The new macroeconometric model of the Polish economy

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Abstract

This paper presents the structural macroeconometric model of the Polish economy, NECMOD, which was developed foremost to facilitate implementation of the monetary policy in Poland through a regular delivery of inflation and GDP projections. The model encompasses all major channels of the monetary policy transmission mechanism and is able to deliver a comprehensive account of factors underlying the main economic developments. With its complex labour market structure, explicit incorporation of inflation expectations, distortionary fiscal policy and heterogeneity of the capital stock, NECMOD is able to describe propagation of a range of macroeconomic shocks. As a forecasting and simulation tool, the model is specifically designed to reflect the dynamic nature of a converging economy.
Non-technical summary

Macroeconomic modelling at central banks is aimed foremost at providing a possibly comprehensive account of the evolution of inflation and GDP. World-wide implementation of the direct inflation targeting strategy (DIT) facilitated development of models delivering not only accurate forecasts of key macrocategories, but also providing policy makers with a thorough description of a forecast scenario. Consistently with these trends, introduction of the DIT framework in Poland necessitated development of models describing the Polish economy in enough detail to be used as story-telling forecasting tools.

NECMOD, the new forecasting model of the National Bank of Poland (NBP), has its origins in a structural model ECMOD that was employed to prepare regular inflation projections at the NBP from 2005 to the beginning of 2008. Both NECMOD and its predecessor were designed to encompass important past and expected future developments in the Polish economy. Not surprisingly, many differences between the two models may be traced back to the processes that have occurred or intensified over the three years since ECMOD was published. Dynamic, far-reaching structural changes in the economy, including accession to the European Union (EU), progressing globalization and continuous fading of labour market transition, affected the NBP staff’s perception of important mechanisms behind monetary and fiscal transmission channels. To a great degree, differences between the two forecasting models reflect accumulated experience and knowledge about currently vital economic forces in Poland.

Compared to other forecasting models of similar type, NECMOD may be distinguished as a model with a comprehensive treatment of supply side forces. The development of the model in this direction was triggered by the nature of economic processes in Poland. While low variability of potential output, the equilibrium rate of unemployment or the real exchange rate are rarely criticized in models built for developed economies, such an assumption would seem starkly unreliable in a country undergoing deep supply side changes. Therefore, much effort was put into modelling institution and path-dependent labour market equilibrium concepts. In particular, the model is tailored to describe high variability of unemployment and participation rates over the post-transformation period.

Another important feature of NECMOD are partially forward looking and model consistent inflation expectations, which bring it closer to recent developments in macromodelling, where great emphasis is put on a role of expectations in shaping decisions of economic agents. The model also allows for a distortionary impact of fiscal policy and inflow of structural funds. In response to observed deepening of economic integration and globalization, domestic prices are relatively closely tied to price developments on the world markets. Moreover, the model takes account of the appreciation trend in the real exchange rate and enables an analysis of equilibrium exchange rate shifts.

The design of the model allows for an analysis of a range of policy and natural experiments, like changes in short-term interest rates, tax reforms or strong migratory outflows in the aftermath of the EU accession. Same developments may be explicitly accounted for in forecast scenarios and published inflation projections.

Finally, the paper gives an insight into technical details of forecast preparation, fan chart construction and data-related issues. As such, it may be treated as a short summary of experience accumulated by the NBP forecasting team and a concise guide into issues related to forecasting in converging economies.
1 Introduction

1.1 Projection models at the NBP

Since 1999, the implementation of the monetary policy in Poland has been conducted within the direct inflation targeting (DIT) framework. Because of the lags between changes to monetary policy instruments and the associated effects on inflation, the new strategy had to be pursued in a forward-looking manner. In particular, it required a much deeper understanding of the monetary transmission mechanism (MTM), clear ideas on propagation of shocks affecting the economy and the ability to formulate conditional forecasts, conventionally named projections. Given the complex nature of any economic structure, formal economic models emerged as indispensable tools for supporting the decision making process in the DIT regime.

Due to the limits and uncertainty related to economic transition, like short time series and lack of well documented stylized facts, the first models developed at the National Bank of Poland (NBP) were relatively simple. In 2000, two structural econometric models emerged as the main tools for preparing regular inflation projections. The first one, NSA, referred to neokeynesian mechanisms and focused on the relation between inflation and changes in aggregate demand, while the second, called MSMI, incorporated also some supply-side elements.\(^1\) In 2004, both models were judged mature enough to use them in official projections. The central inflation projection published in the Inflation Report was then constructed by weighting the solutions obtained from NSA and MSMI.

Despite well acknowledged advantages associated with model averaging, a clear shortcoming of such a combined approach was that its outcome could not always be presented in form of a consistent scenario. This was due to the fact that NSA and MSMI differed substantially in size, structure and variables used. As a result, they could potentially generate qualitatively different predictions for common categories. Given these considerations, a new project was launched, aimed at constructing a new and more comprehensive forecasting tool. The new model, called ECMOD, was completed in 2005 and used henceforth as the main vehicle for generating official NBP projections.\(^2\)

ECMOD had several features that made it particularly useful for preparing regular forecasts.\(^3\) First, it incorporated both supply and demand side of the economy. Second, due to a relatively disaggregated structure, it allowed to analyze economic developments in detail and construct relatively sophisticated scenarios. Third, the set of variables used and forecasted by ECMOD corresponded to the interests of the policy makers and standards prevailing in the European System of Central Banks (ESCB). Finally, the basic structure of the model proved flexible enough to allow future extensions.

1.2 Origins of NECMOD

No model is capable of capturing all forces at work in the economy. By definition, any model is an abstraction and incorporates only those linkages between elements of the underlying economic system that are considered to be most relevant given the interests of final users. Therefore, it should come as no surprise that operating ECMOD involved

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\(^1\) See Klos et al. (2005) for a detailed description of the two models.
\(^2\) Preliminary versions of ECMOD were used in internal forecasting exercises already in 2004.
\(^3\) See Fic et al. (2005) for a documentation of the model structure and its main properties.
making regular refinements, adding new blocks and changing its structure. Most of these changes were aimed at accounting for structural shifts in the Polish and global economy and originated from accumulation of forecasting experience at the NBP. All these changes made ECMOD evolve into a new construct, with hardly any of its elements resembling those from its infancy. This upgrading process can only be fully comprehended and acknowledged by preparing a new documentation of this main forecasting tool used at the NBP. We will refer to the new model as NECMOD. Below we provide a brief overview of the main differences between NECMOD and its predecessor. Whenever possible, we demonstrate how these changes can be seen as responses to new challenges to analysts and forecasters dealing with the Polish economy.

Compared to ECMOD, the new model features a fully-fledged labour market block, with labour participation rates and non-accelerating wage rate of unemployment (NAWRU) endogenously determined. Developing the new model structures was motivated by rapid changes on the labour market following Poland’s EU entry, marked by dynamic drops in unemployment, declining labour force participation and significant outflows of workers, mainly to the old member countries that pursued the open-door policy. The latter phenomenon resulted in growing inflows of remittances, which are now explicitly included as a component of households’ disposable income.

Rebuilding the labour market equations necessitated far-reaching changes in the fiscal sector, with positive spillovers to other blocks of the model. As a result, NECMOD now features a comprehensive system of distortionary taxation and public expenditures, affecting labour demand and supply, firms costs and investment decisions. This turned out particularly useful in view of reductions in the tax wedge implemented in 2007 and 2008. Another important modification to the government sector was switching to the ESA’95 methodology, which allowed a better mapping between public revenues as well as expenditures and the national accounts (i.e. public consumption). Finally, the upgraded structure of the fiscal sector made it possible to incorporate new channels through which structural funds affect the economy and link them to the current account components.

Acknowledging the growing role of global factors in determining food and energy prices as well as their decoupling from other consumer prices, the core inflation was redefined to exclude these two groups of goods. Additionally, new equations were build for food and energy CPI components, with explicit links to trends prevailing in the world commodity markets.

Another new feature of the model is the residential investment block, covering both the demand and supply side. The growing importance of the impulses coming from and feeding into the housing sector in Poland have been particularly visible over the last two years, which saw a rapid increase in housing prices and an unprecedented run for mortgage loans. Needless to say, all these developments, together with the overall financial deepening, suggest also a growing importance of the housing wealth channel.

In ECMOD, fixed assets were assumed to be homogeneous. In other words, there was no separate treatment of residential, private and public investment or the respective capital stocks. Confronted with the housing boom and a growing inflow of structural funds from the EU, this created serious problems for assessment of changes in the economy’s productive capacity. In NECMOD, all three components of gross fixed capital formation are modelled separately and only the last two of them translate (in an imperfectly substitutable way) into accumulation of the productive capital stock. This allowed to
incorporate the effect of structural funds inflow in a more comprehensive and less ad hoc fashion. An important new feature of NECMOD that brings it closer to most recent theoretical developments and central bank practices are forward-looking expectations. These are explicitly incorporated, together with their backward-looking counterparts, into two main price equations, i.e. core CPI and the value-added deflator. These changes to the forecasting model seem to be particularly relevant in a direct inflation targeting regime, which is now well established in Poland and probably translates into better anchoring of inflation expectations. Expectations are also present in the interest rate block of NECMOD. Short-term real interest rates are now obtained from the Fisher rule, with a consistent derivation of long-term rates.

Finally, NECMOD’s structure is supplemented with a fully model consistent fundamental equilibrium exchange rate. This was primarily motivated by most recent forecasting experience, facing rapid appreciation of the exchange rate, accompanied by surprisingly strong export performance. While in the current version of the model the equilibrium exchange rate does not enter directly any of the behavioural equations, it ensures consistency while adding judgement to the forecasts and enriches simulation properties of the model.

### 1.3 Comparison with other forecasting models used in central banks

NECMOD is a large scale quarterly macroeconometric model. Its structure comprises all main groups of macrocategories in an economy, including GDP components, costs and prices, labour market, external sector and fiscal variables, and interest rates. The model is detailed enough to serve a wide range of purposes and address policy relevant questions, including forecasting and simulation exercises.

The model can be classified as a hybrid model, i.e. it strikes a balance between theoretical consistency and coherence with the data. NECMOD is structured to have a long-run equilibrium, broadly consistent with the classical theory, while its short-run dynamics is driven by demand disturbances and adjustment mechanisms. Both features are combined by formulating most of behavioural equations in an error-correction form. Such a structure can be seen as consistent with the presence of implicit adjustment costs, constraining the agents in their optimizing behaviour (see Nickell, 1985).

By construct, NECMOD is an econometric model. This means that, although it was necessary to resort to calibrations for some of the parameters, all behavioural equations are estimated using econometric techniques.

In terms of theoretical coherence and proximity to most recent trends in macromodelling, NECMOD should probably be placed somewhere between the estimated and essentially backward-looking AWM (see Fagan et al., 2005) and a family of third generation models (as classified by Fukac and Pagan, 2006), incorporating forward looking elements

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4See the typology in Pagan (2003).

5It is important to note that the disequilibria triggering error correction mechanisms are written as deviations from relations that should hold between subsets of macrovariables rather than deviations from the full steady-state solution of the model.

6Similar types of models were developed e.g. in the Bank of Spain (MTBE, see Ortega et al., 2007) and the National Bank of Hungary (NEM, see Benk et al., 2006). See also the MCM model of the ECB, documented in Estrada and Willman (2002) or Angelini et al. (2006a, 2006b).
but to large extent calibrated.\textsuperscript{7} Compared to other models sharing a similar overall framework, the main strengths of NECMOD can be seen in its detailed structure, richness of transmission channels and flexibility, which make it a useful tool for comprehensive forecasting exercises and simulation analyses.

In particular, the following elements can be seen as NECMOD’s highlights. First, we are not aware of any other central bank model having such a detailed labour market structure, with endogenous participation rates, NAWRU and linkages to fiscal policy instruments (distortionary taxation and transfers). Although a related framework can be found in MTBE maintained at the Bank of Spain, it is far more stylized there. Second, NECMOD is probably the only central bank model allowing for imperfect substitution between public and private capital stocks, and incorporating a consistent set of channels through which inflow of structural funds affects the economy’s productive capacity. Third, NECMOD joins a still narrow group of macroeconometric models having a fully specified housing block.\textsuperscript{8} Fourth, by incorporating expectations in the key equations, NECMOD gets closer to the modern understanding of the MTM than purely backward-looking models.\textsuperscript{9} Finally, an important novel feature of NECMOD is the derivation and operationalization of the model consistent fundamental equilibrium exchange rate. It is apparent that most of the model’s distinctive characteristics reported above are particularly relevant for the EU new member states. Therefore, we view NECMOD as a useful analytical framework for analyzing economic developments in this very group of countries.

Clearly, what we stress as NECMOD’s main advantages, i.e. high level of detail and a large number of transmission channels, can also be seen as the model’s important weaknesses. Large models are relatively difficult to operate and less transparent than more stylized tools. In particular, given plenty of mechanisms at work, decomposing the model output into their separate contributions is rarely straightforward. Therefore, interpretation of relatively sophisticated simulation exercises require quite a profound knowledge of the model structure, which can be seen as an inconvenient constraint on the policy makers.

More importantly, it has to be underlined that the high level of detail in NECMOD comes at the expense of its theoretical coherence. This drawback of traditional large and medium scale econometric models has been well acknowledged in policy making institutions.\textsuperscript{10} Advances in the economic theory, development of efficient solution algorithms and increasing computational power of computers resulted in emergence of a new generation of constructs, i.e. dynamic stochastic general equilibrium models (DSGE). Since the pioneering work of Smets and Wouters (2003), who demonstrated that a sufficiently specified DSGE model can match the data and forecast quite well, a growing number of central banks have embarked on a process of replacing their more traditional forecasting

\textsuperscript{7}See e.g. FRBUS developed in the Federal Reserve (Brayton and Tinsley, 1996), Canada’s QPM (Black et al., 1994 and Coletti et al., 1996), New Zealand’s FPS (Black et al., 1997) or Japan’s JEM (Fujiiwara et al., 2005).

\textsuperscript{8}See e.g. Bank of Spain’s MTBE or Bank of England’s MM (BoE, 2000).

\textsuperscript{9}Mostly for technical and conceptual reasons, big macroeconomic models with forward-looking expectations are still relatively rare. There have been attempts to incorporate rational expectations into the AWM model, but mostly for policy simulation purposes and not for forecasting. A relatively big macroeconomic model with forward-looking elements is NIGEM, maintained at the National Institute of Economic and Social Research (see NIESR web-page).

\textsuperscript{10}This was to large extent inspired by the critique of Lucas (1976). See also the discussion in Coletti et al. (1996) or a summary of model development evolution in central banks provided by Pagan (2003).
and simulation tools with estimated DSGE models.\footnote{Among the first central bank DSGE models used in the official projections are: Bank of Sweden’s RAMZES (Adolfsson et al., 2005), Bank of Norway’s NEMO (Brubakk et al., 2006) or Bank of Canada’s ToTEM (Murchison and Remmison, 2006). A hybrid model, BEQM (see Harrison et al., 2005), with a DSGE-type core structure, is used by the Bank of England. A DSGE model, NAWM (see Christoffel et al., 2007), is currently under development and is soon to become the main projection model of the European Central Bank.}

However, given the current stage of the DSGE research program, this new generation of models will probably remain complements rather than better substitutes to more traditional, larger and estimated tools for at least another couple of years. This is mainly because most of state-of-art DSGE models are still relatively small and their medium-sized versions are calibrated rather than estimated.\footnote{For instance, large scale calibrated global models are developed at the International Monetary Fund. See Karam et al. (2007) for an overview.} An increasingly popular practice to address less standard policy questions with small and stylized DSGE models, incorporating only the most relevant channels for the questions they are designed to address, is not really a perfectly competitive alternative to having a comprehensive and ready to use model at hand. Therefore, we argue that developing and upgrading large econometric models is still worthwhile and they should remain a part of the suite of models in policy making institutions.

1.4 Methodological aspects

Most of NECMOD’s behavioural equations are expressed in an error-correction form, with the long-run equilibrium relations derived in a consistent way from the model’s theoretical framework and short-term adjustment estimated directly from the data. As regards the long-run relationships, the primary motive underlying their specification is theoretical coherence. The baseline approach is to estimate all free parameters (i.e. those not constrained by the overall structure determined by the underlying theory), but calibration is also used whenever estimations do not yield satisfactory results. Whenever possible, short-run equations are restricted to satisfy the dynamic homogeneity property, which assures that error-correction terms are exactly zero in the steady-state equilibrium.

Like most other models in its class, NECMOD is not estimated as a single system, but rather on an equation-by-equation basis, which is motivated by short time series available. The only exceptions are subsets with cross-equation restrictions imposed on estimated parameters. All equations having a standard error-correction representation are fitted to the data using a two-step procedure, i.e. the long-run relationships estimated (or calibrated) in the first stage enter as an error-correction term in the second stage.

We estimate all equations with forward-looking elements (present in an explicit form or as real interest rates) and those in which endogeneity might be of concern using the generalized method of moments (GMM). The remaining relationships are fitted with the ordinary least squares method (OLS).

The estimation output is interpreted in both economic and econometric terms, with clear priority given to the former. This means that we dismiss the results yielding counterintuitive parameter values or simulation properties, even if it comes at the expense of goodness of fit. Otherwise, we pay attention to standard errors of regressions, autocorrelation in residuals (OLS) and validity of instruments (GMM).
The data used in NECMOD are of quarterly frequency and are seasonally adjusted. They come from official sources, supplemented with estimates done at the NBP. The latter group of variables includes, among others, capital stock series and housing investment. A more detailed description of data preparation procedures can be found in Appendix C and D.

The time series used in NECMOD, and national accounts data in particular, are usually available for the period of 1995-2007. However, to overcome problems related to the transition period and data quality, most of equations are estimated on shorter subsamples, with the earliest observations dropped.

1.5 The structure of this paper

The rest of this paper is organized as follows. Section two provides a detailed description of the model structure. Long-run properties of NECMOD are demonstrated in section three. Section four presents impulse response analysis. The methods used for stochastic simulations, including fan charts, are laid down in section five. Section six concludes.
2 The structure of the model

2.1 Overview

We start the presentation of the model structure by providing a brief and simplified overview of its key relationships, focusing mainly on their long-term components. A more formal description of the equations, including parameter estimates, estimation sample and statistical properties, is postponed to the next subsection.

The central component of the supply side is a production function with two factor inputs: productive capital and labour. The productive capital stock consists of private and public components, treated as imperfect substitutes. Both private and public capital stocks are accumulated in line with the perpetual inventory formula, i.e. the current capital stock is composed of undiscarded past investment flows.

The production function establishes a basis for derivation of factor demands. More specifically, private investment is derived from the profit maximization condition, equating the marginal product of private capital to its real user cost, while labour demand is obtained by inverting the production function. The production function is also used to determine the potential output, which is obtained by substituting the actual labour input with its potential level. The latter is defined as the total labour force, corrected for the equilibrium participation rate and NAWRU.

While output is equal to its potential level in the long-run, it is determined by aggregate demand in the short-run. The aggregate demand definition reflects the main national accounts identity, according to which the gross domestic product (GDP) is a sum of private and public consumption, gross fixed capital formation (including private and public productive investment as well as housing), change in inventories, exports and imports (the latter with a negative sign).

Private consumption and residential investment decisions of the households are founded on a common life-cycle permanent income framework, with an isoelastic and separable utility function. Hence, consumption expenditures can be written as a function of households’ permanent income (proxied by a bundle composed of current disposable income and wealth) and real interest rate, while the housing market is represented by a system incorporating both demand (private consumption) and supply determinants (potential output and real user cost of residential capital).

The labour market block of NECMOD is based on a bargaining model, in which firms and unions negotiate over wages and employment. Bargaining interactions are represented by the wage curve and the price curve. The former is derived from the utility maximization problem of a union and relates wages (corrected for the tax wedge) to the bargaining position of employees. The latter is affected by unemployment rate and relevant replacement rates. The price curve corresponds with profit maximization performed by firms and ties real wages to labour productivity. These two relationships jointly determine the behaviour of wages and can be solved to yield the model consistent estimate of NAWRU.

Participation rates are determined separately for three age groups of the labour force, reflecting heterogeneity of motives governing people’s decisions to remain active on the labour market. These include age-specific replacement rates, the tax wedge, the minimum wage rate and the unemployment rate.

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13It is the same relationship that describes the price-setting behaviour of firms in the long run.
Both producer prices and core consumer prices are set as a mark-up over marginal costs, represented by unit labour costs and import prices, with an explicit role for backward and forward-looking (model consistent) expectations. Food and energy consumer prices are modelled as a bundle of foreign and domestic components, the former proxied by appropriate world commodity prices (exogenous to the model).

Long-run export and import volumes are driven by potential and actual product respectively. Price competitiveness indexes entering export and import volumes equations are defined relative to domestic prices, represented by the value-added deflator. This implies a positive response of exports to relative export prices and a negative reaction of imports to relative import prices. In the short run, export volume additionally depends on external demand, while imports on a bundle of GDP components, weighted by their import content. Foreign trade transaction price setting is modelled as a mixture of producer and local currency pricing, which means that export and import prices are a weighted average of foreign and domestic prices.

The system of four long-run foreign trade relations, together with a set of equilibrium conditions (e.g. optimal net foreign asset position, interest paid on foreign debt, inflow of current transfers) implicitly defines the model consistent fundamental equilibrium exchange rate (FEER), which can be used to assess permanent changes in Poland’s international competitiveness. The current real exchange rate formulation incorporates the nominal convergence hypothesis, according to which Poland’s real convergence is accompanied by a declining price level gaps vis-a-vis its trading partners. The equation is supplemented with a set of behavioural-like variables, including the real interest rate disparity and the net foreign assets position.

The government block is relatively detailed and comprises a wide range of expenditure and revenue components, with a distortionary impact on the rest of the economy. Public debt stabilization is ensured by a fiscal rule, using effective tax rates, transfers and some other expenditures as instruments.

The short-term interest rate is set by the monetary authority following a Taylor-like feedback rule. Long-term interest rates are determined in line with the expectation hypothesis, corrected for risk factors.

The system of behavioural equations is supplemented with a set of identities, including stock-flow relations, like accumulation of net foreign assets or public debt.

2.2 Main equations\textsuperscript{14}

2.2.1 Production and production factors

The core of the supply-side block in the model is the production function. It is used to derive factor demand equations and serves as a basis for potential output assessment.

Production function Production technology in the model, linking output (GDP) to inputs, is represented by a two-factor Cobb-Douglas production function with constant returns to scale and exogenous trend total factor productivity (TFP\_TREND):

\textsuperscript{14}Detailed definitions of variables used in this section can be found in Appendix A. Lower case letters denote natural logarithms. Stars refer to levels of variables consistent with the relevant cointegrating relationship. Standard errors of parameters are reported below their point estimates in parenthesis.
\[ GDP_t = TFP_{\text{TREND}}t \cdot EMP_t^{0.67} \cdot KN_t^{1-0.67} \]  

(1)

It is assumed that labour input (EMP) is homogenous, whereas capital input (KN) consists of two types of productive assets, namely corporate capital (KN_P) and public capital (KN_G).\(^{15}\) These are combined using CES technology, with elasticity of substitution calibrated at \( \frac{1}{3} \):

\[ KN_t = (0.7^{2} \cdot KN_P^{2} - (1 - 0.7)^{3} \cdot KN_G^{2})^{\frac{1}{2}} \]  

(2)

Output elasticity with respect to labour input is set to 0.67, which corresponds to the average labour share according to the national accounts, adjusted for income of self-employed and mark-up estimates for Poland.\(^{16}\) Calibration of the steady-state share of corporate capital in total productive capital stock at 0.7 is based on respective shares observed in developed economies.\(^{17}\)

**Capital accumulation**  Productive capital is accumulated as a result of investment according to the following perpetual inventory formulas:\(^{18}\)

\[ K_P = (1 - DISC_P) \cdot K_{P_{t-1}} + 0.30 \cdot GFCF_P + \\
+ 0.20 \cdot GFCF_{P_{t-1}} + 0.22 \cdot GFCF_{P_{t-2}} + \\
+ 0.09 \cdot GFCF_{P_{t-3}} + 0.08 \cdot GFCF_{P_{t-4}} + \\
+ 0.06 \cdot GFCF_{P_{t-5}} + 0.05 \cdot GFCF_{P_{t-6}} \]  

(3)

\[ K_G = (1 - DISC_G) \cdot K_{G_{t-1}} + 0.14 \cdot GFCF_G + \\
+ 0.15 \cdot GFCF_{G_{t-1}} + 0.18 \cdot GFCF_{G_{t-2}} + \\
+ 0.18 \cdot GFCF_{G_{t-3}} + 0.15 \cdot GFCF_{G_{t-4}} + \\
+ 0.12 \cdot GFCF_{G_{t-5}} + 0.09 \cdot GFCF_{G_{t-6}} \]  

(4)

where GFCF_P (GFCF_G) is corporate (public) investment and DISC_P (DISC_G) stands for the discard rate for gross corporate (public) capital, denoted as \( K_P \) (\( K_G \)).

**Investment demand**  In the long run, corporate investment evolves to equalize the marginal product of corporate capital with its real user cost (RUCC):

\(^{15}\)While productive assets enter the production function in net terms, their accumulation is expressed in gross terms (see equations 3 and 4). It is assumed that these two concepts of capital stock are linked by a constant proportionality coefficient.

\(^{16}\)See Gradzewicz and Hagemejer (2007).

\(^{17}\)The main benchmark country is Spain. We also use data for Portugal, since its recent history (EU accession and accumulation of public capital) may turn out to be similar to the current situation of Poland. Data from USA, Australia and the UK are used as a benchmark for the long-run public to private capital ratio. See e.g. BBVA Foundation (2008) or Kamps (2006).

\(^{18}\)For more details on capital series construction see Appendix C.
The structure of the model

\[ \begin{align*}
0.7^3 \cdot (1 - 0.67) \cdot \frac{GDP\_POT_t}{KN_t} \cdot \left( \frac{KN_t}{KN\_P_t} \right)^3 = RUCC_t 
\end{align*} \]

where \( GDP\_POT \) stands for potential output. The real user cost of corporate capital is defined as a sum of quarterly alternative cost of capital investment \((R\_RATE/4)\), the risk premium \((RISK\_P)\) and the depreciation rate \((DEPR\_P)\), corrected for the effective net tax burden on enterprises \((G\_CORP\_TR)\). The latter takes into account the effective rate of corporate income tax, government subsidies to producers and capital transfers as well as EU investment funds directed to private firms and farmers:

\[ RUCC_t = \frac{R\_RATE_t/4 + RISK\_P_t + DEPR\_P_t}{1 - G\_CORP\_TR_t} \]

In the short run, the investment process is characterised by high inertia and procyclicality. It also depends on the growth rate of the real oil prices and the financial position of enterprises \((FINACC)\), with the latter aimed at capturing the financial accelerator effect:

\[ \Delta gfcf\_p_t = 0.37 \cdot (MPK_{t-1} - RUCC_{t-1}) + 0.13 \cdot \Delta gfcf\_p_{t-1} + \]
\[ +0.08 \cdot \Delta gfcf\_p_{t-2} + (1 - 0.13 - 0.08) \cdot \Delta gdpt_{t-1} + \]
\[ +0.05 \cdot \Delta_4 FINACC_t - 0.006 \cdot \Delta_4 (p\_oil_t + s\_usd\_pln^c - PVA_t) \]

\[ \text{Adjusted } R^2 = 0.04 \]
\[ \text{S.E. of regression} = 0.043 \]
\[ \text{LM test } (p\text{-value}) = 0.80 \]
\[ \text{Estimation sample: 1998q1 - 2007q4} \]

where \( MPK \) is the marginal product of corporate capital defined by the left hand side of equation (5), \( P\_OIL \) is the price of oil, \( S\_USD\_PLN^c \) stands for the dollar-zloty exchange rate, adjusted for the nominal convergence effect (see section 2.2.4), and \( PVA \) is the value-added deflator.

Public investment is a sum of government capital expenditures \((GE\_GFCF)\) and infrastructural investment financed directly from the EU structural funds \((TRANS\_GFCF)\) both expressed in real terms:

\[ GFCF\_G_t = GE\_GFCF_t + TRANS\_GFCF\_G_t \]

Government investment is assumed to grow in line with aggregate output in the short-run, whereas in the long run it is pinned down by the assumption equating the marginal products of public and corporate capital.

Nominal corporate investment \((GFCF\_P\_N)\) is obtained by multiplying its volume with the appropriate deflator \((PGFCF\_P)\), assumed proportional to the value-added deflator. Nominal government investment \((GFCF\_G\_N)\) is defined in a similar manner, except that its deflator \((PGFCF\_G)\) additionally incorporates the effective VAT rate.

\(^{19}\) Alternative cost of capital investment is defined as an average of short-term (3-month) and long-term (5-year) interest rates, deflated by the expected value-added deflator and inflation target, respectively.
Labour demand The long-run demand for labour \((EMP)\) is derived as an inverse of the production function. Short-run fluctuations of employment are explained by an autoregressive component and changes in the effective labour force supply, tied either to changes in the number of labour market participants \((LF)\) or to shifts in the equilibrium unemployment rate \((NAWRU)\). The remaining explanatory factors include changes in the real labour costs incurred by employers, corrected for the trend labour productivity growth, and general government or EU expenditures on the active labour market policy and human capital development \((ALMP\_N)\) relative to nominal GDP \((GDP\_N)\):

\[
\Delta emp_t = \frac{0.05 \cdot (gdp_{t-1} - 0.67 \cdot emp_{t-1} - (1 - 0.67) \cdot kn_{t-1} - tfp\_trend_{t-1}) + (9)}{0.03} + 0.55 \cdot \Delta emp_{t-1} + (1 - 0.55) \cdot \Delta \log(LF_t \cdot (1 - NAWRU_t)) + \]
\[
-0.07 \cdot \Delta (wage\_n_t + GR\_CORP\_TR_t - pwa_t - tfp\_trend_t/0.67)
\]
\[
+2.82 \cdot \Delta \left( \frac{ALMP\_N_t}{GDP\_N_t} \right)
\]

Adjusted \(R^2 = 0.66\)
S.E. of regression = 0.004
LM test (p-value) = 0.14
Estimation sample: 1995Q3 - 2007Q4

Inventories The model does not provide a deep-rooted account of inventory determinants. In the long-run, the ratio of stocks \((STOCK)\) to GDP is constant, so that changes in inventories just reflect fluctuations in the overall economic activity. The assumed procyclicality of inventory investment is motivated by the fact that they consist mainly of materials and unfinished production:

\[
stock^*_t = -0.31 + gdp_t + 0.09 \cdot D04Q2 \quad (10)
\]

The short-run response of inventories to changes in GDP occurs with a delay:

\[
\Delta stock_t = \frac{-0.03 \cdot (stock_{t-1} - stock^*_t) + (11)}{0.02} + 0.54 \cdot \Delta stock_{t-1} + (1 - 0.54) \cdot \Delta gdp_t
\]

Adjusted \(R^2 = 0.68\)
S.E. of equation = 0.004
J-statistic(p-value) = 0.61
Estimation period: 1996Q1 - 2007Q4

Changes in inventories entering the GDP identity are denoted by \(INV_t\) and equal to \(\Delta STOCK_t\). A nominal increase in inventories \((INV\_N)\) is obtained in a similar fashion, with nominal stocks \((STOCK\_N)\) defined as a product of its real counterpart and the eight-period moving average of the value-added deflator. The ratio of \(INV\_N\) and \(INV\) implicitly defines the deflator for the change in inventories \((PINV)\) entering the GDP identity.
2.2.2 Households’ sector

We assume that households maximize utility from consumption and housing services subject to a budget constraint. In the first two paragraphs we provide definitions of income and wealth, which approximate constraints faced by households. The following paragraphs include a description of behavioural equations determining individual consumption, residential investment and housing prices.

**Disposable income**  Households’ nominal disposable income \( YD\_N \) is a sum of remuneration from work (wage fund, \( YD\_WF\_N \)), gross operating surplus generated by micro-firms including farmers (\( YD\_NOS\_N \)), property income (\( YD\_PRO\_N \)), social transfers (\( GE\_SOC\_CASH \)), social relief in kind (\( GE\_RELIEF\_KIND\_N \)), direct transfers to farmers financed from the Common Agricultural Policy (CAP) funds (\( TRANS\_CAP\_N \)) or co-financed by the government (\( GE\_SUB\_FARM\_N \)) and the balance of remittances (\( REM\_BALANCE \)), all net of social security contributions levied either on employed in non-farm sectors (\( GR\_EMP\_N \)) or on farmers (\( GR\_FARM\_N \)), healthcare contributions (\( GR\_HC\_N \)), income and property taxes collected from households (\( GR\_PIT\_N \) and \( GR\_OTAX\_HH\_N \)):

\[
YD\_N_t = YD\_WF\_N_t - GR\_EMP\_N_t + \\
+YD\_NOS\_N_t + YD\_PRO\_N_t + \\
+GE\_SOC\_CASH\_N_t + GE\_RELIEF\_KIND\_N_t + \\
+REM\_BALANCE_t \cdot S\_EUR\_PLN_t + \\
+TRANS\_CAP\_N_t + GE\_SUB\_FARM\_N_t + \\
-GR\_FARM\_N_t - GR\_HC\_N_t - GR\_PIT\_N_t + \\
-GR\_OTAX\_HH\_N_t
\]

The gross operating surplus generated by the household sector is expected to be proportional to GDP in the long run. The property income is modelled in a similar fashion, but additionally depends on the rate of return on wealth (\( WEALTH \)), proxied by the short term (3-month) interest rate \( I\_3M \):

\[
yd\_nos\_n_t^* = -1.32 + gdpr\_n_t \quad \text{(13)}
\]

\[
yd\_pro\_n_t^* = -3.21 + gdpr\_n_t + 0.39 \cdot \frac{WEALTH_t}{GDP_t} \cdot I\_3M_t \quad \text{(14)}
\]

In the short-run, both categories of the disposable income follow simple error-correction processes:

\[
\Delta yd\_nos\_n_t = 0.73 \Delta gdpr\_n_t - 0.46(yd\_nos\_n_{t-1} - yd\_nos\_n_{t-1}^*) + \\
-0.16(I99Q2 - I99Q3) \quad \text{(15)}
\]

*Adjusted $R^2 = 0.74$*
S.E. of equation = 0.028
LM test (p-value) = 0.01
Estimation period: 1997q1 - 2007q4

\[ \Delta yd\_pro\_n_t = 0.01 (0.01) - 0.10 (0.05) (yd\_pro\_n_t \text{\(\_\)} - yd\_pro\_n_t \text{\(\_\)\(-1\)} \] (16)

Adjusted \(R^2\) = 0.08
S.E. of equation = 0.034
LM test (p-value) = 0.00
Estimation period: 1996q1 - 2007q4

Wealth  Households’ nominal wealth (WEALTH\_N) is defined as a sum of corporate and residential capital, both expressed in current prices by multiplying the real stocks (\(K\_P\) and \(K\_H\)) by appropriate price indexes (the GDP deflator PGDP and housing prices PGFCF\_H),\(^{20}\) public debt (G\_DEBT) and Poland’s international investment position (NFA):

\[ WEALTH\_N_t = K\_P \cdot PGDP_t + K\_H \cdot PGFCF\_H_t + \\
G\_DEBT\_N_t + NFA_t \cdot S\_EUR\_PLN_t \] (17)

The real disposable income (YD) and real wealth (WEALTH) are obtained by deflating their nominal counterparts with the consumer price index (CPI).

Consumption  The cointegrating relation for individual consumption is derived from the life-cycle hypothesis, according to which a representative consumer smooths out spending in line with her evaluation of permanent income, after Lettau and Ludvigson (2004) proxied by a weighted average of current disposable income and wealth. A higher level of real short-term interest rates (I\_3MR\_CPI) encourages households to postpone consumption:

\[ \text{comp}_t = -0.25 + 0.92 \cdot yd_t + (1 - 0.92) \cdot \text{wealth}_t - 0.38 \cdot I\_3MR\_CPI_t \] (18)

Short run fluctuations in consumption are driven by fluctuations in disposable income and changes in real interest rates. The former regressor is aimed to capture the fact that not all households have access to financial markets, so their consumption moves in line with their current income:

\[ \Delta \text{comp}_t = 0.008 (0.00) - 0.12 (0.03) (\text{comp}_{t-1} - \text{comp}_{t-1}^*) \] (19)

Adjusted \(R^2\) = 0.28
S.E. of equation = 0.005
J statistic (p-value) = 0.06
Estimation period: 1996q3 - 2007q4

Nominal individual consumption expenditures (CONP\_N) are obtained by multiplying their volume with the appropriate deflator (PCONP), assumed equal to the consumer price index (CPI).

\(^{20}\)The current version of the model does not have a separate price index for corporate capital, so we approximate it with the overall price level, i.e. the GDP deflator.
Residential investments and prices A representative household derives utility from
cconsumption of goods and housing services. If its utility can be expressed in a standard
separable constant relative risk aversion form and intertemporal elasticities of substitution
of consumption and housing are equal, the long run demand for housing services should
be proportional to consumption, with deviations tied to fluctuations in the user cost of
residential capital (RUCC\_H) (see Pain and Westaway, 1997):

\[ k\_h_t^* = 0.38 + comp_t - 0.38 \cdot rucc\_h_t \]  \hspace{1cm} (20)

The user cost of residential capital (written in annual terms) is defined as follows:

\[ RUCC\_H_t = (I\_H_t - INF\_TARGET_t + 4 \cdot DEPR\_H_t) \cdot PGFCF\_H_t/CPI_t \]  \hspace{1cm} (21)

where the interest rate on loan mortgages (\( I\_H \)) is a weighted average of domestic and
foreign interest rates. As the average maturity of a mortgage credit is long, the real interest
rate on housing loans is expressed as the nominal interest rate deflated by the inflation
target (\( INF\_TARGET \)), to which inflation expectations are supposed to converge in
the longer run. Profitability of housing investment also depends on the depreciation rate
(\( DEPR\_H \)) of residential capital.

The supply side of the housing block is represented by a relation linking the share of
residential construction in potential output to relative prices of houses:

\[ gfcf\_h_t^* = -3.36 + gdp\_pot_t + 0.27 \cdot (pgfcf\_h_t - pva_t) \]  \hspace{1cm} (22)

Large variation of housing prices documented for many developed countries (eg. OECD
Economic Outlook 2000, p. 169) indicates at substantial real rigidities in the residential
construction sector. Changes in demand conditions translate mainly into prices and the
housing supply adjusts only with a lag. In order to capture this empirical regularity, we
use the demand equilibrium condition (20) in the short run housing price equation, while
the short run residential investment equation incorporates the supply-side relation (22).
Moreover, the residential deflator fluctuates with changes in real mortgage interest rates
and overall economic activity (proxied by the output gap \( GAP \)), exhibiting a substantial
degree of inertia:

\[
\Delta pgfcf\_h_t = -0.09 \times (k\_h_{t-1} - k\_h_{t-1}^*) + 0.20 \times GAP_t + \\
+0.83 \times \Delta pgfcf\_h_{t-1} + (1 - 0.83) \times \Delta cpi_{t-1} + \\
-0.09 \times \Delta (I\_H_t - \left( \frac{PGFCF\_H_{t-1}}{PGFCF\_H_{t-5}} - 1 \right) + 4 \times DEPR\_H_t)
\]  \hspace{1cm} (23)

Adjusted \( R^2 = 0.81 \)
S.E. of equation = 0.015
LM test (p-value) = 0.16
Estimation period: 1996q3 - 2007q4
Short run changes in residential investment volume are also inertial and limited by the growth of potential GDP, approximating the possibility of extending the building site:

\[
\Delta gcfc_{\_h_t} = -0.26 \cdot (gcfc_{\_h_{t-1}} - gcfc_{\_h_{t-2}}) + 0.62 \cdot \Delta gpc_{\_pot_t} + 0.21 \cdot \Delta gcfc_{\_h_{t-1}} + (1 - 0.62 - 0.21) \cdot \Delta gcfc_{\_h_{t-2}} + 0.13 \cdot I02Q2 - 0.23 \cdot I02Q3 - 0.16 \cdot I03Q4
\]

\(\text{Adjusted } R^2 = 0.60\)
\(\text{S.E. of equation } = 0.043\)
\(\text{LM test (p-value) } = 0.97\)
\(\text{Estimation period: } 1999q1 - 2007q4\)

A product of residential construction volume and housing prices defines nominal residential investment (\(GFCF\_H\_N\)).

### 2.2.3 Labour market

The foundation of specification of labour market block is a wage bargaining model inspired by theoretical model McDonald and Solow (1981) and its empirical implementation by Chamberlain et al (2002). The approach used in NECMOD is described in more detail in Budnik (2008). Bargaining interactions are represented by the wage curve and the price curve. The former is derived from the first order condition for the expected utility maximization problem of a union and can be interpreted as labour supply. The latter is an outcome of profit maximization problem of a firm and can be thought of as representing labour demand. The two relations jointly determine the behaviour of wages, producer prices and the equilibrium rate of unemployment (NAWRU) in the medium and long-run.

Labour force supply is modelled separately for three age groups. The age groups were distinguished in the approach to account for shifts in labour supply tied to demographic factors. The labour force supply is allowed to evolve with economic activity in the economy (as proxied by unemployment rate). In fact, flexible specification of medium-run labour supply for the age groups allows to establish empirically importance of discouraged and marginal worker effects. The course employed to model the labour supply enables as well derivation of the equilibrium participation rate.

**Wages** The wage rate in the long-run is determined by two cointegrating vectors. The first one, interpreted as the wage curve, relates the real after tax wage to the bargaining position of a worker. The real after tax wage is represented by the average gross monthly wage (\(WAGE\_N\)) deflated by the CPI index and adjusted for effective rates of direct income taxes (\(GR\_PIT\_TR\)), social security contributions levied on employees (\(GR\_EMP\_TR\)) and healthcare contribution (\(GR\_HC\_TR\)). The bargaining position of workers is assumed to be affected by unemployment rate (\(UNRATE\)), and a range of institutional factors. Namely, unemployment income replacement rate defined as expected
income when unemployed over the average net wage rate \((RR\_UNEMP)\) and the minimum wage, expressed in relation to the average wage \((MINW)\). As the minimum wage is expected to affect mostly young, low productivity workers with earnings are on the margin of minimum wage, the detrimental impact of the minimum wage regulations on the labour market is assumed to be proportional to the share of population aged 15-24 in the total population \((POP\_Y/POP)\). The wage curve is augmented with the ratio of average level of remittances to wages \((RR\_REM)\), which aims to capture the impact of remittances on softening the budget constraint faced by entrepreneurial workers. Last but not least, aspiration wage of workers are expected to account for growth in total factor productivity:

\[
\text{wage}_n^* = 5.39 + tfp\_trend_t/0.67 + cpi_t - 0.5 \cdot INTAX_t + \\
-1.02 \cdot UNRATE_t + 0.08 \cdot rr\_unemp_t + \\
+0.5 \cdot (GR\_PIT\_TR_t + GR\_HC\_TR_t + GR\_EMP\_TR_t) + \\
-0.5 \cdot GR\_CORP\_TR_t - 0.18 \cdot rr\_rem_t + \\
+0.79 \cdot \left(\frac{POP\_Y}{POP_t}\right) \cdot \text{minw}_t 
\]

The elasticity of wages with respect to all components of the overall tax wedge is calibrated at 0.5 in order to reflect an incomplete pass-through of tax changes to level of employment in the long-run. In line with this calibration, the wage curve includes correction for the level of indirect taxes \((INTAX)\), defined as the sum of effective VAT, excise duty and gambling tax rates \((GR\_VAT\_TR, GR\_EXT\_TR\) and \(GR\_GAM\_TR\) respectively) and social security contributions with \(GR\_CORP\_TR\) denoting the effective rate for social security contributions collected on employers.

The second cointegrating vector, the price curve, relates real producer wages, deflated with a composite index of domestic and import prices \((PVA\) and \(PIMP\), the latter corrected for the effective duty tax rate \(GR\_TAR\_TR\), to labour productivity in the non-farm sector, defined as the ratio of GDP to non-farm employment \((EMPNA)\):

\[
\text{wage}_n^{**} = 1.43 \cdot pva_t + 4.99 - GR\_CORP\_TR_t + \\
+ gdp_t - empna_t + \\
-0.43 \cdot (pimp_t + GR\_TAR\_TR_t) + \\
+0.16 \cdot (1 - D04Q2) 
\]

or equivalently:

\[
pva_t^* = -3.49 + 0.70 \cdot ulcna_t + \\
+(1 - 0.70) \cdot (pimp_t + GR\_TAR\_TR_t) + \\
-0.11 \cdot (1 - D04Q2) 
\]

where \(ULCNA\) denotes non-farm unit labour costs defined below by equation (58).

The formula (27) is used to model long-run evolution of the value-added deflator (see section 2.2.6).
The structure of the model

Equations (25) and (26) jointly determine the long-run behaviour of wages. In the short-run, wages are characterized by certain degree of inertia and depend on inflation, changes in labour productivity and changes in the unemployment rate:

\[
\Delta wage_{-n_t} = -0.05 \cdot (wage_{-n_{t-1}} - wage_{-n_{t-1}}^*) +
\]
\[-0.04 \cdot (wage_{-n_{t-1}} - wage_{-n_{t-1}}^{**}) +
\]
\[+0.59 \cdot \Delta wage_{-n_{t-1}} + (1 - 0.59) \cdot \Delta cpi_{t-1} +
\]
\[+(1 - 0.59) \cdot \Delta (gdp_t - empna_t) +
\]
\[-0.56 \cdot \Delta UNRATET_t
\]

\[
Adjusted R^2 = 0.80
\]
S.E. of equation = 0.002
LM test (p-value) = 0.36
Estimation period: 1996q1 - 2007q4

**NAWRU**

Unemployment is at the equilibrium level when the conditions imposed by the price and wage curves are simultaneously satisfied. Substituting (25) into (26) and taking into account the form of the production function (1) leads to the following formula for the non-accelerating wage rate of unemployment (NAWRU):

\[
NAWRU_t = \left[(cpi_t - pva_t - \frac{0.11}{0.70} \cdot (1 - D04Q2) - 0.5 \cdot INTAX_t) +
\right.
\]
\[+0.5 \cdot (GR\_PIT\_TR_t + GR\_HC\_TR_t +
\]
\[+GR\_EMP\_TR_t + GR\_CORP\_TR_t) +
\]
\[+0.08 \cdot rr\_unemp_t + 0.79 \cdot \left(\frac{POP\_Y_t}{POP_t}\right) \cdot minw_t +
\]
\[+\frac{1 - 0.70}{0.70} \cdot (pimp_t + GR\_TAR\_TR_t - pva_t) +
\]
\[-\frac{EMP\_A_t}{LF_t} + (1 - 0.70) \cdot (lf_t - k_t) +
\]
\[+(1 - 0.70) \cdot tfp\_trend_t - \frac{3.49}{0.70} + 5.39 +
\]
\[-0.18 \cdot rr\_rem_t]/(1 - 0.67 + 1.02)

Thus, the equilibrium level of unemployment increases with the share of young people (aged 15-24) in total population, the minimum wage rate and the tax wedge \((GR\_PIT\_TR GR\_HC\_TR + GR\_EMP\_TR + GR\_CORP\_TR), while inflow of remittances and the share of people working in agriculture \((EMP\_A/LF)) are expected to bring it down. The two key parameters that describe NAWRU sensitivity to changes in its determinants are labour share in GDP \((0.67, see equation (1)) and the semi-elasticity of wages with respect to unemployment \((1.02 from equation (25)). The higher the former and the lower
the latter, the stronger is the reaction of NAWRU, as the adjustment on the labour market takes place mainly through the level of employment and to smaller extent through wages. Moreover, NAWRU is related to the price wedge (CPI/PVA) and the internal terms of trade (PIMP/PVA). Increase in production capacity without accompanying rise in the labour supply lowers the natural unemployment rate. The reverse argument works if labour market participation or working age population build up and adjustment of capital stock lags behind. The above set of variables significantly contribute to hysteresis of unemployment rate in the medium-run. Still, in the long-run NAWRU converges the level dependent only on institutional variables as relative prices and the relation between trend TFP, the capital stock and labour supply are stationary.

**Labour force**  
Labour supply in the model, defined as the number of labour market participants (i.e. employed and unemployed), is divided into three age groups, namely young workers (15-24 years), prime age workers (25-34 years) and elderly workers (45 years or more). The split of labour supply is motivated by significant differences in the participation rates between the age groups and dynamic changes in the age structure of working population of Poland. Due to presence of downward trend in labour force participation rate tied to demographic transition the aggregate approach could therefore lead to biased estimates of economically interpretable parameters of regressions. Second, for each of these groups participation rate is likely be affected by a different set of factors.

Labour market participation is assumed to follow the business cycle. Dependence of labour supply on cyclical swings is a well documented fact in the literature (among recent research on correlation between the two: Briscoe and Wilson (1992), Darby, Hall and Vecchi (1998), Cutler and Tumbull (2001), Schweitzer and Tinsley (2004)). Here, the decision of a person to enter or exit the labour market is assumed to be governed by the discouraged and the marginal worker effects. According to the former, probability of withdrawal from the labour market is decreasing in probability of getting a job. Therefore, high unemployment rate depress participation rate. The marginal worker effect is at work when income effects dominates substitution effect within the business cycle. The effect might be particularly strong for other than breadwinners household members. Once the situation on the labour market improves (i.e. unemployment declines) and the wage rate increases, it might be optimal for them to withdraw from the labour market and engage in household production being economically supported by the head of family. Clearly, these two effects act in opposite directions and it is an empirical issue to establish which outweighs the other in each of the age groups.

**Labour force 15-24**  
The participation rate of the young, defined as the ratio of labour force participants aged 15-24 (LF_Y) to the relevant population (POP_Y), depends negatively on unemployment rate UNRATE which is expressed as a four quarter moving average to reflect the fact that labour supply of the young reacts with a lag to situation on the labour market. Hence, for the young the discouraged worker effect outweighs the marginal worker effect in the business cycle. Because most of young and inexperienced workers are characterized by low productivity and earn at the margin of the minimum wage, an increase in the latter (as compared to the average wage in the economy) in-
creases the number of young workers entering the labour market. Social contributions and direct taxes on labour ($GR\_EMP\_TR+GR\_PIT\_TR+GR\_HC\_TR$) as well as indirect taxes ($INTAX$) influence negatively the participation rate of young. The effect is stronger for the young and elderly than compared to other population groups. Two dummies control for institutional changes in the unemployment benefits scheme ($D97Q1$) and compulsory healthcare insurance ($D99Q1$). Impact on the labour market of growing scholarization over the sample period is captured by inclusion of a trend variable $STUDENT$, defined as a smoothed share of non-extramural students to the size of the population of young. The replacement rate for remittances ($RR\_REM$) captures a negative effect of transfers from abroad on labour market activity of emigrants’ family members who stayed in Poland:

$$
\left( \frac{LF\_Yt}{POP\_Yt} \right)^* = 0.81 - 0.33 \cdot UNRATE_t - 0.3 \cdot INTAX_t + 
-0.3 \cdot (GR\_EMP\_TR_t + GR\_PIT\_TR_t + GR\_HC\_TR_t) + 
+0.21 \cdot minw_t - 0.4 \cdot STUDENT_t - 0.02 \cdot rr\_rem_t + 
-0.03 \cdot (1 - D99Q1) + 0.02 \cdot (1 - D97Q1)
$$

(30)

Short-run fluctuations of the young labour force are determined by factors accounted for in the long run relation and also depend on the real wage growth ($WAGE\_N/CPI$) corrected for the long-run trend in labour productivity growth ($TFP$):

$$
\Delta lf\_yt = -0.45 \cdot \left( \frac{LF\_Yt-1}{POP\_Yt-1} - \left( \frac{LF\_Yt-1}{POP\_Yt-1} \right)^* \right) + 
+0.33 \cdot \Delta lf\_yt-1 + 0.33^2 \cdot \Delta lf\_yt-2 + 
+(1 - 0.33 - 0.33^2) \cdot \Delta pop\_yt + 
+0.15 \cdot \Delta (wage\_nt - cpi_t - tfp\_trend_t/0.67) + 
-0.4 \cdot \Delta STUDENT_t
$$

(31)

$Adjusted \ R^2 = 0.44$

$S.E. \ of \ equation = 0.011$

$LM \ test \ (p-value) = 0.91$

$Estimation \ period: \ 1996q1 - 2007q4$

**Labour force 25-44** The participation rate of prime age workers shows a moderately positive dependence on the average unemployment rate, so the marginal worker effect outweighs the discouraged worker effect for this age group. The impact of tax burden on the participation rate of prime age workers is significantly weaker than for young or elderly. Similarly, social benefits replacement rate for non-participants in the prime age (including disability and retirement benefits as well as social reliefs, $RR\_NLF\_M$) is

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21In the short run, this effect can be offset by a contraction in labour demand, but in the long run both channels act in the same direction.
relatively small. In general, this specification is consistent with a small variability of the participation rate of prime wage workers over the business cycle and its low sensitivity to institutional changes:

\[
\left( \frac{LF_{- M_t}}{POP_{- M_t}} \right)^* = 0.88 + .041 \cdot UNRATE_t - 0.01 \cdot rr_{- nlf_{- m_t}} - 0.1 \cdot INTAX_t + (32)
\]

\[
= -0.1 \cdot (GR_{- EMP\_TR_t} + GR_{- PIT\_TR_t} + GR_{- HC\_TR_t}) + \\
-0.004 \cdot (1 - D99Q1) + 0.003 \cdot (1 - D97Q1)
\]

Short run deviations of prime-age labour force from its equilibrium path are explained by changes in the population and with relatively low inertia:

\[
\Delta l f_{- m_t} = -0.35 \cdot \left( \frac{LF_{- M_t-1}}{POP_{- M_t-1}} \right) - \left( \frac{LF_{- M_t-1}}{POP_{- M_t-1}} \right)^* + (33)
\]

\[
+0.15 \cdot \Delta l f_{- m_t-1} + 0.15^2 \cdot \Delta l f_{- m_t-2} + \\
+(1 - 0.15^2) \cdot \Delta pop_{- m_t}
\]

\[Adjusted \ R^2 = 0.66 \]
\[S.E. \ of \ equation = 0.002 \]
\[LM \ test \ (p-value) = 0.53 \]
\[Estimation \ period: 1996q1 - 2007q4 \]

**Labour force 45+** In the long run, participation rate of workers over 45 depends negatively on the average unemployment rate in the year, with an unemployment rate coefficient estimate being close to that reported for the group of young workers. Labour market activity of elderly depends also on the tax wedge, expected income of non-participant in the age over 45 when out of job to the average wage rate ratio (RR_{- NLF\_O}) and similarly normalized social transfers in kind (RR_{- RELIEF\_KIND}). The variable WORK\_AGE, defined as the ratio of those in the pre-retirement age to the total population aged over 45, is included to capture negative impact on participation rate aging process within this population group:

\[
\left( \frac{LF_{- O_t}}{POP_{- O_t}} \right)^* = 0.62 - 0.31 \cdot UNRATE_t - 0.1 \cdot rr_{- nlf_{- o_t}} - 0.3 \cdot INTAX_t + (34)
\]

\[
= -0.3 \cdot (GR_{- EMP\_TR_t} + GR_{- PIT\_TR_t} + GR_{- HC\_TR_t}) + \\
-0.02 \cdot rr_{- relief\_kind_t} - 0.36 \cdot (1 - WORK\_AGE_t) + \\
+0.01 \cdot (1 - D97Q1) - 0.01 \cdot D07Q1
\]

Short-run fluctuations of the labour force of elderly people reflect changes in its long-run determinants:
\[ \Delta l_f_{o_t} = \frac{1}{0.13} \cdot \left( \frac{LF_{O_t-1}}{POP_{O_t-1}} - \left( \frac{LF_{O_t-1}}{POP_{O_t-1}} \right)^* \right) + 0.11 \cdot \Delta l_f_{o_t-1} + 0.11^2 \cdot \Delta l_f_{o_t-2} + (1 - 0.11 - 0.11^2) \cdot \Delta pop_{o_t} - 0.07 \cdot \Delta rr_{nl f_{o_t}} + -0.01 \cdot (I99Q1 + I99Q3 + I99Q4) \] (35)

Adjusted \( R^2 = 0.55 \)
S.E. of equation = 0.004
LM test (p-value) = 0.96
Estimation period: 1996q1 - 2007q4

Equilibrium participation rate The total labour force (\(LF\)) is obtained as a sum of labour force participants in each of the groups:

\[ LF_t = LF_{Y_t} + LF_{M_t} + LF_{O_t} \] (36)

In this way, demographic factors influencing the evolution of the overall participation rate can be to a great extent accounted for by shifts in the shares of the three age groups in the total population.

The equilibrium participation rate in the economy is defined following:

\[ \left( \frac{LF_t}{POP_t} \right)^* = \left( \frac{LF_{Y_t}}{POP_{Y_t}} \right)^* \left( \frac{POP_{Y_t}}{POP_t} \right) + \left( \frac{LF_{M_t}}{POP_{M_t}} \right)^* \left( \frac{POP_{M_t}}{POP_t} \right) + \left( \frac{LF_{O_t}}{POP_{O_t}} \right)^* \left( \frac{POP_{O_t}}{POP_t} \right) \] (37)

2.2.4 External sector

The external sector of the model consists of five behavioural equations describing the evolution of the exchange rate, export and import prices as well as export and import volumes. In the last paragraph we introduce the concept of fundamental equilibrium exchange rate (FEER), which is the exchange rate level assuring that in the long-run the current account balance to GDP ratio remains on its target level.

Exchange rate The backbone of the exchange rate equation is the purchasing power parity theory (PPP). It states that the nominal exchange rate is determined by the relation of domestic to foreign prices. According to this theory, the real exchange rate, defined as the nominal exchange rate divided by the ratio of domestic to foreign prices \((S_{REER} = S_{NEER} \cdot PVA_{EXT}/PVA)\), should be constant in the long run. The approach followed in NECMOD also refers to the uncovered interest rate parity (UIP)
theory. It predicts that, with free flows of capital, the interest rate disparity between any two countries should be compensated by appropriate change in the exchange rate to assure equality of return on assets denominated in both currencies.

The specification derived from these two theories is augmented with other fundamentals often mentioned in the literature on the subject.\textsuperscript{22} The gap between domestic potential output (GDP\_POT) and external potential output (GDP\_EXT\_POT) is supposed to capture the fact that the price level in developed countries is usually higher than in poorer ones.\textsuperscript{23} In other words, the real effective exchange rate of a country should appreciate if its GDP growth exceeds that of its trade partners, i.e. real convergence should be accompanied by nominal convergence. The parameter for the speed of nominal convergence is calibrated at 0.6, based on cross-section regression of purchasing power parities (PPP) on GDPs for Poland’s main trade partners. An improvement in international investment position, represented by the net foreign assets to GDP ratio (NFA\_GDP), results in the exchange rate appreciation. This mechanism prevents excessive accumulation of foreign assets or liabilities (see paragraph Balance of payments on page 32 for details). Finally, the exchange rate equation includes two step dummies. This is motivated by significant appreciation of the Polish zloty following the EU accession, reflecting a shift in demand for Polish exports (connected with reduction in non-tariff barriers) and inflow of EU funds:

\[
\begin{align*}
    s\_neer_t^* &= 9.12 + pva_t - pva\_ext_t - 0.6 \cdot (gdp\_pot_t - gdp\_ext\_pot_t) + \\
    &\quad -0.25 \cdot NFA\_GDP_t - 1.39 \cdot (I\_3MR\_PV\ A_t - I\_3MR\_EXT_t) + \\
    &\quad -0.07 \cdot D04Q2 - 0.05 \cdot D06Q3 \\
\end{align*}
\]

In the short-run, exchange rate movements are driven by the same set of factors as in the long run:

\[
\begin{align*}
    \Delta s\_neer_t &= -0.34 \cdot (s\_neer_{t-1} - s\_neer_{t-1}^*) + \\
    &\quad + 0.40 \cdot \Delta(pva_t - pva\_ext_t) + \\
    &\quad + 0.41 \cdot \Delta(gdp\_pot_t - gdp\_ext\_pot_t) + \\
    &\quad + 0.49 \cdot \Delta(I\_3MR\_PV\ A_t - I\_3MR\_EXT_t) \\
\end{align*}
\]

\[
\text{Adjusted } R^2 = 0.13 \\
\text{S.E. of equation} = 0.034 \\
\text{J statistic(p-value)} = 0.74 \\
\text{Estimation period: 1996q3 - 2007q4}
\]

\textsuperscript{22}Our framework of modelling the exchange rate is known as the behavioural equilibrium exchange rate approach (BEER), introduced by Clark and MacDonald (1998).

\textsuperscript{23}The most common explanation of this phenomenon is the Balassa-Samuelson hypothesis (see Balassa, 1964; Samuelson, 1964).
Export and import prices Export and core import prices ($P_{EXP}$ and $PIMP\_CORE$) are modelled as a function of domestic prices ($PVA$) and external prices ($PVA\_EXT$). In order to ensure that the convergence effect described in the previous subsection does not change export and import competitiveness, the effective exchange rate used to convert foreign prices to the domestic currency is corrected for the nominal convergence term:\footnote{This is done using the following formula: $s\_neer^e = s\_neer + 0.6 \cdot (gdp\_pot - gdp\_ext\_pot)$.}

\[
\begin{align*}
\text{pexp}_t^* &= -3.42 + 0.37 \cdot (pva\_ext_t + s\_neer_t^e) + (1 - 0.37) \cdot pva_t \\
\text{pimp\_core}_t^* &= -5.94 + 0.61 \cdot (pva\_ext_t + s\_neer_t^e) + (1 - 0.61) \cdot pva_t
\end{align*}
\]

The pass-through of exchange rate fluctuations into export and import prices depends on corresponding demand and supply elasticities.\footnote{See Goldstein and Khan (1985). For instance, under assumption of perfect competition, a small economy is a prices-taker on the world markets, because it faces perfectly elastic demand for exports and perfectly elastic supply of imports (and then the pass-through is complete).} Our estimation results show that in Poland the price-taker effect dominates the price-maker effect for imports, while the opposite is true for exports. It is consistent with the Armington (1969) assumption that imported and domestic goods are imperfect substitutes. As a result, exchange rate appreciation leads to improvement in the terms of trade.

Short-run foreign trade price determinants are the same as long-run ones:

\[
\begin{align*}
\Delta \text{pexp}_t &= - 0.23 \cdot (\text{pexp}_{t-1} - \text{pexp}^*_t) + \\
&+ 0.37 \cdot \Delta (pva\_ext_t + s\_neer_t^e) + \\
&+ (1 - 0.37) \cdot \Delta pva_t
\end{align*}
\]

\[
\begin{align*}
\Delta \text{pimp\_core}_t &= - 0.17 \cdot (\text{pimp\_core}_{t-1} - \text{pimp\_core}^*_t) + \\
&+ 0.46 \cdot \Delta (pva\_ext_t + s\_neer_t^e) + \\
&+ (1 - 0.46) \cdot \Delta pva_t
\end{align*}
\]

Adjusted $R^2 = 0.40$
S.E. of equation = 0.019
$Q$ test ($p$-value) = 0.35
Estimation period: 1996q3 - 2007q4

According to our estimates, the exchange rate pass-through into import prices is lower in the short-run than in the long-run. For export prices, we do not observe differences in their short- and long-run elasticities with respect to exchange rate.
Exports and Imports  The cointegrating relationships for export and import volumes (\(GDP_{EXP}\) and \(GDP_{IMP}\)) are based on the assumption that relative prices affect relative quantities:

\[
gdp_{\text{exp}} = -1.51 + gdp_{\text{pot}} + 0.59 \cdot (p_{\text{exp}} - p_{\text{va}}) + 0.012 \cdot t + 0.06 \cdot D05Q2
\]

\[
gdp_{\text{imp}} = -1.53 + gdp_t - 0.28 \cdot (p_{\text{imp} - \text{core}} + GR_{\text{TAR}_T R} - p_{\text{va}}) + 0.012 \cdot t
\]

The long-run relations are therefore interpreted, respectively, as a supply function of exports and a demand function of imports. The price elasticity of demand for imports is relatively low, consistently with the Armington assumption mentioned above.

In both equations, the linear trend \((t)\) is added to approximate international and, in particular, European trade integration effects. As we mentioned earlier, following the EU accession, there was a shift in demand for Polish exports which is captured by an additional step dummy in the exports equation.

During periods of economic recovery, export and import volumes tend to grow particularly fast. Taking account of a trend in imports and exports shares in GDP, originating from deepening of integration of the Polish economy with its main trading partners, we generalize this regularity to say that during economic booms in growth in export and import volumes tends to outpace the growth resulting only from rise in their relative prices by more than during economic slowdowns. To account for this effect, deviations of demand for exports and imports from their long-run trends were included in the short-run equations. Demand for exports is approximated by external GDP (\(GDP_{EXT}\)) while demand for imports (\(GDP_{IMP}^D\)) by a weighted sum of domestic consumption, exports and gross fixed capital formation, with the weights based on the relative import content of each component.

\[
\Delta(gdp_{\text{exp}} - gdp_{\text{pot}}) = 0.012 - 0.29 \cdot (gdp_{\text{exp}}_{t-1} - gdp_{\text{exp}}_{t-1}) + 2.22 \cdot \Delta(gdp_{\text{ext}} - tfp_{\text{trend}_{\text{ext}}} / 0.67) + 0.03 \cdot \Delta D05Q2
\]

\[Adjusted R^2 = 0.27 \]
\[S.E. of equation = 0.025 \]
\[Q test (p-value) = 0.01 \]
\[Estimation period: 1996q3 - 2007q4 \]

\[
\Delta(gdp_{\text{imp}} - gdp_t) = 0.012 - 0.09 \cdot (gdp_{\text{imp}}_{t-1} - gdp_{\text{imp}}_{t-1}) - 0.24 \cdot \Delta(p_{\text{imp} - \text{core}} + GR_{\text{TAR}_T R} - p_{\text{va}}) + 1.43 \cdot \Delta(gdp_{\text{imp}}^D - tfp_{\text{trend}} / 0.67)
\]
**Adjusted R² = 0.69**
**S.E. of equation = 0.017**
**Q test (p-value) = 0.01**
**Estimation period: 1996q3 - 2007q4**

Nominal exports and imports (GDP\_EXP\_N and GDP\_IMP\_N) are defined as a product of the respective volumes and prices.

**Balance of payments** One of the key variables that influence the exchange rate is the net foreign assets position. As we already mentioned, the presence of this term prevents the NECMOD economy from moving toward a large and persistent external imbalance. Suppose that the current account deficit has been remaining high for a long period. That would lead to the accumulation of Poland’s external assets (NFA, denominated in euro) in line with the following identity:

\[
NFA_t = NFA_{t-1} \cdot \left( \frac{S\_USD\_EUR_t}{S\_USD\_EUR_{t-1}} \right)^{0.4} + CAB_t
\]

where the parameter 0.4 is the fraction of Polish liabilities denominated in U.S. dollars. It follows from equation (38) that the zloty would depreciate, leading to an improvement in exports and reduction of imports in constant prices. Although both exports and imports in current prices rise (since their prices go up), the net effect on the nominal balance of goods and services (CAB\_NT) is positive.\(^{26}\) In consequence, as long as the balance of income (CAB\_INC) and the balance of current transfers (CAB\_TRANS) are treated as exogenous, the current account balance, defined by:

\[
CAB_t = CAB\_NT_t + CAB\_INC_t + CAB\_TRANS_t
\]

improves.

**FEER** The concept of the fundamental equilibrium exchange rate (FEER) derived in this paragraph is closely related to that presented in Bayoumi and Faruqee (1998).\(^{27}\) Using the cointegrating relationships describing export prices (41) and volumes (45), the definition of the real exchange rate and the condition equating real GDP with its potential level, one can write the long-run nominal share of export in GDP as:

\[
EXP\_GDP_t = \exp\{0.37 \cdot (1 + 0.60) \cdot s\_reer_t - 1.50 + 0.012 \cdot t + \\
+0.07 \cdot D05Q2 - 3.20 \cdot (1 + 0.60) + \\
+0.6 \cdot 0.37 \cdot (1 + 0.60) \cdot (gdp\_pot_t - gdp\_ext\_pot_t)\}
\]

Similarly, using equations (42) and (46), the nominal import to GDP ratio can be expressed as:

\(^{26}\)This is because our estimates satisfy the Marshall-Lerner condition. More specifically, if follows from equations (41)-(42) and (45)-(46) that the long-run elasticity of exports in current prices with respect to the exchange rate is 0.37 \cdot (1 + 0.60) = 0.59, while that of imports equals to 0.61 \cdot (1 - 0.28) = 0.44.

\(^{27}\)See Rubaszek (2005) for an extension and application for Poland.
\[ IMP\_GDP_t = \exp\{0.61 \cdot (1 - 0.28) \cdot s\_rer_t - 1.55 + 0.012 \cdot t + 
- 5.27 \cdot (1 - 0.28) + 0.6 \cdot 0.61 \cdot (gdp\_pot_t - gdp\_ext\_pot_t) + 
- 0.28 \cdot GR\_TAR\_TR_t + (0.062 + 0.026) \cdot pimp\_ncore\_r_t \} \]

where \( PIMP\_NCORE\_R \) is defined as the price of non-core import prices (oil and gas) relative to core import prices.

It follows that, if we take the domestic trade policy, world commodity prices and Poland’s development gap vis-a-vis its main trading partners as given, the net trade to GDP ratio is solely a function of the real exchange rate.\(^{28}\) Further, using (50) rewritten in per unit of output terms and treating the shares of current income and transfer balances to GDP as exogenous, we get the relationship linking the real exchange rate with the country’s current account position. This problem can be inverted to yield the real exchange rate as a function of the target external position. Using this transformation, together with a set of equilibrium assumptions on the variables quoted above, we arrive at the model-consistent fundamental exchange rate (FEER).\(^{29}\)

The FEER does not enter directly any of the behavioural equations. In particular, it should not be confused with the long-run level of the exchange rate given by equation (38). Incorporating FEER into the model is aimed to serve two general purposes. First, it ensures consistency while adding judgement to the forecasts. Second, it is helpful in constructing simulations involving permanent shifts in Poland’s external competitiveness.

### 2.2.5 Aggregate demand and supply

**GDP identity** The short run level of real GDP is determined by the standard aggregate demand identity:

\[ GDP_t = CONP_t + CONGOV_t + GFCF\_P_t + GFCF\_G_t + 
+ GFCF\_H_t + INV_t + GDP\_EXP_t - GDP\_IMP_t \]

where \( CONGOV \) is government consumption defined in section (2.2.7).

A similar identity holds in nominal terms:

\[ GDP\_N_t = CONP\_N_t + CONGOV\_N_t + GFCF\_P\_N_t + GFCF\_G\_N_t + GFCF\_H\_N_t + INV\_N_t + +GDP\_EXP\_N_t - GDP\_IMP\_N_t \]

**GDP deflator** The GDP deflator is obtained by dividing nominal GDP \( (GDP\_N) \) by its level expressed in constant prices \( (GDP) \):

\[ PGDP_t = \frac{GDP\_N_t}{GDP_t} \]

\(^{28}\)The Marshall-Lerner condition holds (see footnote 26), so depreciation of the real exchange rate leads to improvement in the trade balance.

\(^{29}\)More specifically, FEER is the level of the real exchange rate ensuring that the long-run net trade to GDP ratio obtained from equations (51) and (52) is such that the current account position is on the target level, given the long-term shares of net income and net current transfers in output.
Potential output  Potential output \((GDP\_POT)\) in NECMOD is based on the production function approach, with full employment defined as total labour force supply corrected for NAWRU:

\[
GDP\_POT_t = TFP\_TREND_t \cdot \left(1 - NAWRU_t\right) \left(\frac{LF_t}{POP_t}\right)^{0.67} \cdot KN_t^{1-0.67}
\]  

(56)

The output gap \((GAP)\) is defined as a percent deviation of actual GDP from its potential level:

\[
GAP_t = \frac{GDP_t - GDP\_POT_t}{GDP\_POT_t}
\]  

(57)

2.2.6 Costs and prices

The cost and prices module of NECMOD consists of equations that represent the behaviour of the value-added deflator, core consumer inflation, as well as consumer energy and food prices. The two key measures of inflation, i.e. the value-added deflator and core inflation, are modelled as a function of non-farm unit labour costs \((ULCNA)\) and import prices \((PIMP)\).\(^\text{30}\) The former are defined as a ratio of gross wages to labour productivity outside private farming, adjusted for the effective rate of social security contributions paid by employers:

\[
ULCNA_t = \frac{WAGE\_N_t \cdot (1 + GR\_CORP\_TR_t) \cdot EMPNA_t}{GDP_t}
\]  

(58)

Import prices are obtained by adjusting core import prices, given by equation (44), for world prices of oil and gas (exogenous in the model), expressed in the national currency:\(^\text{31,32}\)

\[
pimp_t = 0.912 \cdot pimp\_core_t + 0.062 \cdot (p\_oil_t + s\_usd\_pln_t) + 0.026 \cdot (p\_gas_t + s\_usd\_pln_t)
\]  

(59)

Value-added deflator  Consistently with the relationship given by (27), the long-run level of the value-added deflator is determined by unit labour costs and import prices. The presence of a EU pre-accession dummy reflects the level shift and the change in the seasonal pattern of the deflator series.

In the short-run, apart from the growth rate of unit labour costs and import prices, the value-added deflator is determined by inflation expectations. We follow a hybrid

\(^\text{30}\)In the core inflation equations core import prices \((PIMP\_CORE)\) are used.

\(^\text{31}\)The weights are calibrated on the basis of the share of oil and gas in total imports.

\(^\text{32}\)The currency conversion uses the exchange rate corrected for the convergence effect (see section 2.2.4).
approach and consider both forward and backward looking expectations. The former are approximated by the actual values of the deflator, i.e. they are model-consistent.\textsuperscript{33,34}

\[ \Delta pva_t = -0.05 \cdot (pva_{t-1} - pva^*_{t-1}) + 0.14 \cdot \Delta pva_{t-1} + 0.12 \cdot \Delta pva_{t+1} + \\
+ 0.30 \cdot \Delta ulcna_t + 0.13 \cdot \Delta(pimp_t + GR\_TAR\_TR_t) + \\
+ (1 - 0.14 - 0.12 - 0.43) \cdot \left(1 + \frac{INF\_TARGET_t}{100}\right)^{\frac{1}{4}} - 1 \]  \hspace{1cm} (60)

Adjusted \( R^2 = 0.30 \)
S.E. of equation = 0.009
\( J \) statistic(p-value) = 0.98
Estimation period: 1998q2 - 2007q3

\textbf{Core inflation}  Core inflation (\textit{CORECPI}) is modelled using a similar concept to that used for the value-added deflator. In the long-run, the core price level depends on unit labour costs, non-energy import prices and effective rates of indirect taxes:

\[ corecpi^*_t = -3.59 + 0.56 \cdot ulcna_t + \\
+ (1 - 0.56) \cdot (pimp\_core_t + GR\_TAR\_TR_t) + \\
+ BS\_TREND_t + GR\_VAT\_TR_t + \\
+ GR\_GAM\_TR_t + GR\_EXT\_REST\_TR_t \]  \hspace{1cm} (61)

where \( GR\_EXT\_REST\_TR \) is the effective rate of the excise duties on consumption goods excluding fuels and gas and \( BS\_TREND \) is a truncated trend capturing a period of higher growth in core prices relative to their costs determinants in the first part of the sample.

As in the case of the value-added deflator, the short-run dynamics of core inflation is driven by growth rates of non-farm unit labour costs, non-energy import prices, as well as past and expected inflation:

\textsuperscript{33}Expressed this way, the deflator equation resembles the hybrid version of the New Keynesian Phillips Curve (NKPC), with the error correction term corresponding to real marginal costs, taking into account both unit labour costs and production costs proxied by import prices. The NKPC modelling approach proposed by Gali and Gertler (1999) and Sbordone (2002) differs from the traditional approach to Phillips curve estimation by including a direct measure of marginal costs instead proxying them with detrended output or unemployment. Another important feature of NKPC is the use of forward looking expectations in addition to backward looking terms.

\textsuperscript{34}In our choice of instruments we follow Gali et al. (2005) and Gali and Gertler (1999).
\[
\Delta \text{corecpi}_t = -0.03 \cdot (\text{corecpi}_{t-1} - \text{corecpi}_{t-1}^*) + 0.54 \cdot \Delta \text{corecpi}_{t-1} \\
+ 0.33 \cdot \Delta \text{corecpi}_{t+1} + 0.04 \cdot \Delta \text{ulcna}_t + \\
+ 0.03 \cdot \Delta (\text{pimp \_core}_t + \text{GR\_TAR\_TR}_t) + \\
\left(1 - 0.54 \cdot 0.33 - 0.06 \right) \cdot \left(1 + \text{INF\_TARGET}_t \right)^{\frac{1}{2}} - 1
\]

\[
(62)
\]

Adjusted \( R^2 = 0.86 \)
S.E. of equation = 0.003
J statistic (p-value) = 0.98
Estimation period: 1998q2 - 2007q3

\textbf{Food prices} \hspace{1em} \text{In the long-run, food prices (}FOODCPI\text{) are related to the index of world food prices (}P\_FOOD\text{) expressed in the national currency and core inflation, adjusted for the effective rate of VAT:}

\[
\text{foodcpi}_t^* = -1.90 + 0.45 \cdot (p\_food_t + s\_usd\_pln_t^c) + \\
+ (1 - 0.45) \cdot \text{corecpi}_t + 0.45 \cdot \text{GR\_VAT\_TR}_t
\]

\[
(63)
\]

In the short run, food price inflation is determined by its past level, core inflation and dynamics of world food prices:

\[
\Delta \text{foodcpi}_t = -0.08 \cdot (\text{foodcpi}_{t-1} - \text{foodcpi}^*_t) + \\
+ 0.60 \cdot \Delta \text{foodcpi}_{t-1} + 0.35 \cdot \Delta \text{corecpi}_{t-1} + \\
+ (1 - 0.60 - 0.35) \cdot \Delta (p\_food_t + s\_usd\_pln_t^c)
\]

\[
(64)
\]

Adjusted \( R^2 = 0.52 \)
S.E. of equation = 0.011
LM test (p-value) = 0.16
Estimation period: 1996q2 - 2007q4

\textbf{Energy prices} \hspace{1em} \text{Similarly to food prices, long-run energy prices (}ENERCPI\text{) are linked to world prices of energy commodities (}P\_ENER\text{) and core inflation, with an appropriate treatment of the excise duty in energy and non-energy sectors:}

\[
\text{enercpi}_t^* = -2.10 + 0.45 \cdot (p\_ener_t + s\_usd\_pln_t^c) + \\
+ (1 - 0.45) \cdot (\text{corecpi}_t - \text{GR\_EXT\_REST\_TR}_t) + \\
+ \text{GR\_EXT\_ENER\_TR}_t + 0.45 \cdot \text{GR\_VAT\_TR}_t
\]

\[
(65)
\]
The same factors determine the short-run behaviour of energy prices. More specifically, home prices of energy are affected by current changes in prices of energy commodities on the world markets and short-term fluctuations of core inflation:

\[
\Delta \text{enercpi}_t = 0.33 \cdot \Delta \text{enercpi}_{t-1} + 0.07 \cdot \Delta (p_{ener} + s_{usd\_pln}^c) + (66)
\]
\[
+ (1 - 0.33 - 0.07) \cdot \Delta (\text{corecpi}_t - GR\_EXT\_REST\_TR_t) +
\]
\[
- 0.09 \cdot (\text{enercpi}_{t-1} - \text{enercpi}_{t-1}^*) +
\]
\[
+ (1 - 0.45) \cdot \Delta GR\_EXT\_ENER\_TR_t + 0.04 \cdot I98Q1
\]

Adjusted $R^2 = 0.46$

S.E. of equation = 0.012

LM test (p-value) = 0.49

Estimation period: 1996q1 - 2007q4

where $GR\_EXT\_ENER\_TR$ stands for the effective excise duty on fuels and gas.

**Consumer price index** The consumer price index (CPI) is defined as a weighted average of the core price index (CORECPI), food prices (FOODCPI) and energy prices (ENERCPI), with the weights ($W\_CORE$, $W\_FOOD$ and $W\_ENER$) consistent with the CPI basket published by Poland’s Central Statistical Office (CSO). The annual percentage change in CPI is denoted by $INF$.

### 2.2.7 Fiscal sector

NECMOD is characterized by a relatively detailed structure of the fiscal sector. Fine disaggregation of general government revenues and expenditures allows for a comprehensive analysis of various links between the fiscal sector and other blocks of the model. In particular, we are able to consider both demand and distorting supply effects of fiscal policy choices on behaviour of firms and households.

The fiscal module of NECMOD is presented in detail in Appendix B. Here we discuss only the main revenue and expenditure categories.

**General government revenues** General government revenues ($GR\_N$) consist of taxes on production and imports ($GR\_PROD\_TAX\_N$), current taxes on income and wealth ($GR\_INC\_TAX\_N$), social contributions ($GR\_TCONTR\_N$) and property income taxes ($GR\_PROP\_INC\_N$). Non-tax revenues include current transfers ($GR\_OTHER\_CURT\_N$), general government market output ($GR\_OUTPUT\_N$) and capital transfers ($GR\_CAP\_TRANS\_N$):

\[
GR\_N_t = GR\_PROD\_TAX\_N_t + GR\_INC\_TAX\_N_t + (67)
\]
\[
+ GR\_TCONTR\_N_t + GR\_PROP\_INC\_N_t +
\]
\[
+ GR\_OTHER\_CURT\_N_t + GR\_OUTPUT\_N_t +
\]
\[
+ GR\_CAP\_TRANS\_N_t
\]
General government expenditures  General government total expenditures ($GE_N$) is a sum of social transfers in kind ($GE_{RELIEF\_KIND\_N}$), social benefits other than social transfers in kind ($GE_{SOC\_CASH\_N}$), government subsidies to farmers ($GE_{SUB\_FARM\_N}$), government subsidies other than subsidies to farmers ($GE_{SUB\_NOFARM\_N}$), other current transfers ($GE_{OTHER\_TRANS\_N}$), government’s contributions to the EU budget ($GE_{EU\_N}$), interest paid on the public debt ($GE_{FIN\_N}$), government compensation of employees ($GE_{WF\_N}$), government intermediate consumption ($GE_{CON\_N}$), government investment expenditures ($GE_{GFCF\_N}$) and government capital transfers ($GE_{CAP\_TRANS\_N}$):

$$GE_N = GE_{RELIEF\_KIND\_N} + GE_{SOC\_CASH\_N} +$$
$$+GE_{SUB\_FARM\_N} + GE_{SUB\_NOFARM\_N} +$$
$$+GE_{OTHER\_TRANS\_N} + GE_{EU\_N} +$$
$$+GE_{FIN\_N} + GE_{WF\_N} + GE_{CON\_N} +$$
$$+GE_{GFCF\_N} + GE_{CAP\_TRANS\_N}$$  \hspace{1cm} (68)

Government consumption  Nominal government consumption ($CONGOV\_N$) is proportional to expenditures on compensation of employees in the public sector ($GE_{WF\_N}$), intermediate government consumption ($GE_{CON\_N}$), and social transfers in kind ($GE_{RELIEF\_KIND\_N}$) net of general government market output ($GR\_OUTPUT\_N$). In addition, government consumption is influenced by EU transfers for current expenditures of the general government ($TRANS\_EXP\_N$):  $^35$

$$\Delta congov\_n_t = \Delta \ln(GE_{WF\_N} + GE_{CON\_N} +$$
$$+GE_{RELIEF\_KIND\_N} + 0.35 \times TRANS\_EXP\_N +$$
$$-GR\_OUTPUT\_N)$$  \hspace{1cm} (69)

Real government consumption ($CONGOV$) entering the real GDP identity (53) is obtained by deflating its nominal value with the appropriate deflator ($PCONGOV$), based on the value-added deflator and additionally corrected to account for indirect taxes in intermediate government expenditures.

Fiscal policy rule  Fiscal policy rule implemented in the NECMOD model may be called "passive" or "Ricardian", i.e. its only role is to ensure stabilization of the model in the long-term.  $^36$ The rule is backward looking and adjusts all effective tax rates and almost all expenditure components $^37$ in such a way to keep a balanced budget in the long run. Stabilization of the budget deficit ensures, via stabilization of the costs of public debt

$^35$ It is assessed that the share of EU transfers for current expenditures of the general government that can be classified as government consumption is equal to 35%.

$^36$ "Passive" rules contrary to "active" rules do not help the monetary policy to stabilize the business cycle (the output gap). For a comprehensive discussion of the role of fiscal policy in economic models see Wren-Lewis (2007).

$^37$ Except debt servicing cost and expenditure on retirement pensions which are thought to have an obligatory character and therefore cannot be easily adjusted in the conduct of fiscal policy.
service, fixing of the debt level. The parameters of the rule have been set so to stabilize the model in the long-run and have only limited influence on its short-term dynamics. The latter property ensures working of automatic stabilizers in the model which are generally weakened by mechanism of balancing of public sector revenues and expenditures. 38

2.2.8 The monetary authority and interest rates

Monetary policy rule We assume that the central bank can control short-term (3-month) interest rates, so they are determined by a forecast-based monetary policy rule. The specification of the rule is based on a standard Taylor-type reaction function, modified to reflect a forward-looking behaviour of the monetary policy makers. More specifically, the central bank adjusts its short-term nominal interest rates in response to deviations of the expected inflation rate from the target (INF\_TARGET) and the output gap (GAP). Following Clarida et al. (1998), we allow for some interest rate smoothing:

\[
I_{-3}M_t = \frac{0.89 \cdot I_{-3}M_{t-1} + (1 - 0.89) \cdot (I_{-3MR\_EQ} + INF_{t+1} + \\
+ 1.37 \cdot (INF_{t+1} - INF\_TARGET_{t+3}) + 0.81 \cdot GAP_t}{(0.00)}
\]

Adjusted \( R^2 = 0.99 \)
S.E. of equation = 0.005
\( J \) statistic(p-value) = 0.88
Estimation period: 2001q1 - 2007q3

The target level of inflation corresponds to the official targets set by the NBP, smoothed with a four-period forward moving average. In estimation, the value of the equilibrium real rate of interest (\( I_{-3MR\_EQ} \)) is treated as exogenous.

Long-term interest rate Long-term interest rates, represented by a yield on a 5-year government bond (\( I_{-5Y} \)), are modeled in line with the rational expectations hypothesis of the term structure. According to this hypothesis and up to a term premium, there should be no expected difference in the returns from holding a long-term bond or rolling over a series of short-term bonds with a total maturity equal to that of the long bond. As a result, the long-term bond yield can be written as a function of expected short-term interest rates plus a term-premium. In particular, the 5-year interest rate is a weighted moving average of 3 month interest rates (\( I_{-3M} \), expected over the next 20 quarters:

Using the weighting scheme of Shiller et al. (1983), the duration-based transformation from Fuhrer (1996) the long-term interest rate equation can be expressed as follows:

\[
I_{-5}Y_t = 0.94 \cdot I_{-5}Y_{t+1} + (1 - 0.94) \cdot I_{-3}M_t - 0.001
\]

38 Following a permanent shock, automatic stabilisers are not able to bring the economy back to economic equilibrium. In such a situation, a fiscal adjustment is necessary, which is assured in the NECMOD model by the fiscal policy rule. In case of a transitory shock, the automatic stabilisers are able to assure a return to the equilibrium, and the fiscal policy rule actually weakens their impact on the economy. The coefficient controlling the speed of adjustment has been selected to provide a sufficiently strong impact of the rule-following permanent shocks and a reasonably weak impact in response to transitory shock.
Adjusted $R^2 = 0.93$
S.E. of equation = 0.009
$J$ statistic(p-value) = 0.99
Estimation period: 2000q1 - 2007q3

where 0.94 is calibrated using the average duration of the 5-year government bond in Poland, which is approximately 4 years.
3 Long-run equilibrium

In the previous section we provided a description of long run relationships linking the key variables in NECMOD. Now we discuss how they, together with a set of equilibrium conditions, jointly determine the long run solution of the model. This can be done in a relatively transparent way by considering NECMOD’s steady state equilibrium.\footnote{The following exposition is largely inspired by Fagan et al. (2005).}

In order to ensure that the long run model solution features constant ratios of key endogenous variables, it is useful to specify paths for exogenous variables in a consistent fashion. This is basically done using a few simplifying assumptions, which abstract from potentially more realistic scenarios available from external sources. In particular, we make sure that in the long run all foreign nominal variables (commodity price indexes, value-added deflators in Poland’s trading partners) grow at a common and constant rate. A similar technical assumption is made for foreign GDP and TFP growth, as well as nominal interest rates abroad. The fiscal sector is constructed in a way ensuring stabilization of all expenditure and revenue components at a constant proportion to GDP. Population is constant, the age structure fixed and the minimum wage rate is fully indexed to the average wage rate. Other exogenous variables, like transfers from abroad, current account income balance etc., are set either to zero or as a constant proportion of GDP. In what follows, we describe the equilibrium in which both real and nominal convergence processes have been completed, i.e. TFP in Poland and abroad grow at the same constant rate.

The long run real equilibrium of the model is determined by its supply side, the core of which is the production function. Output is consistent with its potential level, set by the productive capital stock, potential labour input and exogenous TFP. In the long run, the marginal product of private capital is equal to its user cost. The first order condition of firms’ profit maximization, together with an assumption that marginal products of private and public capital equate in the long run, allows us to write the steady-state ratio of productive capital to output as a function of the steady-state real interest rate, depreciation rate and the effective tax rate levied on enterprises. The potential labour input equals exogenous population, corrected for the steady-state participation rate and the steady-state NAWRU. It can be shown that in the long run the participation rates and NAWRU converge to levels reflecting institutional and demographic factors, namely replacement rates, effective tax rates, the minimum wage rate and the share of students in the population. The steady-state employment also depends on the steady-state (equilibrium) real interest rate and the steady-state real exchange rate, the latter assumed to coincide with the fundamental equilibrium exchange rate (FEER).\footnote{As discussed in section 2.2.4, FEER is implicitly defined in terms of the target current account position, which in principle can be set at an arbitrary level.}

All these equilibrium conditions imply that the long-run level of output per capita is a function of a range of policy variables and the model specific steady-state real interest rate. It also follows from this discussion that fiscal policy can affect the long run relative intensity of production factors use.

An analogous analysis allows us to derive steady-state expenditure shares in output. Long run relationships presented in section 2.2 imply that the steady-state private consumption share is a function of the steady-state real interest rate and households’ income and wealth to GDP ratios. Gross (i.e. inclusive of employers’ contributions) labour in-
come share is pinned down by the production function labour elasticity (from the first order condition of profit maximization, linking labour productivity to the real wage rate). Gross operating surplus is a constant fraction of GDP. Property income depends on wealth to GDP ratio and as such can be written as a function of the same variables as the consumption share.\textsuperscript{41} The remaining households’ income components, including distortionary transfers and taxes, depend on fiscal policy choices. All in all, these considerations imply that the consumption share in GDP depends on institutional factors related to the labour market and the model specific steady-state real interest rate.

Public consumption share in GDP is obviously a function of government expenditure policies. Determinants of private and public investment shares follow directly from the discussion regarding the steady-state capital-output ratio. The long run proportion of housing capital to private consumption can be shown to depend on the steady-state real interest rate, which pins down the housing investment share in output. Long run inventories are just proportional to GDP. Export and import shares in output can be written as functions of the real exchange rate and import taxes. Summing up, the demand side components are driven by the same set of policy and model specific variables that determine the supply side of the model. It also follows from this discussion that the long run composition of aggregate demand is influenced by the government policy.

The long run relations presented in section 2.2 can also be used to derive equilibrium levels of relative prices defined in the model. For instance, export or import prices relative to the value-added deflator are pinned down by the real exchange rate.

In order to determine the nominal side of the economy, a nominal anchor is needed. In NECMOD, this role is performed by the monetary policy feedback rule, according to which nominal short-term interest rates are assumed to depend on deviations of inflation from the target set by the central bank.\textsuperscript{42}

As noted above, the key model specific variable closing the model in the long run is the steady-state real interest rate. By entering the Taylor rule as a real component of the intercept, it guarantees that the inflation rate is equal to the target set by the monetary authority.\textsuperscript{43}

\textsuperscript{41}This follows from the definition of wealth and the following considerations. As shown above, the private capital to output ratio is fixed by the parameters determining the supply side. The share of public debt in GDP is stabilized by the fiscal rule. As demonstrated below, housing capital to output ratio is pinned down by the determinants of the consumption share in GDP and the steady-state real interest rate. Finally, net foreign assets, being a cumulated flow of current account balances, are pinned down as a share of GDP by the target current account position.

\textsuperscript{42}More specifically, inflation in the steady state is governed by expectations, which can be aligned to the central bank objectives with the interest rate policy.

\textsuperscript{43}The steady-state real interest rate is a complex functions of model parameters and exogenous assumptions. In principle, it is possible to derive it using numerical methods. This makes sense only while considering the steady state equilibrium of the model, accompanied by a set of simplifying assumptions presented above. Therefore, the equilibrium real interest rate is treated as exogenous in estimation, as well as in forecasting and simulation exercises.
4 Impulse response analysis

4.1 Monetary impulse

The monetary impulse is defined as an unexpected one quarter increase in the short-term interest rate by 100 basis points. Following the shock, the interest rate moves in line with the monetary feedback rule given by equation (70), staying above the baseline for 12 quarters due to the presence of the smoothing mechanism. The fiscal rule is switched on. Impulse reaction functions of key macrocategories are given in Table 1.

The interest rate shock influences the economy through five major channels. The appreciation of the zloty due to an increase in the interest rate disparity leads to worsening of net exports and to a decline in import prices (the exchange rate channel). Higher interest rates make future spending relatively more attractive for the households, so they reduce present consumption and residential investment (the substitution-effect channel). The cost of capital channel operates through a rise in the user cost of corporate and residential capital, which brings down both types of investment. Higher interest rates increase income of net lenders (the households sector) and decrease income of net borrowers (the government sector), triggering spending adjustments (the income and cash-flow channel). Finally, a reduction in house prices, driven by increased user cost of residential capital, affects negatively wealth of the households, leading to lower consumption and residential investment (wealth channel).

As a result, GDP growth declines and the strongest impact on the y-o-y growth rate is observed in the fourth quarter after the shock. The main drivers of GDP deceleration are consumption and gross fixed capital formation. A decline in the aggregate demand encourages firms to cut both wages and employment, trimming unit labour costs. This, together with the exchange rate appreciation, leads to a fall in domestic inflation. The maximum effect of the interest rate hike on inflation is observed in the sixth quarter after the shock. Lower inflation and economic activity encourage easing of the monetary policy. A downward correction of short-term interest rates facilitates the economic revival and raises dynamics of prices so that the economy eventually converges to the baseline path.

4.2 Exchange rate impulse

The exchange rate impulse is defined as an unexpected one quarter appreciation of the nominal effective exchange rate by 10%. The simulation is conducted with both monetary and fiscal policy rules. The results are reported in Table 2.

An appreciation of the exchange rate reduces import and export prices. Weaker growth of import prices decreases production cost and, in consequence, both producer and consumer prices. Moreover, the stronger zloty passes through into lower food and energy prices, as both indices depend on world commodity prices expressed in the national currency.

Further, due to higher import price competitiveness and lower export profitability, net exports and the current account balance deteriorate. After the current account balance worsening, moderation of the appreciation trend in the exchange rate already sets in.

Lower contribution of net exports to GDP growth presses down labour and investment demand. As a result, unemployment goes up. However, after a few quarters, a positive impact of lower production cost on the supply side of the economy (via NAWRU) starts
to outweigh the consequences of the negative demand impulse. The initial increase in the unemployment rate is reversed, with wage pressure remaining low. Unit labour costs are allowed to go down, which postpones an increase in inflation following the upswing in aggregate demand, driven by an easing of monetary policy and restoration of the original exchange rate level.

4.3 Social security contribution impulses

Two twin impulses to social security contributions were defined as an expected one percentage point increase in the effective social security contribution rate levied either on employers or employees. The impulse lasts eight quarters. Both monetary and fiscal rules are switched on, except that the social security contribution effective tax rate is exempted from the latter for the period of the impulse. Impulse reaction functions are plotted in Table 3 and 4.

Change in the contribution levied on employers directly affects unit labour costs, translating into higher producer and consumer prices. A hike in the contribution paid on the gross wage by employees influences unit labour cost only indirectly via a fall in net wages, which exacerbates the wage pressure even though unemployment remains initially unchanged. Hence, compared to the hike levied on employers, reaction of inflation is more moderate and lagged.

Adjustment of the aggregate demand is in turn faster if an increase in social security contributions concerns the employees’ share, since it infringes disposable income of the households, depressing consumption and housing investment. In the case of a hike levied on employers, contraction in aggregate demand comes about only as a reaction to weaker labour market conditions.

NAWRU rises after an increase in this quasi-tax burden, reflecting either a reduction in labour supply in response to lower net wages, or demand for labour driven by higher labour costs. To reduce costs, enterprises decrease wages and employment. It results in a cutback of disposable income of the households and a fall in their expenditure on consumption and housing. These developments lead to a prolonged GDP slowdown that hits the bottom about a year after the shock is introduced in the case of changes in employer’s contribution, and with a few years delay in the case of a hike on employees’ part.

4.4 TFP impulse

The TFP impulse is defined as a temporary and unexpected slowdown in TFP growth by one percentage point (in annualized terms) for a period of four quarters. In the simulation, the monetary and fiscal policy rules are switched on. Impulse reaction functions of key macrocategories are given in Table 5.

A contractionary TFP shock leads to an immediate decline in potential output, which translates into lower exports, gross fixed capital formation and, to lesser extent, household’s consumption. Consequently, GDP is reduced as well.

An unexpected decline of TFP dynamics induces faster growth of employment, as firms try to compensate for lower productivity with new hires. However, lower productivity of workers brings wages down, even though labour demand increases. As the positive effect on employment is stronger than the negative one on wages, unit labour costs accelerate. Together with higher growth of import prices, this leads to a surge in inflation.
As the shock dies out, the effects of TFP slowdown are reversed: inflation rate is lower while GDP growth slightly higher than in the baseline scenario. There is a certain persistence of inflation effects tied to a sluggish readjustment of NAWRU and only a gradual closing of the unemployment gap.

4.5 EU structural funds inflow impulse

Impulses depicting an impact of an inflow of the structural funds from the EU are displayed on two distinct examples: one concerning funds directed at augmenting national expenditures on public infrastructure and the other relying on direct transfers to farmers from the Common Agricultural Policy (CAP). In each case, a shock is defined as an expected rise in the inflow of funds, lasting four quarters and amounting to 1% of GDP. After four quarters, the amount of incoming funds returns to the baseline path. A simplifying assumption made is that these additional funds do not require complementary co-financing from the general government sector. Fiscal and monetary rules are allowed to operate. The results are reported in Tables 6 and 7.

A rise in EU infrastructural funds translates directly into higher public gross fixed capital formation. It induces multiplier effects, raising profitability of private investment. The initial impact on potential GDP greatly outweighs the effect on the actual GDP, leading to moderation of prices. This is reinforced by the zloty appreciation, following improvement in the current account balance. Even though growth of labour productivity tied to higher productive capital per worker moves wages up, unit labour cost go down. It takes a year before the upswing in aggregate demand translates into higher inflation.

In contrast, stronger inflow of farm subsidies can be considered foremost as a positive demand shock. A mild reduction of inflation follows from the exchange rate appreciation, following the current account improvement. Higher inflow of CAP funds directly influences individual consumption via the disposable income channel. The rise in individual consumption facilitates GDP growth, triggering higher investment and import demand. The economic upturn leads to an increase in labour demand and acceleration of wages. Developments on the labour market translate into higher unit labour costs and inflation pressure in the economy.

4.6 Government consumption impulse

The public consumption impulse is defined as a one-quarter unexpected cut in the pur-
chases of goods and services by the general government sector, equivalent to 1% of GDP. The simulation is conducted with monetary and fiscal policy rules switched on. The main impulse response functions are plotted in Table 8.

A reduction in purchases of goods and services by the general government sector depresses domestic demand and leads to GDP deceleration. A negative demand shock passes through into a drop in labour demand and wages, with a delay originating from labour market rigidities. Hence, right after the shock we observe a short-lived increase in unit labour cost and slightly higher inflation. However, over the next quarters the labour market adjusts and unit labour costs decelerate, inducing a drop in inflation.

Eventually, lower domestic demand, reinforced by deterioration of labour market conditions lead to easing of the monetary policy. There is a rebound in investments and consumption, followed by the return of inflation to the level consistent with the baseline.
Table 1: Monetary impulse
Table 2: Exchange rate impulse
Table 3: Social contribution levied on employees impulse

CPI inflation

GDP and its components (levels)

Potential GDP [1]

Exchange rates

Interest rates

GDP and its components (y-o-y)

Labour market (1)

Labour market (2)

Investments (levels)

Investments (y-o-y)

External and fiscal equilibrium

Replacement rates

Unit labour costs

Unemployment

Labour market (3)
Table 4: Social contribution levied on employers impulse
Table 5: TFP impulse

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Legend:
- Real GDP (pp)
- Private cons. (pp)
- GFCF (pp)
- Exports (pp)
- Imports (pp)
- Output gap (% of potential GDP)
- Potential GDP (1)
- Potential GDP (pp)
- Gross prod. capital (y-o-y) (pp)
- Potential GDP (2)
- Rank: GDP (pp)
- Gross prod. capital (y-o-y) (pp)
- Real GDP (1)
- NEER (%)
- REER (%)
- 3M interest rate (pp)
- 5Y interest rate (pp)
- EMP (%)
- LF (%)
- Participation rate (pp)
- EMP (pp)
- LF (pp)
- Unemp. (pp)
- Wages (pp)
- Inv. in priv. sec. (%)  
- Inv. in housing sec. (%)  
- Inv. in non-fin. hiring sec. (pp)
- Inv. in housing sec. (pp)
- Inv. def. in housing sec. (pp)
- Trade balance (% of GDP)
- Govrn. balance (% of GDP)
Table 6: EU structural funds inflow impulse - funds on public infrastructure
Table 7: EU structural funds inflow impulse - direct transfers to farmers tied to Common Agricultural Policy
Table 8: Government consumption impulse
5 Fan charts of inflation and GDP growth

5.1 Introduction

Forecasting inflation and GDP growth is an important aspect of contemporary central banking, especially in direct inflation targeting regime. But each forecast is burdened with uncertainty. Estimating and publishing this uncertainty is important for central banks’ communication with the public as it enables to shift the focus from the central path of which probability is zero towards medium-term risks. Also distribution of risks might be relevant if some members of monetary authorities have a non-symmetrical loss function.

Fan charts, which plot the probability density of forecast variable (i.e. an estimate of the probability distribution of possible outcomes at different forecast horizon) have become the most popular method of quantitative representation of forecasts along with their uncertainty.

5.2 Quantifying the uncertainty

Clements and Hendry (1998) distinguish five major sources of uncertainty in large-scale macroeconometric models used for forecasting:

- Future changes in the underlying structure of the economy
- Misspecification of the model
- Inaccuracies in the estimates of the model’s parameters (estimation uncertainty)
- Variable mismeasurement:
  - initial condition uncertainty
  - non-modelled variable uncertainty
  - incorrect categorization of some variable as exogenous
  - lack of invariance to policy changes in exogenous variables
- The cumulation of future shocks to the economy

Unfortunately not all of the abovementioned sources of uncertainty are quantifiable. Moreover, model forecasts are usually corrected by experts and these expert adjustments may decrease forecast errors but still in a non-quantifiable way. For these reasons central banks typically use past forecast errors as a catch-all proxy for future uncertainty. This strategy is often complemented with expert adjustments for anticipated changes in uncertainty, e.g. uncertainty in increased for periods of recession and decreased for periods of steady growth.
5.3 Constructing fan charts at the NBP

At the National Bank of Poland in the period May 2005 - June 2008 uncertainty was estimated on the basis of stochastic simulations of the model, taking into account uncertainty related to the forecasts of exogenous variables and to the error term of the equations. This method allowed for the decomposition of uncertainty and did not require the history of forecasts from the model, which had not been available back in 2005. More details of this method are given in the Appendix E.

In mid-2008, when the history of forecasts from the model became significantly longer, it was decided to modify the method to take into account the information from past forecast errors. In the current NBP’s method the width of the fan chart is consistent with the past errors from the ECMOD/NECMOD model, at the same time depending on the current uncertainty assessment of exogenous variables. The procedure of constructing the fan charts consists of the following steps:

1. **Determining the distributions of historical forecast errors.** On the basis of past forecast errors from the ECMOD/NECMOD model, the forecast errors variance is estimated for every forecasting horizon. In the case of inflation, account is taken of uncertainty of forecasts starting from the quarter in which the Inflation Report is published. In the case of GDP growth forecasts, due to frequent revisions of national accounts, account is also taken of the uncertainty of past values of the variable (up to 7 quarters preceding the publication of the Report inclusive).

2. **Simulation of paths of exogenous variables.** On the basis of the NECMOD model’s multipliers, a set of these exogenous variables was selected, whose uncertainty has a prevailing impact on the uncertainty of inflation and GDP forecasts. Experts forecasting exogenous variables, in every forecasting round present the central path (the expected value) and the uncertainty assessment of the forecast of the given variable (whereas the distribution of risks does not have to be symmetric). The simulation procedure of exogenous variables was chosen in such a way that: 1) the expected value of simulated paths of variables conforms to central paths given by experts, 2) the expected value of stochastic disturbances is zero, 3) the autocorrelation of variables observed in the sample is retained and 3) the cross correlation of shocks among particular variables is retained as well. Exogenous variables are simulated in two versions: with historical uncertainty and with current uncertainty.

In the first case the confidence band of simulated exogenous variables are consistent with the average assessment of uncertainty given by experts in the previous forecasting rounds, while in the second - with the assessment of uncertainty in the current forecasting round.

3. **Simulations from the NECMOD model.** The NECMOD model is used to carry out stochastic simulations in two versions. In the first version, the uncertainty

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44 Technically, model is solved sequentially, quarter after quarter and solution in each period takes into account model-consistent expectations. In each quarter, agents form their expectations about future paths of all variables, relying on the shocks realized up to this period. More specifically, agents assume that in the future endogenous variables evolve according to their equations and exogenous variables follow the laws of motion for the shock processes. In the next quarter shocks for that period are generated and agents update their expectations.
of exogenous variables is set at the average level from the previous forecasting rounds and the interest rate is endogenous, determined by the Taylor rule estimated on the basis of historical data. In the second, the uncertainty of exogenous variables is set at the level from the current forecasting round, and the interest rate does not change in the projection horizon. The simulations do not take into consideration other sources of forecast uncertainty (uncertainty related to the error term, estimators, etc.). The goal of the simulation is to assess the impact of changes in the uncertainty assessment of exogenous variables and of the assumption of the exogeneity of the monetary policy on the uncertainty of GDP and inflation forecasts.

4. Determining the current uncertainty concerning GDP and inflation. The fan chart based only on past forecast errors from the ECMOD/NECMOD model is an adequate estimate of uncertainty of the forecast with the uncertainty of exogenous variables at the level corresponding to the average uncertainty in the previous forecasting rounds. To this fan chart a correction calculated in the previous point is added, by means of which we obtain the uncertainty assessment of the projection with the current level of exogenous variables uncertainty.

5.4 Advantages and disadvantages of the NBP’s method

Advantages:

- Width of the fan charts is consistent with the expert-adjusted, historical forecast errors from the ECMOD/NECMOD model.
- Fan charts reflect changes in uncertainty between forecasting rounds
- Fan chart is constructed under the assumption of exogenous monetary policy
- Revisions of variables (national accounts) are accounted for.
- Method is flexible and can be modified/extended easily.

Disadvantages:

- Changes in ”endogenous” uncertainty are not accounted for in the method (e.g. a rise in uncertainty of investment developments).
- Consequences of improvement/deterioration of the model/forecasts and expert adjustments are not accounted for.
- Fan charts for the model’s endogenous variables other than GDP and inflation are more difficult to construct (as compared with the previous method described in the Appendix E).
6 Concluding remarks

This paper documented the structure, properties and main applications of NECMOD, the NBP’s new structural forecasting model used for preparing regular projections published in the Inflation Report. Built on a similar framework as its predecessor, ECMOD, the new model can be seen as a response to challenges faced in recent years by economists analyzing the Polish economy (and, more generally, transition or catching-up countries). The source of these challenges were structural shifts in the economy, related mainly to EU-accession, globalization, regulatory reforms and dynamic changes on the labour market. The outcome of the project is a comprehensive tool, which can be effectively used for forecasting (with an explicit account for uncertainty) and a range of scenario analyses.

While much effort has been put to make NECMOD as close as possible to the current modeling frontier, set by theoretical advances and best practices of leading central banks, it cannot be expected to stay unmodified over a couple of years. Probably the most desirable extension would be to increase the role of forward-looking behaviour in the model, by adding expectations to investment and consumption equations and the labour market block.
References


Appendix

A NECMOD variables\textsuperscript{45}

- ALMP\_N (EN) – active labour market policy expenditures (sum of the government active labour market policy and firm expenditures on human capital development financed from the EU)
- AVG\_BENEFIT (EN) – average unemployment benefit per unemployed entitled to unemployment benefits
- AVG\_PENSION (EN) – average pension per reitre
- AVG\_RELIEF\_CASH (EN) – average other social relief per not employed (both unemployed and inactive)
- AVG\_RELIEF\_KIND (EN) – average social transfer in kind per per not employed (both unemployed and inactive)
- BS\_TREND (EX) – trend in the core inflation equation, truncated from 2002Q3
- CAB (EN) – current account balance (including the capital account)
- CAB\_INC (EX) – balance of income
- CAB\_NT (EN) – trade balance of goods and services
- CAB\_TRANS (EX) – balance of transfers (with the capital account)
- CONGOV (EN) – government consumption
- CONGOV\_N (EN) – nominal government consumption
- CONP (EN) – individual consumption
- CONP\_N (EN) – nominal individual consumption
- CORECPI (EN) – core CPI index (CPI excluding food and energy prices)
- CPI (EN) – consumer price index
- DEPR\_H (EX) - depreciation rate of residential capital
- DEPR\_P (EX) - depreciation rate of corporate capital
- DISC\_G (EX) - discard rate of public capital
- DISC\_P (EX) – discard rate of private capital
- DxxyQy (EX) – shift dummy variable where xx means year and y denotes quarter when the shift takes place

\textsuperscript{45} Symbols in the brackets following the variable name stand for: EX – exogenous, EN – endogenous.
• EMP (EN) - the number of employed in the economy
• EMP_A (EN) – employment in agriculture
• EMPNA (EN) – non-agriculture employment
• ENER CPI (EN) – index of domestic energy prices
• FIN ACC (EN) – corporate disposable income share in GDP
• FOOD CPI (EN) – index of domestic food prices
• G_BALANCE_GDP (EN) – general government balance as percentage of GDP
• G_BALANCE_N (EN) – general government (nominal) balance
• G_CORP_TR (EN) - effective tax burden levied on enterprises, taking into account the effective tax rate of corporate income tax, EU structural funds directed to firms and farmers and aimed at production, government subsidies to production and government capital transfers
• G_DEBT_DOM_N (EN) – domestic general government debt
• G_DEBT_FOR_N (EN) – foreign general government debt
• G_DEBT_N (EN) – nominal government debt
• G_REF (EX) – share of pensions paid from the state budget (indicating pension reform progress)
• G_OFE (EN) – share of open pension funds in social security contributions
• G_UNEMP (EX)- ratio of unemployed entitled to unemployment benefits to total number of unemployed
• GAP (EN) - output gap
• GDEBT_EUR (EN)- foreign general government debt denominated in euro
• GDP (EN) - gross domestic product
• GDP_EXP (EN) - exports
• GDP_EXP_N (EN) – nominal exports
• GDP_EXT (EX) – foreign GDP (weighted average of the euro area, the UK, and the USA)
• GDP_EXT_POT (EX) – foreign potential output (weighted average of the euro area, the UK, and the USA)
• GDP_IMP (EN) - imports
• GDP_IMP$D$ (EN) - demand for imports equal to weighted sum of domestic consumption, exports, gross fixed capital formation, weights based on the relative import content of each component
• GDP_IMP_N (EN) – nominal imports
• GDP_N (EN) – nominal gross domestic product
• GDP_POT (EN) – domestic potential output
• GE_ALMP_N (EN) – general government expenditures on active labour market policy
• GE_CAP_TRANS_N (EN) – general government capital transfers
• GE_CON_N (EN) – general government intermediate consumption
• GE_EU_N (EN) – general government contribution to the EU budget
• GE_FIN_N (EN) – interest on the general government debt
• GE_GFCF (EN) – real general government investments
• GE_GFCF_N (EN) - nominal general government investments
• GE_INT_DOM_N (EN) – interest on the domestic general government debt
• GE_INT_DOM_COR (EN) – correction factor of interest on the domestic general government debt for components not included in the equation
• GE_INT_FOR_N (EN) – interest on the foreign general government debt
• GE_INT_FOR_N_COR (EN) – correction factor of interest on the foreign general government debt for components not included in the equation
• GE_N (EN) – total general government expenditure
• GE_OTHER_TRANS_N (EN) – other general government transfers without EU budget contribution
• GE_PENSIONS_N (EN) – general government expenditures on pensions
• GE_PRIV_N (EX) – general government income from privatisation
• GE_RELIEF_CASH_N (EN) – general government other social relief expenditures
• GE_RELIEF_KIND_N (EN) – general government social transfers in kind
• GE_SOC_CASH_N (EN) – general government social benefits other than social transfers in kind
• GE_SOC_COMP_N (EN) – general government compensation paid to social relief beneficiaries in 2000-2004
- GE_SUB_FARM_N (EN) – general government subsidies to farmers
- GE_SUB_NOFARM_N (EN) – general government subsidies without subsidies to farmers
- GE_UNEMP_N (EN) – general government unemployment benefits expenditure
- GE_WEU_N (EN) – total general government expenditures including EU funds
- GE_WF_N (EN) – general government compensation of employees
- GFCF_G (EN) – gross fixed public capital formation
- GFCF_G_N (EN) – nominal gross fixed public capital formation
- GFCF_H (EN) – gross fixed residential capital formation
- GFCF_H_N (EN) – nominal gross fixed residential capital formation
- GFCF_P (EN) – gross fixed corporate capital formation
- GFCF_P_N (EN) – nominal gross fixed corporate capital formation
- GR_CAP_TRANS_N (EN) – general government capital transfers revenues
- GR_CIT_N (EN) – taxes on income or profits of firms including holding gains
- GR_CIT_TR (EN) – effective rate of taxes on the income or profits of corporations including holding gains
- GR_CORP_N (EN) – social security contributions paid by employers
- GR_CORP_TR (EN) – effective rate of social security contributions paid by employers
- GR_EMP_CORP_N (EN) – social security contributions paid by both employers and employees
- GR_EMP_N (EN) – social security contributions paid by employees
- GR_EMP_TR (EN) – effective rate of social security contributions paid by employees
- GR_EXT_ENER_N (EN) – revenues from excise duties imposed on goods related to energy (fuels and gas)
- GR_EXT_ENER_TR (EN) – effective tax rate of excise duties imposed on goods related to energy (fuels and gas)
- GR_EXT_N (EN) – total revenues from excise duties and consumption taxes
- GR_EXT_REST_N (EN) – revenues from excise duties on goods other than related to energy (fuels and gas)
• GR_EXT_REST_TR (EN) – effective rate of excise duties on goods other than related to energy (fuels and gas)
• GR_EXT_TR (EN) – effective tax rate of excise duties
• GR_FARM_N (EN) – social security contribution paid by farmers
• GR_FARM_TR (EN) – effective tax rate of social security contribution paid by farmers
• GR_GAM_N (EN) – revenues from gambling taxes
• GR_GAM_TR (EN) – effective rate of gambling taxes
• GR_HC_N (EN) – government revenues from compulsory health care contribution
• GR_HC_TR (EN) - effective rate of compulsory health care contribution
• GR_INC_TAX_N (EN) – revenues from income and wealth taxes
• GR_N (EN) – total general government revenues
• GR_OTAX_CORP_N (EN) – revenues from other taxes on production and products
• GR_OTAX_CORP_TR (EN) - effective tax rate of other taxes on production and products
• GR_OTAX_HH_N (EN) – revenues from taxes on winnings from lottery or gambling and other current taxes
• GR_OTAX_HH_TR (EN) – effective rate of taxes on winnings from lottery or gambling and other current taxes
• GR_OTHER_CURT_N (EN) – revenues from other current transfers
• GR_OUTPUT_N (EN) – general government market output, output for own final use and payments for other non-market output
• GR_PIT_N (EN) – government revenues from personal income tax
• GR_PIT_TR (EN) - effective rate of personal income tax
• GR_PROD_TAX_N (EN) – revenues from taxes on production and imports
• GR_PROP_INC_N (EN) – government’s property income
• GR_TAR_N_EU (EN) – import duties paid to EU budget
• GR_TAR_N (EN) - import duties (including contribution to EU budget)
• GR_TAR_NET_N (EN) - import duties excluding duties paid to EU budget
• GR_TAR_TR (EN) – effective rate of import duties
• GR_TCONTR_N (EN) – total social contributions
• GR_VAT_N_EU (EN) – VAT revenues paid to EU budget
• GR_VAT_N (EN) – VAT revenues (including contribution to EU budget)
• GR_VAT_NET_N (EN) – VAT revenues (excluding contribution to EU budget)
• GR_VAT_TR (EN) – effective rate of VAT
• GR_WEU_N (EN) – total revenues of general government including EU funds
• I_3M (EN) – WIBOR 3M quarterly average
• I_3MR_CPI (EN) – real 3-month interest rate (deflated with CPI)
• I_3MR_EQ (EX) – equilibrium real interest rate
• I_3MR_EXT (EN) – real 3-month foreign interest rate deflated with foreign value-added deflator
• I_3MR_PVA (EN) – real 3-month interest rate deflated with value-added deflator
• I_5Y (EN) – yield on 5-year government bonds
• I_5Y_EUR (EX) – yield on 5-year Bunds
• I_H (EN) – average interest on mortgage loans (weighted average of 5-year domestic and euro zone rates)
• INF (EN) – CPI inflation
• INF_TARGET (EX) – inflation target
• INTAX (EN) – sum of effective rates of indirect taxes
• INV (EN) – change in inventories
• INV_N (EN) - change in inventories (in current prices)
• IxxQy (EX) – one period dummy variable, where xx denotes year and y stands for quarter
• K (EN) – gross productive capital
• K_G (EN) – gross public capital
• K_H (EN) – gross residential capital of households
• K_P (EN) – gross corporate productive capital
• KN (EN) – net productive capital
• KN_G (EN) – net public capital
• KN_P (EN) – net corporate productive capital
• LF (EN) – labour force supply
• LF_M (EN) – middle-aged labour force (25-44 years)
• LF_O (EN) – older labour force (45+ years)
• LF_Y (EN) – younger labour force (15-24 years)
• MINW (EN) – relation of minimum wage to average gross wage in the economy
• MPK (EN) - marginal product of productive capital
• NAWRU (EN) – non-accelerating wages rate of unemployment
• NFA (EN) – net foreign assets in EUR
• NFA_GDP (EN) - net foreign assets to GDP ratio
• OFE_N (EN) – general government transfers to open pension funds
• OPSURP_N (EN) – net operating surplus
• P_ENER (EX) – index of global energy prices
• P_FOOD (EX) – index of global food prices
• P_GAS (EX) – global gas prices (price of Russian gas per 1000 cubic meters)
• P_OIL (EX) – price of oil BRENT
• PCONGOV (EN) – government consumption deflator
• PCONP (EN) – individual consumption deflator
• PEXP (EN) – deflator of exports
• PGDP (EN) – deflator of GDP
• PGFCF_H (EN) – deflator of gross fixed residential capital formation
• PGFCF_G (EN) – deflator of gross fixed public capital formation
• PGFCF_P (EN) – deflator of gross fixed corporate capital formation
• PIMP (EN) – deflator of imports
• PIMP_CORE (EN) – deflator of imports excluding prices of oil and gas
• PIMP_NCORE_R (EN) – relation of non-core import prices to core import prices
• PINV (EN) – deflator of change in inventories
• POP (EX) – total population (including active and inactive on the labour market)
• POP_M (EX) – middle-aged total population (25-44 years)
• POP_O (EX) – older total population (45+ years)
• POP_Y (EX) – younger total population (15-24 years)
• PVA (EN) – deflator of value-added
• PVA_EXT (EX) – deflator of foreign value added
• R_RATE (EN) – real interest rate (equal to the average of 3-month rate deflated with value-added deflator and 5-year rate deflated with inflation target)
• REM_BALANCE (EN) – remittances flow balance
• RETIRED (EX) – number of retired in the economy
• RISK_P (EX) - investment’s risk premium
• RR_NLF_M (EN) – replacement rate for middle-aged labour force (including disability and retirement benefits and social reliefs)
• RR_NLF_O (EN) – replacement rate for older labour force (including disability and retirement benefits and social relief)
• RR_RELIEF_KIND (EN) – replacement rate for social transfers in kind
• RR_REM (EN) – replacement rate for remittances
• RR_UNEMP (EN) – replacement rate for unemployment benefits
• RUCC (EN) – real user cost of capital
• RUCC_H (EN) – real user cost of residential capital
• S_EUR_PLN (EN) – exchange rate EUR/PLN
• S_NEER (EN) – nominal effective exchange rate
• S_REER (EN) – real effective exchange rate
• S_USD_EUR (EX) – exchange rate USD/EUR
• S_USD_PLN (EN) – exchange rate USD/PLN
• STOCK (EN) – level of inventories
• STOCK_N (EN) – nominal level of inventories
• STUDENT (EX) – trend indicating the relation of non-extramural students to younger population
• TFP_TREND (EN) – trend total factor productivity
• TRANS_CAP_N (EN) – Common Agricultural Policy transfers in PLN
- TRANS_EXP_N (EN) – EU transfers for current expenditures of general government in PLN
- TRANS_GFCF_F_N (EN) – EU structural funds for rural development in PLN
- TRANS_GFCF_G_N (EN) – EU structural funds for public capital development in PLN
- TRANS_GFCF_HC_N (EN) – EU funds for human capital development by enterprises in PLN
- TRANS_GFCF_P_N (EN) – other EU transfers mainly for enterprises in PLN
- ULCNA (EN) – unit non-agriculture labour costs
- UNEMP (EN) – number of unemployed
- UNRATE (EN) – unemployment rate
- W_CORE (EX) – weight of core inflation in CPI basket
- W_ENER (EX) – weight of energy prices in CPI basket
- W_FOOD (EX) – weight of food prices in CPI basket
- WAGE_N (EN) – nominal gross average wage
- WEALTH (EN) – wealth
- WEALTH_N (EN) – nominal wealth
- WORK_AGE (EX) – trend indicating the negative influence of gradual ageing of older population on its activity rate (the relation of population between 45+ and 65 years old to total 45+ population)
- YD (EN) – real disposable income
- YD_N (EN) – total nominal disposable income of households
- YD_NOS_N (EN) – nominal income from operating surplus of households
- YD_PRO_N (EN) – nominal income from property of households
- YD_WF_N (EN) – nominal wage bill
B General government sector

B.1 General government revenues

\[
GR_{Nt} = GR_{PROD\_TAX\_Nt} + GR_{INC\_TAX\_Nt} + 
+ GR_{TCONTR\_Nt} + GR_{PROP\_INC\_Nt} + 
+ GR_{OTHER\_CURT\_Nt} + GR_{OUTPUT\_Nt} + 
+ GR_{CAP\_TRANS\_Nt}
\]

\[
GR_{WEU\_Nt} = GR_{Nt} + TRANS\_EXP\_Nt + TRANS\_GFCF\_G\_Nt
\]

Taxes on production and imports

\[
GR_{PROD\_TAX\_Nt} = GR_{VAT\_NET\_Nt} + GR_{EXT\_Nt} + 
+ GR_{GAM\_Nt} + GR_{TAR\_NET\_Nt} + 
+ GR_{OTAX\_CORP\_Nt}
\]

\[
GR_{VAT\_Nt} = GR_{VAT\_TR^R} \cdot (CONP\_Nt + GE\_CON\_Nt + 
+ GE\_GFCF\_Nt + TRANS\_GFCF\_G\_Nt)
\]

\[
GR_{EXT\_Nt} = GR_{EXT\_ENER\_Nt} + GR_{EXT\_REST\_Nt}
\]

\[
GR_{EXT\_ENER\_Nt} = GR_{EXT\_ENER\_TR^R} \cdot 
\cdot W\_ENERt \cdot (CONP\_Nt + GE\_CON\_Nt)
\]

\[
GR_{EXT\_REST\_Nt} = GR_{EXT\_REST\_TR^R} \cdot 
\cdot W\_COREt \cdot (CONP\_Nt + GE\_CON\_Nt)
\]

\[
GR_{GAM\_Nt} = GR_{GAM\_TR^R} \cdot W\_COREt \cdot CONP\_Nt
\]

\[
GR_{TAR\_Nt} = GR_{TAR\_TR^R} \cdot GDP\_IMP\_Nt
\]

\[
GR_{OTAX\_CORP\_Nt} = GR_{OTAX\_CORP\_TR^R} \cdot GDP\_Nt
\]

\[
GR_{TAR\_NET\_Nt} = GR_{TAR\_Nt} \cdot (1 - GR\_EU\_TAR\_Nt)
\]

\[
GR_{VAT\_NET\_Nt} = GR_{VAT\_Nt} \cdot (1 - GR\_EU\_VAT\_Nt)
\]

Taxes on income, wealth, etc.

\[
GR_{INC\_TAX\_Nt} = GR_{PIT\_Nt} + GR_{CIT\_Nt} + GR_{OTAX\_HH\_Nt}
\]

\[
GR_{PIT\_Nt} = GR_{PIT\_TR^R} \cdot ((1 - 0.37) \cdot 0.5 \cdot (1 - G\_REFt)) \cdot 
\cdot GE\_PENSIONS\_Nt + 
+ (1 - G\_REFt) \cdot OFE\_Nt + GE\_UNEMP\_Nt + 
+ (1 - GR\_EMP\_TRt) \cdot YD\_WF\_Nt)
\]

\[
GR_{CIT\_Nt} = GR_{CIT\_TR^R} \cdot OPSURP\_Nt
\]

\[
OPSURP\_Nt = GDP\_Nt - GR_{PROD\_TAX\_Nt} - YD\_WF\_Nt + 
- GR_{CORP\_Nt} - GR_{OTAX\_CORP\_Nt} + 
- KN\_P_t \cdot PGDP_t \cdot DISC\_P_t
\]

\[
GR_{OTAX\_HH\_Nt} = GR_{OTAX\_HH\_TR^R} \cdot GDP\_Nt
\]

---

Superscript \( R \) denotes categories on which the fiscal rule is imposed.
Social contributions

\[ GR\_{ TCONTR\_ N_t} = GR\_ FARM\_ N_t + GR\_ HC\_ N_t + \]
\[ + GR\_ EMP\_ CORP\_ N_t - OFE\_ N_t \]
\[ GR\_ EMP\_ CORP\_ N_t = GR\_ EMP\_ N_t + GR\_ CORP\_ N_t \]
\[ GR\_ EMP\_ N_t = GR\_ EMP\_ TR_t^R \cdot YD\_ WF\_ N_t \]
\[ GR\_ CORP\_ N_t = GR\_ CORP\_ TR_t^R \cdot YD\_ WF\_ N_t \]
\[ GR\_ FARM\_ N_t = GR\_ FARM\_ TR_t^R \cdot GDP\_ N_t \cdot (EMP\_ A_t / EMP_t) \]
\[ GR\_ HC\_ N_t = GR\_ HC\_ TR_t \cdot ((1 - 0.37 \cdot 0.5 \cdot (1 - G\_ REF_t)) \cdot \]
\[ \cdot GE\_ PENSIONS\_ N_t + (1 - G\_ REF_t) \cdot OFE\_ N_t + \]
\[ + GE\_ UNEMP\_ N_t + \]
\[ + (1 - GR\_ EMP\_ TR_t) \cdot YD\_ WF\_ N_t) \]
\[ OFE\_ N_t = G\_ OFE_t \cdot GR\_ EMP\_ CORP\_ N_t \]

Non-tax revenues

\[ GR\_ PROP\_ INC\_ N_t = GR\_ PROP\_ INC\_ N_{t-1} \cdot (GDP\_ N_t + \]
\[ - GR\_ PROD\_ TAX\_ N_t - YD\_ WF\_ N_t + \]
\[ - GR\_ CORP\_ N_t + \]
\[ - KN\_ P_t \cdot PGDP_t \cdot DISC\_ P_t) / (GDP\_ N_{t-1} + \]
\[ - GR\_ PROD\_ TAX\_ N_{t-1} - YD\_ WF\_ N_{t-1} + \]
\[ - GR\_ CORP\_ N_{t-1} + \]
\[ - KN\_ P_{t-1} \cdot PGDP_{t-1} \cdot DISC\_ P_{t-1}) \]
\[ GR\_ OTHER\_ CURT\_ N_t = GR\_ OTHER\_ CURT\_ N_{t-1} \cdot GDP\_ N_t / GDP\_ N_{t-1} \]
\[ GR\_ OUTPUT\_ N_t = GR\_ OUTPUT\_ N_{t-1} \cdot GDP\_ N_t / GDP\_ N_{t-1} \]
\[ GR\_ CAP\_ TRANS\_ N_t = GR\_ CAP\_ TRANS\_ N_{t-1} \cdot GDP\_ N_t / GDP\_ N_{t-1} \]

B.2 General government expenditures

\[ GE\_ N_t = GE\_ SOC\_ CASH\_ N_t + GE\_ RELIEF\_ KIND\_ N_t + \]
\[ + GE\_ SUB\_ NOFARM\_ N_t + GE\_ SUB\_ FARM\_ N_t + \]
\[ + GE\_ OTHER\_ TRANS\_ N_t + GE\_ EU\_ N_t + \]
\[ + GE\_ FIN\_ N_t + GE\_ WF\_ N_t + GE\_ CON\_ N_t + \]
\[ + GE\_ GFCF\_ N_t + GE\_ CAP\_ TRANS\_ N_t \]
\[ GE\_ WEU\_ N_t = GE\_ N_t + TRANS\_ EXP\_ N_t + TRANS\_ GFCF\_ G\_ N_t \]
Social expenditures

\[
\begin{align*}
GE\_SOC\_CASH\_N_t &= (1 - 0.37 \cdot 0.5 \cdot (1 - G\_REF_t)) \cdot \Big( GE\_PENSIONS\_N_t + GE\_UNEMP\_N_t + GE\_RELIEF\_CASH\_N_t + GE\_SOC\_COMP \Big) \\
GE\_UNEMP\_N_t &= (G\_UNEMP_t \cdot UNEMP_t) \cdot \frac{3}{1000} \cdot AVGBENEFIT \\
AVGBENEFIT\_N_t &= AVGBENEFIT\_N_{t-1} \cdot \left( \frac{WAGE\_N_{t-1}}{WAGE\_N_{t-5}} \right)^{0.25} \\
GE\_PENSIONS\_N_t &= RETIRED_t \cdot \frac{3}{1000} \cdot AVGPENSION\_N_t \\
AVGPENSION\_N_t &= \frac{AVGPENSION\_N_{t-1}}{WAGE\_N_{t-1}} \cdot WAGE\_N_t \\
GE\_RELIEF\_CASH\_N_t &= (POP_t - EMP_t) \cdot \frac{3}{1000} \cdot AVGRELIEF\_CASH\_N_t \\
AVGRELIEF\_CASH\_N_t &= AVGRELIEF\_CASH\_N_{t-1} \cdot \left( \frac{WAGE\_N_{t-1}}{WAGE\_N_{t-5}} \right)^{0.25} \\
GE\_RELIEF\_KIND\_N_t &= GE\_RELIEF\_KIND\_N_{t-1} \cdot \left( \frac{WAGE\_N_{t-1} \cdot (POP_t - EMP_t)}{WAGE\_N_{t-5} \cdot (POP_t - EMP\_t)} \right)^{0.25} \\
AVGRELIEF\_KIND\_N_t &= GE\_RELIEF\_KIND\_N_t / ((POP_t - EMP_t) \cdot \frac{3}{1000})
\end{align*}
\]

Subsidies, other current transfers, and contribution to EU budget

\[
\begin{align*}
GE\_SUB\_NOFARM\_N_t &= GE\_SUB\_NOFARM\_N_{t-1} \cdot \frac{GDP\_N_t}{GDP\_N_{t-1}} \\
GE\_SUB\_FARM\_N_t &= GE\_SUB\_FARM\_N_{t-1} \cdot \frac{GDP\_N_t}{GDP\_N_{t-1}} \\
GE\_OTHER\_TRANS\_N_t &= GE\_OTHER\_TRANS\_N_{t-1} \cdot \frac{GDP\_N_t}{GDP\_N_{t-1}} \\
GE\_EU\_N_t &= GE\_EU\_N_{t-1} \cdot \frac{GDP\_N_t}{GDP\_N_{t-1}}
\end{align*}
\]

Compensation of employees, intermediate consumption and investments

\[
\begin{align*}
GE\_WF\_N_t &= GE\_WF\_N_{t-1} \cdot \frac{WAGE\_N_t}{WAGE\_N_{t-1}} \\
GE\_CON\_N_t &= GE\_CON\_N_{t-1} \cdot \frac{GDP\_N_t}{GDP\_N_{t-1}} \\
GE\_GFCF\_N_t &= GE\_GFCF_t \cdot PV_A_t \cdot (1 + GR\_VAT\_TR_t) \\
\Delta ge\_gfcf_t &= \Delta gdp_t - 0.01 \cdot (kn\_g_t - \log(3/T) - kn\_p_t)
\end{align*}
\]
Capital transfers

\[ GE\_CAP\_TRANS\_N_t^R = GE\_CAP\_TRANS\_N_{t-1} \cdot \frac{GDP\_N_t}{GDP\_N_{t-1}} \]

Costs of debt service

\[
\begin{align*}
GE\_FIN\_N_t & = GE\_INT\_DOM\_N_t + GE\_INT\_FOR\_N_t \\
GE\_INT\_FOR\_N_t & = GE\_INT\_FOR\_N\_COR_t \cdot \frac{1}{2000} \cdot \\
& \quad \sum_{i=1}^{5} \left( (G\_DEBT\_FOR\_N_{t-4i} \cdot I\_5Y\_EUR_{t-4i} \cdot \\
& \quad \cdot (G\_DEBT\_EUR_{t-4i} \cdot S\_EUR\_PLN_t + \\
& \quad + (1 - G\_DEBT\_EUR_{t-4i}) \cdot S\_USD\_PLN_t) \cdot \\
& \quad / (G\_DEBT\_EUR_{t-4i} \cdot S\_EUR\_PLN_{t-4i} + \\
& \quad + (1 - G\_DEBT\_EUR_{t-4i}) \cdot S\_USD\_PLN_{t-4i})) \\
GE\_INT\_DOM\_N_t & = GE\_INT\_DOM\_COR_t \cdot \\
& \quad \cdot (W\_SHORT\_DOM_t \cdot I\_3M_t + \\
& \quad + (1 - W\_SHORT\_DOM_t) \cdot I\_5Y_t) \\
\end{align*}
\]

Active labour market policy

\[
\begin{align*}
ALMP\_N_t & = GE\_ALMP\_N_t + TRANS\_GFCF\_HC\_N_t \\
GE\_ALMP\_N_t^R & = GE\_ALMP\_N_{t-1} \cdot (GDP\_N_t/GDP\_N_{t-1}) \\
\end{align*}
\]

B.3 General government balance and debt

\[
\begin{align*}
G\_BALANCE\_N_t & = GR\_N_t - GE\_N_t \\
G\_DEBT\_N_t & = G\_DEBT\_DOM\_N_t + G\_DEBT\_FOR\_N_t \\
G\_DEBT\_FOR\_N_t & = G\_DEBT\_FOR\_N_{t-1} \cdot (G\_DEBT\_EUR_t \cdot S\_EUR\_PLN \\
& \quad + (1 - G\_DEBT\_EUR_t) \cdot S\_USD\_PLN_t) \cdot \\
& \quad / (G\_DEBT\_EUR_t \cdot S\_EUR\_PLN_{t-1} + \\
& \quad + (1 - G\_DEBT\_EUR_t) \cdot S\_USD\_PLN_{t-1}) + \\
& \quad + (-G\_BALANCE\_N_t - GE\_PRIV\_N_t) \cdot \\
& \quad \cdot G\_DEBT\_FOR\_N_{t-1} \\
G\_DEBT\_DOM\_N_t & = G\_DEBT\_DOM\_N_{t-1} + \\
& \quad + (-G\_BALANCE\_N_t - GE\_PRIV\_N_t) \cdot \\
& \quad \cdot (G\_DEBT\_DOM\_N_{t-1}/G\_DEBT\_N_{t-1}) \\
\end{align*}
\]
C Variables

C.1 Residential investment and capital stock series.

Our estimates of residential investment are based on the main indicators taken into account by Poland’s CSO while assessing the level of housing investment:47

- building permits,
- dwellings started,
- dwellings under construction,
- dwellings completed,
- construction output.

The data is adjusted for the usable floor space and the quality growth.48 The period of 2002q4-2003q4 is not taken into account, as the series show artificial peaks, which are due to expected changes in regulations at that time.49

Using common component method for the growth rates of the five series, we obtain weights for each indicator, which are subsequently used to calculate an estimate of the growth rate of residential investment. The level of residential investment pinned down by the data for 2006, published by Eurostat.

Public and private capital series are constructed using the method (modified perpetual inventory) described in Gradziewicz and Kolasa (2004) The initial level of total capital stock in the economy is pinned down by the national accounts data, while that of public capital (defined as assets of the general government sector) is based on the CSO annual publication "Fixed assets in National Economy". For lack of other estimates, the discard rates were calibrated to roughly fit the gross fixed assets series published by the CSO.

C.2 Population, employment and unemployment

The data on population, employment, unemployment and the number of non-participants are based on the Labour Force Survey (LFS). The LFS has been conducted on a quarterly basis, with a break in 1999. The data for the two missing quarters were extrapolated with the TRAMO/SEATS procedure. Since 1995 there have been a few methodological changes introduced in the survey, which we correct for using the expert knowledge.

As described in Budnik (2007), the aggregate LFS data might be severely biased due to failure of the CSO to capture all temporary migration movements in its LFS population calculations. The issue has become particularly worrisome after Poland joined the European Union and migration movements heavily intensified. To correct the bias, we use the

---

47We also use construction output, though it is not officially quoted by the CSO as an important proxy for residential investment activity.
48We assumed 1% annual growth of quality over our sample period.
49In 2002, according to some estimates, over 80% of dwellings completed were inhabited but not formally submitted to registration, which allowed investors to take advantage of the so-called great building tax allowance. Owners also often postponed registration due to higher financial burden which came into effect after introducing the new building law.
data on employment in the national economy collected from enterprises. Although these data do not cover those employed in small enterprises, they are likely to be unbiased, as they are estimated without a reference to the LFS population. The smoothed ratio of employment in the economy to the level of employment according to the LFS was used to correct all time series data originating from the LFS. As the correction does not change proportions between various categories of the LFS data, it does not affect the estimates of the unemployment rate and participation rates.

C.3 Replacement rates for unemployed and inactive

Replacement rates for unemployed and non-participants are defined as expected income of workers when unemployed or inactive, relative to their expected income when employed. Calculation of these categories is hindered by both multiple changes in the social security benefits system and frequent unavailability of data on different benefits in the period considered. The aggregate data on social transfers and the number of persons eligible for different benefits were used to calculate the average level of each benefit. The replacement rate for a particular type of benefit is then calculated as the average benefit to average wage rate, with appropriate corrections for taxes and social security contributions. Next, these individual replacement rates are aggregated using the fractions of unemployed or inactive receiving these particular kinds of benefit, where the fractions are calculated based on declared income of workers in the LFS. In practice, due to data availability constraints, a two step approach has been used. In the first step, four broad categories of benefits are distinguished: retirement benefits, disability benefits, unemployment benefits and other benefits. The last category encompasses family pensions, pre-retirement benefits and allowances, as well as family and social assistance benefits. In the second step, the four replacement rates are used to determine the weighted average replacement rate for unemployed and non-participants, with the weights taken form the LFS survey.

C.4 World indices

C.4.1 World food price index.

The world food price index is based on subindices used by the International Monetary Fund (IMF) for construction of their food price index:

- Wheat prices (24%)
- Sugar prices (9%)
- Olive oil prices (8%)
- Orange prices (5%)
- Bananas prices (4%)
- Pork prices (23%)
- Poultry prices (14%)
- Beef prices (7%)
• Fish prices (5%)

The weights assigned to these types of products reflect the structure of households’ consumption in Poland, in line with the CPI basket.

C.4.2 World energy price index.

The world energy price index consists of three energy products: hard coal (64%), crude oil (22%) and natural gas (14%). The weights assigned to these products were calculated on the basis of the CSO publication “Energy statistics 2005, 2006”.

The individual energy basket components are taken from the following sources:

• hard coal price index from Richard’s Bay,
• worldwide Brent oil index
• natural gas price index Henry Hub (standardized by Reuters).

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50 Following the assumption that energy consumption in Poland concerns mainly these three commodities.
D  Seasonal adjustment

Seasonal adjustment of the time series used in NECMOD is performed using two algorithms, TRAMO\SEATS and X-12-ARIMA. The choice of the particular algorithm is made individually for each variable. To avoid changes in the seasonally corrected time series upon adding new observations between forecasting rounds, optimal seasonal adjustment procedures are fixed between full model re-estimations.

A problem arises when aggregated time series are to be used in the model along with their subcomponents. When seasonal adjustment is performed on each series separately, it rarely happens that the relation between original series holds also for seasonally adjusted ones. In the case of NECMOD, this issue concerns three groups of variables: National Accounts, labour market and fiscal variables.\footnote{In such circumstances, a joint seasonally adjustment procedure is justified.}

To illustrate this procedure, we take GDP and its main expenditure components as an example. The starting point is the following identity, holding for raw (unadjusted series):

\[
\text{GROSS DOMESTIC PRODUCT} = \text{PRIVATE CONSUMPTION} + \\
+ \text{GOVERNMENT CONSUMPTION} \\
+ \text{GROSS FIXED CORP. CAP. FORM.} \\
+ \text{GROSS FIXED PUBLIC CAP. FORM.} \\
+ \text{GROSS FIXED RESID. CAP. FORM.} \\
+ \text{INVENTORIES} + \text{EXPORT - IMPORT}
\]

Each element of the equation above, both in nominal and real terms along with its deflator, is separately corrected for seasonality. After this first step, we get the following system of identities (including error terms).

\[
\begin{align*}
\text{GDP}_N &= \text{CONP}_N + \text{CONGOV}_N + \text{GFCF}_P_N + \text{GFCF}_G_N + \\
&\quad + \text{GFCF}_H_N + \text{INV}_N + \text{GDP}_\text{EXP}_N - \text{GDP}_\text{IMP}_N + \epsilon_1 \tag{73} \\
\text{GDP} &= \text{CONP} + \text{CONGOV} + \text{GFCF}_P + \text{GFCF}_G + \\
&\quad + \text{GFCF}_H + \text{INV} + \text{GDP}_\text{EXP} - \text{GDP}_\text{IMP} + \epsilon_2 \tag{74}
\end{align*}
\]

\[
\begin{align*}
gdp_n &= \text{gdp} + \text{pgdp} + \xi_1 \tag{75} \\
\text{comp}_n &= \text{comp} + \text{pcomp} + \xi_2 \tag{76} \\
\text{congov}_n &= \text{congov} + \text{pcongov} + \xi_3 \tag{77} \\
\text{gfcf}_p_n &= \text{gfcf}_p + \text{pgfcf}_p + \xi_4 \tag{78} \\
\text{gfcf}_g_n &= \text{gfcf}_g + \text{pgfcf}_g + \xi_5 \tag{79} \\
\text{gfcf}_h_n &= \text{gfcf}_h + \text{pgfcf}_h + \xi_6 \tag{80} \\
\text{inv}_n &= \text{inv} + \text{pinv} + \xi_7 \tag{81} \\
\text{gdp}_\text{exp}_n &= \text{gdp}_\text{exp} + \text{peexp} + \xi_8 \tag{82} \\
\text{gdp}_\text{imp}_n &= \text{gdp}_\text{imp} + \text{pimp} + \xi_9 \tag{83}
\end{align*}
\]

\footnote{Additionally, for fiscal variables it is necessary to preserve yearly sums of quarterly observations.}
Next, we run an iterative procedure to impose zero deviation condition on the relations above. Thus all \( \epsilon_i \) and \( \xi_j \) are forced to be possibly close to zero, with all postulated relations between macrocategories preserved.
E  The former method of uncertainty analysis at the NBP

E.1  Introduction

In the period May 2005 - June 2008 uncertainty at the NBP was estimated on the basis of stochastic simulations of the model. Thus, the assumption was made that if the forecasting model was an adequate tool for setting the central projection path, it would also well assess the uncertainty of the projection. As not all of the sources of uncertainty are quantifiable (compare Chapter 5.2) simulations took into account uncertainty related to the forecasts of exogenous variables (as a proxy for variable measurement) and to the error term (as a proxy for estimation uncertainty). In the next paragraphs the details of that procedure are given.

E.2  Procedure

The former procedure of constructing the fan charts consisted of the following steps:

1. Simulation of paths of exogenous variables. Experts forecasting exogenous variables, in every forecasting round presented the central path (the expected value) and the uncertainty assessment of the forecast of the given variable (whereas the distribution of risks had not to be symmetric). The simulation procedure of exogenous variables was designed in such a way that the expected value of simulated paths of variables conformed to central paths given by experts and the autocorrelation of variables observed in the sample was retained.

2. Calculating endogenous variable uncertainty. Endogenous variable uncertainty was proxied by error terms of the estimated equations. These shocks were simulated using historical realizations, as in the 'bootstrap' method. This ensured that shocks to the endogenous variables had not only magnitude observed in the past, but their joint distribution reflected as well the historical cross-correlation structure.

3. Simulations from the model. Fan charts were obtained by running stochastic simulations of the model.\(^2\)

Simulated values of inflation and GDP in each quarter were used to derive percentiles of their distributions. The fan-chart was not the only way to illustrate the uncertainty of the inflation projection. Having had obtained the approximation of its overall probability distribution, it was easy to calculate e.g. the probability of the inflation rate being in a given interval.

\(^2\) Technically, model was solved sequentially, quarter after quarter and solution in each period took into account model-consistent expectations. In each quarter, agents formed their expectations about future paths of all variables, relying on the shocks that had realized up to this period. More specifically, agents assumed that in the future endogenous variables would evolve according to their equations and exogenous variables would follow the laws of motion for the shock processes. In the next quarter shocks for that period were generated and agents updated their expectations.
E.3 Decomposition of uncertainty

The fan chart decomposition allowed to determine the relative importance of uncertainty regarding individual exogenous assumptions and goodness-of-fit of particular equations for the overall forecast uncertainty. It is worth emphasizing that such a breakdown is not possible in a number of other methodologies used to generate fan-charts, including the new NBP’s procedure.

The fan-chart decomposition enabled to better understand the sources of forecast uncertainty and to better communicate them to the public. This procedure allowed the central bank to concentrate on economic indicators which are essential for the monetary policy conduct. Besides, it could help to identify these equations and exogenous variables, which were crucial to the quality of the forecasts.

Generally, to decompose uncertainty, the following procedure was carried out. Stochastic simulations proceeded exactly the same way as described above, with one exception: shocks were switched on one at time or in groups. Then, the forecast variance attributed to each shock was calculated and divided by the total variance of the forecast (when all shocks were considered) to obtain the relative contributions.

The determinants of forecast uncertainty for inflation and output growth not only differed from each other, but also were different over short and long forecast horizons. In the version of NECMOD from June 2008, the uncertainty for GDP growth over the whole forecast horizon came mainly from error accumulation, while the uncertainty related to exogenous variables played a less important role. The situation was similar for the short-run inflation uncertainty. However, in the case of inflation, the role of exogenous variables was increasing significantly with the forecast horizon, and both groups had a similar contribution to uncertainty in the last quarters of projection.

With regard to individual factors, inflation uncertainty arose mainly from shocks in equations forming the price block of the model (food prices, energy prices and core inflation). However, in the long run their impact decreased significantly. The error term in the exchange rate equation also played an important role both in the short and in the long run. The impact of shocks to the labour market, the value-added deflator and oil prices was relatively small in the short run, but played a significant role in the long run. The contribution of uncertainty related to foreign food prices grew gradually with the forecast horizon, having become the most important source of uncertainty. At the end of the forecasting period, one could also observe a growing impact of errors in the import equation.

Factors which determine the aggregate demand were mainly responsible for output uncertainty both in the short and in the long run. This was particularly true for shocks to import, stock and corporate investment equations, but also, though to lesser extent, for consumption and export equations.

E.4 Advantages and disadvantages

Advantages:

- Uncertainty is estimated on the basis of the same model which is used for constructing the projection.
• Sources of uncertainty of the projection are precisely determined.
• Decomposition of uncertainty is possible.
• Uncertainty depends on the current uncertainty of exogenous variables.
• No history of forecasts from the model is needed.

Disadvantages:

• Significant sensitivity to the form of the forecasting model.
  Uncertainty should depend on the choice and the specification of the model. However, excessive sensitivity of results obtained on the basis of the model to endogenization or exogenization of selected variables or to the change in the form of the equation, is undesirable.

• Overestimation of uncertainty.
  Methods based on stochastic simulations with shocks applied both to endogenous and exogenous variables, may overestimate uncertainty. First, uncertainty is estimated only on the basis of the model, i.e. the role of experts in adjusting forecasts is ignored. Secondly, uncertainty obtained in stochastic simulations increases if some parameters of the model equations are calibrated. Calibration of parameters (based on the knowledge about the economy) should, to the contrary, reduce the actual uncertainty of forecasts.

• Disregarding data-related uncertainty.
  Data are burdened with uncertainty due, among others, to revisions of national accounts and the lack of certain variables at the starting point of the projection (they are estimated by experts). This uncertainty is not taken into consideration in stochastic simulations.

• Independence of stochastic simulation results from past forecasting errors