price and debt) and not real variables (consumption and investment). Therefore, the reactions of land price and debt are not affected by this type of misspecification whereas the responses of consumption, capital and output are. A possible interpretation behind such a view could be that agents hold the belief that the effect of financial shocks are smoothed out through aggregation so that they do not matter for aggregate real variables.

We set parameter values as in Table 2 and now experiment with structural change

⁴This feature could possibly be enforced by appropriate regulation of credit markets. Alternatively, Appendix A.3 shows how it arises if government uses procyclical taxes.

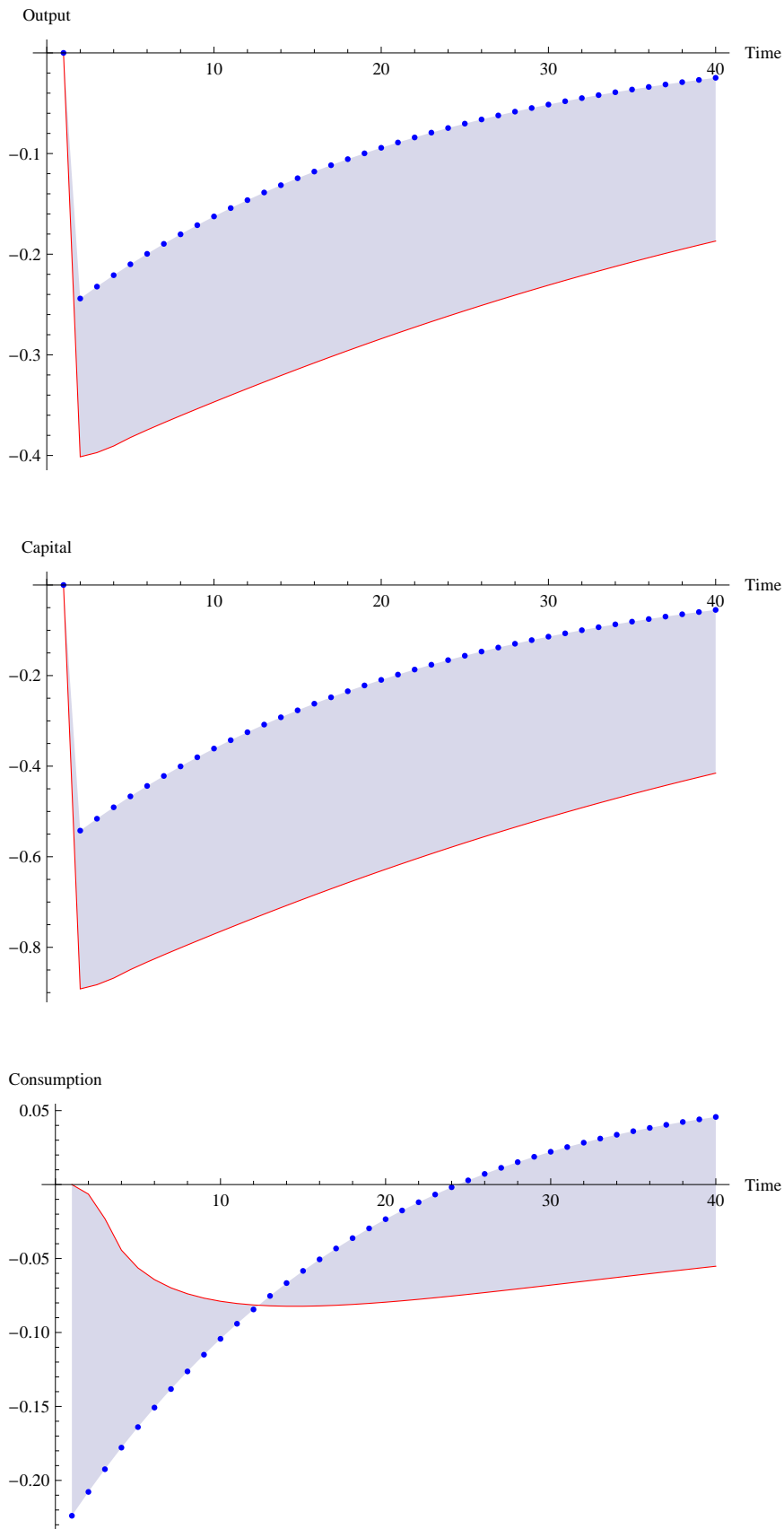
that decreases ε from 0.5 to 0 and ρ_θ from 1 to 0.98. That is, agents incorrectly believe both that leverage follows a unit root and that land price is procyclical when structural change strikes. The responses are reported in Figure 7, which differs from Figure 3 in two important ways. First, not surprisingly, the reaction of consumption is now hump-shaped and exhibits more persistence. This is because agents do not take into account that leverage shocks affect consumption directly. In consequence, investment is more volatile. Second, there is no more overshooting and the recession is more persistent: the recovery occurring in Figure 3 after about 24 quarters does not show up in Figure 7. Under our formulation of model misspecification, consumption is more sluggish so that investment is more volatile when the economy is hit by a leverage shock. In that way, the impact of leverage shocks on output is amplified and more persistent under learning.

5 Robustness Analysis

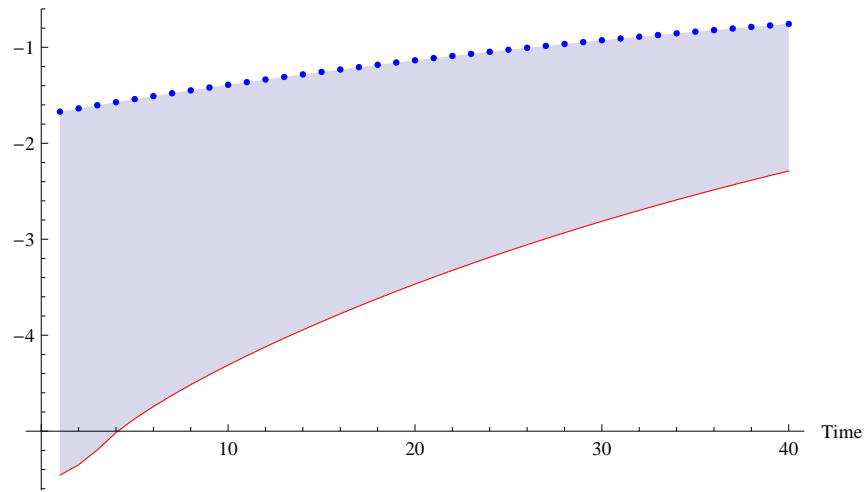
5.1 Alternative Assumptions

To assess the robustness of the findings reported in Section 4.1, we now relax two assumptions. First, we depart from logarithmic utility and we allow σ to take on values that are larger or smaller than one. Second, we adopt the timing assumption that is implied by the margin requirement interpretation of the borrowing constraint (Aiyagari and Gertler [3]). That is, borrowing is limited to the current market value of collateral, as opposed to tomorrow's market value. In other words, we replace (3) by $\Theta_t Q_t L_{t+1} \geq (1 + R)B_{t+1}$. In addition, we go back to the case such that $\varepsilon = 0$.

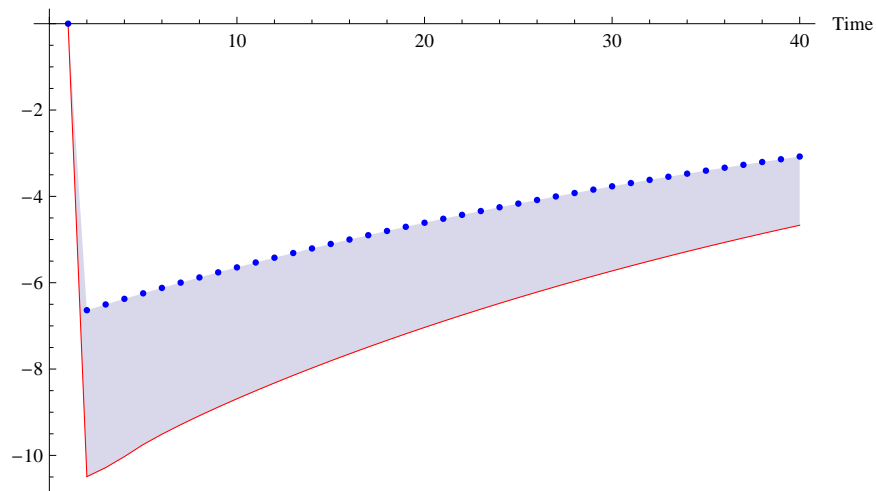
Figure 7: Responses to a -5% Leverage Shock under Model Misspecification (Learning: Red Solid Line; Rational Expectations: Blue Dotted Line; Parameter Values as in Table 2)



Land Price



Debt



In Table 3, we report the output amplification variation that obtains under learning, compared with the rational expectations equilibrium. For example, the impact of a -5% leverage shock on output's deviation (from its steady-state value, in percentage terms) is about -1.01 percentage points under learning and -0.24 percentage points under RE (see Figure 3) when parameters are set according to Table 2. Therefore, the first column of Table 3 reports that the difference is, in absolute value, $|\Delta y| \approx 0.77$. Similarly, the second and third columns report $|\Delta y|$ when all parameter values are set according to Table 2, except for risk aversion σ which equals 0.5 and 3, respectively. Finally, the last column in Table 3 reports $|\Delta y|$ in the margin requirement model.

Table 3. Output Amplification Gain Under Learning

Benchmark	$\sigma = 0.5$	$\sigma = 3$	Margin Model
0.77 pp	0.75 pp	0.79 pp	0.70 pp

Direct inspection of Table 3 shows that our main findings are robust both to changes in the utility function's curvature and to an alternative timing assumption. Output amplification is quantitatively similar across all different models and this turns out to be the case for the other variables as well. In addition, how the numbers change in Table 3 accords with intuition. First, under the timing assumed in (3), incorrect beliefs about the economy further amplify shocks because land price forecasts are temporarily deviating from RE. In the margin model where the borrowing limit depends on today's collateral market values, forecast errors are slightly less important during deleveraging episodes. In addition, larger risk aversion implies that consumption will fall by less and, therefore, that investment will fall by more at impact, which means that output will also fall by more.

5.2 Interest Rate News Shocks

We finally focus on interest rate movements as an alternative source of financial shocks (e.g. Uribe and Yue [23]). Keeping the leverage ratio constant and allowing the interest rate to follow an AR(1) process leads to results that qualitatively mimic those obtained under leverage shocks. The only difference is quantitative, as the learning dynamics differ by an even bigger margin from rational expectations equilibria under interest rate shocks. We do not report those similar results and explore a new avenue by supposing, instead, that the interest rate R_t is now subject to news shocks, as follows. In period t , agents expect next period's interest rate to go down. However, such news does not materialize and the interest rate that effectively holds at $t + 1$ stays constant at $R = 1/\mu - 1$, the same level as before. In contrast to previous sections, leverage is set to its steady-state value $\bar{\Theta}$ and it is no longer subject to random shocks so that the borrowing constraint becomes:

$$\bar{\Theta}Q_tL_{t+1} \geq E_t[1 + R_{t+1}]B_{t+1} \quad (17)$$

We focus on the margin requirement formulation such that the current land price appears in the collateral constraint (17). In that formulation, financial news shocks that are not realized still affect aggregate variables at t because they affect the land price Q_t and the borrowing limit in (17). The linearized equations of the model under interest rate news shocks are given in Appendix A.5.

Figure 8 reports the pseudo-impulse response functions after an interest rate news shock, when parameter values are set according to Table 2. In period 1, a news that the interest factor $1 + R$ will go down by -1% in period 2 hits the economy. However, such news does not materialize. Instead, agents do not know that structural change operates a permanent increase of steady-state leverage $\bar{\Theta}$ from 1.26 to 1.6 at the time the news is spread. This means that agents' beliefs incorrectly embody a steady-state

level of leverage that is lower than what it actually is.⁵ Figure 8 shows that such a news shock under structural change affect consumption and investment in period 1, because land price goes up and the borrowing constraint is relaxed. This implies that output increases in period 2 though there is no temporary shock to the interest rate. The main differences between learning and rational expectations dynamics are as follows. Under learning, consumption spikes in period 2 when agents incorrectly believe that leverage is low. As a consequence, they curtail investment and a recession follows after period 3, when agents know for sure that the good news does not materialize. Such a recession does not occur under rational expectations. Note that consumption and investment comove, as in Jaimovich and Rebelo [13].

6 Conclusion

In this paper, we attempt to capture the idea that the financial sector dynamics may originate structural change that has large macroeconomic effects. In a simple business-cycle model with collateral constraint, we replace rational expectations with adaptive learning. Following a structural change in the economy agents gradually learn that this structural change took place rather. We focus on financial shocks that drive up and down the leverage ratio determining how much household and firms can borrow.

Our main finding is that leverage shocks are amplified when agents gradually learn that the structural economy reverts to a regime where the changes in the leverage level are no longer permanent. We show that the magnitude of the consequent recession may in part be attributed to the high level of leverage and the correspondingly high level of the debt-to-GDP ratio. Our model indicates that financial shocks to leverage cause larger

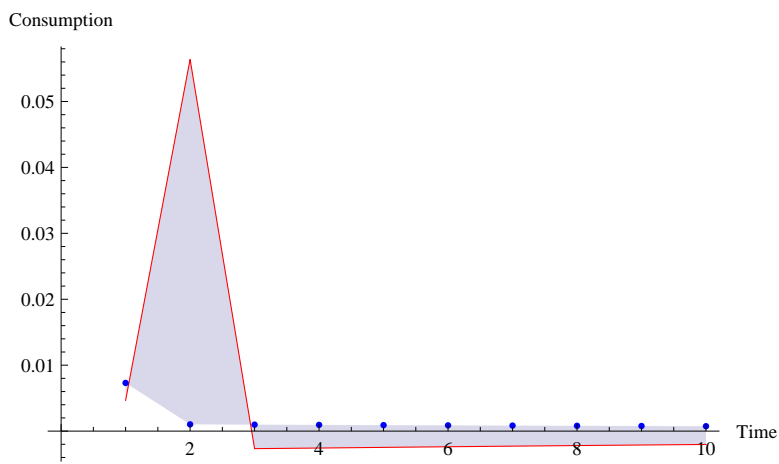
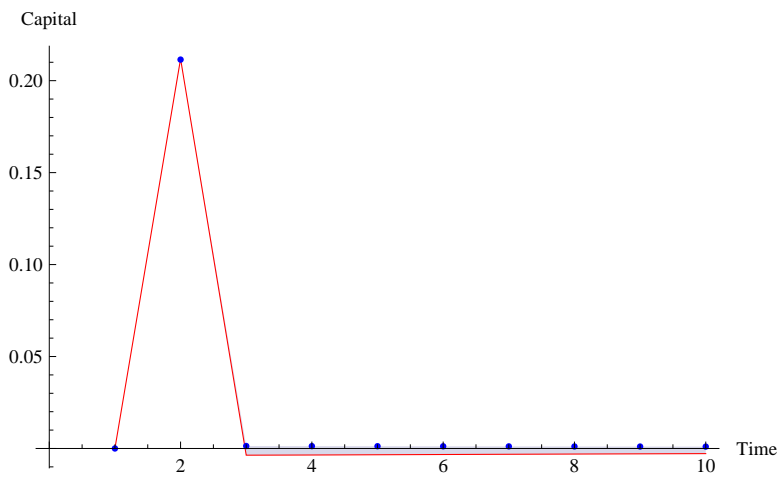
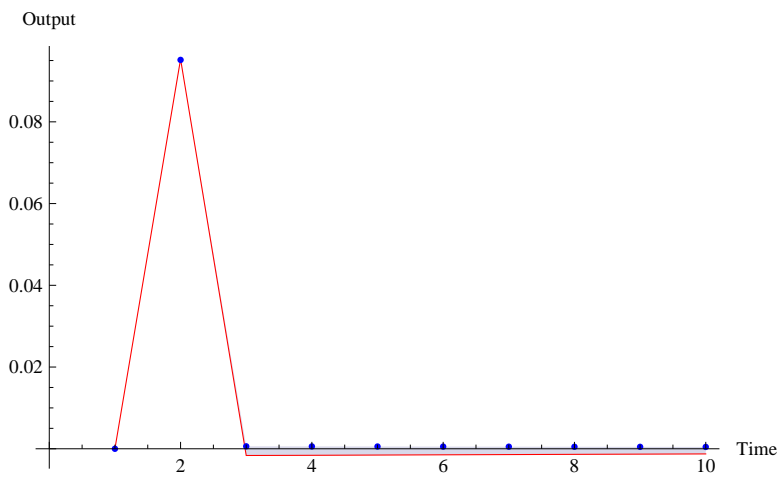
⁵As learning is applied to deviations from steady state, agents do not learn levels.

aggregate volatility in economies that are more levered.

In addition, we also show that procyclical leverage may act as an aggravating factor: under the assumption that leverage responds positively to increases in land price, the recession is further worsened. In contrast, the counterfactual experiment with countercyclical leverage shows dampened effects of leverage shocks. One possible interpretation of this finding is that macro-prudential policies enforcing countercyclical leverage have potential stabilizing effects on the economy in the face of financial shocks, at zero cost provided that non-distortionary policies are implemented (e.g. through regulation).

Finally, we illustrate both how learning with a misspecified model that ignores real/financial linkages also contributes to magnify financial shocks and how interest rate news shocks are propagated under learning.

Rational Expectations: Blue Dotted Line; Parameter Values as in Table 2)



A Appendix

A.1 Intertemporal Equilibria around Steady State

This section derives the linearized version, in percentage deviations from steady-state values, of the set of equations (2)-(7) defining, together with the leverage law of motion $\Theta_t = \bar{\Theta}^{1-\rho\theta} \Theta_{t-1}^{\rho\theta} \Xi_t$, local intertemporal equilibria. In all equations below, x_t denotes the deviation of X_t from its steady-state value in percentage terms. For example, $k_t \equiv (K_t - K)/K$, where K is the steady-state capital stock. Eliminating ϕ_t by using (7), one gets the following linearized equations corresponding to (2)-(7), respectively:

$$\frac{K}{Y}k_t - \frac{B}{Y}b_t = -\frac{C}{Y}c_{t-1} - \frac{(1+R)B}{Y}b_{t-1} + \left(\alpha + (1-\delta)\frac{K}{Y}\right)k_{t-1} \quad (18)$$

$$b_t = E_{t-1}[q_t] + \theta_{t-1} \quad (19)$$

$$c_t = -\lambda_t/\sigma \quad (20)$$

$$\begin{aligned} q_t + \lambda_t(1 - \mu\bar{\Theta}) &= E_t[\lambda_{t+1}] \left(\beta(1 - \bar{\Theta}) + \gamma\beta\frac{Y}{Q} \right) + E_t[q_{t+1}](\beta + \bar{\Theta}(\mu - \beta)) \\ &+ \alpha\gamma\beta\frac{Y}{Q}E_t[k_{t+1}] + \theta_t\bar{\Theta}(\mu - \beta) \end{aligned} \quad (21)$$

$$\lambda_t = E_t[\lambda_{t+1}](\beta(1 - \delta) + \alpha\beta\frac{Y}{K}) + \alpha\beta(\alpha - 1)\frac{Y}{K}E_t[k_{t+1}] \quad (22)$$

$$\theta_t = \rho\theta_{t-1} + \xi_t \quad (23)$$

Define $P_t \equiv (b_t \ k_t \ \theta_t)'$ and $S_t = (c_t \ q_t \ \lambda_t)'$ the vectors of predetermined and jump variables, respectively. Then equations (18)-(23) can be decomposed into two subsystems, each pertaining to P_t and S_t . The first block is composed of (18), (19) and (23) and can be written:

$$M_0P_t = M_1S_{t-1} + M_2E_{t-1}[S_t] + M_3P_{t-1} + V\xi_t \quad (24)$$

where:

$$M_0 = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{B}{Y} & \frac{K}{Y} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{C}{Y} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$M_3 = \begin{pmatrix} 0 & 0 & 1 \\ -(1+R)\frac{B}{Y} & \alpha + (1-\delta)\frac{K}{Y} & 0 \\ 0 & 0 & \rho_\theta \end{pmatrix} \text{ and } V = (0 \ 0 \ 1)'$$

The second block (20)-(22) can be written:

$$M_4 S_t = M_5 E_t[S_{t+1}] + M_6 P_t + M_7 E_t[P_{t+1}] \quad (25)$$

where:

$$M_4 = \begin{pmatrix} 0 & 1 & 1 - \mu \bar{\Theta} \\ 0 & 0 & 1 \\ \sigma & 0 & 1 \end{pmatrix}, \quad M_5 = \begin{pmatrix} 0 & \beta + \bar{\Theta}(\mu - \beta) & \beta(1 - \bar{\Theta}) + \gamma\beta\frac{Y}{Q} \\ 0 & 0 & \beta(1 - \delta) + \alpha\beta\frac{Y}{K} \\ 0 & 0 & 0 \end{pmatrix},$$

$$M_6 = \begin{pmatrix} 0 & 0 & \bar{\Theta}(\mu - \beta) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M_7 = \begin{pmatrix} 0 & \alpha\gamma\beta\frac{Y}{Q} & 0 \\ 0 & \alpha\beta(\alpha - 1)\frac{Y}{K} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Finally, substituting the expression of P_t from (24) in (25) and piling up the resulting two block of equations allows one to rewrite the system as:

$$X_t = \mathbf{A}X_{t-1} + \mathbf{B}E_{t-1}[X_t] + \mathbf{C}E_t[X_{t+1}] + \mathbf{D}\xi_t \quad (26)$$

where $X_t = \text{vec}(S_t \ P_t)$ and:

$$\mathbf{A} = \begin{pmatrix} M_4^{-1}M_6M_0^{-1}M_1 & M_4^{-1}M_6M_0^{-1}M_3 \\ M_0^{-1}M_1 & M_0^{-1}M_3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} M_4^{-1}M_6M_0^{-1}M_2 & O_3 \\ M_0^{-1}M_2 & O_3 \end{pmatrix},$$

$$\mathbf{C} = \begin{pmatrix} M_4^{-1}M_5 & M_4^{-1}M_7 \\ O_3 & O_3 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} M_4^{-1}M_6M_0^{-1}.V \\ M_0^{-1}.V \end{pmatrix}$$

where O_3 is a 3-by-3 zeroes matrix.

A.2 Closed-Economy Model with Constant Interest Rate

The purpose of this appendix is to show that, similar to the open-economy model developed in Section 2, the debtor interest rate is constant over time in a closed-economy version with domestic borrowers and lenders, when the preferences of the latter are appropriately chosen.

Let us now assume that lenders are domestic agents (instead of foreign countries as in Section 2), whose unique role is to provide loans to borrowers. Following Iacoviello [12], lenders derive utility from consumption and land holdings, and they get interest income from last period's loan payments. As discussed in Pintus and Wen [22], lenders may be interpreted as financial intermediaries. The representative lender take decisions that solve:

$$\max E_0 \sum_{t=0}^{\infty} \mu^t \left\{ \frac{(C_t^l)^{1-\sigma_c} - 1}{1-\sigma_c} + \psi \frac{(L_t^l)^{1-\sigma_l} - 1}{1-\sigma_l} \right\} \quad (27)$$

with σ_c, σ_l, ψ all strictly greater than zero and $\mu \in (0, 1)$, subject to the budget constraint:

$$C_t^l + Q_t(L_{t+1}^l - L_t^l) + B_{t+1} = (1 + R_t)B_t \quad (28)$$

where C_t^l and L_t^l denotes the lender's consumption and land holdings, respectively, Q_t is the land price, B_{t+1} is the new loan. The interest rate R_t is now endogenous and it is determined by the equality between the demand and supply of loans.

The first-order conditions obtained from (27)-(28) with respect to consumption, land, and lending are, respectively:

$$(C_t^l)^{-\sigma_c} = \chi_t \quad (29)$$

$$\chi_t Q_t = \mu E_t[\chi_{t+1} Q_{t+1}] + \mu \psi (L_{t+1}^l)^{-\sigma_l} \quad (30)$$

$$\chi_t = \mu E_t[\chi_{t+1}(1 + R_{t+1})] \quad (31)$$

where χ_t is the Lagrange multiplier of constraint (28) in period t .

Assuming that lenders' utility is linear in consumption (that is, $\sigma_c = 0$), one gets from

(29) that in any rational expectations equilibrium $\chi_t = 1$ for all $t \geq 0$ so that, in view of (31), the interest factor is constant and given by $1 + R = 1/\mu$. As in the small-open economy model developed in Section 2, the interest rate is constant over time.

The borrower side of the model is still described by (1), (2) and (3), as in Section 2, with the addition that the total amount of land is now divided between lenders and borrowers according to:

$$L_t + L_t^l = \bar{L}.$$

where \bar{L} is the fixed supply of land. How exactly is land divided depends on both the sequence of land price and the lender's preferences, as reflected in the first-order condition (30). In addition, the representative borrower's first-order conditions are given by (4)-(7). As in Section 2, if $\mu \in (\beta, 1)$, then the borrower's credit constraint (3) is binding. Therefore, the main difference is that the closed-economy model allows some reallocation of land from lenders to borrowers when a shock hits the economy. Under our calibration (see Table 2), however, the effect of land reallocation is quantitatively unimportant because the land share γ is reasonably small. We have run simulations for the rational expectations versions of the open and closed economies and we have confirmed that the impulse-response functions of the variables involved in Section 2 are quantitatively similar under TFP shocks. In particular, the land price and debt behaves in the same way in both economies.

A.3 Model with Elastic Leverage

This section derives some simple micro-foundations for the assumption of elastic leverage captured in (16). The case when leverage is procyclical (that is, $\varepsilon > 0$) obtains in a setting with ex-post moral hazard and costly monitoring similar to Aghion et al. [2, p.1391]. Suppose that the borrower has wealth QL and has access to investment oppor-

tunities, which can be financed by credit in the amount B . If the borrower repays next period, his income is $I - (1 + R)B$, where I is whatever income was generated by investing. If the borrower defaults next period, his income is now $I - pQL$, assuming that he loses his collateral with some probability p , which represents for example the frequency of foreclosures. Strategic default is avoided provided that $I - (1 + R)B \geq I - pQL$, that is, if $pQL \geq (1 + R)B$. The lender incurs a cost $C(p)L$ of going to court, with $C'(p) > 0$ and $C''(p) > 0$, and he chooses the optimal monitoring policy by solving:

$$\max_p pQL - C(p)L \quad (32)$$

which gives $Q = C'(p)$. The higher the land price, the larger the incentives to increase effort to collect collateral. Assuming now that the cost function is $C(p) = \phi p^{1+1/\varepsilon} / (1+1/\varepsilon)$, with $\varepsilon > 0$, gives that $p = (Q/\phi)^\varepsilon$. Setting the scaling parameter $\phi = Q^* \Theta^{-1/\varepsilon}$, where Q^* is steady-state land value and Θ is leverage, gives (16). Therefore, ex-post moral hazard leads to procyclical leverage.

In contrast, countercyclical leverage obtains if government implements procyclical taxes as follows. Suppose now that the lender gets $(1 - \tau)pQL - C(p)L$ when monitoring, where $1 \geq \tau \geq 0$ is the tax rate. Under the assumption that the cost function is isoelastic, the optimal p is now $p = ((1 - \tau)Q/\phi)^\varepsilon$. If government set time-varying taxes such that $1 - \tau = (Q/\phi)^{-\eta/\varepsilon - 1}$, for some $\eta \geq 0$, then it follows that $p = (Q/\phi)^{-\eta}$ and that leverage is countercyclical. Note that this happens provided that the tax rate goes up when the land price goes up.

The first-order conditions of the model with elastic leverage are derived from (1), (2) and (15)-(16) (which replaces (3) as the new borrowing constraint). Because only the borrowing constraint is modified, it follows that the first-order condition (5) with respect to land is modified and becomes:

$$\lambda_t Q_t = \beta E_t[\lambda_{t+1} Q_{t+1}] + \beta \gamma E_t[\lambda_{t+1} Y_{t+1} / L_{t+1}] + \phi_t \tilde{\Theta}_t E_t[Q_{t+1}] \quad (33)$$

As a consequence, only (19) and (21) are modified in the linearized equations (18)-(23) and become:

$$b_t = (1 + \varepsilon)E_{t-1}[q_t] + \theta_{t-1} \quad (34)$$

$$\begin{aligned} q_t + \lambda_t(1 - \mu\bar{\Theta}) &= E_t[\lambda_{t+1}] \left(\beta(1 - \bar{\Theta}) + \gamma\beta\frac{Y}{Q} \right) + E_t[q_{t+1}](\beta + \bar{\Theta}(1 + \varepsilon)(\mu - \beta)) \\ &+ \alpha\gamma\beta\frac{Y}{Q}E_t[k_{t+1}] + \theta_t\bar{\Theta}(\mu - \beta) \end{aligned} \quad (35)$$

Accordingly, only the following matrices appearing in (24)-(25) are modified:

$$M_2 = \begin{pmatrix} 0 & 1 + \varepsilon & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, M_5 = \begin{pmatrix} 0 & \beta + \bar{\Theta}(1 + \varepsilon)(\mu - \beta) & \beta(1 - \bar{\Theta}) + \gamma\beta\frac{Y}{Q} \\ 0 & 0 & \beta(1 - \delta) + \alpha\beta\frac{Y}{K} \\ 0 & 0 & 0 \end{pmatrix}.$$

The matrices **A**, **B**, **C**, **D** in (26), however, are unchanged.

A.4 Model with Interest Rate News Shocks

The first-order conditions of the model with interest rate news are derived from (1), (2) and (17) (instead of (3)). It follows that the first-order condition (5) with respect to land is modified and becomes:

$$\lambda_t Q_t = \beta E_t[\lambda_{t+1} Q_{t+1}] + \beta \gamma E_t[\lambda_{t+1} Y_{t+1} / L_{t+1}] + \bar{\Theta} \phi_t Q_t \quad (36)$$

Finally, replacing $\phi_t = \lambda_t / E_t[1 + R_{t+1}] - \beta \lambda_{t+1}$ and noting r the percentage deviation of the gross interest rate $1 + R$, it follows that (18), (19) and (21) are modified in the linearized equations (18)-(23) and become:

$$\frac{K}{Y}k_t - \frac{B}{Y}b_t = -\frac{C}{Y}c_{t-1} - \frac{(1+R)B}{Y}(r_{t-1} + b_{t-1}) + \left(\alpha + (1 - \delta)\frac{K}{Y} \right) k_{t-1} \quad (37)$$

$$E_{t-1}[r_t] + b_t = q_{t-1} \quad (38)$$

$$\begin{aligned} q_t(1 - \mu\bar{\Theta}) + \lambda_t(1 - \mu\bar{\Theta}) &= E_t[\lambda_{t+1}] \left(\beta(1 - \bar{\Theta}) + \gamma\beta\frac{Y}{Q} \right) + E_t[q_{t+1}]\beta(1 - \bar{\Theta}) \\ &+ \alpha\gamma\beta\frac{Y}{Q}E_t[k_{t+1}] - \mu\bar{\Theta}E_t[r_{t+1}] \end{aligned} \quad (39)$$

Only the following matrices appearing in (24)-(25) are modified:

$$M_0 = \begin{pmatrix} 1 & 0 & 1 \\ -\frac{B}{Y} & \frac{K}{Y} & 0 \\ 0 & 0 & 1 \end{pmatrix}, M_3 = \begin{pmatrix} 0 & 0 & 0 \\ -(1+R)\frac{B}{Y} & \alpha + (1-\delta)\frac{K}{Y} & -(1+R)\frac{B}{Y} \\ 0 & 0 & \rho_R \end{pmatrix}$$

$$M_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, M_7 = \begin{pmatrix} 0 & \alpha\gamma\beta\frac{Y}{Q} & -\mu\bar{\Theta} \\ 0 & \alpha\beta(\alpha-1)\frac{Y}{K} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Defining now $P_t \equiv (b_t \ k_t \ r_t)'$, the matrices **A**, **B**, **C**, **D** in (26) are left unchanged.

A.5 Time-Varying Persistence of Leverage Shocks in the Data

To be written.

References

- [1] Adam, K., Kuang, P., Marcet, A. (2011). House price booms and the current account. NBER Working Paper 17224.
- [2] Aghion, P., Banerjee, A., Piketty, T. (1999). Dualism and macroeconomic volatility. *Quarterly Journal of Economics* 114: 1359-1397.
- [3] Aiyagari, R., Gertler, M. (1999). "Overreaction" of asset prices in general equilibrium. *Review of Economic Dynamics* 2: 3-35.
- [4] Boz, E., Mendoza, E. (2010). Financial innovation, the discovery of risk, and the U.S. credit crisis. NBER Working Paper 16020.

- [5] Bullard, J., Duffy, J. (2004). Learning and structural change in macroeconomic data. St Louis Fed Working Paper 2004-016.
- [6] Cao, D. (2011). Collateral shortages, asset price and investment volatility with heterogeneous beliefs. Mimeo Georgetown University.
- [7] Chaney, T., Sraer, D., Thesmar D. (2010). The collateral channel: how real estate shocks affect corporate investment. NBER Working Paper 16060 and *American Economic Review*, forthcoming.
- [8] Cordoba, J.-C., Ripoll, M. (2004). Credit cycles redux. *International Economic Review* 45: 1011-1046.
- [9] Eusepi, S., Preston, B. (2011). Expectations, learning and business cycle fluctuations. *American Economic Review*, forthcoming.
- [10] Evans, G., Honkapohja, S. (2001). *Learning and expectations in macroeconomics*. Princeton University Press.
- [11] Geanakoplos, J. (2009). The leverage cycle. In Acemoglu, D., Rogoff, K., Woodford, M., (Eds.), *NBER Macroeconomic Annual* 24: 1-65.
- [12] Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review* 95: 739-764.
- [13] Jaimovich, N., Rebelo, S. (2008). News and business cycles in open economies. *Journal of Money, Credit and Banking* 40: 1699-1711.
- [14] Jermann, U., Quadrini, V. (2009). Macroeconomic effects of financial shocks. NBER Working Paper 15338 and *American Economic Review*, forthcoming.
- [15] Kiyotaki, N., Moore, M. (1997). Credit cycles. *Journal of Political Economy* 105: 211-248.

- [16] Kocherlakota, N. (2000). Creating business cycles through credit constraints. *Federal Reserve Bank of Minneapolis Quarterly Review* 24: 2-10.
- [17] Kuang, P. (2011). Imperfect knowledge about asset prices and credit cycles. Mimeo University of Frankfurt and University of Mannheim.
- [18] Marcet, A., Sargent, T. (1989). Convergence of least squares learning mechanisms in self-referential linear stochastic models. *Journal of Economic Theory* 48: 337-368.
- [19] Mendoza E. (2010). Sudden stops, financial crises and leverage. *American Economic Review* 100: 1941-1966.
- [20] Mian, A., Sufi, A. (2009). House prices, home equity-based borrowing, and the U.S. household leverage crisis. NBER Working Paper 15283 and *American Economic Review*, forthcoming.
- [21] Midrigan, V., Philippon, T. (2011). Household leverage and the recession. NBER Working Paper 16965.
- [22] Pintus, P., Wen, Y. (2010). Leveraged borrowing and boom-bust cycles. St Louis Fed Working Paper 2010-027.
- [23] Uribe, M., Yue, V. (2006). Country spreads and emerging countries: who drives whom? *Journal of International Economics* 69: 6-36.
- [24] Williams, N. (2003). Adaptive learning and business cycles. Princeton University mimeo.