Policy Distortions and Aggregate Productivity with Endogenous Establishment-Level Productivity[†]

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Abstract

The large differences in income per capita across countries are mostly explained by differences total factor productivity (TFP). What explains these differences in TFP across countries? Evidence suggests that the (mis)allocation of factors of production across heterogenous production units is an important factor. We study factor misallocation across countries in a model with an endogenously determined distribution of establishment-level productivity. In this framework, policy distortions not only misallocate resources across a given set of productive units, but also worsens the distribution of establishment-level productivity as observed in the cross-country data. We show that in this model compared to the model with an exogenously specified distribution of establishment-level productivity, the quantitative effect of a given policy distortion is substantially amplified, by a 6-fold factor for empirically plausible policy-distortion configurations. Moreover, the implications of the model for average establishment size and the dispersion of establishment-level productivity are more in line with cross-country evidence.

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1 Introduction

A crucial question in economic growth and development is why are some countries rich and others poor. A consensus has emerged in the literature whereby the large differences in income per capita across countries are for the most part explained by differences in labor productivity and in particular total factor productivity (TFP).[1] What explains differences in TFP across countries? A recent literature has emphasized the (mis)allocation of factors across heterogenous production units as an important factor.[2] We study factor misallocation across countries in a model with an endogenously determined distribution of establishment-level productivity. In this framework, policy distortions not only misallocate resources across a given set of productive units, but also worsens the distribution of establishment-level productivity as observed in the cross-country data. We show that in this model compared to the model with an exogenously specified distribution of establishment-level productivity, the quantitative effect of a given policy distortion is substantially amplified, by a 6-fold factor for empirically plausible amounts of misallocation. Moreover, the implications of the model regarding establishment size and the dispersion of establishment-level productivity are more in line with cross-country evidence.

We develop a framework with heterogeneous production units that builds on Hopenhayn (1992) and Restuccia and Rogerson (2008). The framework is extended to allow for an endogenous determination of the distribution of establishment-level productivity. We use this framework to study the impact of policy distortions on misallocation and aggregate measured productivity. There is a continuum (mass one) of homogeneous households with standard preferences over consumption goods. Households accumulate physical capital and supply inelastically to the market their endowment of one unit of labor time. The key

[1]See for instance Klenow and Rodriguez-Clare (1997), Prescott (1998), and Hall and Jones (1999).

[2]See Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Guner, Ventura, and Xu (2008), and Hsieh and Klenow (2009). See also surveys of the literature in Hopenhayn (2011), Restuccia and Rogerson (2013), and Restuccia (2013b).

elements of the model are on the production side. One good is produced in each period. The production unit is the establishment. An establishment has access to a decreasing returns to scale production function with capital and labor as inputs. Establishments are heterogenous with respect to total factor productivity, but differently from the standard framework, the distribution of establishment-level productivity is not exogenously specified, rather it is determined by endogenous investment by the establishment and the properties of the economic environment such as policy distortions. In other words, entering establishments draw their initial level of productivity from an invariant distribution which is determined endogenously in the model. Establishments are subject to an exogenous exit rate of λ. Following the literature, the economy faces a set of policy distortions which we assume for simplicity take the form of output taxes τ on individual producers. That is, each producer faces an idiosyncratic tax and it is the properties of the policy distortions that generate misallocation in the model. Revenues collected from these taxes are rebated back to the households as a lump-sum transfer.

We characterize the solution of this model in continuous time in closed form. To assess the quantitative properties of the model relative to the existing and related literature, we calibrate the model and provide a set of relevant quantitative experiments. We consider a benchmark economy that faces no distortions, i.e., an economy whose policy distortion process implies no taxes on individual producers ($\mu_\tau = 0, \sigma_\tau = 0$), and calibrate the parameters of this economy to data for the United States. Most parameters share the usual targets in the literature, for instance in Restuccia and Rogerson (2008). The parameters that require some discussion are the productivity investment cost, the variance in the distribution of establishment level productivity, and the value added growth rate of establishments. These parameters are targeted to data on the aggregate growth rate of TFP, the average employment growth of establishments, and the right tail index of the share of employment distribution in the US data. We then perform quantitative analysis by exploring the implications of this model with different specification for the policy distortions, i.e., different values for (μ_τ, σ_τ). Our main result is that the quantitative effect of policy distortions on aggregate TFP is substantially larger than that of the model with an exogenous distribution of establishment-level productivity. That is, the endogenous distribution of establishments amplifies the quantitative effect of distortions on aggregate TFP. This amplification effect is more than 6-fold. In particular, configurations of policy distortions that are reasonable compared to empirical measures of misallocation such as those estimated for China, India, and Mexico in Hsieh and Klenow (2009, 2012) generate an aggregate TFP that is 14 percent of that of the benchmark economy, whereas the same policy distortions imply an 87 percent aggregate TFP in the model with an fixed exogenous distribution of productivity. In addition, we show both analytically and quantitatively, that the model generates implications for the distribution of establishments productivity, the employment growth of establishments over time, the number and size of establishments in the economy that are consistent with the available cross-country data. To contrast with the standard framework, we show that these implications are only generated when the distribution of establishment productivity is endogenously determined.

Our paper is related to a large and growing literature on misallocation and productivity. By studying the aggregate impact of policy distortions across countries our paper is closely linked to that of Restuccia and Rogerson (2008) and the related literature. By endogenizing the distribution of establishment productivity our paper is closely related to the analysis in Restuccia (2013a), Bello et al. (2011), Ranasinghe (2013), Bhattacharya et al. (2013), Gabler and Poschke (2013), and Hsieh and Klenow (2012). The key difference between all these papers and ours is that the initial distribution of productivity is exogenous in the previous literature whereas it is endogenous in our paper.

The paper proceeds as follows. In the next section, we describe the model. Section 4 calibrates a benchmark economy with no distortions to data for the United States. In section 5 we perform a series of quantitative experiments to assess the impact of policy distortions on aggregate TFP and other relevant statistics. We conclude in section 6.

2 Economic Environment

We consider a standard version of the neoclassical growth model as in Restuccia and Rogerson (2008), however, we allow establishments to invest in their own productivity as in Bhattacharya et al. (2013). Time is continuous and the horizon is infinite. Establishments have access to a decreasing return to scale technology, they pay an one time fixed cost to entry and they die at an exogenous rate. Establishments hire labor and rent capital in a competitive market. New entrants draw their productivity from an endogenous distribution. We study a balanced growth path, in which the economy grows at an exogenous rate. We then analyze policy distortions that affect the allocation of factors across establishments, the investment on productivity of establishments, and therefore, aggregate measured TFP and output. We also compare the effect of the same policy distortions in an environment where the initial distribution of productivity is exogenous. In what follows we describe the environment in more details.

2.1 Baseline Model

There is an infinity-lived representative household with preferences over consumption goods described by the utility function,

$$\max \int_0^\infty e^{-\delta t} u(c) dt$$

where c is consumption and δ is the discount rate. The household is endowed with one unit of productive time and $k_0 > 0$ units of the capital stock at date 0.

The unit of production in the economy is the establishment. Each establishment is described by a production function $f(z, k, n)$ that combines capital service k and labor service n to produce output. The function f is assumed to exhibit decreasing returns to scale in capital and labor jointly and to satisfy the usual Inada conditions. The production function is given by:

$$y = z^{2(1-\alpha-\gamma)} k^{\alpha} n^{\gamma}, \quad \alpha, \gamma \in (0,1), \quad , \quad 0 < \gamma + \alpha < 1.$$

Establishment's productivity z follows a Brownian motion. Establishments can invest in upgrading their productivity z by choosing the drift of the Brownian motion x_z. The establishement productivity evolves according to the Brownian motion:

$$dz = x_z dt + \sigma_z z dw_z,$$

where ${x_z}$[3] is the drift and it is endogenous, σ_z is the standard deviation, and $dz = \sqrt{dt}$ is the standard Wiener process of the Brownian motion. We assume that investing in upgrading productivity is costly to establishments, and this cost is in output and is described by a cost function $q(\cdot)$, that is increasing and convex in productivity upgrading x_z. Establishments also face an exogenous probability of death λ.

New establishment can also be created. Entrants must pay an entry cost n_e measured in labor units. After paying this cost a realization of the initial establishment level productivity z is drawn from a distribution, where the pdf is represented by $g(\cdot)$. We assume that there exist an unlimited mass of potential entrants. Feasibility in the model requires:

$$C + I = Y - Q$$

[3] x_z is the policy function and we will show later that z will follow a Geometric Brownian Motion.

where C is aggregate consumption, I is aggregate investment, Y is aggregate output, and Q is the total cost of investing in productivity.

2.2 Policy Distortions

We next introduce policies that create idiosyncratic distortions to establishment-level decisions. These policies distort both the allocation of resources and the investment in productivity. Specifically, we assume that each establishment faces its own policy distortion. In what follows next, we will simply refer to this distortion as output tax. We will use τ to refer to the establishment-level output tax rate.

We assume that the output taxes τ follows a Brownian motion,

$$d\tau = \mu_\tau \tau dt + \sigma_\tau \tau dw_\tau$$

where μ_τ is the drift, σ_τ is the standard deviation, and $d\tau = \sqrt{dt}$ is the standard Wiener process of the Brownian motion. For simplicity, we assume that the output tax process and the productivity process are uncorrelated, that is $E(dw_\tau, dw_z) = 0$.

The output tax affects the establishment decision of investing in productivity by changing the drift of the Brownian motion that governs the productivity process. In the presence of output taxes the establishment level productivity follows the resulting Brownian motion:

$$dz = \frac{x_z}{\tau} z dt + \sigma_z z dw_z.$$

At the time of entry, the establishment-level output tax rate τ is not known, its value is revealed after the establishment draws its productivity z. From the entering establishment perspective, the relevant information is the joint distribution over these pairs, that is, over the output-tax and over the productivity. We denote $g(z, \tau)$ the join distribution, where we define $s = z \times \tau$.

A given distribution of establishment-level output tax and productivity may not lead to a balanced budget for the government. As a result, we assume that budget balance is achieved by either lump-sum taxation or redistribution to the representative household. We denote the lump-sum tax by T.

3 Equilibrium

We focus on the stationary balanced growth path where the number of establishments grows at an exogenous rate $\eta = \frac{dM/dt}{M}$. The stationary balanced growth path is characterized by an invariant joint distribution of establishments' productivity z and output tax τ, $g(\cdot, \cdot)$, and a mass of establishments M. In equilibrium the mass of establishments grows over time, however; the rank of the establishments' size distribution is constant. In the stationary balanced growth path, the rental prices for labor and capital services are constant, and we denote them by w and r, respectively. Before defining the stationary balanced growth path formally, it is useful to consider the decision problems faced by incumbents, entrants, and consumers. We start describing the incumbent's problem.

3.1 Incumbent establishment's problem

Incumbent establishments maximize the present value of their profit. They make two decisions: a static one and a dynamic one. The static decision is to hire labor services and to rent capital, and the dynamic decision is to invest in upgrading their productivity. In what follows next, we describe the static problem and then the dynamic one.

3.1.1 Incumbent establishment's static problem

At any instant of time an establishment chooses how much capital to rent k and how much labor to employe n. This decision is static, and solely depends on establishment's productivity level z, the establishment's output tax τ_y, the rental rate of capital r and wages w.

Formally, the instant profit function $\pi(z, \tau_y)$ is defined by:

$$\pi(z, \tau_y) = \max_{k,n} (1 - \tau_y) y - wn - rk.$$

From which we obtain the optimal demand for capital and labor:

$$n(z, \tau) = \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\gamma}{w}\right)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\gamma}} \tau^2 z^2 \tag{1}$$

$$k(z, \tau) = \left[\left(\frac{\alpha}{r}\right)^{1-\gamma} \left(\frac{\gamma}{w}\right)^{\gamma} \right]^{\frac{1}{1-\alpha-\gamma}} \tau^2 z^2 \tag{2}$$

where the output tax τ is defined as $\tau = (1 - \tau_y)^{\frac{1}{2(1-\alpha-\gamma)}}$. For future calculations, we redefine the instant profit function as a function of the optimal factors demanded:

$$\pi(z, \tau) = m(w, r) \tau^2 z^2 \tag{3}$$

where $m(w, r) = (1 - \alpha - \gamma) \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\gamma}{w}\right)^{\gamma} \right]^{\frac{1}{1-\alpha-\gamma}}$. In what follows we describe the incumbent establishments' dynamic problem.

3.1.2 Incumbent establishment's dynamic problem

Incumbent establishments choose how much to invest in upgrading their productivity x_z. The cost of investing in upgrading productivity is in output and is described by the cost function

$q(\cdot)$, this function is increasing and convex in the productivity parameter x_z, specifically $q(x_z) = c_\mu \frac{x_z^2}{2}$. The optimal decision of upgrading productivity is characterized by maximizing the present value of profits, subject to the Brownian motion governing the evolution of productivity and the Brownian motion governing the evolution of output taxes. Formally, incumbent establishments solve the following dynamic problem:

$$W(z,\tau) = \max_{x_z} \left\{ m(w,r)z^2\tau^2 - q(x_z) + \frac{1}{1+\lambda+R} E_{z,\tau} W(z+dz, \tau+d\tau) \right\}$$

$$\begin{aligned} s.t. \quad dz &= \frac{x_z}{\tau} dt + \sigma_z z dw_z \\ d\tau &= \mu_\tau \tau dt + \sigma_\tau \tau dw_{\tau,} \end{aligned}$$

where λ is the exogenous probability of death and R is the rental rate of capital r minus the depreciation rate δ_k. Next, we define the Hamilton-Jacobi-Bellman of the stationary solution,

$$(\lambda+R)W(z,\tau) = \max_{x_z} \left\{ m(w,r)\tau^2 z^2 - \frac{c_\mu}{2} x_z^2 + \frac{x_z}{\tau} W_z' + \frac{\sigma_z^2}{2} z^2 W_{zz}'' + \mu_\tau \tau W_\tau' + \frac{\sigma_\tau^2}{2} \tau^2 W_{\tau\tau}'' \right\}.$$

From the first order conditions, we find that the optimal investment rate x_z is a function of the output tax τ, the investment cost c_μ, and the marginal present value profits W_z',

$$x_z = \frac{W_z'}{c_\mu \tau}.$$

By guessing and verifying, we find that the optimal Hamilton-Jacobi-Bellman equation is given by $W(z,\tau) = A(w,r)z^2\tau^2$, where the constant $A(w,r)$ is the solution of the polynomial:

$$\frac{A(w,r)^2}{c_\mu} - \left[\frac{\lambda + R}{2} - \frac{\sigma_z^2}{2} - \mu_\tau - \frac{\sigma_\tau^2}{2}\right] A(w,r) + \frac{m(w,r)}{2} = 0. \tag{4}$$

There are two possible solutions of this polynomial, and below we restrict the solution to the negative root. This root has the desired property that an increase in profits $m(w,r)$ increases the present value of an operating establishment. In the following Lemma 1 we characterize formally the value function of operating incumbents.

Lemma 1. *Given an output tax τ, a productivity level z, and operating profits $m(r,w)$, the value function that solves the establishment dynamic problem is given by $W(z,\tau) = A(w,r)\tau^2 z^2$, where the constant $A(w,r)$ is characterized by the following expression:*

$$A(w,r) = \left(\Theta - \sqrt{\Theta^2 - \frac{2m(r,w)}{c_\mu}}\right)\frac{c_\mu}{2}$$

where,

$$\Theta = \frac{\lambda + R}{2} - \frac{\sigma_z^2}{2} - \mu_\tau - \frac{\sigma_\tau^2}{2}.$$

Then expected "innovation" growth rate, follows Gibrath's law

$$\frac{dz}{z} = \frac{2A(w,r)}{c_\mu}dt + \sigma_z dw.$$

The proof of Lemma 1 is straightforward from equation (4). From Lemma 1, we find that the growth rate of productivity does not depend on the intrinsic characteristics of each individual establishment, that is it does not depend on the establishment productivity z neither on the output tax τ. As a consequence Gilbrath's law holds and productivity does not depend on establishment size, which is supported by empirical evidences.[4] Now that we have fully characterized the incumbent establishment's problem, we are ready to discuss the entrant's

[4]For more detail Luttmer (2010) is a good reference.

establishment problem.

3.2 Entering establishment's problem

Potential entering establishments make their entry decision knowing that they face a distribution over potential draws for the pairs (z, τ), and the entry cost n_e, which is measured in labor units. Therefore, the expected value of an entering establishment is equal to

$$W_e = \int_{z\times\tau} W(z,\tau)g(z\times\tau)d(z\times\tau) - wn_e.$$

In an equilibrium with entry, W_e must be equal to zero, otherwise additional establishments would enter. The condition $W_e = 0$ is thus referred to as free-entry condition.

3.3 Invariant distribution of establishments

Now that we have characterized the optimal decision of incumbents and entering establishments, we are ready to find the invariant joint distribution of productivity level z and output tax τ, $g(\cdot,\cdot)$. The first step to characterize this distribution is to rewrite the productivity z and the output tax τ Brownian motions as a function of s, where $s = \tau \times z$. The resulting s Brownian motion is given by:

$$\frac{ds}{s} = \mu dt + \sigma dw_s, \tag{5}$$

where the drift μ is equal to the sum of the drift of the output tax Brownian Motion μ_τ and the drift of the productivity Brownian motion μ_z. It is important to remember that the drift of the productivity Brownian motion μ_z is an endogenous object and is given by the solution of the incumbent establishment's dynamic problem, $\mu_z = \frac{2A(w,r)}{c_\mu}$. The variance σ^2

of the s Brownian motion is the sum of the variance of the output tax Brownian Motion σ_τ^2 and the variance of the productivity Brownian motion σ_z^2

In order to characterize the invariant distribution, it is useful to rewrite the model in *logs*. Let x denote the logarithm of the establishment size s, that is $x = log(s)$. Now we can rewrite the Geometrican Brownian motion (5) as a Brownian motion in the logarithm of the establishment size$s$,

$$dx = \hat{\mu} dt + \hat{\sigma} dw_x,$$

where $\hat{\mu} = (\mu_z + \mu_\tau) - \frac{1}{2}(\sigma_z^2 + \sigma_\tau^2)$, and $\hat{\sigma}^2 = \sigma_z^2 + \sigma_\tau^2$. Let $F(x,t)$ be the measure of producing establishments at time t with characteristics x. $F(x,t)$ satisfies the Kolmogorov forward equation (also known as the Fokker-Planck equation), associated with the Brownian process given by:

$$\frac{\partial F(x,t)}{\partial t} = -\hat{\mu} \frac{\partial F(x,t)}{\partial x} + \frac{\hat{\sigma}^2}{2} \frac{\partial^2 F(x,t)}{\partial x^2} - \lambda F(x,t). \tag{6}$$

The Kolmogorov forward equation describes the evolution over time of the distribution of operating establishments, taking into account the exogenous probability of death λ. Let $M(\cdot)$ denote the mass of establishments operating at time t, that is $M(\cdot)$ is the integral of all establishment operating at time t,

$$M(t) = \int_0^\infty F(x,t) dx.$$

$M(t)$ describes the evolution of the mass of operating establishments over time. Next, we rewrite the measure of operating establishment with characteristic x at time t, as the product of the mass of establishment operating at time t, $M(t)$, and the probability density of establishments with characteristic x at time time t, $f(x,t)$, that is

$$F(x,t) = M(t) f(x,t), \tag{7}$$

where the probability density satisfies the usual property that $\int_0^\infty f(x,t)dx = 1$.

We are interested in finding the invariant distribution, that is the probability density function that is independent of time, let $f_\infty(x)$ denote such invariant distribution, and note that on the stationary balanced growth path we have that:

$$F_\infty(x) = Mf_\infty(x).$$

That is the measure of establishment with characteristic x is a linear function of the mass of establishment M and the time invariant probability density $f_\infty(x)$. As a result, on the balanced growth path the mass of establishment grows, however the rank is fixed. Now we are ready to characterize the invariant distribution $g(\cdot)$.

Lemma 2. *Given wages w and the rental rate of capital r, the stationary distribution $g(s)$ is a Pareto distribution*

$$g(s) = (\xi - 1)s^{-\xi},$$

where the tail index of the pareto distribution is given by

$$\xi = -\left(\frac{\mu}{\sigma^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{4}\right)^2 + \frac{2(\eta+\lambda)}{\sigma^2}},$$

and the net entry rate is given by $\varepsilon = \eta + \lambda - (\mu - \frac{1}{2}\sigma^2 + \frac{3\sigma^2}{2}\xi)\xi$.

The poof of Lemma 2 is on the appendix, from this Lemma we characterize the invariant joint distribution of productivity z and output tax τ. Next, we can characterize all other aggregate variables and their dynamics as a function of the invariant distribution. Aggregate output is given by the mass of establishments multiplied by the establishment output probability density's. The establishment output's probability density can be found using the same methodology as in Lemma 2. From our definition of output, we have $y = z^{2(1-\alpha-\gamma)}k^\alpha n^\gamma$, substituting the demand for labor, equation (1), and for capital equation (2), and integrating over all operating establishment, we obtain aggregate output:

$$Y = M\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\int_1^{\infty} s_Y ds_Y$$

where $s_Y = z^2 \times \tau^{2(\alpha+\gamma)}$. Let $F_Y(s_Y, t)$ denote the measure of establishment with characteristic s_Y at time t. We can follow exactly the same methodology as in lemma 2 to characterize the invariant distribution of output. This invariant distribution is also a Pareto distribution,

$$g_Y(s_Y) = (\xi_Y - 1)s^{-\xi_Y},$$

where the tail index ξ_Y is given by,

$$\xi_y = -\left(\frac{\mu_y}{2\sigma_y^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu_y}{2\sigma_y^2} - \frac{1}{4}\right)^2 + \frac{(\eta+\lambda)}{2\sigma_y^2}}.$$

It is important to notice that the tail index of the invariant distribution of output has the same functional form of the tail index of the invariant distribution of s. However, the drift μ_Y and the standard deviation σ_Y are now a function of the brownian motion of productivity z and output tax τ.[5] Next, we present the consumer's problem.

[5]Using Ito lemma's we find that the Brownian motion that governs the process s_Y is given by $\frac{dF}{F} = (\alpha+\gamma)\left[\mu_\tau - (1-\alpha-\gamma)\sigma_\tau + \frac{\mu_z}{\alpha+\gamma}\right]dt + \left[(\alpha+\gamma)\sigma_\tau + \sigma_z\right]dw_F$. As a result, it is straightforward to see that the drift of this brownian motion μ_Y is equal to $\mu_y = (\alpha+\gamma)\left[\mu_\tau - (1-\alpha-\gamma)\sigma_\tau + \frac{\mu_z}{\alpha+\gamma}\right]$ and the standard deviation is given by $\sigma_y = (\alpha+\gamma)\sigma_\tau + \sigma_z$.

3.4 Consumer's problem

The consumer in our model seeks to maximize lifetime utility subject to the law of motion of their wealth given by:

$$(RK + w + T + \Pi - c)\, dt,$$

where w is the rental price of labor, R is the rental price of capital minus capital depreciation ($R = r - \delta_k$), T is the lump-sum tax levied by the government, Π is the total profit from the operations of all establishment, and c is consumption.

We assume that consumers have log utility ($u(c) = log(c)$), and we characterize the equilibrium interest rate by solving the consumer's problem. We define total wealth as:

$$a = K + \frac{w}{R} + \frac{T}{R} + \frac{\Pi}{R}$$

and we rewrite the wealth law of motion as:

$$da = (Ra - c)dt.$$

The consumer solves the following Hamilton-Jacobi-Bellman equation:

$$\delta V(a) \quad = \quad \max_{c} \left\{ \log(c) + \left[Ra - c \right] V'(a) \right\}.$$

In Lemma 3 we show that in the balanced growth path the interest rate R is equal to the discount rate δ.

Lemma 3. *In the balanced growth path the interest rate is equal to the discount rate $R = \delta$.*

In the next section, we define the equilibrium.

3.5 The stationary balanced growth path

Definition A stationary balanced growth path, with an exogenous growth rate of establishment $\eta = \frac{dM/dt}{M}$, for this economy is an invariant productivity distribution, $g(\cdot)$, a mass of firms M, prices, r and w, a productivity growth rate μ_z, a value function for incumbents $W(s)$, a value function for entrants $W_e(s)$, and policy functions k, n and y, such that:

i) (Consumer optimization) $\delta = c/a$, and $r - \delta_k = \delta$.

ii) (Plant optimization) $W(s, w, r) = A(w, r)s^2$ solve incumbent problem, and $k(s, w, r)$, $n(s, w, r)$ and $\tilde{y}(s, w, r) = (1 - \tau_y)y$ and $x_z(s, w, r)$ are optimal policy functions.

iii) (Endogenous productivity growth rate) equal to

$$\mu_z = \frac{2A(w, r)}{c_\mu}$$

iv) (Free-entry)

$$\int_1^\infty W(s, w, r)g(s)ds = w \times n_{entry}$$

v) (Endogenous entry) rate equal to

$$\varepsilon = \eta + \lambda - (\mu - \frac{1}{2}\sigma^2 + \frac{3\sigma^2}{2}\xi)\xi$$

vi) (Market clearing)

$$1 - \varepsilon \times M \times n_{entry} = M \int_1^\infty n(s, w, r)g(s)ds$$

$$K = M \int_1^\infty k(s, w, r)g(s)ds$$

$$C + I + T = Y - \int_1^\infty c_\mu \frac{x_z^2(s)}{2} g(s)ds$$

vii) (Government budget balance)

$$T = M \int_1^{\infty} \tau_y y g(s) ds$$

viii) ($g(s)$ is an invariant distribution)

$$g(s) = (\xi - 1)s^{-\xi},$$

and $s = \tau \times z$.

The equilibrium is a fix point in measures. The interest rate r is constant and equal to sum of the discount rate δ and the depreciation of capital δ_k. Therefore, solving the equilibrium is to find the productivity μ_z, the right tail index ξ, the entry rate ε, the mass of firms M and wage w that solve the optimal decision of investing in upgrading productivity, satisfy the invariant distribution, satisfy net entry and free entry, and clear the labor market.

3.6 Exogenous TFP distribution

The main difference between our model and the previous models in the literature is that in our model new entrants draw their productivity from the invariant distribution, which is endogenous, while in the previous literature new entrants draw their productivity from an exogenous distribution. In order to quantify how much the endogenous distribution contributes to explain differences in TFP, we need to develop a version of the model where the distribution of TFP is exogenous. We follow Restuccia and Rogerson (2008) and Bhattacharya et al. (2013), and set the exogenous distribution as the distribution generated by the model without policy distortions, that is when $\mu_\tau = \sigma_\tau = 0$. Applying Lemma 2 to the case where there is no policy distortions, $\mu_\tau = \sigma_\tau = 0$, we obtain the endogenous distribution of the undistorted economy, which is a Pareto distribution with tail index ξ_z given by:

$$\xi_z = -\left(\frac{\mu_z}{\sigma_z^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu_z}{\sigma_z^2} - \frac{1}{4}\right)^2 + \frac{2(\eta+\lambda)}{\sigma_z^2}}.$$

We assume that the distribution of productivity z and the distribution of output taxes τ are uncorrelated. Again from the point of the view of the establishment the important distribution is the joint distribution of productivity z and output taxes τ. However, now the distribution of TFP is fixed. In order to calculate the invariant joint distribution $h(\cdot)$ this time, we apply a similar methodology as in Lemma 2. The following Lemma characterize the invariant joint distribution. of output taxes.

Lemma 4. *Let η be the plants growth rate along the balance growth path and let λ be the exit rate. Then, given, $\tau \in (0, 1]$, the stationary distribution, $h(\tau)$, is a Pareto distribution,*

$$h(\tau) = (\xi_\tau + 1)\tau^{\xi_\tau},$$

where the tail index ξ_τ is given by:

$$\xi_\tau = \left(\frac{\mu_\tau}{2\sigma_\tau^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu_\tau}{2\sigma_\tau^2} - \frac{1}{4}\right)^2 + \frac{(\eta+\lambda)}{2\sigma_\tau^2}}.$$

Now, that we have characterized both the invariant distribution of productivity $g(\cdot)$ and the invariant distribution of output tax $h(\cdot)$, we can characterize the joint distribution of productivity z and output tax τ, $f(\cdot)$.

Lemma 5. *Assume that productivity distribution is taken as exogenous, Then rank size firm distribution is exogenous and given by ξ_z.*

Proof Given that the productivity z and output tax τ are independent stochastic variables, we know that the join distribution of productivity z and output tax τ, $f(\cdot)$ is the product of the distribution of productivity $g(\cdot)$ and the distribution of output tax $h(\cdot)$. The joint distribution $f(\cdot)$

is given by:

$$f(\tau, z) = z^{-\xi_z} \tau^{\xi_\tau}.$$

Now we can find the probability density function of $s = \tau \times z$ by changing variables:

$$f(s) = \int_0^1 \left(\frac{s}{\tau}\right)^{-\xi_z} \tau^{\xi_\tau} d\tau = \frac{s^{-\xi_z}}{(\xi_z + \xi_\tau) + 1}.$$

The joint invariant distribution is also a Pareto distribution with tail index ξ_z. ■

From Lemma 5 the Gini coefficient of the establishment size distribution is independent of output tax. The definition of equilibrium for this economy is almost the same as the previous one. However, the main distribution is that the distribution of productivity is exogenous.

4 Calibration

We calibrate the benchmark economy with no distortions (i.e., $\mu_\tau = \sigma_\tau = 0$) to data for the United States. Our main objective is to study the quantitative impact of policy distortions on aggregate TFP and GDP per worker in an economy that is relatively more distorted than the United States in the same spirit of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).

We start by selecting a set of parameters that are standard in the literature so these parameters have either well known targets which we match or the values have been well discussed in the literature. Following the literature we assume decreasing returns in the establishment-level production function and set $\alpha + \gamma = 0.85$ (e.g., Restuccia and Rogerson, 2008). Then we split it between α and γ by assigning 1/3 to capital and 2/3 to labor, implying $\alpha = 0.283$ and $\gamma = 0.567$. We select the annual exit rate λ to 10 percent from the estimates of exit rates in the literature (e.g., Davis and Haltiwanger (1996)).

We calibrate all other parameters by solving the equilibrium of the model and making sure the equilibrium statistics match some targets. All these parameters are selected simultaneously but some parameters have first order impact on some targets so we discuss them in turn. The establishment exogenous growth rate η is calibrated to match the growth rate of TFP, which in the United States is equal to 2 percent in the last 100 years. We use the property of the model that the aggregate growth rate of TFP over time is proportional to the growth rate of establishment, which is given by:

$$\eta = \frac{dM/dt}{M} = \frac{dTFP/dt}{TFP} \frac{1}{(1-\alpha-\gamma)} = \frac{0.02}{0.15} = 0.0946.$$

We calibrate the investment cost c_μ to match the employment growth rate, that we calculate from the model. Employment in the model is given by the solution of the establishment static problem, more precisely equation (1),[6] as a result, in the model the growth rate of employment also follows a Brownian motion:

$$\frac{dn}{n} = (2\mu + \sigma^2)dt + 2\sigma dw,$$

where in the case with no distortions $\mu = \mu_z$ and $\sigma = \sigma_z$. The drift μ_z is a function of the cost of investing in productivity c_μ. We calibrate c_μ to match the establishment's employment growth rate, which is approximately equal to two percent, according to Hsieh and Klenow (2012).

The volatility of the productivity process σ_z is calibrated to match the Gini coefficient of the

[6] From the static problem of establishments, we find the labor demand, equation 1, which is given by $n(z,\tau) = m_n \tau^2 z^2$ where $m_n = \left[\left(\frac{\alpha}{r}\right)^\alpha \left(\frac{\gamma}{w}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha-\gamma}}$. So, n follows a geometric Brownian motion: $\frac{dn}{n} = (2\mu + \sigma^2)dt + 2\sigma dw$.

The employment stationary distribution $g_N(\cdot)$ is a Pareto distribution. From the invariant distribution $g(s) = (\xi - 1)s^{-\xi}$ we obtain the distribution of employment $g_N(\cdot)$. Let a be equal $a = (\xi+1)/2 - 1$, $g_N(\cdot)$ is a monotonic transformation of the invariant distribution $g(\cdot)$. Thus, employment stationary distribution is given by: $f(n) = \frac{(\xi+1)}{2} n^{-(\xi+1)/2}$.

Therefore, we can calculate the Gini index of the establishment distribution, which is equal to $Gini(n) = \frac{1}{(\xi-2)}$.

establishment size distribution in the United States. We calculate the Gini coefficient from the employment invariant distribution $f(\cdot)$. Applying the same methodology as in Lemma 2, we calculate the invariant distribution of employment:

$$f(n) = \frac{(\xi+1)}{2} n^{-(\xi+1)/2}.$$

Therefore, we can calculate the Gini index, which is equal to $Gini(n) = \frac{1}{(\xi-2)}$ and Luttmer (2010) finds that is equal to 1.06 to the Unite State.

We calibrate the discount rate δ to match a long term interest rate of four percent. The depreciation of capital δ_k is calibrated to match the capital output ratio to 2.3, and we calibrate the entry cost η_e to normalize the mass of establishments to one. The parameters value are summarized at Table 1, and in the next section we perform the comparative static exercise with respect to policy distortions.

Table 1: Benchmark Calibration to US Data

Parameter	Value	Target
σ_z	0.03	Gini coefficient
c_μ	21,280	Employment growth rate
n_e	21.548	Normalization of M
η	0.09	TFP growth rate
δ	0.04	Interest rate
δ_k	0.0069	Capital output ratio

5 Quantitative Experiments

We quantify the impact of policy distortions on aggregate TFP, aggregate output, and productivity investment and other relevant statistics by comparing these statistics in each distorted economy relative to the no distortions benchmark economy. We highlight the quantitative impact of policy distortions in our model relative to the model where the distribution of TFP is exogenous as this is the basic framework in the existing literature. We show that endogenous establishment-level TFP not only amplifies the quantitative impact of policy distortions in aggregate TFP, but also generates differences in average establishment size across economics that is broadly consistent with cross country evidence.

The quantitative experiments are divided in three parts. First, we study the impact of policy distortions on the economy by comparing statistics of this economy relative to the benchmark economy. More precisely, we quantify the impact of changes in the deterministic component of policy distortions μ_τ and of changes in the stochastic component σ_τ, where the stochastic component is misallocation. Second, we quantify the contribution to these results of the endogenous component of the distribution of establishment-level TFP by comparing the predictions of our model to the model where the distribution of TFP is exogenous. Third, we quantify the contribution in our results of general equilibrium effects on wages relative to the benchmark economy.

5.1 Policy Distortions

We study the quantitative impact of changes in the deterministic component of policy distortions μ_τ and in the stochastic component σ_τ on the economy and report statistics relative to the undistorted benchmark economy.

To start, we set μ_τ to 0 and consider variations in σ_τ. We report the results in Table 2 for a number of statistics such as aggregate output, TFP, mass of entry, average establishment size, investment, wages, and measures of the dispersion of establishment level productivity (TFPQ)[7] as well as a summary measure of distortions (TFPR).[8]

All statistics reported are relative to the benchmark economy in percent except for the

[7] The analytical expression of TFPQ is on the paper appendix

[8] The analytical expression of TFPR is on the paper appendix

standard deviation of log TFPR which is 0 in the economy with no distortions. For this statistic we compare the difference in the standard deviation of log TFPR across economies as a gauge of the extent of distortions in that economy relative to the available evidence for some countries.

Table 2: Effects of changes in σ_τ ($\mu_\tau = 0$)

	σ_τ					
Variable	0.05	0.10	0.25	0.50	0.75	1.00
Relative Y	81.6	69.0	47.2	30.1	20.7	14.0
Relative TFP	81.6	69.0	47.3	30.3	21.0	14.5
Relative E	99.98	99.9	99.6	98.4	96.5	93.8
Relative establishment size s	100.1	100.4	102.7	111.7	130.4	168.2
Relative investment μ_z	99.9	99.7	97.9	91.5	80.8	65.8
Relative wages w	100.0	100.1	100.5	102.2	105.3	111.6
Relative SD(logTFPQ)	100.0	100.0	99.9	99.4	98.7	97.7
SD(logTFPR) percent	0.1	0.3	0.7	1.4	2.1	2.9

As is apparent from Table 2, policy distortions that generate misallocation through variations in σ_τ have a substantial impact on aggregate output and aggregate TFP. For instance, the economy with $\sigma_\tau = 1$ has an aggregate output and TFP that is around 14 percent of that of the benchmark economy (a 7-fold difference). Differences in output and TFP are increasing in the amount of distortions. More distorted, poorer economies have a higher average employment size since the mass of entry falls. We note that the substantial effect of policy distortions on TFP does not arise from unreasonable amounts of misallocation. To gauge that, following the literature we have computed the dispersion in TFPR as a summary measure of distortions in the economy. This measure of distortions is increasing in σ_τ but the most distorted economy has distortions that are 2.9 percentage points higher than the benchmark economy. The difference in the SD of log TFPR between China or India and the United States is between 14 and 29 percentage points depending on the year analyzed as reported by Hsieh and Klenow (2009).

We next look at economies that differ in σ_τ for a negative value of μ_τ. In particular, we consider $\mu_\tau = -0.31$. In Table 3 we report the results in this case and as a reference for our choice of μ_τ, we note that the combination of μ_τ with σ_τ=2.5 generates an economy that reproduces the GDP per worker and the establishment size distribution of Mexico.[9]

Table 3: Effects of changes in σ_τ ($\mu_\tau = -0.31$)

	σ_τ					
Variable	0.05	0.10	0.25	0.50	0.75	1.00
Relative Y	86.6	84.2	77.9	69.5	62.8	57.2
Relative TFP	82.8	80.6	74.5	66.6	60.4	55.1
Relative E	25.7	25.6	25.4	24.6	23.2	21.4
Relative establishment size s	16.2	16.3	16.3	16.6	17.0	17.7
Relative investment μ_z	398.8	398.6	397.1	392.0	383.44	371.58
Relative wages w	64.8	64.8	64.9	65.0	65.4	65.9
Relative SD(logTFPQ)	122.1	122.1	121.9	121.5	120.8	119.9
SD(logTFPR) percent	8.6	8.6	8.7	8.9	9.2	9.6

Comparing Table 2 and Table 3 we observe a surprising result with respect to changes in the deterministic component of the output tax μ_τ. For each σ_τ, a smaller μ_τ generates a higher output and TFP. We observe this by comparing output and TFP from Table 2 and Table 3, a smaller μ_τ generates an increase in the SD of log TFPR, consequently, more distortions, but a hight output and TFP. The main force behind this result is that a decreasing in μ_τ reduces the average size of operating establishments for each σ_τ, since establishments face a decreasing returns to scale technology, output is greater than in the economy in which establishments are on average larger.

[9]Notice that μ_τ is negative because τ is defined as $\tau = (1-\tau_y)^{\frac{1}{2(1-\alpha-\gamma)}}$, as result a smaller τ implies a more distorted economy.

5.2 Endogenous TFP Distribution

Next, we quantify the importance of the endogenous distribution of establishment-level productivity in our framework. We consider two variations of our model in order to better place our results relative to the existing literature. First, we consider the model with an exogenous initial distribution of TFP which is fixed to the distribution of establishment-level TFP generated by the model for the benchmark economy (i.e., the economy without policy distortions). In this economy, incumbent establishments invest in their productivity and this investment can differ across economies with different policy distortions, but all new entrants draw from the same distribution of productivities as in the benchmark economy. This version of the model broadly captures the recent literature that has endogenized productivity investment across establishments in the life cycle (see for instance Ranasinghe (2012); Bhattacharya et al. (2013); Gabler and Poschke (2013); among others). We report statistics for this version of the model in the columns identified as Exo1 in Table 4. Second, we consider the model with an exogenous distribution of establishment-level productivity by in addition to fixing the initial distribution, fixing the amount of productivity investment by establishments (i.e. we set μ_z exogenously to the level of the benchmark economy). This implies that the invariant distribution of establishment-level TFP is the same in all economies regardless of the policy distortions. This case is comparable to the framework in Restuccia and Rogerson (2008) where the distribution of productive units is kept constant in all economies. We denote this version of the model as Exo2 in Table 4.

In Table 4 we report the results for $\mu_\tau = 0.00$, which is the same value as in Table 2, and a range of σ_τs for our economy with endogenous distribution End, and the two versions of the model with exogenous distributions Exo1 and Exo2.

The main result from Table 4 is that economies with an exogenous distribution are not able to generate significant changes in output and in TFP. While the model with an endogenous distribution on the more distorted case, when σ_τ is equal to 1 percent, generates output

Table 4: Effects of changes in σ_τ ($\mu_\tau = -0.00$)

	$\sigma_\tau = 0.5$			$\sigma_\tau = 1.0$		
Variable	End	Exo1	Exo2	End	Exo1	Exo2
Relative Y	30.1	92.4	92.4	13.9	86.5	86.3
Relative TFP	30.3	92.5	92.5	14.4	86.9	86.8
Relative E	98.4	98.4	98.4	93.8	93.8	93.8
Relative establishment size s	111.7	100.0	100.0	168.2	100.0	100.0
Relative investment μ_z	91.5	100.0	100.0	65.8	100.2	100.0
Relative wages w	102.2	100.0	100.0	110.65	100.2	100.0
Relative Gini	90.3	89.3	89.3	93.5	89.3	89.3
Relative SD(logTFPQ)	99.4	100.0	100.0	97.7	100.0	100.0
SD(logTFPR) percent	8.9	8.9	8.9	9.6	9.6	9.6

per worker that is 14 percent of the benchmark economy, the model with the exogenous distribution and the same policy distortion generates an economy that is only 93 percent of the benchmark economy. The same is true to relative TFP, while the model with endogenous distribution generates economies that have 14 percent of the TFP of the benchmark economy, the model with exogenous distribution generates only differences of 87 percent. As a result, the endogenous distribution is a fundamental mechanism that amplifies the impact of policy distortions, and this amplification effect is substantial, a 3-fold in the case of $\sigma_\tau = 0.5$ and 6-fold in the case of $\sigma_\tau = 1$.

Another important result from Table 4 is that the difference between the outcome of the model Exo1 and the model Exo2 is negligible, which indicates that investment in TFP alone is not able to generate a significant impact on output and TFP. However, when the distribution of TFP is endogenous this channel becomes more relevant, and the difference in output and TFP are greater. Table 5 presents the result of the three models to σ_τ fixed to 1 percent and to two values of $\mu_\tau = -0.05$ and $\mu_\tau = -0.10$.

From Table 4 we observe the same result as in Table 5, both models Exo1 and Exo2 are not able to generate significant differences in output and in TFP when we vary μ_τ. However,

Table 5: Effects of changes in μ_τ ($\sigma_\tau = 1.00$)

	$\mu_\tau = -0.05$			$\mu_\tau = -0.10$		
Variable	End	Exo1	Exo2	End	Exo1	Exo2
Relative Y	26.5	82.7	83.6	36.3	78.9	80.7
Relative TFP	26.5	83.9	84.6	35.5	80.7	82.4
Relative E	86.2	86.2	86.2	77.2	77.2	77.2
Relative establishment size s	58.0	100.0	100.0	44.8	100.0	100.0
Relative investment μ_z	128.7	98.4	83.1	185.3	96.7	100.0
Relative wages w	92.3	98.3	100.0	83.1	96.7	100.0
Relative Gini	85.3	89.3	89.3	77.6	89.3	89.3
Relative SD(logTFPQ)	101.9	99.9	100.0	105.9	99.8	100.0
SD(logTFPR) percent	3.6	3.6	3.6	4.6	4.6	4.6

one important difference that arises from the End model compared to the model Exo1 and the model Exo2 is that a reduction in μ_τ in the End increases output and TFP, while, in the models Exo1 and Exo2 it decreases output and TFP. The main reason for this difference is that in both Exo1 and Exo2 model changes in μ_τ do not affect the average size of establishment, because they are fixed, while in the End model reductions in μ_τ decrease the average size of establishments. As a result, in both models Exo1 and Exo2 more distortions only decrease output and TFP, while in the End model, the source of distortions are relevant to the final outcome.

5.2.1 General Equilibrium Effects

Next, we quantify the importance of changes in prices, more precisely wages, in our framework, and we also quantify the impact of these changes with respect to the model with an exogenous distribution but with investment in TFP (Exo1), and the model with no investment in TFP (Exo2), exactly as we did in Table 4 and Table 5. In order to make this comparison, we fix wages in all models to the wages of the benchmark undistorted economy. In Table 6 we report the results for $\mu_\tau = 0.00$ and to $\sigma_\tau = 0.5$ and $\sigma_\tau = 1.0$, which are the same values as in Table 4.

Table 6: Effects of changes in σ_τ when wages are fixed ($\mu_\tau = -0.00$)

	$\sigma_\tau = 0.5$			$\sigma_\tau = 1.0$		
Variable	End	Exo1	Exo2	End	Exo1	Exo2
Relative Y	26.0	92.3	92.3	00.2	86.3	86.3
Relative TFP	26.6	92.4	92.4	00.2	86.4	86.4
Relative E	99.7	99.7	99.7	98.7	98.7	98.7
Relative establishment size s	133.1	100.0	100.0	300.1	100.0	100.0
Relative investment μ_z	100.2	100.2	100.0	101.0	101.0	100.0
Relative wages w	100.0	100.0	100.0	100.0	100.0	100.0
Relative Gini	91.8	89.3	89.3	99.8	89.3	89.3
Relative SD(logTFPQ)	100.0	100.0	100.0	100.0	100.0	100.0
SD(logTFPR) percent	8.9	8.9	8.9	9.6	9.6	9.6

The main result of Table 6 compared to Table 4 is that the general equilibrium effect on wages attenuates the impact of policy distortions. We can observe this by comparing output and TFP for the same set of policy distortions in Table 6 and in Table 4. Although in the three models End, Exo1, and Exo2 wages attenuate the effect of policy distortions, the magnitude of the effect of wages is different. In the model with an endogenous distribution this effect is significant, while in the models with an exogenous distribution Exo1 and Exo2 is negligible. This distinction is mainly attributed to the fact that in both models Exo1 and Exo2 establishments have a constant size. As a result, even when wages are flexible, policy distortions have a small impact on labor demand and consequently on wages. However, in the model with an endogenous distribution, policy distortions have a significant impact on establishment size, and consequently on labor demand and wages. When wages are flexible, an increase in the average size of establishments, increases labor demand, and wages. Since wages are higher, establishments invest less in productivity. When wages are fixed, establishments do not adjust their investment in productivity as a response to changes in wages, as a result, establishments are on average larger than in the model with flexible wages, and because of decreasing to returns to scale technology output and TFP are even lower than in the model with flexible wages.

In table 7 we report the result for $\sigma_\tau = 1.00$ per cent, and $\mu_\tau = -0.05$ and $\mu_\tau = -0.10$, which are the same values as in Table 5.

Table

Table 7: Effects of changes in μ_τ when wages are fixed ($\sigma_\tau = 1.00$)

	$\mu_\tau = -0.05$			$\mu_\tau = -0.10$		
Variable	End	Exo1	Exo2	End	Exo1	Exo2
Relative Y	35.9	83.7	83.7	55.1	81.0	81.0
Relative TFP	34.2	85.2	85.2	50.0	84.1	84.1
Relative E	80.4	80.4	80.4	59.4	59.4	59.4
Relative establishment size s	51.4	100.0	100.0	27.9	100.0	100.0
Relative investment μ_z	91.6	91.6	100.0	83.7	83.7	100.0
Relative wages w	100.0	100.0	100.0	100.0	100.0	100.0
Relative Gini	80.2	89.3	89.3	66.3	89.3	89.3
Relative SD(logTFPQ)	99.4	99.4	100.0	100.0	100.0	100.0
SD(logTFPR) percent	3.6	3.6	3.6	4.6	4.6	4.6

Comparing Table 7 and Table 5 we arise in a surprising result, now the general equilibrium effect of wages aggravates the impact of policy distortions on output and TFP. This result is the opposite of what we have found in the comparison between Table 6 and Table 4. As in the comparison of Table 6 and Table 4, we also observe that the general equilibrium impact of wages are only significant for the model with an endogenous distribution. The intuition behind the opposite result is very similar to the intuition of the comparison of changes in σ_τ. An decreases in the deterministic component of the output taxes μ_τ reduces the average size of establishments, smaller establishments demand less labor, and consequently wages decrease, when this happen, investing in productivity become more profitable, which reduces the impact of a reduction in μ_τ on establishment size. When wages are fixed, establishments do not adjust their investment in productivity as a response to changes in wages, as a result, the average size of establishments is larger than in the model with flexible wages. As establishment become larger output and TFP become smaller.

5.3 Discussion

To be written.

6 Conclusions

We developed a tractable dynamic model that endogenizes the distribution of establishment-level TFP across economies (what Hsieh and Klenow, 2009 and the related literature call TFPQ). The model tractability allows us to find close-form solutions that can be used to identify distortions from establishment-level data. In this framework, policy distortions not only generate differences in productivity investment across establishments as in the recent literature endogenizing life-cycle investments, but also the distribution of establishment-level productivity where new establishments draw their productivity from. We showed that empirically-reasonable policy distortions have substantial negative effects on aggregate TFP in this economy, an effect that is orders of magnitude larger than in the same model with exogenous distributions of establishment-level TFP. Moreover, we showed that in this framework policy distortions can potentially reconcile the observed differences in average establishment size across countries.

We have considered policy distortions that are uncorrelated to establishment-level productivity and nevertheless have found that these distortions have substantial negative effects on aggregate TFP. Since Restuccia and Rogerson (2008) have emphasized the substantially larger productivity impact of correlated idiosyncratic distortions, it would be interesting to explore the implications of correlated distortions in our framework. This requires a non-trivial extension of the theory and for this reason we leave this interesting and important exploration for future work. We also think that it would be interesting to explore specific policies or institutions such as size-dependent policies, firing taxes, financial frictions, among many others in the literature in the context of this model with endogenous establishment-level TFP. These explorations of specific policies in this framework may reconcile the puzzle in the literature of the apparent disparity in finding between indirect and direct quantitative assessments of misallocation and productivity emphasized in Hopenhayn (2013). We also leave these explorations for future work.

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7 Appendix

This appendix presents the proof of Lemma 1, Lemma 2, and Lemma 5, and the defition of TFPQ and TFPR in the context of the model.

Proof of Lemma 1

From the Kolmogorov forward equation (6), from our guess of the probability density function (7), and after some algebraic manipulation, we find that the dynamics of the probability density of operating establishments, $f(\cdot,\cdot)$ satisfies the following Kolmogorov forward equation:

$$\frac{\partial f(x,t)}{\partial t} = -\hat{\mu}\frac{\partial f(x,t)}{\partial x} + \frac{\sigma^2}{2}\frac{\partial^2 f(x,t)}{\partial x^2} + \left[\frac{(\Delta J(t) - dM(t)/dt)}{M(t)} - \lambda\right] f(x,t)$$

where

$$J(t) = J(x,t)|_0^\infty = \left(-\hat{\mu}F(x,t) + \frac{\sigma^2}{2}\frac{\partial F(x,t)}{\partial x} + \sigma^2\frac{\partial f(x,t)}{\partial x}\frac{F(x,t)}{f(x,t)}\right)\Bigg|_0^\infty .$$

From the expression above, we observe that the dynamics of the probability density of operating establishments is a function of the rate of departure of firms (first two terms in the right-hand side), and net rate of new entrants (last right-hand-side term), where both are a function of the expected profitability and the aleatory shocks captured in the Kolmogorov forward process.

In the stationary balanced growth path, the density of operating establishments, $f_\infty(x)$, does not vary over time, i.e. $\frac{\partial f(x,t)}{\partial t} = 0$. As a result we can rewrite the Kolmogorov forward equation in the balanced growth path as follows:

$$[\eta+\lambda-\frac{\Delta J}{M}]f_\infty(x)=-\hat{\mu}\frac{\partial f_\infty(x)}{\partial x}+\frac{\hat{\sigma}^2}{2}\frac{\partial^2 f_\infty(x)}{\partial x^2}$$

where

$$\frac{\Delta J}{M}=-\hat{\mu}f_\infty(x)+\frac{3\hat{\sigma}^2}{2}\frac{\partial f_\infty(x)}{\partial x}\bigg|_0^\infty.$$

The Kolmogorov forward equation in the stationary balanced growth path implies that the rate of departure of establishments is equal to the rate of new entrants at each profitability level x.

Now we guess that the invariant probability distribution has the following functional form $f_\infty(x)=Ke^{-\xi x}$, consequently the first derivative is equal to $f'_\infty(x)=-\xi f_\infty(x)$ and the second is equal to $f''_\infty(x)=\xi^2 f_\infty(x)$. Since f is a probability distribution, it must satisfies that its measure is equal to one, i.e. $\int_0^\infty f_\infty(x)dx=1$, which implies that $K=\xi$. Therefore, $\frac{\Delta J}{M}=-\hat{\mu}f_\infty(x)+\frac{3\hat{\sigma}^2}{2}\frac{\partial f_\infty(x)}{\partial x}\bigg|_0^\infty=(\hat{\mu}+\frac{3\hat{\sigma}^2}{2}\xi)\xi$ and the tail index ξ is found by solving the following polynomial,

$$\xi^2+\frac{\hat{\mu}}{\hat{\sigma}^2}\xi-\frac{(\eta+\lambda)}{2\hat{\sigma}^2}=0.$$

Substituting $\hat{\mu}$, from its definition, $\hat{\mu}=\mu-\frac{1}{2}\sigma^2$, and $\hat{\sigma}$ from its definition, $\hat{\sigma}=\sigma$, we find the following polynomial,

$$\xi^2+\left(\frac{\mu}{2\sigma^2}-\frac{1}{4}\right)\xi-\frac{(\eta+\lambda)}{2\sigma^2}=0. \tag{8}$$

We restrict ξ, which is the tail index of the invariant distribution, to the positive root that solves equation (8), which is $\xi=-\left(\frac{\mu}{2\sigma^2}-\frac{1}{4}\right)+\sqrt{\left(\frac{\mu}{2\sigma^2}-\frac{1}{4}\right)^2+\frac{2(\eta+\lambda)}{\sigma^2}}$.■

Proof of Lemma 2

We first guess the steady state distribution, $f_{\infty}(y) = Ke^{\xi_\tau y}$, $y = e^{\tau}$, as a result $f'_{\infty}(y) = \xi_\tau f_{\infty}(y)$ and $f''_{\infty}(y) = \xi_\tau^2 f_{\infty}(y)$. Moreover, $\int_{-\infty}^{0} f_{\infty}(y)dy = 1$ implies that $K = \xi_\tau$.

The invariant distribution $f_{\infty}(\cdot)$ must satisfy the Kolmogorov forward equation:

$$[\eta + \lambda - \frac{\Delta J}{M}]f_{\infty}(x) = -\hat{\mu}_\tau \frac{\partial f_{\infty}(x)}{\partial x} + \frac{\hat{\sigma}_\tau^2}{2}\frac{\partial^2 f_{\infty}(x)}{\partial x^2}.$$

By substituting our guess of the invariant distribution into the Kolmogorov equation, we find a polynomial in ξ_τ given by:

$$\xi_\tau^2 - \frac{\hat{\mu}_\tau}{\hat{\sigma}_\tau}^2 \xi - \frac{(\eta + \lambda)}{2\hat{\sigma}_\tau{}^2} = 0.$$

Substituting $\hat{\mu} = \mu - \frac{1}{2}\sigma^2$ and $\hat{\sigma} = \sigma$, and selecting the positive root of the polynomial, we find $\xi_\tau = \left(\frac{\mu_\tau}{2\sigma_\tau^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu_\tau}{2\sigma_\tau^2} - \frac{1}{4}\right)^2 + \frac{(\eta+\lambda)}{2\sigma_\tau^2}}$ and this concludes the proof. ■

Proof of Lemma 5

Proof Given that the productivity z and output tax τ are independent stochastic variables, we know that the join distribution of productivity z and output tax τ, $f(\cdot)$ is the product of the distribution of productivity $g(\cdot)$ and the distribution of output tax $h(\cdot)$. The joint distribution $f(\cdot)$ is given by:

$$f(\tau, z) = z^{-\xi_z}\tau^{\xi_\tau}.$$

Now we can find the probability density function of $s = \tau \times z$ by changing variables:

$$f(s) = \int_0^1 \left(\frac{s}{\tau}\right)^{-\xi_z} \tau^{\xi_\tau} d\tau = \frac{s^{-\xi_z}}{(\xi_z + \xi_\tau) + 1}.$$

The joint invariant distribution is also a Pareto distribution with tail index ξ_z. ■

Definition of TFPQ

We calculate TFPQ at establishment level as in Hsieh and Klenow (2009),

$$TFPQ(z) = \frac{y}{k^{\alpha} n^{\gamma}} \propto \frac{z^2 \tau^{2(\alpha+\gamma)}}{z^{2(\alpha+\gamma)} \tau^{2(\alpha+\gamma)}} = z^{2(1-\alpha-\gamma)}$$

Therefore, TFPQ at establishment level depends on distortions given and on μ_z, which is an endogenous object. The log of TFPQ follows an exponential distribution $f(x) = \xi_z e^{-\xi_z x}$, where ξ_z is defined as:

$$\xi_z = -\left(\frac{1}{8} + \frac{\mu_z}{2\sigma_z^2}\right) + \sqrt{\left(\frac{1}{8} + \frac{\mu_z}{2\sigma_z^2}\right)^2 + \frac{(\eta+\lambda)}{2\sigma_z^2} + \frac{\mu_z}{2\sigma_z^2} - \frac{1}{4}},$$

Therefore, the satandard deviation of the $\log TFPQ$ is equal to the variance of an exponential distribution, which is given by:

$$SD[\log TFPQ] = \frac{1}{\xi_z},$$

Definition of TFPR

We calculate TFPR in the model as in Hsieh and Klenow (2009):

$$TFPR(z,\tau) = \frac{1}{(1-\tau_y)} = \tau^{-2(1-\alpha-\gamma)}$$

Therefore $\log TFPR(z,\tau)$ follows an exponential distribution $f(x) = \xi_{TFPR} e^{-\xi_{TFPR} x}$ with

$$\xi_{TFPR} = -\left(\frac{1}{8} + \frac{\mu_{TFPR}}{2\sigma_{TFPR}^2}\right) + \sqrt{\left(\frac{1}{8} + \frac{\mu_{TFPR}}{2\sigma_{TFPR}^2}\right)^2 + \frac{(\eta+\lambda)}{2\sigma_{TFPR}^2} + \frac{\mu_{TFPR}}{2\sigma_{TFPR}^2} - \frac{1}{4}},$$

where (applying Ito's calculus)

$$
\begin{aligned}
\mu_{TFPR} &= -2(1-\alpha-\gamma)\left[\mu_\tau - \frac{1}{2}(3-2\alpha-2\gamma)\sigma_\tau^2\right], \\
\sigma_{TFPR}^2 &= 4(1-\alpha-\gamma)^2\sigma_\tau^2.
\end{aligned}
$$

Therefore, SD of the $\log TFPR$ is equal to

$$
SD[\log TFPR] = \frac{1}{\xi_{TFPR}},
$$