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Monetary policy in a non-representative
agent economy: A survey

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Abstract

It is well-known that central bank policies affect not only macroeconomic aggregates, but also their distribution across economic agents. Similarly, a number of papers demonstrated that heterogeneity of agents may matter for the transmission of monetary policy on macro variables. Despite this, the mainstream monetary economics literature has so far been dominated by dynamic stochastic general equilibrium (DSGE) models with representative agents. This article aims to tilt this imbalance towards heterogeneous agents setups by surveying the main positive and normative findings of this line of the literature, and suggesting areas in which these models could be implemented. In particular, we review studies that analyze the heterogeneity of (i) households' income, (ii) households' preferences, (iii) consumers' age, (iv) expectations, and (v) firms' productivity and financial position. We highlight the results on issues that, by construction, cannot be investigated in a representative agent framework and discuss important papers modifying the findings from the representative agent literature.

Keywords: Heterogeneous Agents; Monetary Policy.

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Non-technical summary

It is well-known that central bank policies affect not only macroeconomic aggregates, but also their distribution across economic agents. Similarly, a number of papers demonstrated that heterogeneity of agents may matter for the transmission of monetary policy on macro variables. Despite this, the mainstream monetary economics literature has so far been dominated by dynamic stochastic general equilibrium models with representative agents (RA). This article aims to tilt this imbalance towards heterogeneous agents (HA) setups by surveying the main positive and normative findings of this line of the literature, and suggesting areas in which these models could be implemented. In particular, we review studies that analyze the heterogeneity of (i) households' income, (ii) households' preferences, (iii) consumers' age, (iv) expectations, and (v) firms' productivity and financial position.

We find six major topics in monetary policy for consideration of which models with heterogeneous agents are required. First, models with heterogeneity of income or age offer new findings on the cost of inflation. They show that RA models, since they neglect distributional effects, may give biased measures of the cost of inflation, which in turn affects the optimal monetary policy design. Second, in models featuring income heterogeneity and uninsurable idiosyncratic uncertainty the equilibrium real interest rate is lower than that found in the RA framework due to precautionary savings. Hence, this line of HA models helps to resolve the “risk-free rate puzzle”, i.e. a well-known result that the observed real rates are lower than those obtained in standard RA models. Third, taking into account heterogeneity of expectations changes their policy implications. In particular, it strengthens propagation of shocks, alters the determinacy regions or generates chaotic dynamics. Fourth, unlike RA models, HA setups can be used to study redistributive effects of inflation. What is more important, however, if this redistribution generates asymmetric responses, it will also affect aggregate macroeconomic variables. Fifth, heterogeneity of preferences alters substantially the findings from the RA literature on the cost of business cycle fluctuations. In HA models, these costs can be up to 100 times higher than in models with representative agents. Sixth, heterogeneity of preference or firm productivity is crucial for introduction of imperfect financial intermediation. Such models can be used i.a. to show the impact of financial

sector on the economy or for examination of the working of monetary and regulatory policy in the environment of financial imperfections.

Our study makes two important general points. First, HA models should become an important ingredient of monetary economics. Not only do they provide information that is unavailable in the absence of heterogeneity, but also change our view on several classical results from the RA literature. Second, despite the recent effort in the area of monetary policy research to take into account heterogeneity, there is still a lot of work to be done. Several research topics, including the investigation of optimal monetary policy and the analysis of dynamic effects of monetary policy shocks in various HA frameworks or the consequences of introducing heterogeneity in the banking sector should (and most likely will) be put on the research agenda soon.

1 Introduction

In the last decades, with the development of numerical methods and increased power of computers, macroeconomic models have incorporated different types of heterogeneity. The seminal papers by Bewley (1980), Auerbach and Kotlikoff (1987), Hopenhayn (1992), Aiyagari (1994), Huggett (1996), Krusell and Smith (1998), Melitz (2003), Branch and McGough (2009), and other works surveyed by Heathcote et al. (2009), show that heterogeneity is important in the analysis of the long run effects of different economic policies and helps in understanding the dynamics of macroeconomic aggregates. Despite this, the representative agent New Keynesian framework is still the dominant tool used for quantitative policy analysis at central banks and other policy making institutions. The most prominent examples are the following models: SIGMA (Federal Reserve), NAWM (ECB), ToTEM (Bank of Canada), BEQM (Bank of England) or RAMSES (Sveriges Riksbank).

The aim of this paper is to explore the heterogeneous agents literature that is relevant to monetary policy. We do it along two margins. The first one divides the literature by the type of investigated heterogeneity. In the real world, it can manifest in various forms, many of which are not pertinent to monetary policy-making. We focus only on those aspects of heterogeneity which are most relevant to monetary policy. In particular, we consider the heterogeneity of households in terms of income, preferences, age and expectations, as well as the heterogeneity of firms' productivity and financial position.

The second margin emphasizes two dimensions where the HA literature can contribute. First, it can help to address questions that the representative agents models cannot take up by construction, like the redistributive role of inflation or the consequences of heterogeneous expectations on monetary transmission. Second, it has the potential to change the results obtained in a standard RA setting. These include lowering the equilibrium real interest rate, increasing the cost of business cycle fluctuations or modifying the optimal policy prescriptions.

Given the limited size of this survey and the rapid development of HA models related to monetary policy, we do not aim at covering every relevant topic or paper. Instead,

within each type of heterogeneity analyzed, we concentrate on two or three areas of research that seem most relevant from a central banks' perspective.

The rest of the article is structured as follows. Section 2 outlines the standard New Keynesian RA model, which currently dominates the discussion on the shape of monetary policy. This model is presented to serve as a benchmark for the discussion offered in the remaining sections. Section 3 investigates the effects of households' income heterogeneity on the equilibrium value of the real interest rate and reviews the main findings related to the welfare costs of inflation. In section 4 we show how heterogeneity of preferences affects the cost of business cycle fluctuations and how it allows for the inclusion of financial frictions in monetary models. In section 5 we examine how age heterogeneity affects the optimal design of monetary policy, the welfare costs of inflation, and the monetary transmission mechanism. Moreover, we look at the distributional effects of inflation and their impact on aggregate macro variables. Section 6 investigates how heterogeneity of expectations influences determinacy regions of monetary policy rules and the dynamic effects of stochastic shocks. Finally, section 7 examines how heterogeneity of firms' productivity affects the link between real activity and inflation. It also discusses the positive and normative effects of monetary policy when idiosyncratic productivity shocks affect firms' access to external financing. The last section concludes.

2 Representative agent model

Currently, the New Keynesian RA model is the dominant tool used for quantitative policy analysis and forecasting at most central banks around the world. It is based on a paradigm that the dynamics of aggregate variables is determined by the behavior of infinitely lived, rational, representative agents, who solve their inter- and intratemporal optimization problems. Below we outline the basic structure of such an RA model.

A continuum of households indexed by i maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{it}, L_{it} \dots), \quad (1)$$

where β is the discount factor, $u(\cdot)$ denotes the utility function, C_{it} stands for consumption, L_{it} is the labour input and \mathbb{E}_0 describes the expectation operator conditional on information at period 0. Three dots describe other factors, including stochastic aggregate shocks.

Expression (1) is maximized subject to the budget constraint:

$$P_t C_{it} + B_{i,t+1} \leq R_t B_{it} + W_{it} L_{it} + R_{it}^k K_{it} + D_{it}, \quad (2)$$

and the capital law of motion:

$$K_{i,t+1} = (1 - \delta) K_{it} + (1 - S(I_{it}/I_{i,t-1})) I_{it}. \quad (3)$$

In the two formulas above, K_{it} is the capital stock, W_{it} and R_{it}^k denote nominal wages and rental rate on capital, D_{it} are dividends, whereas B_{it} stands for net worth. The gross nominal interest rate and the price level are denoted by R_t and P_t , respectively. $S(I_{it}/I_{i,t-1})$ is the investment adjustment cost function such that $S(1) = S'(1) = 0$ and $S''(1) > 0$.

A continuum of firms indexed by j use labor L_{jt} and capital K_{jt} in the production process:

$$Y_{jt}^S = f(A_t, L_{jt}, K_{jt}, \dots), \quad (4)$$

where A_t is productivity. The demand for output produced by firm j decreases with its price P_{jt} and rises with the aggregate demand Y_t , so that:

$$Y_{jt}^D = g(P_{jt}/P_t, Y_t, \dots). \quad (5)$$

Moreover, it is assumed that adjusting prices is costly:

$$\psi_{jt} = \psi(P_{jt}/P_{j,t-1}, Y_{jt}, \dots), \quad (6)$$

where $\psi(\cdot)$ is the price adjustment cost function. Given that labor and capital inputs are chosen so that $Y_{jt}^S = Y_{jt}^D = Y_{jt}$, the value of dividends amounts to:

$$D_{jt} = P_{jt}Y_{jt} - W_{jt}L_{jt} - R_{jt}^k K_{jt} - P_t\psi_{jt}, \quad (7)$$

Where P_t is the aggregate price level.

The optimization problem of firm j is to set its price P_{jt} at a level maximizing the present value of expected dividends:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Theta_t D_{jt}, \quad (8)$$

where Θ_t is the stochastic discount factor, consistent with households' optimization problem.

Finally, the last actor of the model is the central bank that sets the nominal interest rate according to the monetary policy rule:

$$R_t = R(R_{t-1}, \Pi_t, \hat{Y}_t, \dots), \quad (9)$$

where $\Pi_t = P_t/P_{t-1}$ is the inflation rate and \hat{Y}_t stand for the output gap, defined as the deviation of output from its target (usually steady-state) level.

In the New Keynesian RA model it is assumed that either households and firms are identical, or there exist complete markets for state-contingent claims allowing them to insure against idiosyncratic risk. Hence, all relevant individual variables do not differ across the agents. As a result, aggregation is straightforward and economy-wide

shocks are the only source of uncertainty in the model. This assumption limits the computational costs: standard perturbation techniques, Kalman filtering and Bayesian inference can be applied to solve, simulate and estimate the New Keynesian RA model. Moreover, all calculations can be performed in user friendly programs such as Dynare.

In the case of HA models, the entry costs are substantially higher. Developing a model usually requires writing one's own, model-specific computer codes, collecting microeconomic data needed to parametrize the model, and many CPUs to solve and simulate it. Even the calculation of a stationary equilibrium, which can be done with a pencil and paper for the standard New Keynesian RA model, is computationally and time demanding for a simple HA model (see Rios-Rull, 1997, for an extended exposition of the problem). For this reason, most papers using HA models restrict their attention to the long-term implications of different economic policies, and relatively few studies investigate the dynamic effects of aggregate shocks. Finally, it should be noted that the use of HA models for forecasting is so far limited, since (according to our best knowledge) no efficient algorithms have been developed to fit their dynamics to the dynamics of macroeconomic aggregates observed in reality. This might change in the near future, given that much research is currently done to create computationally tractable HA models and to develop efficient numerical methods of solving and fitting them to the data.

3 Heterogeneity of households' income

The first type of heterogeneity we investigate is that related to households' income. According to the US Census Bureau, income inequality in the US is sizable. In 2008, the share of aggregate income received by the lowest and highest quintiles stood at 3.4% and 50.0%, respectively. The Gini coefficient for wages amounted to 0.466 (DeNavas-Walt et al., 2009) and for net wealth 0.77 (Heathcote et al., 2010). Inequality is also substantial in other countries around the world, as evidenced by the special issue of the Review of Economic Dynamics on "Cross Sectional Facts for Macroeconomists" (see Krueger et al., 2010, for an introduction to this issue). This inequality, which is absent in the RA setup, can be relatively well explained by HA models with individual income uncertainty and incomplete financial markets. This section discusses the main insights from this class of models for two issues related to monetary policy: (i) implications of income heterogeneity for the equilibrium level of the real interest rate and (ii) new channels through which inflation influences welfare.

We start with discussing the equilibrium level of the real interest rate. In the RA model this rate is given by the consumption Euler equation, which states that the marginal utility of consumption follows:

$$u'(C_t) = \beta \mathbb{E}_t(r_t u'(C_{t+1})), \quad (10)$$

where $r_t = R_t \times \mathbb{E}_t(\Pi_{t+1}^{-1})$ is the real interest rate. For the constant relative risk aversion (CRRA) utility function, this implies the steady-state level of r_t given by

$$\bar{r} = \frac{1}{\beta} \gamma_C^\phi, \quad (11)$$

where ϕ is the coefficient of relative risk aversion and γ_C is the expected growth rate of consumption on the equilibrium path. The data for the US and many other economies show that real interest rates implied by the RA model tend to be significantly higher than those observed in reality.¹

¹In the period 1961-2008, the average per capita real consumption growth rate in the US was 1.9% and the short-term real interest rate averaged 1.5%. Fitting equation (10) to these numbers is impossible unless one assumes unreasonable values of β or ϕ .

The attempts to explain this “risk-free rate puzzle” within the RA model have been largely unsuccessful (see Weil, 1989; Canzoneri et al., 2007). This is not the case for the HA model with idiosyncratic income risk, against which individuals cannot insure due to financial markets incompleteness. In this kind of setup individuals accumulate so-called precautionary savings to self-insure against future negative income shocks (see Zeldes, 1989 for a theoretical model and Carroll and Samwick, 1998 for empirical evidence). Higher savings increase the stock of capital, which decreases the real interest rate below the level implied by the RA model.

To illustrate the importance of precautionary savings for the level of the real interest rate we simulate two popular HA models, which were proposed by Huggett (1993) and Aiyagari (1994). Both models consider a flexible-price exchange economy in which individuals experience idiosyncratic income shocks. This is the only source of heterogeneity, in all other aspects agents are identical. The equilibrium value of the interest rate is given by the condition stating that the supply of and demand for assets are equal. In the Huggett model (without capital), this means that the aggregate value of households' deposits is equal to the aggregate value of households' loans. In the Aiyagari model (with capital), this condition states that the aggregate value of capital must be equal to the difference between aggregate households' deposits and loans.

In both models, the optimization problem faced by individuals is given by (1), subject to the budget constraint (2), the borrowing limit constraint:

$$B_{i,t+1} \geq -\underline{B} \quad (12)$$

and the individual income process:

$$\ln W_{it} = \rho \ln W_{i,t-1} + (1 - \rho^2)^{1/2} \sigma \epsilon_{it} \quad (13)$$

where $\epsilon_{it} \sim N(0, 1)$ and σ stands for the unconditional standard deviation of $\ln W_{it}$.

To explore the effects of income heterogeneity and financial markets imperfections on the equilibrium level of interest rates, we solve both models for various values of \underline{B} and σ . The chosen grid $\underline{B} \times \sigma \in \{0, 1, 2, 4, 8\} \times \{1/4, 1/2, 3/4, 1\}$, where the average annual income is normalized to unity, covers various estimates for the United States and

other countries (see Krueger et al., 2010). The remaining parameters, set at an annual frequency, are as follows. The discount factor β is fixed at 0.96, which means that the equilibrium real interest rate implied by the RA model is $\bar{r} = 1.0417$ (see equation (11) for $\gamma_C = 1$). The utility function is of the CRRA form, with the coefficient of relative risk aversion ϕ equal to 2. In the case of the Aiyagari model, we follow the source article and assume that the production function is of the Cobb-Douglas form with the capital share α equal to 0.36, and that capital depreciates at a rate $\delta = 0.08$. Next, we set the persistence parameter ρ of the individual income process (13) to 0.96, in line with the estimates of Huggett (1996), Floden and Lindé (2001) and Storesletten et al. (2004). Finally, we approximate the individual income process by a seven-state Markov chain using the algorithm of Tauchen (1986).

The results of our simulations are presented in Table 1. They show that the equilibrium level of the interest rate increases with the credit limit. This is because, at a given interest rate, a loosening of the borrowing constraint means a higher aggregate value of loans. To ensure the equilibrium on the asset market, this requires an increase in the aggregate value of deposits, hence a higher level of the interest rate. In the limit, i.e. when $\underline{B} \rightarrow \infty$, individuals can perfectly insure against individual income risk and thus the real interest rate converges to the RA model value of 4.17%. The results also show that an increase in the volatility of the individual income process leads to a decrease in the level of the real interest rate. The interpretation is that at a given level of the interest rate, an increase in the risk of future income creates additional demand for precautionary savings. This must be offset by a decrease in the level of the interest rate so that asset markets clear. In the limit, i.e. when $\sigma \rightarrow 0$, income of all individuals is the same and thus the real interest rate converges to the RA model value. Finally, one can note that in the Huggett model, when no borrowing is allowed, the level of the real interest rate is deeply negative. This is because the asset market clearing condition requires that no household, even the most productive one, holds deposits. In other words, the interest rate needs to be low so that everybody spends his or her entire income on consumption.

Table 1: Levels of the real interest rate in the Huggett and Aiyagari models (in %)

$\downarrow \underline{B}$	$\sigma \rightarrow$	Huggett model				Aiyagari model			
		1/4	1/2	3/4	1	1/4	1/2	3/4	1
0		0.82	-5.10	-14.9	-29.1	3.71	2.74	1.50	0.41
1		3.29	1.44	-1.29	-4.58	3.74	2.82	1.62	0.57
2		3.45	1.93	-0.12	-1.99	3.76	2.85	1.66	0.65
4		3.60	2.30	0.70	-0.10	3.79	2.87	1.73	0.78
8		3.68	2.40	1.18	0.53	3.81	2.88	1.79	1.12

Notes: In the RA model the equilibrium real interest rate is 4.17%. \underline{B} is measured in terms of average annual wage.

The second important topic we discuss in this section is related to the cost of inflation. In the RA model, there are three sources of this cost. First, the traditional approach developed by Bailey (1956) and Friedman (1969) treats inflation, through its impact on nominal interest rates, as a tax on money holding. Therefore, the optimal level of inflation is consistent with the zero-nominal interest rate, as implied by the Friedman rule. In this approach, the welfare cost of inflation is measured by the area under the money demand curve, which is called the “welfare triangle”. Lucas (2000) estimates that the value of this “triangle” for an annual inflation rate of 10% is about 0.5% of GDP. Similar estimates are obtained by Cooley and Hansen (1989). The second reason why inflation is costly in the RA model, thoroughly discussed by Wolman (2001), is related to the existence of price stickiness, constraining firms in setting optimal prices in each period. This creates a dispersion of relative prices, hence the economy operates inside its production possibility frontier. The more steady-state inflation deviates from zero, the more severe becomes the cost of this dispersion. According to various estimates, the welfare cost of 10% inflation due to the presence of nominal rigidities amounts to over 5% of GDP (Casares, 2004; Guerron-Quintana, 2010; Aruoba and Schorfheide, 2011).² The last channel through which inflation affects welfare in the RA framework, investigated by Lagos and Wright (2005), explores the role of money in facilitating exchange on the goods market. In their search model, an increase in inflation leads to a reduction in money holdings and thus discourages market activity. In other words, this “money hold-up problem” causes that some of transactions that would take place when

²It might be argued that these figures overestimate the true cost of inflation since they rely on the assumption that the frequency of price adjustments does not depend on the level of inflation, which is inconsistent with the empirical evidence (see e.g. Gagnon, 2009).

inflation is low are not made in a high-inflation environment. The estimates of this channel show that the cost of 10% inflation is around 2% of GDP (Lagos and Wright, 2005).

In HA models with uninsurable idiosyncratic income risk, inflation affects welfare because money serves as a means of self-insurance against future income shocks. In this setup, which was introduced to the literature by Bewley (1980), the Friedman rule might not be feasible. In particular, if the rate of money supply contraction is equal to the inverse of the discount rate β (which is what the Friedman rule implies), households would like to hold an infinite amount of money and hence a monetary equilibrium does not exist. On the other hand, if money is the only asset held by households, then positive inflation increases the cost of precautionary savings and thus raises the volatility of individual consumption. The cost of inflation due to this “self-insurance” channel was quantified by Imrohoroglu (1992), who estimates that compensating individuals for the loss of utility due to 10% inflation requires an increase in income by over 1%.

Obviously, money is not the only asset in the economy that can be used for self-insurance purposes, which has an impact on the estimates of the optimum inflation rate. Akyol (2004) investigates an economy with money and government bonds, where the latter asset can be traded only before idiosyncratic income shocks are observed. Due to this liquidity constraint, in equilibrium high-income agents hold money and bonds, whereas money holding of low-income agents is null (there are only two income states in the model). The effects of inflation are twofold. First, it redistributes income from high-earners to low-earners, which improves welfare.³ Second, it limits savings of high-earners and thus diminishes consumption smoothing, which is welfare detrimental. The numerical results indicate that this trade-off is optimized for a rate of inflation ranging between 5% and 10%, depending on the model parameterization. In a similar setup, Algan and Ragot (2010) analyze the economy in which individuals can self-insure by accumulating money and capital. Due to the “money in the utility” assumption, money holdings are positive and depend on the nominal interest rate. As a result, an increase in inflation induces households to substitute money for capital, which raises the aggregate stock of capital and output. The simulations of the model show that a 10% increase

³A more detailed discussion about the redistribution effects of inflation is presented in section 5.

3

in inflation raises the capital stock by 1.0-3.3% and output by 0.4-1.5%, depending on the assumptions related to taxes and functioning of the labour market. The cost of inflation is also analyzed in a recent paper by Chiu and Molico (2010). They extend the search model of Lagos and Wright (2005) by endogenizing participation in the centralized market, which makes money holdings heterogeneous. According to their results, moderately positive inflation relaxes liquidity constraints of non-participating agents, which can be welfare improving. Due to this redistribution effect, the welfare costs of moderate inflation are about twice lower than in the RA model of Lagos and Wright.

The reviewed literature shows that the heterogeneity of households' income, which is substantial in the real world, has a sizeable impact on the monetary policy design. First, the existence of uninsurable idiosyncratic risk leads to a build-up of precautionary savings, which lowers the level of the real interest rate. Second, this kind of setup allows for investigation of new channels through which inflation affects welfare. On the one hand, since money is used to self-insure against individual income fluctuations, the cost of inflation is higher than in the traditional approach. On the other hand, inflation redistributes income from the rich to the poor, which, for moderate levels of inflation, is welfare improving. This topic will be discussed in more detail in Section 5.

4 Heterogeneity of households' preferences

A second important source of heterogeneity that has been analysed in the literature is related to preferences. This type of heterogeneity modifies the household's lifetime utility function (1) by assuming heterogeneity of the rate of time preference β or of the shape of the utility function $u(\cdot)$. In the latter case a popular assumption is varying the parameter of risk aversion ϕ in a CRRA utility function.

These sources of heterogeneity have strong empirical support. Several studies document substantial heterogeneity in risk aversion. To mention but a few, Holt and Laury (2002) use experimental evidence, Cohen and Einav (2007) study data from insurance contracts and Barsky et al. (1997) analyse survey evidence. For example, in the sample surveyed in the latter study the parameter ϕ varies between 1.5 and infinity.⁴ The same applies to the rate of time preference. Lawrance (1991) uses data from the Panel Study of Income Dynamics to document wide differences in time preference rates across households. Barsky et al. (1997) show that, while on average respondents prefer consumption today, there is substantial variability in β , including several cases of reversed preferences ($\beta > 1$).

The idea that agents may differ in their tastes has been applied in several papers. Most of them study issues not related to monetary policy (e.g. Krusell and Smith, 1998; Hendricks, 2007; Kaplow, 2008). Regarding topics relevant for monetary policy, the HA literature deals mainly with two issues: (i) the welfare cost of business cycle fluctuations and (ii) the workings and consequences of financial imperfections.

The first question - the welfare cost of business cycle fluctuations - is an issue of key importance for monetary policy.⁵ Traditional calculations based on the RA framework show a negligible cost of cyclical fluctuations. In particular, in a path-breaking article Lucas (1987) calculated the cost of economic fluctuations to be 0.008% of consumption.

This contrasts with central banking practice, which shows that monetary authorities care about stabilising the real economy (e.g. Taylor, 1993). While a study examining the role of business cycle fluctuations for optimal monetary policy in a heterogeneous preference environment is (to our knowledge) still missing, the papers analysed below

⁴To be precise Barsky et al. (1997) report the intertemporal elasticity of substitution to range between 0 and 0.65.

⁵The treatment of business cycle costs is not limited to heterogeneous preference models, see e.g. Imrohoroglu (1989).

give sufficient evidence to expect that a HA setting would most probably alter the standard results obtained from RA models.

Krusell and Smith (1999) and Krusell et al. (2009) assume that agents can draw different values of the time preference rate. At the same time, households face idiosyncratic unemployment uncertainty. As is common in the HA literature, all agents face a borrowing constraint as in (12). Patient agents (high β) save more and accumulate more assets. As a result, they become wealthy and less credit constrained. Impatient agents save less and own fewer assets. Thus, they are more heavily affected by the borrowing constraint. Most gains from eliminating fluctuations are achieved by the least and most patient consumers. The former gain because they are the most credit constrained group. Eliminating fluctuations reduces their risk and improves welfare. The richest gain for a different reason. As described in section 3, uncertainty leads to precautionary savings and, hence reduces the real interest rate. Eliminating fluctuations reduces uncertainty and raises the equilibrium rate, which benefits primarily those who own most assets.

Krusell and Smith (1999) show that eliminating business cycle fluctuations raises aggregate welfare by approximately 0.1% (in terms of consumption). This may not seem large, but is still by an order of magnitude more than found by Lucas (1987). Krusell et al. (2009) improve these calculations by changing the way individual data is aggregated. As an effect, they find aggregate welfare gains from stabilizing the economy of approximately 1% of steady-state consumption.

We use selected results from Krusell et al. (2009) as an illustration to the workings of a model with and without preference heterogeneity. The calibration features log utility and three values of $\beta \in \{0.9823, 0.9879, 0.9935\}$ together with the assumption that 80% of agents are endowed with the medium, 10% with the high and 10% with the low value. Furthermore, Krusell et al. assume three states for employment (employed, short-term unemployed and long-term unemployed). The assumptions on β together with the transition matrices between the employment states ensure a wealth distribution roughly consistent with US data. Table 2 shows the distribution of welfare gains from eliminating business cycles by agents with different time preference. As explained above, the largest gains, exceeding 3% of consumption, are achieved by patient (i.e. rich) households. Also impatient (poor) agents gain substantially (1.7%) due to relaxation

of the credit constraint. The average gain is slightly less than 1% of consumption. The numbers can be contrasted with the baseline Lucas's calculations, where, under identical assumptions on the utility function, the welfare gain is just 0.008%.

Table 2: Welfare gains from eliminating business cycles in the Krusell et al. model (in % of consumption)

Patience level			All agents	Representative agent model
low β	medium β	high β		
1.708	0.597	3.309	0.974	0.008

Source: Krusell et al. (2009).

Schulhofer-Wohl (2008) also deals with the cost of business cycles, but introduces heterogeneity by assuming that people differ in their risk aversion. This leads to different findings with respect to the cost of business cycles. In particular, the simultaneous presence of risk-averse and risk-neutral agents opens the door to an insurance market where the latter insure the former from the consequences of business cycle fluctuations at a small cost. As a result, the intuitive finding that very risk-averse households suffer a lot from cyclical fluctuations does not hold in this framework and the welfare effects of business cycles become smaller than in the RA framework.

These examples show that HA models have the potential to improve our knowledge on both margins analysed in this survey. First, they change the estimates of aggregate effects of cyclical fluctuations. This is certainly the more important margin for monetary policy and, as our excellence in treating heterogeneity advances, it may even lead to reformulations of central bank objectives. Second, somewhat less importantly for central bankers, HA models also improve our understanding of who gains and who loses from stabilization policy. Nevertheless, as the presented examples show, the way heterogeneity is introduced may substantially influence the results.

The second issue analysed within models with preference heterogeneity - the workings and consequences of financial imperfections - has recently gained substantial attention. The main specific questions are related to the impact of financial sector shocks on the economy, the way financial frictions modify the working of monetary policy and the optimal monetary and regulatory policies under financial system imperfections.⁶

⁶Similar questions are also analysed in frameworks with firm heterogeneity. These are described in section 7.

In this literature, preference heterogeneity plays a specific role - it allows for a simultaneous introduction of savings (deposits) and borrowing (loans). Introducing agents with various propensity to consume not only brings the model closer to reality, but also, more importantly, allows for the introduction of financial intermediation between savers and borrowers. As a next step, this intermediation is made imperfect, hence opening the floor for frictions generated by the financial sector, making it possible to analyse topics that could not be handled under the RA assumption.

Technically, heterogeneity is usually introduced by varying β , which allows for the presence of less and more patient agents. The former, endowed with a low β , take loans from the financial sector. The latter, with a high β , generate deposits.⁷ The financial sector intermediates between depositors and borrowers in an imperfect fashion. The most common financial friction present in the heterogeneous preference literature are collateral constraints - impatient agents are allowed to borrow only up to a certain fraction of the value of their collateral (capital or housing stock). To make the model relatively easy to solve, the collateral constraint is usually assumed to be binding at all times.

Most of this literature originates from the seminal paper of Kiyotaki and Moore (1997) and deals with three problems important from the central bank's perspective. First, it analyses the impact of financial shocks on the economy. This topic has been undertaken i.a. by Iacoviello (2005), who modified the standard Kiyotaki and Moore model to include housing and showed how the presence of collateral constraints amplifies the impact of housing market shocks on the economy. The financial crisis of 2007-09 created a natural environment to use such models to examine the consequences of banking sector shocks. Gerali et al. (2010) brought a collateral constraint model to the data using Bayesian techniques in order to estimate the impact of financial sector disturbances on the euro area during the financial crisis.

This strand of the literature also asks how the presence of financial market imperfections changes the working of monetary policy. The generally found answer is that financial frictions amplify the effects of central bank's actions (Gerali et al., 2010;

⁷One important exception is Curdia and Woodford (2008), where heterogeneity is driven by a Markov process and affects the marginal utility of consumption, without specifying which deep parameter is changed.

Brzoza-Brzezina et al., 2011). For example, a monetary tightening lowers the value of collateral, thus decreasing the amount of credit available to households and entrepreneurs. As a result, consumption and investment decline by more than it would be the case without financial imperfections.

Finally, models with heterogeneous preferences are used to speak normatively about monetary and regulatory policies in the presence of financial market imperfections. One important reference is Curdia and Woodford (2008), who show that monetary policy should react to fluctuations in interest rate spreads. Another application is related to optimal macroprudential policy - a new concept intended to make a connection between financial and macroeconomic stability. In particular, macroprudential policy is supposed to limit the risk of systemic financial sector crises having significant macroeconomic costs (Borio and Drehmann, 2009; Galati and Moessler, 2011). Financial friction models are currently used to assess the impact of various regulatory policies on the business cycle and some of them (e.g. Angelini et al., 2010) feature heterogeneous preferences and collateral constraints. In this respect, an important and rapidly growing line of research analyses the consequences of occasionally binding collateral constraints (He and Krishnamurthy, 2008; Jeanne and Korinek, 2010). Unlike models with eternally binding constraints discussed above, this framework allows to study optimal policy reactions to low probability - high impact, nonlinear developments in the financial system.⁸ All these studies can prove helpful in distinguishing between policies that stabilise or amplify the cycle, and to draw a connection between regulatory and monetary policies.

Summing up, models with heterogeneous preferences have already expanded our knowledge in two important directions. First, they shed new light on the costs of business cycle fluctuations and hence, on the way optimal monetary policy should be conducted. While HA models clearly have the potential to change the RA results that business cycles do not matter, we are probably still far away from a consensus in this area. Second, preference heterogeneity allowed us to introduce imperfect financial intermediation into DGSE models. This made it possible to analyze i.a. the consequences of various monetary and regulatory policies on the economy.

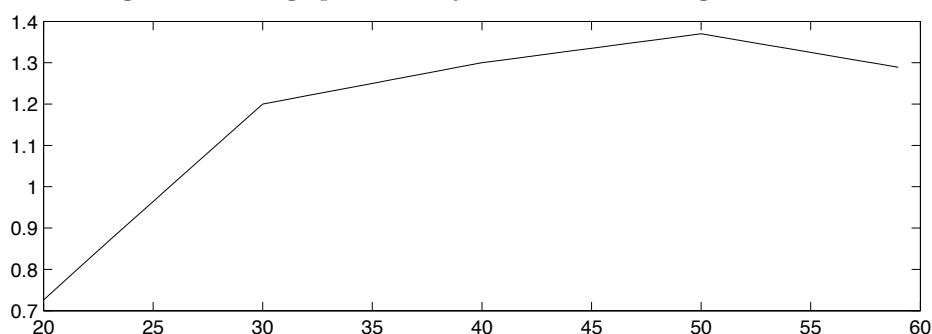
⁸This comes at the expense of making the model solution far more demanding since standard linear perturbation techniques cannot be applied.

5 Heterogeneity of consumers' age

One of the most important sources of heterogeneity in economics is related to the age of consumers. In standard RA models, consumers live infinitely and consequently this aspect of reality is neglected in the mainstream discussion.

While thinking about age heterogeneity, it is important to distinguish between exogenous heterogeneity that is intrinsically associated with age, and endogenous heterogeneity that is a result of the exogenous heterogeneity. The two exogenous sources of heterogeneity among agents in different ages are life expectancy (or survival probability) and productivity. It is well documented that life expectancy declines with age.⁹ Second, people of different ages have different productivity. The distribution of productivity over age is hump-shaped and reaches a peak around the age of 50, as shown in Figure 1.

Figure 1: Average productivity distribution over age in the US.



Source: Hansen (1993). Value for each age as a ratio of average productivity of the respective age group to average productivity.

The exogenous heterogeneity creates natural endogenous heterogeneity in terms of income, wealth, and portfolio structure, which is documented in numerous studies. For example, Diaz-Giménez et al. (1997) describe the distribution of earnings, income and wealth, while Doepke and Schneider (2006a) show the distribution of portfolio structure in the US. In a recent paper, Heer et al. (2011) document several facts about the lifecycle distribution of money, which are: (i) hump-shaped distribution of money holdings over the lifecycle, (ii) no clear-cut relation between the variation of money holdings and age, (iii) low bivariate correlations between money and income, money and wealth, and

⁹See life tables for different countries prepared e.g. by the World Health Organization, available at http://www.who.int/healthinfo/statistics/mortality_life_tables/en/.

money and holdings of the interest bearing assets, and (iv) income, wealth and age as regressors explaining only a small share of the variation of money holdings.

The standard way of introducing consumers' age into economic models is to use the overlapping generations (OLG) framework, initially proposed by Diamond (1965). The OLG concept was extended to a multi-age cohorts setup, where the age groups are heterogeneous in terms of productivity, income or wealth (Auerbach and Kotlikoff, 1987; Huggett, 1996; Rios-Rull, 1996). In this section, we discuss how the inclusion of lifecycle behaviour in economic models affects the monetary policy design. In particular below we show how age heterogeneity affects (i) optimal monetary policy and costs of inflation, (ii) distributional effects of inflation and its impact on aggregate variables, and (iii) the monetary transmission mechanism.

The first issue we address is the impact of heterogeneity on the optimal design of monetary policy. In standard RA models, there is a natural welfare criterion for policy evaluation, namely the representative consumer's utility. It is well known that in a wide range of this class of models, in the absence of uncertainty and nominal frictions, optimal monetary policy satisfies the Friedman rule, which implies zero nominal interest rates and implicitly deflation. This may no longer be the case in OLG models. Since they do not have a representative agent, one concept that can be used is Pareto optimality. McCallum (1983) shows that when one allows for the transaction-facilitating role of money, the Friedman rule is usually necessary for Pareto optimality.¹⁰ It should be noted, however, that in OLG models it is possible that the Pareto efficient allocation delivers lower welfare than other feasible allocations for all but a measure zero subset of agents, namely the initial old. This is why economists often use the stationary equilibrium welfare as a criterion for optimality in this setup.¹¹ With this criterion, the Friedman rule might no longer be the best outcome. For example, Freeman (1993) finds that zero inflation is optimal. Smith (2002), using a different OLG model, shows that an optimal policy generates inflation higher than the Friedman rule. Bhattacharya et al. (2005) indicate that, depending on how money is introduced into the OLG framework, zero or positive inflation rate is optimal. The main reason for all these deviations from

¹⁰For the exceptions, see McCallum (1990) and Brock (1990).

¹¹Which, as some people argue, means equal weight on each generation's welfare (Freeman, 1993).

the Friedman rule is the fact that transfer of wealth from old and rich to poor and young generated by monetary injections increases utility in the stationary equilibrium, but obviously lowers utility of the initial old.

Another aspect of monetary policy that is closely related to its optimal design is the cost of inflation. Most studies in this area focus on income (or wealth) heterogeneity and were described in Section 3. Gomme (2008) analyzes the cost of inflation in an OLG framework with a cash-in-advance constraint and finds important differences. First, he finds that the optimal inflation rate (using the stationary equilibrium utility as a criterion) could be as high as 95% and that the welfare benefit of deviating from zero inflation is around 1% of income. The main reason is that money injections imply a transfer of wealth from old to young and hence flattening of the life-cycle consumption profile that agents find desirable. However, if other taxes are allowed, and so the inflation tax is not the only option to transfer resources across generations, the optimality of the Friedman rule is restored, with welfare costs similar to the ones obtained in a standard RA model. For example, a 10% inflation rate generates a welfare cost ranging from 0.5% to 0.9% of income. If the Gomme model is extended to include endogenously determined credit goods, the optimal inflation rate becomes lower, falling to 5% in the model with no other taxes. This is because introduction of endogenous credit goods makes it easier for each agent to escape the inflation tax. If other taxes are included, noting that in this model for technical reasons the stationary equilibrium money growth rate cannot be negative, the optimal inflation rate turns out to be 0%.

The second issue that we analyze in this section is the distributional effect of inflation. It has been long argued that inflation affects welfare because it redistributes wealth in an arbitrary manner. Galli and van der Hoeven (2001) provide a survey of the empirical literature. In most studies, inflation increases income inequality. For example, Erosa and Ventura (2002) consider a model with costly credit services. In their model, inflation represents a flat tax on monetary transactions, but agents can evade this tax by using costly credit services. In the presence of economies of scale in credit costs, richer agents buy a higher proportion of goods with credit, therefore their inflation tax rate is lower. As a result, the welfare costs of inflation for high income individuals are lower than for low income individuals.

Only recently the generational aspect of redistribution has been included in the discussion. In a recent paper, Heer et al. (2011) check whether an OLG model with three different ways of introducing money, i.e. money-in-utility, costly credit services and limited participation, can replicate the lifecycle distribution of money. They find that all three models can explain a hump-shaped distribution of money over age. However, all three models fail to predict weak explanatory power of income, wealth and age for the money holdings. Only the limited participation model can account for the low bivariate correlations between income and money as well as between income and holdings of the interest bearing assets.

There are several papers that attempt to explain the effects of unexpected inflation. Doepke and Schneider (2006a) assess the impact of an unexpected rise in inflation. Since the redistribution may depend on how fast agents adjust this shock, the authors consider two extreme scenarios. In the first one, which they call the full surprise (FS), there is a one-off increase in the price level by 5%, that leaves the interest rates unchanged. This leads to redistribution since the real value of future nominal payments falls. In the second scenario, which they call indexing ASAP (IA), there is a surprising one-time announcement that inflation will be five percentage points higher than expected in the next 10 years. Since the bond markets immediately adjust the nominal yield curve, redistribution occurs due to higher expected future nominal rates. The results of this experiment are presented in Table 3. The main losers are rich, old households which hold fixed-interest rate bonds, while the biggest winners are young, middle class households with fixed interest rate mortgage debt. Additionally, when comparing across sectors, the main beneficiary is the government, whose debt is valued in nominal terms, while foreign agents holding the government's debt lose most.

In a follow-up paper Doepke and Schneider (2006b) extend their framework into a general equilibrium model to allow for adjustment of macroeconomic aggregates. They find that, since retirees adjust labor supply less than workers, an unexpected moderate rise in inflation leads to a reduction of labour supply by the young, which is not offset by an increase in labour supply by the old. Moreover, such a shock has an impact on aggregate consumption and savings. Young consumers, who have relatively long life expectancy, smooth their gain out over many periods, and hence increase their con-

sumption marginally and savings substantially. Old consumers, having relatively short life expectancy, spread their loss over relatively few periods, and so their consumption decreases substantially and savings go down only marginally. As a result, inflation can have persistent effects on the stock of capital, labour supply and thereby on output.

In another article on the subject, Meh et al. (2010) analyse how the distributional effects of unexpected inflation differ under two types of monetary policy, namely inflation and price level targeting. Using Canadian data, they show that the scale of redistribution after an unexpected 1% price level increase under inflation targeting is about three times larger than under price level targeting. The reason is the fact that after an unexpected increase of prices, they return to the original level under price level targeting, whereas under inflation targeting they remain permanently higher. Additionally, the redistribution has a positive impact on the aggregate labour supply and thus output, since the increase in labour supply of losing workers dominates the decline in labour supply of the gaining households. Again, the effect is more pronounced under inflation targeting than under price level targeting.

Table 3: 1989 Wealth redistribution across households after a 5% inflation experiment (in % of average net worth in each group)

Age Cohort	Poor		Middle Class		Rich	
	FS	IA	FS	IA	FS	IA
≤ 35	+14.4	+0.2	+44.9	+18.9	+5.5	+2.1
36 – 45	+13.3	+4.0	+12.4	+5.8	-1.5	-0.9
46 – 55	+2.2	+0.6	+1.9	+1.4	-2.6	-1.6
56 – 65	-2.9	-0.5	-5.5	-1.4	-6.4	-2.4
66 – 75	-6.9	-1.3	-9.9	-2.7	-6.6	-2.9
> 75	-10.4	-1.0	-15.0	-2.6	-10.8	-4.7

Source: Doepke and Schneider (2006a).

A different possible channel for redistributive effects of inflation is considered by Heer and Sussmuth (2007). In their OLG model with money-in-the-utility, households can participate in the stock market, but face three different types of transaction costs: (i) a fixed entry cost, (ii) a proportional costs of maintaining their asset position, and (iii) a proportional costs of changing their asset position. This cost structure is crucial for the results, since it generates asymmetry in the response of money holdings to an increase of inflation. The authors analyze an increase in average inflation, taking into

account two effects: the 'Feldstein effect' and the 'portfolio composition effect'. Due to the 'Feldstein effect', in a nominally based capital tax system, higher inflation leads to an increase in the real tax burden. Therefore, the after tax real interest rate is lower, which depresses savings and the stationary equilibrium capital stock. Because of the 'portfolio composition effect', higher inflation leads to lower money holdings by households. However, due to costly assets market participation, agents of different age do not adjust their money proportionally (as in the standard RA model). For example, in response to a rise in average inflation from 3.06% (P. Volker's chairmanship at the Fed) to 6.43% (A. Greenspan's terms), the 2-year old and the 59-year old decrease their money by 35.2% and 36.2%, respectively. Therefore, an increase in inflation (contrary to the standard RA model) lowers the stationary equilibrium real interest rate and hence increases capital stock.

Yet another effect of unanticipated inflation is analyzed by Heer and Maussner (2011), who consider a standard New Keynesian model with an OLG structure and progressive income taxation. They find three channels through which inflation may affect the economy. First, real wages increase after a monetary expansion, especially for the younger and less productive workers. Second, since tax brackets are adjusted to actual inflation with a lag, there is a 'bracket creep' effect of unanticipated inflation that redistributes income to the poor. Third, since pensions are indexed to actual inflation with a lag, an unanticipated inflation redistributes income from old to young. When all three effects are taken into account, surprise inflation increases the inequality of total income, but decreases inequality of disposable income. Interestingly, while these redistributive effects are sizable, the aggregate business cycle properties of the Heer and Maussner HA model are very similar to those of a corresponding RA setup.

The third research area analyzed in this section is related to the impact of age heterogeneity on the monetary policy transmission mechanism. The literature on this topic is relatively scarce. Fujiwara and Teranishi (2008) show that responses to an unexpected monetary policy shock in a deterministic lifecycle model could be different than in a RA model. The main reason is the fact that responses of the retirees are different from those of the workers. Since the former rely more on financial assets, a positive interest rate shock increases their consumption (although on impact it falls). Therefore,

an overall drop in consumption is smaller than in a RA setup. A similar question is addressed by Heer and Maussner (2011), who consider business cycle properties of the New Keynesian model with progressive taxation, finding almost no differences between the RA and HA frameworks. Two notable exceptions are the responses of consumption and investment to a monetary shock, which are approximately 30% stronger in the HA variant. This is in contrast to Fujiwara and Teranishi (2008) who assume flat income tax rates.

The above review shows that there are several important issues in monetary policy that require taking into account age heterogeneity. First, the optimal design of monetary policy and the welfare costs of inflation might be different. Second distributional effects of inflation affect agents of different age asymmetrically therefore affecting aggregate macro variables. Third these asymmetries do matter when the monetary transmission mechanism is concerned, hence responses to monetary policy shock may change if age heterogeneity is taken into account. Unfortunately, due to high complexity of multi-generational dynamic general equilibrium models, these questions are addressed by the literature only to limited extent.

6 Heterogeneity of expectations

One of the assumptions imposed in standard RA models is rationality of the representative agent, which also concerns the way in which she forms expectations. In the New Keynesian framework, both expected inflation and expected output play a central role for monetary policy transmission, therefore assumptions on expectation formation are of crucial importance for models' dynamics as well as for its policy implications. If one assumes expectations to be rational, whereas in fact it is not the case, one can get a wrong idea about policy issues. Similarly, if there is heterogeneity in the way agents form expectations, one needs a model which is capable of accounting for a wider range of expectation formation mechanisms.

A growing number of empirical articles work in favour of the heterogeneous expectations hypothesis, which, by definition, excludes full rationality. Using survey data, Carroll (2003), Mankiw et al. (2004) and Branch (2004) provide evidence that economic agents, both professional economists and consumers, have heterogeneous expectations and that distribution of expectations among agents tends to fluctuate over time in response to economic developments. Mankiw et al. (2004) obtain evidence on time-varying dispersion of beliefs using surveys of inflation expectations. Using also survey data Kokoszczynski et al. (2010) find evidence that consumers' inflation expectations do not fulfill the unbiasedness condition which contradicts the rational expectations hypothesis.

From the theoretical point of view, conditions underlying the rational expectations hypothesis undermine this concept at its core because rationality requires agents to possess a great deal of knowledge and skills. Evans and Honkapohja (2001) note that even econometricians must use proxies for economic reality, therefore it is hard to expect that average economic agents, not skilled in economics, are fully rational. This would require them to know and to process the true structure and state of the world they live in, or at least to behave as if they knew it. On the other hand, rational expectations do not constitute an *ad hoc* assumption. A conditional expected value operator arises naturally when agents' decision problems are formulated mathematically as dynamic optimization problems. Therefore, when bringing other expectations formation mechanism into the picture, one should justify them.

Acknowledging this fact, some articles discuss settings in which departure from rationality can constitute an optimal (read rational) choice, even if rationality is an option. Evans and Ramey (1992, 1998) and Brock and Hommes (1997, 1998, 2000) show that if information is costly, it may be optimal to choose a not fully rational way of forming expectations. This is the case because when agents form expectations, they consider both costs and benefits of available predictors (expectation formation mechanisms). As a result they may behave optimally by choosing not the rational expectations operator. In this context Brock and Hommes (1997) introduce a concept of adaptively rational equilibrium dynamics – agents choose predictors from a set of available functions, each of which has a cost which increases in its sophistication, and the probability that a given predictor is chosen depends on its cost and on its performance measure. The authors analyze a cobweb setting with rational and naive agents and show that time-variation of agents' choices can become quite complicated. Branch (2002) extends this framework to the case in which also adaptive expectations are available.

Branch (2004) makes a further extension of the rationally heterogeneous expectations setup by investigating a dynamic predictor selection in a discrete choice setting. In his model, agents choose among vector autoregression, adaptive and naive expectations. The probability that an agent chooses a given predictor depends on its relative mean squared error and on its cost. Estimation is conducted on the survey data from the University of Michigan's Survey Research Center on expected inflation. The data supports the model of rationally heterogeneous expectations – agents switch predictors as the relative mean squared error changes over time which suggests that survey respondents are distributed across rational and adaptive expectations and that their fractions change over time. As an example, in the periods of high economic fluctuations, such as the 1970s, a higher proportion of agents used the rational expectations operator than during the periods of lower volatility. Hence, variation in distribution of agents across predictors seems to be structural in the sense that high volatility encourages agents to adopt rational expectations more often (Branch, 2004) or in the sense that it tends to shrink dispersion among employed expectations operators (Mankiw et al., 2004).

All these considerations and empirical findings can have strong implications for monetary policy. We discuss the most important issues in the rest of this section. In

particular, we focus on how heterogeneous expectations affect (i) the propagation of monetary and other economic shocks, (ii) the data fit of a standard NK setup, and (iii) the area of determinacy region for the standard monetary policy feedback rules.

Kurz et al. (2002, 2005) introduce heterogeneity of beliefs as a mechanism which helps to propagate the macroeconomic shocks through the real business cycle (RBC) setting. They show that diversity of beliefs yields business cycle dynamics that looks realistic, even when shocks are significantly smaller than in the standard RBC model. In fact, in a rational belief equilibrium heterogeneous expectations constitute a driving force of economic fluctuations, introducing the role for the monetary policy to stabilize the real sphere of the economy (e.g. Motolese, 2001, 2003). Also Branch and McGough (2010a) investigate the implications of heterogeneous expectations for business cycle dynamics. Within a stochastic growth model, where a fraction of agents forms rational expectations and the remaining ones employ more parsimonious forecasting models (therefore, a fraction of agents is rationally bounded), authors demonstrate that heterogeneous expectations can lead to a substantial improvement in the internal propagation of the business cycle. Moreover, they show that this propagation depends on the degree of heterogeneity, and that the calibrated model with heterogeneity provides a closer fit to the data than its RA counterpart.

Only very recently have heterogeneous expectations been introduced into the New Keynesian model, which is extensively used by central banks in monetary policy making. In the early study of this strand of the literature Branch and McGough (2009) derive aggregate demand and supply equations from a micro-founded sticky price model in which agents have heterogeneous, possibly boundedly rational expectations. The authors assume that a proportion of agents use rational expectations and the remaining ones use simple adaptive rules. The dynamic properties of the model depend crucially on the distribution of expectation operators across agents and differ from those implied by models with purely rational expectations. The major contribution of Branch and McGough is that they provide necessary conditions for expectation operators, under which the first-order conditions for consumption and prices in a heterogeneous expectations economy obtain a form which is analogous to the standard rational-expectations case with aggregate supply and demand equations, with the exception that conditional

expected value is replaced by a linear combination of individual expectation operators.

Moreover, they show that equilibrium determinacy is affected by heterogeneity of expectations in an ambiguous way. We will now take a closer look at these results.

Branch and McGough consider the following model, which can be thought of as a heterogeneous expectations extension to the canonical New Keynesian RA setup (e.g. Clarida et al., 1999):¹²

$$y_t = \mathcal{E}_t(y_{t+1}) - \Phi(i_t - \mathcal{E}_t(\pi_{t+1})) + \eta_t^y \quad (14a)$$

$$\pi_t = \beta \mathcal{E}_t(\pi_{t+1}) + \lambda \mathcal{E}_t(y_{t+1}) + \eta_t^\pi \quad (14b)$$

$$i_t = \phi_y \mathbb{E}_t(y_{t+1}) + \phi_\pi \mathbb{E}_t(\pi_{t+1}) + \eta_t^i \quad (14c)$$

where $\mathcal{E}_t(\psi_{t+1}) = \alpha \mathbb{E}_t(\psi_{t+1}) + (1 - \alpha)\theta^2(\psi_{t-1})$ for $\psi \in \{y, \pi\}$, $\mathbb{E}_t(\cdot)$ denotes the expected value operator conditioned upon the information available to agents at time t , whereas α and $1 - \alpha$ denote fractions of rational and boundedly rational agents in the economy, respectively. Parameter θ defines the nature of autoregression in the boundedly rational agents expectation formation mechanism, who either discount past data ($\theta \leq 1$) or extrapolate it ($\theta > 1$).

For $\beta = 0.99$, $\Phi = 0.157$ and $\lambda = 0.024$, Branch and McGough (2009) conduct a sensitivity analysis of equilibrium determinacy with respect to parameters $\alpha \in [0, 1]$ and $\theta > 0$. We replicate their calculations and report the results for $\alpha \in \{1, 0.9, 0.8\}$ and $\theta \in \{0.9, 1.1\}$ in Figure 2. The regions of determinacy are plotted on the $[0, 2] \times [0, 2]$ square, whose elements (points) correspond to various values of ϕ_π and ϕ_y . Determinacy is defined as lack of sunspots, which happens if both eigenvalues associated with the forward-looking variables of system (14) are larger than one in absolute terms.

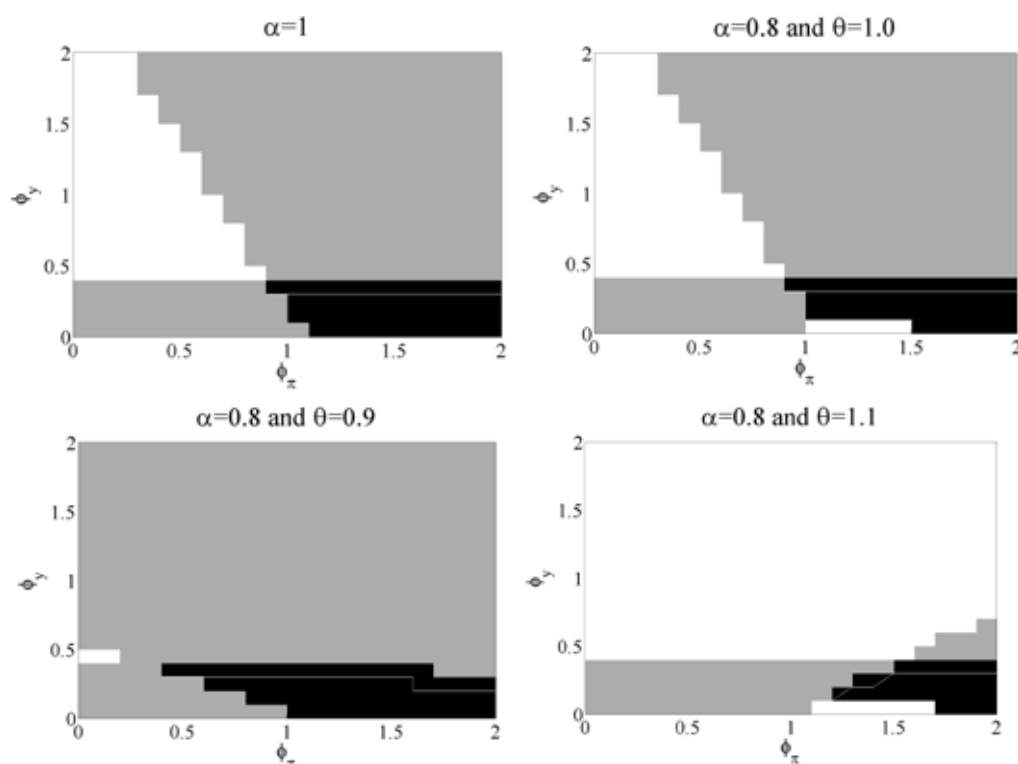
Depending on the way in which boundedly rational agents form expectations (i.e. depending on the value of θ), an increase in their share in the economy (i.e. decrease of α) expands or shrinks the determinacy area, hence it expands or limits the space within which monetary policy rule parametrization is feasible (in the sense that it rules out sunspot equilibria). More specifically, if adaptive agents form expectations in a

¹²We have added stochastic disturbances to the original Branch and McGough (2009) model in order to examine the impulse responses. Inclusion of these disturbances does not influence determinacy areas discussed below.

discounting way ($\theta < 1$), the determinacy region expands, which indicates a stabilizing force of non-rational discounting expectations. On the other hand, if adaptive agents extrapolate past trends ($\theta > 1$), the determinacy area shrinks and monetary policy should react more aggressively to inflation expectations in order to ensure determinacy of the equilibrium.

We also use the Branch and McGough model to illustrate how different parametrizations of the economy influence its dynamic properties as captured by the impulse responses. Model (14) includes three shocks: a monetary shock η^y , a shock to inflation η^π and a shock to output η_i^y . The consecutive rows of Figure 3 report the responses of output, inflation and of the interest rate to these three shocks. We assume $\phi_y = 0.125$ and $\phi_\pi = 1.5$. As in the determinacy area analysis, we account for four cases. These are: $\alpha = 1$; $\alpha = 0.8$ and $\theta = 0.9$; $\alpha = 0.8$ and $\theta = 1$; $\alpha = 0.8$ and $\theta = 1.1$. The main conclusion which can be drawn from inspecting Figure 3 is that departures from

Figure 2: Equilibrium determinacy under heterogeneous expectations

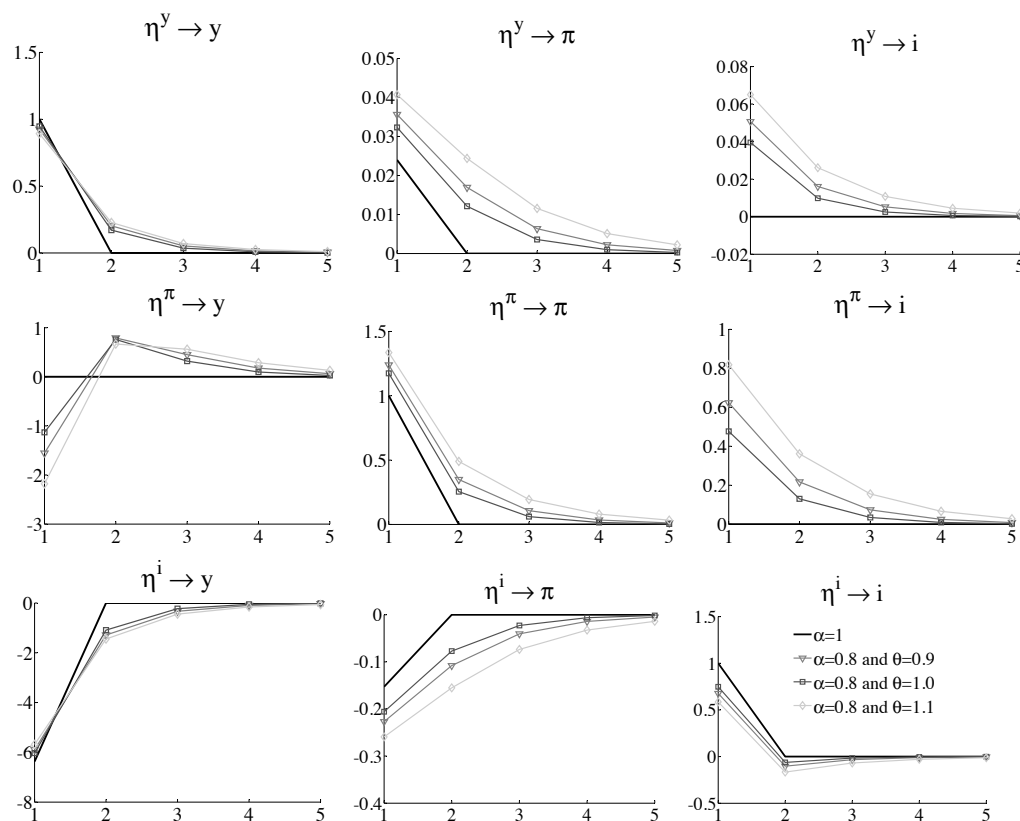


Notes: Black areas represent determinacy, grey areas represent indeterminacy of order one, i.e. the case when one eigenvalue is explosive and the second one is nonexplosive, white areas represent indeterminacy of order two, i.e. when both eigenvalues are located inside the unit circle.

rationality, as measured by a decrease in α , tend to amplify transmission of shocks in the economy. This is true both for $\theta = 0.9$ and for $\theta = 1.1$, i.e. independent of the way in which boundedly rational agents form expectations.

In the policy part of the paper Branch and McGough consider rational and adaptive agents. Also Geiger and Sauter (2009) allow for a dichotomy in expectation formation within the NK setting. More specifically, they assume a special role of money in the process of forming expectations. It is assumed that a fraction of agents, called co-integration observers, forms inflation expectations by observing the past money growth trend and using simple empirically-based forecasting rules. The rest of agents is rational and thereby make model-consistent forecasts. Authors advocate that monetary beliefs

Figure 3: Impulse responses under heterogeneous expectations



help to stabilize macroeconomic dynamics if the economy is hit by an aggregate demand or by an interest rate shock. The return to equilibrium is smoother and less bumpy than in the case of fully rational expectations and this is especially true for contractionary shocks. Authors also conclude, that monetary policy, when faced with heterogeneous

expectations, operates best under a Taylor rule which responds to contemporaneous inflation and output rather than to expected future ones.

Finally, Branch and McGough (2010b) consider dynamic predictor selection within the New Keynesian model with heterogeneous expectations. They extend their earlier framework (Branch and McGough, 2009) by incorporating endogenous movements in the shares of predictors along the lines of Brock and Hommes (1997). Agents choose between using a costly perfect-foresight predictor or an adaptive forecasting model. Authors find that, depending on the cost of the perfect-foresight predictor, the models' steady state can be stable or the system can bifurcate. Moreover, the qualitative nature of models' non-linear dynamics turns out to be driven by the central banks' policy stance and by the way non-rational agents form expectations. These results have serious implications for monetary policy implementation and suggest that standard monetary policy rules – evaluated in a rational expectations environment – may need to be revised in a heterogeneous expectations structure. Complex dynamic behaviour and thus possibly excess volatility may occur in a heterogeneous expectations economy even when an active monetary policy rule (i.e. one satisfying the Taylor principle, which usually yields equilibrium determinacy under rational expectations) is implemented.

To assess the empirical performance of the standard three-equation New Keynesian model under non-rationality Paloviita (2007) uses European panel data and, instead of making explicit assumptions about expectations formation, she uses observed expectations, for which proxies are taken from the survey. The real time version of the model also includes a perceived output gap in the Taylor rule instead of the revised output gap estimates. The observed expectations may, but do not have to be rational and results suggest they are not. Contrary to the assumption of rational expectations hypothesis, the author finds that expectational errors with respect to inflation and output gap are positively autocorrelated. It is also found that inclusion of observed expectations and of real-time variables improves the empirical performance of the model as compared with the baseline rational expectations formulation.

The reviewed literature makes at least two central points. First, the empirical studies give support to the thesis that agents form expectations in a heterogeneous way. Second, the way in which the heterogeneous expectation formation is accounted

for in macroeconomic models affects their policy implications. In particular, models with heterogeneous expectations can yield significantly different insights than their rational-expectations counterparts, including not only stronger shock propagation, but also altered determinacy regions or chaotic dynamics.

7 Heterogeneity of firms' productivity and financial position

It is well known from the empirical literature relying on micro data that firms differ substantially in many important characteristics, including productivity. As summarized by Bartelsman and Doms (2000), a high degree of heterogeneity across establishments and firms in terms of productivity is a stylized fact, confirmed by case studies and more comprehensive research for various countries and industries.¹³ Evans (1987) and Hall (1987) show that the dynamics of manufacturing firms (growth and its volatility) is negatively related to firms' size and age. A number of papers explore the link between individual firm characteristics and their international status. They are largely inspired by an influential study by Bernard et al. (1995), who documented that firms of diverse size, factor intensity and productivity coexist even in narrowly defined industries and only some of them export.

These results were paralleled by an outbreak of theoretical models explicitly dealing with firm heterogeneity. Jovanovic (1982) introduced an equilibrium model with firms facing idiosyncratic productivity shocks, which leads to selection through exit and entry. Another important milestone was Hopenhayn (1992), who made this framework more tractable by introducing a stationary equilibrium concept. Building on the Hopenhayn structure, Melitz (2003) demonstrated how an interaction of productivity differences across firms and sunk costs determines firms' decisions to enter foreign markets, instigating an outbreak of research addressing a wide range of important topics in international economics. The contributions mentioned above used a static framework and so focused on stationary equilibrium effects, ignoring dynamic adjustment. This gap was filled by Ghironi and Melitz (2005), who used the Melitz model as the microeconomic underpinning of an open economy dynamic stochastic business cycle model with flexible prices. Differences in productivity across firms or sectors are also addressed by the lumpy investment literature, pioneered by Caballero and Engel (1999) and developed into a general equilibrium framework by Thomas (2002).

¹³Important contributions include Baily et al. (1992) for the US manufacturing, and Roberts and Tybout (1996) for developing countries.

This line of models has already produced some results deepening our understanding of (i) the monetary policy transmission mechanism and (ii) the link between real activity and inflation. Bachmann et al. (2006) demonstrate that microeconomic non-convexities in adjusting the productive capacity can substantially smooth the response of investment to aggregate shocks and generate important history dependence in business cycles. Sveen and Weinke (2007) find that lumpiness of plant-level investment helps to render the standard New Keynesian model capable of explaining the dynamic effects of monetary policy shocks. The potential relevance of the literature stressing sunk entry costs for monetary policy can be appreciated by studying the contribution of Bilbiie et al. (2008). They show that allowing for producer entry in a model with price rigidities adds endogenous persistence to the New Keynesian Phillips curve, bringing it closer to the data. Also, endogenous price of equity, i.e. firms' market value, creates an additional monetary policy transmission channel, affecting the model's indeterminacy region. Overall, however, it is fair to say that interactions between heterogeneous firm productivity, sunk entry or adjustment costs, and monetary policy remains a relatively unexplored area.

In contrast, there is a vast and growing number of studies dealing with monetary general equilibrium models in which idiosyncratic shocks faced by firms affect their access to credit. Seminal works in this field include Bernanke et al. (1999) and Carlstrom and Fuerst (1997). The key ingredient in this generation of models is asymmetry of information between borrowers and lenders. While the former observe their realized returns, the latter can do it only after paying monitoring costs. Contrary to the standard RA framework, where there is only one interest rate, fully controlled by a central bank, the costly state verification drives a wedge between the interest paid on funds raised externally and the risk-free rate. As this wedge depends positively on borrowers' leverage, the framework exhibits a financial accelerator property, i.e. endogenous movements in the external finance premium can propagate and amplify macroeconomic shocks. The strength of this additional channel depends in particular on the degree to which financial contracts are indexed to inflation. As demonstrated by Christiano et al. (2004) or (in a different framework) by Meh et al. (2009), if indexation is low, redistribution between debtors and creditors due to price level changes can substantially magnify the economy's response to some shocks.

This class of models has been extensively used to (i) provide a richer description of the monetary policy transmission mechanism and (ii) revisit the optimal monetary policy design. In the rest of this section, we take a closer look at these two areas.

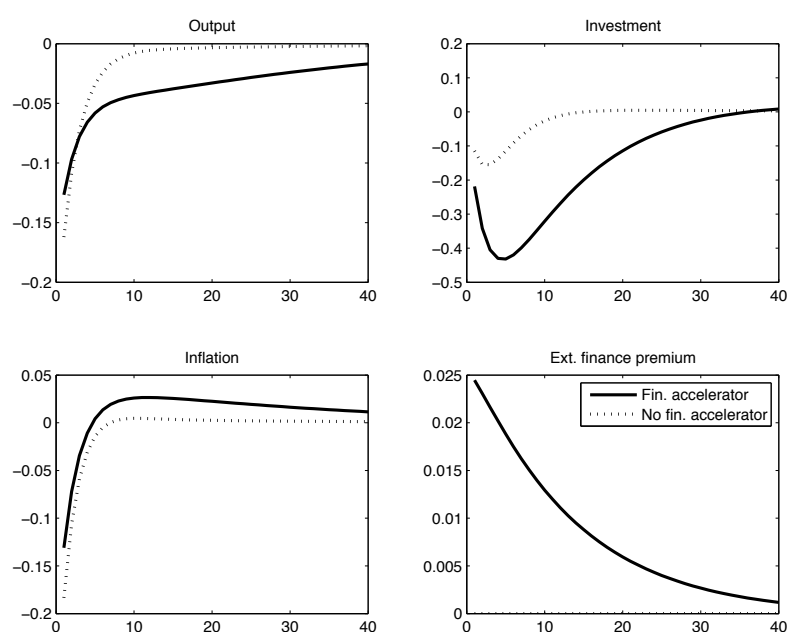
To illustrate the first point, we add the financial accelerator setup developed by Bernanke et al. (1999) to the framework sketched out in section 2 and demonstrate how it modifies the economy's responses to a monetary shock. The model frequency is quarterly. We assume standard values for the common parameters, i.e. we set the discount factor β to 0.995, the depreciation rate δ to 0.025, the coefficient of relative risk aversion ϕ to 2, the Frisch elasticity of labour supply to 2, the capital share α to 0.33, and the investment adjustment cost curvature $S''(1)$ to 5. The price adjustment cost is consistent with the average period between price adjustments in the Calvo scheme of 4 quarters. We parametrize the policy rule given by (9) so that the coefficient on the lagged interest rate is 0.9 and the long-run response to deviations of inflation and the output gap from their steady-state levels are 1.5 and 0.5, respectively. The parametrization of the financial block is such that it implies the same steady-state leverage, bankruptcy rate and external finance premium as in Bernanke et al. (1999).

Figure 1 compares the impulse responses to a contractionary monetary shock, defined as a 10 basis points innovation in the monetary policy rule, with and without the financial accelerator. Allowing for credit frictions substantially amplifies the response of investment, doubling their response on impact and tripling at the trough. This is because an increase in the interest rate depresses the price of capital, which negatively affects borrowers' net worth and so their borrowing conditions, as reflected in an increase in the external finance premium. As a result, the response of output is far more persistent than in a standard RA setup, where the premium is zero by assumption.

The literature on the interactions between idiosyncratic shocks and firms' access to credit is not restricted to positive analyses, so in the rest of this section we summarize its main implications for optimal monetary policy design. A number of papers have investigated how differences in access to credit affect the performance of various monetary regimes, usually nested within a family of generalized Taylor-like feedback rules. Using a closed economy model with sticky prices, Faia and Monacelli (2007) find that in the presence of financial frictions central banks should respond to changes in asset

prices, which is in contrast to the implications of a standard New Keynesian framework. However, the marginal gain of such responses vanishes with a growing anti-inflationary stance. In a two-country setup, Faia (2010) finds that the presence of credit frictions strengthens the case for floating exchange rate regimes in economies facing external shocks. Using a similar model, Gilchrist et al. (2002) point out that the enhanced desirability of multiple currencies is not so obvious as it depends on shock asymmetry as well as financial integration and cross-country heterogeneity of financial structures.

Figure 4: Impulse responses to a monetary shock



A related line of research considers small open economy models in which firms differ in their access to credit and their debt is denominated in the foreign currency. Gertler et al. (2007) show that in such an environment fixing the exchange rate exacerbates the contraction caused by an adverse shock on foreign borrowing terms. A similar result is obtained by Curdia (2007) in a model of sudden stops driven by changes in the perceptions of foreign lenders. Devereux et al. (2006) analyze the propagation of external shocks to interest rates and the terms of trade under CPI targeting, nontradable goods price targeting and the exchange rate peg. According to their results, credit frictions do not affect the ranking of alternative policy rules, with the fixed exchange rate regime performing worst. Elekdag and Tchakarov (2007) show that this result is

not robust to alternative model parameterizations. In particular, at a certain level of leverage (which turns out to be close to the average for emerging market economies) the peg starts to dominate the float if shocks originate abroad.

The literature analyzing (Ramsey) optimal monetary policy in the presence of heterogeneous access to external finance has been relatively scarce and started to grow only recently in response to the 2008 financial crisis. Using a simple closed economy model in which firms need to borrow to hire labour (the only production factor), De Fiore and Tristani (2009) or Demirel (2009) show that, unlike in the frictionless case, strict price stabilization is not an optimal response to productivity shocks. In practice, however, the resulting welfare losses are rather small. A more complex investigation is offered by Kolasa and Lombardo (2011), who consider a two-country model with endogenous capital accumulation and find non-negligible deviations of strict inflation targeting from the optimal policy, especially in the presence of nontradable production. They also show that optimal responses crucially depend on debt denomination.

All the contributions discussed above are based on the financial accelerator mechanism developed by Bernanke et al. (1999), which relies on several simplifying assumptions, one of which being the constant returns to scale in production. These assumptions facilitate aggregation across heterogeneous firms, but come at the expense of limiting the number of questions such models can address. A more sophisticated and important extension to this line of literature is offered by Cooley and Quadrini (2006), who combine idiosyncratic productivity shocks with borrowing limits as in Kiyotaki and Moore (1997) and sunk entry costs. Contrary to the models surveyed above, this setup allows for differences in leverage across firms. Since the production technology exhibits decreasing returns to scale, small firms are relatively more levered. As a result, their response to monetary shocks is substantially larger than that of big firms. While Cooley and Quadrini find that the real consequences of these distributional effects turn out to be quantitatively small, their model can be considered as a promising first step towards further research along these lines.

To summarize, while the monetary policy implications of the literature allowing for interactions between heterogeneity in firm productivity and entry or adjustment costs are yet to be developed, this line of research has already proved useful in explaining

persistence in inflation and transmission of monetary shocks. A number of positive and normative results were produced by studies linking idiosyncratic firm productivity to credit frictions. In particular, they provide a description of a strong and empirically plausible mechanism propagating and amplifying monetary shocks, and suggest that the optimal monetary policy should deviate from the standard price stability objective.

Table 4: Main findings from heterogeneous agent models

Topic	Main conclusions
The cost of inflation	Models with heterogeneity of income or age offer new findings on the cost of inflation. First, they allow for the analysis of redistribution effects of inflation, which is not possible with RA models. Second, they enable to investigate the “self-insurance” channel through which inflation affects welfare. As a result, they offer new insights on the optimal monetary policy design.
Transmission of monetary shocks	Models with heterogeneity of expectations and firm productivity provide a richer and more realistic description of the monetary transmission mechanism, and of the link between real activity and inflation. In particular, they add inertia to the responses of key macroeconomic aggregates to monetary shocks.
The equilibrium real interest rate	Models featuring income heterogeneity offer new insights on the level of the equilibrium real interest rate. Due to uninsurable idiosyncratic risk, agents hold precautionary savings, which lowers the equilibrium real interest rate. This helps to solve the “risk-free rate puzzle”: observed real rates are lower than those derived from RA models.
Determinacy of monetary policy	Heterogeneity of expectations is used to study the conditions for monetary policy rules to yield a determinate solution of the new Keynesian model. Models with heterogeneous expectations can yield significantly different implications than their RA counterparts, including chaotic dynamics and altered determinacy regions.
Redistributive effects of monetary policy	Heterogeneity of age allows to study the redistributive effects of inflation, which is an interesting topic in itself. Furthermore, redistribution may, like in the case of age heterogeneity, generate asymmetric responses and therefore have an impact on macro aggregates.
Cost of business cycle fluctuations	Heterogeneity of preferences extends and changes substantially the findings from the RA literature on the cost of cyclical fluctuations. Business cycles affect in different ways wealthy (unconstrained) and poor (financially constrained) agents. In a HA setting the aggregate welfare cost of cyclical fluctuations can be even 100 times higher than under the RA assumption.
Effects of financial frictions	Heterogeneity of preferences or of firm productivity allows for the introduction of imperfect financial intermediation. Such models are used i.a. to explain the impact of financial sector shocks on the economy or to analyze the working of monetary and regulatory policy under financial imperfections. They also modify the implications of RA models on the optimal volatility of inflation.

8 Conclusions

In this article we review the literature applying models with heterogeneous agents to monetary policy related issues. This literature accounts nowadays for an increasing (though still small) part of monetary policy research. We divide the existing literature into five categories, related to heterogeneity of (i) income, (ii) preferences, (iii) age, (iv) expectations, and (v) firms' characteristics.

This research brings a number of new results, which can be divided into two groups. First, we analyze issues that, by construction, cannot be investigated in a RA framework. In particular, we discuss the redistributive role of inflation, the impact of financial frictions on business cycle dynamics, the consequences of demographic trends for monetary policy or the impact of heterogeneous expectations on monetary transmission.

Second, HA models change several important findings from the RA literature. We show how heterogeneity, through its impact on precautionary savings, lowers the equilibrium real interest rate. We explain and document, how heterogeneity changes the estimates of the cost of business cycle fluctuations. Last, but not least, we explain how heterogeneity may affect the optimal rate and volatility of inflation. The main findings are summarized in Table 4.

All in all, two important conclusions can be made. First, inferring only from the results described in this study, it is clear that HA models must become an important ingredient of monetary economics. Not only do they provide information that could not be obtained without the introduction of heterogeneity, but also change our view on several classical results from the RA literature. Second, despite the effort that has recently been made in the area of monetary policy research with HA models, this field seems still underdeveloped. Several research topics, including the investigation of optimal monetary policy and the analysis of dynamic effects of monetary policy shocks in various HA frameworks or the consequences of introducing heterogeneity in the banking sector should (and probably will) be put on the research agenda soon.

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