The Shape of Aggregate Production Functions: Evidence from Estimates of the World Technology Frontier

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Abstract. The article provides multifaceted evidence on the shape of the aggregate country-level production function, derived from the World Technology Frontier, estimated on the basis of annual data on inputs and output in 19 highly developed OECD countries in the period 1970–2004. A comparison of its estimates based on the Data Envelopment Analysis approach and the Bayesian Stochastic Frontier Analysis approach uncovers a number of departures from the Cobb–Douglas as well as the translog benchmark production function. Partial elasticities derived from the translog production function estimation are variable across countries and time, and are strongly correlated with stocks of inputs used for production. We also find notable departures from perfect substitutability between unskilled and skilled labor. Tests for constancy of returns to scale provide mixed evidence on this property.

Keywords and Phrases: World Technology Frontier, aggregate production function, Data Envelopment Analysis, Stochastic Frontier Analysis, returns to scale

JEL Classification Numbers: E23, O11, O14, O33, O47

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1 Introduction

It is paradigmatic in contemporary macroeconomics to assume that the production process can be summarized by an aggregate production function, mapping the stocks of appropriately specified aggregate inputs onto the unique aggregate output. When the precise shape of this function is not the central question of the economic problem at hand, it is also frequently assumed to take the Cobb–Douglas form, valued for its simplicity, analytical tractability, and agreement with most broad patterns observed in the data.

Seen from an empirical viewpoint, there is however no consensus on the preferred functional form of the aggregate (country-level) production function.[1] The estimation of aggregate production functions is notoriously difficult due to multiple empirical issues: endogeneity of input use, measurement uncertainty of input and output aggregates such as GDP, physical capital and human capital, problems with comparability across countries and time, just to name a few. Another important issue that we deal with in the current paper is that the production function is a *technological* concept, one of a technical relationship between inputs and outputs, whereas in reality, country-level productivity may also be affected by *non-technological* variables such as taxation, presence of various barriers to doing business (such as corruption, crime, complicated bureaucratic procedures, etc.), sectoral composition of production, labor market institutions, or financial constraints. To obtain reliable estimates of the production function itself, one thus ought to control for differences in these institutional conditions. We achieve this goal by taking the World Technology Frontier approach.

The objective of the current paper is then to estimate the aggregate, country-level production function as a relationship between countries' aggregate inputs and their maximum attainable output, computed on the basis of the World Technology Frontier (WTF hereafter) – where the WTF is the best-practice frontier at each moment in time. By doing so, we are able to single out technological aspects of the production processes from their institutional background, at least up to a multiplicative constant. Such

[1] Taking aggregation issues seriously, it is even dubious if such an aggregate production function exists at all (see e.g., Felipe and Fisher, 2003). The ability to aggregate local input–output relationships into an aggregate function where total output depends on total stocks only and not on the distribution of factor endowments across plants, requires strong homogeneity assumptions imposed on the individual production processes – which are very unlikely to hold. Keeping this caveat in mind, the "aggregate production function", which we refer to, can then be viewed only as an approximate relationship between aggregate inputs and output, which could be altered due to shifts in factor distribution. See Temple (2006) for a discussion of this interpretation.

estimates of the aggregate production function will be then used as a convenient starting point for further analyses, aimed at deriving this function's crucial characteristics, and discussing which parametric form agrees most with the available empirical evidence. As most important features of the estimated aggregate production function, we shall investigate the constancy of its returns to scale and the pattern of potential dependence of its partial elasticities on factor endowments.

We estimate the aggregate production function with two alternative methods. First, we apply the nonparametric Data Envelopment Analysis (DEA) approach,[2] augmented with the Simar and Wilson (1998, 2000) bootstrap procedure to adjust for the bias of DEA efficiency estimates as well as to compute standard errors and confidence intervals for the estimates. The advantage of this approach is that it does not require us to make a priori assumptions on the functional form of the aggregate production functions and yields testable predictions instead. Unfortunately, the DEA approach cannot provide precise predictions on the function's differential features such as its partial elasticities. Second, we also use the Stochastic Frontier Analysis (SFA) methodology[3] which allows us to estimate the production function directly, under certain predefined (parametric) functional specifications. This is done on the basis of Bayesian techniques which are particularly suited to production function estimation due to their relative robustness under collinearity and measurement error. The advantage of the SFA approach is that it allows to test several specifications directly. It is also useful for drawing precise conclusions on the aggregate production function's partial elasticities and returns to scale.

Based on these methods, we obtain the following principal results:

- departures from the Cobb–Douglas and the translog production function are pronounced,
- partial elasticities of the aggregate production function are correlated with inputs, providing further evidence against the Cobb–Douglas specification,
- unskilled and skilled labor are not perfectly substitutable,
- several tests of returns to scale provide mixed evidence on this property.

[2]For applications in macroeconomics, see e.g., Färe et al. (1994), Kumar and Russell (2002), Henderson and Russell (2005), Jerzmanowski (2007), Badunenko, Henderson and Zelenyuk (2008), and Growiec (2010a).

[3]For applications in macroeconomics, see e.g., Koop, Osiewalski and Steel (1999, 2000) and Bos et al. (2010). See Kumbhakar and Knox Lovell (2000) for general reference.

The remainder of the paper is structured as follows. Section 2 presents the dataset and methodology. Section 3 offers an overview of the World Technology Frontier, viewed through the lens of our data. Section 4 discusses the properties of the aggregate, country-level production function, inferred from the WTF. Section 5 concludes.

2 Data and methodology

2.1 Data sources for the construction of our variables

The macroeconomic dataset used in the current study covers 19 highly developed OECD economies in the period 1970–2004. The WTF will be computed with taking GDP or GDP per worker as output, and as inputs – the aggregate stocks of physical capital, human capital (subdivided into unskilled and skilled labor), and sometimes also the "raw" number of employees.

2.1.1 The primary data source

To construct the primary dataset for all our further analyses, we have gathered international data on GDP and GDP per worker from the Penn World Table 6.2 (cf. Heston, Summers and Aten, 2006), available for 1960-2004. The unit of measurement is the PPP converted US dollar under constant prices as of year 2000.

Physical capital stocks, in turn, have been calculated using the perpetual inventory method (cf. Caselli, 2005). We have used country-level private investment shares as well as government shares from the Penn World Tables 6.2. When doing this, we have taken an intermediate stance between the two polar standpoints as for the role of government in capital accumulation – one is that government spending is all consumption, and the other one is that it is all investment – and consequently we assumed that the government invests the same percentage of its GDP share as the private economy does. Under this assumption, the overall (private and public) investment share is $s/(1-g)$ where s is the private investment share and g is the government share. Furthermore, following Caselli (2005), we assumed an annual depreciation rate of 6%.

Country-level human capital data have been taken from de la Fuente and Doménech (2006). The raw variables provided in this contribution are shares of population aged 25 or above having completed primary, some secondary, secondary, some tertiary, tertiary, or postgraduate education. The considered dataset is of 5-year frequency only and ends in 1995. Nevertheless, the de la Fuente–Doménech dataset has been given priority among all possible education attainment databases due to its presumed superior quality. The original de la Fuente–Doménech data have then been extrapolated forward in the time-series dimension until the year 2000 using Cohen and Soto (2007) schooling data as a predictor for the trends. Neither Barro and Lee (2001) nor Cohen and Soto (2007) data could be used directly for this purpose because neither of them is (even roughly) in agreement with the de la Fuente–Doménech dataset – nor with each other – in the period where all datasets offer data points. Furthermore, the human capital data have been extrapolated to all intermediate years as well. This was possible due to the fact that human capital variables are, in general, very persistent and not susceptible to business cycle variations.

Human capital aggregates have been constructed from these educational attainment data using the Mincerian exponential formula with a concave exponent, following Hall and Jones (1999), and more directly, Caselli (2005):

$$H^U = \sum_{i \in S_U} \psi_i e^{\phi(s_i)}, \qquad H^S = \sum_{i \in S_S} \psi_i e^{\phi(s_i)}, \tag{1}$$

where S_U is the set of groups of people who completed less than 12 years of education (less than elementary, elementary, less than secondary), S_S is the set of groups of people who completed 12 years of education or more (secondary, less than college, college or more), ψ_i captures the share of i-th education group in total working-age population of the given country, s_i represents years of schooling in i-th education group (cf. de la Fuente and Doménech, 2006), and $\phi(s)$ is a concave piecewise linear function:

$$\phi(s) = \begin{cases} 0.134s & s < 4, \\ 0.134 \cdot 4 + 0.101(s-4) & s \in [4, 8), \\ 0.134 \cdot 4 + 0.101 \cdot 4 + 0.068(s-8) & s \geq 8. \end{cases} \tag{2}$$

The overall human capital index may be computed as the sum of unskilled and skilled labor: $H = H^U + H^S$.[4] We have however allowed these two types of labor to be imperfectly substitutable and thus enter the production function separately. The perfect substitution case where only total human capital matters is an interesting special case of our generalized formulation; the data do not support this assumption, however.

The raw number of workers has been taken from the Penn World Table 6.2 (cf. Heston, Summers and Aten, 2006).

[4] The cutoff point of 12 years of schooling, delineating unskilled and skilled labor, seems adequate for the relatively highly developed OECD economies in our sample, though it might be set too high if developed economies were to be considered as well (cf. Caselli and Coleman, 2006).

While employing the aforementioned panel dataset in parametric analyses such as the SFA, one faces two critical problems. Firstly, nearly all macroeconomic level-variables are non-stationary in the time series dimension. Hence, either a co-integrating (preferably a baseline) relation between the targeted variables ought to be found, or the variables need to be transformed into stationary ones. The former solution, in the case of panel data (see e.g., Pedroni, 1999; Larsson, Lyhagen and Löthgren, 2001), is still regarded as highly controversial and frequently criticized in the literature, whereas the latter one dramatically changes the scope of the analysis. To avoid both options, we have narrowed down the time dimension of our SFA analyses, using data covering entire decades instead of single years. One further potential advantage of this approach is that original human capital data are readily available at decadal frequency.

Secondly, should significant outliers be found within our sample, the final results are likely to be biased. The same problem could also appear due to business-cycle fluctuations. Escaping short- and medium-term disturbances appears particularly important in a growth analysis such as ours. Thus, the HP filter has been applied to our data as to exclude the outliers and cyclical features in the data.

2.2 Methodological issues

The primary objective of the current paper is to draw conclusions on the shape of the aggregate, country-level production function, based on estimates of the World Technology Frontier. As mentioned in the introduction, our WTF estimation procedure is twofold. First, we shall use the deterministic nonparametric DEA procedure, augmented with the Simar–Wilson bootstrap in order to eliminate estimation bias and compute standard errors of measures of each country's distance to the frontier. Second, we shall also use the parametric stochastic frontier approach (SFA), where parameters of the explicitly specified aggregate production function as well as distance measures are computed using a Bayesian procedure.

As a rule, the WTF will be estimated *sequentially*, so that for computing the WTF in each period t, data from periods $i = 1, 2, ..., t$ will be used. This corresponds to the assumption that technologies, once developed, remain available for use forever (see e.g., Henderson and Russell, 2005).

2.2.1 Data Envelopment Analysis

The idea behind the DEA method is to construct the best-practice production function nonparametrically, as a convex hull of production techniques (input–output configurations) currently used in countries present in the data.

The production function is then inferred indirectly as a fragment of the boundary of this convex hull for which is output is maximized given inputs. More precisely, for each observation i, output y_i is decomposed as:

$$y_i = E_i f(\mathbf{x}_i) \tag{3}$$

i.e., into a product of the maximum attainable output given inputs $y_i^* \equiv f(\mathbf{x}_i)$ and the Shephard distance function $E_i \in (0, 1]$. In other words, the efficiency index E_i measures (vertical) distance to the technology frontier, while the frontier itself is computed nonparametrically as $y_i^* = f(\mathbf{x}_i)$. It should be noted that the vector of inputs, $\mathbf{x}_i$, could in principle be of any length $n \in \mathbb{N}$, but if one distinguishes too many types of inputs then (i) the DEA could run into numerical problems due to the "curse of dimensionality" (cf. Färe et al., 1994), and (ii) the efficiency levels could be overestimated due to too small a sample size.

Formally, the (output-based) DEA method is a linear programming technique allowing one find the Shephard distance function E_j for each unit $j = 1, 2, ..., I$ in the sample such that its reciprocal – the efficiency index θ_j is maximized given a series of feasibility constraints (cf. Fried, Knox Lovell and Schmidt, 1993):

$$\begin{aligned} &\max_{\{\theta_j, \lambda_1, \ldots, \lambda_I\}} \theta_j \\ \text{s.t.} \quad &\theta_j y_j \leq \sum_{i=1}^{I} \lambda_i y_i, \\ &\sum_{i=1}^{I} \lambda_i x_{1i} \leq x_{1j}, \\ &\sum_{i=1}^{I} \lambda_i x_{2i} \leq x_{2j}, \\ &\vdots \\ &\sum_{i=1}^{I} \lambda_i x_{ni} \leq x_{nj}, \\ &\lambda_i \geq 0, \qquad i = 1, 2, ..., I. \end{aligned} \tag{4}$$

It is also additionally assumed that $\sum_{i=1}^{I} \lambda_i = 1$ for VRS (variable returns to scale), and $\sum_{i=1}^{I} \lambda_i \leq 1$ for NIRS (non-increasing returns to scale). Under the CRS (constant returns to scale) assumption, no further restriction on λ_i's is necessary.

The Shephard distance function $E_j = D_j(x_j, y_j)$ is computed as the reciprocal of the (output-oriented Debreu–Farell) efficiency index θ_i (that is, $E_i = 1/\theta_i$).

Since the data contain a finite number of data points, one for each country and each year, by construction the DEA–based production function will be piecewise linear and its vertices will be the actually observed *efficient* input–output configurations (i.e. non-dominated by any linear combination of other observed input–output configurations).

2.2.2 Advantages and limitations of the DEA approach

The DEA is a deterministic, data-driven approach to deriving the production function from observed input–output pairs. Its unquestionable strength lies in the fact that it does not require any particular functional form of the aggregate production function (provided that it satisfies the free-disposal property), and provides testable predictions on its shape instead. Indeed, the usual assumption of a Cobb–Douglas aggregate production function may lead to marked biases within growth accounting or levels accounting exercises leading to an overestimation of the role of total factor productivity (TFP), as argued by Caselli (2005) and Jerzmanowski (2007). As for the predicted shape of the production function, one obvious fact is that due to the characteristics of the method, it will be piecewise linear for any finite data sample. With reasonably large data samples, however, certain parametric forms could be tested formally against the DEA-based nonparametric benchmark, such as the Cobb–Douglas, CES, or translog.

There are important limitations to the DEA approach as well. First, its deterministic character makes it silent on the estimation precision of the aggregate production function and of the predicted efficiency levels if inputs and outputs are subject to stochastic shocks. This weakness is however removed in the current study by using bootstrap techniques due to Simar and Wilson (1998, 2000b).

Second, the DEA provides a biased proxy of the actual technological frontier. Certainly, even the most efficient units in the sample could possibly operate with some extra efficiency: they are themselves aggregates of smaller economic units and must therefore have some internal heterogeneity. Taking account of that, the frontier could easily be shifted upwards; efficiency in nevertheless normalized to 100% for the most efficient units in the sample. Again, the bootstrap method due to Simar and Wilson (1998, 2000b) helps in this respect by allowing for corrections in the bias as well as for estimating confidence intervals for the actual efficiency levels and the technological frontier.

Third, the DEA constructs the production function basing on the efficient data points. This makes it naturally sensitive to outliers and measurement error. This cannot be fully accounted for by using bootstrap techniques. However, on the one hand, outliers characterized by obvious errors are easily spotted because they spoil the whole subsequent analysis. Systematic mismeasurement associated with some units could be left unnoticed, however, if these units fall short of the frontier. The data have been carefully checked, though, so that one can be confident that the risk of errors has been minimized.

2.2.3 Simar and Wilson's bootstraps

The main objective of bootstrap procedures is to replicate the true Data Generating Process (DGP) when it is unknown. In particular, in the stochastic extension of the deterministic DEA, applied in the current paper, it is assumed that the true frontier f is unknown and consequently E_i (for $i = 1, ..., I$) are unknown, too. Simar and Wilson's bootstraps are then used to formulate an approximation of the process generating values of Shephard distance functions[5] E_i. We shall apply the procedure described by Simar and Wilson (1998) which assumes that random variables $E_1, ..., E_I$ are i.i.d. with an unknown density function g on $(0, 1]$ (the output-oriented case). In particular, it means that the values of E_i are independent of the random variables generating observed inputs and output, (X_i, Y_i).[6] It is called the *homogeneity* assumption (see Simar and Wilson, 2000a) and thus the procedure is called the homogeneous Simar and Wilson (SW) bootstrap.[7]

As the outcome of the homogenous SW bootstrap, we received, for each unit i, the bootstrap estimate of the Shephard distance function $\hat{E}_i$ and a set of bootstrap realizations $E_{ib}, b = 1, ..., B$, where B is the number of bootstrap iterations.[8] Consequently, we also obtained the bootstrap bias, estimates of variance of $\hat{E}_i$, and respective confidence intervals. Estimates $\hat{E}_i$ may be (additionally) bias-corrected. If the bootstrap procedure is consistent, then asymptotically, these estimates may be used for E_i. Some Monte Carlo experiments conducted in Simar and Wilson (1998, 2000a) suggest that this SW bootstrap is consistent.[9]

[5] Or, equivalently, Debreu–Farrell efficiency indices θ_i.

[6] Vectors (X_i, Y_i), for $i = 1, ..., I$, are assumed to be i.i.d., too. Their realizations are the observed input-output pairs $\{(x_i, y_i), i = 1, ..., I\}$.

[7] We used the procedure *boot.sw98* contained in the free software package FEAR (written in R).

[8] See Simar and Wilson (1998). Usually, $B = 2000$ is considered sufficient in the literature.

[9] However, there is no rigorous proof so far that the homogenous SW bootstrap is consistent (cf. Simar and Wilson, 2000a).

It is important to emphasize that the homogeneity assumption may be relaxed. When one relaxes this assumption, then the inefficiency of a unit depends on the observed values of inputs and outputs, i.e., on the pair (x_i, y_i) $(i = 1, ..., I)$. Such procedures are called *heterogeneous* bootstraps. Such a procedure was first proposed in the paper by Simar and Wilson (2000b), where the pairs (x_i, y_i) were expressed in cylindrical coordinates.[10] In the papers by Kneip, Simar and Wilson (2008, 2009) as well as Park, Jeong and Simar (2009), generalized procedures were proposed, allowing for:

- orthonormal coordinates, with one of them being connected with E_i,
- the CRS case.

These authors have also proposed a formal proof of consistency of certain bootstrap procedures.

Unfortunately, these procedures generate a lot of additional computational burden which limits their practical applicability (see the comments in Kneip, Simar and Wilson, 2008, 2009). For example, these procedures depend on unknown constants whose values are established arbitrarily. Moreover, for large numbers of units in the sample, complexity of the algorithm blows up calculation times beyond acceptable limits. For these reasons, the software is still being developed (see Kneip, Simar and Wilson, 2009) and could not be used for the purposes of the current study.

2.2.4 Testing local and global returns to scale

In order to test the extent of returns to scale in the production technology on the basis of DEA estimates of the WTF, we have used the resampling algorithm suggested by Simar and Wilson (1998) and have adopted the method introduced by Löthgren and Tambour (1999) and by Simar and Wilson (2002).

As far as tests of *local* returns to scale are concerned, we use procedures based on bootstrap confidence intervals proposed by Löthgren and Tambour (1999). The returns to scale test is performed using the following nested testing procedure:

Test 1:

$H_0 : S^{C-NIRS} = 1$ (scale-efficient or increasing returns to scale),

$H_1 : S^{C-NIRS} > 1$ (decreasing returns to scale).

If H_0 in Test 1 is not rejected, we proceed with the second test:

[10] In DEA, inefficiency has a radial character, so (x_i, y_i) is strictly connected with E_i.

Test 2:

$H_0 : S^{CRS} = 1$ (scale-efficient),

$H_1 : S^{CRS} > 1$ (increasing returns to scale),

where

$$S^{CRS} = \frac{\sum_{j=1}^{T} \theta_j^{CRS}(x_j, y_j)}{\sum_{j=1}^{T} \theta_j^{VRS}(x_j, y_j)}, \quad S^{C-NIRS} = \frac{\sum_{j=1}^{T} \theta_j^{CRS}(x_j, y_j)}{\sum_{j=1}^{T} \theta_j^{NIRS}(x_j, y_j)},$$

and $\theta_j^{CRS}(x_j, y_j), \theta_j^{VRS}(x_j, y_j)$ and $\theta_j^{NIRS}(x_j, y_j)$ are output-oriented Farell distance functions under the assumption of constant, variable, and non-increasing returns to scale, respectively.

Let $\hat{S}^{*C-NIRS}(\alpha)$ and $\hat{S}^{*CRS}(\alpha)$ denote the lower bound of the bootstrap (1-α)-confidence interval for S^{C-NIRS} and S^{CRS}, respectively. The test procedure is straightforward. If $\hat{S}^{*C-NIRS}(\alpha) > 1$, then H_0 in Test 1 is rejected and we conclude that the technology features decreasing returns to scale. If $\hat{S}^{*C-NIRS}(\alpha) = 1$, then H_0 in Test 1 cannot be rejected and we perform Test 2. Next, if $\hat{S}^{*CRS}(\alpha) > 1$, then the hypothesis of scale efficiency is rejected by Test 2 and we conclude that the technology exhibits increasing returns to scale. Finally, if $\hat{S}^{*CRS}(\alpha) = 1$, we conclude that the technology is scale-efficient.

In turn, the statistical test of *global* returns to scale is based on two nested tests proposed by Simar and Wilson (2002). In Test 1, the null hypothesis is tested that the technology (production frontier) exhibits globally constant returns to scale (CRS) against an alternative hypothesis that the technology is characterized by variable return to scale (VRS). That is:

Test 1:

H_0: technology is globally CRS,

H_1: technology is VRS.

If H_0 is rejected, we shall perform Test 2 with H_0 stating that the technology exhibits globally non-increasing returns to scale (NIRS) against H_1 that the technology is VRS:

Test 2:

H_0 : technology is globally NIRS,

H_1 : technology is VRS.

Simar and Wilson (2002) discussed various statistics for testing these hypotheses; among these, we have selected the following ratios of means:

$$\hat{S}^{CRS} = \frac{\sum_{j=1}^{T} \hat{\theta}_j^{CRS}(x_j, y_j)}{\sum_{j=1}^{T} \hat{\theta}_j^{VRS}(x_j, y_j)} \text{ in Test 1, and } \hat{S}^{C-NIRS} = \frac{\sum_{j=1}^{T} \hat{\theta}_j^{CRS}(x_j, y_j)}{\sum_{j=1}^{T} \hat{\theta}_j^{NIRS}(x_j, y_j)} \text{ in Test 2,}$$

where $\hat{\theta}_j^{CRS}(x_j, y_j), \hat{\theta}_j^{VRS}(x_j, y_j)$ and $\hat{\theta}_j^{NIRS}(x_j, y_j)$ are estimators of the (output-oriented) Farrell distance function under the assumption of constant, variable, and non-increasing return to scale, respectively.

By construction $\hat{S}^{CRS} \geq 1$ because $\hat{\theta}_j^{CRS}(x_j, y_j) \geq \hat{\theta}_j^{VRS}(x_j, y_j)$. The null hypothesis in Test 1 is rejected when $\hat{S}^{CRS}$ is significantly greater than 1. The p-value of the null hypothesis is derived from bootstrapping (see Simar and Wilson, 2002):[11]

$$p - \text{value} = \sum_{b=1}^{B} \frac{I_{[0,+\infty)}(\hat{S}^{CRS,b} - \hat{S}_{obs}^{CRS})}{B},$$

where $B = 2000$ is the number of bootstrap replications, $I_{[0,+\infty)}$ is the indicator function, $\hat{S}^{CRS,b}$ is the b-th bootstrap sample, and $\hat{S}_{obs}^{CRS}$ is the original observed value. The same methodology is used in Test 2.

2.2.5 Stochastic frontier analysis

To take a broader picture of the (in)efficiency of aggregate production processes in highly developed countries, the results obtained with the DEA approach will be compared against estimates resulting from stochastic frontier analysis (SFA). In this alternative approach, stochastic disturbances may be explicitly taken into account, and the potential biases caused by outliers or measurement error should be minimized. On the other hand, these advantages are conditional on finding the appropriate parametric representation of the aggregate, WTF-based production function.

In its simplest, log-linear form, the stochastic frontier model for panel data, employed in this paper, can be written as:

$$y_{it} = x_{it}^{'}\beta + v_{it} - u_{it}, \tag{5}$$

where y_{it} represents the logarithm of output in country $i = 1, ..., N$ and period $t = 1, ..., T$, β represents the vector of estimated parameters, and x_{it} carries information about K factors of production expressed in logarithms, plus a constant term. Given

[11] To test the hypotheses regarding global returns to scale of the technology we use suitably modified codes written by O. Badunenko (see http://sites.google.com/site/obadunenko/codes).

this notation, the case $x_{it} = (1, \ln K_{it}, \ln H_{it}^U, \ln H_{it}^S)$ represents our benchmark Cobb–Douglas specification with physical capital, unskilled labor and skilled labor as inputs, and no restrictions on returns to scale. One could however extend this vector to accommodate cross-terms such as

$$\begin{aligned} x_{it} &= \Big(1, \ln K_{it}, \ln H_{it}^U, \ln H_{it}^S, \ln^2 K_{it}, \ln^2 H_{it}^U, \ln^2 H_{it}^S, ... \\ & \quad ... \ln K_{it} \ln H_{it}^U, \ln K_{it} \ln H_{it}^S, \ln H_{it}^U \ln H_{it}^S\Big) \end{aligned}$$

in which case it becomes the translog production function. Constant returns to scale may also be imposed, wherever necessary, by writing down the production function in its intensive form. We shall do this in some of our estimated specifications along with introducing the regularity conditions, which serve as a source of prior information and depend on the specification of the frontier.

The basic theoretical framework for stochastic frontier (SF) models employed in this research was originally proposed by Aigner, Lovell and Schmidt (1977).[12] In their seminal paper, the authors assumed the total, "composed" error to be a sum of two components: a symmetric, normally distributed variable (the idiosyncracy, v_{it}) and the absolute value of a normally distributed variable (the inefficiency, u_{it}). Ever since, the main stream of research on stochastic frontier models appears to have focused upon the various distributional assumptions on the latter inefficiency term. Single-parameter distributional specifications of v_{it} and u_{it} (for instance, normal and half-normal, respectively) have produced some skepticism in the subsequent literature, though. To resolve this problem, Gamma-density has been applied (Greene, 2003). Nevertheless, as shown by Ritter and Simar (1997), these attempts have been hardly successful so far, mainly due to computational difficulties and identification problems.[13]

In sum, the crux of the SFA approach lies with the decomposition of the error term into two components: the country- and time-specific idiosyncratic shock (or measurement error) v_{it}, and the technical inefficiency component u_{it} which is assumed to be non-negative. Both components are assumed to be independent of one another. In our analysis, we shall employ several different assumptions concerning the distribution of u_{it}.

[12]There is also another seminal paper in this field – by Meeusen and van den Broeck (1977).

[13]These problems become especially apparent in relatively small data sets, such as the one that has been applied and presented in this paper. Therefore, we shall not discuss the case of an unrestricted Gamma-distribution.

The next section proceeds with details of the estimation procedure and assumptions made in the course of our SFA analysis.

2.2.6 Bayesian estimation framework

When applying the Stochastic Frontier methodology to panel data, one ought to keep in mind that production possibilities are strongly affected by technological progress. Consequently, it would be "unfair" to observations from the past to evaluate them against a frontier estimated with a dataset including more recent data as well, since at earlier times, production processes could not enjoy the possibilities offered by technologies developed later on. Even when computing the WTF in a sequential manner, akin to the one used in our DEA analysis, which should partially alleviate this problem, one ought to control for technological progress at the WTF to obtain a fair evaluation of the evolution of technical inefficiency across countries and time.

To address this issue, we have employed Battese and Coelli's (1992, 1995) decomposition of the inefficiency term u_{it}. It takes the following form:

$$u_{it} = u_i \cdot z_t,$$

where u_i is either a truncated normal or exponential distribution[14] and $z_t = \exp[-\eta(t - T)]$, where positive (or negative) η indicates increasing (or decreasing, respectively) inefficiency over time. Hence, the Battese–Coelli methodology urges the modeler to assume that the random part of u_{it} is time-invariant, and its temporal evolution is described by a deterministic function z_t with an estimated parameter η. This rigidity is partly overcome when the WTF is estimated sequentially, so that for each period t, data from periods $i = 1, 2, ..., t$ are used. In such case, temporal shifts in u_{it} appear not only due to changes in z_t, but also due to the consecutive re-estimations of the WTF. The inefficiency term u_{it}, the Debreu–Farrell efficiency measure θ_{it} and the Shephard distance measure are interrelated via the equality $\theta_{it} = 1/E_{it} = \exp(-u_{it})$.

An alternative approach allowing one to deal with stochastic frontiers with time-varying inefficiencies was offered by Cornwell, Schmidt and Sickles (1990). Regrettably, this approach has much in common with DEA (in fact, it is a "deterministic" frontier model, and it is distribution-free in terms of u_{it}), so it was not used for the purposes of the current analysis.[15]

[14] Robustness tests have been done upon these two different distributional assumptions, though in terms of our final results, choosing any of them makes little difference. The results are available from the authors upon request.

[15] Given the purposes of the current study, there are two major disadvantages of Cornwell, Schmidt

From the computational perspective, two different approaches have been employed in SFA to isolate the inefficiency component. The first one is based upon the stright-forward application of maximum likelihood methods, as presented by Jondrow et al. (1982) or (for a more general class of distributions) by van den Broeck et al. (1994). In this case, given the parameters of the model θ, the likelihood function takes the form of:

$$L(y_{it}, \theta) = \prod p(y_{it}|x_{it}, \theta) \tag{6}$$

and requires derivation of $p(y_{it}|x_{it}, \theta) = \int_{\mathbb{R}_+} p(y_{it}|x_{it}, u_{it}, \theta)p(u_{it}|\theta)du_{it}$.

The other (Bayesian) approach relies on a posterior simulator, such as Gibbs sampling (cf. e.g., Koop, Steel and Osiewalski, 1995), applied to determine the distribution of the inefficiency component u_{it} via draws from the posterior distribution $p(\theta|y_{it}, x_{it})$. Hence, as opposed to Jondrow et al.'s approach, no explicit analytical formula for the likelihood function is needed.

In our study, the stochastic frontier will be estimated with Bayesian techniques that naturally correspond to the latter approach. Thus, all structural parameters of the production function $y_{it} \sim N(x'_{it}\beta - u_{it}, \sigma^2)$, contained in the vector β, as well the variance of disturbances v_{it} and u_{it}, the mean of the inefficiency term u_{it}, and the pace of technological progress η, will be estimated with a Bayesian procedure. The first step of this procedure consists in making appropriate assumptions on the considered shapes of parameter distributions and endowing them with priors. For example, β is assumed to take the multivariate normal distribution (possibly truncated, to depict the regularity conditions), $\beta \sim N(\mu, \Sigma)$.

The complexity of stochastic frontier models makes numerical integration methods inevitable. In the current study, as in most recent Bayesian literature, this procedure is based upon Markov chain Monte Carlo (MCMC), as introduced by Koop, Steel, and Osiewalski (1995). To evaluate the convergence of the MCMC estimation procedure, the following tests were done:

- assessment of the history plot (which plots the estimated parameter value against the iteration number),
- autocorrelation tests: high autocorrelation might imply slow exploration of the entire posterior distribution,
- evaluation of posterior kernel density plots.

and Sickles' (1990) model: (i) it labels all omitted time-invariant effects as inefficiency, and (ii) it can only measure countries relative to each other, not relative to the frontier, set up in absolute terms.

Due to the obvious multi-collinearity present in our data, consisting of HP-filtered, constructed time series of annual frequency, this Bayesian procedure suffers from low estimation efficiency and may run into risk of leading to spurious results. We have therefore limited our SF analysis to data of decadal frequency. As we shall see shortly, this is enough to show significant departures of the Cobb–Douglas and translog parametric results from the non-parametric DEA benchmark, and to characterize a number of intriguing properties of the estimated production function.

2.2.7 Advantages and limitations of the SFA approach

A large amount of work has been devoted in the literature to the development of Bayesian methods suitable for making inference in stochastic frontier models. Some of the important advantages of this approach include: (i) the possibility of exact inference on technical efficiency, (ii) the possibility of using some a priori knowledge on the shape of aggregate production functions as well as (iii) relatively easy incorporation of ideas and restrictions such as regularity conditions, or the optimal treatment of parameter and model uncertainty.

Although applications of Bayesian approaches to SFA are popular in the empirical literature, some competing methods, such as the aforementioned deterministic DEA, have also been strongly advocated. Undoubtedly, SFA makes it possible to account for the stochastic disturbances or measurement errors to which DEA method seems quite sensitive (cf. Koop and Steel, 2001). However, while choosing the SFA approach (based upon either classical or Bayesian econometrics), any researcher has to make far more assumptions than in the case of DEA. The utmost objective of comparing these two approach is thus to make these assumptions testable.

3 The World Technology Frontier: 1980–2004

Before turning to our benchmark specification of the aggregate production function:

$$Y = F(K, H^U, H^S),$$

where output is produced using physical capital and the stocks of unskilled and skilled labor as (mutually imperfectly substitutable) inputs, let us first offer a simple illustration why, according to our data, the aggregate production function specification is an issue which cannot be resolved by a simple imposition of a Cobb–Douglas relationship.

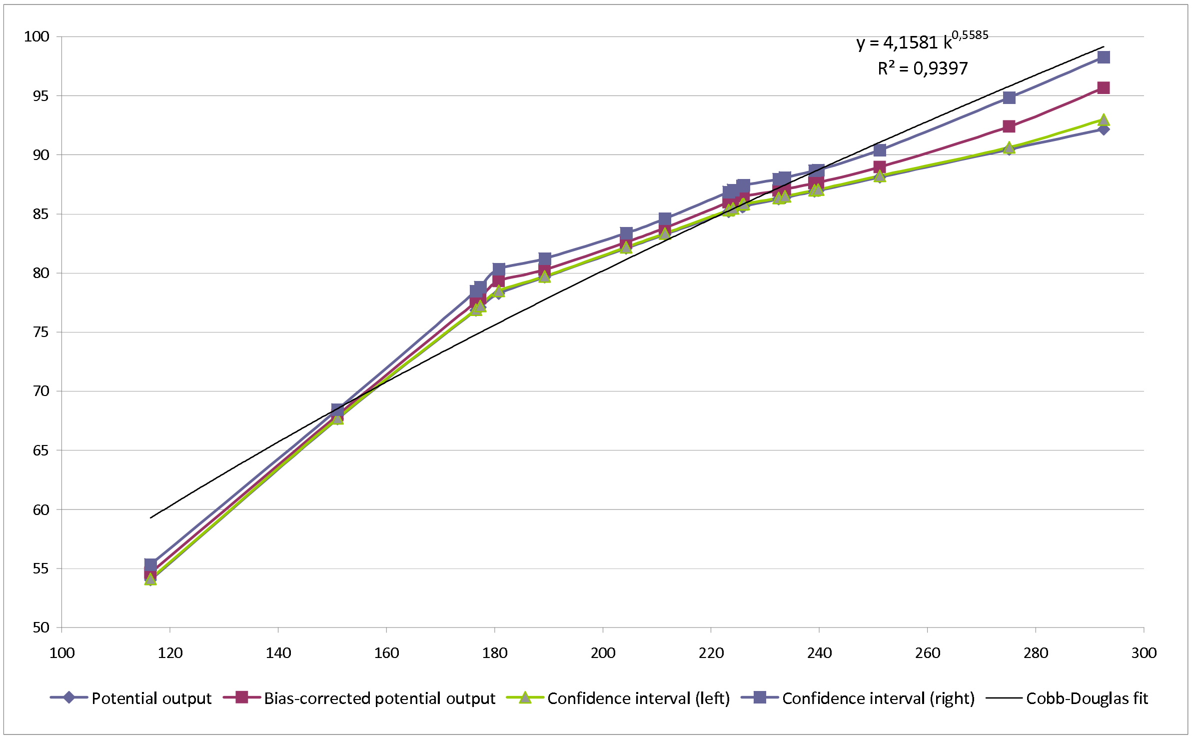

Figure 1: Potential output given inputs in 2000 – estimates of a two-factor production function with constant returns to scale. DEA vs. Cobb-Douglas.

In Figure 1 we see that even when human capital is not included in the production function, and the function itself is assumed to have constant returns to scale to capital and (unaugmented) labor: $Y/L = f(K/L)$, the deviations from Cobb–Douglas are already visible. The production function estimated with the DEA has visibly more curvature, leading to a conclusion that as far as countries' potential product (as captured by the WTF) is concerned, the Cobb–Douglas function will systematically overestimate productivity for extremely low and high capital endowments, and underestimate it in the intermediate range. This stylized result does not depend on whether DEA bias is eliminated with the Simar and Wilson's bootstrap or not.

As we will see shortly, however, neglecting human-capital augmentation of labor, assuming perfect substitutability between its unskilled and skilled part, and imposing constant returns to scale, can lead to serious misspecification problems. Hence, our benchmark specification will not impose such restrictions.

Characterizing the WTF in 1980–2004, we shall first concentrate on the results obtained with the DEA approach. These results are presented in Tables 1–3. Table 1 presents Debreu-Farrell efficiency measures computed for each country and year (1 represents 100% efficiency, and the larger is the number, the more inefficient is the data unit), measuring distance to the WTF. Table 2 presents bootstrap–corrected efficiency measures. As opposed to the original distances, these ones have been corrected for the inherent bias in DEA estimates. Table 3 presents “potential” (WTF-based, bias-corrected) output per worker, denominated in thousands of PPP converted US dollars under constant prices as of year 2000. Naturally, it is the product of each country’s actual GDP and the Debreu-Farrell efficiency measure, capturing the distance to the WTF.

Table 1: Distances to the World Technology Frontier. Debreu-Farrell efficiency measures.

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Greece	Ireland	Italy	Japan	Netherlands	Norway	Portugal	Spain	Sweden	Switzerland	UK	USA
1980	1,2022	1,1343	1,0686	1,0702	1,1995	1,2634	1,1237	1,2057	1,0085	1,0000	1,1874	1,0496	1,0000	1,0192	1,0000	1,1711	1,0000	1,0000	1,0000
1981	1,2034	1,1245	1,0765	1,0707	1,2032	1,2587	1,1241	1,1908	1,0125	1,0000	1,1882	1,0505	1,0000	1,0359	1,0000	1,1734	1,0000	1,0000	1,0000
1982	1,2195	1,1287	1,0800	1,1246	1,1879	1,2517	1,1192	1,2212	1,0163	1,0000	1,1851	1,0753	1,0000	1,0504	1,0000	1,1733	1,0000	1,0000	1,0000
1983	1,2484	1,1294	1,0850	1,1324	1,1733	1,2488	1,1188	1,2265	1,0212	1,0000	1,1708	1,1083	1,0000	1,0350	1,0000	1,1670	1,0000	1,0000	1,0000
1984	1,2315	1,1260	1,0810	1,1277	1,1489	1,2419	1,1207	1,2305	1,0224	1,0000	1,1636	1,1115	1,0000	1,0301	1,0000	1,1566	1,0000	1,0000	1,0000
1985	1,2158	1,1266	1,0749	1,1266	1,1244	1,2410	1,1242	1,2265	1,0000	1,0000	1,1522	1,1238	1,0000	1,0192	1,0021	1,1485	1,0000	1,0000	1,0000
1986	1,2064	1,1222	1,0727	1,1357	1,1075	1,2438	1,1249	1,2305	1,0000	1,0000	1,1410	1,1319	1,0000	1,0097	1,0058	1,1518	1,0000	1,0000	1,0000
1987	1,2168	1,1256	1,0707	1,1506	1,1068	1,2378	1,1216	1,2309	1,0000	1,0000	1,1373	1,1597	1,0000	1,0082	1,0000	1,1557	1,0000	1,0000	1,0000
1988	1,2162	1,1343	1,0674	1,1597	1,1150	1,2326	1,1206	1,2280	1,0000	1,0000	1,1260	1,1632	1,0000	1,0100	1,0000	1,1570	1,0000	1,0000	1,0000
1989	1,2089	1,1404	1,0679	1,1774	1,1216	1,2321	1,1125	1,2372	1,0000	1,0000	1,1128	1,1519	1,0000	1,0108	1,0000	1,1571	1,0000	1,0000	1,0000
1990	1,2161	1,1439	1,0739	1,2069	1,1382	1,2549	1,1110	1,2477	1,0000	1,0000	1,1010	1,1391	1,0000	1,0000	1,0000	1,1625	1,0000	1,0042	1,0000
1991	1,2421	1,1391	1,0757	1,2486	1,1380	1,2997	1,1125	1,2827	1,0000	1,0000	1,0909	1,1412	1,0000	1,0000	1,0000	1,1719	1,0000	1,0128	1,0000
1992	1,2472	1,1391	1,0832	1,2706	1,1432	1,3534	1,1199	1,2895	1,0000	1,0047	1,0904	1,1443	1,0000	1,0148	1,0211	1,1992	1,0000	1,0093	1,0000
1993	1,2421	1,1469	1,0960	1,2749	1,1533	1,3924	1,1358	1,3019	1,0000	1,0149	1,1008	1,1587	1,0000	1,0280	1,0512	1,2232	1,0334	1,0001	1,0000
1994	1,2277	1,1670	1,1127	1,2749	1,1526	1,3951	1,1492	1,3094	1,0000	1,0199	1,1240	1,1763	1,0000	1,0443	1,0680	1,2145	1,0941	1,0000	1,0000
1995	1,2044	1,1816	1,1168	1,2689	1,1632	1,3780	1,1514	1,3214	1,0000	1,0204	1,1450	1,1726	1,0000	1,0621	1,0767	1,1945	1,2231	1,0000	1,0000
1996	1,2034	1,2025	1,1278	1,2729	1,1684	1,3617	1,1617	1,3268	1,0000	1,0222	1,1681	1,1721	1,0000	1,0717	1,0881	1,1888	1,2887	1,0000	1,0000
1997	1,2115	1,2117	1,1401	1,2747	1,1741	1,3432	1,1795	1,3403	1,0000	1,0298	1,1988	1,1712	1,0000	1,0815	1,0983	1,1758	1,3383	1,0000	1,0000
1998	1,2145	1,2256	1,1507	1,2694	1,1901	1,3484	1,1812	1,3307	1,0000	1,0399	1,2498	1,1762	1,0000	1,1033	1,1111	1,1608	1,3693	1,0000	1,0000
1999	1,2216	1,2261	1,1608	1,2615	1,2047	1,3430	1,1820	1,3364	1,0000	1,0549	1,3074	1,1806	1,0000	1,1317	1,1063	1,1476	1,4011	1,0000	1,0000
2000	1,2244	1,2268	1,1629	1,1851	1,2158	1,3383	1,1811	1,3314	1,0000	1,0699	1,3567	1,1915	1,0000	1,1541	1,0987	1,1355	1,4318	1,0000	1,0000
2001	1,2317	1,2249	1,1623	1,0000	1,2206	1,3340	1,1798	1,3245	1,0000	1,0814	1,3856	1,1930	1,0000	1,1697	1,0964	1,1240	1,4393	1,0000	1,0000
2002	1,2309	1,2277	1,1722	1,0000	1,2265	1,3294	1,1815	1,3135	1,0000	1,0917	1,4128	1,2028	1,0000	1,1826	1,0990	1,1128	1,4516	1,0000	1,0000
2003	1,2296	1,2270	1,1812	1,0000	1,2320	1,3238	1,1850	1,3065	1,0000	1,0985	1,4302	1,2160	1,0000	1,1957	1,0881	1,1000	1,4686	1,0000	1,0000
2004	1,2288	1,2421	1,1881	1,0000	1,2376	1,3177	1,1875	1,3084	1,0000	1,1045	1,4384	1,2217	1,0000	1,2067	1,0683	1,0880	1,4848	1,0000	1,0000

Table 2: Distances to the World Technology Frontier. Bootstrap-based bias-corrected Debreu-Farrell efficiency measures.

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Greece	Ireland	Italy	Japan	Netherlands	Norway	Portugal	Spain	Sweden	Switzerland	UK	USA
1980	1,2087	1,1391	1,0730	1,0869	1,2161	1,2721	1,1362	1,2169	1,0389	1,0364	1,1976	1,0609	1,0340	1,0331	1,0293	1,1769	1,0543	1,0125	1,0589
1981	1,2095	1,1282	1,0812	1,0876	1,2202	1,2671	1,1348	1,2032	1,0358	1,0299	1,1980	1,0609	1,0379	1,0485	1,0289	1,1789	1,0590	1,0173	1,0576
1982	1,2265	1,1323	1,0852	1,1440	1,2047	1,2595	1,1293	1,2302	1,0359	1,0251	1,1952	1,0856	1,0353	1,0625	1,0293	1,1782	1,0593	1,0209	1,0530
1983	1,2573	1,1330	1,0908	1,1502	1,1894	1,2560	1,1285	1,2340	1,0425	1,0233	1,1809	1,1201	1,0348	1,0476	1,0288	1,1716	1,0570	1,0228	1,0602
1984	1,2384	1,1293	1,0868	1,1448	1,1643	1,2493	1,1299	1,2371	1,0473	1,0217	1,1732	1,1221	1,0430	1,0415	1,0247	1,1611	1,0569	1,0227	1,0745
1985	1,2208	1,1295	1,0803	1,1442	1,1411	1,2481	1,1332	1,2325	1,0310	1,0258	1,1611	1,1343	1,0571	1,0313	1,0210	1,1526	1,0596	1,0237	1,0741
1986	1,2116	1,1251	1,0785	1,1576	1,1241	1,2502	1,1347	1,2360	1,0575	1,0291	1,1498	1,1428	1,0627	1,0205	1,0233	1,1557	1,0608	1,0243	1,0776
1987	1,2220	1,1285	1,0758	1,1768	1,1203	1,2437	1,1326	1,2354	1,0891	1,0284	1,1493	1,1719	1,0686	1,0183	1,0223	1,1595	1,0605	1,0239	1,0809
1988	1,2215	1,1372	1,0723	1,1874	1,1253	1,2390	1,1324	1,2336	1,1055	1,0313	1,1392	1,1746	1,0614	1,0189	1,0276	1,1611	1,0604	1,0241	1,0830
1989	1,2148	1,1434	1,0727	1,2060	1,1302	1,2389	1,1266	1,2432	1,1128	1,0293	1,1263	1,1609	1,0444	1,0214	1,0330	1,1613	1,0612	1,0214	1,0796
1990	1,2222	1,1471	1,0787	1,2363	1,1476	1,2641	1,1256	1,2552	1,1130	1,0312	1,1139	1,1469	1,0399	1,0226	1,0338	1,1676	1,0612	1,0221	1,0757
1991	1,2482	1,1420	1,0796	1,2822	1,1461	1,3070	1,1268	1,2915	1,1188	1,0346	1,1037	1,1493	1,0453	1,0297	1,0260	1,1768	1,0614	1,0281	1,0690
1992	1,2545	1,1421	1,0868	1,3016	1,1515	1,3584	1,1323	1,2991	1,1163	1,0294	1,1022	1,1518	1,0575	1,0376	1,0415	1,2037	1,0484	1,0220	1,0784
1993	1,2491	1,1503	1,0996	1,3008	1,1632	1,4018	1,1472	1,3104	1,1200	1,0317	1,1119	1,1662	1,0740	1,0441	1,0670	1,2338	1,0831	1,0113	1,0902
1994	1,2339	1,1706	1,1165	1,2996	1,1623	1,4076	1,1627	1,3161	1,1288	1,0345	1,1370	1,1850	1,0916	1,0588	1,0829	1,2241	1,1446	1,0182	1,1014
1995	1,2097	1,1856	1,1211	1,2889	1,1752	1,3879	1,1656	1,3277	1,1386	1,0325	1,1605	1,1811	1,1078	1,0760	1,0905	1,2028	1,3005	1,0217	1,1093
1996	1,2093	1,2070	1,1326	1,2946	1,1828	1,3707	1,1771	1,3334	1,1410	1,0338	1,1865	1,1813	1,1369	1,0850	1,1014	1,2032	1,3542	1,0238	1,1078
1997	1,2189	1,2180	1,1457	1,3042	1,1916	1,3499	1,1984	1,3508	1,1430	1,0411	1,2225	1,1826	1,1427	1,0959	1,1114	1,1889	1,3898	1,0260	1,1105
1998	1,2263	1,2360	1,1588	1,3075	1,2134	1,3563	1,2038	1,3395	1,1541	1,0514	1,2797	1,1912	1,1527	1,1230	1,1284	1,1692	1,4172	1,0261	1,1093
1999	1,2368	1,2389	1,1716	1,3119	1,2315	1,3576	1,2076	1,3470	1,0939	1,0660	1,3414	1,1978	1,1583	1,1517	1,1288	1,1566	1,4446	1,0247	1,0985
2000	1,2421	1,2397	1,1750	1,2685	1,2447	1,3589	1,2097	1,3419	1,0835	1,0809	1,3949	1,2088	1,1603	1,1743	1,1203	1,1470	1,4726	1,0243	1,0887
2001	1,2489	1,2377	1,1738	1,0985	1,2491	1,3557	1,2068	1,3340	1,0708	1,0934	1,4226	1,2094	1,1595	1,1918	1,1165	1,1367	1,4776	1,0224	1,0774
2002	1,2486	1,2400	1,1833	1,1576	1,2533	1,3519	1,2062	1,3241	1,0670	1,1053	1,4506	1,2181	1,1675	1,2034	1,1188	1,1263	1,4838	1,0215	1,0707
2003	1,2487	1,2401	1,1936	1,1603	1,2538	1,3458	1,2062	1,3148	1,0668	1,1128	1,4677	1,2320	1,1753	1,2153	1,1090	1,1134	1,4943	1,0226	1,0703
2004	1,2479	1,2543	1,2016	1,1558	1,2579	1,3392	1,2071	1,3149	1,0774	1,1173	1,4754	1,2348	1,1719	1,2283	1,0869	1,1010	1,5080	1,0241	1,0718

Table 3: Potential output, computed based on bias-corrected Debreu-Farrell efficiency measures.

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Greece	Ireland	Italy	Japan	Netherlands	Norway	Portugal	Spain	Sweden	Switzerland	UK	USA
1980	62,26	57,83	63,57	61,09	56,70	49,47	62,35	57,10	42,13	55,47	50,58	62,50	61,24	30,47	50,57	52,34	62,27	45,18	65,99
1981	63,12	58,26	65,42	61,36	57,85	50,35	63,38	56,51	43,19	56,01	51,78	61,64	62,64	31,62	52,35	52,93	62,74	46,35	66,44
1982	64,88	59,50	66,95	65,08	58,10	51,15	64,20	57,77	44,43	56,54	52,81	62,53	63,70	32,74	54,13	53,55	62,89	47,67	66,89
1983	67,45	60,62	68,48	66,29	58,33	52,21	65,34	58,10	46,05	57,27	53,38	64,51	64,95	32,97	55,83	54,04	62,93	49,04	68,33
1984	67,35	61,49	69,34	67,01	58,07	53,28	66,68	58,66	47,72	58,16	54,40	65,10	66,80	33,46	57,26	54,44	63,15	50,32	70,42
1985	67,21	62,59	70,00	68,00	57,86	54,74	68,22	59,09	48,54	59,53	55,37	66,60	69,02	33,81	58,55	54,93	63,50	51,63	71,62
1986	67,40	63,46	70,98	69,68	57,94	56,49	69,76	60,07	51,52	60,99	56,51	68,01	70,71	34,13	59,99	55,94	63,66	52,82	73,03
1987	68,60	64,87	71,97	71,55	58,69	57,94	71,16	60,97	55,01	62,27	58,27	70,71	72,49	34,73	61,09	56,91	63,67	53,75	74,35
1988	69,20	66,68	73,00	72,76	59,93	59,45	72,73	61,91	57,96	63,69	59,53	71,83	73,55	35,42	62,53	57,70	63,69	54,47	75,53
1989	69,56	68,39	74,32	74,33	61,21	61,08	73,87	63,42	60,52	64,67	60,48	71,87	74,14	36,18	64,07	58,40	63,79	54,86	76,27
1990	70,97	69,86	76,04	76,61	63,24	63,89	75,19	64,95	62,67	65,74	61,15	71,78	75,84	36,90	65,47	59,52	63,85	55,47	76,94
1991	73,81	70,65	77,36	80,03	64,31	67,76	76,49	67,52	65,05	66,88	61,60	72,63	78,47	37,83	66,48	61,06	63,96	56,61	77,45
1992	75,79	71,66	79,10	82,09	65,83	72,50	77,99	68,32	66,90	67,62	62,25	73,46	81,78	38,80	69,11	63,90	63,42	57,41	79,23
1993	77,21	73,23	81,26	83,17	67,76	77,30	80,09	69,12	69,18	69,08	63,39	75,15	85,51	39,73	72,41	67,33	65,96	58,19	81,32
1994	78,03	75,78	83,79	84,39	69,01	80,26	82,26	69,66	71,95	70,72	65,43	77,23	89,32	40,99	74,84	68,85	70,33	60,08	83,49
1995	78,23	78,31	85,43	85,08	71,10	81,66	83,55	70,75	75,06	72,00	67,41	77,90	92,94	42,36	76,31	69,72	80,75	61,77	85,59
1996	79,97	81,57	87,62	86,93	72,88	83,01	85,51	71,86	77,89	73,30	69,53	78,89	97,54	43,43	77,61	71,77	85,05	63,31	87,16
1997	82,42	84,34	89,94	89,18	74,74	83,92	88,24	73,94	80,83	74,77	72,18	79,97	100,00	44,59	78,55	72,80	88,34	64,80	89,24
1998	84,68	87,68	92,24	91,12	77,43	86,33	89,79	74,77	84,49	76,21	76,11	81,57	102,68	46,43	79,75	73,28	91,14	66,14	91,12
1999	87,02	89,92	94,48	93,12	79,92	88,26	91,14	76,99	82,83	77,73	80,52	82,98	104,95	48,37	79,63	73,95	93,89	67,37	92,19
2000	88,85	91,83	95,89	91,50	82,11	90,04	92,26	78,79	84,81	79,06	84,75	84,61	106,95	50,10	78,79	74,65	96,58	68,65	93,22
2001	90,70	93,35	96,86	80,24	83,76	91,42	92,92	80,60	86,59	80,00	87,70	85,45	108,83	51,63	78,23	75,29	97,62	69,83	94,00
2002	91,99	95,06	98,73	85,36	85,42	92,76	93,81	82,29	89,09	80,80	90,92	86,95	111,74	52,93	78,04	76,12	98,67	71,07	95,09
2003	93,31	96,52	100,68	86,21	86,88	94,07	94,88	83,92	91,93	81,40	93,67	89,15	114,89	54,26	76,96	77,03	100,07	72,47	96,70
2004	94,59	99,03	102,41	86,51	88,64	95,42	96,17	85,95	95,83	82,08	95,92	90,91	117,18	55,65	75,00	78,12	101,77	73,87	98,43

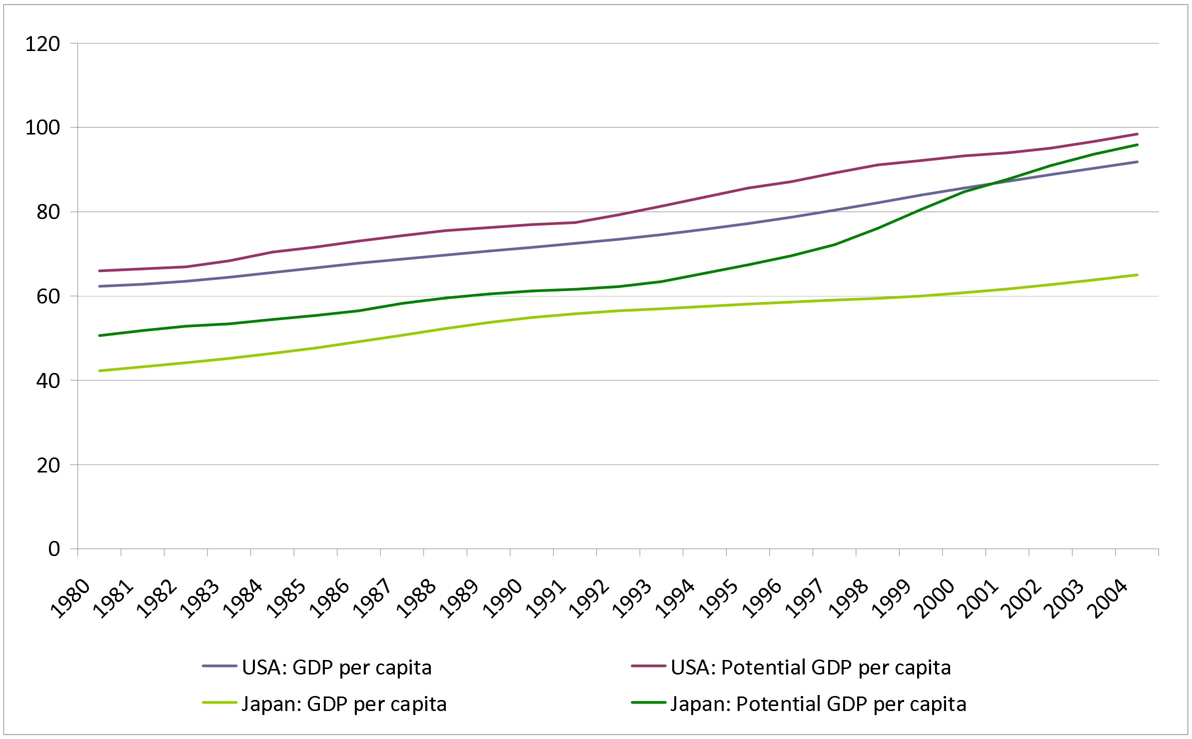

Figure 2: Growth in output and potential output. USA and Japan.

What cannot be seen too clearly in Tables 1–3 is that there are interesting regularities visible in the observed trends. For example, as illustrated in Figure 2, growth in actual and potential productivity can be parallel, but there also can be clear departures from the parallel pattern: while the USA maintained a relatively stable distance to the WTF across time, in Japan it diverged, especially in the last years.

Diverging stories can also be told about Greece and Ireland. In the former country, distance to the WTF in terms of technical efficiency was sizeable and increasing throughout the period; in the latter, it was much smaller, and distance to the WTF first increased but then decreased again.

After a brief presentation of our results, let us pass to the presentation of their corollaries for the shape of the aggregate production function. In particular, we shall dwell on the discrepancies between the nonparametric DEA outcomes discussed above and their SFA counterparts obtained under the parametric assumption of a Cobb–Douglas or translog production function. This will be done in the following section.

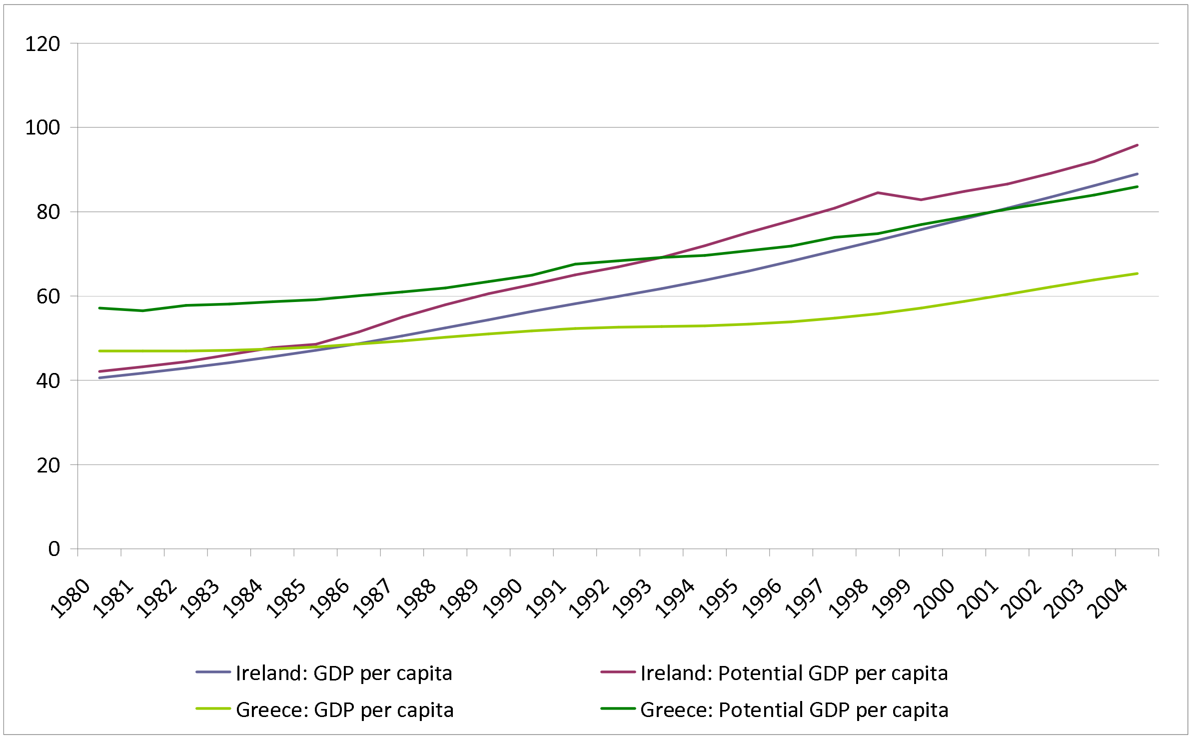

Figure 3: Growth in output and potential output. Greece and Ireland.

4 The shape of aggregate production functions: evidence from estimates of the World Technology Frontier

In the current section, we will address the following questions:

1. Does the Cobb–Douglas aggregate production function correctly capture the properties of the WTF estimated with the bias-corrected DEA method?
2. How do translog production function estimates compare against the Cobb–Douglas and DEA-based ones?
3. Are the estimated parameters of aggregate production functions economically plausible? In the translog case, what do they imply for partial elasticities and their dependence on factor endowments?
4. Is the standard assumption of (both locally and globally) constant returns to scale valid?

We will see shortly that the answer to the first question is generally negative. Indeed, our study provides several arguments against the validity of Cobb–Douglas aggregate production functions, corroborating the preliminary evidence illustrated in Figure 1.

Regarding the second and third question, however, we are not able to offer an alternative *parametric* form of the function that would be in sufficiently good agreement with the nonparametric (bias-corrected) DEA benchmark and thus could be used as a worthy subsititute of the Cobb–Douglas in subsequent parametric analyses. In particular, our SFA-based estimates of translog production functions indicate marked departures of this particular functional specification from the DEA benchmark, too. On the other hand, these translog estimations provide a strong argument why the Cobb–Douglas is too simple a specification to match the complex patterns present in the data: partial elasticities inferred from the translog production function are varying substantially across countries, and they are heavily correlated with factor endowments.

Turning to the last question, the available evidence on constant vs. variable returns to scale is ambiguous. In a series of DEA-based tests of *local* returns to scale (in a given country and year), the null of their constancy is rarely rejected. In a test of *global* constancy of returns to scale, the null of constant returns to scale can be rejected against the alternative of variable returns to scale with 99% confidence.

Let us first document the notable differences between Cobb-Douglas and translog parametric specifications of the aggregate production function and their nonparametric counterparts.

4.1 DEA vs. SFA

Table 4 presents a comparison of eight different characterizations of the World Technology Frontier in the year 2000. In consecutive columns, we document Farrell efficiency measures θ_i (such that potential output of country i at WTF is $Y_i^* = \theta_i Y_i$) computed according to the following methodologies:

1. Bias-corrected DEA with constant returns to scale, and aggregate capital and labor taken as inputs.

2. Bias-corrected DEA with variable returns to scale, and aggregate physical and human capital as inputs.

3. Bias-corrected DEA with variable returns to scale, and aggregate physical capital as well as unskilled and skilled labor as inputs (the difference between these estimates and the aforementioned ones capture the degree of imperfect substitutability between unskilled and skilled labor).

4. SFA under the assumption of a Cobb–Douglas production function with variable returns to scale and aggregate physical capital as well as unskilled and skilled labor as inputs.

5. SFA under the assumption of a Cobb–Douglas production function with variable returns to scale and aggregate physical capital as well as unskilled and skilled labor as inputs, with partial elasicities restricted to the range $(0, 1)$.[16]

6. SFA under the assumption of a Cobb–Douglas production function with constant returns to scale and aggregate physical capital as well as unskilled and skilled labor as inputs.

7. SFA under the assumption of a translog production function with aggregate physical capital as well as unskilled and skilled labor as inputs, estimated in an intensive form: $\frac{Y}{H^U} = F\left(\frac{K}{H^U}, \frac{H^S}{H^U}\right)$.

8. SFA under the assumption of a translog production function with variable returns to scale and aggregate physical capital as well as unskilled and skilled labor as inputs.

Table 4: Technical efficiency – comparison of alternative measurements for the year 2000.

	DEA(K,L)	DEA(K,H)	DEA(K,Hu,Hs)	SFA-CD (K,Hu,Hs)	SFA-CD (K,Hu,Hs) [Restricted]	SFA-CD (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs) [intensive]	SFA-Translog (K,Hu,Hs)
Australia	1,2021	1,2381	1,2421	1,1315	1,1041	1,1317	1,0921	1,0704
Austria	1,1628	1,2407	1,2397	1,1635	1,1335	1,1893	1,1450	1,1440
Belgium	1,0734	1,1972	1,1750	1,3205	1,2798	1,3277	1,3019	1,2625
Canada	1,2075	1,2700	1,2685	1,1236	1,1115	1,1202	1,0565	1,0389
Denmark	1,2168	1,2248	1,2447	1,0868	1,0714	1,1254	1,0970	1,1452
Finland	1,2469	1,3526	1,3589	1,0708	1,0549	1,1011	1,0590	1,0826
France	1,1406	1,2041	1,2097	1,2657	1,2151	1,2025	1,1781	1,1747
Greece	1,3207	1,3532	1,3419	1,1878	1,1521	1,1999	1,0945	1,0702
Ireland	1,0133	1,0635	1,0835	1,3736	1,3398	1,4596	1,4552	1,6770
Italy	1,1824	1,2216	1,0809	1,4633	1,4041	1,3716	1,3163	1,3259
Japan	1,4641	1,4780	1,3949	1,0636	1,0339	1,0196	1,0239	1,0707
Netherlands	1,2522	1,2683	1,2088	1,1689	1,1467	1,1497	1,1592	1,1414
Norway	1,0377	1,0574	1,1603	1,1164	1,1140	1,1513	1,3704	1,4399
Portugal	1,2806	1,2104	1,1743	1,8440	1,7765	1,8699	1,4997	1,4533
Spain	1,1920	1,1944	1,1203	1,7358	1,6628	1,6664	1,4765	1,3906
Sweden	1,1964	1,1791	1,1470	1,1159	1,0961	1,1374	1,1010	1,1052
Switzerland	1,4088	1,4592	1,4726	1,0140	1,0151	1,0185	1,0761	1,0820
UK	1,0140	1,0263	1,0243	1,4592	1,3845	1,3990	1,2945	1,2237
USA	1,0104	1,1210	1,0887	1,4015	1,3678	1,2999	1,2129	1,4271

Table 5 complements Table 4 by presenting a comparison of alternative estimates of potential output in per worker terms (for international comparability). Marked

[16] Without this restriction, the Bayesian estimation procedure leads to a negative estimate of the partial elasticity of unskilled labor.

departures of parametric SFA results (both Cobb–Douglas and translog) from the DEA benchmark are clearly visible. Most importantly, though, the discrepancy is the strongest with respect to the estimates of the Cobb–Douglas function where potential output is under- or overestimated by far in certain countries such as Ireland, Spain, and the USA. The translog production function alleviates some of this discrepancy, but does not go the full way.

To make sure, stochastic estimates included in Table 5 are computed as

$$Y_{it}^{*} = \theta_{it} Y_{it} = \theta_{it} F(K_{it}, H_{it}^{U}, H_{it}^{S}) \exp\left(v_{it}\right),$$

where $\theta_{it} = \exp(-u_{it})$ is the Debreu-Farrell efficiency measure reported above. Hence, by definition they contain the idiosyncratic disturbance term v_{it} as well, which could dominate the result if the postulated functional form of the aggregate production function provides a bad fit to the data. To get rid of this feature of our results, we have also computed potential output according to:

$$Y_{it}^{**} = \theta_{it} F(K_{it}, H_{it}^{U}, H_{it}^{S}),$$

so that the idiosyncratic disturbance term is not included. The results are presented in Table 6 where it is observed that the differences between the DEA and SFA results are not that large anymore if idiosyncratic disturbances are not included. One has to keep in mind, though, that in likely case of production function misspecification, numbers reported in SFA columns of Table 6 will be biased. The reason is that they do not represent inefficiency-corrected measures of *actual* output, but of output *as if* the estimated production function provided a perfect fit to the data (which it does not).

Differences across different production function specifications, documented in Tables 4–6, suggest that the parametric functional forms used in our SFA analyses, especially the Cobb–Douglas ones, are likely to be misspecified. They also constitute suggestive evidence that allowing for imperfect substitutability between unskilled and skilled labor helps obtain significantly different (and thus certainly better, since this step allows for more generality) results, supporting the earlier findings by Growiec (2010a,b).

Having said that, let us now discuss the SFA results in more detail.

4.2 Parametric estimates of the aggregate production function

Our parametric, SFA-based estimates of aggregate production functions have been contained in Table 7. The estimated parameter η is the technical change parameter

Table 5: Potential output – comparison of alternative estimates. Stochastic estimates include inefficiency and idiosyncratic errors.

	DEA(K,L)	DEA(K,H)	DEA(K,Hu,Hs)	SFA-CD (K,Hu,Hs)	SFA-CD (K,Hu,Hs) [Restricted]	SFA-CD (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs) [intensive]	SFA-Translog (K,Hu,Hs)
Australia	85,99	88,57	88,85	80,94	78,98	80,96	78,12	76,57
Austria	86,14	91,91	91,83	86,18	83,97	88,10	84,81	84,75
Belgium	87,60	97,69	95,89	107,76	104,43	108,34	106,24	103,02
Canada	87,10	91,61	91,50	81,05	80,17	80,80	76,21	74,94
Denmark	80,28	80,81	82,11	71,70	70,68	74,24	72,37	75,55
Finland	82,61	89,62	90,04	70,95	69,89	72,95	70,16	71,73
France	86,99	91,83	92,26	96,52	92,67	91,71	89,85	89,59
Greece	77,54	79,45	78,79	69,74	67,64	70,45	64,26	62,84
Ireland	79,31	83,25	84,81	107,52	104,87	114,25	113,90	131,27
Italy	86,48	89,35	79,06	107,03	102,70	100,32	96,28	96,98
Japan	88,96	89,80	84,75	64,62	62,82	61,95	62,21	65,05
Netherlands	87,65	88,77	84,61	81,82	80,26	80,47	81,14	79,90
Norway	95,65	97,47	106,95	102,91	102,68	106,12	126,32	132,73
Portugal	54,63	51,63	50,10	78,67	75,79	79,77	63,98	62,00
Spain	83,84	84,00	78,79	122,08	116,94	117,20	103,84	97,80
Sweden	77,87	76,74	74,65	72,63	71,34	74,02	71,65	71,93
Switzerland	92,40	95,70	96,58	66,50	66,58	66,80	70,57	70,96
UK	67,96	68,79	68,65	97,80	92,79	93,77	86,76	82,02
USA	86,52	95,99	93,22	120,01	117,12	111,31	103,85	122,20

of the Battese and Coelli (1995) intertemporal component of the inefficiency term z_t, whereas λ is the mean of the distribution of its time-invariant component u_i.

We find that under any of the three considered Cobb–Douglas specifications, the partial elasticity with respect to capital is consistently estimated at $0.6-0.7$ which is a large number. The partial elasticity with respect to unskilled labor, on the other hand, is typically close to zero, or even negative (if the parameter range is left unrestricted). Measures of scale elasticity are generally less than unity, suggesting (mildly) decreasing returns at the aggregate level; standard errors of estimation suggest that they might be indistinguishable from unity (representing constant returns), though.

Turning to the estimates of the translog production function reported in Table 7, we observe that at least some of the quadratic terms are statistically significant, indicating non-negligible departures from the Cobb–Douglas benchmark. In such case, aggregate returns to scale, when not restricted to be constant, can be country-specific. The same applies to partial elasticities – their magnitude will vary across countries and time. We shall document these meaningful variations in the following subsection.

One may also draw a few conclusions on the preferred shape of the aggregate (WTF-based) production function by comparing potential output excluding the idiosyncratic disturbance term, Y^{**}, to each country's actual output Y. Ratios of form Y^{**}/Y have been presented in Table 8, allowing us to see where departures of output from the assumed functional form (controlling for inefficiency) are most pronounced. What is especially notable there, is that there are some strong correlations between these

Table 6: Potential output – comparison of alternative estimates. Stochastic estimates include inefficiency but not the idiosyncratic errors.

	DEA(K,L)	DEA(K,H)	DEA(K,Hu,Hs)	SFA-CD (K,Hu,Hs)	SFA-CD (K,Hu,Hs) [Restricted]	SFA-CD (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs) [intensive]	SFA-Translog (K,Hu,Hs)
Australia	85,99	88,57	88,85	68,94	68,44	69,46	64,00	70,75
Austria	86,14	91,91	91,83	71,99	71,28	72,04	63,30	73,58
Belgium	87,60	97,69	95,89	80,91	80,72	80,99	64,03	84,03
Canada	87,10	91,61	91,50	74,70	72,68	74,52	73,25	80,48
Denmark	80,28	80,81	82,11	59,06	58,86	58,64	55,33	62,26
Finland	82,61	89,62	90,04	61,98	62,22	61,26	59,79	63,87
France	86,99	91,83	92,26	76,07	75,61	76,27	66,14	77,44
Greece	77,54	79,45	78,79	58,80	59,19	58,58	54,62	60,98
Ireland	79,31	83,25	84,81	67,26	66,25	67,54	47,77	75,39
Italy	86,48	89,35	79,06	74,31	75,20	74,75	57,79	77,67
Japan	88,96	89,80	84,75	64,33	64,46	67,34	64,32	61,62
Netherlands	87,65	88,77	84,61	70,48	70,42	70,12	59,69	70,30
Norway	95,65	97,47	106,95	80,48	78,71	79,38	58,01	82,64
Portugal	54,63	51,63	50,10	48,91	49,01	49,23	30,99	47,50
Spain	83,84	84,00	78,79	83,77	84,22	84,76	54,16	78,52
Sweden	77,87	76,74	74,65	59,89	59,45	59,04	58,25	64,58
Switzerland	92,40	95,70	96,58	73,19	72,35	71,51	64,75	71,46
UK	67,96	68,79	68,65	64,02	63,61	64,15	53,61	67,49
USA	86,52	95,99	93,22	81,39	79,49	82,30	68,21	85,11

departures, and factor endowments. In particular, for all-but-one tested production functions, departures from the function are positively correlated with the unskilled labor stock and negatively correlated with the skilled labor stock. Correlation with output is obvious; it is however notable that its magnitude varies across proposed specifications, being largest for the Cobb–Douglas.

4.3 Nonconstancy of partial elasticities

Another test, aiming at defining the desirable properties of aggregate production functions, is to check if their partial elasticities tend to vary across countries and time if they are not restricted against such behavior.[17] Table 9, based on the translog specification, shows that it is indeed the case. Similar patterns are observed across countries in our translog estimates both in the CRS case (estimated according to an intensive form of the translog production function), and in the VRS case.

The sheer variability is not the key in documenting deviations from the Cobb–Douglas, though. What is particularly notable here is that partial elasticities are strongly correlated with factor endowments, suggesting that these devations are *systematic*. Even though the translog might not be the optimal specification for the

[17]Bernanke and Gürkaynak (2001) as well as Gollin (2002) have documented substantial variability of capital and labor income shares across countries. Our current exercise, documenting the variability of implied partial elasticities, is complementary to theirs: partial elasticities and factor shares coincide under the Cobb–Douglas specification but the former depend on factor endowments otherwise.

Table 7: Parameters of the estimated production functions (SFA).

	SFA-CD (K,Hu,Hs)	s.e.	SFA-CD (K,Hu,Hs) [Restricted]	s.e.	SFA-CD (K,Hu,Hs) [CRS]	s.e.	SFA-Translog (K,Hu,Hs) [intensive form]	s.e.	SFA-Translog (K,Hu,Hs)	s.e.
Constant	0,8143	0,5760	0,8088	0,4791	0,0143	0,2714	-0,7068	2,0330	-14,6600	7,1070
LNK	0,6524	0,0576	0,6160	0,0559					2,3670	1,0660
LNHU	-0,0120	0,0345	0,0226	0,0197					-0,0436	0,8460
LNHS	0,3180	0,0480	0,3322	0,0511					0,0807	0,7220
LNK_INT					0,6807	0,0575	1,2850	0,8984		
LNHS_INT					0,3088	0,0496	0,2876	0,5878		
LNK2									-0,2053	0,1873
LNHU2									0,0937	0,0554
LNHs2									0,1710	0,0924
LNK2_INT							-0,1922	0,1998		
LNHS2_INT							0,1888	0,0976		
LNKHS_INT							0,0212	0,1280		
LNKHU									0,0995	0,0978
LNKHS									0,0399	0,1204
LNHUHS									-0,2022	0,0666
eK	0,6524		0,6160		0,6807		country-specific		country-specific	
eHu	-0,0120		0,0226		0,3088		country-specific		country-specific	
eHs	0,3180		0,3322				country-specific		country-specific	
Scale elast.	0,9584		0,9708							
η	-0,0673	0,0383	-0,0621	0,0447	-0,0711	0,0406	0,2308	0,1523	0,2527	0,1120
λ	4,5830	1,3380	5,0930	1,4040	4,6760	1,3320	5,4410	1,4920	5,0990	1,3610

aggregate production function (see the discussion in the preceding subsection), it has sufficient flexibility to accomodate such systematic variations in partial elasticities. Table 10 shows that these correlations may reach levels as high as 88%. This same finding is also depicted in Figures 4–6.

4.4 Results on returns to scale

Apart from the issues discussed above, our WTF estimates also provide interesting conclusions on local and global returns to scale. One advantage of methods used in the current analysis is that they do not require the researcher to impose a priori restrictions whether returns to scale be decreasing, increasing, or constant. This property is then only obtained as a result and can be statistically tested against the null of constant returns. We have conducted such tests for our DEA-based estimates of the aggregate production funcion, according to Löthgren and Tambour (1999) and Simar and Wilson (2002) procedures. Results of tests carried out for all units in the sample separately are summarized in Table 11. Comparing the bias-corrected DEA-based efficiency estimates under variable, non-increasing, and constant returns to scale leads, in most cases, to the conclusion that local returns to scale are constant. It is not always the case, though.

Table 8: Departures from the parametric production function in 2000, controlling for technical inefficiency.

	SFA-CD (K,Hu,Hs)	SFA-CD (K,Hu,Hs) [Restricted]	SFA-CD (K,Hu,Hs) [CRS]	SFA-Translog (K,Hu,Hs) [intensive form]	SFA-Translog (K,Hu,Hs)	mean
Australia	0,9638	0,9567	0,9710	0,8947	0,9890	0,9550
Austria	0,9718	0,9623	0,9725	0,8545	0,9933	0,9509
Belgium	0,9915	0,9892	0,9925	0,7846	1,0298	0,9575
Canada	1,0356	1,0075	1,0330	1,0155	1,1158	1,0415
Denmark	0,8953	0,8922	0,8889	0,8387	0,9438	0,8918
Finland	0,9355	0,9391	0,9246	0,9024	0,9640	0,9331
France	0,9975	0,9914	1,0000	0,8672	1,0154	0,9743
Greece	1,0014	1,0081	0,9977	0,9302	1,0386	0,9952
Ireland	0,8593	0,8464	0,8629	0,6103	0,9631	0,8284
Italy	1,0159	1,0282	1,0219	0,7901	1,0619	0,9836
Japan	1,0588	1,0609	1,1084	1,0586	1,0141	1,0602
Netherlands	1,0069	1,0060	1,0018	0,8528	1,0044	0,9744
Norway	0,8731	0,8539	0,8611	0,6293	0,8965	0,8228
Portugal	1,1464	1,1488	1,1541	0,7265	1,1135	1,0579
Spain	1,1911	1,1976	1,2052	0,7701	1,1165	1,0961
Sweden	0,9202	0,9134	0,9071	0,8950	0,9923	0,9256
Switzerland	1,1160	1,1032	1,0904	0,9873	1,0897	1,0773
UK	0,9552	0,9491	0,9572	0,7998	1,0070	0,9337
USA	0,9505	0,9283	0,9611	0,7966	0,9940	0,9261
mean	0,9940	0,9885	0,9953	0,8423	1,0180	0,9676
corr.with K/L	-0,0338	-0,0697	-0,0410	0,2097	-0,1821	0,0005
corr.with Hu/L	0,3229	0,4014	0,3186	-0,3301	0,2893	0,1950
corr.with Hs/L	-0,2911	-0,3453	-0,2868	0,5985	-0,2014	-0,0704
corr.with Y/L	-0,5131	-0,5485	-0,5074	-0,3745	-0,4870	-0,5675

Table 9: Partial elasticities estimated from the translog production function.

	CRS - eK	s.e.	CRS - eHs	s.e.	VRS - eK	s.e.	VRS - eHu	s.e.	VRS - eHs	s.e.
Australia	0,4558	0,0942	0,3839	0,0553	0,4175	0,0822	0,2349	0,0643	0,3976	0,0521
Austria	0,5204	0,1246	0,3543	0,0809	0,5099	0,1155	0,2276	0,0876	0,3601	0,0767
Belgium	0,5187	0,0850	0,2232	0,0455	0,4690	0,0754	0,3560	0,0780	0,2436	0,0425
Canada	0,4058	0,1161	0,5311	0,0885	0,3738	0,1033	0,1187	0,0583	0,5384	0,0808
Denmark	0,4441	0,0989	0,4195	0,0625	0,4668	0,0963	0,2148	0,0857	0,4237	0,0679
Finland	0,4898	0,0841	0,2171	0,0408	0,4906	0,0855	0,3876	0,0947	0,2341	0,0588
France	0,4845	0,0833	0,2728	0,0391	0,3334	0,0980	0,3137	0,0724	0,3064	0,0544
Greece	0,5122	0,0865	0,1603	0,0501	0,4705	0,0781	0,4249	0,0879	0,1847	0,0516
Ireland	0,6289	0,1459	0,1829	0,0962	0,6733	0,1493	0,3393	0,1316	0,1831	0,1004
Italy	0,4706	0,1309	0,0998	0,0845	0,2960	0,1296	0,4945	0,1065	0,1500	0,0798
Japan	0,4478	0,0869	0,3587	0,0440	0,2677	0,1183	0,2462	0,0772	0,3936	0,0755
Netherlands	0,4028	0,1869	0,1119	0,1174	0,3102	0,1680	0,5370	0,1182	0,1569	0,1117
Norway	0,4462	0,0913	0,2633	0,0391	0,4527	0,0923	0,3695	0,0931	0,2804	0,0616
Portugal	0,6643	0,1065	-0,0682	0,1009	0,6150	0,1050	0,5581	0,1158	-0,0413	0,0879
Spain	0,5696	0,1082	-0,0131	0,0916	0,4291	0,1053	0,5504	0,1037	0,0310	0,0770
Sweden	0,4432	0,0848	0,3368	0,0378	0,4212	0,0744	0,2930	0,0721	0,3528	0,0431
Switzerland	0,2869	0,1245	0,5312	0,0740	0,3119	0,1197	0,2025	0,0805	0,5423	0,0789
UK	0,5557	0,1454	0,3333	0,0957	0,4161	0,1569	0,2085	0,0859	0,3543	0,1049
USA	0,3593	0,1303	0,6378	0,1120	0,2052	0,1514	0,0236	0,0696	0,6581	0,1265
mean	0,4793		0,2809		0,4174		0,3211		0,3026	
std. dev.	0,0891		0,1828		0,1158		0,1481		0,1753	

Table 10: Partial elasticities vs. stocks.

	CRS	VRS
Capital	-0,6879	-0,5471
Unskilled labor		0,8082
Skilled labor	0,3905	0,8855
RTS vs Y/L		-0,1484

Figure 4: Physical capital stock vs. the capital elasticity of the aggregate production function.

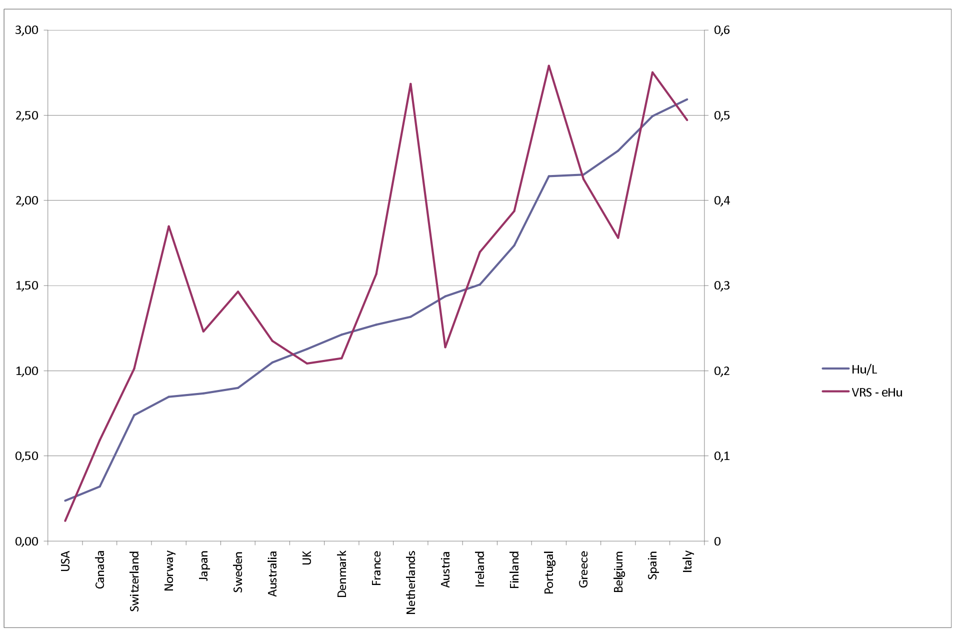

Figure 5: Stock of unskilled labor vs. the unskilled labor elasticity of the aggregate production function.

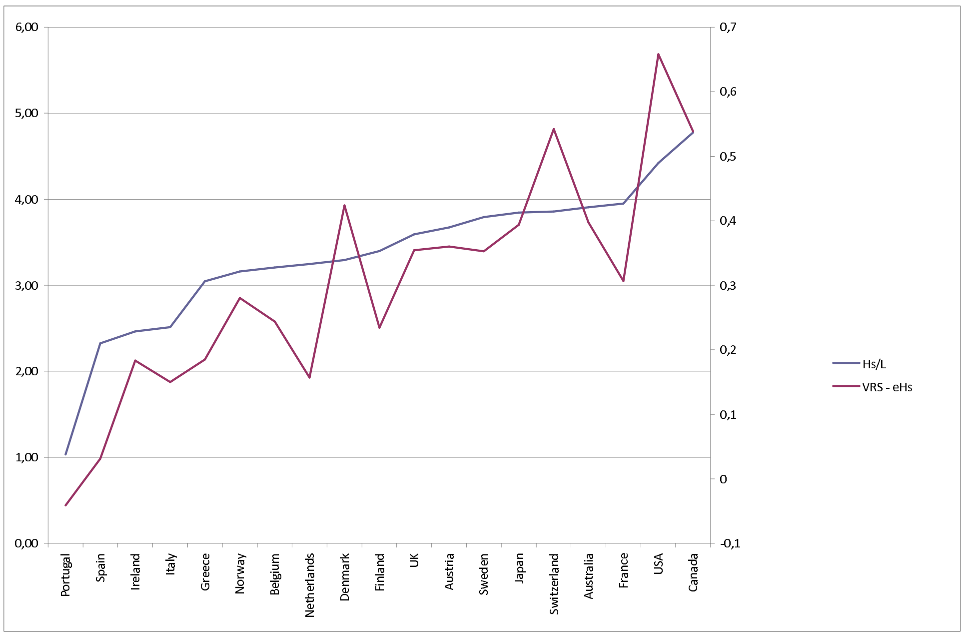

Figure 6: Stock of skilled labor vs. the skilled labor elasticity of the aggregate production function.

Table 11: Statistical tests of local returns to scale (Löthgren and Tambour, 1999). Results based on bootstrap–based DEA estimates.

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Greece	Ireland	Italy	Japan	Netherlands	Norway	Portugal	Spain	Sweden	Switzerland	UK	USA
1980	CRS	DRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	DRS	CRS	DRS	DRS	DRS	CRS
1981	CRS	CRS	CRS	CRS	DRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	DRS	CRS	DRS
1982	CRS	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS
1983	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS
1984	CRS	DRS	CRS	CRS	DRS	IRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS
1985	CRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	IRS	DRS	CRS	DRS	DRS	CRS	CRS
1986	DRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	DRS	CRS	DRS	CRS	DRS	CRS	DRS
1987	CRS	CRS	CRS	DRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	DRS	CRS	DRS	DRS
1988	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	DRS
1989	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	CRS
1990	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	IRS
1991	CRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	DRS	DRS	CRS	CRS	CRS
1992	CRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	IRS	CRS	DRS	CRS	DRS	CRS	DRS	DRS
1993	CRS	CRS	DRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	DRS	CRS	DRS	DRS	DRS
1994	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	DRS	CRS
1995	IRS	CRS	CRS	CRS	CRS	IRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	IRS	DRS	CRS	CRS	DRS
1996	CRS	CRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS
1997	CRS	CRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	DRS	DRS	CRS	CRS	CRS	DRS
1998	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	DRS	CRS	DRS	DRS	CRS
1999	CRS	DRS	CRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	DRS	CRS	DRS	DRS	DRS	CRS
2000	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	DRS	CRS	DRS
2001	CRS	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	IRS	DRS	CRS	CRS	DRS	CRS
2002	CRS	DRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS
2003	CRS	DRS	CRS	CRS	DRS	IRS	CRS	CRS	CRS	IRS	CRS	CRS	DRS	DRS	CRS	CRS	CRS	DRS	CRS
2004	IRS	DRS	CRS	CRS	DRS	CRS	CRS	DRS	CRS	CRS	CRS	DRS	CRS	DRS	CRS	DRS	DRS	CRS	CRS

Going beyond testing local returns to scale for individual observations, one could also carry out the Simar–Wilson (2002) statistical test of global returns to scale, based on all data. The results are illustrated in Figure 7. Plotted against the timeline, this graph shows how each of the quartiles of Shephard distance ratios shifts across time. The most important feature of this Figure is that all these lines are located above one. Therefore, under all conventional significance levels (including $\alpha = 1\%$), the null of constant returns to scale has to be rejected for the alternative of variable returns to scale.

In sum, DEA-based returns-to-scale tests provide mixed evidence on this property. On the one hand, the aggregate production function is often locally indistinguishable from CRS; on the other hand, it is also robustly identified as globally VRS.

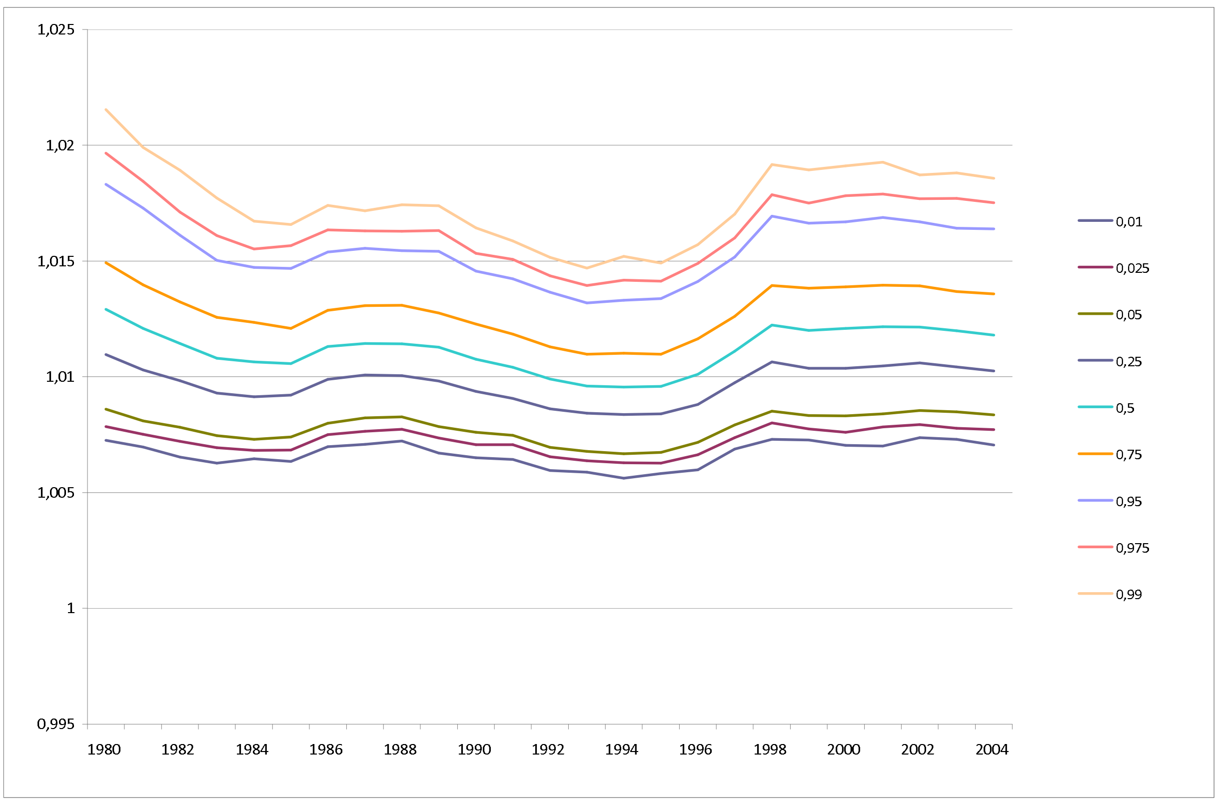

Figure 7: Testing global returns to scale.

Some inference on returns to scale can also be done using our SFA results. As is visible in Table 12, results of estimations of the Cobb–Douglas and the translog production function without the CRS restriction lead to a conclusion that returns to scale are generally country-specific, yet globally close to constant. When computed for the entire sample of countries, the scale elasticity is not distinguishable from unity in the statistical sense. Country-specific translog production function estimates indicate, however, that returns to scale might depend, e.g., on the size of the economy – they seem to be decreasing in the US, and slightly increasing in smaller economies such as Portugal and Ireland. This result might also reflect the misspecification of the estimated translog production function, though, so it should be treated with care.

Table 12: Returns to scale – evidence from stochastic frontier estimates.

	returns to scale (IRS >1, DRS <1)	s.e.
SFA-CD(K,Hu,Hs)	0,958	
SFA-CD(K,Hu,Hs) [Restricted]	0,971	
Australia	1,050	0,035
Austria	1,098	0,053
Belgium	1,069	0,036
Canada	1,031	0,038
Denmark	1,105	0,067
Finland	1,112	0,064
France	0,954	0,044
Greece	1,080	0,045
Ireland	1,196	0,094
Italy	0,941	0,062
Japan	0,907	0,064
Netherlands	1,004	0,055
Norway	1,103	0,066
Portugal	1,132	0,056
Spain	1,011	0,045
Sweden	1,067	0,045
Switzerland	1,057	0,067
UK	0,979	0,045
USA	0,887	0,066
Translog(K,Hu,Hs) mean	1,041	0,055

5 Conclusion

Summing up, the objective of the current paper has been to investigate the shape of the aggregate (country-level) production function based on estimates of the World Technology Frontier (WTF). Using annual data on inputs and output in 19 highly developed

OECD countries in 1970–2004, we have estimated the WTF both non-parametrically and parametrically (using the bias-corrected DEA and SFA approach, respectively) and then used these estimates to assess several properties of the implied production function.

We have obtained the following principal results:

- departures from the Cobb–Douglas and the translog production function are pronounced,
- partial elasticities of the aggregate production function are correlated with inputs,
- unskilled and skilled labor are clearly not perfectly substitutable,
- tests for constancy of returns to scale provide mixed evidence on this property.

References

[1] Aigner, D., K. Lovell, K., P. Schmidt (1977), "Formulation and Estimation of Stochastic Frontier Function Models", *Journal of Econometrics* 6, 21-37.

[2] Badunenko, O., D.J. Henderson, V. Zelenyuk (2008), "Technological Change and Transition: Relative Contributions to Worldwide Growth During the 1990's", *Oxford Bulletin of Economics and Statistics* 70(4), 461-492.

[3] Barro, R.J., J.-W. Lee (2001), "International Data on Educational Attainment: Updates and Implications", *Oxford Economic Papers* 53(3), 541-563.

[4] Battese, G., T. Coelli, (1992), "Frontier Production Functions, Technical Efficiency and Panel Data: With Application to Paddy Farmers in India", *Journal of Productivity Analysis* 3(1-2), pp. 153-169.

[5] Battese, G., T. Coelli, (1995), "A Model for Technical Inefficiency Effects in a Stochastic Frontier Production Function for Panel Data", *Empirical Economics* 20, 325-332.

[6] Bernanke, B.S., R.S. Gürkaynak (2001), "Is Growth Exogenous? Taking Mankiw, Romer and Weil Seriously" [In:] B.S. Bernanke, K. Rogoff, eds., *NBER Macroeconomics Annual 2001*. Cambridge, MA: MIT Press, 11-57.

[7] Bos, J.W.B., C. Economidou, M. Koetter, J.W. Kolari (2010), "Do All Countries Grow Alike?", *Journal of Development Economics* 91(1), 113-127.

[8] Caselli F. (2005), "Accounting for Cross-Country Income Differences" [In:] P. Aghion, S. Durlauf (eds.), *Handbook of Economic Growth*. Elsevier, Amsterdam.

[9] Caselli F., W. J. Coleman (2006), "The World Technology Frontier", *American Economic Review* 96(3), 499-522.

[10] Cohen, D., M. Soto (2007), "Growth and Human Capital: Good Data, Good Results", *Journal of Economic Growth* 12(1), 51-76.

[11] Cornwell, C., P. Schmidt, R.C. Sickles (1990), "Production Frontiers With Cross-Sectional and Time-Series Variation in Efficiency Levels", *Journal of Econometrics* 46(1-2), 185-200.

[12] de la Fuente, A., R. Doménech (2006), "Human Capital in Growth Regressions: How Much Difference Does Data Quality Make?", *Journal of the European Economic Association* 4(1), 1-36.

[13] Färe, R., S. Grosskopf, M. Noriss, Z. Zhang (1994), "Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries", *American Economic Review* 84(1), 66-83.

[14] Felipe, J., Fisher, F. M. (2003), "Aggregation in Production Functions: What Applied Economists Should Know", *Metroeconomica* 54(2-3), 208–262.

[15] Gollin, D. (2002), "Getting Income Shares Right", *Journal of Political Economy* 110(2), 458-474.

[16] Greene, W. (2003), "Maximum Simulated Likelihood Estimation of the Normal-Gamma Stochastic Frontier Function", *Journal of Productivity Analysis* 19(2-3), 179-190.

[17] Growiec, J. (2010a), "Productivity Differences Across OECD Countries and US States, 1970–2000: The World Technology Frontier Revisited", Warsaw School of Economics, submitted paper.

[18] Growiec, J. (2010b), "On the Measurement of Technological Progress Across Countries", National Bank of Poland Working Paper 73.

[19] Hall, R.E., C.I. Jones (1999), "Why Do Some Countries Produce So Much More Output Per Worker Than Others?", *Quarterly Journal of Economics* 114(1), 83-116.

[20] Henderson, D. J., R. R. Russell (2005), "Human Capital and Convergence: A Production–Frontier Approach", *International Economic Review* 46(4), 1167-1205.

[21] Heston, A., R. Summers, B. Aten (2006), "Penn World Table Version 6.2", Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania.

[22] Jerzmanowski, M. (2007), "Total Factor Productivity Differences: Appropriate Technology Vs. Efficiency", *European Economic Review* 51, 2080-2110.

[23] Jondrow, J., C.A. Knox Lovell, I.S. Materov, P. Schmidt (1982), "On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model", *Journal of Econometrics* 19(2-3), 233-238.

[24] Kneip, A., L. Simar, P.W. Wilson (2008), "Asymptotics and Consistent Bootstraps of DEA Estimators in Non-parametric Frontier Models", *Econometric Theory* 24, 1663-1697.

[25] Kneip A., L. Simar, P.W. Wilson (2009), "A Computationally Efficient, Consistent Bootstrap for Inference with Non-paramteric DEA Estimators", Discussion Paper 0903, Institut de Statistique, Université catholique de Louvain.

[26] Koop, G., J. Osiewalski, M.F.J. Steel (1999), "The Components of Output Growth: A Stochastic Frontier Analysis", *Oxford Bulletin of Economics and Statistics* 61(4), 455-487.

[27] Koop, G., J. Osiewalski, M.F.J. Steel (2000), "Measuring the Sources of Output Growth in a Panel of Countries", *Journal of Business and Economic Statistics* 18(3), 284-299.

[28] Koop, G., M.F.J. Steel, J. Osiewalski (1995), "Posterior Analysis of Stochastic Frontier Models Using Gibbs Sampling", *Computational Statistics* 10, 353-373.

[29] Koop, G., M.F.J. Steel (2001), "Bayesian Analysis of Stochastic Frontier Models" [In:] Baltagi, B., ed., *A Companion to Theoretical Econometrics*, Blackwell, Oxford, pp. 520-573.

[30] Kumar, S., R. R. Russell (2002), “Technological Change, Technological Catch-up, and Capital Deepening: Relative Contributions to Growth and Convergence”, *American Economic Review* 92(3), 527-548.

[31] Kumbhakar, S.C., C.A. Knox Lovell (2000), *Stochastic Frontier Analysis*. Cambridge University Press, Cambridge.

[32] Larsson, R., J. Lyhagen, M. Löthgren (2001), “Likelihood-Based Cointegration Tests in Heterogeneous Panels”, *Econometrics Journal* 4, 109-142.

[33] Löthgren M., M. Tambour (1999), “Testing Scale Efficiency in DEA Models: A Bootstrapping Approach”, *Applied Economics* 31, 1231-1237.

[34] Meeusen, W., J. van den Broeck (1977), “Efficiency Estimation from Cobb-Douglas Production Functions with Composed Error”, *International Economic Review* 18(2), 435-444.

[35] Park, B.U., S.O. Jeong, L. Simar (2009), “Asymptotic Distribution of Conical-Hull Estimators of Directional Edges”, Discussion Paper 0907, Institut de Statistique, Université catholique de Louvain.

[36] Pedroni, P. (1999), “Critical Values for Cointegration Tests in Heterogeneous Panels With Multiple Regressors”, *Oxford Bulletin of Economics and Statistics* 61, 653-670.

[37] Ritter, C., L. Simar (1997), “Pitfalls of Normal-Gamma Stochastic Frontier Models”, *Journal of Productivity Analysis* 8, 167-182.

[38] Simar, L., P.W. Wilson (1998), “Sensitivity of Efficiency Scores: How to Bootstrap in Nonparametric Frontier Models”, *Management Sciences* 44(1), 49-61.

[39] Simar, L., P.W. Wilson (2000a), “Statistical Inference in Nonparametric Frontier Models: State of the Art”, *Journal of Productivity Analysis* 13, 49-78.

[40] Simar, L., P.W. Wilson (2000b), “A General Methodology for Bootstrapping in Non-parametric Frontier Models”, *Journal of Applied Statistics* 27(6), 779-802.

[41] Simar, L., P.W. Wilson (2002), “Non-parametric Tests of Returns to Scale”, *European Journal of Operational Research* 139(1), 115-132.

[42] Temple, J. (2006), “Aggregate Production Functions and Growth Economics”, *International Review of Applied Economics* 20(3), 301-317.

[43] van den Broeck, J., G. Koop, J. Osiewalski, M.F.J. Steel (1994), “Stochastic Frontier Models: A Bayesian Perspective,” *Journal of Econometrics* 61, 273-303.