

NATIONAL BANK OF POLAND WORKING PAPER №. 141

Uncertainty as commitment

Jaromir Nosal, Guillermo Ordoñez

We thank Mark Aguiar, Marco Bassetto, Philip Bond, V.V. Chari, Emmanuel Farhi, Mike Golosov, Gary Gorton, Oleg Itskhoki, Bob King, Nobu Kiyotaki, Svetlana Pevnitskaya, Richard Rogerson, Esteban Rossi-Hansberg, Alp Simsek, Jean Tirole, Jonathan Vogel and Warren Weber for helpful comments, as well as seminar participants at Boston College, Boston University, Chicago Fed, Columbia, Florida State University, Harvard, National Bank of Poland, Minneapolis Fed, Notre Dame, Penn State, Princeton, St. Louis Fed, Toulouse School of Economics, Wharton, the Roma Macro Junior Conference at the EIEF, the SED 2012 Annual Meetings in Cyprus and the LAEF Conference at Santa Barbara. The usual waiver of responsibilities applies.

Jaromir Nosal – Columbia University.

Guillermo Ordoñez – University of Pennsylvania and NBER.

Design:

Oliwka s.c.

Layout and print:

NBP Printshop

Published by:

National Bank of Poland
Education and Publishing Department
00-919 Warszawa, 11/21 Świętokrzyska Street
phone: +48 22 653 23 35, fax +48 22 653 13 21

© Copyright by the National Bank of Poland, 2013

ISSN 2084-624X

<http://www.nbp.pl>

Contents

1	Introduction	4
2	Simple Example	11
2.1	Full Information	13
2.1.1	No Commitment	13
2.1.2	Commitment	13
2.2	Imperfect Information	15
3	Full Model	20
3.1	Environment	20
3.1.1	Banks	20
3.1.2	Households and Government	21
3.1.3	Timing	22
3.2	Preliminaries	23
3.3	Full Information	27
3.3.1	Commitment	27
3.3.2	No Commitment	31
4	Imperfect Information in the Full Model	33
4.1	Ex-post Shocks to Cash Holdings	35
5	Policy Implications	40
5.1	Financial Innovation	40
5.1.1	Extended setting with shocks to cash holdings	42
5.2	Number of banks	44
5.3	Asymmetric Bank Sizes	46
6	Conclusions	48
A	Appendix	51
A.1	Proof of Proposition 1	51

A.2	Proof of Lemma 3	52
A.3	Proof of Lemma 4	53
A.4	Proof of Lemma 5	55

Abstract

Time-inconsistency of no-bailout policies can create incentives for banks to take excessive risks and generate endogenous crises when the government cannot commit. However, at the outbreak of financial problems, usually the government is uncertain about their nature, and hence it may delay intervention to learn more about them. We show that intervention delay leads to *strategic restraint*: banks endogenously restrict the riskiness of their portfolio relative to their peers in order to avoid being the worst performers and bearing the cost of such delay. These novel forces help to avoid endogenous crises even when the government cannot commit. We analyze the effect of government policies from the perspective of this new result.

JEL: G21, G28, E61.

Keywords: bailouts, commitment, liquidity, banking, government policy, regulation.

1 Introduction

Few would disagree that bailouts are socially costly. Yet, they are commonly used to help banks and financial institutions during crises in most countries, dating as far back as the 1800s.¹ In the recent 2008-09 financial crisis, for example, the U.S. government used a variety of instruments to bail out, on an unprecedented scale, many financial entities that were exposed to systemic risk.

An extensive literature, most recently represented by Farhi and Tirole (2012), provides an explanation for this phenomenon based on moral hazard–driven excessive leverage of banks, triggered by the time inconsistency of no-bailout policies. If the government faces certain losses due to bank failures in case of a shock – there may not be enough liquidity in the system because of a systemic nature of the shock, for example – then it may be tempted to bail out banks in distress *ex post*. Without commitment, banks internalize this *ex-post* reaction and hence have no incentive to avoid exposing themselves to such shocks *ex ante*, effectively creating crises endogenously. This leads to equilibrium outcomes that are away from the social optimum (in which the government commits to no bailouts, hence disciplining bank actions and avoiding endogenous crises).

Two important assumptions underlying these results are perfect observability of the state of the economy – the shocks are public information, and degenerate timing of events – all banks fail or survive at the same time. However, casual observation of crises episodes reveals that neither is satisfied in practice. The outbreak of most financial crises is rarely characterized by all financial institutions suddenly and simultaneously showing distress. As a result, policymakers are usually uncertain whether they are facing an isolated incident of distress, which can be solved internally in the financial system through mergers and acquisitions, or a more systemic event, in which output may be lost without government intervention. Hence, at least at the onset of a financial crisis, bailout decisions are taken under conditions of uncertainty, with government observing only imperfect signals of the severity of the problem: we call this situation *government uncertainty*.

In the most recent crisis, for example, when U.S. policymakers decided not to bail out Lehman Brothers on September 14, 2008, allowing the company to file for bankruptcy in the hopes that another company would take over, they

¹In 1857, after the *Livingston vs. Bank of New York* case, for example, courts ordered that ‘the mere fact of suspension of specie payments when it is general is not of itself sufficient proof of fraud or injustice’, officially sanctioning suspensions of specie payments in case of aggregate crises and systemic events.

were criticized for putting the financial system on the brink of a collapse. However, this decision was in part motivated by uncertainty about the nature of the underlying problem and by the hope that the financial system would restore normality without relying on costly public intervention.² The bailout of Continental Illinois Bank and Trust Company in 1984 provides another example. The FDIC chairman at the time, William Isaac, explained that ‘the best estimates of our staff, *with the sparse numbers we had at hand*, were that more than 2,000 banks might be threatened or brought down by a Continental collapse.’³ This anecdotal evidence suggests that bailout decisions are usually made under conditions of uncertainty, fueled by governments’ limitations to acquire or process information.

The main result of this paper is that *government uncertainty* has the potential for sustaining commitment outcomes even when the government lacks commitment. Intuitively, at the onset of financial problems, governments are uncertain about their systemic nature, and hence about the actual need for a bailout. In order to observe more signals and *learn* about the nature of the shock, the government may want to *delay bailout* and let the first bank(s) in distress fail – with learning allowing it to avoid an unnecessary and potentially costly intervention. Crucially, intervention delay makes the relative performance of banks’ portfolios critical for individual banks, since no bank wants to be amongst the first in line for government help. We call this effect *strategic restraint*, as banks endogenously restrict the riskiness of their portfolio relative to their peers in order to avoid being amongst the worst performers, inducing a sort of competition to reduce excessive risk-taking.

In the model, bankers borrow short-term from households to finance projects that are illiquid. Projects may suffer shocks over time, in which case they require extra funds to bring them to fruition. The shock hitting a project may be idiosyncratic, hitting only certain banks, or aggregate, hitting all banks. High levels of short-term debt allow banks to invest in large projects, but at the same time hinder their ability to refinance if a shock hits. We study the problem of a central authority, which we call the *government*, which maximizes total welfare (bankers’ plus households’) using an interest rate policy that affects the cost of borrowing to refinance in case of shocks. An intervention that reduces the cost of borrowing to bankers, which we call a *bailout*, is financed through taxes on

²Kelly, Lustig, and Van Nieuwerburgh (2011) provide a thorough discussion of the timing of events and the evolution of government’s announcements of bailouts during the recent crisis.

³Our emphasis in italics.

households in a way that is socially costly (e.g. due to distortions). The benefits of bailouts, on the other hand, are naturally given by bringing banks' projects to fruition, and thus increasing output.

When the government observes a bank in distress – which we define as the bank running out of cash and options for refinancing on the market – it does not observe whether the shock is idiosyncratic or aggregate, information that is critical for taking appropriate action. If the shock is idiosyncratic, other banks have enough liquidity to take over the distressed bank, and no intervention is needed. If the shock is aggregate, intervention is the only way to avoid a project failure. Hence, the government's decision to bail out the bank or not depends on its beliefs about the nature of the shock.

We show that, if the government is initially relatively optimistic it is not facing an aggregate shock, then it chooses to learn more by delaying intervention, not bailing out the first distressed bank(s). By delaying the bailout, the government maintains the option of introducing the bailout at a later time – after observing subsequent signals (further bank distress) – under a more precise belief about the true state. For the banks, on the other hand, delay introduces an incentive to avoid being the worst performers. In the model, this happens through banks leveraging less and carrying more cash reserves than its peers for the eventuality of being hit by the refinancing shock, at the cost of downsizing its project. This is the *strategic restraint* effect, which gives Bertrand-style competition among banks for lower leverage. We show that in the unique equilibrium of the economy, banks compete away all excessive leverage, and the allocation coincides with the one which obtains under commitment, here driven by government uncertainty and strategic restraint instead.

In our benchmark, we consider a stark case in which banks can guarantee not being the worst performer by choosing slightly lower leverage relative to other banks. Our results, however, are robust to relaxing this assumption. In an extended environment, we consider shocks to cash holdings of individual banks, such that small deviations in leverage do not guarantee outperforming other banks. In this case, we show that the *government uncertainty* and *strategic restraint* forces still operate, moving the equilibrium allocation towards the optimal (commitment) outcome. In this case, however, the unique equilibrium achieves leverage that is intermediate between the commitment (optimal) and non-commitment (inefficient) equilibrium, the benchmark model being a limiting case.

Given that *government uncertainty* and *strategic restraint* forces robustly implement allocations that dominate the non-commitment equilibrium in terms of welfare (the benchmark achieving the optimum), natural policy questions arise regarding the effect of regulation and economic environment on the effectiveness of these effects. How does financial innovation affect uncertainty and the likelihood of endogenous crises? Is government uncertainty more effective when there are many banks? Is it more effective when banks are of similar size? We address these questions in the paper.

We model financial innovation as affecting the ability of banks to insure away part of their idiosyncratic risk, for example by trading securitized products or over-the-counter derivatives. In the benchmark model, any level of insurance introduces differences in the cash position of banks which depend on the type of shocks: when the shock is idiosyncratic, the affected bank has more cash to refinance because it has claims on healthy projects of other banks. This difference in cash translates into a clear difference between idiosyncratic and aggregate shocks in when the bank is going to run out of cash and financing options, and hence when it is going to show distress. In the benchmark, cash and time of distress are deterministically connected, and hence the government can perfectly infer the shock from the time the bank becomes distressed. Hence, perhaps surprisingly, we show that in the benchmark model, for *any* level of insurance, imperfect information plays no role and the equilibrium of the economy coincides with one with no government uncertainty, leading to an inefficient equilibrium allocation (the non-commitment allocation).

This result hinges on the ability to perfectly infer the underlying state from the time the bank becomes distressed, which is relaxed in our extended environment discussed earlier. With shocks to cash holdings, the government knows only the *average* time of distress under idiosyncratic and aggregate shocks, and hence the inference from the actual time of distress to the underlying shock is imperfect. This preserves the government uncertainty and strategic restraint forces in economies with sufficiently imperfect insurance. In particular, we derive a cap on the fraction of idiosyncratic risk that can be insured away, which still allows for *government uncertainty* to lead to welfare-superior allocations. We find the cap to be an increasing function of the variance of the shocks to cash holdings. These results point to an unexplored effect of financial innovation, such as securitization, in that it hinders the effectiveness of government uncertainty and strategic restraint in implementing commitment outcomes, by reducing the

government's value of delaying the bailout and learning. This suggests a new rationale for regulating financial innovation.

We also explore the role of industry concentration, measured by the number of banks. A larger number of banks strengthens our two forces that induce governments to delay intervention. First, governments become more optimistic that there exists a healthy bank able to take over a bank in distress. Second, there is a larger option value of learning the true state, by allowing the first bank in distress to fail, and making better decisions later for a larger number of banks. The overall result is that decreasing industry concentration makes commitment outcomes more easily sustainable.

Finally, we study the effect of bank size heterogeneity. When a single bank is asymmetrically large in the industry, it is less likely that it can be acquired by its peers and hence more likely that the government must bail it out in case of distress, even if it is the first bank experiencing problems. Hence, with sufficient asymmetry in bank size, large banks do not have any concern for their relative performance and choose excessive leverage. Given this behavior, smaller banks have a strategic incentive to increase their leverage relative their leverage in the symmetric case: they choose to expose themselves only slightly less than the large bank. In our setting, the 'too big to fail' problem shows up very differently than in the rest of the literature because large banks become shields for smaller banks to take excessive risk, exerting a negative externality on the economy by inducing endogenous systemic crises of larger magnitude.

Related Literature There is a large literature on the time-consistency of the no bailout policies and moral hazard behavior of banks, to which this paper contributes. A number of papers, extensively reviewed in Stern and Feldman (2004), argue that the existence of 'too big to fail' banks is the source of the time inconsistency of policies, and at the root of crises. Another strand of the literature, most recently represented by Acharya and Yorulmazer (2007), Pasten (2011) and especially Farhi and Tirole (2012) argue that 'too big to fail' banks are not necessary and coordinated actions by smaller banks give rise to endogenous crises too. The results in this paper apply in both environments, always working towards putting more discipline on bank actions and achieving welfare-superior allocations.

Our setting builds on Holmström and Tirole (1998) and Farhi and Tirole (2012). Relative to their work, we introduce idiosyncratic shocks and the possibility of government uncertainty about the nature of the shock. We additionally

allow for efficient takeovers of distressed banks by healthy banks, making the true nature of the shock crucial for the government. In contrast to their work, the timing of bank distress at the onset of a crisis is critical for us.

Our work also relates to Acharya and Yorulmazer (2007), who develop a model of ‘too-many to fail’ in an environment where bank takeovers are also possible and technologically superior to bailouts, as in our paper. In our model, the ‘wait and see’ strategy of the government has the additional gain of providing information to the government about the nature of the shocks, which creates *strategic restraint* and hinders the possibility of herding that they highlight.

Recently, Green (2010) and Keister (2011) argue that bailouts may be optimal to avoid excessive hoarding of liquidity. In a similar vein, Cheng and Milbradt (2010) suggest bailouts can instill confidence on credit markets. In our setup, whatever the optimal level of liquidity is, it can be attained as long as government uncertainty and strategic restraint forces are at work, even in the absence of commitment.

Bianchi (2012) concludes that moral hazard effects of bailouts are significantly mitigated by making bailouts contingent on the occurrence of a systemic financial crisis. In contrast, in our framework shocks are unobservable and hence the government cannot make bailouts contingent upon them. This gives rise to a positive option of delay and learning, which is exactly what mitigates the moral hazard problem.

Davila (2012) also argues, as in our extension to asymmetric bank sizes, that large banks allow small banks to take more risks, making the economy-wide leverage and probability of bailouts larger when large banks are present. While his results are based on banks’ uncertainty about bailout policies, ours depend on governments’ uncertainty about the nature of the shocks.

A recent strand of the literature highlights the effects of policy uncertainty in inducing crises and delaying recoveries. Cukierman and Izhakian (2011), for example, show that uncertainty about policymakers’s actions can induce sudden financial collapses when investors follow a max-min behavior. Baker, Bloom, and Davis (2012) argue that uncertainty about future policies delays recoveries since individuals prefer to ‘wait and see.’ In our case, it is the government who is uncertain about the nature of refinancing shocks and may like to ‘wait and see’ before intervening, reducing the likelihood of endogenous crises.

Finally, there is a previous literature that explores the ability of imperfect information to improve equilibrium outcomes under time inconsistency. Cremer

(1995) shows in a static game that the inability of a principal to observe workers' types can serve as a commitment device to punishing low output realizations, while Carrillo and Mariotti (2000) show that an agent with time-inconsistent preferences might optimally choose not to learn in order to restrict future selves. The effect of imperfect information in our model has a similar flavor, but in our banking setting, the competition that arises from the banks' concerns for their relative position is critical for our results, and is absent in their settings. Furthermore, our setting is dynamic, and the government not only delays because of imperfect information but also because there are gains from learning and resolving the imperfect information.

In what follows, we first set up a simple analytical example in order to illustrate the main forces behind our arguments. Then, we provide a micro-founded model of these forces, first analyzing the model under full information as a benchmark and then introducing imperfect information to derive our main results. Next, we discuss policy implications. Finally, we conclude and discuss avenues for future research.

2 Simple Example

Consider an economy with two banking entrepreneurs (banks or firms more generally) that live for three periods. At $t = 0$, each bank is endowed with a project and a unit of numeraire, that we will call *cash*, which can be used for consumption or savings. At $t = 1$, each project can either be successful, paying $Y > 1$, or in distress, needing additional funds to continue and to pay Y at $t = 2$. We assume both projects are in distress (*aggregate shock*) with probability P_2 , only one project is in distress (*idiosyncratic shock*) with probability $2P_1$ and no project is in distress with probability P_0 . Refinancing the whole distressed project requires a unit of cash and it is not possible to increase the size of the project.

At $t = 0$ the bank can save a fraction c of the endowed cash to refinance up to a fraction c of a distressed project. We assume cash can only be consumed at $t = 0$ or used for refinancing the project at $t = 1$. Hence, conditional on saving c , the expected payoff for the bank is $Y - c$ if the project is successful and $cY - c$ if the project is in distress.

We assume that, in case of an idiosyncratic shock, the *healthy bank* (the one with a successful project) can make a take-it-or-leave-it offer for the distressed project, using the proceedings from its own project. Since the healthy bank receives $Y > 1$ of cash at $t = 1$, it has enough resources to refinance the distressed project fully. In this situation, the healthy bank pays $1 - c$ to reap the full benefit equal to $(1 - c)Y$ from the part of the project the distressed bank cannot refinance with own savings.

There is also a government in the economy that can contribute to refinance an additional fraction c^G of the project, such that $c + c^G \leq 1$. We call this contribution a *bailout*. A bailout is implemented by transferring funds with social costs that are proportional to the size of the transfer, $c^G T$. We also assume bailouts are *undirected* – once offered, healthy banks also have access to the bailout and can consume c^G from a transfer that socially costs $c^G T$.

We will denote by $y = Y - 1$ the social gain from refinancing with private funds, by $x = Y - T$ the social gain from refinancing with public funds (*bailouts*), and by $\hat{x} = 1 - T$ the social gain from a transfer to a healthy bank. We assume that bailouts are socially costly (raising public funds introduces distortions) but beneficial (bailouts save socially useful projects). This is summarized in Assumption A, which implies that $y > x > 0 > \hat{x}$.

Assumption A *Bailouts are socially costly, but beneficial: $1 < T < Y$.*

Below, we first study the case in which the government has *full information* about the nature of the shocks (whether both, one or no projects are in distress). This benchmark highlights how the government's inability to commit reduces welfare. Then, we assume *imperfect information* about the nature of shocks and show that the economy can achieve commitment outcomes, even when the government is still unable to commit.

2.1 Full Information

2.1.1 No Commitment

In this section we assume the government is unable to commit to any policy announced at $t = 0$ and that it observes the status of the two projects at $t = 1$.

The government never intervenes in the case of an idiosyncratic shock. Since $y > x$, the distressed project can be taken over and refinanced by the healthy bank without distortions. In contrast, the government always intervenes in the case of an aggregate shock. Since $x > 0$ it is preferred to refinance the project at a social cost than let it disappear.

How much do banks save at $t = 0$ knowing that this is how the government behaves at $t = 1$? Conditional on the other bank saving c' , a bank's ex-ante payoff from saving c is

$$V(c) = (P_0 + P_1 + P_2)Y + P_1(1 - c')(Y - 1) + c(P_1Y - 1). \quad (1)$$

The first term captures the benefits that the bank obtains from its own project independently of c , except when it is the only bank in distress. The second term shows the gains from taking over the distressed project of the other bank. The last term shows the net gains from saving c . The benefit of saving c is refinancing the own distressed project if the shock is idiosyncratic, avoiding being taken over by the other, healthy, bank. The cost of saving c is giving up its consumption at $t = 0$. Assumption B below guarantees that without commitment, ex-ante payoffs decline with c and then the unique equilibrium has no savings (i.e. $c = 0$), which implies that bailouts occur on the equilibrium path when the shock is aggregate.

Assumption B *Without commitment, banks prefer not to save: $P_1Y < 1$.*

The solid line in Figure 1 shows the expected payoffs for a bank under this assumption and shows why no savings (i.e., $c = 0$) is the unique non-commitment equilibrium.

2.1.2 Commitment

Assume now the government has a commitment device that makes any policy announcement at $t = 0$ binding and credible. As argued above, the government never wants to announce a bailout for the case of the idiosyncratic shock. Hence,

we consider two alternative policies: (i) bailout if the shock is aggregate, and (ii) never bailout.

When the government commits to bailout if the shock is aggregate we are basically in the situation we studied in the previous section, without commitment. As argued, under Assumption B banks choose $c = 0$. In contrast, when the government commits to never bailout, conditional on the other bank saving c' , a bank's ex-ante payoff from saving c is

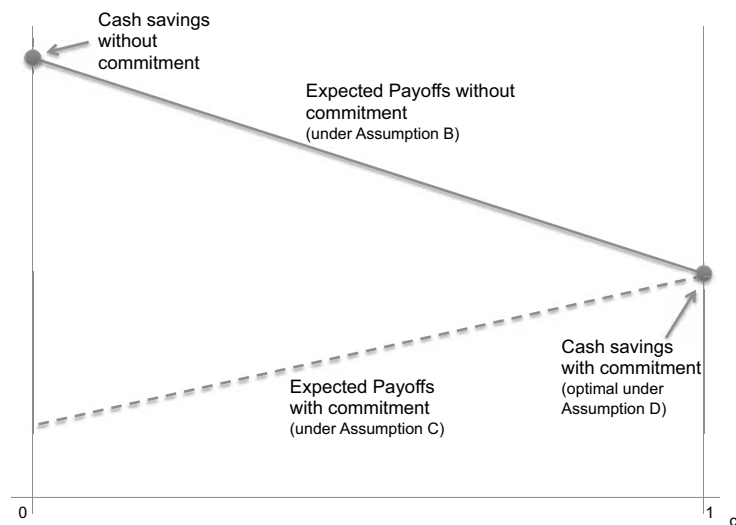
$$\widehat{V}(c) = (P_0 + P_1)Y + P_1(1 - c')(Y - 1) + c((P_1 + P_2)Y - 1). \quad (2)$$

This expression is the same as equation (1), except that now the bank obtains the benefits from its own project, independently of c , only if its own project is successful. The net benefits of holding c increase, since now savings are useful to refinance the project both when the shock is idiosyncratic and when it is aggregate. Assumption C guarantees that with commitment, ex-ante payoffs increase with c and then the unique equilibrium features enough savings to fully refinance (this is, $c = 1$), which implies that bailouts never occur on the equilibrium path.

Assumption C *With commitment to no bailouts, banks prefer to save: $(P_1 + P_2)Y > 1$.*

The dashed line in Figure 1 shows the expected payoffs for a bank under this assumption and shows why full savings (i.e., $c = 1$) is the unique non-commitment equilibrium.

Figure 1: Expected Payoffs in the Simple Model



When the government is able to commit, it has the ability to select one of these two equilibria using a welfare maximization criterion. If the government commits to bailout if the shock is aggregate, banks do not save ($c = 0$) and ex-ante welfare is

$$W^{ea} = 2[P_0Y + P_1(2Y - 1) + P_2(Y - T)].$$

If the government commits to never bailout, banks save to full refinancing ($c = 1$) and ex-ante welfare is

$$\widehat{W}^{ea} = 2(Y - 1).$$

Hence, the government would like to commit to never bailout when $\widehat{W}^{ea} > W^{ea}$, which is the case under the following assumption:

Assumption D *It is ex-ante optimal to commit to no bailouts: $P_2(y - x) > P_0 + P_1$.*

In words, the government wants to commit to never bailout if the gains from commitment (private refinancing by savings rather than public refinancing by distortionary transfers when the shock is aggregate) are larger than the losses from commitment (each bank gives up consumption, which is a waste when its own project is successful). If this assumption holds, there is a welfare loss from lack of commitment.

The following lemma summarizes the discussion for the full information environment.

Lemma 1 *With full information and under Assumptions A - D, the equilibrium is unique and characterized as follows: (i) when governments can commit, banks save to full refinancing ($c = 1$) and bailouts never happen on the equilibrium path, (ii) when governments cannot commit, banks do not save ($c = 0$) and bailouts happen in case of aggregate shocks. Moreover, (i) dominates (ii) in terms of welfare.*

2.2 Imperfect Information

In this section, we maintain the assumption of no commitment, relax the assumption of government's full information about the nature of shocks at $t = 1$, and derive our main result. First, we assume banks show distress *in sequence*, with the bank with the lowest cash savings showing distress first if suffering a need of

refinancing.⁴ Second, the government only observes whether a given project is in distress, but not the nature of the shock (aggregate versus idiosyncratic).

When the government observes a bank's project in distress, it does not know whether the project of the other bank is also in distress (such that both projects will eventually need refinancing), or not (such that the healthy bank has enough resources to take over the distressed project and efficiently refinance it with private funds). However, observing a bank's project in distress is a valuable signal, and it leads to the government updating the probability of an aggregate shock, using Bayes' rule, to

$$P'_2 = \frac{P_2}{P_1 + P_2} > P_2. \quad (3)$$

If the government bails out the first bank in distress, the expected welfare is

$$P'_2 2x + (1 - P'_2)(x + \hat{x}).$$

In words, if the shock is aggregate the government saves both projects but if the shock is idiosyncratic, the government saves the distressed project with public funds (which is inefficient, since it would have been taken over in the absence of a bailout), and inefficiently provides public funds to the healthy bank (since bailouts are assumed to be undirected).

If the government does not bail out the first bank in distress, the expected welfare is

$$P'_2 x + (1 - P'_2)y.$$

In words, if the shock is aggregate the government loses the first project (the government can always bailout the second project if also showing distress) but if the shock is idiosyncratic the government induces a takeover (which is the most efficient refinancing alternative) and avoids inefficient public transfers to the healthy bank. This expression then captures the static and dynamic gains from delaying governments' intervention. The static part is avoiding a mistake with the first distressed bank. The dynamic part is the value of learning the nature of the shock and not providing inefficient transfers to healthy banks.

If the government is certain that the shock is aggregate (i.e. $P'_2 = 1$), then it always prefers to bail out the first bank in distress (since $2x > x$). On the other

⁴When banks initially save the same c , we assume as a tie breaking rule that each bank has a probability $1/2$ of showing distress first. The relationship between cash c and the order of showing distress is assumed in the simple economy, and later derived in the full model.

hand, if the government is certain that the shock is idiosyncratic (i.e. $P'_2 = 0$), then it never prefers to bail out the first bank in distress (since $y > x + \hat{x}$).

In general, there is a cutoff \bar{P} , such that for all $P'_2 < \bar{P}$, the government assigns a relatively low posterior probability to aggregate shocks such that it always prefers to delay intervention. The *delayed bailout condition* for the government is therefore

$$P'_2 < \bar{P} \equiv 1 - \frac{x}{y - \hat{x}}. \quad (4)$$

Factors that increase the cutoff, making it more likely that governments delay intervention are (i) low gains from public refinancing (low x), (ii) high gains from private refinancing (high y), or (iii) high costs of unnecessary transfers to healthy banks (more negative values of \hat{x}).

An important assumption here is that governments cannot help banks little by little, maintaining them alive until learning the nature of the shock. Hence, if the shock is aggregate and governments do not help banks immediately to refinance fully, the project is lost.⁵

How do the banks react to the fact that the government is uncertain about the nature of shocks? If the delayed bailout condition (4) is *not* satisfied, then banks know they would be bailed out whenever their project is in distress, always obtaining Y regardless of c . Furthermore, in case of being healthy under an idiosyncratic shock they receive a windfall of public transfers that depends on when the other bank shows distress. This implies that, conditional on the other bank saving c' , a bank's ex-ante payoff from saving c is,

$$Y + P_1(1 - c')(Y - 1) - c,$$

clearly decreasing in c . Hence, with imperfect information and no intervention delay, banks never save anything for refinancing ($c = 0$), which replicates the result we obtained with full information and no commitment.⁶

If the delayed bailout condition (4) holds, when the shock is aggregate, the first bank showing distress fails while the second bank showing distress is bailed

⁵This assumption is certainly realistic for financial intermediaries with very high leverage on overnight debt. In thinking about non-financial corporations, however, this assumption seems less plausible, which is consistent with more targeted and gradual bailouts in real activity, such as the reorganization of car companies or airlines, for example.

⁶Note, however, that this result is independent of Assumption B when information is imperfect. If Assumption B does not hold banks would save when there is full information, while they would still not save when there is imperfect information and no intervention delay. This highlights the importance of the delayed bailout condition in the absence of full information.

out. Intervention delays then introduce banks' concerns about their performance relative to their peer. In particular, there is a discontinuity in the the expected payoffs from being first versus second in distress. If the sequence of showing distress depends on the cash holdings of the bank (which is true by assumption in this simple case), the value for a bank of saving the same amount as the other bank, (i.e., $c = c'$) is

$$V(c = c'|c') = (P_0 + P_1 + \frac{P_2}{2})Y + P_1(1 - c')(Y - 1) + c((P_1 + \frac{P_2}{2})Y - 1)$$

and the value of deviating and choosing slightly more savings $\hat{c} = c' + \varepsilon$ is

$$V(\hat{c} = c' + \varepsilon|c') = (P_0 + P_1 + P_2)Y + P_1(1 - c')(Y - 1) + \hat{c}(P_1Y - 1).$$

This deviation is preferred as long as

$$\Delta V \equiv V(\hat{c}) - V(c') = P_2Y \frac{(1 - c')}{2} - \varepsilon > 0, \quad (5)$$

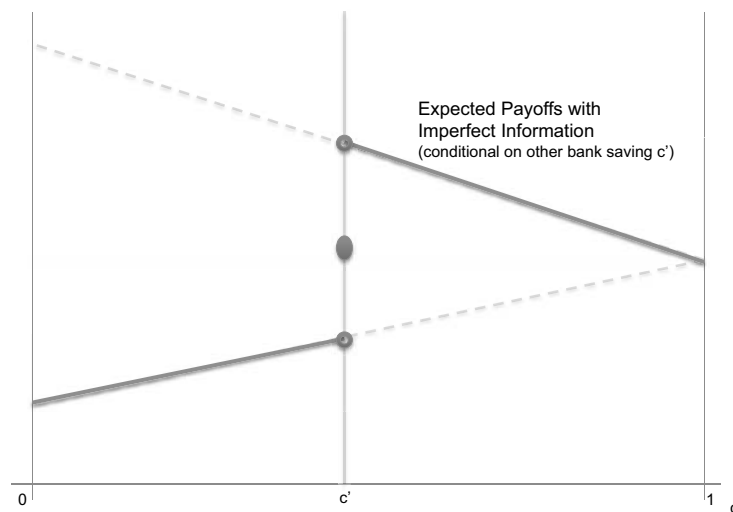
which is strictly satisfied for any small enough deviation ($\varepsilon \rightarrow 0$) for any $c' < 1$ and equal to zero at $c' = 1$. We call this condition *strategic restraint*.

Figure 2 shows in the solid discontinuous line the expected payoff of a bank with cash c , when the other bank's cash is c' . For any value of $c \leq c' < 1$, the bank would like to deviate upwards, and in particular, this is true for any tie-break at which $c = c'$, where the referred discontinuity in expected payoffs is located. Competition for the relative position moves the reference c' upwards, until $c = c' = 1$. The only point at which the incentives for deviation cease to exist is at the corner, $c = 1$. Hence, strategic restrains, reminiscent of Bertrand competition, induce a unique equilibrium in which both banks save fully to refinance projects in distress ($c = 1$)

In essence, even if the government is unable to commit, it can achieve commitment outcomes if it faces uncertainty about the nature of shocks. The main condition for this to happen is that the government remains relatively confident that the shock is idiosyncratic, even after observing a project in distress, and hence delays intervention. This result is summarized in Lemma 2 below.

Lemma 2 *With imperfect information, no commitment and under Assumptions A - D, the equilibrium is unique. If $P'_2 \leq \bar{P}$, banks save to full refinancing ($c = 1$) and bailouts never happen on the equilibrium path, replicating the outcome of full information with commitment. If $P'_2 > \bar{P}$, banks do not save ($c = 0$) and*

Figure 2: Expected Payoffs with Imperfect Information



bailouts happen when shocks are aggregate, which is welfare inferior, replicating the outcome of full information without commitment.

The simple example of this section illustrates the main mechanism behind the effectiveness of uncertainty in implementing commitment outcomes. However, all functional forms and assumptions were taken for clarity and tractability. In what follows, we provide a micro-founded model of these forces. In particular, we model the benefits and costs of private and public liquidity provision and we endogenize both the size of projects and the order in which banks show distress.

3 Full Model

The model environment builds on Holmström and Tirole (1998) and Farhi and Tirole (2012), with several important modifications. First, we introduce two types of shocks, aggregate and idiosyncratic, and allow for imperfect information about the nature of the shock. Second, we allow for a non-degenerate timing of events, in which banks with higher leverage ratios endogenously show distress earlier. Third, we admit the possibility of healthy banks takeover distressed banks.

3.1 Environment

Time is continuous and finite, $t \in [0, 2]$, and there is no discounting. There are three types of agents in the economy: two banking entrepreneurs (banks hereafter), a continuum of households and a government. Banks borrow short-term to finance illiquid projects which either pay off at $t = 1$ or need refinancing and pay off at $t = 2$. A bank's project needs refinancing because of an aggregate shock (both banks need refinancing) or an idiosyncratic shock (only one of the two banks needs refinancing). These shocks hit only at date $t = 1$, and for the rest of time the economy is deterministic. Households are risk neutral providers of loans to banks. The government maximizes total welfare, using interest rates and taxes on households as its only policy instruments.

3.1.1 Banks

The two banks in the economy have the objective of maximizing their individual net worth, V . At $t = 0$, they choose the size i of an investment project, which is financed using own initial assets A and funds borrowed from the households. The size i also determines the speed of expense outflows (to pay suppliers, workers, etc), which happens at a rate idt during the period, such that all projects run out of funds at $t = 1$ and larger projects have a larger outflow rate than smaller projects.

The payoff from each project consists of two parts. The first part is deterministic, πi at time $t = 1$. The second part is random. If the project does not suffer any shock, it returns $(\rho_0 + \rho_1)i$ at time $t = 1$, and the bank has the choice to extend the project to size $j \leq i$ which returns $\hat{\rho}j$ at time $t = 2$. If the project suffers a shock, it only pays πi at time $t = 1$, and the bank has the choice to refinance the project to size $j \leq i$ which returns $(\rho_0 + \rho_1)j$ at time $t = 2$.

Refinancing a project, however, does not change its intrinsic rate of expenses outflow – if only half of a large project is refinanced ($j = i/2$), for example, the bank would run out of cash to pay expenses at $t = 1.5$. We introduce financial frictions by assuming that from the total output of the project, ρ_1 is a benefit that can only be captured by bankers, and hence it is not pledgeable.⁷ This and other parametric assumptions that make the model economically interesting are summarized below.

Assumption 1 *Assumptions about projects's payoffs*

1. *Binding pledgeability:* $\pi < 1$ and $\rho_0 < 1$.
2. *Efficient projects:* $\pi + \rho_0 + \rho_1 > 1 + P_1 + P_2$.
3. *Efficient refinancing:* $\rho_0 + \rho_1 > 1$.
4. *Inefficient expansion:* $\rho_0 < \hat{\rho} < 1$.

The first part of Assumption 1 guarantees that investment in period 0 is finite, and that refinancing depends on retained earnings. The second part guarantees that running the project is ex-ante socially efficient. The third part guarantees that refinancing the project is also ex-post socially efficient. The last part assumes that expansions are inefficient, but privately profitable if the cost of expansion is ρ_0 .

3.1.2 Households and Government

A continuum of households born at $t = 0$ or $t = 1$, consume at $t + 1$ and are risk neutral, with utility given by $U_t = x_{t+1}$. They are endowed with assets S_t when born, which they allocate between holding cash (or storing at a return 1) and lending to banks. The return on their savings is consumed in period $t + 1$. We assume perfect competition of households as lenders, which imply their return is always 1, and denoting government taxes as T (which may serve to finance potential bailouts), utility for each generation is given by,

$$U_t = S_t - T_{t+1}$$

⁷This can be derived from first principles as in Holmström and Tirole (1998), for example by moral hazard within the bank, in which the banker exerts hidden efforts that affect the outcome of the project.

The government is benevolent and maximizes welfare W , which is equal to the weighted sum of the banks' surplus, V , and households' surplus, U .

$$W = \beta V + U_0 + U_1$$

To maximize W the government may need to transfer resources from households to banks to refinance projects in distress. This is what we call a *bailout*. The weight $\beta < 1$ introduces the idea that transfers between households and banks is costly from a welfare perspective.⁸

We assume the only policy instrument governments can use to bailout banks, transferring funds from households to bankers is an interest rate policy that determines borrowing costs of bankers (i.e. $R(t)$, $t \in [0, 2]$) and taxes on households to implement such interest rates. We assume this policy is *undirected*, which means that the government cannot reduce interest rate only for certain banks and not others. Once a bailout is implemented, all banks can borrow at a lower rate, regardless of whether they had a refinancing shock or not.

3.1.3 Timing

At $t = 0$, the government announces a bailout policy as a function of the time the first and second banks show distress, by which we mean the time $t \geq 1$ at which they eventually run out of refinancing opportunities in the market. In the commitment case, it then just executes the announcement, whereas in the non-commitment case it has a chance to deviate from the announced policy ex-post.

At $t = 1$, either both banks suffer a refinancing shock (*aggregate shock*) with probability P_2 , only one bank suffers a refinancing shock (*idiosyncratic shock*) with probability $2P_1$, or no bank suffers a refinancing shock with probability P_0 .

- If no bank suffers the shock, banks choose whether to expand or not, payoffs are realized at $t = 2$ and the game ends.
- If only one bank suffers a shock and it shows distress (it may never show distress if it retained enough cash to fully refinance the project), the government decides whether to bail out or not.
 - If the government does not provide a bailout, the healthy bank decides whether to take over the distressed project or not.

⁸We could additionally include distortionary effects of transfers, as in Farhi and Tirole (2012). This is a straightforward extension that does not bring anything new to our analysis, so we omit it this from our analysis for expositional purposes.

- If the government provides a bailout, the healthy bank decides whether to use the cheaper funds to expand its project.⁹
- If both banks suffer a shock, at the time that the first bank shows distress the government decides whether to bail out or not. If the government does not provide a bailout and the second bank shows distress, the government decides again whether to bailout or not, payoffs are realized at $t = 2$ and the game ends.

The above timing of events applies to both the full information and the imperfect information cases. With full information, the government knows how many projects need refinancing when deciding whether to provide a bailout or not. With imperfect information the government does not know how many projects need refinancing when observing one project in distress and is deciding whether to provide a bailout or not.

3.2 Preliminaries

Here we derive notation and basic results that are used in the rest of the paper. We first describe the banks' borrowing decisions at $t = 0$ and $t = 1$. Then we describe the government's bailout decision when observing a bank in distress at time t . Finally we develop notation similar to that of the simple model to derive the main forces behind our results.

Bank borrowing We assume bank borrowing is non-contingent.¹⁰ At $t = 0$ banks promise to repay b per unit of investment independently of the realized state at $t = 1$, which implies $b \leq \pi$. Limited liability, together with risk-neutrality and competitiveness of the lenders (households) imply that the amount owed, $R(i - A)$, should be equal to the repayment that lenders require, bi . Since the alternative use of cash for households is storage, with return 1, the market interest rate is just $R = 1$. This implies

$$i - A = bi, \quad \text{and hence} \quad i = A/(1 - b). \quad (6)$$

⁹Here the government is the leader in reacting when a bank is in distress. We also discuss later the results when the timing allows for healthy banks to takeover before governments decide bailouts, and still have the possibility of taking over or expanding after governments decide bailouts. This last possibility is particularly relevant when information is imperfect.

¹⁰This assumption is not critical, just convenient. If repayment is conditional on success, the optimal level of investment will increase, but the liquidity choice considerations will remain.

Since, in case of a shock, the cash available at $t = 1$ for reinvestment purposes is equal to $c = (\pi - b)$ per unit of investment, banks face a tradeoff between increasing the initial size of the project and holding some cash to face refinancing if needed.

As in our simple example, the reinvestment scale j depends on the cash carried at $t = 1$ that can be levered, equal to $ci = (\pi - b)i$, with the restriction that the reinvestment cannot increase the size of the project, i.e. $j \leq i$. The second period payoff in this case is $(\rho_0 + \rho_1)j$, of which, crucially, only $\rho_0 j$ is pledgeable by the lenders. If the required market rate of return on bank lending is R , then the maximum the bank can raise at $t = 1$ is

$$R(j - ci) = \rho_0 j, \quad \text{which implies} \quad j = \min \left\{ \frac{c}{1 - \rho_0/R}, 1 \right\} i. \quad (7)$$

This clearly implies that at the market rate, $R = 1$, banks need to save cash $c = 1 - \rho_0$ per unit of investment if they want to self-refinance the whole project in case of a shock. If banks hold $c = 0$, then they can invest in the largest project size feasible, $i = A/1 - \pi$, but will not be able to refinance anything of it at the market rate in case of a shock.

Denote by $\bar{t}(c)$ the calendar time of distress after refinancing the maximum possible at market rate. Then, we have

$$\bar{t}(c) = \min \left\{ 1 + \frac{c}{1 - \rho_0}, 2 \right\}. \quad (8)$$

Government Policy The government can modify the interest rate that banks face to refinance, and hence the level of refinancing. For example, if the government sets $R = \rho_0$, the interest rate is exactly equal to the pledgeable amount ρ_0 and banks are able to refinance any amount of reinvestment, up to the feasible level i , with zero cash holdings. Without loss of generality, we assume that the policy interest rate takes only two values: (i) a no intervention *market rate* of $R = 1$ and (ii) a *bailout rate* of $R = \rho_0$.¹¹

¹¹This restriction gives us a natural way of modeling banks' distress - it is when they run out of money (i.e. cash holdings go to zero), and cannot continue the project unless intervention or takeover take place. For a more general set of policies (i.e. $R > \rho_0$), intervention would have to take place earlier, and for some strictly positive level of cash holdings of a bank. In a more general environment, a number of forces can break the government's indifference between different ways of generating the same average interest rate towards backloading intervention. For example, if there is a chance of a stochastic shock that nullifies the bank's distress, it would generate a strictly positive option value of waiting until the last possible moment before intervention. We do not explicitly incorporate these forces here, but we view our restriction

The government can implement a rate $R = \rho_0$ by taxing a fraction of $1 - \rho_0$ of households' storage and reducing its return to ρ_0 . This implies households are indifferent between lending banks at a rate $R = \rho_0$ or storing. An alternative is that governments issue bonds to provide money directly to banks against the pledgeable amount and later cover those bonds by taxing households. In both cases taxes have to be equal to $T = (1 - \rho_0)(i - j)$, which is the difference between what the bank cannot refinance itself ($i - j$) and the maximal return the government can recover from the pledgeable part of the project, $\rho_0(i - j)$.

As in the motivating simple model of Section 2, under full information the government does not want to introduce a bailout rate if the shock is idiosyncratic because the healthy bank can take over the distressed project without the need of taxes. Denote the government's belief that both banks need refinancing by p . Under full information p is either 0 (idiosyncratic shock) or 1 (aggregate shock). Under imperfect information $p = P'_2$ after the first bank shows distress.

The decision of the government is going to be a binary one: whether or not to introduce the bailout rate, given its belief that both banks are in need of refinancing p and after observing at least one bank running out of cash at time $t > 1$. We summarize it by a function $\Pi(t, p), t \in [1, 2]$ which takes values in $\{0, 1\}$.

For the purposes of banks' optimization, it is going to be crucial what is the *earliest* time that the government is willing to introduce a bailout when observing a bank in distress.

Definition 1 *The **earliest bailout time** t_p^* is the minimum time of government bailout when the probability that both banks need refinancing is p and the government observes a bank with zero funds:*

$$t_p^* = \min\{t | \Pi(t, p) = 1\}. \quad (9)$$

We are restricting the set of government policies Π to ones that guarantee that t_p^* is well defined. When it does not generate confusion, we just call t^* the policy when an aggregate shock is certain, t_1^* .

Takeovers and Expansions In case of the idiosyncratic shock, if a government does not bail out the first bank in distress, the healthy bank gains $\rho_0 + \rho_1 - 1 > 0$ per unit of investment from taking over the distressed project, on policies as motivated by such considerations.

which is feasible using the proceeds from its successful project. If a government bails out the first bank in distress, since bailouts are undirected, healthy banks borrow at the bailout interest rate $R = \rho_0$, expanding their successful projects and gaining $\hat{\rho} - \rho_0 > 0$ per unit of expansion. However, by Assumption 1, this expansion is socially inefficient because it generates $\hat{\rho} < 1$ per unit of investment. Hence, the cost of bailing out a bank when the shock is idiosyncratic is not only that the bailout prevents socially efficient takeovers, but also that it induces healthy banks to invest in socially inefficient expansions.

If the timing is expanded to allow for healthy banks to take over distressed projects *before* governments decision about bailouts, then they would do so provided that $\rho_0 + \rho_1 - 1 \geq \hat{\rho} - \rho_0$, i.e. when the gains from taking over privately are larger than the gains from expanding at a subsidized rate. In such case, government knows that a project showing distress is a sure sign of an aggregate shock (since otherwise, the healthy bank would have taken over), and then the full information analysis that we provide next applies. In contrast, if $\rho_0 + \rho_1 - 1 < \hat{\rho} - \rho_0$, healthy banks would prefer to avoid taking over in order to push governments to believe they are facing an aggregate shock and then exploit the benefits from expanding at a subsidized bailout rate. In this situation, governments would not be able to infer the health of the other bank when a projects shows distress, and then the imperfect information analysis that we provide next applies.

This implies that our results are robust to such changes in the timing, provided that bailouts are undirected, there are private gains for healthy banks from using subsidized rates and those gains are large enough so that healthy banks do not want to reveal any information.¹²

Returns To use the intuition we developed in the simple model, it is convenient to define the social gain from refinancing with private funds as $y = \beta\rho_1 - \beta(1 - \rho_0)$ (both the gains ρ_1 and the costs $1 - \rho_0$ per unit of refinancing is weighted by the bankers' β), the social gain from refinancing with public funds as $x = \beta\rho_1 - (1 - \rho_0)$ (the gains of the *bailouts* are weighted by the banker's β but the costs are weighted by the households' 1) and the social gains from providing public funds to healthy banks as $\hat{x} = \beta(\hat{\rho} - \rho_0) - (1 - \rho_0)$ (the gains for bankers $\hat{\rho} - \rho_0$ are lower than the costs for households).

¹²In short, our results remain as long as governments' uncertainty about the nature of shocks persists. Thinking about other channels of communication between governments and banks is interesting but outside the scope of this paper. Still, given our result that governments' uncertainty leads to commitment outcomes in the absence of commitment, governments want to avoid communication channels that provide too much information.

As with Assumption A, we impose that bailouts are socially costly (raising public funds introduce distortions) but beneficial (bailouts save socially useful projects), which together with Assumption 1, implies $y > x > 0 > \hat{x}$.

Assumption 2 *Bailouts are socially costly, but beneficial: $\beta\rho_1 > 1 - \rho_0$.*

3.3 Full Information

Here, we assume, as a full information benchmark, that the government observes at $t = 1$ how many projects are in distress.

3.3.1 Commitment

Assume the government is able to commit to a policy announced at $t = 0$. We first solve the optimal reaction of banks given a policy announcement and then we compute the optimal policy announcement.

At $t = 0$, the bank chooses the project size, and then how much cash c to retain at $t = 1$, conditional on the government's policy and the refinancing problem described in Section 3.2. The value function of the bank as a function of the cash choice c depends on whether $\bar{t}(c)$ is larger or smaller than the government's policy t^* . In particular,

$$V(c) = \begin{cases} V_s(c) + P_2[c + (\rho_0 + \rho_1 - 1)(\bar{t} - 1)]i & \text{if } \bar{t}(c) < t^* \\ V_s(c) + P_2[c + \rho_0 + \rho_1 - (t^* - 1) - \rho_0(2 - t^*)]i & \text{if } \bar{t}(c) \geq t^* \end{cases} \quad (10)$$

where

$$V_s(c) = (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1\rho_1j + P_1V_{TO}$$

is the expected value when there are no shocks or when the shock is idiosyncratic and hence are independent of the government's policy, t^* . V_{TO} is the value of taking over a distressed project, $V_{TO} = (\rho_0 + \rho_1 - 1)(i' - j')$, which only depends on the other bank's choice of i' and c' and then is a constant from the perspective of choosing c . Note that a bank can takeover only if it does not suffer any shock itself, and it has enough proceedings from a healthy project to take over a distressed project to full scale. Hence, when deciding c it does not consider how this can help in taking over a larger fraction of a distressed project.

The jump in the value function for banks is generated by the government's policy. Since there are no bailouts before t^* , if the bank does not hold cash to refinance until t^* (this is $\bar{t}(c) < t^*$), then it will need to scale down the project

to $j = (\bar{t}(c) - 1)i < i$. In this case, the payoff for the bank is ci plus the returns $(\rho_0 + \rho_1)j$ minus the cost j of refinancing at a cost 1 per unit of reinvestment. Since $j - ci = \rho_0 j$, the value function in this case can be rewritten simply as $V_s(c) + P_2 \rho_1 j$.

In contrast, when $\bar{t}(c) \geq t^*$, there is bailout, which implies banks can borrow at a rate ρ_0 at time t^* . This implies banks refinance as little as possible at interest rate $R = 1$, and then refinance up to full scale at rate $R = \rho_0$: a fraction $(t^* - 1)i$ is refinanced at cost of 1 per unit of refinancing and the rest (a fraction $(2 - t^*)i$ at cost ρ_0 per unit of refinancing. Naturally in this case, the gains are given by $(\rho_0 + \rho_1)i$, for the full project, and are independent of c as long as $\bar{t}(c) > t^*$.

We now impose two natural assumptions that give our problem economic bite in terms of the government effect on leverage choices. First, Assumption 3 below guarantees that banks care about refinancing scale j – when faced with a tradeoff between increasing investment i and sacrificing reinvestment j , they choose not to sacrifice reinvestment. It guarantees that banks will always choose c such that $\bar{t} \geq t^*$, (as defined by equation 8).

Assumption 3 *Banks care about reinvestment scale ($\bar{t} \geq t^*$):*

$$(P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1) - (P_1 + P_2) \frac{\rho_1}{1 - \rho_0} (1 - \pi) < 0.$$

This condition is given by the derivative of the value function when $\bar{t}(c) < t^*$ with respect to i . The first term on the left is the benefit of less c and a larger project. This benefit accrues when there is no shock (P_0) or when only the other bank needs refinancing (P_1). The second term is the payoff lost due to downscaling j , which happens when the bank needs refinancing (P_2 and P_1). When Assumption 3 is violated, banks sole objective is to maximize i , independent of the government policy.

Next, Assumption 4 ensures that if the government provides a bailout, i.e. $t^* < 2$, then it is not optimal for banks to carry a cash level such that the implied \bar{t} is greater than t^* .

Assumption 4 *The promise of a bailout increases leverage ($\bar{t} \leq t^*$):*

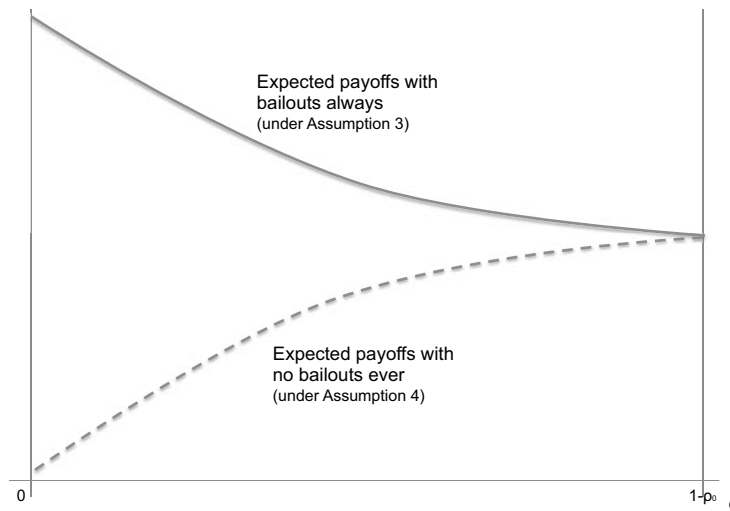
$$(P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) - P_1 \frac{\rho_1}{1 - \rho_0} (1 - \pi) - P_2 > 0$$

This condition is given by the derivative of the value function when $\bar{t}(c) \geq t^*$ with respect to i , and evaluating it at the most stringent condition, $t^* = 2$. The

first term on the left is again the benefit of less c and higher i . Compared with Assumption 3, this benefit also accrues in case both banks fail (there is no change in reinvestment scale j if $\bar{t} > t^*$). The second term is the cost of downsizing the project, which only happens when an idiosyncratic shock pushes the bank to fail. The third term captures the cost of foregone consumption of extra liquidity in case of a bailout.

Figure 3 shows the banks' expected payoffs when the government commits to never bailout (solid line under Assumption 3) and when the government commits to bailout if the shock is aggregate (dashed line under Assumption 4). The intuition is the same as in the simple model and Figure 1.

Figure 3: Expected Payoffs in the Full Model



Proposition 1 below establishes that, under the stated assumptions, the optimal cash choice of banks is a level that allows them to refinance fully in case of an aggregate shock, given a government policy t^* , but not high enough to refinance fully in case of an idiosyncratic shock (i.e. no bailout policy).

Proposition 1 *Under Assumptions 1-4, given government policy t^* , the optimal choice of cash is characterized by*

$$c^*(t^*) = (1 - \rho_0)(t^* - 1),$$

where $t^* \in [1, 2]$ is the earliest bailout time, as defined in Definition 1.

Proof In appendix.

Given Proposition 1, and the solution to the bank's maximization problem, $c^*(t^*)$, the only characteristic that matters for welfare in terms of choosing a policy rule $\Pi(t, p)$, is the earliest bailout time t^* , which under commitment is like choosing c^* directly from the set $[0, 1 - \rho_0]$. We will therefore express ex-ante welfare in terms of the cash choice of banks. Ignoring constants, welfare can be expressed as

$$W^{ea}(c) = \beta[\pi + \rho_0 + \rho_1 - 1 - P_1 - P_2]2i(c) - (1 - \beta)P_2((1 - \rho_0) - c)2i(c). \quad (11)$$

where $i(c) = \frac{A}{1 - \pi + c}$ and $ci(c) = (\pi - 1)i(c) + A$.

Clearly, the optimal policy depends on the welfare weight on bankers, β . For $\beta = 1$, which implies equal weights in the welfare function, the fact that banks are subsidized does not change welfare per se, because utility is transferrable one to one between households and banks. In that case, the government only cares about output, and ex-ante wants to transfer resources from households to bankers, implying an optimal government policy of $t^* = 1$ and $c^* = 0$. In contrast, when β is low, the weight governments put on producing output is low, since households gain nothing from it.

Definition 2 defines equilibrium under commitment and Proposition 2 characterizes equilibrium under commitment, showing equilibrium outcomes depend on β .

Definition 2 (Commitment Equilibrium) *A symmetric equilibrium of the economy under commitment is a cash level c^* and policy of the government $\Pi(t, p = 1)$, such that c^* is the optimal response of the banks to policy, i.e. it maximizes (10) given $\Pi(t, p = 1)$, and $\Pi(t, p = 1)$ is such that c^* maximizes welfare (11).*

Proposition 2 (Optimal Policy with Commitment) *Given other parameters, define*

$$\beta^* = \frac{P_2(2 - \rho_0 - \pi)}{(\pi + \rho_0 + \rho_1 - 1 - P_1 - P_2) + P_2(2 - \rho_0 - \pi)} < 1.$$

Then,

(i) *If $\beta < \beta^*$, $\frac{dW^{ea}(c)}{dc} > 0$ for all $c \in [0, 1 - \rho_0]$. The equilibrium cash holding is $c^* = 1 - \rho_0$, which corresponds to welfare maximizing policy choice of no bailout, this is $t^* = 2$.*

(ii) If $\beta > \beta^*$, $\frac{dW^{ea}(c)}{dc} < 0$ for all $c \in [0, 1 - \rho_0]$. The equilibrium cash holding is $c^* = 0$, which corresponds to a welfare maximizing policy of immediate bailout, this is $t^* = 1$.

(iii) For $\beta = \beta^*$, the equilibrium government policy is indeterminate. $t^* \in [1, 2]$ and $c^*(t^*)$ is determined as in Proposition 1.

From $W^{ea}(c)$ evaluated at $c = 0$ (bailout in aggregate shocks) versus $c = 1 - \rho_0$ (never bailout), the benefits of committing to no bailouts are given by the social gains from private refinancing (using own savings) relative to the social gains from public refinancing when shocks are aggregate, this is

$$P_2(y - x),$$

while the costs of committing to no bailouts are given by the necessary reduction in the scale of the project times the benefits per unit of investment,

$$\beta \left[\frac{1 - \rho_0}{2 - \rho_0 - \pi} \right] [\pi + \rho_0 + \rho_1 - 1 - P_1 - P_2].$$

The cost is adjusted by β , which is the weight governments assign to bankers. The equation for β^* in the proposition comes from equalizing the above costs and benefits.

3.3.2 No Commitment

In this section we assume that the government is unable to commit to its policy announcements. The banks internalize the government's optimal ex-post actions in their optimization problem, effectively making them first-movers and giving them the ability to choose the time of the bailout t^* to maximize equation (10).

Definition 3 (Non-Commitment Equilibrium) *A symmetric equilibrium without commitment is a cash choice of banks c^* and a policy of the government $\Pi(t, p = 1)$ such that given the banks' choice of cash, the policy $\Pi(t, p = 1)$ is the ex-post best response, and hence banks maximize (10) given the government's reaction to both banks' cash choices.*

As in the simple model, the government never intervenes in the case of an idiosyncratic shock. This is because $y > x$, and, in a symmetric equilibrium, takeover is feasible and socially preferable. In contrast, the government always

intervenes when the shock is aggregate. This is because $x > 0$, and it is preferred to refinance the remainder of the project at a social cost than let it fail.

How much do banks save at $t = 0$ knowing this reaction of the government? Under Assumption 4, the unique non-commitment equilibrium is when all banks hold zero cash.

Proposition 3 (Optimal Policy without Commitment) *Under Assumptions 2 and 4, the unique equilibrium without commitment is characterized by banks choosing $c^* = 0$, and the government immediately intervening when the shock is aggregate.*

In what follows, we focus on the parameter space subset under which it is ex-ante optimal for governments to commit not bailing out banks in the aggregate state, but it is ex-post optimal for them to bail out banks in such a state, which implies:

Assumption 5 *Inefficient excessive leverage: $\frac{1-\rho_0}{\rho_1} < \beta < \beta^*$.*

This assumption, which combines Assumption 2 and Proposition 2, introduces into our model the tradeoff we set out to study: the time-inconsistency of government policies. Ex-ante the government would like to commit to no bailouts, but without commitment, there is excessive inefficient leverage in the economy, with large projects but no liquidity to refinance in case both banks fail, with inefficient bailouts on the equilibrium path. This assumption is more likely to hold when P_2 is low, ρ_1 is low and π is relatively high with respect to ρ_0 .¹³

¹³There are several ways this assumption may not hold. On the one hand, if β is either lower or higher than both cutoffs β^* and $\frac{1-\rho_0}{\rho_1}$, then lack of commitment does not introduce any inefficiency, rendering the problem irrelevant. On the other hand, if $\frac{1-\rho_0}{\rho_1} > \beta > \beta^*$, it is ex-ante optimal for governments to commit to bailout banks when the shock is aggregate, but it is ex-post optimal not bailing out banks in such a state. In this last case there is inefficient insufficient liquidity and the strategic restraint mechanism we highlight in the next sections are not effective to eliminate such an inefficiency.

4 Imperfect Information in the Full Model

In contrast to the previous section, here we assume the government does not observe the realization of the shocks at $t = 1$. Specifically, at some time $1 \leq t < 2$, the government may observe a bank in distress: not having any liquidity to continue the project. In such case, the government has to decide whether to bail out – introduce the low interest rate ρ_0 immediately, or do nothing, in which case the remainder of the project gets lost if not taken over.¹⁴ The government that decides not to bail out the first bank in distress, however, always faces the concern that both banks suffered a shock and *there is not enough liquidity in the system*. The posterior probability of both banks in distress, conditional on one bank in distress, is given by P'_2 in equation (3).

In case of bailing out the first bank in distress, interim welfare is

$$[(1 - P'_2)(x + \hat{x}) + P'_2(2x)](2 - \bar{t})i.$$

In case of *not* bailing out the first bank in distress, interim welfare is

$$[(1 - P'_2)y + P'_2x](2 - \bar{t})i.$$

Ex-post, the government decides to delay the bailout of the first bank in distress if

$$P'_2 < \bar{P} \equiv 1 - \frac{x}{y - \hat{x}}, \quad (12)$$

which is exactly the same as the *delayed bailout condition* in equation (4) in the simple model, but with y , x and \hat{x} redefined to correspond to the full model's setup. If this condition does not hold, the first bank in distress is bailed out regardless of the nature of the shock. As in the simple model, in this case, banks have even less incentives to reduce the scale of the project than under full information and no commitment, since the bank expects to be bailed out regardless whether the shock is aggregate or idiosyncratic.

In contrast, if the condition (12) is satisfied, the first bank in distress is not bailed out, but the second bank in distress is. This implies that the banks value

¹⁴An important assumption is that a bank that discontinues the project due to a missing flow of needed reinvestments, cannot restart it at a later date if finding additional funds.

functions become:

$$V(c) = \begin{cases} V_s(c) + P_2[c + (\rho_0 + \rho_1 - 1)(\bar{t}(c) - 1)]i & \text{if } \bar{t}(c) < \bar{t}(c') \\ V_s(c) + P_2[c + (\rho_0 + \rho_1 - 1)(\bar{t}(c) - 1)]i + \frac{1}{2}P_2\rho_1(2 - \bar{t}(c))i & \text{if } \bar{t}(c) = \bar{t}(c') \\ V_s(c) + P_2[c + (\rho_0 + \rho_1 - 1)(\bar{t}(c) - 1)]i + P_2\rho_1(2 - \bar{t}(c))i & \text{if } \bar{t}(c) > \bar{t}(c') \end{cases} \quad (13)$$

where V_s is the same as before in equation (10).

Now, as in the full information case, there is a jump in the value function – in fact there are two. The additional one is the midpoint between being bailed out or not when the banks hold the same cash, i.e. $\bar{t}(c) = \bar{t}(c')$. The difference between equations (13) and (10) is that now what matters is whether the bank runs out of cash before or after its competitor (summarized by $\bar{t}(c)$). Under (12), what determines being bailed out when the shock is aggregate is whether the bank is the first or the second in showing distress. The value function has the same jumps as in Figure 2 for the simple model, but for the shapes depicted in Figure 3 for the full model.

Definition 4 (Non-Commitment Equilibrium with Delay) *A symmetric equilibrium without commitment in case of delay (condition (12) holds) is a policy $\Pi(t, p)$ and the cash choice of banks c^* , such that $\Pi(t, P'_2) = 0 \ \forall t$ after observing the first bank in distress, and $\Pi(t, 1) = 1 \ \forall t$ after observing the second bank in distress and the cash choice of banks c^* is such that given the other bank's choice of cash, each bank maximizes (13).*

Next, we solve the ex-ante optimal cash choice of a bank, c , taking as given the cash choice of the other bank, c' . In particular, we ask whether it is optimal for a bank to deviate from a symmetric strategy $c = c'$ (which implies $\bar{t}(c) = \bar{t}(c')$). The crucial part of the argument is how any deviation $c \neq c'$ affects the probability that the bank is the first one showing distress, and hence the one failing when the condition (12) holds.

Note that a marginal deviation upwards from $c = c'$ (i.e. carrying slightly more liquidity than the other bank), has the benefit of increasing discontinuously the probability of a bailout (we relax this assumption later), at the cost of downsizing the project slightly, from $i(c')$ to $i(c) < i(c')$. For any marginal change, the first effect dominates, and there are always incentives to deviate as long as

$$\frac{1}{2}P_2\rho_1[2 - \bar{t}(c')]i(c') > 0, \quad (14)$$

which holds for all $\bar{t}(c') < 2$. The fraction $1/2$ is the change in the probability of being bailed out, which is multiplied by the probability of an aggregate shock and the benefit of financing the project until completion. We will refer to equation (14) as the *strategic restraint condition*.

Proposition 4 (Equilibrium: No Commitment and Government Uncertainty)

If $P'_2 < \bar{P}_2$, there is a unique symmetric equilibrium where $c^ = 1 - \rho_0$ (i.e. $\bar{t}(c^*) = 2$), which coincides with the optimal solution under commitment. The equilibrium policy of the government is $\Pi(t, P'_2) = 0 \forall t$ after observing the first bank in distress, and $\Pi(t, 1) = 1 \forall t$ after observing the second bank in distress.*

The statement of the proposition follows from applying the *strategic restraint condition* to all cases in which the *delayed bailout condition* holds. In all such cases, the value of being the second bank in distress is discontinuously higher than the value of being the first bank in distress. Following a Bertrand-style undercutting argument, banks want to deviate from a symmetric strategy in order to avoid being the first. At $\bar{t} = 2$, there is a corner solution and no more incentives to deviate, since banks can self-finance completely.

The starkness of the above result relies on the discrete change in probabilities driven by a continuous change in the action (cash c). This is certainly an extreme specification as it requires that banks can perfectly control their time of distress by their cash choice. In the next subsection, we generalize our result to an environment with ex-post shocks to cash holdings, such that small deviations in ex-ante cash choices do not guarantee a bank being second in showing distress, only changing the relative position in a probabilistic way. We derive an analogous result, showing that the Bertrand type competition still plays a role in bringing the equilibrium allocation closer to the optimum with commitment.

4.1 Ex-post Shocks to Cash Holdings

Here we consider a shock to the cash position of the bank at $t = 1$, after the refinancing shock has been realized. A positive shock implies that the bank holds more cash than planned (for example the certain return was higher than expected) while a negative shock implies the bank holds less cash than planned (for example there was an unexpected expense to cover during the previous period). Formally, the cash available for refinancing is

$$c(h)i = ci + hi,$$

where $h \sim \mathcal{N}(0, \sigma_h^2)$. Given the shocks, cash maps into time of distress analogously to (8):

$$t(h|\bar{t}) = \begin{cases} 1 & \text{if } \frac{h}{1-\rho_0} < -(\bar{t} - 1) \\ \bar{t} + \frac{h}{1-\rho_0} & \text{if } -(\bar{t} - 1) < \frac{h}{1-\rho_0} < (2 - \bar{t}) \\ 2 & \text{if } (2 - \bar{t}) < \frac{h}{1-\rho_0} \end{cases} \quad (15)$$

where \bar{t} is the expected time of distress in case of a shock, given by equation (8). Since h follows a normal distribution with zero mean, $t(h)$ is distributed according to

$$f(t|\bar{t}) = \begin{cases} \Phi\left(-\frac{1-\rho_0}{\sigma_h}(\bar{t} - 1)\right) & \text{for } t = 1 \\ \phi\left(\frac{1-\rho_0}{\sigma_h}(t - \bar{t})\right) & \text{for } 1 < t < 2 \\ 1 - \Phi\left(\frac{1-\rho_0}{\sigma_h}(2 - \bar{t})\right) & \text{for } t = 2 \end{cases} \quad (16)$$

where Φ denotes the standard cumulative normal distribution and ϕ denotes the density of the standard normal distribution.

Let's suppose that a bank is considering holding cash c such that, in case of a shock, distress happens in expectation at moment t . Conditional on the other bank holding cash such that, in case of a shock, its distress happens in expectation at moment \bar{t} , the probability of being the first in distress is

$$\eta(t|\bar{t}) \equiv \Pr(t > \bar{t}) = \Phi\left(\frac{1-\rho_0}{\sigma_h}(t - \bar{t})\right).$$

Define as η_t the marginal change in this probability when a bank decides to increase cash to survive longer in expectation (increasing t), conditional on the other bank still showing distress in expectation at \bar{t} . Then,

$$\eta_t(t, \bar{t}) = \frac{\partial \eta(t|\bar{t})}{\partial t} = \left(\frac{1-\rho_0}{\sigma_h}\right) \phi\left(\frac{1-\rho_0}{\sigma_h}(t - \bar{t})\right).$$

In a symmetric equilibrium, in which both banks hold in expectation the same amount of cash, the probability of showing distress second is 50% when the shock is aggregate ($\eta(\bar{t}|\bar{t}) = 0.5$) and the change in such probability from holding more cash is $\eta_t(\bar{t}, \bar{t}) = \left(\frac{1-\rho_0}{\sigma_h}\right) \phi(0)$, where $\phi(0)$ the value of the density of the standard normal distribution evaluated at 0.

We now derive the analog of the strategic restraint condition in the benchmark

model. The value of a bank from deviating from a symmetric strategy \bar{t} to t is

$$V(t|\bar{t}) = [(P_0 + P_1)(\rho_0 + \rho_1 + (1 - \rho_0)(t - 1)) + P_1\rho_1(t - 1) + P_2\rho_1[(t - 1) + \eta(t|\bar{t})(2 - t)]]i(t) + V_{TO}$$

where

$$i(t) = \frac{A}{1 - \pi + (t - 1)(1 - \rho_0)}, \quad (17)$$

and $V_{TO} = (\rho_0 + \rho_1 - 1)(i' - j')$, with i' and j' denoting the choices of the other bank, (and hence irrelevant from the maximization perspective).

The derivative of V with respect to t is

$$\begin{aligned} V_t(t|\bar{t}) &= [(P_0 + P_1)(\rho_0 + \rho_1 + (1 - \rho_0)(t - 1)) + P_1\rho_1(t - 1) + P_2\rho_1[(t - 1) + \eta(t|\bar{t})(2 - t)]]i_t(t) \\ &\quad + [(P_0 + P_1)(1 - \rho_0) + P_1\rho_1 + P_2\rho_1[1 - \eta(t|\bar{t}) + \eta_t(t, \bar{t})(2 - t)]]i(t) \end{aligned}$$

where

$$i_t(t) = -\frac{A(1 - \rho_0)}{(1 - \pi + (t_1 - 1)(1 - \rho_0))^2} = -\frac{(1 - \rho_0)}{A}i^2(t)$$

Multiplying by $A/i^2(t)$ and rearranging terms gives

$$\begin{aligned} \frac{A}{i^2(t)}V_t(t|\bar{t}) &= \overbrace{(P_1 + P_2)\rho_1(1 - \pi) - (P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1)(1 - \rho_0)}^{C > 0} + \\ &\quad P_2\rho_1 \left\{ \underbrace{\eta_t(t|\bar{t})(2 - t)(1 - \pi + (1 - \rho_0)(t - 1))}_{\text{increasing chance of a bailout}} - \underbrace{\eta(t|\bar{t})(1 - \rho_0 + 1 - \pi)}_{\text{reducing size of the project}} \right\} \quad (18) \end{aligned}$$

where the constant term C is positive by Assumption 3.

To put this expression in perspective, if we assume full information and no bailouts, such that $\eta(t|\bar{t}) = 0$ and $\eta_t(t|\bar{t}) = 0$, we have

$$\frac{A}{i^2(t)}V_t(t|\bar{t}, \eta = 1, \eta_t = 0) = C > 0,$$

which implies that banks always want to increase cash reserves (i.e. decrease leverage) to refinance full scale in case of a shock. In contrast, if we assume full information and guaranteed bailouts, such that $\eta(t|\bar{t}) = 1$ and $\eta_t(t|\bar{t}) = 0$, we have

$$\frac{A}{i^2(t)}V_t(t|\bar{t}, \eta = 1, \eta_t = 0) = C - P_2\rho_1(2 - \rho_0 - \pi) < 0,$$

which implies banks always want to reduce cash reserves (i.e. increase leverage)

to maximize the size of the project, given the certainty of a bailout.¹⁵

This implies that, if $\eta_t = 0$, there is a cutoff $\bar{\eta}$, such that, for all probabilities of bailout low enough (this is, $\eta < \bar{\eta}$), banks would like to reduce leverage (increase t) to have the possibility of refinancing the project fully in case of a shock. We further assume, quite naturally, that banks do not restrict leverage when they do not have control over the probability of a bailout.

Assumption 6 *If there is a probability 50% of bailout under aggregate shocks, banks take excessive leverage, i.e. $\bar{\eta} < 0.5$.*

Effectively, this assumption is a complement of Assumptions 3 and 4 for continuous distributions, which together imply

$$\frac{A}{i^2(t)} V_t(t|\bar{t}, \eta = 1, \eta_t = 0) \equiv Z = C - \frac{1}{2} P_2 \rho_1 (2 - \rho_0 - \pi) < 0.$$

In general, when expression (18) is positive, there are individual incentives to deviate by increasing t above \bar{t} , increasing enough the chance of a bailout when the shock is aggregate to justify the reduction in the size of the project. We focus on symmetric strategies, and consider deviations evaluated at $t = \bar{t}$, which implies replacing $\eta(\bar{t}|\bar{t}) = \frac{1}{2}$ and $\eta_t(\bar{t}|\bar{t}) = \frac{(1-\rho_0)}{\sigma_h} \phi(0)$ in expression (18).

Consider the possible values for σ_h . If $\sigma_h = \infty$, $\eta_t = 0$, the randomness of cash holding is so large that banks cannot change their relative position by reducing the leverage. In this case, under Assumption 6 there is no strategic restraint. For $\sigma_h < \infty$, a reduction in leverage increases the probability of not being the first bank in distress when the shock is aggregate, which is captured by $\eta_t > 0$. The smaller the σ_h , the larger the marginal increase in such probability and the more likely that expression (18) is positive, inducing a reduction in leverage. In the limit, when $\sigma_h = 0$, $\eta_t = \infty$, and we obtain the same conclusion as in the benchmark model. The gains from deviating are so large that we recover the commitment outcome with full refinancing in case of shocks.

Proposition 5 *Under Assumption 6, there exists $\underline{\sigma}_h$, such that for all $\sigma_h < \underline{\sigma}_h$, the equilibrium is unique and characterized by $c^*(\sigma_h) = (1 - \rho_0)(t^*(\sigma_h) - 1)$, where*

$$\underline{\sigma}_h \equiv - \frac{P_2 \rho_1 (1 - \pi) (1 - \rho_0) \phi(0)}{Z},$$

¹⁵This equation has a negative sign since, by Assumption 4, $C - P_2 \rho_1 (2 - \rho_0 - \pi) < P_2 (1 - \rho_0) (\rho_0 + \pi - 2) < 0$.

and $t^*(\sigma_h)$ solves

$$\frac{A}{i^2(t)} V_t(t|t) = Z + \eta_t P_2 \rho_1 (2 - t)(1 - \pi + (1 - \rho_0)(t - 1)) = 0, \quad (19)$$

where

$$\eta_t = \frac{(1 - \rho_0)}{\sigma_h} \phi(0).$$

Furthermore, $t^*(\sigma_h)$ is decreasing in σ_h and equilibrium cash holdings converge to the commitment outcome when the volatility of the shock goes to zero, i.e.

$$\lim_{\sigma_h \rightarrow 0} c^*(\sigma_h) = 1 - \rho_0.$$

Proof. For a given σ_h , strategic restraint for symmetric strategies t is equivalent to requiring a positive value for (19). (i) For the first part of the proposition, we want to show that for all $\sigma_h < \underline{\sigma}_h$, (19) has a unique root in the interval $t \in [1, 2]$, and is positive for $t < t^*(\sigma_h)$. To see that, note that (19) is quadratic, and strictly negative and decreasing at $t = 2$. Second, under our definition of $\underline{\sigma}_h$, (19) is positive at $t = 1$. That means that it has exactly one root in $[1, 2]$, equal to $t^*(\sigma_h)$ above, and that the strategic restraint $((19) > 0)$ is satisfied for all $t < t^*(\sigma_h)$. (ii) For the limiting result of the proposition, note that the function (19) is decreasing in σ_h , and hence $t^*(\sigma_h)$ is an increasing function. Furthermore, as $\sigma_h \rightarrow 0$ then $\eta_t \rightarrow \infty$ and $\bar{t}^* \rightarrow 2$. ■

Intuitively, from expression (18), the benefits from deviating and holding more cash than the other bank is to increase the chances of being second in distress and obtain a bailout when the shock is aggregate. In a symmetric equilibrium the benefits are decreasing in σ_h and \bar{t} . In contrast, the costs from deviating come from downsizing the project. In a symmetric equilibrium, the costs are independent of σ_h or t . Since the bank is indifferent between deviating or not when expression (18) is zero, the larger is σ_h the lower the benefits from deviating. Hence the indifference is recovered when \bar{t} is lower, or equivalently, when banks hold cash in equilibrium to refinance a smaller fraction of the distressed project in case of an aggregate shock.

5 Policy Implications

In the previous section, we showed that *government uncertainty* together with *strategic restraint* has the potential to implement efficient commitment outcomes, even in the absence of commitment. Below, we study the impact of financial innovation, industry concentration, and asymmetric bank sizes on the potential of government uncertainty to induce more efficient outcomes in the absence of commitment.

5.1 Financial Innovation

This section analyzes how the information available to governments and the incentives of banks change with the level of financial innovation in the economy that allows banks to insure away part of their idiosyncratic risk. We allow banks to diversify their risk by holding claims on other banks' projects and selling a part of their own project. There are a number of financial arrangements that would achieve that goal. One is that banks sell a fraction of claims on their project to a pool, and then hold the pool of the industry in proportions equal to their relative contributions by investing in *securitized assets*. More generally one could consider many other *swaps and over-the-counter derivatives* that allow banks to cross-insure each other's cash flows, in order to get rid of some of the idiosyncratic risk of their project.

We call s the fraction of a project's risk a bank can diversify away by using financial instruments. We derive a cap on s , which can then be more generally thought of as a cap on the individual amount traded on financial instruments, that allow the government uncertainty channel highlighted in this paper to keep operating to circumvent lack of commitment. We use a securitization example to fix ideas, but the logic of this section can be applied more broadly to any financial arrangement or innovation that diversify risk.

Formally, the level of securitization is summarized by the fraction s of the project that a bank contributes to the pool and invests in the pool. For the case of two banks, for example, $s = 1/2$ implies that the bank diversifies idiosyncratic risks completely, getting the average return of the industry. Each bank is then responsible for financing the remaining fraction $1 - s$ of its own project.

High level of securitization in the economy has the benefits of providing more diversification to banks, which results in less need for accessing the market for additional funds. It has, however, the cost of reducing the level of government

uncertainty. In our setup, banks are risk neutral and hence there are no benefits from diversification, but only costs in terms of possibly inducing inefficient, non-commitment, outcomes.¹⁶

First, we show that in our benchmark, *any* level of securitization completely removes government uncertainty (Proposition 6). Then, we extend our benchmark setup to include ex-post shocks to cash holdings, as in Section 4.1. In the extended setup, we show that government uncertainty remains for a set of positive securitization levels, and derive a cap on securitization that maintains the forces of strategic restraint and commitment-like outcomes.

In the benchmark model, for *any* level of securitization $s > 0$, the time at which distress occurs differs between idiosyncratic and aggregate shocks, sending a clear signal of the source of distress to the government. In a symmetric equilibrium, in case of an idiosyncratic shock, a bank in distress refinances $(1 - s)j$, using $(1 - s)ci$ from the claims on the cash holding of the distressed project, $s(\rho_0 + \rho_1 + c)i$ from the claims on the healthy project and $(1 - s)\rho_0 j$ from the pledgeable output of the refinanced project. From the lenders' break even condition this implies that

$$(1 - s)j - (1 - s)ci - s(\rho_0 + \rho_1 + c)i = (1 - s)\rho_0 j$$

Defining \hat{t} as the time of distress of a bank after an idiosyncratic shock, then

$$\hat{t} = \min \left\{ 1 + \frac{c + s(\rho_0 + \rho_1)}{(1 - s)(1 - \rho_0)}, 2 \right\}$$

In contrast, in case of an aggregate shock, securitization does not affect the total liquidity in the economy and the time of distress of a bank is \bar{t} , the same as derived in equation (8). This implies that the time of distress differs depending on the shock, and

$$\hat{t} - 1 = \frac{c + s(\rho_0 + \rho_1)}{(1 - s)(1 - \rho_0)} > \bar{t} - 1 \equiv \frac{c}{(1 - \rho_0)}, \quad (20)$$

An immediate implication is

Proposition 6 (Financial Innovation) *In the benchmark model, for any se-*

¹⁶Benefits of diversification are clearly important, but modeling them is outside the scope of this paper. Here, we choose to focus instead on analyzing the costs which securitization imposes by leading to non-commitment outcomes – which are much less studied and understood in the literature.

curitization level $s > 0$, the unique equilibrium is the non-commitment equilibrium under full information.

The above proposition is very intuitive: any level of securitization introduces a difference in the time of distress of a bank when the shock is idiosyncratic and when it is aggregate, since in the first case the distressed bank has more cash, from the claim on the successful project, to face the refinancing needs. It follows, then, that the government can infer the nature of the shock from the timing of distress, restoring full information.¹⁷

The above result depends crucially on the ability of the government to perfectly observe all the banks' choices, particularly leverage, which determines the time of distress when both banks need refinancing. Introducing unobserved heterogeneity ex-post allows positive securitization that still preserves the positive effects of government uncertainty. Below, we consider such an extension.

5.1.1 Extended setting with shocks to cash holdings

We extend the previous setting along the lines of Section 4.1. Consider a shock to the cash position of the bank, $h \sim \mathcal{N}(0, \sigma_h^2)$, that hits at $t = 1$, after the refinancing shock has been realized (either aggregate or idiosyncratic). The cash available for refinancing in the case of an idiosyncratic shock is then

$$c(h)i = ci + s(\rho_0 + \rho_1)i + h(1 - s)i$$

Note the shock to cash, h , is proportional to the size of the project that needs refinancing.

Define the time of distress given an aggregate shock as $t^a(h)$ such that $(t^a(h) - 1)i$ is the amount of the investment that can be refinanced at market rate $R = 1$. When the shock is aggregate this is equivalent to Section 4.1, and hence $t^a(h) = t(h)$ is given by equation (15), and its density is given by $f^a(t|\bar{t}) = f(t|\bar{t})$ in equation (16).

Define the time of distress given an idiosyncratic refinancing shock as $t^i(h)$, where $(t^i(h) - 1)i$ is the amount of the investment that can be refinanced at the market rate $R = 1$, equal to $c(h)/[(1 - \rho_0)(1 - s)]$. We can write $t^i(h)$ as function

¹⁷Since the government observes A and i it can infer \bar{t} . Whenever the actual time of distress is higher than \bar{t} the government is certain that only one bank needs funds and then intervention is not needed.

of the expected time of distress \bar{t} as

$$t^i(h|\bar{t}) = \begin{cases} 1 & \text{if } \frac{h(1-s)+s(\rho_0+\rho_1)}{1-\rho_0} < -(\bar{t}-1) \\ \frac{\bar{t}-s}{(1-s)} + \frac{s(\rho_0+\rho_1)}{(1-s)(1-\rho_0)} + \frac{h}{1-\rho_0} & \text{if } -(\bar{t}-1) < \frac{h(1-s)+s(\rho_0+\rho_1)}{1-\rho_0} < (2-\bar{t})-s \\ 2 & \text{if } (2-\bar{t})-s < \frac{h(1-s)+s(\rho_0+\rho_1)}{1-\rho_0} \end{cases} \quad (21)$$

Given the distribution of h , $t^i(h)$ is distributed according to following density $f^i(t|\bar{t})$

$$f^i(t|\bar{t}) = \begin{cases} \Phi\left(-\frac{1-\rho_0}{\sigma_h} \frac{(\bar{t}-1)}{(1-s)} - \frac{s(\rho_0+\rho_1)}{(1-s)}\right) & \text{for } t = 1 \\ \phi\left(\frac{1-\rho_0}{\sigma_h} \left(t - \frac{\bar{t}-s}{(1-s)} - \frac{s(\rho_0+\rho_1)}{(1-s)(1-\rho_0)}\right)\right) & \text{for } 1 < t < 2 \\ 1 - \Phi\left(\frac{1-\rho_0}{\sigma_h} \frac{(2-\bar{t}-s)}{(1-s)} - \frac{s(\rho_0+\rho_1)}{(1-s)}\right) & \text{for } t = 2 \end{cases} \quad (22)$$

where ϕ and Φ are the pdf and cdf of the standard normal distribution, and the mean of $t^i(h)$ (this is, setting $h = 0$) is equal to $\frac{\bar{t}-s}{(1-s)} + \frac{s(\rho_0+\rho_1)}{(1-s)(1-\rho_0)}$.

Clearly, when $s = 0$, $f^a(h) = f^i(h)$. When $s > 0$, $f^i(1|\bar{t}) < f^a(1|\bar{t})$ and $f^i(2|\bar{t}) > f^a(2|\bar{t})$, which implies it is more likely to see aggregate shocks earlier than idiosyncratic shocks. The updated probability of an aggregate shock, after observing a bank in distress, is then

$$P'_2 \equiv P(Agg|t) = \frac{P(t^a(h) = t|Agg)P_2}{P(t^a(h) = t|Agg)P_2 + P(t^i(h) = t|Id)P_1}.$$

The government does not bailout the first bank in distress as long as the probability of an aggregate shock is smaller than the cutoff \bar{P}_2 from the delayed bailout condition (12)

$$P'_2 = \frac{\frac{f^a(t)}{f^i(t)} \frac{P_2}{P_1}}{1 + \frac{f^a(t)}{f^i(t)} \frac{P_2}{P_1}} \leq \bar{P}_2, \quad (23)$$

Note that, when $s = 0$, $f^a(t) = f^i(t)$, and P'_2 is the one obtained in the benchmark without securitization, $P'_2 = \frac{P_2}{P_1+P_2}$. Additionally, for $s > 0$, the likelihood ratio under normality is declining in t . A sufficient condition for (23) to hold at any moment t is that it holds at $t = 1$, which gives

$$\left(\frac{\bar{t}-s}{(1-s)} + \frac{s(\rho_0+\rho_1)}{(1-s)(1-\rho_0)} - 1\right)^2 - (\bar{t}-1)^2 \leq 2\sigma_h^2 \ln\left(\frac{\bar{P}_2}{(1-\bar{P}_2)} \frac{P_1}{P_2}\right). \quad (24)$$

Since the left hand side of (24) is strictly increasing in s and the right hand side is a constant, there is a strictly positive \bar{s} that is the minimum between $1/2$

(the maximum possible securitization) and the value of s that satisfies (24) with equality.

Any level of securitization lower than or equal to \bar{s} guarantees an outcome more efficient than the non-commitment one, as described in Proposition 5. Finally, it is straightforward to see that \bar{s} is weakly increasing in σ_h^2 . There is a σ_h^2 large enough such that $\bar{s} = 1/2$ and full securitization does not prevent uncertainty to implement the commitment outcome. In contrast, if $\sigma_h^2 = 0$, we are back in the benchmark case, in which $\bar{s} = 0$ and any level of securitization eliminates government uncertainty.

5.2 Number of banks

In what follows we study the government incentives to bail out the first bank in distress when $N > 2$. For any number of banks, not bailing out the *first* bank in distress is enough to trigger strategic restraints and obtain commitment-like outcomes. Below, we show that having more banks in the economy reduces the incentives to bailout the first bank in distress due to two forces. First, more banks increase the option value of waiting and learning about the true nature of the shock. Second, more banks may reduce the likelihood there is not a healthy bank able to save the first bank in distress. We analyze these forces in turn below.

Denote by $p_{(N,d)|s}$ the probability of having d banks in distress conditional on having N banks in total and observing s bank already showing distress. For example, $p_{(2,2)|1} \equiv P'_2$ is the updated probability of an aggregate shock after observing one bank in distress from the previous discussion when $N = 2$.

We first consider a case in which the probability of an aggregate shock (all N banks have a liquidity shock) is independent of N , and a counterfactual policy of bailing out for sure the second bank in distress. This case highlights an option value element in the incentives of the government to delay intervention.

Lemma 3 *Let $\hat{x} = 0$, $p_{(N,N)|1} = p$ for all N . If the government bails out for sure the second bank in distress, then the condition for delayed bailout is $p < 1 - \frac{x}{y}$ for all N .*

This Lemma is an important benchmark to understand the effects of N in the condition for delay. Under the assumptions in the Lemma the number of banks N does not affect the conditions for delaying intervention on the first bank. This result, however depends on two restrictions. First, the second bank in distress is

bailed out. Second, the probability of an aggregate shock is independent on the number of banks.

In the next two lemmas we separately relax these two restrictions and isolate the two forces for which more banks induce governments to delay intervention more likely. The first force is the *option value of waiting and seeing*, which works through making the cutoff of the delayed bailout condition increasing in the number of banks. The second force is more mechanical, a *lower likelihood of an aggregate shock*, which works through making governments more optimistic that there is at least some healthy bank able to take over the first bank in distress when there are many banks.

Lemma 4 *For all N , let $\hat{x} = 0$, $p_{(N,N)|1} = p$, $p_{(N,N-1)|2,to} = p'$ conditional on the first bank being taken over (to) and $p_{(N,N-1)|2,nto} = 1$ conditional on the first bank being not taken over (nto). The delayed bailout condition of the first bank is more easily satisfied for larger N .*

Lemma 5 *Assume $\hat{x} = 0$. Let $1 - \alpha$ be the probability of an aggregate shock, and if there is no aggregate shock, let each bank have a liquidity shock with probability $1 - \lambda$. Also, assume governments always bailout second banks in distress. Then the probability to delay the bailout of the first bank in distress weakly increases with N .*

The proofs of these three lemmas are in the Appendix. Their combination leads to the following Proposition

Proposition 7 *The larger the number of banks in the economy, the more likely governments delay interventions when banks start showing distress.*

In words, a larger number of banks introduce two forces that delay government intervention and lead to strategic restraints. Lemma 3 shows a benchmark, under which the number of banks does not affect the probability of an aggregate shock and does not affect the option value of delaying, since we impose bailouts of a second bank shows distress. In this case, the government's incentives to delay bailouts is independent of the number of banks.

Lemma 4 relaxes the assumption that governments have to intervene if a second bank is in distress. This introduces the option value of not bailing out the first bank and having the chance to make the optimal decision of not bailing out if other banks show distress. This Lemma shows that more banks makes more valuable the option value of learning and making better decisions ex-post.

Lemma 5 relaxes the assumption that the probability of an aggregate shock is independent of N . Effectively, it is less likely that no bank is healthy when there are many banks, and then less likely that the first bank in distress is not taken over and fails in case of not being bailed out by the government.

These last two forces complement each other. When there are many banks in the industry and a bank shows distress, it is less likely it fails if not bailed out and it is more valuable for governments to wait and see.

5.3 Asymmetric Bank Sizes

This section studies the effects of asymmetric bank sizes on governments' incentives to delay bailouts. Our goal is to analyze the impact of *too big to fail banks*, this is banks whose balance sheets are very large relative to the next biggest bank in the industry.

Formally, we modify the benchmark by assuming that Bank 1 has higher initial assets than Bank 2, i.e. $A_1 > A_2$. Such ex-ante asymmetry will imply ex-post asymmetry in investment size and consequently a healthy Bank 2 may not have enough funds to take over a distressed Bank 1. Specifically, Bank 2's available cash, which potentially can be used to refinance Bank 1's project, is equal to $(\rho_0 + \rho_1 + c_2)i_2$. Hence, the reinvestment scale in case Bank 2 needs to take over Bank 1's project is

$$I = \min \left\{ \frac{(\rho_0 + \rho_1 + c_2)i_2}{1 - \rho_0}, (2 - \bar{t}_1)i_1 \right\}.$$

The reinvestment scale is either equal to the part of the project that needs refinancing, $(2 - \bar{t}_1)i_1$, or to the maximal amount of money that Bank 2 can raise by leveraging up its cash. Clearly, for large enough asymmetry, the latter is going to be strictly smaller than the former and the project will be scaled down even under takeover.

The delayed bailout condition becomes

$$(1 - P'_2)[yI - \hat{x}(2 - \bar{t}_1)i_2] \geq x(2 - \bar{t}_1)i_1, \quad (25)$$

which is the same as

$$P'_2 \leq \bar{P}_2 \equiv \frac{y \left(\frac{I}{(2 - \bar{t}_1)i_1} \right) - \hat{x} \left(\frac{i_2}{i_1} \right) - x}{y \left(\frac{I}{(2 - \bar{t}_1)i_1} \right) - \hat{x} \left(\frac{i_2}{i_1} \right)} = 1 - \frac{x}{y \left(\frac{I}{(2 - \bar{t}_1)i_1} \right) - \hat{x} \left(\frac{i_2}{i_1} \right)}. \quad (26)$$

When A_2/A_1 goes to zero, i_2/i_1 also goes to zero and hence both terms in the denominator approach zero, which makes the cutoff \bar{P}_2 approach zero as well. This implies that there is a level of asymmetry large enough such that \bar{P}_2 is small enough and the government always bails out the large bank in distress, regardless of the updated belief about the probability the second bank is successful or not. In contrast, as A_2/A_1 goes to one, \bar{P}_2 converges to the original cutoff shown in equation (12). Any level of asymmetry makes the cutoff smaller, such that delay is more difficult to occur.

If condition (26) does not hold, Bank 1 has no incentive to restrain leverage, since it would be bailed out anyways, then choosing $c_1 = 0$. This implies that it is optimal for Bank 2, conditional on Bank 1 holding no cash, to hold slightly positive amount of cash to guarantee showing distress in second place when the shock is aggregate. The large bank becomes a ‘shield’ for the small bank to engage in inefficient levels of leverage. This points to a new and unique negative externality of ‘too big to fail’ banks for households, who may not only need to bailout large banks but also excessive risk exposure of small banks.¹⁸

¹⁸Since the payoff function for Bank 2 is discontinuous at $c_2 = 0$, its supremum is not attainable. Still, if cash is discretely undivisible, then the cash holding of Bank 2 is given by one unit of cash, regardless of how small the unit of measurement is.

6 Conclusions

At the onset of financial crises, banks usually tend to show distress sequentially. Then, initially, when deciding whether to intervene or not, governments are uncertain about the nature of the problem at hand. We show that such *government uncertainty* leads governments to delay bailouts in order to learn further about the nature of the underlying situation from market outcomes. Crucially, such delays introduce incentives for banks to compete, in Bertrand-style fashion, for their relative performance, giving rise to endogenous *strategic restraint* of their levels of risk taking and exposure to liquidity shocks.

We show that these novel forces have dramatic effects on the equilibrium outcomes in the economy. In seminal models of banking and liquidity choices, Holmström and Tirole (1998) and Farhi and Tirole (2012) show that no commitment can lead to endogenous crises and inefficient bailouts. Modelling *government uncertainty* radically changes these results, moving the economy from inefficiently high levels of leverage to efficient levels, even in the absence of commitment.

Based on these insights we provide a novel discussion of how financial innovations, banking concentration and asymmetric bank sizes can induce endogenous systemic events. In our case this works through the effects of these aspects of the banking industry on the inference problem of the government and the ensuing strategic behavior of banks.

The literature has identified the time-inconsistency of governments' policies as an important justification for macro-prudential regulation and direct overseeing of banks' activities. However, historically regulators have been incapable to design macro-prudential regulation that prevents crises without choking off growth. Our work suggests that it may be optimal for governments to design political structures that delay bailouts decisions, or regulatory standards that maintain an optimal level of uncertainty, imposing lower burdens on preventive regulation and giving more room to facilitate optimal self-regulation. Contrary to the common view that information and speed of action are desirable characteristics of policymakers, it may be the case that banks' perception about policymakers reacting fast to systemic events give them incentives to coordinate on those, endogenously generating crises.

Our framework can be extended in several directions to address other interesting questions regarding bank bailouts, information frictions and time consistency. One natural extension of our analysis is to include the possibility of contagion, where a failure of one bank may *trigger* the need for refinancing of other banks.

Our benchmark model can be easily extended to include an exogenous probability of contagion, interpreted as an increase in the probability of an aggregate shock, conditional on one project failing. This possibility lowers the probability that there will be enough liquidity in the system if the first bank in distress is allowed to fail, which reduces the incentives of the government to delay bailouts, making commitment outcomes harder to obtain.¹⁹ This is naturally a very crude way of thinking about contagion. Modeling its micro-foundations may be important in evaluating the welfare costs of delay.

Another avenue to explore is the solvency versus liquidity nature of banks' distress. This particular uncertainty always haunts governments when there are bank runs, since their optimal response depends on the nature of the distress. In case of insolvency, the bank should be wound down; in case of illiquidity, its assets should be replenished. Our model in this paper is not equipped to address this issue. By assumption, the project size deterministically determines the speed of cash outflows of the project, and every time the project needs refinancing, it is still a positive net present value investment. Hence, all shocks are liquidity shocks. In ongoing work, we explore an alternative environment in which the inference problem of the government is about finding out whether the underlying problem of the bank is insolvency or illiquidity, and then deciding whether to save the project at all or not.

Finally, in this paper we focus solely on risk-free loans, where banks cannot default on their debt. Hence, all bailouts in the model are bailouts of equity holders and not debt holders. In an extension that allows for default, interest rates would include a premium for the expected probability of default, which clearly depends on whether a bailout to lenders is expected in case of default. If banks and lenders expect bailouts in case of a systemic event, the premium is low and there is effectively a subsidy to risk taking, possibly inducing excessive leverage and endogenous crises. This extension needs further analysis, but it clarifies that restricting the model to bailing out only equity holders is not critical for our results.

¹⁹Formally, we can modify the benchmark model's stochastic structure by introducing a probability χ that the situation is contagious. This can be modeled as an increase in the probability of an aggregate shock from P_2 to $P_2 + \chi$, conditional on one project failing. If the contagion shock is independent of the other refinancing shocks, then the delayed bailout condition becomes $\hat{P}'_2 = \frac{P_2 + \chi}{P_1 + P_2 + \chi} > P'_2$, meaning that the delayed bailout condition is more difficult to satisfy than in the absence of contagion.

References

- Acharya, Viral, and Tanju Yorulmazer. 2007. “Too Many to Fail - An Analysis of Time-inconsistency in Bank Closure Policies.” *Journal of Financial Intermediation* 16:1–31.
- Baker, Scott, Nicholas Bloom, and Steven Davis. 2012. “Measuring Economic Policy Uncertainty.” Working Paper, Stanford University.
- Bianchi, Javier. 2012. “Efficient Bailouts?” NBER Working Paper 18587.
- Carrillo, Juan, and Thomas Mariotti. 2000. “Strategic Ignorance as a Self-Disciplining Device.” *Review of Economic Studies* 67:529–544.
- Cheng, Ing-Haw, and Konstantin Milbradt. 2010. “The Hazards of Debt: Rollover Freezes, Incentives and Bailouts.” Working Paper, MIT Sloan.
- Cremer, Jacques. 1995. “Arm’s Length Relationships.” *Quarterly Journal of Economics* 110:275–296.
- Cukierman, Alex, and Yehuda Izhakian. 2011. “Bailout Uncertainty in a Microfunded General Equilibrium Model of the Financial System.” Working Paper, Tel-Aviv University.
- Davila, Eduardo. 2012. “Does Size Matter? Bailouts with Large and Small Banks.” Working Paper, Harvard University.
- Farhi, Emmanuel, and Jean Tirole. 2012. “Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts.” *American Economic Review* 102:60–93.
- Green, Edward. 2010. “Bailouts.” *Economic Quarterly, Richmond Fed*, no. 1.
- Holmström, Bengt, and Jean Tirole. 1998. “Private and Public Supply of Liquidity.” *Journal of Political Economy* 106:1–40.
- Keister, Todd. 2011. “Bailouts and Financial Fragility.” Working Paper, New York Fed.
- Kelly, Bryan, Hanno Lustig, and Stijn Van Nieuwerburgh. 2011. “Too-Systemic-To-Fail: What Option Markets Imply About Sector-wide Government Guarantees.” *NBER Working Paper* 17149.
- Pasten, Ernesto. 2011. “Time-Consistent Bailout Plans.” Working Paper, Central Bank of Chile.
- Stern, Gary, and Ron Feldman. 2004. *Too Big to Fail: The Hazard of Bank Bailouts*. The Brookings Institution.

A Appendix

A.1 Proof of Proposition 1

Fix t^* and consider c such that $\frac{c}{1-\rho_0} < t^* - 1$. The bank's value function on this part of the domain is

$$V(c) = P_2(c + (\rho_0 + \rho_1 - 1)(\bar{t}(c) - 1))i + (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1\rho_1j + P_1V_{TO}$$

where

$$j = ci/(1 - \rho_0)$$

$$V_{TO} = (\rho_0 + \rho_1 - 1)(i' - j') \text{ and}$$

$$i = \frac{A}{1 - \pi + c}.$$

Replacing with $ci = (\pi - 1)i + A$ and $(\bar{t}(c) - 1)i = \frac{ci}{1-\rho_0}$

$$V'(i) = (P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1) - P_1\rho_1\frac{1 - \pi}{1 - \rho_0} - P_2\rho_1\frac{1 - \pi}{1 - \rho_0}$$

and $V'(c) = V'(i)i'(c)$. Since $i(c) = \frac{A}{1-\pi+c}$, $i'(c) = -\frac{A}{(1-\pi+c)^2} < 0$, then $V'(c) > 0$ if and only if $V'(i) < 0$, which is the case if

$$(P_0 + P_1)(\rho_0 + \rho_1 + \pi - 1) - (P_1 + P_2)\frac{\rho_1}{1 - \rho_0}(1 - \pi) \underbrace{\leq}_{\text{Assumption 3}} 0.$$

The interpretation of this result is that whenever leverage is too high to refinance fully under interest rate 1 (i.e. c is too low), it is optimal for the bank to increase cash holdings to assure fuller refinancing scale.

For $1 > \frac{c}{1-\rho_0} > t^* - 1$, the bank always refinances fully on the market and the value function is

$$V(c) = P_2(c + \rho_0 + \rho_1 - (t^* - 1) - \rho_0(2 - t^*))i + (P_0 + P_1)(c + \rho_0 + \rho_1)i + P_1\rho_1j + P_1V_{TO}.$$

again, $V'(c) = V'(i)i'(c)$ and $i'(c) < 0$, which implies that $V'(c) < 0$ if and only if $V'(i) > 0$

$$V'(i) = (P_0 + P_1 + P_2)(\pi - 1 + \rho_0 + \rho_1) - P_1 \rho_1 \frac{(1 - \pi)}{1 - \rho_0} - P_2((1 - \rho_0)(t^* - 1) + \rho_0)$$

Then $V'(i) > 0$ if and only if

$$\begin{aligned} & (P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) - P_1 \frac{\rho_1}{1 - \rho_0}(1 - \pi) - P_2(\rho_0 + (1 - \rho_0)(t^* - 1)) \\ & > (P_0 + P_1 + P_2)(\rho_0 + \rho_1 + \pi - 1) - P_1 \frac{\rho_1}{1 - \rho_0}(1 - \pi) - P_2 \underbrace{\hspace{1cm}}_{\text{Assumption 4}} > 0. \end{aligned}$$

Hence, on this part of the domain, it is optimal to decrease cash holdings and increase leverage.

Third for $\frac{c}{1 - \rho_0} > 1$, the value of the bank is

$$V(c) = (P_2 + P_1)(c + \rho_1 + \rho_0 - 1)i + P_0(c + \rho_0 + \rho_1)i + P_1 V_{TO},$$

taking the derivative with respect to i and considering $V'(c) < 0$ if $V'(i) > 0$

$$V'(i) = (\pi + \rho_1 + \rho_0) - (1 + P_1 + P_2) \underbrace{\hspace{1cm}}_{\text{Assumption 1}} > 0.$$

Therefore, here also it is optimal to decrease cash. This completes the proof. ■

A.2 Proof of Lemma 3

For $N = 2$ the condition for not bailing out the first bank in distress is given by condition (12) with $P'_2 = p_{(2,2)|1} = p$. In the case of $N = 3$, the social gains from bailing out the first bank in distress are

$$p3x + (1 - p) \left[\frac{p_{(3,2)|1}}{(1 - p)} 2x + \frac{p_{(3,1)|1}}{(1 - p)} x \right],$$

while the social gains from not bailing out the first bank in distress but bailing out the second bank in distress are

$$p2x + (1 - p) \left[\frac{p_{(3,2)|1}}{(1-p)}(y + x) + \frac{p_{(3,1)|1}}{(1-p)}y \right]$$

Since $\frac{p_{(3,2)|1}}{(1-p)} + \frac{p_{(3,1)|1}}{(1-p)} = 1$, the condition to delayed bailout is also $p_{(3,3)|1} = p < 1 - \frac{x}{y}$. More generally, for any arbitrary N , the social gains from bailing out the first bank in distress are

$$pNx + (1 - p) \sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1-p)} dx$$

while the social gains from not bailing out the first bank in distress but bailing out the second bank in distress are

$$p(N-1)x + (1 - p) \sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1-p)} (y + (d-1)x).$$

Since $\sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1-p)} = 1$, the condition to delayed bailout is again

$$p_{(N,N)|1} = p < 1 - \frac{x}{y}.$$

■

A.3 Proof of Lemma 4

For the $N = 3$ case, the delayed bailout condition is exactly as in the two bank case. For $N = 4$, the social gains from bailing out the first bank in distress are

$$p4x + (1 - p)[x + q(p'2x + (1 - p')x)]$$

while the social gains from not bailing out the first bank in distress are

$$p3x + (1 - p)[y + q \max\{p'2x + (1 - p')x, p'x + (1 - p')y\}] \quad (27)$$

where $q = \frac{p_{(4,3)|1} + p_{(4,2)|1}}{(1-p)}$, $p' = p_{(4,3)|2,to} = \frac{p_{(4,3)|1}}{q(1-p)}$ and $(1-p') = p_{(4,2)|2,to} = \frac{p_{(4,2)|1}}{q(1-p)}$.

Then it is optimal to not bail out the first bank in distress if

$$(1-p)(y-x) + (1-p)q \max\{-p'x + (1-p')(y-x), p'x - (1-p')(y-x)\} > px \quad (28)$$

The last term in equation (27) is the value of keeping the option of introducing the bailout at a later time, under new posterior p' which is necessarily more accurate. Since this number is at least weakly bigger than the value of immediate bailout, the actual cutoff value for delayed bailout condition \bar{P} is weakly higher than for the two bank case. For values of p' for which it is optimal to not bail out the second bank in distress, the increase is strictly positive. This shows that the cutoff probability for delayed bailout in the $N = 4$ case is equal or larger than the two or three banks case.

For the more general N bank case the condition for delayed bailout is exactly (28) if we restrict p' and q to be the same across N , and we restrict government policies to always bail out the third bank in distress. More generally, incentives to delay depend on the sequence of Bayesian updates in response to observing takeovers. Let $p'_{(N,N-k)|k,to}$ be the posterior probability that $N - k$ banks are in distress conditional on observing k takeovers (well defined for $2k < N$). Suppose that for two different numbers of banks in the economy, $M > N$, $p'_{(M,M-k)|k,to} = p'_{(N,N-k)|k,to}$, whenever both are well defined. In this case, giving the government more chances to introduce the bailout always has an effect of relaxing the delayed bailout condition. It is a sum of nonnegative numbers for both M and N , and these numbers are the same in both cases up to the point where you cannot delay further with N . However, in the M case, it has at least one more nonnegative term, just like in the comparison between $N = 3$ and $N = 4$. Since the government will have at least weakly more such chances for M rather than N , the left hand side of the analog of (28) will be weakly larger. ■

A.4 Proof of Lemma 5

The probability of an aggregate shock after observing a first bank in distress are

$$p(N) \equiv p_{(N,N|1)} = \frac{(1 - \alpha) + \alpha(1 - \lambda)^N}{1 - \alpha\lambda}$$

The social gains from bailing out the first bank in distress is

$$p(N)Nx + (1 - p(N)) \sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1 - p(N))} dx$$

while the social gains from not bailing out the first bank in distress but bailing out the second bank in distress are

$$p(N)(N - 1)x + (1 - p(N)) \sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1 - p(N))} (y + (d - 1)x)$$

Since $\sum_{d=1}^{N-1} \frac{p_{(N,d)|1}}{(1 - p(N))} = 1$, the condition to delay intervention is

$$p(N) < 1 - \frac{x}{y}$$

While the cutoff for delay does not change, the updated probability makes it more difficult to bail out the first bank for a larger number of firms, then triggering strategic restraints more likely for larger firms. ■