The Welfare Cost of Monetary Policy Loss after the Euro Adoption in Poland*

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Abstract

One of the most important aspects associated with the Eurozone accession of Poland is the macroeconomic impact of the loss of autonomous monetary policy. In order to analyze this issue, we build a two country DSGE model with sticky prices. We begin by evaluating performance of our model. Next, we investigate how joining the Eurozone will affect the business cycle behavior of the main macroeconomic variables in Poland. We find that the Euro adoption will have a noticeable impact on the Polish economic fluctuations. In particular, volatility of domestic output increases and volatility of inflation decreases. Also, in order to quantify the effect of the Euro adoption, we compute the welfare effect of this monetary policy change. Our findings suggest that the welfare cost is not large.

1 Introduction

After accession to the European Union in 2004, Poland, as well as the other New Member States, is going to adopt the Euro as a national currency and become a member of the Euro area. However, the accession to the Euro area implies important changes in both

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macroeconomic policy and behavior of the accessing economy. The most important of these are:

- fixing of the exchange rate against the other participants of the Euro area,
- resignation from the autonomous monetary policy in favor of the common monetary policy conducted by the ECB.

The consequence of these changes generate additional benefits for the accessing country, but they may also induce additional costs to the economy.

To a large extent, the existing literature focused on welfare benefits associated with the membership in the monetary union. Rose (2000) and Frankel and Rose (2002) argued that accession to the monetary union boosts trade, creating welfare gains. But their estimate of the magnitude of this effect met some criticism - see e.g. Faruqee (2004). In case of the Polish economy, the important contribution of Daras and Hagemejer (2008) shows the benefits of the Euro adoption, taking into account the trade creation affect and a decline of a long-term real interest rate through elimination of the risk premium. Their results were calculated using dynamic Computable General Equilibrium model (CGE) framework.

Our contribution to the discussion on the consequences of Euro adoption concentrates on the costs associated with abandoning the autonomous monetary policy. So, after accession to the Euro area monetary policy will be conducted by the European Central Bank and will be responding, in the first place, to the events affecting the whole Euro area. In the presence of asymmetric shocks, affecting mainly a given member country, the common monetary policy will be less suited to the current situation of this country, giving rise to additional volatility of economic development and associated costs of these fluctuations.

Thus, our analysis aims at assessing the impact of resignation from autonomous monetary policy on the volatility of economic development, measured by a set of main macroeconomic indicators. We are also going to provide an estimate of the welfare costs associated with the monetary policy change.

In order to perform this calculation, we propose a Dynamic Stochastic General Equilibrium model (DSGE[1]) of two economies (Poland and the Euro area) linked through trade in goods and services and incomplete international assets market. In order to specify properly the parameters of the model, we decided to estimate a relatively large number of parameters on the basis of the data from both economies. It allows us to develop a model that mimics closely the behavior of both economies.

[1]This framework builds on the seminal paper of Kydland and Prescott (1982) and the Real Business Cycle school, which was enhanced with Keynesian-type of nominal rigidities, like in the important work of Smets and Wouters (2003), resulting in neoclassical synthesis - see e.g. Goodfriend and King (1998).

The DSGE methodology is relatively well suited to analyze the consequences of monetary policy regime switch. The model describes the laws of motion of the economy which are derived from microeconomic foundations. In other words, all agents populating the economy solve well specified decision problems and respond in an optimal way to changes in economic environment. As the economy is described in terms of preferences, technologies and the rules of market clearing, the model is parametrized in terms of so called "deep" parameters. It implies that these parameters are invariant to changes in policies and changing environment and it is possible to analyze the consequences of these changes in a way that is immune to the Lucas critique (see Lucas, 1976). Additionally, agents populating the economy, while making their decisions, form expectations (under the assumption on rationality) about future path of economic development, so the model incorporates expectations in an internally coherent way. All these features make the DSGE framework a parsimonious tool to analyze the consequences of resignation from the autonomous monetary policy in Poland.

According to the standard economic theory, monetary policy is neutral in the long run, so the change of the monetary policy regime should not affect the economy in the long run. Thus the monetary policy regime change should influence the volatility of the economy, rather than the level of economic development. As households are usually assumed to be risk averse, they dislike high variation in incomes and consumption, thus higher volatility of economic growth generates the welfare costs for households. Since households preferences are directly specified in the DSGE framework, we are able to assess the consequences of Euro adoption not only in terms of volatility of economic growth (as in Karam, Laxton, Rose, and Tamirisa (2008)), but also in terms of welfare (as in Lucas, 1987).

Our approach is directly related to the literature on the costs of business cycle fluctuations, which starts from the seminal contribution of Lucas (1987). Lucas finds that the costs of economic fluctuations are very small - his estimate for the US economy implies that individuals would sacrifice at most 0.1% of lifetime consumption (his point estimate under reasonable calibration of the prototype economy amounts to 0.008%). The result of Lucas was quite controversial and launched subsequent research. Barlevy (2004a) reviews the literature on this topic and finds that the literature finds the costs of business cycles ranging from 0.01% to 2% of lifetime consumption. The higher estimates are usually obtained with either non-standard preferences (e.g. Epstein-Zin) or high risk aversion (calibrated to match household micro data, which is inconsistent with the macro evidence for the class of CRRA preferences).

The standard models of the business cycle fluctuations generate relatively low estimates of the business cycle costs due to two considerations. On the one hand, when thinking about fluctuations one is usually thinking of recessions, as times when people are worse off, but the lack of economic fluctuations also means that there are no economic expansions - the long periods when individuals are actually better off. On the other hand, economic fluctuations in a DSGE models are optimal responses of economic agents to changes in the economic environment. If these are optimal, so it is relatively hard to make people much more happy without fluctuations. Additionally, the standard models of business cycle fluctuations imply that there are no long-run consequences of these, so there are no level effect of fluctuations (and thus the magnitude of the welfare loses is of the second order).

There is also a literature that argues that there is the level effect. The so called endogenous business cycle literature (see e.g. Barlevy, 2004b) assumes that there are long run effects of economic fluctuations, so fluctuations have the level effect on welfare, which results in much higher costs of cyclical fluctuations - an order of magnitude higher, like 8% in Barlevy (2004b). This approach builds mainly on the empirical evidence showing that higher volatility of economic growth is usually associated with lower average growth rates. But the tools of this approach are mostly econometric, thus the causality still remains unsolved. As far as we know there does not exist a general equilibrium model incorporating these relationship, which is well grounded in the data. Due to this shortcoming and the fact that literature on endogenous cycle is not in the mainstream of economic thinking, we choose the first approach to the business cycle.

The literature on the costs of the Euro adoption (or more broadly - monetary union) is rather limited. Ca' Zorzi, De Santis, and Zampolli (2005) argue that adopting the Euro is more likely to be welfare enhancing the higher the relative volatility of supply shocks across the participating countries, the smaller the correlation of countries' supply shocks and the larger the variance of real exchange rate shocks. Additionally, the welfare effects do not depend on deterministic factors influencing the real exchange rate (such as Balassa-Samuelson effect), but rather on variances and covariances of shocks. They also claim, that the Euro area accession decreases the effectiveness of the monetary policy response to the stochastic shocks, so it creates a cost of monetary union. However, the Euro adoption could be beneficial if the positive influence on potential output (through the trade creation channel) is higher than the negative effect of lower monetary policy effectiveness.

Karam, Laxton, Rose, and Tamirisa (2008) use a DSGE model developed in IMF to assess the implications of Euro adoption on the volatility of an accessing country (the model is roughly calibrated to data from the Czech Republic - the authors argue that it is a typical New Member economy[2]). Their results show that the accession of a small country to the Euro area increases volatility of both inflation and output, as the exchange rate no longer

[2]This argument is very stylized as there are many features that distinguish Eastern European countries, especially in terms of volatility of GDP components - for some discussion, see Choueiri, Murgasova, and Székely (2005).

buffers a part of the volatility generated by economic shocks[3]. The increase of the volatility of economic fluctuations gets smaller with the fiercer competition in the goods market, the smaller rigidities and the greater trade integration with the Euro area. The approach of Karam, Laxton, Rose, and Tamirisa (2008) provides many interesting answers, although it focuses on the volatility effects of Euro adoption, emphasizing the inflation-output volatility faced by the central bank. They do not try to quantify the welfare results of the Euro area accession.

Lopes (2007) uses a framework that is the most closely related to ours. She also uses a symmetric two-country DSGE model, with capital and nominal rigidities[4], but she calibrates all of the model parameters. The latter is relatively hard in case of New Member States as the research on technologies, preferences and market structures in these economies is rather scarce. She finds the welfare costs of loosing monetary policy independence of 0.25%of lifetime consumption (in case of Poland). We believe that her result might be biased, since it is based only on one draw of random shocks for about 1000 periods and our experiments show that one needs a large number of draws to for convergence of the welfare result. She also points into the existence of the level effects, which in our computations disappear as the number of draws increases.

The rest of the paper is organized as follows. Next section discusses the model we are going to use in order to assess the consequences of loosing monetary policy independence. Then we discuss calibration and estimation issues and show the performance of the model, both in terms of impulse response functions and moments of the model variables against the data. Next, we present and discuss the results of the model both in terms of volatilities of economic aggregates, as well as in terms of welfare. Then we conclude the paper.

2 Model

We employ the standard dynamic general equilibrium model of business cycle with nominal rigidities. In our model, monopolistic producers set prices in a style proposed by Calvo (1983). We build a two country model in the tradition of Chari, Kehoe, and McGrattan (2002). In the model we call Poland the home country and Eurozone the foreign country.

[3]The authors argue that "...flexible exchange rate plays an important buffering role that facilitates macroeconomic adjustment to shocks in small, emerging economies, which allows the central bank to achieve better outcomes in terms of domestic volatility. In general, the results show that there is a cost to a small, emerging economy in joining a common currency area when this flexibility is lost. The essential reason is that there are rigidities in domestic adjustment, and when the burden of macroeconomic adjustment is forced onto domestic nominal variables under the common currency, macroeconomic volatility generally increases..." - Karam, Laxton, Rose, and Tamirisa (2008), page 354.

[4]Although she uses the staggered price setting in the spirit of Taylor (1980) and we choose a framework of Calvo (1983), that is more frequently used in the DSGE literature.

Monetary policy is modeled as an interest rate rule similar to the one proposed by Taylor (1993). Our model shares many features with closed economy models (including Erceg, Henderson, and Levin (2000), Smets and Wouters (2003)), or small open macro economy models (including , Altig, Christiano, Eichenbaum, and Linde (2005), Christiano, Eichenbaum, and Evans (2005), Adolfson, Laséen, Lindé, and Villani (2005)), or other two country models (including Lopes (2007) and Rabanal and Tuesta (2007)).

Households can save in domestic bonds and/or international bonds. We assume that domestic bonds markets are complete but international bonds markets are incomplete. Introduction of this market structure leads to the arbitrage condition that after log-linearized version takes a form of the uncovered interest rate parity (UIP) condition. Households also decide how much capital to rent to producers (utilization rate) and choose how much to invest in new capital. Furthermore, households supply labor in the competitive labor market.

There are three stages of the production process. In the first stage producers offer differentiated products to both domestic and foreign second stage producers. They set their prices according to the Calvo scheme. By including the nominal rigidities in the buyer's currency, we obtain the incomplete exchange rate pass-through. In the remaining two stages, perfectly competitive producers combine those differentiated goods into a single consumption/investment good with domestic and foreign component.

Next, we describe in detail the optimization problems of consumers and producers as well as the behavior of fiscal and monetary authorities.

2.1 Households

There is a continuum of households of measure 1. The fraction ω of households reside in home country and the fraction $1-\omega$ of households reside in the foreign country. Households choose consumption level, as well as their labor supply, domestic bonds holdings (complete markets), international bonds holdings (incomplete markets). They also choose the capital utilization rate and the investment level. The representative domestic household's preferences are of the form [5]

$$W_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t, \zeta_t) \right], \tag{1}$$

[5] The convention employed in this paper is that asterisk denotes the counterpart in the foreign country of a variable in the home country (for example c_t is consumption in the home country, and c_t^* is consumption in the foreign country). The same applies to the model's parameters. Whenever we see potential for confusion we explicitly clarify notation.

where c_t and l_t denote the representative household's level of consumption and labor supply, respectively. ζ_t denotes the labor supply shock that follows AR(1) process

$$\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} + \varepsilon_{\zeta,t}, \tag{2}$$

where $E_t[\zeta_t] = 1$ and $\hat{\zeta}_t = \zeta_t - 1$ (in the steady state $\bar{\zeta} = 1$). Throughout the whole paper we assume the following form of the instantaneous utility function

$$u(c_t, l_t, \zeta_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \psi \zeta_t \frac{l_t^{1+\gamma}}{1+\gamma}. \tag{3}$$

We restrict our analysis to the case of a representative consumer by assuming complete domestic financial markets. In period t there is a complete set of state contingent one-period nominal bonds B_{t+1}, each worth $\Upsilon_{t,t+1}$. Households can also trade bonds with abroad. We assume that there is one, internationally traded, uncontingent nominal bond, nominated in foreign currency D^*_{t+1} (denote home country holdings of this bond as $D^*_{H,t+1}$, and foreign country holdings of this bond as $D^*_{F,t+1}$). From the point of view of the home country the interest rate on this bond is $R^*_t \kappa_t$, where κ_t denotes risk premium. The risk premium is a function of the domestic debt (as in Schmitt-Grohe and Uribe (2003))

$$\kappa_t = \exp\left(-\chi\left(\frac{e_t D^*_{H,t+1}}{P_t GDP_t} - d\right)\right)\epsilon_{\kappa,t}, \tag{4}$$

where e_t, P_t and GDP_t denote the nominal exchange rate, the price of the consumption good and GDP, respectively. The constand d is calibrate so as there is no risk premium in the steady state. $\epsilon_{\kappa,t}$ denotes the risk premium shock that follows AR(1) process

$$\hat{\epsilon}_{\kappa,t} = \rho_\kappa \hat{\epsilon}_{\kappa,t-1} + \varepsilon_{\zeta,t}, \tag{5}$$

where $E_t[\epsilon_t] = 1$ and $\hat{\epsilon}_t = (\epsilon_t - 1)/1$ (in the steady state $\bar{\varepsilon} = 1$).

Moreover, households own the capital stock. The dynamics of the physical stock follows the law of motion

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \left(1 - S\left(\frac{x_t}{x_{t-1}}\right)\right)x_t, \tag{6}$$

where $\tilde{k}_t$ and x_t denote the capital stock and investments. $S(x_t/x_{t+1})$ is a function which transforms investments into physical capital. We adopt the specification of Christiano, Eichenbaum, and Evans (2005) and assume that $S(1) = S'(1) = 0$, and $\iota = 1/S''(1) > 0$. Households choose also the capital utilization rate u_t, and have to pay the capital utilization rate adjustment costs, according to the following cost function $\Psi(u_t)$, satisfying $\Psi(1) = 0$, $\Psi'(1) > 0$ and $\Psi''(u_t) > 0$. Moreover, households rent capital to producers. Denote the capital stock employed by producers as k_t, then

$$k_t = u_t \tilde{k}_t \tag{7}$$

and the rental rate of capital is denoted r_t.

All households face the same budget constraint in each period

$$\begin{aligned} c_t + x_t + \frac{E_t\left[\Upsilon_{t,t+1} B_{t+1}\right]}{P_t} + \frac{D^*_{H,t+1} e_t}{P_t R^*_t \kappa_t} = \\ w_t l_t + \left(r_t u_t - \Psi\left(u_t\right)\right) \tilde{k}_t + \frac{B_t}{P_t} + \frac{e_t D^*_{H,t}}{P_t} - T_t + \Pi_t, \end{aligned} \tag{8}$$

where P_t, w_t, T_t and Π_t denote the price of the consumption good, real wage, real lump sum tax and real profits from all producers, respectively.

The representative household maximizes (1) subject to the budget constraint (8) (denote the Lagrangian multiplier on budget constraint as λ_t) and the law of motion of capital (6) (denote the Lagrangian multiplier on budget constraint as $\lambda_t Q_t$). Solving, we get the following first order conditions

$$c_t: \quad \beta^t u_{c,t} = \lambda_t, \tag{9}$$

$$l_t: \quad \beta^t u_{l,t} = -\lambda_t w_t, \tag{10}$$

$$x_t: \quad \begin{aligned} &\lambda_t = \lambda_t Q_t \left(1 - S\left(\tfrac{x_t}{x_{t-1}}\right) - S'\left(\tfrac{x_t}{x_{t-1}}\right)\tfrac{x_t}{x_{t-1}}\right) \\ &+ E_t\left[Q_{t+1}\lambda_{t+1}\left(S'\left(\tfrac{x_{t+1}}{x_t}\right)\left(\tfrac{x_{t+1}}{x_t}\right)^2\right)\right], \end{aligned} \tag{11}$$

$$\tilde{k}_{t+1}: \quad E_t\left[\lambda_{t+1}\left(r_{t+1}u_{t+1} - \Psi\left(u_{t+1}\right)\right)\right] = Q_t\lambda_t - E_t\left[Q_{t+1}\lambda_{t+1}\left(1-\delta\right)\right], \tag{12}$$

$$u_t: \quad r_t = \Psi'\left(u_t\right), \tag{13}$$

$$B_{t+1}: \quad E_t\left[\Upsilon_{t,t+1}\right] = E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\right], \tag{14}$$

$$D^*_{H,t+1}: \quad \frac{1}{R^*_t \kappa_t} = E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\frac{e_{t+1}}{e_t}\right], \tag{15}$$

and the transversality conditions. Define the nominal interest rate in home country as

$$\frac{1}{R_t} = E_t\left[\Upsilon_{t,t+1}\right] \tag{16}$$

The log-linearized equations can be found in Appendix A.

2.2 Producers

There are three stages of production process in both economies. Next, we describe the actions of producers in home country, producers in foreign country act analogously. In the last stage final domestic goods producers buy home and foreign intermediate goods and combine them into domestic consumption/investment goods that are sold to consumers. In the second stage there are two sectors: H and F. In sector H producers buy home country heterogeneous intermediate goods and aggregate it into sector H homogeneous intermediate goods that are sold to the domestic final goods producers. Similarly in sector F, producers buy foreign country heterogeneous intermediate goods and aggregate them into sector F homogeneous intermediate goods that are sold to domestic final goods producers. In the first stage, heterogeneous intermediate goods producers use capital and labor to produce heterogeneous intermediate goods that are sold both in home country and foreign country. Next we describe the problems of producers in home country in more detail. In terms of notation, goods produced at home are subscripted with an H (excluding home county sector F homogeneous intermediate goods which are subscripted with an F), while those produced abroad are subscripted with an F (excluding foreign country sector H homogeneous intermediate goods which are subscripted with an H). Moreover, goods that are sold to agents in home are not superscripted, while those sold in foreign are superscripted with an asterisk.

2.2.1 Final Goods Producers

Final goods producers operate in a perfectly competitive market. They buy sector H homogeneous intermediate goods $Y_{H,t}$ and sector F homogeneous intermediate goods $Y_{F,t}$ at competitive prices $P_{H,t}$ and $P_{F,t}$, respectively. They use those goods to produce a final goods Y_t using the following technology

$$Y_t = \left[\eta^{\frac{\mu}{1+\mu}} \left(Y_{H,t}\right)^{\frac{1}{1+\mu}} + (1-\eta)^{\frac{\mu}{1+\mu}} \left(Y_{F,t}\right)^{\frac{1}{1+\mu}}\right]^{1+\mu}, \tag{17}$$

which are sold to home country consumers. Since markets are competitive producers take prices as given and, in each period t, choose inputs and output to maximize profits given by

$$P_t Y_t - P_{H,t} Y_{H,t} - P_{F,t} Y_{F,t} \tag{18}$$

subject to the production function (17).

Solving the problem in (18) gives the inputs demand functions

$$Y_{H,t} = \eta \left(\frac{P_{H,t}}{P_t} \right)^{\frac{-(1+\mu)}{\mu}} Y_t, \tag{19}$$

$$Y_{F,t} = (1-\eta) \left(\frac{P_{F,t}}{P_t} \right)^{\frac{-(1+\mu)}{\mu}} Y_t. \tag{20}$$

Using the zero profit condition, we construct the index of domestic prices

$$P_t = \left[(1-\eta) \left(P_{F,t}\right)^{\frac{-1}{\mu}} + \eta \left(P_{H,t}\right)^{\frac{-1}{\mu}} \right]^{-\mu}. \tag{21}$$

2.2.2 Homogeneous Intermediate Goods Producers

Homogeneous intermediate goods producers in sector $d \in \{H, F\}$ operate in a competitive market. Producers in sector H, buy heterogeneous intermediate goods $y_H(i)$, $i \in [0,1]$, produced in home country, while the producers in sector F buy heterogeneous intermediate goods $y_F(i)$, $i \in [0,1]$, produced in foreign country. Prices of these heterogeneous intermediate goods are set in currency of home country, $p_{d,t}(i)$, $i \in [0,1]$ and $d \in [H, F]$. The technology for producing homogeneous intermediate goods in sector $d \in \{H, F\}$ is as follows

$$Y_{d,t} = \left(\int_0^1 y_{d,t}(i)^{\frac{1}{1+\mu_d}} di \right)^{1+\mu_d}. \tag{22}$$

Given prices, in each period t, producers choose inputs and output to maximize profits

$$P_{d,t} Y_{d,t} - \int_0^1 p_{d,t}(i) y_{d,t}(i) di \tag{23}$$

subject to to the production function (22).

Solving the producers problem in (23) we obtain the following input demands for heterogeneous intermediate goods

$$y_{d,t}(i) = \left(\frac{p_{d,t}(i)}{P_{d,t}} \right)^{\frac{-(1+\mu_d)}{\mu_d}} Y_{d,t}. \tag{24}$$

From the zero profit condition we also get the price index

$$P_{d,t} = \left[\int p_{d,t}(i)^{\frac{-1}{\mu_d}} di \right]^{-\mu_d}. \tag{25}$$

2.2.3 Heterogeneous Intermediate Goods Producers

The technology for producing each heterogeneous intermediate good $i \in [0,1]$ is the standard constant returns to scale production function

$$y_{H,t}(i) + \frac{1-\omega}{\omega} y^*_{H,t}(i) = y_t(i) = Ak_t(i)^{\alpha} (z_t l_t(i))^{1-\alpha} \tag{26}$$

where $k_t(i)$ and $l_t(i)$ are the inputs of capital and labor, respectively, while $y_H(i)$ and $y^*_H(i)$ are the amounts of the intermediate heterogeneous good i sold in home country and in foreign country[6], respectively. Moreover z_t denotes the stationary technology shock that follows an AR(1) process

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}, \tag{27}$$

where $E_t[z_t] = 1$ and $\hat{z}_t = z_t - 1$ (in the steady state $\bar{z} = 1$).

Next we find the cost function to simplify notationally the profit maximization problem. The cost minimization problem of producer i in period t is

$$c(y_t(i)) = \min_{k_t(i), l_t(i)} [r_t k_t(i) + w_t l_t(i)] \tag{28}$$

subject to the (26), where r_t and w_t are the gross nominal rental rate of capital and real wage, respectively. Solving (28) we get the following cost function

$$c(y_t(i)) = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{1}{(z_t)^{1-\alpha}} r_t^{\alpha} w_t^{1-\alpha} y_t(i) = mc_t y_t(i), \tag{29}$$

where mc_t denotes marginal cost, $mc_t(y_t(i)) = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{1}{(z_t)^{1-\alpha}} r_t^{\alpha} w_t^{1-\alpha}$. Moreover, from the cost minimization problem we get the following condition for the optimal inputs employment ratio

$$\frac{r_t}{w_t} = \frac{\alpha}{1-\alpha} \frac{l_t}{k_t}. \tag{30}$$

The heterogeneous intermediate good i producer sells her products both to home country and foreign country intermediate homogeneous goods producers. Heterogeneity of the intermediate goods, since producers have market power, introduces monopolistic competition, thus it allows us to add the price stickiness to the model. The producers set their prices according to the Calvo scheme. We assume that the prices of heterogeneous intermediate goods are sticky in the buyers currency, which is consistent with the incomplete short term pass-through. Since the marginal cost function is constant in $y_t(i)$, we can write down the separate profit maximization problems for goods sold in home country and foreign country.

[6] Note that $y^*_{H,t}(i)$ is expressed in per ca pita units of foreign country thus it has to be multiplied by $\frac{1-\omega}{\omega}$.

In each period, the heterogeneous good i producer, while selling her product in home country $y_H(i)$, receives a signal to adjust her price with probability $(1-\theta_H)$, otherwise the price evolves according to the following formula $p_{H,t+1}(i) = p_{H,t}(i)\,\bar{\pi}$, where $\bar{\pi}$ denotes the steady state inflation in home country. If the producer receives the signal for price reoptimalization it chooses the reoptimalized price $p_{H,t}^{new}(i)$ and the production level in each period until the next reoptimization $\{y_{H,t+j}(i)\}_{j=0}^{\infty}$ that maximize profits given by the following function

$$E_t \sum_j (\beta\theta_H)^j \Lambda_{t,t+j} \left(\frac{(1+\tau_H)\, p_{H,t}^{new}(i)\, \bar{\pi}^j}{P_{t+j}} - mc_{t+j} \right) y_{H,t+j}(i) \tag{31}$$

subject to the demand function (24), where $\Lambda_{t,t+j}$ denotes the intertemporal discount factor (consistent with the home country households problem), while τ_H denotes production subsidy .

Similarly, the same producer, while selling her product in foreign country, in each period with probability $(1-\theta_H^*)$ reoptimizes her price and with probability θ_H^* the price evolves according to the following formula $p_{H,t+1}^*(i) = p_{H,t}^*(i)\,\bar{\pi}^*$, where $\bar{\pi}^*$ denotes the steady state inflation in foreign country. While reoptimizing the producer chooses $p_{H,t}^{new,*}(i)$ and $\left\{y_{H,t+j}^*(i)\right\}_{j=0}^{\infty}$ that maximize the following profit function

$$E_t \sum_j (\beta\theta_H^*)^j \Lambda_{t,t+j} \left(\frac{e_{t+j}\,(1+\tau_H^*)\, p_{H,t}^{new,*}(i)\, (\bar{\pi}^*)^j}{P_{t+j}} - \frac{1-\omega}{\omega} mc_{t+j} \right) y_{H,t+j}^*(i) \tag{32}$$

subject to the demand function (24), where e_{t+j} denotes the nominal exchange rate (price of foreign currency in home currency), while τ_H denotes production subsidy .

Define the real exchange rate as $q_t = \frac{e_t P_t^*}{P_t}$. Solving the problems (31) and (32) we get the following first order conditions

$$\begin{aligned}
E_t \sum_j (\beta\theta_H)^j \Lambda_{t,t+j} \left(\frac{p_{H,t}^{new}(i)\, \bar{\pi}^j}{P_{t+j}} - \frac{1+\mu_H}{1+\tau_H} mc_{t+j} \right) y_{H,t+j}(i) &= 0, && (33) \\
E_t \sum_j (\beta\theta_H^*)^j \Lambda_{t,t+j} \left(\frac{q_{t+j} p_{H,t}^{new,*}\, (\bar{\pi}^*)^j}{\pi_{t,t+j}^*} - \frac{1+\mu_H^*}{1+\tau_H^*} \frac{1-\omega}{\omega} mc_{t+j} \right) y_{H,t+j}^*(i) &= 0. && (34)
\end{aligned}$$

Thus the producer sets its price so that discounted real marginal revenue is equal to discounted real marginal cost, in expected value. We assume that the subsidy is set to eliminate the monopolistic distortion associated with positive markups, thus for $d \in \{H, F\}$ we have $\tau_d = \theta_d$ and $\tau_d^* = \theta_d^*$. Note that if $\theta = 0$ we obtain the standard condition that price equals marginal cost. The log-linearized equations can be found in Appendix A.

2.3 Government

The government uses lump sum taxes to finance government expenditure and production subsidies. The government's budget constraint in this economy equals

$$G_t + \int_0^1 \tau_H p_{H,t}(i) y_{H,t}(i) di + \int_0^1 \tau_F p_{F,t}(i) y_{F,t}(i) di = T_t.$$

Since in our framework Ricardian equivalence holds there is no need to introduce government debt. Moreover, we assume that government expenditures are driven by a simple autoregressive process

$$G_{t+1} = \left(1 - \rho_g\right) \mu_g + \rho_g G_t + \varepsilon_{g,t+1}. \tag{35}$$

2.4 Central Bank

We assume that monetary policy is conducted according to an extended Taylor rule (similar to the one assumed by Smets and Wouters (2003)) that targets deviations from the steady state of CPI inflation, GDP, growth rate of inflation and growth rate of GDP. We also allow for interest rate smoothing

$$R_t = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\gamma_R} \left(\left(\frac{\pi_t}{\bar{\pi}}\right)^{\gamma_\pi} \left(\frac{GDP_t}{G\bar{D}P}\right)^{\gamma_y} \left(\frac{GDP_t}{GDP_{t-1}}\right)^{\gamma_{dGDP}} \left(\frac{\pi_t}{\pi_{t-1}}\right)^{\gamma_{d\pi}}\right)^{1-\gamma_R} e^{\varphi_t} \tag{36}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$.

2.5 Market Clearing.

In equilibrium, the goods markets, the assets markets and the production factors markets must clear.

The market clearing condition in the final goods market takes the form

$$c_t + x_t + G_t + \Psi\left(u_t\right) = Y_t. \tag{37}$$

Note that we have included the market clearing condition in the intermediate goods markets through notation. The market clearing condition in the production factors markets are

$$\begin{aligned} \int_0^1 l_t(i) &= l_t \\ \int_0^1 k_t(i) &= k_t \end{aligned}$$

The market clearing condition in the assets markets are

$$B_{t+1} = B^*_{t+1} = 0$$

and

$$D^*_{H,t+1} + D^*_{F,t+1} = 0$$

2.6 Balance of Payments and GDP

Using the market clearing conditions and the budget constraint we get the balance of payments equation

$$q_t \frac{1-\omega}{\omega} \frac{P^*_{H,t} Y^*_{H,t}}{P^*_t} + \frac{e_t}{P_t} \frac{D^*_{H,t+1}}{R^*_t \kappa_t} = \frac{P_{F,t} Y_{F,t}}{P_t} + \frac{e_{t-1} D^*_{H,t}}{P_{t-1}} \frac{q_t}{q_{t-1} \pi^*_t} \tag{38}$$

Furthermore, to close the model, since there is GDP in the Taylor rule we need the formula for GDP, which has the following form

$$GDP_t = Y_t + \frac{1-\omega}{\omega} \frac{e_t P^*_{H,t} Y^*_{H,t}}{P_t} - \frac{P_{F,t} Y_{F,t}}{P_t} \tag{39}$$

The log-linearized equations can be found in Appendix A.

3 Calibration and estimation

In order to evaluate the properties of the model against the data describing Polish and the Eurozone economies, we applied a mixture of calibration and estimation procedure. First, we calibrated a subset of parameters that can be easily extracted from the raw data or those resulting from the steady state considerations. Afterwords, we performed a Bayesian estimation of the other parameters, that mainly govern the business cycle volatility of the model.

3.1 Calibration Procedure

The calibration of the parameters was based mainly on the data from the quarterly National Accounts, issued either by the Eurostat (in case of the Eurozone[7]), or by the Polish Central Statistical Office (GUS). As a measure of exports and imports we used the data from the

[7]The Eurozone is defined as EA-15 and includes: Belgium, Cyprus, Denmark, Ireland, Greece, Spain, France, Italy, Luxembourg, Malta, Netherlands, Austria, Portugal, Slovenia and Finland)

Table 1: The most important calibrated parameters of the model

Parameter	Value
β	0.99
β^*	0.99
δ	0.017
δ^*	0.017
$\frac{1+\mu}{\mu}$	2
$\frac{1+\mu^*}{\mu^*}$	2
η	0.614
η^*	0.99
α	0.33
α^*	0.33

Polish National Accounts, then we adjusted the Eurozone data, treating the resulting additional net trade with the rest of the world as government consumption[8]. Due to the lack of data on average hours worked, we use employment as a proxy for a measure of labor in the model. We used data on total employment (domestic concept) form the Eurostat in case of Eurozone, and data on employment form the Labor Force Study, in case of Poland. As a measure of wages, we used quarterly data on average wages in the national economy, in case of Poland (due to the lack of data on the compensation of employees for the whole period) and compensation of employees per person employed in case of the Eurozone.

The most important calibrated parameters of the model can be found in Table 1. The discount factors β and β^* were set at the same levels[9] of 0.99, which implies the annual long-term real interest rate of 4%, consistent with the average real interest rate (3-months interest rate deflated by the expected inflation, under assumption of a perfect foresight) in the Eurozone for the period 1995-2007. Physical capital depreciation rate δ was set at 7% annually in both countries. The elasticity of production with respect to capital, α was set at 0.33 in both countries, in line with most of the DSGE literature. The inverse of the elasticity of the capital utilization cost function $\Psi = \frac{\Psi'(1)}{\Psi''(1)}$ was set at 0.1, roughly in line with the estimate in Smets and Wouters (2003) (as the data on capacity utilization are hardly reliable and usually cover only manufacturing, we decided not to use them in the estimation step and we calibrated the elasticity from the literature). The long-term inflation rate was set at 2.5% annually for both economies. The share of the population of Poland in the total

[8]In a a two country framework one need to decide how to deal with the trade with rest of the world. Treating it as government consumption in our framework is relatively nondistortionary, as it only affect the steady state government consumption share and is roughly in line with the approach of Chari, Kehoe, and McGrattan (2007).

[9]In order to avoid the steady state effects of monetary policy regime change, we used the same discount factor for both economies at set it at the level consistent with the Eurozone.

population of the Eurozone and Poland, ω was set at 0.107, on the basis of the data from the Eurostat.

The parameters μ and μ^* were set at 1 in both countries, implying the Armington elasticity of substitution $\frac{1+\mu}{\mu} = 2$, consistently with the evidence given by Ruhl (2005) and discussion presented in McDaniel and Balistreri (2003). The home bias parameters, η and η^* were set at 0.614 and 0.9903 respectively, reflecting the export to absorption ratio for the period $2004 - 2007$ in case of Poland and $1995 - 2007$ in case of the Eurozone. The steady state consumption shares (in absorption) were set at 0.609 and 0.573 in Poland and the Eurozone, respectively. The corresponding figures for investment shares are 0.206 and 0.2095. As there is no direct measure of costs of adjusting capacity utilization, in the stead state we set them at 1% of absorption in both countries and treat the government expenditures share as a residual. The absorption to GDP ratio were set at 1.03 and 0.99 in Poland and the Eurozone. We set the debt share at 0.46, in line with the average ratio of total external debt of the Polish economy to GDP for the years $2005-2007$. Some other parameters of the model, that were not estimated were calculated on the basis of the steady-state relations of the model.

3.2 Estimation Procedure

All data used in the estimation of the model were expressed in quarterly frequency and adjusted for seasonality (except for the interest rates) using Demetra package and expressed in constant prices of the year 2000. As the model does not distinguish between different price indicators and monetary policy is aimed at stabilizing consumer price inflation, we decided to express all real variables in terms of consumption prices either in Poland or in the Eurozone. We also normalized the variables by the population shares ω and $1 - \omega$ in each quarter[10]. Then, all variables in logs were detrended using Hodrick-Prescott filter (see Hodrick and Prescott (1997)), with the standard parameter for the quarterly data $\lambda = 1600$, and expressed as a log-difference of a given variable and HP-trend, which is consistent with the log-linearization of the model.

Our approach was to keep the basic structure of the model relatively simple and to use relatively small number of fundamental stochastic shocks to describe the cyclical fluctuations of the economy (we used a technology, a government consumption, a monetary policy and a labor supply[11] shocks for both countries and one risk premium shock). But this approach

[10] As there are no consistently measured quarterly data for the Eurozone, we extrapolated the annual data using constant quarterly growth rates within a year, assuring that the data for the beginning of the year are the same as measured by annual data.

[11] Especially in case of Poland, it is very hard to model the labor market variables using a relatively simple determination of labor market equilibrium, as in the presented model. So we decided to add a labor supply

would limit the information that is used in the estimation, as the number of observed variables needs to be equal to the number of stochastic shocks. So, in the estimation of the parameters, we included additional information form other variables, assuming that theory are observed with an iid noise. As primarily variables (observed without noise), we used the following time series for the period $IIIQ1996 - IVQ2007$: GDP, government expenditure, consumer price inflation, employment for both Poland and the Eurozone, and the interest rate differential (gross, as in the model denoted as R). The set of additional variables (observed with noise) includes (for both economies): consumption, investments and trade indicators (both exports and imports, expressed in Polish currency).

As the model is log-linearized, it can be expressed as a state-space representation and it's likelihood can be evaluated against data via the Kalman filter. We decided to use the Bayesian approach to estimate the model parameters, since it allows to use additional information, that can be provided via the prior distribution of the parameters. The expression for the likelihood function cannot be found analytically, however the posterior distribution of the model parameters can be estimated via Monte Carlo Markov Chain (MCMC) algorithm, proposed by Metropolis and Teller (1953) and Hastings (1970). We used the Dynare package in order to solve, estimate and simulate the model (see e.g. Juillard (1996)).

After finding the mode of the posterior distribution (using the Chris Sims csminwel procedure that applies a quasi-Newton method with BFGS update of the estimated inverse Hessian, robust against "cliffs", i.e. hyperplane discontinuities) we applied the Metropolis-Hastings MCMC algorithm with 50000 replications (to assure that the MCMS algorithm have converged) in two blocks in order to compute the posterior distribution of the model parameter. The acceptation rates were ca. 24%, within the band 23%-25% recommended in the literature. On the basis of the the univariate and multivariate convergence diagnostic of Brooks and Gelman (1998) indicating that the MCMC algorithm converged, we dropped 10% of draws for the purpose of construction of the posterior probability distributions.

When choosing the parameters of the prior distributions, we used the results of Smets and Wouters (2003), Adolfson, Laséen, Lindé, and Villani (2005) and Kolasa (2008) and, to some extend, some pre-estimation exercises. The chosen parameters of the prior distribution are presented in Table 2. For most cases we have not distinguished between Poland and the Eurozone, except for ι's, Calvo probabilities θ's and parameters of the Taylor rule. As the installation of the new investments goods in Poland could be more costly, due to more restricted regulations we choose to set lower value for ι than for ι^*. Also, due to more stringent product market regulations in Poland, we choose to set slightly lower values for θ_H and θ_F. Additionally, taking into account the results of Kolasa (2008), we picked such

shock, in order to allow the model to have a chance to describe the labor market behavior in line with the data.

Table 2: Basic description of the prior and posterior distributions

Parameter	Prior distribution			Posterior distribution		
	type	Mean	St. Dev.	Mode	Mean	St. Dev.
ι	norm	0.10	0.025	0.101	0.097	0.023
ι^*	norm	0.30	0.025	0.337	0.338	0.023
γ	norm	4.00	0.250	4.070	4.039	0.245
γ^*	norm	4.00	0.250	4.129	4.096	0.245
θ_H	beta	0.60	0.080	0.504	0.516	0.066
θ_F^*	beta	0.70	0.080	0.760	0.755	0.034
θ_F	beta	0.60	0.080	0.806	0.771	0.066
θ_H^*	beta	0.70	0.080	0.755	0.735	0.064
χ	norm	0.05	0.030	0.081	0.084	0.025
σ	norm	2.00	0.300	3.117	3.107	0.242
σ^*	norm	2.00	0.300	1.912	1.941	0.214
γ_R	beta	0.82	0.020	0.721	0.720	0.020
γ_π	norm	1.25	0.070	1.274	1.291	0.080
γ_{GDP}	norm	0.40	0.050	0.457	0.459	0.050
$\gamma_{d\pi}$	norm	0.20	0.030	0.198	0.208	0.030
γ_{dGDP}	norm	0.10	0.020	0.109	0.109	0.020
γ_R^*	beta	0.80	0.020	0.766	0.766	0.018
γ_π^*	norm	1.30	0.080	1.420	1.419	0.075
γ_{GDP}^*	norm	0.30	0.060	0.333	0.332	0.061
$\gamma_{d\pi}^*$	norm	0.20	0.030	0.203	0.206	0.030
γ_{dGDP}^*	norm	0.10	0.020	0.097	0.093	0.020
ρ_z	beta	0.95	0.010	0.945	0.944	0.010
ρ_z^*	beta	0.95	0.010	0.949	0.947	0.010
ρ_G	beta	0.85	0.100	0.624	0.656	0.113
ρ_G^*	beta	0.85	0.100	0.850	0.832	0.068
ρ_l	beta	0.60	0.100	0.773	0.761	0.054
ρ_l^*	beta	0.60	0.100	0.717	0.714	0.060
ρ_{a_κ}	beta	0.60	0.100	0.554	0.533	0.062

prior distributions of the parameters of the Taylor rule, that the policy rule in the Eurozone is slightly more inflation oriented than in Poland. The standard deviations of the stochastic shocks were set on the basis of the pre-estimation exercise. For the standard deviations of the noise components of additional observed variables, we assumed that the noise component variability constitutes 1% of the overall variability of a given variable.

3.3 Parameters

The results of the estimation procedure are shown in Table 2. Additionally, we plotted the prior and posterior distributions of all estimated parameters - see Figure 1, including standard deviations of noisy components of additional variables[12].

The results of the estimation indicate that our assumption of the relative size of the parameter governing the cost of adjusting capital stock was correct. Also, the additional information from the data have not changed strongly the prior distribution of labor supply elasticity. The estimated coefficients of the Calvo parameters indicate that the degree of nominal rigidity in Poland is higher than in the Eurozone, especially in case of goods sold domestically. The estimated Calvo probabilities for the Eurozone are roughly in line with the results of Adolfson, Laséen, Lindé, and Villani (2005).

The information in data series significantly changed the estimate of the inverse of the elasticity of intertemporal substitution in Poland - the estimated parameter $\sigma = 3.1$ is much higher than the mean of the prior, which was set at 2.

The estimated interest rate smoothing coefficient in the Taylor rule for Polish economy proved to be lower than expected, $\gamma_R = 0.72$, although within the range usually obtained in the literature. Additionally, the responsiveness of the interest to the consumer price inflation and GDP is slightly higher than assumed, both in case of Poland and the Eurozone. The estimated degree of interest rate smoothing of the ECB $\gamma_{R^*} = 0.77$, is slightly lower than the usual estimates (see e.g. Smets and Wouters (2003) or Adolfson, Laséen, Lindé, and Villani (2005)). The estimated Taylor rule in case of the NBP is as follows:

$$\hat{R}_t = 0.72\hat{R}_{t-1} + (1-0.72)[1.29\hat{\pi}_t + 0.46G\hat{D}P_t + 0.11dG\hat{D}P_t + 0.21d\hat{\pi}_t] + \varphi_t \tag{40}$$

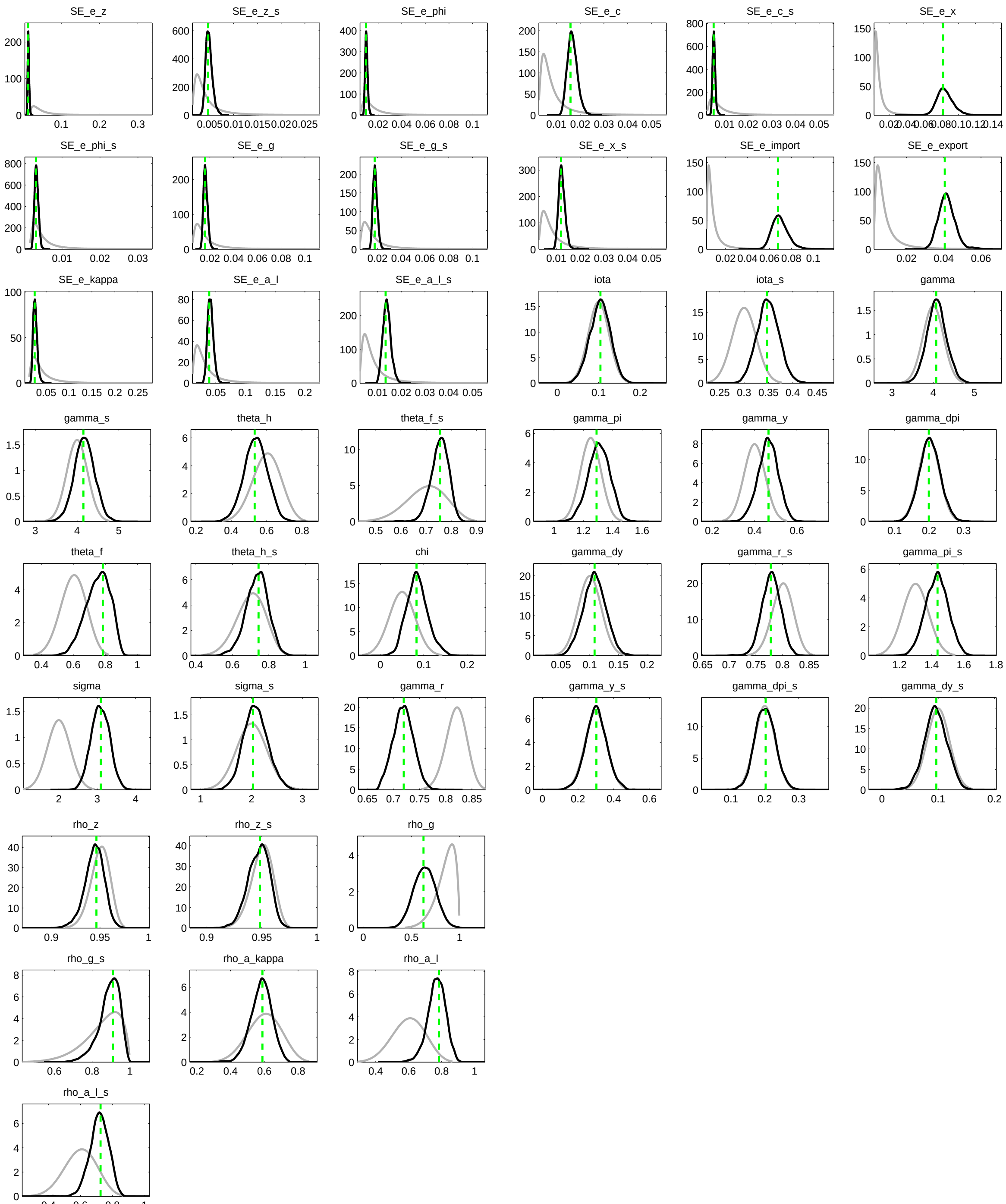
and

$$\hat{R}^*_t = 0.77\hat{R}^*_{t-1} + (1-0.77)[1.42\hat{\pi}^*_t + 0.33G\hat{D}P^*_t + 0.09dG\hat{D}P^*_t + 0.21d\hat{\pi}^*_t) + \varphi^*_t \tag{41}$$

in case of the ECB.

[12] These are the plots of: SE_e_c, SE_e_c_s, SE_e_x, SE_e_x_s, SE_e_import, SE_e_export.

Figure 1: Prior and posterior distributions of the model parameters



The estimated persistence of the technology shocks is very similar in Poland and in the Eurozone, roughly equal to $0.94 - 0.95$. The estimation revealed the the persistence of the government spending shock is much higher in the Eurozone, than in Poland, in line with the economic intuition. Also, the persistence of the labor supply shock proved to be higher in case of Poland, reflecting the features of the transforming economy. The persistence of the risk premium shock proved to be relatively small, also in line with the economic intuition[13].

The estimated volatilities of the shocks governing the evolution of the economy (not reported in Table 2, but depicted in Figure 1) reveal that the volatility of the technology shock is much larger in Poland, again in line with the economic intuition and in line with the evidence from the relative volatility of output of the Polish economy compared to the Eurozone. The same applies for the monetary policy shock and the labor supply shock. On the other hand, the volatility of the government spending shock is slightly higher in the Eurozone, than in Poland.

3.4 Model's Data Fit

In order to evaluate the ability of the model to replicate the features of the data from the Polish and the Eurozone economies, we compared the moments of the model generated variables against the moments of the data. Instead we could have used the theoretical moments of the model's variables, but these describe the large sample properties of the model, whereas when calculating the moments of the data we are using only ca. 50 observations. In order to overcome this issue we decided to simulate the model behavior in short sample. So, we simulated the model using random draws of the stochastic shock[14] for 152 periods and then dropped the first 100 observations[15], calculating the moments for only 52 observations. We replicated this procedure 10000 times (for different, independent draws of stochastic shocks) and averaged the resulting correlations in order to assure that our calculated moments are history-independent.

Table 3 shows the results of our procedure - the upper panel shows the results for the Polish economy, the lower panel - for the Eurozone economy. The first two columns show the volatilities of the model generated variables against the data (measured by the standard deviations), the middle two columns show the cyclicality of variables, as measured by correlations with GDP (GDP^* in case of the Eurozone). The last columns show the persistence

[13] Taking into account the relatively fast adjustment and efficiency of the exchange rate market, it is not surprising that the agents respond relatively quickly to the exogenous changes in the interest rate disparities.

[14] As was mentioned earlier, the additional shocks of the model - noise in additional observables - were used only for the purpose of the estimation, so in the simulations performed on the model, we turned off these shocks.

[15] As the simulation starts from the steady state, we dropped some observations to assure that we calculate the moments of variables not being biased by being too close to the steady state.

Table 3: Moments of the model generated variables against the data

	Volatility		Correlation with GDP		Persistence	
	data	model	data	model	data	model
			Poland			
GDP	0.020	0.025	1.000	1.000	0.403	0.639
c	0.010	0.010	0.611	0.324	0.806	0.678
x	0.089	0.070	0.874	0.597	0.800	0.990
π	0.010	0.026	-0.276	-0.353	-0.279	0.790
w	0.012	0.026	0.464	0.492	0.662	0.603
R	0.021	0.023	0.282	-0.571	0.849	0.891
l	0.015	0.012	0.601	0.195	0.928	0.757
q	0.061	0.044	0.118	0.655	0.813	0.385
$export$	0.048	0.048	0.337	0.761	0.812	0.421
$import$	0.062	0.015	0.517	0.158	0.765	0.810
			Eurozone			
GDP^*	0.006	0.010	1.000	1.000	0.817	0.886
c^*	0.006	0.005	0.842	0.522	0.868	0.620
x^*	0.027	0.040	0.758	0.784	0.935	0.981
π^*	0.003	0.007	0.288	-0.464	0.215	0.832
w^*	0.004	0.016	0.617	0.571	0.646	0.631
R^*	0.006	0.006	0.635	-0.831	0.896	0.867
l^*	0.005	0.004	0.805	0.305	0.948	0.724

of the model generated variables and the data - measured by autocorrelation.

The model generates too much volatility of GDP, both in Poland and the Eurozone. The same is true for inflation and real wages (and investments in case of the Eurozone economy). The model correctly reproduces, for both economies, the volatility of consumption nominal interest rates and exports. The volatilities of investments in Poland and employment in both economies, as well as imports were underestimated by the model.

Our model generates too little comovement with output in case of consumption, investments, employment and imports in case of Poland. The same is true for consumption and employment in case of the Eurozone. The cyclicality of the real exchange rate is largely exaggerated by the model. The model predicts countercyclical inflation in the Eurozone, whereas it is slightly procyclical in the data. Also the cyclicality of interest rates are different in the model and data for both economies - in the data interest rates are procyclical (slightly procyclical in case of Poland), whereas the model predicts that monetary policy is countercyclical (the procyclically of interest rates in the data is rather not intuitive and could be an artifact of the short sample used in the analysis). The model reproduces generally well the persistence of analyzed variables - excluding the real exchange rate, exports, and especially inflation.

Summing up, the model fits the data relatively well, although not perfectly. But taking into account the short sample used in the estimation and evaluation of the model properties, its performance is rather good. The main problems are: exchange rate (even with the equity premium puzzle shock the model cannot generate the properties of this variable, but this is rather common to this kind of methodology - see e.g. Chari, Kehoe, and McGrattan (2002)), cyclicality of the interest rates, and cyclicality of inflation (the latter in case of the Eurozone).

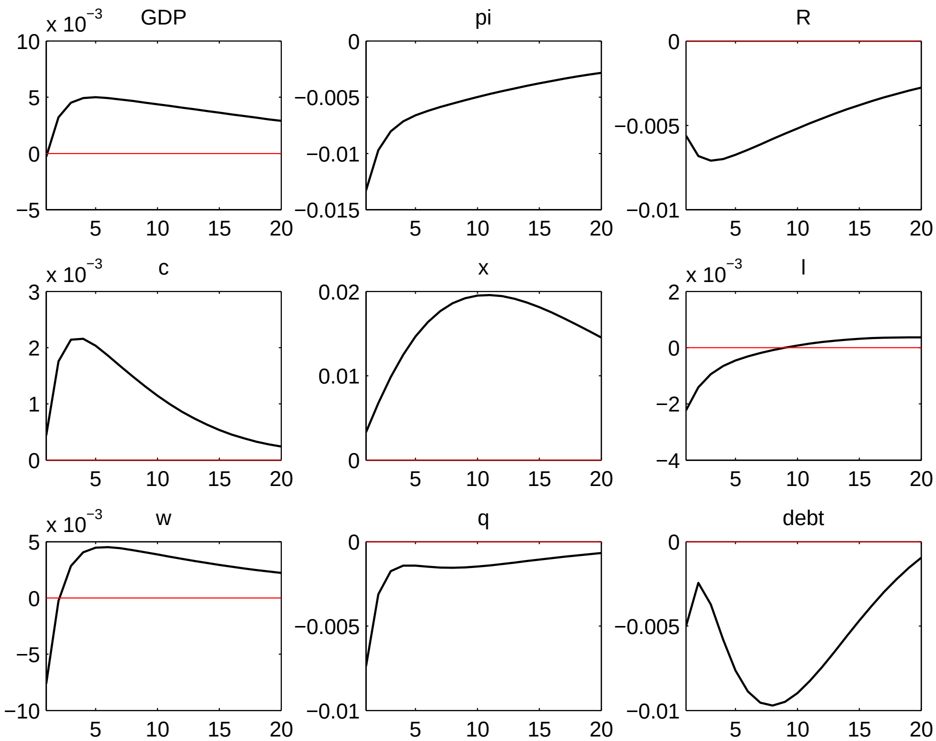
3.5 Impulse Response Functions

In order to understand the dynamic properties of the model, we calculated the impulse response of the model to the most important shocks of the model - asymmetric technology shocks and asymmetric monetary policy shock (that occurred in the Polish economy).

3.5.1 Asymmetric Technology Shock

After an unanticipated asymmetric technology shock (hitting the Polish economy, see Figure 2) the model predicts the prolonged increase of both consumption and investments, the latter generating the increase of capital stock. The decrease of the domestic prices translates into falling inflation and a decline of the nominal interest rates. Real wages decrease on impact, but then quickly rise above the steady state and stay there for a while. The technology shock

Figure 2: Impulse response to the asymmetric technology shock



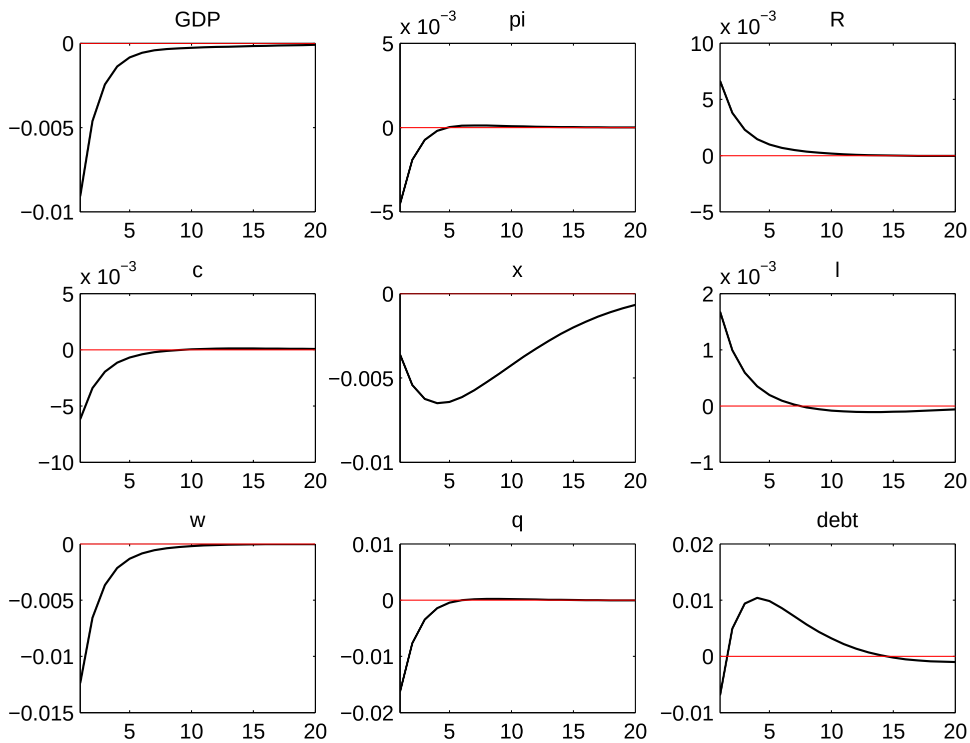
proved to be labor-saving, resulting in drop of employment for the first 10 quarters. This effect is due to the fact that the income effect dominates the substitution effect. After a couple of quarters employment recovers and at some point it even goes over its steady state value. With lower employment (and higher labor productivity) and higher capital, the GDP rises for the whole analyzed period.

The higher level of activity in the Polish economy translates into a decrease of foreign debt - domestic agents use the period of higher activity of the economy and higher income to pay back some of the debt they have against the foreign agents. As a result of higher productivity in the domestic economy, we observe a decline of the real exchange rate (i.e. exchange rate appreciates in real terms). After about $6-8$ years after the shock the economy converges back to the steady state.

3.5.2 Asymmetric Monetary Policy Shock

Figure 3 presents the response of the model economy to an unanticipated asymmetric monetary policy shock. Initially, the domestic interest rates increase, then the Taylor rule kicks in and the interest rates slowly revert to the steady state.

Figure 3: Impulse response to the asymmetric monetary policy shock



In response to the higher nominal interest rates, we observe a decline of inflation, lower households consumption and investments. Although real wages also decrease, we observe an increase of employment, as again income effect dominates the substitution effect. In spite of the latter effect, due to the decline of capital, the GDP declines. After the initial decline of debt, households start to borrow from abroad to smooth out consumption. In reaction to an increase in interest differential, the exchange rate appreciates (i.e. drops) in real terms.

When we compare the reaction of the model to the technology shock (see Figure 2) and the monetary policy shock (see Figure 3), we can observe that the economy stabilizes much faster in the latter case. This feature is relatively intuitive - the technology shock is an example of a supply shock, generating more persistent response of the economy. On the other hand - monetary policy shock, as a pure demand shock, generates much less persistence of economic fluctuations.

4 Results

In this section we analyze business cycle behavior of the most important macroeconomic variables of the Polish economy in the presence of shocks in two regimes:

- *Autonomous Monetary Policy* (denoted as OUT), monetary policy in Poland is conducted by the National Bank of Poland and the Taylor rule describing this policy is given by equation (40), while monetary policy in the Eurozone is conducted by the ECB and the Taylor rule that describes this policy is given by equation (41). The nominal exchange rate is not fixed and adjusts freely to the market conditions.

- *Common Monetary Policy* (denoted as IN), monetary policy both in Poland and Eurozone is conducted by the ECB according to the augmented Taylor rule that is the same as the ECB Taylor rule estimated from the data, except for it assigns weight $\omega_T = \frac{GDP^{Poland}}{GDP^{Eurozone}+GDP^{Poland}} = 2,97\%$ to the Polish variables:

$$\hat{R}_t^* = 0.77\hat{R}_{t-1}^* + (1-0.77)[1.42(\omega_T\hat{\pi}_t + (1-\omega_T)\hat{\pi}_t^*) + 0.33(\omega_T G\hat{D}P_t + (1-\omega_T)G\hat{D}P_t^*) \\ + 0.09(\omega_T dG\hat{D}P_t + (1-\omega_T)dG\hat{D}P_t^*) + 0.21(\omega_T d\hat{\pi}_t + (1-\omega_T)d\hat{\pi}_t^*)] + \varphi_t^*.$$

  Nominal exchange rate is fixed and cannot be changed $e_t = \bar{e}$. We want to stress that in this simulation we do not eliminate the risk premium volatility as a source of the interest rate differential between domestic and foreign households. In our model the risk is associated rather with fluctuations of the domestic debt than fluctuations of the exchange rate. Thus the accession to the Eurozone does not eliminate the effect that this risk has on the interest rate differential.

Given the differences between the two regimes there are four important factors that may affect the Polish economy after joining Eurozone: (1) the Taylor rule in the Eurozone is more inflation oriented than in Poland; (2) there will be less variability hitting the Polish economy, since the Polish monetary policy shock is replaced with the Eurozone monetary policy shock (which has 3 times smaller standard deviation); (3) the monetary policy rule will be focused on the whole Eurozone economy rather than the Polish economy; and (4) the nominal exchange rate will be fixed.

To analyze the differences between the two regimes we run 50,000 simulations for 1000 periods each. Using the results from the simulations we compare the business cycle behavior of the most important macroeconomic variables and we compute by how much consumer's welfare in Poland decreases after joining the Eurozone.

4.1 GDP and Inflation Variability

In our simulations we calculated standard deviations of the main macroeconomic variables. The results are presented in Table 4. Most variables are more volatile under *Common Monetary Policy*, which is mostly due to the fact that *Common Monetary Policy* fits Polish economy less than *Autonomous Monetary Policy*. Furthermore, the nominal exchange rate under *Common Monetary Policy* no longer cushions some shocks. These effects out-weight the effect of disappearance of the domestic monetary policy shock. The only difference is inflation, which is less volatile under *Common Monetary Policy* than *Autonomous Monetary Policy*. In this case there are three factors that decrease volatility of inflation. First, the Taylor rule under *Common Monetary Policy* reacts to inflation stronger than the Taylor rule under rule *Autonomous Monetary Policy*. Second, in *Autonomous Monetary Policy* regime there is the domestic monetary policy shock, which is more volatile than the foreign monetary policy shock, thus adds more volatility to the economy and it mostly affects inflation, whereas under *Common Monetary Policy* regime this shock is replaced with the foreign country monetary policy shock. Third, fixing the nominal exchange rate stabilizes the inflation in the home country (at the same time increases volatility of GDP)[16]. This result is somewhat different from Karam, Laxton, Rose, and Tamirisa (2008), who find that volatility of both GDP and inflation will increase after a new member country joins the Eurozone, whereas we find that volatility of GDP will increase but inflation will become more stable.

[16]If the exchange rate adjusts freely, different international shocks hitting an economy (in a world with sticky prices) might be cushioned by the exchange rate movements, thus the shocks partially affect output and partially prices. If the exchange rate is fixed than the external shocks impact mostly GDP not prices.

Table 4: Variability of the Polish business cycle.

Variables	*Autonomous MonetaryPolicy*	*Common MonetaryPolicy*	Change
GDP	0.0276	0.0327	+18%
Inflation	0.0288	0.0098	-66%
Interest Rate	0.0261	0.0300	+15%
Consumption	0.0106	0.0208	+96%
Labor	0.0119	0.0134	+13%

One might also wonder why the interest rate after the Euro adoption is more volatile. This is mostly due to the fixing of the nominal exchange rate. Note, that in *Common Monetary Policy* regime the differential of the interest rates (the UIP condition, see equation (A.13)) depends on the risk premium, the expected change in the real exchange rate and the difference in the expected inflation. But, in *Common Monetary Policy* regime the UIP condition is replaced with the following condition (in loglinearized version)

$$\hat{R}_t - \hat{R}_t^* = \hat{\kappa}_t \tag{42}$$

thus the shocks to the risk premium affect only the interest rate differential, whereas previously some cushion was provided by the exchange rate (which would absorb some the volatility). Also greater volatility of the interest rate increases volatility of consumption, which explains why volatility of consumption increases by more than volatility of GDP.

4.2 Welfare Cost of Joining Eurozone

The results presented in the previous subsection do not provide quantitative measure of the costs of joining the Eurozone. Thus we expressed the cost in terms of consumer welfare. We ask how much consumption, consumers would be willing to give up in order to stay indifferent between joining and not joining the Eurozone. This corresponds to calculating the compensating variation associated with the full elimination of *Autonomous Monetary Policy* regime. Welfare analysis follows the method of Lucas (1987).

We use our simulations to compute the consumers welfare in both regimes of

$$W^{OUT} = E_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(c_t^{OUT}, l_t^{OUT}, \zeta_t\right) \right], \tag{43}$$

$$W^{IN} = E_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(c_t^{IN}, l_t^{IN}, \zeta_t\right) \right]. \tag{44}$$

Next we compute what percentage (denoted by λ) of every period consumption, consumers would have to give up to be indifferent between regimes. To find λ we solve

$$W^{OUT}(\lambda) = W^{IN}, \tag{45}$$

where $W^{OUT}(\lambda) = E_0\left[\sum_{t=0}^{\infty} \beta^t u\left((1-\lambda)c_t^{OUT}, l_t^{OUT}, \zeta_t\right)\right]$. The details on how we compute λ are presented in Appendix B. We find that $\lambda = 0.051\%$. This means that joining the Eurozone would have the same effect as a decrease in consumption by 0.051% in every period. Note, that it does not mean that authors expect consumption in Poland to decrease permanently by 0.051% after the Eurozone accession.

This result is a little bit smaller that the 0.25% estimate of Lopes (2007), but her result might be biased since, as far as we understand, she run only one simulation in each regime (one realization of random shocks). It seems that in order to deal with this problem she extended the simulation for many periods (1000), but we are not entirely convinced that extending the length of the simulation solves the problem of the bias. Thus, instead, we run 50000 simulations, as we noticed that only a large number of simulation guaranties convergence of the welfare result.

We can also express this welfare cost in period zero consumption rather than permanent consumption. In order to do this we compute what percentage (denoted as λ_0) of period zero consumption consumer would have to give up to be indifferent between regimes. To find λ_0 we solve

$$W^{OUT}(\lambda_0) = W^{IN},$$

where $W^{OUT}(\lambda_0) = E_0\left[u\left((1-\lambda_0)c_0^{OUT}, l_0^{OUT}, \zeta_0\right) + \sum_{t=1}^{\infty} \beta^t u\left(c_t^{OUT}, l_t^{OUT}, \zeta_t\right)\right]$. We find that $\lambda_0 = 4.73\%$. This means that joining the Eurozone would have the same effect as a 4.73% (one time) decrease in period zero consumption. Again, note that it does not mean that authors expect consumption in Poland to fall by 4.73% in the period after the Eurozone accession.

To get the grasp on the results note, that welfare is computed using the utility function in (3), thus it depends on consumption and labor. Hence, we need to analyze behavior of consumption and labor in both regimes. Note from Table 4, that the volatility of both consumption and labor is higher in *Common Monetary Policy* regime than in *Autonomous Monetary Policy* regime. So given our utility function it clearly must translate into lower welfare in *Common Monetary Policy* regime than in *Autonomous Monetary Policy* regime. We also computed the welfare cost of business cycle (in percentage of permanent change in the steady state consumption). We found that the welfare cost of business cycle in *Autonomous Monetary Policy* regime is equal to -0.017%, whereas in *Common Monetary Policy* regime is equal to -0.068%. These results are just a little bit higher than the estimates of Lucas (1987) for the US economy, who calculates the cost of business cycle at 0.01%, but the US economy is more stable. Furthermore, Storesletten, Telmer, and Yaron (2001) use OLG framework to point out that the welfare cost of business cycle might be sensitive to the persistence of the variables. We do not use the OLG framework, nevertheless, in our model the persistence of both consumption and labor is roughly in line with the data.

We want to stress that in our model wages are not sticky, which translates into lower welfare cost of business cycles. Including the sticky wages in our model could increase the welfare cost of the Eurozone accession. Unfortunately, our way of simulating the welfare effect of joining Eurozone precludes adding wage stickiness to our model. This would be an interesting extension of our work. Another extension worth considering might be the extension beyond the representative agent framework, which has a potential for generating higher costs of business cycles. The heterogeneity alone might not be enough, since the results of Schulhofer-Wohl (2008) show that adding risk averse heterogeneity among consumers with complete markets (full insurance against risk) actually decreases the costs of business cycle fluctuations.

4.3 Decomposition of Volatility Changes

In this subsection we decompose the change in variance of the main macroeconomic variables into separate factors related to the Eurozone accession presented on page 27. We stress that the simulations run here are used only for purposes of the decomposition of the volatility changes in order to isolate the effects of different factors that are involved in joining the Eurozone. In order to achieve the desired isolation, while making those simulations, we make counter-factual assumptions.

First, we compute the effect of the change of the parameters of the Taylor rule. This change impacts the economy in two important ways: (1) monetary policy is more inflation oriented than the policy implied by the estimated Taylor rule for Poland; and (2) under the new monetary policy rule the extend of the of the interest rate smoothing is higher. To isolate this effect we run the following simulation (call it simulation 2): everything is exactly like in simulation OUT from the previous subsection (in this subsection we call it simulation 1), but the Taylor rule of the NBP has the same parameters as the ECB's Taylor rule. This experiment will allow us to say how the standard deviations in question would change if the NBP run autonomous monetary policy (responding to the Polish variables only), but with the same parameters as the ECB. The results of this simulation are presented in Table 6 in the column denoted as "Simulation 2". To see the effects compare the results from simulation 1 and simulation 2. This change leads to lower variability of inflation and greater variability of GDP, which is expected since the change means that the central bank is more focused on inflation and less on GDP. Greater variability of GDP translates into greater variability of real side of the economy, i.e. the standard deviation of consumption and labor increases.

Also, greater interest rate smoothing combined with more inflation oriented monetary policy and smaller variability of inflation results in the reduction of the standard deviation of the interest rate.

Table 5: The decomposition of the volatility changes.

Variables	Simulation 1	Simulation 2	Simulation 3	Simulation 4
GDP	0.0276	0.0317	0.0289	0.0327
Inflation	0.0288	0.0210	0.0200	0.0098
Interest rate	0.0261	0.0191	0.0173	0.0300
Consumption	0.0106	0.0123	0.0088	0.0208
Labor	0.0119	0.0120	0.0118	0.0134

Second, we analyze the effect of the replacement of the domestic monetary policy shock with the foreign monetary policy shock. Thus, we run simulation 3, in which everything is exactly like in simulation 2 with one exception. We replace the value of the variance of the monetary policy shock (estimated for the Polish economy) with the value from the Eurozone. The purpose of this exercise is to isolate the effect of smaller volatility of the monetary policy shock in the Eurozone than in Poland. The results of this simulation are presented in Table 6 in the column denoted as "Simulation 3". This change reduces the volatility of the economy, which is intuitive, since the standard deviation of the monetary policy shock declines from 0.0101 to 0.0033.

Finally, we isolate the effects of the fixing of the exchange rate combined with the adoption of the common monetary policy (due to technical reasons - violation of the Blanchard-Khan conditions - we were not able to separate those two effects). Intuitively, here we analyze the effect of the disappearance of the cushion for some shocks in the form of the exchange rate and the effect of the reorientation of monetary policy from domestic variables to the aggregate variables for the extended Eurozone. The results of this simulation are presented in Table 6 in the column denoted as "Simulation 4". In fact this is the final change thus this simulation is exactly the same as simulation IN in the previous subsection (here we call it simulation 4). This change increases the standard deviations of GDP and labor due to the fact the focus of monetary policy is on the whole Eurozone rather than Poland. Second, fixing the exchange rate decreases the volatility of inflation. Third, greater volatility of GDP combined with the lack of cushion in the form of the exchange rate increases volatility of the interest rate differential (due to risk premium), which in turn translates into higher volatility of the domestic interest rate. Higher volatility of GDP together with higher volatility of the interest rate generate higher volatility of consumption and labor.

4.4 Sensitivity Analysis

In this subsection we analyze the effect of an increase of the weight of Polish variables in the *Common Monetary Policy* regime to the weight of Polish population in the Eurozone $\omega_T^{'} = 10.76\%$). The results of the new simulation are presented in Table 6. Basically the result is pretty much the same as previously. Furthermore, the welfare cost of joining the Eurozone drops only slightly from $\lambda = 0.051\%$ to $\lambda^{'} = 0.0474\%$ of the lifetime consumption, or differently from $\lambda_0 = 4.73\%$ to $\lambda_0^{'} = 4.42\%$ of the period zero consumption. This only slightly lessens the effects of joining the Euro area.

Table 6: The weight of Poland and variability of the Polish business cycle.

Variables	*Autonomous Monetary Policy*	*Common Monetary Policy* ($\omega_T = 2.97\%$)	*Common MonetaryPolicy* ($\omega_T^{'} = 10.76\%$)
GDP	0.0276	0.0327	0.0325
Inflation	0.0288	0.0098	0.0097
Interest Rate	0.0261	0.0300	0.0299
Consumption	0.0106	0.0208	0.0206
Labor	0.0119	0.0134	0.01335

5 Conclusion

The Eurozone accession for Poland might be costly, since it means loosing monetary policy as a tool to smooth out the business cycle. We build a two country dynamic stochastic general equilibrium model to quantify the effect of the loss of the autonomous monetary policy. In our model there are nominal rigidities in the form of sticky prices, which introduces a short run non-neutrality of monetary policy. This study focuses on the effects of the Eurozone accession on the business cycle behavior of the main macroeconomic variables. There are also other effects that must be analyzed in order to obtain a complete picture of the effects of the Euro adoption.

Since there is scarce evidence that higher fluctuations cause lower economic growth, we assumed that changes in the business cycle behavior will not have any long run effect on the Polish economy. We focused on the change in the business cycle behavior of the main macroeconomic variables. We found that after Poland joins the Eurozone, the volatility of GDP will increase - the standard deviation of GDP will go up from 2.76% to 3.27%, and the volatility of inflation will decrease - the standard deviation of inflation will go down from 2.88% to 0.97%. We additionally present how different factors involved with the Eurozone accession affect these outcomes.

We also computed the effect of this change on the consumers welfare and we found that the regime change will have the same effect on welfare as the decline of consumption in period after the Eurozone accession by 4.73% (or a lifetime consumption decline by 0.051%). We also checked how the weight of the Polish economy in the decisions of the ECB affects the results and we found that the increase in weight from 2.93% (which is equal to the GDP share) to 10.76% (which is equal to the population share) lessens the cost of joining the Eurozone, but the difference is tiny.

We want to stress that in our model wages are flexible, which, most likely, means that our result underestimates the cost of joining the Eurozone. Another restriction that might affect the results is the existence of the representative agent - relaxing this restriction could increase the cost of business cycle fluctuations and might affect the costs of the Eurozone accession. Additionally, the introduction of unemployment into the concept of equilibrium (like e.g. in the search-matching framework) might also positively affect the costs of business fluctuations. We leave these extensions for the future research.

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Appendix A The log-linearized model

Denote the log deviation of a variable from its steady state as $\hat{x}_t = \log\left(\frac{x_t}{\bar{x}}\right)$. To derive the first order approximation we use the following method

$$x_t = \bar{x}x_t\bar{x}^{-1} = \bar{x}e^{(\log x_t - \log\bar{x})} = \bar{x}e^{\log\frac{x_t}{\bar{x}}} = \bar{x}e^{\hat{x}_t} \approx \bar{x}\left(1+\hat{x}_t\right)$$

A.1 Households

Capital accumulation. From (6) we obtain

$$\hat{\bar{k}}_{t+1} = (1-\delta)\,\hat{\bar{k}}_t + \delta\hat{x}_t \quad (A.1)$$

$$\hat{\bar{k}}^*_{t+1} = (1-\delta^*)\,\hat{\bar{k}}^*_t + \delta^*\hat{x}^*_t \quad (A.2)$$

Capital supply. From (7) we obtain

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_t \quad (A.3)$$

$$\hat{k}^*_t = \hat{u}^*_t + \hat{\bar{k}}^*_t \quad (A.4)$$

Tobin's Q. From (11) we obtain

$$\hat{x}_t = \frac{\iota}{1+\beta}\hat{Q}_t + \frac{\beta}{1+\beta}E_t\hat{x}_{t+1} + \frac{1}{1+\beta}\hat{x}_{t-1} \quad (A.5)$$

$$\hat{x}^*_t = \frac{\iota^*}{1+\beta^*}\hat{Q}^*_t + \frac{\beta^*}{1+\beta^*}E_t\hat{x}^*_{t+1} + \frac{1}{1+\beta^*}\hat{x}^*_{t-1} \quad (A.6)$$

Euler equation. From (12) we obtain

$$\sigma E_t\left(\hat{c}_{t+1} - \hat{c}_t\right) = E_t\left[\hat{r}_{t+1} - \hat{Q}_t - \beta(1-\delta)\left(\hat{r}_{t+1} - \hat{Q}_{t+1}\right)\right] \quad (A.7)$$

$$\sigma^* E_t\left(\hat{c}^*_{t+1} - \hat{c}^*_t\right) = E_t\left[\hat{r}^*_{t+1} - \hat{Q}^*_t - \beta^*(1-\delta^*)\left(\hat{r}^*_{t+1} - \hat{Q}^*_{t+1}\right)\right] \quad (A.8)$$

Capital utilization. From (13) we obtain

$$\hat{r}_t = \frac{\Psi''(1)}{\beta^{-1} - (1-\delta)}\hat{u}_t \quad (A.9)$$

$$\hat{r}^*_t = \frac{\Psi''(1)^*}{(\beta^*)^{-1} - (1-\delta^*)}\hat{u}^*_t \quad (A.10)$$

No arbitrage, capital and nominal assets. From (15) we obtain

$$\sigma E_t\left(\hat{c}_{t+1}-\hat{c}_t\right)=E_t\left(\hat{R}_t-\hat{\pi}_{t+1}\right) \tag{A.11}$$

$$\sigma^* E_t\left(\hat{c}^*_{t+1}-\hat{c}^*_t\right)=E_t\left(\hat{R}^*_t-\hat{\pi}^*_{t+1}\right) \tag{A.12}$$

No arbitrage, home and international assets. From (15) we obtain

$$\hat{R}_t-\hat{R}^*_t=E_t\left[\left(\hat{q}_{t+1}-\hat{q}_t\right)+\left(\hat{\pi}_{t+1}-\hat{\pi}^*_{t+1}\right)\right]+\hat{\kappa}_t \tag{A.13}$$

Labor market. From (9) and (10) we obtain

$$\sigma\hat{c}_t+\gamma\hat{l}_t=\hat{w}_t \tag{A.14}$$

$$\sigma^*\hat{c}^*_t+\gamma^*\hat{l}^*_t=\hat{w}^*_t \tag{A.15}$$

A.2 Producers

Denote $p_{H,t}=\frac{P_{H,t}}{P}$, $p_{F,t}=\frac{P_{F,t}}{P}$, $p_{H,t}=\frac{P^*_{H,t}}{P^*}$, $p^*_{F,t}=\frac{P^*_{F,t}}{P^*}$, and $d_t=\frac{e_t D^*_{H,t+1}}{P_t}$

Demand for homogeneous intermediate goods. From (19) and (20) we obtain

$$\hat{Y}_{H,t}=-\frac{1+\mu}{\mu}\hat{p}_{H,t}+\hat{Y}_t \tag{A.16}$$

$$\hat{Y}_{F,t}=-\frac{1+\mu}{\mu}\hat{p}_{F,t}+\hat{Y}_t \tag{A.17}$$

$$\hat{Y}^*_{H,t}=-\frac{1+\mu^*}{\mu^*}\hat{p}^*_{H,t}+\hat{Y}^*_t \tag{A.18}$$

$$\hat{Y}^*_{F,t}=-\frac{1+\mu^*}{\mu^*}\hat{p}^*_{F,t}+\hat{Y}^*_t \tag{A.19}$$

The inflation of intermediate goods prices. From the definition of relative price $p_{d,t}=\frac{P_{d,t}}{P_t}$ and inflation of sector d intermediate good prices $\pi_{d,t}=\frac{P_{d,t}}{P_{d,t-1}}$, $d\in\{H,F\}$ we obtain.

$$\hat{\pi}_{H,t}=\hat{\pi}_t+\hat{p}_{H,t}-\hat{p}_{H,t-1} \tag{A.20}$$

$$\hat{\pi}_{F,t}=\hat{\pi}_t+\hat{p}_{F,t}-\hat{p}_{F,t-1} \tag{A.21}$$

and

$$\hat{\pi}^*_{F,t}=\hat{\pi}^*_t+\hat{p}^*_{F,t}-\hat{p}^*_{F,t-1} \tag{A.22}$$

$$\hat{\pi}^*_{H,t}=\hat{\pi}^*_t+\hat{p}^*_{H,t}-\hat{p}^*_{H,t-1} \tag{A.23}$$

Final goods producers. From (21) we obtain

$$\hat{\pi}_t = (1-\eta)\left(\bar{p}_F\right)^{\frac{-1}{\mu}}\left(\hat{\pi}_{F,t} + \hat{p}_{F,t-1}\right) + \eta\left(\bar{p}_H\right)^{\frac{-1}{\mu}}\left(\hat{\pi}_{H,t} + \hat{p}_{H,t-1}\right) \tag{A.24}$$

$$\hat{\pi}^*_t = \eta^*\left(\bar{p}^*_F\right)^{\frac{-1}{\mu}}\left(\hat{\pi}^*_{F,t} + \hat{p}^*_{F,t-1}\right) + (1-\eta^*)\left(\bar{p}^*_H\right)^{\frac{-1}{\mu}}\left(\hat{\pi}^*_{H,t} + \hat{p}^*_{H,t-1}\right) \tag{A.25}$$

Marginal costs of heterogeneous intermediate goods. From (29) we obtain

$$\hat{mc}_t = \alpha\hat{r}_t + (1-\alpha) \tag{A.26}$$

$$\hat{mc}^*_t = \alpha^*\hat{r}^*_t + (1-\alpha^*)\left(\hat{w}^*_t - \hat{z}^*_t\right) \tag{A.27}$$

Optimal production factors employment. From (30) we obtain

$$\hat{r}_t - \hat{w}_t = \hat{l}_t - \hat{k}_t \tag{A.28}$$

$$\hat{r}^*_t - \hat{w}^*_t = \hat{l}^*_t - \hat{k}^*_t \tag{A.29}$$

Prices of heterogeneous intermediate goods. From (33) and (34) we get home goods prices:

$$\begin{aligned}\hat{p}_{H,t} =& \frac{\theta_H}{1+\beta\theta_H^2}\left(\hat{p}_{H,t-1} - \hat{\pi}_t\right) + \frac{\beta\theta_H}{1+\beta\theta_H^2}E_t\left(\hat{p}_{H,t+1} + \hat{\pi}_{t+1}\right)\\ &+ \frac{(1-\theta_H)(1-\beta\theta_H)}{1+\beta\theta_H^2}\hat{mc}_t\end{aligned} \tag{A.30}$$

$$\begin{aligned}\hat{p}^*_{H,t} =& \frac{\theta^*_H}{1+\beta(\theta^*_H)^2}\left(\hat{p}^*_{H,t-1} - \hat{\pi}^*_t\right) + \frac{\beta\theta^*_H}{1+\beta(\theta^*_H)^2}E_t\left(\hat{p}^*_{H,t+1} + \hat{\pi}^*_{t+1}\right)\\ &+ \frac{(1-\theta^*_H)(1-\beta\theta^*_H)}{1+\beta\theta_H^2}(\hat{mc}_t - q_t)\end{aligned} \tag{A.31}$$

and foreign goods prices

$$\begin{aligned}\hat{p}^*_{F,t} =& \frac{\theta^*_F}{1+\beta^*(\theta^*_F)^2}\left(\hat{p}^*_{F,t-1} - \hat{\pi}^*_t\right) + \frac{\beta^*\theta^*_F}{1+\beta^*(\theta^*_F)^2}E_t\left(\hat{p}^*_{F,t+1} + \hat{\pi}^*_{t+1}\right)\\ &+ \frac{(1-\theta^*_F)(1-\beta^*\theta^*_F)}{1+\beta^*(\theta^*_F)^2}\hat{mc}^*_t\end{aligned} \tag{A.32}$$

$$\begin{aligned}\hat{p}_{F,t} =& \frac{\theta_F}{1+\beta^*(\theta_F)^2}\left(\hat{p}_{F,t-1} - \hat{\pi}_t\right) + \frac{\beta^*\theta_F}{1+\beta^*(\theta_F)^2}E_t\left(\hat{p}_{F,t+1} + \hat{\pi}_{t+1}\right)\\ &+ \frac{(1-\theta_F)(1-\beta^*\theta_F)}{1+\beta^*(\theta_F)^2}\left(\hat{mc}^*_t + \hat{q}_t\right)\end{aligned} \tag{A.33}$$

Aggregated Production function. From (26) and (22) we obtain

$$\frac{\bar{Y}_H}{A\bar{k}^{\alpha}\left(\bar{z}\bar{l}\right)^{1-\alpha}}\hat{Y}_{H,t}+\left(1-\frac{\bar{Y}_H}{A\bar{k}^{\alpha}\left(\bar{z}\bar{l}\right)^{1-\alpha}}\right)\hat{Y}^*_{H,t}=\alpha\hat{k}_t+(1-\alpha)\left(\hat{z}_t+\hat{l}_t\right) \tag{A.34}$$

$$\frac{\bar{Y}^*_F}{A^*\left(\bar{k}^*\right)^{\alpha}\left(\bar{z}^*\bar{l}^*\right)^{1-\alpha}}\hat{Y}_{F,t}+\left(1-\frac{\bar{Y}^*_F}{A^*\left(\bar{k}^*\right)^{\alpha}\left(\bar{z}^*\bar{l}^*\right)^{1-\alpha}}\right)\hat{Y}^*_{F,t}=\alpha^*\hat{k}^*_t+(1-\alpha^*)\left(\hat{z}^*_t+\hat{l}^*_t\right) \tag{A.35}$$

A.3 Government

Government expenditures. From (35) we obtain

$$\hat{G}_t=\rho_G\hat{G}_{t-1}+\hat{\varepsilon}_{G,t} \tag{A.36}$$

$$\hat{G}^*_t=\rho^*_G\hat{G}^*_{t-1}+\hat{\varepsilon}^*_{G,t} \tag{A.37}$$

A.4 Central Bank

Taylor rule. From (36) we obtain

$$\hat{R}_t=\gamma_R\hat{R}_{t-1}+(1-\gamma_R)\left(\gamma_\pi\hat{\pi}_t+\gamma_y\hat{y}_t+\gamma_{dGDP}d\hat{GDP}_t+\gamma_{d\pi}\hat{d\pi}_t\right)+\varphi_t \tag{A.38}$$

$$\hat{R}^*_t=\gamma^*_R\hat{R}^*_{t-1}+(1-\gamma^*_R)\left(\gamma^*_\pi\hat{\pi}^*_t+\gamma^*_y\hat{y}^*_t+\gamma^*_{dGDP}d\hat{GDP}^*_t+\gamma^*_{d\pi}\hat{d\pi}^*_t\right)+\varphi^*_t \tag{A.39}$$

A.5 Closing the model

Market clearing. From (37) we obtain

$$\frac{\bar{C}}{\bar{Y}}\hat{C}_t+\frac{\bar{X}}{\bar{Y}}\hat{X}_t+\frac{\bar{G}}{\bar{Y}}\hat{G}_t+\frac{\Psi'(1)}{\bar{Y}}\hat{u}_t=\hat{Y}_t \tag{A.40}$$

$$\left(\frac{\bar{C}}{\bar{Y}}\right)^*\hat{C}^*_t+\left(\frac{\bar{X}}{\bar{Y}}\right)^*\hat{X}^*_t+\left(\frac{\bar{G}}{\bar{Y}}\right)^*\hat{G}^*_t+\left(\frac{\Psi'(1)}{\bar{Y}}\right)^*\hat{u}^*_t=\hat{Y}^*_t \tag{A.41}$$

Balance of Payments. From (38) we obtain

$$\begin{aligned}\frac{1-\omega}{\omega}\left(\frac{\bar{q}\bar{p}^{*\,\frac{1-\omega}{\omega}}_H\bar{Y}^*_H}{G\bar{D}P}\right)\left(\hat{q}_t+\hat{p}^*_{H,t}+\hat{Y}^*_{H,t}\right)+\left(\frac{\bar{d}}{G\bar{D}P}\right)\frac{\beta^*}{\bar{\pi}^*}\left(\hat{d}_t-\hat{\Re}^*_t-\hat{\kappa}_t\right)=\\=\left(\frac{\bar{p}_F\bar{Y}_F}{G\bar{D}P}\right)\left(\hat{p}_{F,t}+\hat{Y}_{F,t}\right)+\left(\frac{\bar{d}}{G\bar{D}P}\right)\frac{1}{\bar{\pi}^*}\left(\hat{d}_{t-1}+\hat{q}_t-\hat{q}_{t-1}-\hat{\pi}^*_t\right)\end{aligned} \tag{A.42}$$

GDP. From (39) we obtain

$$G\hat{D}P_t = \left(\frac{\bar{Y}}{G\bar{D}P}\right)\hat{Y}_t + \frac{1-\omega}{\omega}\left(\frac{\bar{q}\bar{p}_H^* \frac{1-\omega}{\omega}\bar{Y}_H^*}{G\bar{D}P}\right)(\hat{p}_H^* + \hat{Y}_H^* + \hat{q}_t) - \left(\frac{\bar{p}_F\bar{Y}_F}{G\bar{D}P}\right)(\hat{p}_F + \hat{Y}_F) \quad (A.43)$$

$$G\hat{D}P_t^* = \left(\frac{\bar{Y}^*}{G\bar{D}P^*}\right)\hat{Y}_t^* + \left(\frac{\bar{p}_F \frac{\omega}{1-\omega}\bar{Y}_F}{\bar{q}G\bar{D}P^*}\right)(\hat{p}_F + \hat{Y}_F - \hat{q}_t) - \frac{\bar{p}_H^*\bar{Y}_H^*}{G\bar{D}P^*}(\hat{p}_H^* + \hat{Y}_H^*) \quad (A.44)$$

Appendix B Welfare Cost Computation

Here we describe the details of computing λ. First notice that it is much more convenient to solve

$$\frac{W^{OUT}(\lambda)}{\bar{c}^{1-\sigma}} = \frac{W^{IN}}{\bar{c}^{1-\sigma}}, \quad (B.1)$$

rather than (45). Next we split total utility into utility from consumption and disutility from work. Then for any regime $\Theta \in \{IN, OUT\}$

$$\begin{aligned}\frac{W^{\Theta}(\lambda)}{\bar{c}^{1-\sigma}} &= E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left((1-\lambda)c_t^{\Theta}\right)^{1-\sigma}}{1-\sigma}\right] - E_0\left[\sum_{t=0}^{\infty}\beta^t\psi\zeta_t\frac{\left(l_t^{\Theta}\right)^{1+\gamma}}{1+\gamma}\right]\\ &= \frac{W^{C,\Theta}(\lambda)}{\bar{c}^{1-\sigma}} - \frac{W^{L,\Theta}}{\bar{c}^{1-\sigma}}\end{aligned}$$

where $\frac{W^{C,\Theta}(\lambda)}{\bar{c}^{1-\sigma}} = E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left((1-\lambda)c_t^{\Theta}\right)^{1-\sigma}}{1-\sigma}\right]$ and $\frac{W^{L,\Theta}}{\bar{c}^{1-\sigma}} = E_0\left[\sum_{t=0}^{\infty}\beta^t\psi\zeta_t\frac{\left(l_t^{\Theta}\right)^{1+\gamma}}{1+\gamma}\right]$. It is convenient to denote $W^{C,\Theta}(1) \equiv W^{C,\Theta}$, than $\frac{W^{C,\Theta}(\lambda)}{\bar{c}^{1-\sigma}} = (1-\lambda)^{1-\sigma}\frac{W^{C,\Theta}}{\bar{c}^{1-\sigma}}$. To find $\frac{W^{C,\Theta}}{\bar{c}^{1-\sigma}}$ and $\frac{W^{L,\Theta}}{\bar{c}^{1-\sigma}}$ first notice that (9) and (10) imply that in the steady state

$$\varrho\bar{c}^{1-\sigma} = \psi\bar{l}^{1+\gamma} \quad (B.2)$$

After the straightforward computations of the equilibrium conditions in the steady state we get that

$$\varrho = \frac{(1-\alpha)\left[1-\beta(1-\delta)\right]}{\alpha\delta\beta}\frac{\left(\frac{\bar{x}}{\bar{y}}\right)}{\left(\frac{\bar{c}}{\bar{y}}\right)}$$

which we calibrated to match the steady state relationships and we found that $\varrho = 1.09$. Next we used (B.2) to find the following

$$\begin{aligned}\frac{W_C}{\bar{c}^{1-\sigma}} &= \sum\beta^t\frac{\left(\frac{c_t}{\bar{c}}\right)^{1-\sigma}}{1-\sigma} = \sum\beta^t\frac{\left(exp(\hat{c}_t)\right)^{1-\sigma}}{1-\sigma}\\ \frac{W_L}{\bar{c}^{1-\sigma}} &= \sum\beta^t\psi\zeta_t\frac{\frac{l_t^{1+\gamma}}{\bar{c}^{1-\sigma}}}{1+\gamma} = \sum\beta^t\psi\zeta_t\frac{\frac{l_t^{1+\gamma}}{\frac{\psi}{\varrho}\bar{l}^{1+\gamma}}}{1+\gamma} = \sum\beta^t\varrho\zeta_t\frac{\left(\frac{l_t}{\bar{l}}\right)^{1+\gamma}}{1+\gamma} = \sum\beta^t\varrho\zeta_t\frac{\left(exp(\hat{l}_t)\right)^{1+\gamma}}{1+\gamma}\end{aligned}$$

Notice that with this formula it is not necessary to find the steady state values of $\bar{c}$ or $\bar{l}$.

To find the welfare cost expressed in period zero consumption λ_0 we use analogous method. The only difference is that (B.1) is replaced with

$$\frac{W^{OUT}(\lambda_0)}{\bar{c}^{1-\sigma}} = \frac{W^{IN}}{\bar{c}^{1-\sigma}},$$

where

$$\begin{aligned} \frac{W^{\Theta}(\lambda_0)}{\bar{c}^{1-\sigma}} &= E_0\left[\frac{\left((1-\lambda_0)c_0^{\Theta}\right)^{1-\sigma}}{1-\sigma}\right] + E_0\left[\sum_{t=1}^{\infty}\beta^t\frac{\left(c_t^{\Theta}\right)^{1-\sigma}}{1-\sigma}\right] - E_0\left[\sum_{t=0}^{\infty}\beta^t\psi\zeta_t\frac{\left(l_t^{\Theta}\right)^{1+\gamma}}{1+\gamma}\right] \\ &= (1-\lambda_0)(1-\beta)\frac{W^{C,\Theta}}{\bar{c}^{1-\sigma}} + \beta\frac{W^{C,\Theta}}{\bar{c}^{1-\sigma}} - \frac{W^{L,\Theta}}{\bar{c}^{1-\sigma}}. \end{aligned}$$