Equilibrium foreign currency mortgages

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Abstract

This paper proposes a novel explanation for why foreign currency denominated loans to households have become so popular in some emerging economies. Our argument is based on what we call the debt limit channel, which arises when multi-period contracts are offered to financially constrained borrowers against collateral that is established on newly acquired assets. Whenever the difference between domestic and foreign interest rates is positive, this effect biases borrowers’ choices towards foreign currency, even if the exchange rate is known to depreciate as implied by the interest parity condition. We demonstrate in a simple macroeconomic framework that the debt limit channel is quantitatively important and can result in dollarization of debt also when borrowing in foreign currency is risky. We next use a small open economy DSGE model and show that, if first-order effects related to the debt limit channel are neutralized by appropriate adjustment in debt contracts, the equilibrium share of foreign currency loans is small.

JEL: D58, E32, E44, F41, G11, G21

Keywords: foreign currency loans, mortgages, portfolio choice, general equilibrium models.
1 Introduction

Despite ongoing development of local financial markets and progress in inflation control achieved during the last decades, foreign currency lending remains an important feature of the financial structure in many countries, raising concerns among policy makers. According to the Financial Soundness Indicators published by the IMF, the (unweighted) average share of foreign currency and foreign-currency-linked borrowing in total gross outstanding loans amounted in 2014 to nearly 30% in the 66 surveyed countries (see Table 1). While this proportion is non-negligible even among developed euro area economies, it is particularly high in Central and Eastern Europe (CEE), where it comes close to a half. As Table 2 reveals, the currency substitution in this region is not restricted to the sector of non-financial firms, but also prevalent among households, and in particular related to financing house purchases. While the euro adoption by Slovenia, Slovakia and the three Baltic states effectively solved this problem in these economies, it took strong actions by the government or financial supervision in other countries to curb lending in foreign currency to households.\footnote{See Rosenberg and Tirpak (2008) for early evidence of regulation aimed at restricting foreign currency lending in CEE countries. More recently, some tougher measures have been taken. In 2013, the financial supervision authority in Poland issued a recommendation to banks against offering foreign currency mortgage loans to households that do not earn in that currency, which effectively shut down new originations of this type of lending. In Hungary, banks were forced to convert foreign currency housing loans to domestic currency in 2014. In 2015, a law was passed in Croatia that facilitated the conversion of loans denominated in Swiss francs into loans denominated in euro.}

A number of papers have tried to rationalize the observed widespread and persistent dollarization of lending. Most of this literature focuses on the supply side factors related to the functioning of international financial markets and banks.\footnote{In the context of international debt, the literature has stressed the so-called original sin, i.e. the inability of emerging economies to borrow internationally in its own currency, see e.g. Terrones and Catao (2000), Calvo (2001), Ize and Yeyati (2003) or Eichengreen et al. (2004). Several demand-side explanations have also been offered, usually focusing on borrowing by firms, see e.g. Goswami and Shrikhande (2001), Jeanne (2003) or Ranciere et al. (2003).} While all these channels might help understand the currency composition of loans to firms, their relevance for borrowing by households is limited. Household debt is predominantly domestic and not international, which means that the original sin argument does not apply. Based on the bank-level survey data, Brown and Haas (2012) argue that foreign currency lending to households in emerging Europe has not been driven by foreign banks due to their easier access to foreign wholesale funding, which suggests that supply-side explanations might not be relevant in this context.
If anything, lending in foreign currency should be rather less attractive to banks because of higher credit risk. However, the demand-side rationalizations existing in the literature are also not very appealing in this context. In particular, households usually earn in domestic currency and hence natural hedging cannot be the major motive in choosing the foreign currency. As a result, popular explanations for dollarization of household debt are based on some irrational behavior or speculative motives, like those created by the empirically observed violation of the uncovered interest rate parity (UIP) or implicit government bailout guarantees in case of severe exchange rate depreciation (Ranciere et al., 2010).

This paper offers an alternative and novel explanation for why foreign currency loans, and foreign currency mortgages in particular, have become so popular in some countries whenever banks made them available at non-discriminatory terms, as it was the case in the CEE region. At the heart of our argument is what we call the debt limit channel, which, whenever the exchange rate is expected to depreciate, allows credit constrained agents to backload the real payments on their loans (i.e. effectively hold more debt) if they choose to borrow in foreign currency. This effect arises only if loans are multi-period and collateralized on newly purchased assets. Such financial arrangements resemble those characteristic for new mortgage originations, and especially in the CEE countries, where multiple mortgages on a single property are very rare.

The debt limit channel operates in a perfect foresight environment and also affects choices under uncertainty. To convey the intuition of how it works in the former case, we use the following simple deterministic example. Consider a financially constrained agent that at period 0 can take a 2-period adjustable-rate loan denominated either in local or foreign currency. The amount that can be borrowed is independent of denomination and equal to 100 units of either domestic or foreign currency, assuming that the nominal exchange rate at time 0 equals unity. Loans are repaid in equal principal payments of 50 units of either local or foreign currency, depending on the chosen denomination. The real interest rate is constant and normalized to zero. The nominal interest rate charged on outstanding debt in the domestic currency is 1% for period 1 and 0% for period 2, while in the case of a foreign currency loan it equals zero in both periods. Financial markets, at which the agent cannot directly participate, ensure no arbitrage between one-period holdings of either of the two

\[ \text{In the CEE countries, vast majority of housing loans are adjustable rate mortgages.} \]
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Currencies, which implies that the nominal exchange rate depreciates by 1% in period 1 and then remains constant. With these assumptions, the cash flows related to loan repayment, expressed in local currency units, are $50 + 0.01 \times 100 = 51$ in period 1 and $50 + 0 \times 50 = 50$ in period 2 for domestic currency borrowing, and $50 \times 1.01 = 50.5$ in both period 1 and 2 for foreign currency borrowing. Note that the present discounted value of both streams of payments is the same and equal to $101/1.01 = 100$, so what makes them distinct is their different distribution over time: exchange rate depreciation effectively backloads real loan payments.\(^4\) If our agent is impatient in a way that makes her credit constrained, she will strictly prefer to borrow in foreign currency as it effectively allows to transfer some of the financial burden into the future.

Another way of explaining the essence of the debt limit channel is by noting that a multi-period adjustable rate loan is equivalent to a sequence of one-period loans with a special nominal rollover commitment. This commitment is different for local and foreign currency borrowing because exchange rate movements affect the local currency value of the latter. As a result, exchange rate depreciation acts as if the agent has a higher debt limit for the debt rollover if she borrows in foreign currency. This can be also seen in our illustrative example, in which outstanding debt at the end of period 1, expressed in local currency units, is equal to 50 in the case of local currency borrowing, and 50.5 if borrowing is denominated in the foreign currency.

One of the insights from this simple example is that foreign and domestic currency loans granted at apparently equal terms (loan-to-value ratio, repayment schedule) are not perfect substitutes to relatively impatient (and hence credit constrained) agents, even if there is no risk and the interest rate parity holds exactly, i.e. the exchange rate depreciates whenever there is a positive difference between domestic and foreign interest rates. Hence, our argument does not rely on some speculative behavior of (usually considered risk-averse) households, like betting on an appreciation of the local currency.

Our analysis implies that, for the debt limit channel to bias household mortgage choices towards foreign currency, the following conditions must hold: (i) the difference between domestic and foreign interest rates is positive, (ii) borrowers are relatively impatient, (iii)

\(^4\)In this respect, the debt limit channel has some resemblance to the tilt effect of inflation studied by Modigliani (1974).
mortgage debt is long-term, (iv) collateral constraint is established on newly purchased assets. We argue that the available cross-sectional evidence indicates that these conditions fit very well to the mortgage markets in the CEE region. First of all, as can be seen from Table 2, the only two countries where foreign currency loans are virtually non-existent are the Czech Republic and Slovakia (the latter observed in the year prior to euro adoption), which are also the only economies for which the interest rate differential is close to zero. Second, according to empirical evidence presented in Fidrmuc et al. (2013), foreign currency loans in the region were chosen mainly by young households, who are found to be relatively impatient (Read and Read, 2004). From a macroeconomic perspective, Dohmen et al. (2016) find that agents in the CEE economies are less patient than in more developed countries, whose currencies dominate in the world financial markets. Turning to the last two conditions, it is clear that mortgages are usually issued at longer maturities than other loans to households. It also has to be noted that home equity lines and multiple mortgages on a single property are very rare in the CEE region, and hence mortgage loans are effectively collateralized only when a property is purchased. Therefore, according to our analysis, these last two features of mortgages may help explain why foreign currency became such a popular choice for this type of contracts, but not necessarily for consumer credit, see Table 2.

In this paper we formalize the simple example and discussion offered above by embedding it into a simple general equilibrium model where agents can borrow from abroad either in domestic or foreign currency. This allows us to study how the debt limit channel operates under risk and how it interacts with what may be called the standard balance sheet channel. According to the latter, riskiness of foreign currency borrowing increases with the correlation between exchange rate depreciation and marginal utility. We show that the risk considerations of the debt limit channel lead to preference for borrowing in foreign currency if the exchange rate depreciation is positively correlated with tightness of the borrowing constraint as it usually implies an increase in the effective debt limit exactly when it is needed most. We confirm that the debt limit channel can be quantitatively important, even under specific risk structure that emphasizes the balance sheet effect, which makes foreign currency borrowing

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5 Unfortunately, time series evidence on the share of foreign currency loans is not very useful to validate our theory. This is because this type of contracts quickly dominated new mortgage originations in the selected CEE countries soon after becoming available, and then were under heavy influence of regulatory measures rather than shaped by other macroeconomic factors. See footnote 1.
very risky.

We next show how the bias towards foreign currency can be eliminated by appropriate adjustment in the loan contracts so that agents are indifferent to currency denomination in the certainty equivalence case. To provide quantitative assessment of equilibrium currency composition of household debt after such an adjustment, we use a medium-sized small open economy DSGE model, augmented with a housing sector and collateral constraints as in Iacoviello (2005), and with multi-period mortgages similar to Garriga et al. (2015). The model allows us to study quantitatively how mortgage dollarization depends on the economic structure, including the types of shocks hitting the economy. When we calibrate our model for Poland, we find that for domestic monetary policy shocks, only foreign currency loans are offered in equilibrium. However, if one considers productivity or risk premium shocks only, the latter defined as a disturbance to the uncovered interest parity condition, the equilibrium dollarization is close to zero. These findings are consistent with the empirical literature, which usually suggests that dollarization is positively related to domestic monetary volatility and negatively related to the exchange rate volatility, see e.g. Barajas and Morales (2003), Luca and Petrova (2008), Cuaresma et al. (2011) or Fidrmuc et al. (2013). If we consider all shocks together, including those coming from abroad, the share of foreign currency loans in equilibrium is positive but small. Finally, our model implies that the equilibrium dollarization of mortgages is increasing in the volume of openness to international trade, inflow of remittances and volume of external debt.

From a methodological perspective, this paper is related to several other works. Korinek (2011) uses a general equilibrium small open economy model to analyze currency composition of borrowing by emerging market economies, but his focus is on international rather than domestic debt, with contracts lasting only one period. While solving for the equilibrium currency composition of household debt under uncertainty in our DSGE model, we use the second-order approximation to the optimality conditions for portfolio choice as in Tille and van Wincoop (2010), Devereux and Sutherland (2011) or Evans and Hnatkovska (2012). Our contribution to this literature is to provide insights on portfolio choices made by financially constrained agents, which makes it similar to Devereux and Yetman (2010) and Dedola and Lombardo (2012), but with a special attention to the debt limit channel. Finally, Brzoza-Brzezina et al. (2017) use a similar DSGE model to that considered in this paper and
examine the effect of foreign currency borrowing on monetary and macroprudential policy transmission, but assume that contracts last only one period and treat the composition of household debt as exogenous.

The rest of this paper is organized as follows. Section two presents how the debt limit channel arises in a simple general equilibrium setup. Section three introduces a more elaborate DSGE framework with housing and collateral constraints. In Section four we present the perturbation-based solution to the portfolio problem under uncertainty, focusing on how the debt limit channel modifies the outcomes. Section five discusses calibration of the DSGE model that is subsequently used in Section six to show how composition of shocks and economic structure affect the equilibrium debt structure. Section seven concludes.
2 Simple model with foreign borrowing

To demonstrate the working of the debt limit channel in a more formal way, we first embed it into a very simple general equilibrium model. In this model, the only way in which nominal exchange rate movements affect the economy is through their impact on the balance sheets of agents holding long-term loans. This will allow us to show that the debt limit channel can be quantitatively important and bias choices of financially constrained agents towards foreign currency debt even if it is much more risky than that denominated in domestic currency. We will also discuss how appropriate adjustment in the contracts can eliminate this bias in the certainty equivalence case.

2.1 Environment

Let us consider a simple small open endowment economy with nominal long-term collateralized debt and fully flexible prices.

A representative agent (household) maximizes the expected value of her discounted period utility flows

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}$$

where $c_t$ is consumption, $u$ is a continuously differentiable, strictly increasing and concave function, and $0 < \beta < 1$. We assume that $\beta r^* < 1$, with $r^*$ denoting (gross) world real interest rate so that agents in the home economy are impatient relative to the rest of the world.

Every period each household receives a constant real endowment $y$ and has access to long-term debt instruments denominated either in domestic or foreign currency. This endowment can be freely traded so that $P_t = S_t P_t^*$, where $P_t$ and $P_t^*$ are the home and foreign price levels, and $S_t$ is the nominal exchange rate expressed as the home currency price of one unit of foreign currency. Following Woodford (2001), we model a long-term debt contract as a perpetuity, with period principal payments equal to a constant fraction $0 < \delta \leq 1$ of outstanding debt and variable interest cost. The representative household’s budget constraint

\[This means that $\delta$, and hence effectively debt maturity, is treated as a (constant) contractual constraint rather than a choice parameter. Otherwise, since households are impatient, they would choose the lowest possible $\delta$. The constant value of this parameter can be motivated by the fact that foreign currency mortgage]
can be written as

\[
P_t c_t + (R_{t-1} - 1 + \delta)L_{H,t-1} + (R^*_t - 1 + \delta)S_t L_{F,t-1} \leq P_t y + L^\text{new}_{H,t} + S_t L^\text{new}_{F,t} \tag{2}\]

where \( R_t \) and \( R^*_t \) denote the (gross) one-period nominal rates of interest on, respectively, domestic and foreign currency debt, \( L^\text{new}_{H,t} \) is new borrowing in domestic currency (net of pre-payment of past loans), \( L^\text{new}_{F,t} \) is net new borrowing in foreign currency, and debt accumulation is given by

\[
L_{H,t} = L^\text{new}_{H,t} + (1 - \delta)L_{H,t-1} \tag{3}
\]

\[
L_{F,t} = L^\text{new}_{F,t} + (1 - \delta)L_{F,t-1} \tag{4}
\]

We restrict \( L_{H,t} \) and \( L_{F,t} \) to be non-negative, but do not impose such restrictions on \( L^\text{new}_{H,t} \) and \( L^\text{new}_{F,t} \). This implies that each type of loans can be prepaid at no additional cost, but cannot turn into a deposit. Note that, since interest payments depend on the current rates, the budget constraint (2) can be rewritten using (3) and (4) in a compact form

\[
P_t c_t + R_{t-1} L_{H,t-1} + R^*_t S_t L_{F,t-1} \leq P_t y + L_{H,t} + S_t L_{F,t} \tag{5}
\]

so that it does not depend on \( \delta \) and hence is identical to the case of one-period debt.

Additionally, households face the following borrowing limit

\[
L^\text{new}_{H,t} + S_t L^\text{new}_{F,t} \leq m P_t \tag{6}
\]

where \( m \geq 0 \). This restriction can be interpreted as a collateral constraint, with the real value of assets that can be used to secure new lending normalized to unity and \( m \) denoting the loan-to-value (LTV) ratio.

The nominal interest rates \( R_t \) and \( R^*_t \) are set by the domestic and foreign monetary contracts in CEE countries, at least during the period of their expansion and until the regulators stepped in, were granted at very similar terms as those denominated in domestic currency.
Simple model with foreign borrowing

authorities, respectively, according to the simple feedback rules

\[ R_t = r^* \bar{\pi} \left( \frac{\pi_t}{\bar{\pi}_t} \right)^\nu \]  

(7)

\[ R^*_t = r^* \bar{\pi}^* \left( \frac{\pi^*_t}{\bar{\pi}^*_t} \right)^\nu \]  

(8)

where \( \pi_t = \frac{P_t}{P_{t-1}} \) and \( \pi^*_t = \frac{P^*_t}{P^*_{t-1}} \), \( \bar{\pi}_t \) and \( \bar{\pi}^*_t \) are the exogenous inflation targets at home and abroad, and variables without time subscripts indicate the steady state values. We assume that \( \nu > 1 \) so that the price levels in both regions are determinate, see Woodford (2003).

We also assume that financial markets, in which households do not participate, ensure no-arbitrage pricing. Then we have the Fisher equation

\[ \frac{R_t}{\mathbb{E}_t \{ \pi_{t+1} \}} = \frac{R^*_t}{\mathbb{E}_t \{ \pi^*_{t+1} \}} = r^* \]  

(9)

that postulates equalization of the (ex ante) real interest rates in both regions.

An equilibrium in this economy is a sequence \( \{c_t, L_{H,t}, L_{F,t}, L_{H,t}^{\text{new}}, L_{F,t}^{\text{new}}, R_t, R^*_t, P_t, P^*_t, S_t\}_{t=1}^{\infty} \), for given sequence \( \{\bar{\pi}_t, \bar{\pi}^*_t\}_{t=1}^{\infty} \), initial debt \( L_{H,0} \) and \( L_{F,0} \), and initial price levels \( P_0 \) and \( P^*_0 \), such that households maximize their utility (1) subject to constraints (2)-(4) and (6), the monetary authorities follow their feedback rules (7) and (8), the Fisher relationship (9) holds, \( P_t = S_t P^*_t \) for every \( t \), and all markets clear.

2.2 Debt limit channel under certainty

Let us assume for a moment that all future inflation target shocks are fully anticipated by agents. Then not only ex ante, but also ex post real interest rates are the same in both regions and equal to \( r^* \). The budget constraint (5) simplifies to

\[ c_t + r^* l_{t-1} = y + l_t \]  

(10)

where \( l_t = \frac{L_{H,t}}{P_t} + \frac{L_{F,t}}{P^*_t} = l_{H,t} + l_{F,t} \) is total real debt. The collateral constraint in real terms can be written as

\[ l_t \leq m + (1-\delta) \left( \frac{l_{H,t-1}}{\bar{\pi}_t} + \frac{l_{F,t-1}}{\bar{\pi}^*_t} \right) \]  

(11)
It is clear from this equation that past debt composition matters for the level of current debt (and consequently for current consumption) unless loans are one period ($\delta = 1$), even though the real cost of foreign and domestic currency borrowing is exactly the same.

To see how the household’s optimal choice of currency denomination depends on expected exchange rate movements, let us rewrite the collateral constraint (11) as

$$l_t \leq m + \frac{1 - \delta}{\pi_t} l_{t-1} \left( 1 + \gamma_t \frac{l_{F, t-1}}{l_{t-1}} \right)$$

(12)

where $\gamma_t = \frac{S_t - S_{t-1}}{S_t} = \frac{S_t}{S_{t-1}} - 1$ is the rate of change in the nominal exchange rate. Consider the empirically relevant case $R_t > R_t^*$ for all $t$ (nominal interest rates in emerging economies are usually higher than in developed countries), which implies that $\gamma_t > 0$ for all $t$. Then, the higher is the share of foreign currency debt taken in the past, the larger is the total amount of debt that households can hold today. Since households are financially constrained, their choice will be to borrow only in foreign currency. As discussed in the simple example from the introduction, the reason is that if the nominal exchange rate is expected to depreciate, borrowing in foreign currency increases the effective debt limit for given value of $m$.

The working of the debt limit channel can be also intuitively explained as follows. Note that a nominal multi-period adjustable rate loan can be thought of as a sequence of one-period loans, but with a special nominal rollover commitment that is different for local and foreign currency borrowing because exchange rate movements affect the local currency value of the latter. For one unit worth of local currency initially borrowed at time $t = 0$, these rollover commitments are $1 - \delta, (1 - \delta)^2, (1 - \delta)^3, \ldots$ for a domestic currency loan, and $(1 - \delta)\frac{S_i}{S_0}, (1 - \delta)^2\frac{S_i}{S_0}, (1 - \delta)^3\frac{S_i}{S_0}, \ldots$ if a loan is taken in the foreign currency. No-arbitrage pricing implies that the interest cost per unit borrowed at every refinancing stage is the same for domestic and foreign currency loans. However, if the exchange rate depreciates after the loan is taken so that $S_t > S_0$ for $i > 0$, the rollovers are higher for foreign currency loans. This matters because households are financially constrained and hence, in contrast to more patient financial markets, attach a positive value to an additional unit of debt.\(^7\)

\(^7\)Note that the debt limit channel described above is independent of the repayment plan, as long as it is the same for local and foreign currency loans. For example, the latter will be strictly preferred by credit constrained borrowers if the exchange rate is expected to depreciate and repayment can be described by any sequence of positive principal payments $\{\delta_{i+1}\}_{i=1}^n$, where $n$ is loan maturity. Naturally, whenever $R_t < R_t^*$, the debt limit channel can work in a way that favors domestic rather than foreign currency borrowing.
Simple model with foreign borrowing

As we already mentioned in the introduction, the debt limit channel does not arise under all types of long-term contracts. In addition to multi-periodicity, the key assumption is that, like in equation (6), the collateral constraint restricts new borrowing rather than total debt. Hence, as in Garriga et al. (2015), our modeling of financial arrangements is valid for new mortgages (i.e. loans for house purchase), and perhaps also for credit-financed purchase of fixed assets by firms, so that the collateral is effectively established on newly acquired assets. The framework does not necessarily fit other types of lending, including those resembling home equity lines of credit as considered in a DSGE framework by Brzoza-Brzezina et al. (2014). However, home equity lines and multiple mortgages on a single property are very rare in the CEE region, hence our modeling choices adequately describe the mortgage market structure in the countries we focus on.

2.3 Debt limit channel under risk

We now explain how the bias towards foreign currency can be modified by the presence of risk. To this end, it is instructive to look at the following two Euler equations associated with, respectively, local and foreign currency borrowing\(^8\)

\[
\begin{align*}
    u_t' - \Theta_t &= \beta E_t \left\{ u_{t+1}' \frac{R_t}{\pi_{t+1}} \right\} - \beta (1 - \delta) E_t \left\{ \frac{\Theta_{t+1}}{\pi_{t+1}} \right\} \\
    u_t' - \Theta_t &= \beta E_t \left\{ u_{t+1}' \frac{R^*_t}{\pi_{t+1}} \right\} - \beta (1 - \delta) E_t \left\{ \frac{\Theta_{t+1}}{\pi_{t+1}} \right\}
\end{align*}
\]

(13) \hspace{1cm} (14)

where \(u_t'\) is the marginal utility and \(\Theta_t\) denotes the Lagrange multiplier on the collateral constraint (expressed in real terms). The left-hand sides of each of these equilibrium conditions are the same for both types of loans and can be interpreted as the current period net benefit of one additional unit of borrowing. The right-hand sides describe the expected net cost, and each is a difference between two components. The first one is related to the financial cost of repaying a loan and we will call it the balance sheet effect. The second term corresponds to the debt limit channel and can be interpreted as the value of the rollover commitment that the loan contract guarantees. This effect shows up only if loans are multi-period (\(\delta < 1\))

\(^8\)In these first-order conditions we omit the terms related to the non-negativity constraint on \(L_{H,t}\) and \(L_{F,t}\).
and the gain it describes is strictly positive as long as the collateral constraint is binding, i.e. $\Theta_t > 0$.

It is straightforward to confirm that, under certainty equivalence (or up to first order of approximation), agents will choose to borrow in foreign currency if the nominal exchange rate is expected to depreciate. This is because, since equation (9) holds, the balance sheet effect does not bias households’ choices towards any of the currency, while the gain associated with the debt limit channel is higher for foreign currency borrowing as expected exchange rate depreciation implies $E_t \{\pi_{t+1}\} > E_t \{\pi^*_{t+1}\}$.

If uncertainty is taken into account, the balance effect channel is no longer neutral and favors borrowing in the currency that is financially less risky, i.e. the one that implies lower covariance of its associated rate of return with borrower’s marginal utility. As regards the risk implications of the debt limit channel, it makes the foreign (local) currency more attractive if the comovement between exchange rate depreciation and tightness of the collateral constraint is positive (negative). The intuition is analogous to that developed before: unexpected depreciation of the domestic currency helps increase the effective level of debt, which is especially valued when the borrowing constraint is relatively tight.

### 2.4 Is the debt limit channel quantitatively important?

Since in many countries exchange rates tend to be volatile, borrowing in foreign currency is financially very risky. Hence, one might argue that the balance sheet channel is likely to dwarf the working of the debt limit channel, making the bias created by the latter empirically irrelevant.

To show that this is not the case, we offer the following quantitative evaluation based on the simple model introduced above. Since we want to emphasize the implications of exchange rate risk, we assume that loans denominated in the domestic currency are riskless. This is achieved by shutting down any fluctuations in domestic inflation target shocks so that $\tilde{\pi}_t = \bar{\pi}$, in consequence of which $\pi_t = \bar{\pi}$ and $R_t = R = r^* \bar{\pi}$. The only source of uncertainty in the economy are foreign inflation shocks $\tilde{\pi}^*_t$, which we assume to be i.i.d., and hence from equations (8) and (9) we have $\pi^*_t = \tilde{\pi}^*_t$ and $R^*_t = R^* = r^* \tilde{\pi}^*$. As a result, the real return on foreign currency $R^*_t / \tilde{\pi}^*_{t-1}$ is also i.i.d., in line with empirical evidence that we discuss while
Let us assume that our model works at a quarterly frequency and use the following calibration. The size of endowment $y$ is normalized to unity and preferences are logarithmic so that $u(c) = \ln c$. We set $r^* = 1.005$ and $\pi^* = 1.005$, which implies the steady state value of the world real interest rate and inflation of 2% annually. For the debt limit channel to be operational, we need: impatient home agents ($\beta^{-1} > r^*$), multi-period debt ($\delta < 1$) and positive nominal interest rate differential ($R > R^*$). We choose $\beta = 0.99$, in line with what is typically assumed in the context of mortgage debt (Campbell and Hercowitz, 2009). Loan amortization $\delta$ is calibrated at 0.015, which implies (Macaulay) debt duration in our model equal to that of a 25-year adjustable rate loan (typical mortgage contract in Poland). As regards the interest rate differential, we pick three values that represent its variation across CEE countries included in Table 2: 1%, 2.5% and 5% (all annualized). We also consider three values for the standard deviation of the foreign inflation target: 0.01, 0.03 and 0.04. These numbers are representative of the region and roughly consistent with the observed exchange rate volatility in Croatia, Hungary and Poland, respectively. Finally, we set $m = 0.064$, which for the 2.5% interest rate differential and 50% share of foreign currency loans implies the steady state debt-to-output ratio $\frac{l^*}{4y} = 0.7$, coinciding with the average ratio of mortgage loans to yearly GDP in Poland.

We solve the model using value function iteration over discretized state space, see the Appendix for details. The resulting mean shares of foreign currency loans for different model parametrizations are reported in Table 3, where we also include the results for zero risk that can be established theoretically and do not require numerical solution. Clearly, the model is very simple and stylized as it focuses only on the demand side of credit and exchange rate risk so that, when interest rates at home and abroad are the same, households want to hold only debt denominated in the home currency. Despite this, for many empirically relevant combinations of the interest rate differential and exchange rate volatility, the model implies that agents would choose borrowing in the foreign currency as the benefits of holding more debt are not offset by increased fluctuations in their balance sheets. We conclude that the debt limit channel is quantitatively important and warrants a more detailed study that we offer in the next section using a much richer theoretical framework.
2.5 Eliminating bias in choice of currency denomination

One way of interpreting the existence of the debt limit channel is that offering foreign and domestic currency loans at the same terms (i.e. LTV, repayment schedule) is “wrong” as, under non-zero interest rate differential, one of them will be strictly preferred over the other by borrowers, even if these are risk neutral. A natural question then arises: How should the loan contracts be adjusted to become perfect substitutes under certainty equivalence?

Probably the most natural option is to make the cap on new lending \( m \) dependent on the currency of loan denomination. To this end, let us replace the borrowing constraint (6) by

\[
\frac{1}{m_H} L_{H,t}^{\text{new}} + \frac{1}{m_F} S_t L_{F,t}^{\text{new}} \leq P_t \tag{15}
\]

Then, as we show in the Appendix, agents are indifferent between borrowing in local and domestic currency in the steady state as long as

\[
\frac{m_F}{m_H} = \frac{\pi^* - \beta(1 - \delta)}{\pi^* - \beta(1 - \delta) \frac{R^*}{R}} \tag{16}
\]

Hence, if there is a positive difference between domestic and foreign interest rate (and the exchange rate is expected to depreciate), we must have \( m_F < m_H \), i.e. more collateral should be required for a unit of foreign currency loans compared to a unit borrowed in local currency. To get an idea on how big this discrimination should be, let us use the calibration from our quantitative exercise described before. Then, for the interest rate differential of 2.5% (average for Poland), \( m_F \) should be around 17% lower than \( m_H \).

An alternative way of eliminating the debt limit channel is to appropriately modify the interest payment schedule. Consider a debt contract, in which the interest cost is due only on the principal repayment, with the remainder added to next period debt. Such a contractual arrangement can be thought of as automatic rollover of some part of interest payments, or a special case of teaser loans. A representative household budget constraint can now be written as

\[
P_t c_t + R_{t-1} \delta L_{H,t-1} + R_{t-1} \delta S_t L_{F,t-1} \leq P_{t} y + L_{H,t}^{\text{new}} + S_t L_{F,t}^{\text{new}} \tag{17}
\]

where

\[
L_{H,t} = L_{H,t}^{\text{new}} + (1 - \delta) R_{t-1} L_{H,t-1} \tag{18}
\]
Simple model with foreign borrowing

\[ L_{F,t} = L_{F,t}^{new} + (1 - \delta) R_t^* L_{F,t-1} \quad (19) \]

Note that by substituting (18) and (19) into (17), we still obtain the same budget constraint as in equation (5). However, as we show in the Appendix, the two first-order conditions associated with the choice of foreign and domestic currency loans are now identical up to first order. As a result, absent risk considerations, agents are indifferent between these two types of mortgages.
Chapter 3

3 Full model with mortgages

The key insight from the simple model was that the debt limit channel is quantitatively important and can make financially constrained agents strictly prefer foreign currency loans over those denominated in the domestic currency whenever both are granted at similar terms, even if the former are much more risky. This model is arguably very stylized. In particular, it assumes that the exchange rate affects only the balance sheets of households (and not their incomes), and that its movements are unrelated to what happens in the economy. Such a way of modeling makes foreign currency loans extremely risky and ignores their potential hedging properties. Hence, one can argue that after accounting for these additional features, the debt limit channel is even more likely to prevail.

We have also seen how appropriate adjustment in the two debt contracts can eliminate the bias towards one of the currencies, at least in the certainty equivalence case. Then, according to our simple model, agents will strictly prefer domestic over foreign currency loans when risk is taken into account. However, this does not imply that agents will choose to take only domestic currency loans once one considers a more realistic economic environment. Therefore, to offer quantitative insights on the equilibrium choice of currency denomination of debt under uncertainty, we need to move to a richer framework.

In this section we introduce a fully-fledged DSGE model that modifies and extends the simple setup described above in several directions. First of all, for reasons explained before, we concentrate on one specific type of debt, i.e. housing mortgages, and assume that contracts are formulated such that under certainty equivalence borrowers are indifferent between domestic and foreign currency debt.\(^9\) While we keep the open economy setup to allow for exchange rate movements and the impact of foreign disturbances, all mortgage contracts are now between agents populating the home economy, and hence our focus is on the currency composition of domestic rather than international debt.\(^10\) We also move from endowment to production economy and model the borrowing limit explicitly as a collateral constraint, with house prices affecting its tightness. By introducing home bias in preferences, we allow for endogenous movements in the real exchange rate, and by assuming local currency pricing

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\(^9\) As we discuss later, this adjustment also allows us to solve the model with perturbation techniques, which makes it possible to consider a much richer theoretical structure.

\(^10\) As argued by Jeanne (2003), the currency composition of international debt mainly reflects the practices in the international financial centers, which are exogenous from a small economy’s perspective.
Full model with mortgages

we introduce its incomplete pass through.

Since our aim is to eventually perform a meaningful quantitative exercise, we incorporate several frictions that have been found important in the DSGE literature, such as habits in consumption, investment adjustment costs, and price rigidity. Finally, we consider several types of stochastic shocks that affect productivity, monetary policy, foreign variables and, most importantly for the exchange rate dynamics, the risk premium on international financial markets.

In this section we describe the problems faced by agents populating our model economy. A full list of equations making up the model can be found in the Appendix. Throughout, the notation is the same as in section 2 so only new variables, functions and parameters are defined.

3.1 Households

Our model economy is populated by two types of households, differing in the degree to which they discount future utility flows. Based on their equilibrium behavior, we name the relatively patient agents savers, while the other type will be called borrowers. We denote the measure of borrowers by $\omega$ and normalize the mass of households to unity so that the measure of savers is $1 - \omega$.

Each agent of type $i \in \{s, b\}$ maximizes her expected lifetime utility over consumption $c_{i,t}$, housing $\chi_{i,t}$ and labor supply $n_{i,t}$

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_t^i u(c_{i,t}, \chi_{i,t}, n_{i,t}) \right\}$$  

(20)

where $\beta_s > \beta_b$.

Since savers are relatively patient, they own all capital stock $k_t$ and firms in the economy so that they earn dividends $\Pi_t$. They also have access to domestically and internationally traded one-period bonds $D_t$ and $D_t^*$, the latter denominated in the foreign currency. Each period savers offer to borrowers adjustable rate mortgage contracts either in local or foreign currency, specified as in section 2.5. A sequence of their budget constraints can be written in a compact form as
\[
P_t c_{s,t} + P_{\chi,t}[\chi_{s,t} - (1 - \delta_\chi)\chi_{s,t-1}] + P_{k,t}[k_t - (1 - \delta_k)k_{t-1}] + D_t + S_tD^*_t + L_{H,t} + S_tL_{F,t} + T_t
\leq W_t m_{s,t} + R_{k,t}k_{t-1} + R_{H,t-1}L_{H,t-1} + S_t R_{F,t-1}L_{F,t-1} + R_{t-1}D_{t-1} + S_t \rho_{t-1}R^*_tD^*_{t-1} + \Pi_t
\]

(21)

where \(\delta_k\) and \(\delta_\chi\) are the depreciation rates for physical capital and housing, respectively, \(P_{k,t}\) is the price of capital, \(W_t\) is the nominal wage, \(T_t\) denotes lump sum taxes, \(R_{H,t}\) and \(R_{F,t}\) are interest rates applied to domestic and foreign currency mortgages, respectively, while

\[
\rho_t = \left(1 + \frac{D^*_t S_t}{P_t y_t}\right) \varepsilon_{\rho,t}
\]

(22)

is the risk premium, with \(\varrho > 0\) to ensure stationarity (see Schmitt-Grohe and Uribe, 2003) and \(\varepsilon_{\rho,t}\) denoting a risk premium shock, which we introduce to allow for exogenous deviations from the UIP. Note that, since savers are not constrained in taking a position in bonds, in equilibrium we must have \(R_{H,t} = R_t\) and \(R_{F,t} = \rho_tR^*_t\).

Mortgages are the only financial contracts available for borrowers so their period budget constraint is

\[
P_t c_{b,t} + P_{\chi,t}[\chi_{b,t} - (1 - \delta_\chi)\chi_{b,t-1}] + R_{H,t-1}L_{H,t-1} + S_t R_{F,t-1}L_{F,t-1}
\leq W_t m_{b,t} + L_{H,t} + S_t L_{F,t}
\]

(23)

where we also assume for simplicity that all taxes are paid by savers. Additionally, borrowers face a collateral constraint

\[
L^\text{new}_{H,t} + S_t L^\text{new}_{F,t} \leq m\mathbb{E}_t \{P_{\chi,t+1}\chi^\text{new}_{b,t}\}
\]

(24)

so that new mortgage originations \(L^\text{new}_{H,t}\) and \(L^\text{new}_{F,t}\), related to total mortgage debt \(L_{H,t}\) and \(L_{F,t}\) by (18) and (19), are secured by new housing purchases \(\chi^\text{new}_{b,t}\), which augment undepreciated housing from the previous period to constitute current housing stock according to the following law of motion

\[
\chi_{b,t} = \chi^\text{new}_{b,t} + (1 - \delta_\chi)\chi_{b,t-1}
\]

(25)
3.2 Firms

Several types of firms operate in our small open economy. Monopolistically competitive intermediate goods producers, of unit measure and indexed by $\nu$, use the following production function

$$y_{H,t}(\nu) + y^*_{H,t}(\nu) = \varepsilon_{z,t}k_{t-1}(\nu)^\alpha[\omega n_{b,t}(\nu) + (1 - \omega)n_{s,t}(\nu)]^{1-\alpha}$$

(26)

to supply domestic and foreign markets, where $\varepsilon_{z,t}$ is exogenous productivity. Their profit maximization is subject to the demand sequences derived from the following Dixit-Stiglitz aggregators

$$y_{H,t} = \left[\int_0^1 y_{H,t}(\nu)^{\frac{1}{\mu}} d\nu\right]^{\mu}$$

(27)

$$y^*_{H,t} = \left[\int_0^1 y^*_{H,t}(\nu)^{\frac{1}{\mu}} d\nu\right]^{\mu}$$

(28)

where $\mu > 1$. Additionally, intermediate goods firms face a Calvo-like price rigidity so that with exogenous probability $1 - \theta_H$ they are allowed to reoptimize their price $P_{H,t}(\nu)$ while supplying the domestic market, and with probability $1 - \theta^*_H$ they can change their price $P^*_{H,t}(\nu)$ charged on foreign customers. Prices of firms that do not reoptimize in the current period are indexed to steady state inflation.

Each of the monopolistically competitive importers indexed by $\iota$ purchase $y_{F,t}(\iota)$ foreign goods at price $S_tP^*_t$ and sells them domestically at price $P_{F,t}(\nu)$. These firms are subject to the demand sequences implied by

$$y_{F,t} = \left[\int_0^1 y_{F,t}(\iota)^{\frac{1}{\mu}} d\iota\right]^{\mu}$$

(29)

and a similar Calvo price rigidity as domestic producers, with probability of reoptimization $1 - \theta_F$.

Capital and housing production is undertaken by perfectly competitive firms owned by patient households. They purchase undepreciated stocks from the previous period and produce according to

$$k_t = (1 - \delta_k)k_{t-1} + \left(1 - \Gamma_k\left(\frac{i_{k,t}}{i_{k,t-1}}\right)\right)i_{k,t}$$

(30)
\[ \chi_t = (1 - \delta_\chi)\chi_{t-1} + \left(1 - \Gamma_\chi \left(\frac{i_{\chi,t}}{i_{\chi,t-1}}\right)\right)i_{\chi,t} \]  

where \(i_{k,t}\) and \(i_{\chi,t}\) are final goods used for capital and housing investment while the adjustment costs functions are parametrized such that \(\Gamma_j(1) = \Gamma'_j(1) = 0\) and \(\Gamma''_j(1) = \kappa_j \geq 0\) for \(j = \{k, \chi\}\).

### 3.3 Government

The fiscal authority finances its expenditures, fixed at \(g\) in real terms, with lump sum taxes levied on savers such that the government budget is balanced every period

\[ P_t g = (1 - \omega)T_t \]  

The monetary authority responds to deviations of inflation from its target (steady state) level according to the feedback rule

\[ \frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\frac{\pi_t}{\pi}\right)^{(1-\gamma_R)\gamma_\pi} e^{\varepsilon_{R,t}} \]  

where \(0 \leq \gamma_R < 1\), \(\gamma_\pi > 1\) and \(\varepsilon_{R,t}\) is a monetary policy shock.

### 3.4 Foreign block

Export demand is given by a standard function used in small open economy models and consistent with imperfect substitutability between domestic and foreign goods

\[ y_{H,t}^* = \left(\frac{p_{H,t}^*}{\phi_y^* y_t^*}\right)^{-\frac{1}{\phi_y^*}} \]  

where \(\phi_y^* > 0\) and \(y_t^*\) is output produced abroad.

All three foreign variables entering the model, i.e. foreign output \(y_t^*\), foreign inflation \(\pi_t^*\) and foreign interest rate \(R_t^*\) are treated as exogenous.
3.5 Market clearing

The model is closed with a standard set of market clearing conditions. We assume that final goods used domestically are a CES aggregate of goods produced at home and imports. Hence we have

\[
\left(1 - \eta\right)^{\frac{1}{\phi}y_{F,t}^\phi + \eta^{\frac{1}{\phi}y_{H,t}^\phi}} = \omega c_b,t + (1 - \omega)c_s,t + i_{k,t} + i_{\chi,t} + g_t
\] (35)

The aggregate resource constraint is

\[
y_{H,t}\Delta_{H,t} + y_{H,t}\Delta_{H,t}^* = \varepsilon_{z,t}k_{t-1}^{\alpha} \left(\omega n_{b,t} + (1 - \omega)n_{s,t}\right)^{1-\alpha} \equiv y_t
\] (36)

where \(y_t\) is output, \(\Delta_{H,t} = \int_0^1 \left(\frac{P_{H,t}(\nu)}{P_{H,t}^*}\right)^{\frac{\mu}{\phi}} d\nu\) and \(\Delta_{H,t}^* = \int_0^1 \left(\frac{P_{H,t}(\nu)}{P_{H,t}^*}\right)^{\frac{\mu}{\phi}} d\nu\) are the measures of price dispersion for domestic production and exports, with \(P_{H,t}\) and \(P_{H,t}^*\) denoting the price indexes consistent with aggregation defined by (27) and (28).

The evolution of net foreign debt can be written as

\[
D_t^* = \Delta_{F,t} P_{F,t} y_{F,t} - P_{H,t}^* y_{H,t}^* + \theta_{t-1} R_{t-1}^* D_{t-1}^* \] (37)

where \(\Delta_{F,t} = \int_0^1 \left(\frac{P_{F,t}(\nu)}{P_{F,t}^*}\right)^{\frac{\mu}{\phi}} d\nu\) is the measure of price dispersion for imports, with \(P_{F,t}\) denoting the price index consistent with aggregation defined by (29).

Finally, housing market clearing implies

\[
\chi_t = \omega \chi_{b,t} + (1 - \omega) \chi_{s,t}
\] (38)

3.6 Functional forms

We assume that household utility takes the following separable form (for \(i \in \{s, b\}\))

\[
u(c_{i,t}, \chi_{i,t}, n_{i,t}) = \log(c_{i,t} - \xi \bar{c}_{i,t-1}) + A_{\chi} \log(\chi_{i,t}) - \frac{n_{i,t}^{1+\sigma_n}}{1+\sigma_n}
\] (39)

where \(\bar{c}_{i,t}\) is aggregate consumption (equal to \(c_{i,t}\) in equilibrium) that households take as given during optimization, \(A_{\chi} > 0, \sigma_n > 0\) and \(\xi \in [0, 1]\) is the degree of external habit formation. For savers we additionally assume that their housing demand is fixed at \(\chi_{s,t} = \chi_s\).
This assumption implies that housing is effectively priced by leveraged households, which is consistent with Geanakoplos (2010), and, since it shuts down trade between savers and borrowers, can also be interpreted as housing market segmentation between the two types of agents.
4 Solving for equilibrium portfolio

The model described in Section 3 is too large to be solved using global methods so we resort to perturbation techniques. In this section we explain how we use them to derive the steady state share of local and foreign currency loans. Our method follows closely Tille and van Wincoop (2010) in that we use the second-order accurate portfolio choice conditions together with the first-order approximation to the remaining equilibrium conditions to calculate the composition of the loan portfolio.

Let us define the one-period gross real rates of return on local and foreign currency from \( t \) to \( t + 1 \)

\[
r_{H,t+1} = \frac{R_t}{\pi_{t+1}}
\]

\[
r_{F,t+1} = \frac{q_{t+1}R_t^*}{q_t\pi_{t+1}}
\]

where \( q_t \equiv \frac{S_tP_t^*}{P_t} \) is the real exchange rate. Since savers are not constrained while taking positions on the domestic and international bond markets or while offering mortgages to borrowers, in equilibrium they must be indifferent between holding local and foreign currency assets, which leads to the UIP condition

\[
E_t \{ u_{s,t+1}(r_{H,t+1} - r_{F,t+1}) \} = 0
\]

(42)

where \( u_{i,t} \equiv (c_{i,t} - \xi c_{i,t-1})^{-1} \) for \( i \in \{s, b\} \) is marginal utility consistent with (39). The portfolio decisions by borrowers can be described by the following two Euler equations for, respectively, local and foreign currency borrowing

\[
u_{b,t} - \Theta_t = \beta_bE_t \{ u_{b,t+1}r_{H,t+1} \} - \beta_b(1 - \delta)E_t \{ \Theta_{t+1}r_{H,t+1} \}
\]

(43)

\[
u_{b,t} - \Theta_t = \beta_bE_t \{ u_{b,t+1}r_{F,t+1} \} - \beta_b(1 - \delta)E_t \{ \Theta_{t+1}r_{F,t+1} \}
\]

(44)

where \( \Theta_t \) denotes the Lagrange multiplier on the collateral constraint (24) written in real terms. These equilibrium conditions are analogous to equations (13) and (14), except that now they incorporate the adjustment of mortgage contracts that eliminates the debt limit

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111This means that the collateral constraint (24) is assumed to hold in equality, i.e. we restrict our attention to sufficiently small deviations from the non-stochastic steady state.
channel under certainty equivalence as explained in Section 2.5.

It is easy to verify that, once one takes into account the UIP condition (42), equations (43) and (44) are equivalent to first (and zero) order, and hence the portfolio problem is not determinate at this level of approximation. Therefore, higher-order expansions have to be used to obtain the equilibrium shares of domestic and foreign currency lending. In the Appendix we show that applying the second-order approximation to equations (42)-(44) results in the following condition

$$
\mathbb{E}_t \left\{ (\hat{r}_{H,t+1} - \hat{r}_{F,t+1}) \left( (1 + s_\Theta) \hat{u}_{b,t+1} - s_\Theta \hat{\Theta}_{t+1} - \hat{u}_{s,t+1} \right) \right\} = 0 \tag{45}
$$

where $s_\Theta = \frac{(1-\delta)(b_s-b_b)}{s_\Theta'}$ and hats over variables denote log-deviations from the non-stochastic steady state. Note that $s_\Theta = 0$ for $\delta = 1$, in which case we obtain the standard portfolio equilibrium condition, stating that excess return on any asset must be conditionally uncorrelated with the ratio of trading agents’ marginal utilities. If $\delta < 1$ so that mortgage contracts are multi-period, $s_\Theta > 0$ and hence what matters on the part of borrowers is their marginal utility corrected for tightness of the collateral constraint. This correction is a second-order manifestation of the debt limit channel under risk that we discussed in Section 2.3. It can be quantitatively important as $s_\Theta = 0.33$ for our benchmark calibration described later.

Note that since equation (45) includes only products, it can be evaluated to second-order accuracy using the first-order accurate solution for the variables entering it. We show in the Appendix that the first-order approximation of our model is affected only by the steady state composition of domestic and foreign currency loans, but not by their individual dynamics, and that the conditional expectations on the right-hand side of equation (45) are time invariant. Therefore, given the model parameters and the stochastic properties of shocks hitting the economy, we can find the steady state portfolio numerically by applying an iterative procedure, in which we start from some arbitrary steady state portfolio composition, and then iterate to find one in which equation (45) is satisfied.
5 Calibration

We calibrate the model to Poland, one of the CEE economies with a high share of mortgages denominated in foreign currency. The unit of time is one quarter. The calibrated structural parameter values are reported in Table 4.

We follow the standard practice and chose most of parameter values from the previous literature or to match selected means observed in the data. The discount factor of savers is calibrated at 0.995 to yield the steady-state annual real interest rate of 2%, while that of borrowers is set to 0.99 so that the relative impatience of this type of agents is similar used by Campbell and Hercowitz (2009). The inverse of the Frisch elasticity of labor supply is set to a standard value of 2. Habits in consumption are calibrated at 0.75, in line with evidence from estimated DSGE models for the Polish economy (see e.g. Brzoza-Brzezina et al., 2016). The weight on housing services in utility is selected to obtain the housing wealth to annual GDP ratio of 1.3. The relative share of borrowers is set to 0.25 in order to match the observed average mortgage debt to GDP ratio in Poland of 0.7 (annualized).

We set the mortgage contract parameters to reflect the properties of a typical housing loan taken in Poland. This gives an LTV ratio of 0.85 and the principal decay factor equal to 0.015. The latter is chosen such that it implies the Macaulay duration of a housing loan in our model’s steady state of 10 years, as for a standard 25-year mortgage.

The depreciation rates of physical capital and housing are calibrated at 0.015 and 0.007, respectively, implying the shares of non-residential and residential investment in GDP of 0.187 and 0.037. As regards the residential investment adjustment parameter, we consider a special case in which it approaches infinity, so that the housing stock is fixed, which is a fairly common assumption in the business cycle literature. For physical capital, we chose the investment adjustment cost curvature of 0.6 to match the volatility of total investment observed in the data. The capital share in production function is set to a conventional value of 0.3. The calibrated steady state markup in the goods market of 1.2 is also standard. As regards the degree of price stickiness, we set the Calvo probabilities for all markets to 0.75, implying the average price duration of one year, in line with the micro data-based estimates for many countries, including Poland.

The elasticity of substitution between domestically produced and foreign goods is cali-
brated at a level of 1.5, which is a typical value used in the international macroeconomics literature (Bodenstein, 2010). The home bias in the basket of final goods is set to 0.7 so that it implies the share of imports in GDP of 0.3, consistently with openness of the Polish economy once one corrects for the import content of exports. The debt elasticity of the risk premium is chosen at a small level of 0.001, just to render the model stationary.

As regards the policy parameters, the steady state share of government spending in output matches its data counterpart of 0.2. The annual steady state inflation rate is assumed to be 2%, i.e. close to the inflation target in Poland. Finally, the parameters of the interest rate feedback rule are estimated using Polish data, implying a substantial degree of smoothing (0.845) and quite aggressive response to deviations of inflation from the target (1.85), both numbers being similar to earlier Taylor rule estimates for Poland.

We calibrate the properties of the stochastic shocks outside of the model. The data used and estimation details can be found in the Appendix. As it is standard in the literature, the log of productivity $\varepsilon_{z,t}$ is assumed to follow an AR(1) process that we fit to the Solow residual series for Poland. The standard deviation of the i.i.d. monetary policy shock $\varepsilon_{R,t}$ is taken from the econometric estimates of the monetary policy rule mentioned before. The foreign block $\{\pi^*, y^*, R^*_t\}$ is modeled as a structural VAR(1). Finally, the log of the risk premium shock $\varepsilon_{\rho,t}$ is assumed to follow an AR(1) process, with parameters estimated using the log-linearized version of the UIP condition (42) and the Kalman filter to sort out the impact of other disturbances, see Appendix for details.\footnote{We have also considered other sources of aggregate uncertainty used in the DSGE literature, including shocks to exogenous spending, investment efficiency and housing preference. While they help to better match some of the moments implied by the model to the data, they turn out to be of little importance for the steady state portfolio.}

Table 5 compares the moments implied by our calibrated model to the data, with a special attention paid to the rates of return $r_{H,t}$ and $r_{F,t}$ as they are the key observable variables for the portfolio problem described in the previous section. While calculating the model-based moments, we treat the share of foreign currency loans as exogenous and assume it to be equal to a half. This value is not necessarily consistent with the second-order accurate solution to the portfolio problem faced by borrowers, but in line with the Polish data. Taking into account that only one structural parameter (investment adjustment cost) but none of the shocks was left free to match any of the moments displayed in the table, the fit is remarkably
good. Admittedly, the model fails to generate sufficient persistence in the real exchange rate and procyclical nominal interest rate. However, and more importantly for our analysis, it matches very well the volatility and autocorrelation of the real rates of return on the local and foreign currency.
6 Simulation results

6.1 Equilibrium currency composition of loans

In this section we discuss the quantitative implications of the model described above for the equilibrium share of foreign currency loans, with a special focus on the working of the debt limit channel under risk. One of the aims is to check whether this type of loans would still be traded if they were offered at terms making them perfect substitutes to domestic currency loans under certainty equivalence.

The first column in Table 6 presents the results by considering one shock at a time, and for the full stochastic model. According to our model, when all shocks are taken into account, we obtain a moderate share of mortgage dollarization of around 15%. This is clearly below the levels observed in those CEE countries for which the interest rate differential is significantly positive (recall Table 2). It hence indicates that the dollarization of housing loans in this region can be largely attributed to the lack of or insufficient adjustments in mortgage contracts offered in the foreign currency for the (first-order accurate) effects of the debt limit channel.

If business cycle fluctuations were driven exclusively by productivity shocks, only local currency loans would be offered in equilibrium. In contrast, there would be no trade in this type of loans in a world with monetary shocks only.\textsuperscript{13} If foreign shocks were the only source of aggregate uncertainty, the equilibrium portfolio of mortgages would consist of both foreign and local currency denominated contracts, with some bias towards the former. For risk premium shocks, which are the main drivers of exchange rate movements, the equilibrium share of foreign currency loans is very small, or just under 5%.

To gain some insight on these results and understand better the role of the debt limit channel, let us take a closer look at the risk premium shock. This shock is of particular interest as it accounts for the majority fluctuations in the difference between the returns on the two currencies and has the biggest impact on the equilibrium portfolio. Figure 1 presents the impulse responses of the terms entering the portfolio formula (45) to a positive

\textsuperscript{13}More specifically, equation (45) is satisfied for the share of foreign currency loans far below zero if all fluctuations are driven by productivity shocks and far above unity for the case of monetary shocks only. Since we do not allow any of the loans to be negative (borrowers cannot use one mortgage to offer another to savers), we report the corner solution.
Simulation results

risk premium shock, for two alternative assumptions regarding the steady state share of foreign currency loans: one consistent with the portfolio solution when risk premium shocks are the only source of aggregate risk (4.9%, dashed lines), the other corresponding to the value observed in the data (50%, solid lines). In the latter case, we treat the share as exogenous, i.e. we do not explain what other sources of risk could result in such an equilibrium value.

Let us concentrate first on the second variant as this tells us in which direction the equilibrium forces push a portfolio that is initially balanced between the two currencies. An unexpected risk premium shock results in depreciation of the exchange rate, and so the real rate of return on foreign currency increases. The weaker exchange rate raises inflation, to which the monetary authority responds by increasing the short-term interest rate. Because of the inertia in the policy rule (33), the nominal rate increases only moderately and hence the real rate of return on domestic currency falls on impact. Overall, excess return on foreign currency goes up so that, ex post, borrowing in local currency turns out to be a better financial decision than borrowing in foreign currency and, symmetrically, offering foreign currency loans pays more than lending in local currency.

From the risk perspective, it is crucial how these payoffs correlate with their valuation by the trading agents. Since exchange rate depreciation boosts the country’s international price competitiveness, it results in a boom that pushes wages up. As a result, borrowers find themselves financially less constrained despite increased cost of servicing foreign currency loans, and their marginal utility goes down. Hence, from the balance sheet perspective, borrowing in the foreign currency provides a good hedge against risk premium shocks. As regards savers, the monetary policy response to inflation implies that their marginal utility increases following a positive risk premium shock. Hence, contracts denominated in foreign currency turn out to be also good insurance for this group of agents as it gives them extra return when they value it most.

Overall, if only the balance sheet effects mattered, the equilibrium forces would push the share of foreign currency loans over 50%. However, with multi-period lending, borrowers care not only about income effects of their portfolio decisions, but also on how they affect the amount of debt that can effectively be held. As discussed before, the existence of this debt limit channel manifests itself in the adjustment to the portfolio valuation that takes also into account tightness of the collateral constraint, as indicated by formula (45). Note that,
due to improvement in the economic conditions, a positive risk premium shock generates a temporary relaxation of the collateral constraint. Hence, the exchange rate depreciation that helps effectively increase the debt limit comes at a time when it is not so much needed, meaning that, from this perspective, lending in the foreign currency is not a good hedge. As can be seen from Figure 1, the net effect of the balance sheet and debt limit channels, which we show under the label “adjusted marginal utility of borrowers”, is such that borrowers prefer to bias their portfolio towards domestic currency debt.

The equilibrium effect combines savers’ preference for foreign currency mortgages and the opposite force on the part of borrowers. Under our calibration, this results in their share of 4.9%. For this value, and in line with formula (45), excess return on either currency is uncorrelated with the ratio of marginal utilities of savers and borrowers, the latter adjusted for the debt limit effect.

6.2 Structural determinants of foreign currency loans

One of the advantages of having a structural model is that it can be used to analyze how the equilibrium share of foreign currency loans depends on selected macroeconomic characteristics, and confront these predictions with existing empirical evidence. This exercise can be also treated as a sensitivity analysis that aims to check if our results are robust to alternative modeling choices or calibration. Note first that the model’s implications that follow from the shock decomposition presented in Table 6 are consistent with the empirical literature on determinants of dollarization. In particular, our model predicts that the equilibrium share of foreign currency loans is declining in the volatility of risk shocks (main drivers of exchange rate movements) and increasing in the volatility of monetary shocks (important for fluctuations in the interest rate). This is in line with the empirical findings surveyed in the introduction, according to which dollarization is particularly high in countries where exchange rate is stable and local monetary policy is volatile.

The impact of other structural characteristics on the equilibrium share of foreign currency loans as implied by our model is presented in Table 7. As already mentioned, the estimated monetary feedback rule for Poland features a rather aggressive response to deviations of inflation from the target. When we reduce this coefficient to a more conventional value of
Simulation results

1.5, which also coincides with the standard calibration of the Taylor rule, inflation volatility implied by our model goes up from 0.43 to 0.50, and the equilibrium share of mortgage loans denominated in the foreign currency goes up, reflecting an increase in the volatility of the real rate of return on local currency. Thus, our model is consistent with a positive relationship between dollarization and inflation volatility found in the empirical literature, see e.g. Cuaresma et al. (2011).

Using household-level survey data from the CEE countries, Fidrmuc et al. (2013) find that households’ plans to borrow in foreign currency are positively associated with remittances they receive. To check if this is also the case according to our model, we add an exogenous source of income, denominated and fixed in the foreign currency, to borrowers’ budget constraint (23) and to the balance of payments equation (37). We calibrate the steady state value of these transfers from abroad to 2.3% of output, which corresponds to the average GDP share of remittances in 2013 of the CEE countries listed in Table 2 according to the World Bank. Indeed, consistently with evidence from the survey data, the equilibrium share of foreign currency loans predicted by our model is now higher. This result is intuitive as income received in the foreign currency provides a natural hedge against exchange rate-induced fluctuations in payments associated with borrowing in this currency, and hence borrowers consider this type of contracts less risky.

In our benchmark calibration we assumed for simplicity that the steady state net foreign assets position of the home economy is zero. In fact, foreign debt in the CEE region is quite significant, which may have important consequences for equilibrium portfolio. According to our model, this effect is moderately positive as setting the steady state ratio of net foreign assets to output to −54.7% annually, which is in line with the 2014 estimates of the international investment position to GDP ratio for the CEE countries listed in Table 2 according to the IMF, increases the equilibrium share of foreign currency mortgages, consistently with empirical evidence presented by Skibinska (2018). Again, this effect can be explained with our model using the natural hedging argument. Since foreign debt is denominated in foreign currency and held by savers, they are exposed to exchange rate risk. Offering foreign

\[ \text{Expression} \]

14The reason why remittances are included only in borrowers’ budget constraint is that emigration in the CEE region has mainly economic motives and is concentrated in the group of relatively young people that can be considered financially constrained. The assumption on the remittances being fixed in foreign currency is made for simplicity. Making it dependent on foreign output to capture its possibly positive correlation with foreign business cycles does not significantly affect the results.
currency loans to borrowers helps them reduce this exposition.

Since our benchmark calibration is based on Poland, which is relatively closed compared to other countries in the region, we also check our model’s predictions for a higher steady state share of imports in output. To this end, we increase this ratio to 50% by resetting the home bias parameter \( \eta \) from 0.7 to 0.5. The equilibrium share of foreign currency loans goes up as, with higher openness, foreign shocks become more important and, as we have seen from Table 6, their presence favors dollarization.

Finally, we show the outcomes under the assumption that savers’ consumption preferences are linear so that their marginal utility is constant, and hence it drops from the equilibrium portfolio formula (45).\(^{15}\) This case makes the model assumptions closer to those considered in the simple setup described in Section 2, where agents borrowed directly from the rest of the world. Recall from our discussion in Section 6.1 that, for our baseline model with risk premium shocks only, borrowers push for a lower share of foreign currency loans while savers would like it to be higher. Therefore, if the equilibrium portfolio reflects only the preferences of borrowers, as it is effectively the case if savers’ marginal utility is constant, the degree of mortgage dollarization would be smaller.

\(^{15}\)To make these results comparable to the benchmark case, we adjust the weight on consumption in saver’s utility so that their allocations in the steady state are the same as under benchmark parametrization.
7 Conclusions

In this paper we have offered a novel explanation of why foreign currency loans to households have become so popular in some CEE countries. Our argument is based on what we call the debt limit channel, which arises when loans are multi-period and collateralized on newly purchased assets. This channel makes borrowing in domestic and foreign currency imperfect substitutes even under certainty equivalence. We show that offering these two types of loans at apparently equal terms (i.e. at the same LTV ratio and with the same distribution of principal repayments) biases the choice made by borrowers towards foreign currency if domestic interest rates are higher than abroad as this allows them to backload the real repayments and hence effectively increase the limit of debt imposed by the collateral constraint. When first-order effects of this bias are eliminated by adjustment in the contractual environment, the equilibrium share of foreign currency loans is positive but low. This suggests that banning this type of contracts, as it was done by the financial supervision authorities in several CEE countries, can be a fairly reasonable substitute for more sophisticated regulation that would force banks to appropriately modify their lending practices in a way that takes the debt limit channel into account.

We have arrived at these conclusions by using a very stylized general equilibrium framework, and confirmed them within a richer DSGE model calibrated for Poland. To focus on the working of the debt limit channel, we abstracted away from several potentially relevant features of the mortgage market, including the possibility of default.\textsuperscript{16} This choice is additionally motivated by the fact that, at least in Poland, debt write-offs due to default on housing loans are very low, averaging to merely 0.35\% of outstanding mortgage debt over the period 2009-2014 (NBP, 2015). Importantly, this ratio has never exceeded 0.5\%, despite massive depreciation of the Polish zloty against the Swiss frank (in which most foreign currency mortgages in Poland are denominated) during this period. This is not so surprising given that mortgages in the CEE region are recourse loans.

However, credit risk may be more important in other countries, with different contractual arrangements, so extending our analysis in this direction is a promising research avenue. This extension may be particularly important if one tries to adapt our analysis to firms, where

\textsuperscript{16}See Elenev et al. (2016) for a recent general equilibrium model that allows households to default on their mortgage debt.
long-term loans for purchases of new assets are prevalent and their currency composition has not yet been fully understood, especially from the debt limit channel perspective.
References


Tables and figures

Table 1: Share of foreign currency debt in total outstanding debt

<table>
<thead>
<tr>
<th>Region</th>
<th>Number of countries</th>
<th>Foreign currency loans (% of total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>66</td>
<td>29.4</td>
</tr>
<tr>
<td>Euro area</td>
<td>14</td>
<td>16.2</td>
</tr>
<tr>
<td>Non-euro area CEE</td>
<td>12</td>
<td>49.4</td>
</tr>
<tr>
<td>Latin America</td>
<td>5</td>
<td>26.0</td>
</tr>
<tr>
<td>Africa</td>
<td>13</td>
<td>20.7</td>
</tr>
<tr>
<td>Asia</td>
<td>14</td>
<td>34.4</td>
</tr>
</tbody>
</table>

Notes: This table presents foreign currency and foreign-currency-linked part of gross loans to residents and nonresidents as a percentage share of total gross loans, taken from the IMF Financial Soundness Indicators (FSIs) database. The numbers are for 2014, except for Uruguay and Zambia, for which 2013 data are used.

Table 2: Share of foreign currency debt in CEE countries by institutional sector

<table>
<thead>
<tr>
<th>Country (year)</th>
<th>Total private non-financial</th>
<th>Non-financial corporations</th>
<th>Households</th>
<th>Households (for housing)</th>
<th>Interest rate differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-euro area members (2014)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bulgaria</td>
<td>58.9</td>
<td>70.6</td>
<td>37.0</td>
<td>53.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Croatia</td>
<td>71.4</td>
<td>68.8</td>
<td>73.2</td>
<td>92.6</td>
<td>2.0</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>9.0</td>
<td>21.4</td>
<td>0.2</td>
<td>0.0</td>
<td>-0.1</td>
</tr>
<tr>
<td>Hungary</td>
<td>51.9</td>
<td>50.1</td>
<td>53.6</td>
<td>52.9</td>
<td>4.9</td>
</tr>
<tr>
<td>Lithuania</td>
<td>72.2</td>
<td>75.1</td>
<td>69.3</td>
<td>.</td>
<td>1.4</td>
</tr>
<tr>
<td>Poland</td>
<td>28.8</td>
<td>26.1</td>
<td>30.3</td>
<td>46.3</td>
<td>2.5</td>
</tr>
<tr>
<td>Romania</td>
<td>58.3</td>
<td>53.6</td>
<td>63.3</td>
<td>85.1</td>
<td>5.2</td>
</tr>
<tr>
<td>Euro area members prior to euro adoption</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estonia (2010)</td>
<td>88.5</td>
<td>92.0</td>
<td>85.2</td>
<td>93.9</td>
<td>1.4</td>
</tr>
<tr>
<td>Latvia (2013)</td>
<td>86.6</td>
<td>84.7</td>
<td>88.8</td>
<td>.</td>
<td>2.9</td>
</tr>
<tr>
<td>Slovakia (2008)</td>
<td>19.0</td>
<td>32.2</td>
<td>2.7</td>
<td>2.9</td>
<td>0.4</td>
</tr>
<tr>
<td>Slovenia (2006)</td>
<td>59.4</td>
<td>66.2</td>
<td>42.8</td>
<td>68.6</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Notes: The percentage shares of foreign currency outstanding debt presented in the table are based on the official statistics published by each country’s central bank. For non-euro area countries the shares are for 2014, while for the euro area members they cover the year prior to euro adoption. The interest rate differential is calculated as the difference between the 3-month money market interest rate in a given country and that in the euro area, except for Latvia, for which the 6-month money market rate differential is used. The numbers are averages over the 2005-2014 period for the non-euro area countries, or averages from 2005 to the last year prior to euro adoption for the euro area member states.
Table 3: Mean share of foreign currency loans - simple model

<table>
<thead>
<tr>
<th>4(R - R*)</th>
<th>σ = 0.00</th>
<th>σ = 0.01</th>
<th>σ = 0.025</th>
<th>σ = 0.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1%</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.5%</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>5%</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes: This table presents the ergodic mean share of foreign currency loans, for various levels of interest rate differential (annualized) and foreign inflation target volatility, according to the model used in the quantitative exercise described in Section 2.4.

Table 4: Calibrated structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ω</td>
<td>0.25</td>
<td>Share of borrowers in population</td>
</tr>
<tr>
<td>βs</td>
<td>0.995</td>
<td>Discount factor of savers</td>
</tr>
<tr>
<td>βb</td>
<td>0.99</td>
<td>Discount factor of borrowers</td>
</tr>
<tr>
<td>ξ</td>
<td>0.75</td>
<td>Habit formation</td>
</tr>
<tr>
<td>σn</td>
<td>2</td>
<td>Inverse Frisch elasticity</td>
</tr>
<tr>
<td>Aχ</td>
<td>0.46</td>
<td>Weight of housing in utility</td>
</tr>
<tr>
<td>m</td>
<td>0.85</td>
<td>LTV ratio on mortgage originations</td>
</tr>
<tr>
<td>δ</td>
<td>0.015</td>
<td>Loan decay parameter</td>
</tr>
<tr>
<td>δχ</td>
<td>0.007</td>
<td>Housing depreciation rate</td>
</tr>
<tr>
<td>δk</td>
<td>0.015</td>
<td>Capital depreciation rate</td>
</tr>
<tr>
<td>κχ</td>
<td>∞</td>
<td>Housing investment adjustment cost</td>
</tr>
<tr>
<td>κk</td>
<td>1</td>
<td>Non-housing investment adjustment cost</td>
</tr>
<tr>
<td>α</td>
<td>0.3</td>
<td>Capital share in output</td>
</tr>
<tr>
<td>µ</td>
<td>1.2</td>
<td>Steady-state product markup</td>
</tr>
<tr>
<td>θH, θH*, θF</td>
<td>0.75</td>
<td>Calvo probabilities</td>
</tr>
<tr>
<td>η</td>
<td>0.7</td>
<td>Home bias</td>
</tr>
<tr>
<td>φ, φ*</td>
<td>1.5</td>
<td>El. of subst. btw. domestic and foreign goods</td>
</tr>
<tr>
<td>ρ</td>
<td>0.001</td>
<td>Risk premium elasticity wrt. foreign debt</td>
</tr>
<tr>
<td>g/y</td>
<td>0.2</td>
<td>Steady-state share of gov. spending in output</td>
</tr>
<tr>
<td>π</td>
<td>1.005</td>
<td>Steady-state inflation</td>
</tr>
<tr>
<td>γR</td>
<td>0.845</td>
<td>Interest rate smoothing in monetary policy rule</td>
</tr>
<tr>
<td>γπ</td>
<td>1.85</td>
<td>Response to inflation in monetary policy rule</td>
</tr>
</tbody>
</table>
Table 5: Moment matching

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard dev.</th>
<th>Autocorrelation</th>
<th>Corr. with output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Output</td>
<td>1.12</td>
<td>1.28</td>
<td>0.93</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.98</td>
<td>1.13</td>
<td>0.88</td>
</tr>
<tr>
<td>Investment</td>
<td>5.62</td>
<td>5.63</td>
<td>0.94</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.43</td>
<td>0.50</td>
<td>0.41</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.69</td>
<td>0.48</td>
<td>0.57</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>4.58</td>
<td>5.85</td>
<td>0.18</td>
</tr>
<tr>
<td>Return on local currency</td>
<td>0.52</td>
<td>0.69</td>
<td>0.55</td>
</tr>
<tr>
<td>Return on foreign currency</td>
<td>4.46</td>
<td>4.08</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: The standard deviations are expressed in percent.

Table 6: Equilibrium share of foreign currency loans

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Share of foreign currency loans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity</td>
<td>0.0</td>
</tr>
<tr>
<td>Monetary</td>
<td>100.0</td>
</tr>
<tr>
<td>Foreign</td>
<td>55.7</td>
</tr>
<tr>
<td>Risk premium</td>
<td>5.2</td>
</tr>
<tr>
<td>All</td>
<td>15.5</td>
</tr>
</tbody>
</table>

Table 7: Equilibrium share of foreign currency loans - alternative specifications

<table>
<thead>
<tr>
<th>Model variant</th>
<th>Share of foreign currency loans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>15.5</td>
</tr>
<tr>
<td>Less aggressive mon. policy</td>
<td>20.5</td>
</tr>
<tr>
<td>Remittances</td>
<td>21.4</td>
</tr>
<tr>
<td>Foreign debt</td>
<td>19.0</td>
</tr>
<tr>
<td>High openness</td>
<td>17.8</td>
</tr>
<tr>
<td>Risk-neutral savers</td>
<td>8.9</td>
</tr>
</tbody>
</table>

Notes: The model variant with less aggressive monetary policy is obtained by resetting the monetary policy repose to inflation from $1.85$ to $\gamma = 1.5$. The model with remittances features exogenous transfers from the foreign economy to borrowers, denominated and fixed in foreign currency, and equal to $2.3\%$ of home economy’s annual output in the steady state. In the model with foreign debt we allow for a non-zero net foreign assets position, calibrating it to $-54.7\%$ of annual output in the steady state. The case of higher openness considers the share of imports in aggregate demand $1 - \eta$ equal to 0.5.
Figure 1: Impulse responses to risk premium shock

Note: The figure plots the impulse responses to a positive risk premium shock assuming the steady state share of foreign currency loans of 50% (solid lines) or 4.9% (dashed lines). The responses are presented in percent deviations from the steady state. The adjusted marginal utility of borrowers is defined as $(1 + s_{O})\hat{u}_{b,t+1} - s_{O}\hat{\theta}_{t+1}$ while the adjusted relative marginal utility is $(1 + s_{O})\hat{u}_{b,t+1} - s_{O}\hat{\theta}_{t+1} - \hat{u}_{s,t+1}$, see the equilibrium portfolio formula (45).
Appendix

A.1 Solving the model from Section 2

The simple model described in section 2.1 with additional assumptions made in subsection 2.4 can be written as the solution to the Bellman equation

\[ V(l_{H,t-1},l_{F,t-1},\bar{\pi}_t^*) = \max_{l_{H,t},l_{F,t}} \left\{ u \left( y + l_{H,t} + l_{F,t} - r^* \left[ l_{H,t-1} + \frac{\bar{\pi}_t^*}{\bar{\pi}_t} l_{F,t-1} \right] \right) + \mathbb{E}_t V(l_{H,t},l_{F,t},\bar{\pi}_{t+1}) \right\} \]  

subject to

\[ l_{H,t} + l_{F,t} - (1 - \delta) \left( \frac{l_{H,t-1}}{\bar{\pi}_t} + \frac{l_{F,t-1}}{\bar{\pi}_t^*} \right) \leq m \]  

(A.1)

It is convenient to rewrite the model in the following state variables: \( l_t \equiv l_{H,t} + l_{F,t} \) and \( s_{H,t} \equiv l_{H,t}/l_t \). Then the Bellman equation becomes

\[ V(l_{t-1},s_{H,t-1},\bar{\pi}_t^*) = \max_{l_t,s_{H,t}} \left\{ u \left( y + l_t - r^* l_{t-1} \left[ s_{H,t-1} + \frac{\bar{\pi}_t^*}{\bar{\pi}_t} (1 - s_{H,t-1}) \right] \right) + \mathbb{E}_t V(l_t,s_{H,t},\bar{\pi}_{t+1}) \right\} \]  

subject to

\[ l_t - (1 - \delta) l_{t-1} \left( \frac{s_{H,t-1}}{\bar{\pi}_t} + \frac{1 - s_{H,t-1}}{\bar{\pi}_t^*} \right) \leq m \]  

(A.3)

To find a solution to this problem, we use a grid of 2,000 equally spaced points for \( l_t \in [1,5] \) and a 2-point grid for \( s_{H,t} = \{0,1\} \). Hence, at a given point in time, agents can hold either domestic or foreign debt, but there are no constraints on prepayment or currency conversion. The foreign inflation target shock is discretized with 7 points distributed symmetrically around \( \bar{\pi}_t^* \), where the nodes and probability transition matrix are obtained with the Tauchen (1986) method. The solution is obtained by iterating value function \( V \) on such defined discretized state space.

A.2 Discrimination between foreign and domestic currency loans

Assume that the collateral constraint is as in equation (15). Then the Euler equations associated with domestic and foreign currency loans can be written as

\[ u'_t = \beta \mathbb{E}_t \left\{ u'_{t+1} \frac{R_t}{\bar{\pi}_{t+1}} \right\} + \Theta_t \frac{\Theta_{t+1}}{m_H} - \beta(1 - \delta) \mathbb{E}_t \left\{ \frac{\Theta_{t+1}}{m_H \bar{\pi}_{t+1}} \right\} \]  

(A.5)
\[ u'_t = \beta \mathbb{E}_t \left\{ u'_{t+1} \frac{R_t^*}{\pi_{t+1}^*} \right\} + \Theta_t \frac{\Theta_{t+1}}{m_F \pi_{t+1}^*} - \beta(1 - \delta) \mathbb{E}_t \left\{ \frac{\Theta_{t+1}}{m_F \pi_{t+1}^*} \right\} \]  \hspace{1cm} (A.6)

For agents to be indifferent between the two contracts, the right-hand sides of these two equations must be equal. If agents are risk neutral (or under certainty equivalence), this implies

\[ \frac{\Theta_t}{m_H} - \beta(1 - \delta) \mathbb{E}_t \left\{ \frac{\Theta_{t+1}}{m_H \pi_{t+1}^*} \right\} = \frac{\Theta_t}{m_F} - \beta(1 - \delta) \mathbb{E}_t \left\{ \frac{\Theta_{t+1}}{m_F \pi_{t+1}^*} \right\} \]  \hspace{1cm} (A.7)

In the steady state, after using the Fisher equation, we have

\[ \frac{m_F}{m_H} = \frac{1 - \beta(1 - \delta)\frac{1}{\pi^*}}{1 - \beta(1 - \delta)\frac{1}{\pi^*}} = \frac{\pi^* - \beta(1 - \delta)\pi^*}{\pi^* - \beta(1 - \delta)\pi^*} \]  \hspace{1cm} (A.8)

Alternatively, consider now a modified contracting structure summarized by equations (17)-(19), keeping the collateral constraint in its original form (6). Then the Euler equations associated with domestic and foreign currency loans become

\[ u'_t = \beta \mathbb{E}_t \left\{ u'_{t+1} \frac{R_t}{\pi_{t+1}} \right\} + \Theta_t - \beta(1 - \delta) \mathbb{E}_t \left\{ R_t \frac{\Theta_{t+1}}{\pi_{t+1}} \right\} \]  \hspace{1cm} (A.9)

\[ u'_t = \beta \mathbb{E}_t \left\{ u'_{t+1} \frac{R_t^*}{\pi_{t+1}^*} \right\} + \Theta_t - \beta(1 - \delta) \mathbb{E}_t \left\{ R_t^* \frac{\Theta_{t+1}}{\pi_{t+1}^*} \right\} \]  \hspace{1cm} (A.10)

It is easy to see by using the Fisher equation (9) that these two equations are identical in the certainty equivalence case.

### A.3 Equilibrium conditions for the model from Section 3

In this section of the Appendix we present a full list of 39 equations making up the model described in section 3. These equations determine the equilibrium evolution of 39 variables \( \{c_s,t, c_b,t, u_s,t, u_b,t, n_s,t, n_b,t, \chi_s,t, \chi_b,t, \chi,t, l_{H,t}, l_{F,t}, k_t, i_t, i_k,t, y_{H,t}, y_{F,t}, y_t, \tilde{y}_t, d_t^*, \rho_t, \Theta_t, \Delta_H,t, \Delta_F,t, m_c,t, r_{k,t}, w_t, p_{H,t}, p_{F,t}, p_H,t, p_F,t, p_{H,t}, p_{F,t}, q_t, \pi_t, R_t \}_{t=1}^{\infty} \) for given initial conditions \( \{c_{s,0}, c_{b,0}, k_0, \chi_0, i_{k,0}, i^{*}_{t,0}, l_{H,0}, l_{F,0}, \Delta_{H,0}, \Delta_{*H,0}, \Delta_{F,0}, p_{H,0}, p_{F,0}, R_0, R_0^* \} \) and for given exogenous sequence \( \{\varepsilon_{z,t}, \varepsilon_{\chi,t}, \varepsilon_{p,t}, \varepsilon_{R,t}, y_t, \pi_t, R_t^* \}_{t=1}^{\infty} \). Lower-case letters are the real counterparts of the nominal variables defined in the main text.
Appendix

Households

Marginal utility (for $i \in \{s, b\}$)

$$u_{i,t} = (c_{i,t} - \xi c_{i,t-1})^{-1}$$  \hspace{1cm} (A.11)

Euler equation for savers

$$u_{s,t} = \beta_s E_t \{u_{s,t+1} \pi_{t+1}^{-1} R_t \}$$  \hspace{1cm} (A.12)

Uncovered interest rate parity

$$E_t \{ u_{s,t+1} (R_t \pi_t - q_t \rho_t R_t^* \pi_t^*)^{-1} l_{F,t-1} \} = 0$$  \hspace{1cm} (A.13)

Risk premium

$$\rho_t = \left( 1 + \rho \frac{d_t q_t}{y_t} \right) \varepsilon_{\rho,t}$$  \hspace{1cm} (A.14)

Borrowers’ budget constraint

$$c_{s,t} + p_{\chi,t}(\chi_{s,t} - (1-\delta)\chi_{s,t-1}) + R_{t-1} l_{H,t-1} \pi_t^{-1} + q_t \rho_{t-1} R_{t-1}^* (\pi_{t-1}^*)^{-1} l_{F,t-1} = w_t n_{s,t} + l_{H,t} + q_t l_{F,t}$$  \hspace{1cm} (A.15)

Collateral constraint

$$l_{H,t} + q_t l_{F,t} - (1-\delta) \left( \frac{R_{t-1} l_{H,t-1} \pi_t^{-1} + q_t \rho_{t-1} R_{t-1}^* \pi_{t-1}^* l_{F,t-1}}{\pi_t} \right) = m E_t \{ p_{\chi,t+1} \pi_{t+1} [\chi_{s,t} - (1-\delta)\chi_{s,t-1}] \}$$  \hspace{1cm} (A.16)

Euler equations for impatient households

$$u_{b,t} = \beta_b E_t \left\{ u_{b,t+1} \frac{R_t}{\pi_{t+1}^*} \right\} + \Theta_t - \beta_b (1-\delta) E_t \left\{ \Theta_{t+1} \frac{R_t}{\pi_{t+1}^*} \right\}$$  \hspace{1cm} (A.17)

$$u_{b,t} = \beta_b E_t \left\{ u_{b,t+1} \frac{q_{t+1} \rho_t R_t^*}{q_t \pi_{t+1}^*} \right\} + \Theta_t - \beta_b (1-\delta) E_t \left\{ \Theta_{t+1} \frac{q_{t+1} \rho_t R_t^*}{q_t \pi_{t+1}^*} \right\}$$  \hspace{1cm} (A.18)

Housing demand

$$\chi_{s,t} = \chi_s$$  \hspace{1cm} (A.19)
\[ u_{b,t} p_{\chi,t} = \varepsilon_{\chi,t} A_{\chi} A_{b,t}^{-1} + \beta_b (1 - \delta_{\chi}) \mathbb{E}_t \{ u_{b,t+1} p_{\chi,t+1} \} \]
\[ + \Theta_t m \mathbb{E}_t \{ p_{\chi,t+1} \pi_{t+1} \} - \beta_I (1 - \delta_{\chi}) m \mathbb{E}_t \{ \Theta_{t+1} p_{\chi,t+2} \pi_{t+2} \} \]  
(A.20)

Capital Euler equation

\[ u_{s,t} p_{k,t} = \beta \mathbb{E}_t \{ u_{s,t+1} [(1 - \delta_k) p_{k,t+1} + r_{k,t+1}] \} \]  
(A.21)

Labor supply (for \( i \in \{s, b\} \))

\[ w_t u_{i,t} = n_i^{\sigma_n} \]  
(A.22)

Capital and housing producers

Capital accumulation

\[ k_t = (1 - \delta_k) k_{t-1} + \left( 1 - \frac{\kappa_k}{2} \left( \frac{i_{k,t}}{i_{k,t-1}} - 1 \right) \right) i_{k,t} \]  
(A.23)

Investment demand

\[ 1 = p_{k,t} \left( 1 - \frac{\kappa_k}{2} \left( \frac{i_{k,t}}{i_{k,t-1}} - 1 \right) \right)^2 - \kappa_k \left( \frac{i_{k,t}}{i_{k,t-1}} - 1 \right) \left( \frac{i_{k,t}}{i_{k,t-1}} \right) \]
\[ + \beta_k \kappa_k \mathbb{E}_t \left\{ \frac{u_{s,t+1}}{u_{s,t}} p_{k,t+1} \left( \frac{i_{k,t+1}}{i_{k,t}} - 1 \right) \left( \frac{i_{k,t+1}}{i_{k,t}} \right)^2 \right\} \]  
(A.24)

Housing accumulation

\[ \chi_t = (1 - \delta_\chi) \chi_{t-1} + \left( 1 - \frac{\kappa_\chi}{2} \left( \frac{i_{\chi,t}}{i_{\chi,t-1}} - 1 \right) \right) i_{\chi,t} \]  
(A.25)

Housing investment demand

\[ 1 = p_{\chi,t} \left( 1 - \frac{\kappa_\chi}{2} \left( \frac{i_{\chi,t}}{i_{\chi,t-1}} - 1 \right) \right)^2 - \kappa_\chi \left( \frac{i_{\chi,t}}{i_{\chi,t-1}} - 1 \right) \left( \frac{i_{\chi,t}}{i_{\chi,t-1}} \right) \]
\[ + \beta_k \kappa_\chi \mathbb{E}_t \left\{ \frac{u_{s,t+1}}{u_{s,t}} p_{\chi,t+1} \left( \frac{i_{\chi,t+1}}{i_{\chi,t}} - 1 \right) \left( \frac{i_{\chi,t+1}}{i_{\chi,t}} \right)^2 \right\} \]  
(A.26)
Appendix

Final goods aggregation

Aggregate demand

\[ \tilde{y}_t = \left( 1 - \eta_c \right)^{1} \frac{\tilde{y}_{F,t}}{1} + \eta_c \frac{\tilde{y}_{H,t}}{1} \]  
(A.27)

Demands

\[ y_{F,t} = (1 - \eta) \tilde{p}_{F,t} \tilde{y}_t \]  
(A.28)

\[ y_{H,t} = \eta \tilde{p}_{H,t} \tilde{y}_t \]  
(A.29)

Domestic producers

Marginal cost

\[ mc_t = \frac{1}{\alpha^\alpha} \frac{1}{(1 - \alpha)^{\alpha \epsilon} z_t} k_t w_t^{\alpha} \]  
(A.30)

Optimal factor proportions

\[ \frac{r_{k,t}}{w_t} = \frac{\alpha}{1 - \alpha} \frac{\omega k_t + (1 - \omega) n_{s,t}}{k_{t-1}} \]  
(A.31)

Optimal prices set by reoptimizing firms for domestic market and exports

\[ \tilde{p}_{H,t} = \mu \left\{ \sum_{j=0}^{\infty} \left( \beta_s \theta_{H} \right)^j u_{s,t+j} mc_t + j \left( \frac{\pi_{\pi_t}}{\pi_{t-\pi_{t+j}}} \right)^{\frac{1}{\eta}} \tilde{y}_{H,t+j} \right\} \]  
(A.32)

\[ \tilde{p}_{H,t}^* = \mu \left\{ \sum_{j=0}^{\infty} \left( \beta_s \theta_{H} \right)^j u_{s,t+j} mc_t + j \left( \frac{\pi_{\pi_t}}{\pi_{t-\pi_{t+j}}} \right)^{\frac{1}{\eta}} \tilde{y}_{H,t+j} \right\} \]  
(A.33)

Price indexes for goods sold domestically and for exports

\[ \tilde{p}_{H,t}^{1/\eta} = \theta_{H} \left( \tilde{p}_{H,t-1}^{1/\pi_t} \right)^{\frac{1}{\eta}} + (1 - \theta_{H}) \tilde{p}_{H,t}^{1/\eta} \]  
(A.34)

\[ \left( \tilde{p}_{H,t}^{*} \right)^{1/\eta} = \theta_{H} \left( \tilde{p}_{H,t-1}^{*} \right)^{\frac{1}{\eta}} + (1 - \theta_{H}) \left( \tilde{p}_{H,t}^{*} \right)^{1/\eta} \]  
(A.35)
Foreign trade

Optimal prices set by reoptimizing importers

\[
\hat{p}_{F,t} = \mu - E_t \left\{ \sum_{j=0}^{\infty} (\beta_s \theta_F)^j u_{s,t+j} p_{F,t+j}^{\frac{1}{1-\pi}} \left( \frac{\pi_{n_t}}{\pi_{t+j}} \right)^{\frac{\pi_t}{1-\pi}} y_{F,t+j} \right\} (A.36)
\]

Price index for imports

\[
p_{F,t}^{1/\pi} = \theta_F \left( p_{F,t-1}^{\frac{1}{1-\pi}} + (1 - \theta_F) p_{F,t}^{\frac{1}{1-\pi}} \right) (A.37)
\]

Export demand

\[
y_{H,t}^* = (p_{H,t}^*)^{-\phi} y_t^* (A.38)
\]

Monetary authority

Monetary policy rule

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left( \frac{\pi_t}{\pi} \right)^{(1-\gamma_R)\gamma} \varepsilon_{R,t} (A.39)
\]

Market clearing

Aggregate demand

\[
\bar{y}_t = \omega c_{b,t} + (1 - \omega) c_{s,t} + i_{k,t} + i_{\chi,t} + g (A.40)
\]

Definition of aggregate output

\[
y_t = y_{H,t} \Delta_{H,t} + y_{H,t}^* \Delta_{H,t}^* (A.41)
\]

Resource constraint

\[
y_t = \varepsilon_{z,t} k_{t-1}^{\alpha} (\omega n_{b,t} + (1 - \omega) n_{s,t})^{1-\alpha} (A.42)
\]

Net foreign debt

\[
d_t^* = \Delta_{F,t} y_{FA} - p_{H,t}^* y_{H,t}^* + \sigma_{t-1} R_{t-1}^{\ast} \frac{d_{t-1}^*}{\pi_{t-1}} (A.43)
\]
Price dispersion indexes
\[
\Delta_{H,t} = \left( \frac{p_{H,t}}{p_{H,t-1}} \right)^{\mu/p} \theta_H \Delta_{H,t-1} \left( \frac{\pi}{\pi_t} \right)^{\mu/p} + (1 - \theta_H) \left( \frac{\tilde{p}_{H,t}}{p_{H,t}} \right)^{\mu/p} \tag{A.44}
\]
\[
\Delta_{H,t}^* = \left( \frac{p_{H,t}}{p_{H,t-1}} \right)^{\mu/\pi} \theta_{H}^* \Delta_{H,t-1}^{*} \left( \frac{\pi^*}{\pi_t^*} \right)^{\mu/\pi} + (1 - \theta_H^*) \left( \frac{\tilde{p}_{H,t}}{p_{H,t}} \right)^{\mu/\pi} \tag{A.45}
\]
\[
\Delta_{F,t} = \left( \frac{p_{F,t}}{p_{F,t-1}} \right)^{\mu/p} \theta_{F} \Delta_{F,t-1} \left( \frac{\pi}{\pi_t} \right)^{\mu/p} + (1 - \theta_F) \left( \frac{\tilde{p}_{F,t}}{p_{F,t}} \right)^{\mu/p} \tag{A.46}
\]

Housing market clearing
\[
\chi_t = \omega \chi_{b,t} + (1 - \omega) \chi_{s,t} \tag{A.47}
\]

A.4 Deriving equilibrium portfolio condition (45)

The portfolio choice of savers is given by the UIP condition (42), repeated below for convenience
\[
E_t \{ u_{s,t+1}(r_{H,t+1} - r_{F,t+1}) \} = 0 \tag{A.48}
\]

while that of borrowers can be described by the following condition derived by combining equations (43) and (44)
\[
E_t \{ (u_{b,t+1} - (1 - \delta) \Theta_{t+1})(r_{H,t+1} - r_{F,t+1}) \} = 0 \tag{A.49}
\]

Approximating these two equations to second order yields
\[
E_t \{ (\hat{r}_{H,t+1} - \hat{r}_{F,t+1}) + 0.5(\hat{r}_{H,t+1}^2 - \hat{r}_{F,t+1}^2) + \hat{u}_{s,t+1}(\hat{r}_{H,t+1} - \hat{r}_{F,t+1}) \} = 0 \tag{A.50}
\]
\[
\begin{align*}
\hat{u}_{b}E_t \{ (\hat{r}_{H,t+1} - \hat{r}_{F,t+1}) + 0.5(\hat{r}_{H,t+1}^2 - \hat{r}_{F,t+1}^2) + \hat{u}_{b,t+1}(\hat{r}_{H,t+1} - \hat{r}_{F,t+1}) \} \\
- (1 - \delta) \Theta E_t \{ (\hat{r}_{H,t+1} - \hat{r}_{F,t+1}) + 0.5(\hat{r}_{H,t+1}^2 - \hat{r}_{F,t+1}^2) + \hat{\Theta}_{t+1}(\hat{r}_{H,t+1} - \hat{r}_{F,t+1}) \} \} = 0 \tag{A.51}
\end{align*}
\]

By merging these two conditions to eliminate the first-order terms, using the steady state relation \( u_{b}(1 - \frac{\partial \hat{u}}{\partial \hat{\pi}}) = \Theta(1 - \frac{\partial \hat{\Theta}}{\partial \hat{s}} (1 - \delta)) \) implied by either of the two Euler equations (43) or
(44), and rearranging, we obtain

\[
\mathbb{E}_t \left\{ (\hat{r}_{H,t+1} - \hat{r}_{F,t+1}) \left( (1 + s_\Theta)\hat{u}_{b,t+1} - s_\Theta \hat{\Theta}_{t+1} - \hat{u}_{s,t+1} \right) \right\} = 0
\]  

(A.52)

where \( s_\Theta = \frac{(1-\delta)(\beta P - \beta I)}{\delta \beta P} \), which is formula (45) in the main text.

To see that the conditional expectations in equation (A.52) are time-invariant, note first that the second-order approximation to the UIP condition (A.50) implies that excess return \( \hat{r}_{x,t} \equiv \hat{r}_{H,t} - \hat{r}_{F,t} \) is i.i.d. up to first order as it depends only on second-order terms. Then the first-order solution to the model implies

\[
\hat{a}_{t+1} = A_a \hat{s}_t + B_a \epsilon_{t+1}
\]  

(A.53)

\[
\hat{r}_{x,t+1} = B_r \epsilon_{t+1}
\]  

(A.54)

where \( \hat{a}_{x,t} \equiv (1 + s_\Theta)\hat{u}_{b,t+1} - s_\Theta \hat{\Theta}_{t+1} - \hat{u}_{s,t+1} \), \( \hat{s}_t \) is a vector of state variables, \( \epsilon_t \) is a vector of structural shocks with covariance matrix \( \Sigma \), while \( A_a, B_a \) and \( B_r \) are the model solution matrices of appropriate dimensions. This allows us to write

\[
\mathbb{E}_t \{ \hat{r}_{x,t+1} \hat{a}_{t+1} \} = B_a \mathbb{E}_t \{ \epsilon_{t+1} \epsilon_{t+1}' \} B_r' = B_a \Sigma B_r'
\]  

(A.55)

and hence, for given coefficients showing up in the model equilibrium conditions approximated to first order, the conditional expectations in formula (45) are constant.

### A.5 First-order model dynamics given the steady-state portfolio

In this appendix we show that first-order accurate equilibrium dynamics of all model variables other than \( l_{H,t} \) and \( l_{F,t} \) do not depend on individual time-variation in these variables, but only on the steady state portfolio.

First note that the only equations where local and foreign currency loans show up is borrowers’ budget constraint (A.15) and collateral constraint (A.16). Approximating them
to first-order yields

$$
\frac{c_b}{l} \dot{b}_{b,t} + \chi_b l \left[ \dot{x}_{b,t} - (1 - \delta_x) \dot{x}_{b,t-1} + \delta_x \dot{p}_{\chi,t} \right] + \frac{s_H}{\beta_s} \dot{r}_{H,t-1} + \frac{1 - s_H}{\beta_s} \dot{r}_{F,t-1} + \frac{1}{\beta_s} \dot{l}_{t-1} \\
= \frac{wn_b}{l} (\ddot{w}_t + \ddot{n}_{b,t}) + \hat{l}_t \tag{A.56}
$$

$$
\dot{l}_t - \frac{1}{\beta_s} \left[ s_H \dot{r}_{H,t-1} + (1 - s_H) \dot{r}_{F,t-1} + \dot{l}_{t-1} \right] \\
= \frac{m \chi_b \pi}{l} \left[ \dot{x}_{b,t} - (1 - \delta_x) \dot{x}_{b,t-1} + \delta_x (\dot{m}_t + E_{t} \{ \dot{p}_{\chi,t+1} + \pi_{t+1} \}) \right] \tag{A.57}
$$

where $l_t \equiv l_{H,t} + q_t l_{F,t}$ denotes total loans and $s_H \equiv \frac{l_H}{l}$ describes the steady state composition of the loan portfolio. Hence, while approximating to first order the model derived in section 3 and summarized in Appendix A.3, we can get rid of $\dot{l}_{H,t}$ and $\dot{l}_{F,t}$ and replace them with $\dot{l}_t$. Note that since equations (A.17) and (A.18) are equivalent to first order and hence one of them can be dropped, the number of independent model equations matches the number of endogenous variables.

### A.6 Data sources

To estimate the processes driving stochastic shocks and evaluate how well the model matches the key moments observed in the data, we use the following quarterly macroeconomic time series for Poland (home economy) and the euro area (foreign economy):

- **Output** - real gross domestic product at market prices, chain linked volumes
- **Consumption** - household and NPIHS final consumption expenditure, chain linked volumes
- **Investment** - gross fixed capital formation, chain linked volumes
- **Inflation** - log-difference of all items HICP
- **Interest rate** - 3-month money market interest rate
- **Real exchange rate** - bilateral exchange rate of the Polish zloty against the euro, divided by the ratio of all items HICP in Poland versus the euro area
Employment - total employment according to the Labor Force Survey

The data are for the period 1996-2014, come from the Eurostat and, where applicable, are seasonally adjusted using the Tramo/Seats procedure. To make the time series compatible with the log-linearized version of the model, all of them are logged and then filtered with the Hodrick-Prescott filter, using the standard smoothing parameter 1600.

A.7 Estimation of shock processes

To estimate the shock processes, we use the transformed data described in the previous section of the Appendix and, where applicable, the calibrated values of structural parameters as in Table 4.

Productivity shock We first construct the capital stock series using the log-linearized version of equation (30) and Polish data on investment. We use the thus obtained series together with data on output and employment for Poland to calculate the Solow residual according to the log-linearized version of production function (36), thus obtaining our estimate of $\hat{\varepsilon}_{z,t}$. Fitting an AR(1) process to this series gives the autoregressive coefficient $\rho_z = 0.83$ and standard deviation of innovation $\sigma_z = 0.0059$. Since quarterly employment data for Poland are available only from 2000, the estimation sample is 2000-2014.

Monetary policy shock By fitting the log-linearized version of the monetary feedback rule (33) to the Polish data on the interest rate and inflation we obtain $\gamma_R$ and $\gamma_\pi$ reported in Table 4, as well as the standard deviation of the monetary policy shock $\hat{\varepsilon}_{R,t}$ equal to $\sigma_R = 0.0021$.

Foreign variables We use the euro data on inflation, output and the interest rate to estimate a structural VAR(1) model for the logs of $\pi^*_t$, $y^*_t$ and $R^*_t$. To structuralize the system, we use the ordering in the Cholesky decomposition that is standard in the VAR literature, implying no contemporaneous response of inflation and output to interest rate shocks, and no current response of inflation to output shocks.

Risk premium shock Estimation of the process driving $\hat{\varepsilon}_{\rho,t}$ uses the log-linearized version
of the UIP condition (42) and risk premium (22), lagged by one period

\[
\hat{R}_{t-1} - \hat{\pi}_t - \hat{R}^*_t - \hat{\pi}^*_t + \hat{q}_{t-1} - \varrho \hat{d}_{t-1} = \hat{\varepsilon}_{\rho, t-1} \tag{A.58}
\]

Since stochastic shocks are the only source of ex post errors in expectations, we can write

\[
\hat{R}_{t-1} - \hat{\pi}_t - \hat{R}^*_t + \hat{\pi}^*_t - \hat{q}_{t-1} - \hat{q}_{t-1} = \hat{\varepsilon}_{\rho, t-1} + a_\rho \varepsilon_{\rho, t} + \sum_{i \in \Upsilon \setminus \{\rho\}} a_i \varepsilon_{i, t} \tag{A.59}
\]

where \( \Upsilon = \{ \rho, z, R, \pi^*, y^*, R^* \} \) indexes the i.i.d. innovations to stochastic shocks driving our model, coefficients \( a_i \) for \( i \in \Upsilon \) provide mapping from shocks to errors in expectations, and we dropped the foreign debt term as \( \varrho \) is very close to zero under our calibration. Equation (A.59) together with an AR(1) process for unobservable \( \hat{\varepsilon}_{\rho, t} \) constitute the state-space system that can be estimated using the Kalman filter and the data on the interest rates, inflation and real exchange rate. To sort out the impact of shocks other than risk premium, we fix the variance of the last term on the right-hand side of equation (A.59) during estimation to that implied by the model solution absent risk premium shocks. This estimation yields the autoregressive coefficient of risk premium shocks \( \rho_{\rho} = 0.17 \), and the standard deviation of innovation \( \sigma_{\rho} = 0.038 \).