A Forecasting Metric for Evaluating DSGE Models for Policy Analysis

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Introduction

- DSGE models are widely used for forecasting and policy analysis at central banks and other policy institutions
- e.g. Federal Reserve, RBNZ, Bank of England, Riksbank, ECB among others have their own working versions of DSGE models
- No clear consensus on what criteria to be used to select a model
  - some broad measure of overall fit is preferred
  - hyperparameter λ given by Del Negro et. al. is a statement of overall misspecification/ fit in a DSGE-VAR framework
- I argue overall fit measure as being both inappropriate and uninformative
- I propose a “metric” that evaluates the models suitability for practical monetary policy analysis

My Metric: New Framework for Evaluating Model Misspecification

- My evaluation metric uses standard existing Bayesian tools of prior and posterior predictive analysis.
- It introduces some new Bayesian tools
- It emphasizes the link between:
  - misspecification in the model and
  - the effect on the structural interpretation of the model for a specific task
- I focus on the task of:
  - improving the conduct of monetary policy analysis and
  - getting the structure of one step forecast errors correct

This paper is not about a horse race

- This paper is not about achieving lower RMSPE
- DSGE model should not be used to enter into a horse race for producing lower RMSPE
- Problem with sole focus on achieving lower RMSPE:
  - if both models are badly misspecified, wrong to ask which is more accurate
  - if both horses run in the wrong direction, doesn't matter which one runs faster
  - neither will reach its destination
- However, not implying that one should not ride horses
  - DSGE models are an important tool in the toolbox of central bankers and
  - can help improve our understanding of the economy if handled correctly
- Correct implication would be:
  - one should ride horses carefully
  - emphasis should be on technique (structural analysis) and not speed (lower RMSPE)

- This paper provides formal tools for doing this structural analysis carefully
- The general framework for applying this new approach laid out in a joint paper with Jon Faust
  - "Bayesian Evaluation of Misspecified DSGE Models"

Motivating the Metric

- Policy Meetings: At any meeting, policy makers:
  - Look at the intended policy rate for that period, $i_{t|t-1}$
  - Observe new available information in that period, ν_t.
  - Decide changes to policy in light of the structural interpretation of this "News"

Policy Function

- Example: Consider a simple Taylor rule
  - $i_t = a + b\pi_t + cy_{t|t}$
    - π is inflation gap, observed state variable
    - y is output gap, unobserved state variable
- Update in policy rule:
  - $i_t - i_{t|t-1} = b(\pi_{t|t} - \pi_{t|t-1}) + c(y_{t|t} - y_{t|t-1})$
    - first term on the RHS is one step ahead FE in π
    - second term is obtained via the Kalman filter
  - Kalman filter
    - $y_{t|t} - y_{t|t-1} = \gamma(Z_t - Z_{t|t-1})$
    - where Z_t is the vector of observed variables
    - γ is the Kalman gain

Update in Policy Instrument

- In a linear and Gaussian world, therefore:
  - The revision in policy rate, $i_t - i_{t|t-1}$, is a function of:
    (a) policy rule parameters
    (b) news or one step ahead forecast errors in observed variables: $Z_t - Z_{t|t-1}(\nu_t)$
  - The structural interpretation of this revision is given by:
    - structural shocks, ε_t from the DSGE model
  - Link between news, ν_t and structural shocks, ε_t
    - ν_t is a linear function of ε_t and
    - the updated or estimated structural shocks, $\varepsilon_{t|t}$, depend on:
    ν_t and ν_{t+1} via the Kalman smoother algorithm

Outline of this paper

- For practical monetary policy analysis, in this paper,I, therefore:
  (a) carefully study the first and second order moments of one step forecast errors and
  (b) analyze the properties of structural shocks relative to (a)

- I do the following:
  - compare the variance covariance matrix of one step FE in observed sample to random samples from the DSGE model

  - explain this disconnect using the correlations between estimated structural shocks for observed sample

  - compare the mean of the one step FE in observed sample with Survey of Professional Forecasters

  - explain this disconnect using the mean of the estimated structural shocks for observed sample

DSGE Model Evaluated: Smets-Wouters(AER, 2007)

- Smets-Wouters model shown to be "good" in a lot of different aspects
  - for instance, model forecasts as well as Bayesian VAR's.
- Want to check for potential misspecification that affects policy making in order to further improve on the model
- Observed Variables: seven macro series
  - GDP growth, consumption growth, investment growth, inflation, real wage growth, labor hours and the nominal interest rate
- Time period: Quarterly data, 1966 to 2004 (156 observations)
- Estimation Method: Bayesian estimation

Comparing Structure of Forecast Errors

- The variance-covariance of 1 step ahead forecast errors:
  - $\Omega = vcov(\nu_t)$
- Compare Ω implied the DSGE model for random samples to Ω implied by the DSGE model for the observed sample.
  - Posterior Predictive distribution: Ω implied for random samples
  - Posterior In-sample distribution: Ω implied for the observed sample
  - Population Mode Value: Ω implied by the mode of the estimated parameter vector θ
- First I will report the point estimates at the posterior mode and later look at the uncertainty around these point estimates

Results

	Standard Deviations	
Variable	DSGE Model (population)	DSGE Model (observed sample)
ΔGDP	0.85	0.74
ΔC	0.56	0.62
ΔI	1.75	1.84
Hours	0.61	0.54
ΔW	0.52	0.55
Inflation	0.29	0.28
Interest-Rate	0.24	0.24

Evaluated at posterior mode

Table: The Main Diagonal of Ω: 1 period ahead FE standard deviations

- Table: One period ahead forecast error correlation
- A lot of numbers, will highlight a few
  - For example:
    - One step ahead FE Corr(ΔC, Inflation)
    - One step ahead FE Corr(Hours, Δw)

Correlations

Variable	DSGE Model (population)	DSGE Model (observed sample)
ΔGDP,ΔC	0.50	0.56
ΔGDP,ΔI	0.59	0.48
ΔGDP,Hours	0.63	0.49
ΔGDP,ΔW	0.19	0.12
ΔGDP,Inflation	-0.05	-0.16
ΔGDP,Interest-Rate	0.04	0.14
ΔC,ΔI	0.22	0.36
ΔC,Hours	0.38	0.22
ΔC,ΔW	0.04	0.26
ΔC,Inflation	-0.08	-0.28
ΔC,Interest-Rate	0.05	0.07
ΔI,Hours	0.41	0.48
ΔI,ΔW	0.14	0.02
ΔI,Inflation	0.01	-0.04
ΔI,Interest-Rate	0.03	0.14
Hours,ΔW	-0.01	-0.30
Hours,Inflation	0.12	0.11
Hours,Interest-Rate	0.29	0.38
ΔW,Inflation	-0.09	-0.20
ΔW,Interest-Rate	-0.03	-0.09
Inflation,Interest-Rate	0.31	0.17

Evaluated at posterior mode

One period ahead forecast error correlations

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Evaluated at posterior mode

One period ahead forecast error correlations

Posterior Density Analysis

- I will be using the following terminology:

- Posterior predictive distribution
  - take a draw from the posterior distribution of θ
  - generate a random sample
  - calculate the feature of interest on this random sample
  - repeat for all draws from the posterior distribution of θ
  - you get posterior predictive distribution for the feature of interest

- Posterior in-sample distribution
  - same as above except
  - now you use the actual observed data instead of random samples
  - there is no sampling uncertainty involved in this disribution
  - you are treating the observed data as the truth

- Population mode value
  - this value of the feature of interest is based entirely on parameter vector θ
  - often times co-insides with the mode of the posterior predictive distribution

Semi Formal Metrics

- Where does the in-sample distribution lie relative to the predictive distribution
- If lies in tail, one can take either of following two views:
  - Treat the observed sample as a freak from the standpoint of the DSGE model - that is, DSGE model unlikely to produce random samples similar to the observed sample or
  - If you treat the observed sample as a fair characterization of the world we are in: - that says, DSGE model is misspecified or simply put, incorrect.

Example 1: One step ahead FE Corr(Hours, Δw)

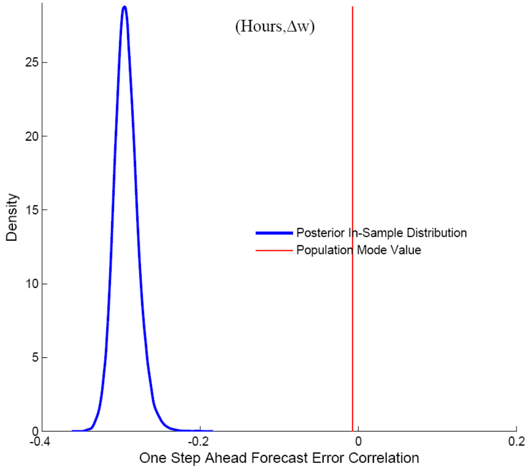

Figure: One Step Ahead Forecast Error Correlation: Hours and Wage Growth.

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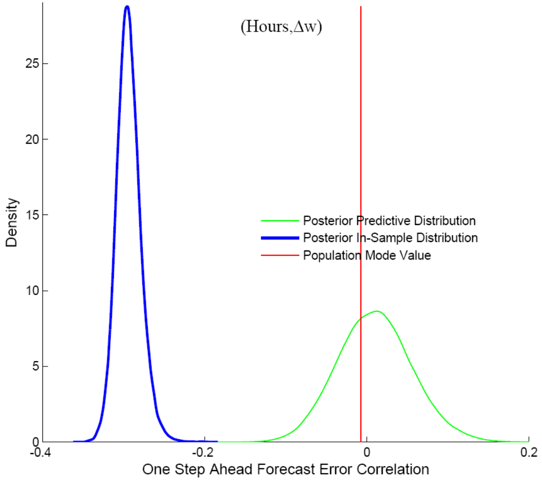

Figure: One Step Ahead Forecast Error Correlation: Hours and Wage Growth.

How the model explains this one step ahead FE Corr(Hours, Δw)

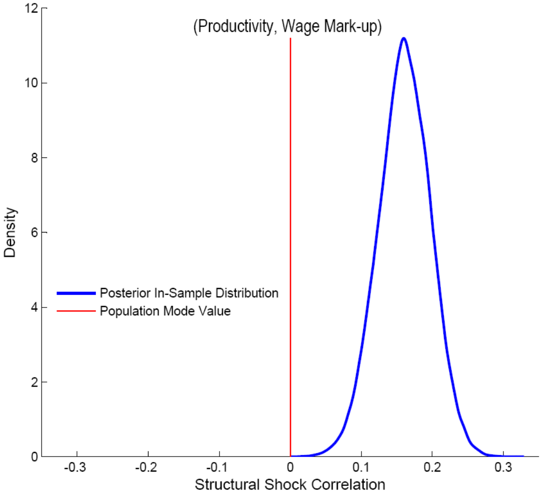

Figure: Structural Shock Correlation: Productivity and Wage Mark-up.

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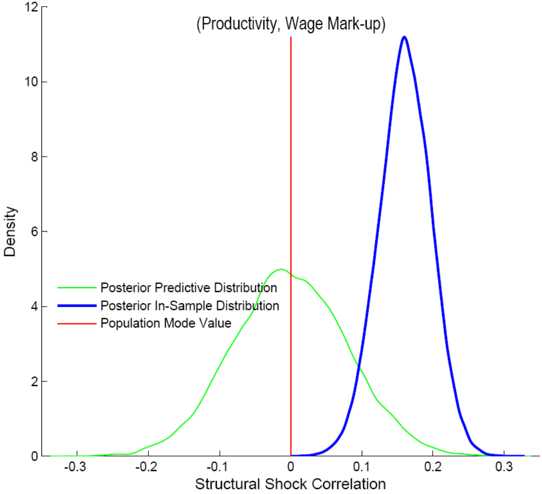

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Example 2: One step ahead FE Corr(ΔC, Inflation)

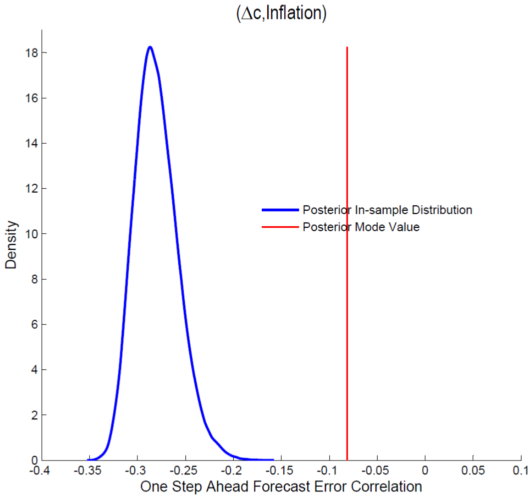

Figure: One Step Ahead Forecast Error Correlation: Consumption Growth and Inflation.

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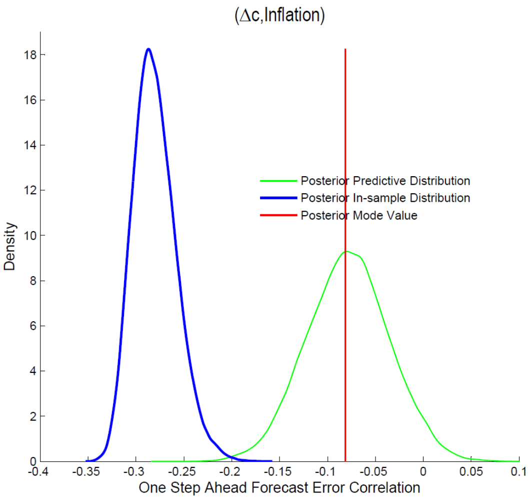

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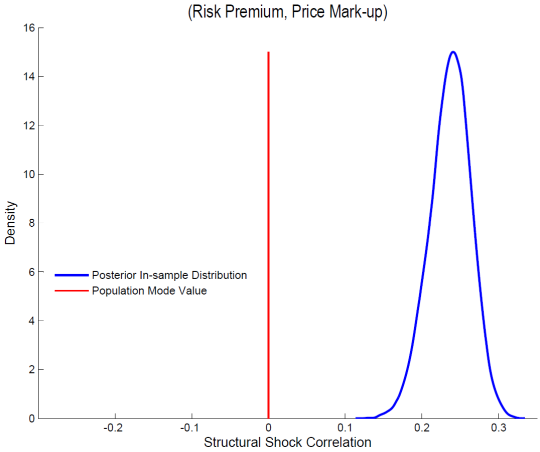

Figure: Structural Shock Correlation: Risk Premium and Price Mark-up.

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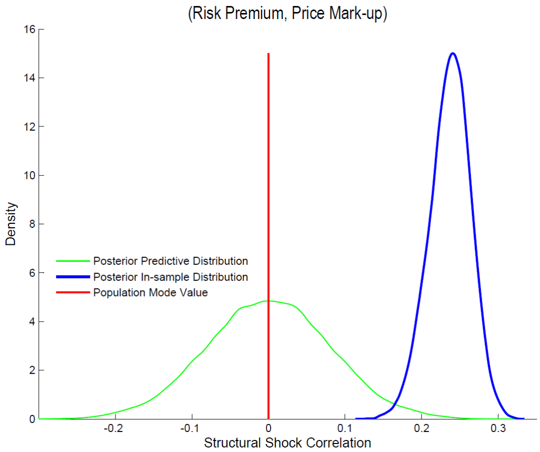

Figure: Structural Shock Correlation: Risk Premium and Price Mark-up.

Variance Covariance Matrix of One step Forecast Errors

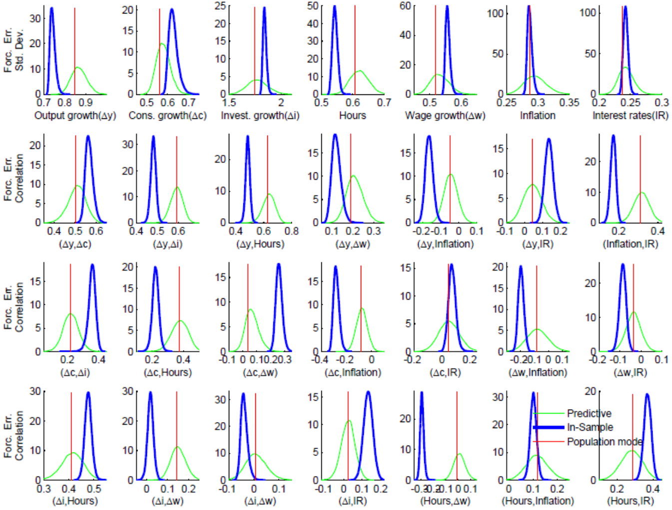

Figure: Standard deviation and correlations of one step ahead forecast errors.

Variance Covariance Matrix of Estimated Structural Shocks

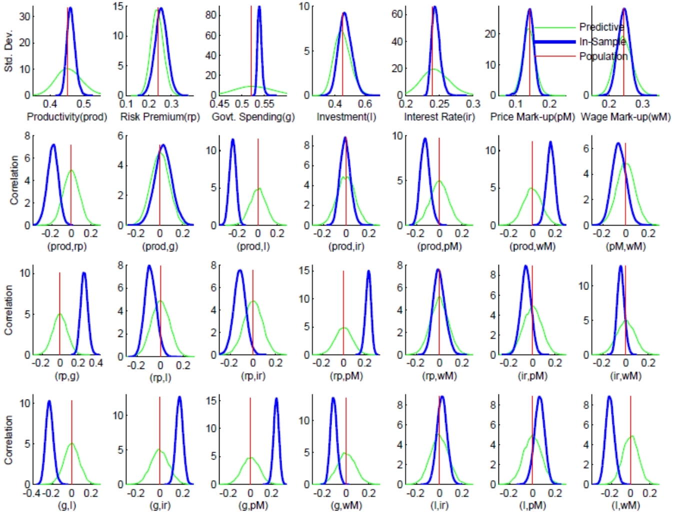

Figure: Standard deviation and correlations of estimated structural shocks, $\varepsilon_{t|t}$

Comparison of One Step In-sample FE with SPF

- I compare the mean one step error FE from the DSGE model for the observed sample with Survey of Professional Forecasters
- Main finding:
  - I find mean error from DSGE model to be much larger compared to the median SPF forecast error
- Explain this disconnect of the DSGE model using the estimated mean value of the structural shocks for the observed sample

One step Forecast Errors: Δy, Δc, Δi

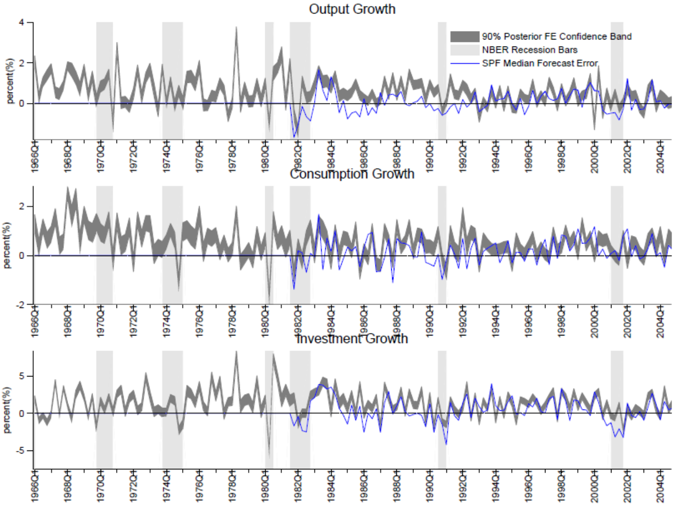

Figure: One step ahead FE's in output, consumption and investment growth

One step Forecast Errors: Inflation and Interest Rate

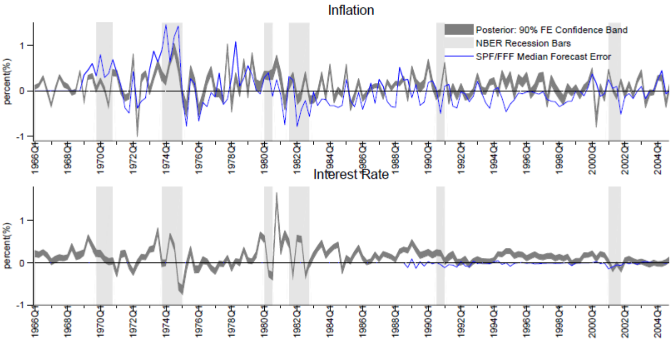

Figure: One step ahead FE's in inflation and interest rate

Mean Error for One Step Ahead FE

Variable	Period	SPF	DSGE	DSGE(full sample)
ΔGDP	81Q4-04Q4	0.03	0.40	0.56
ΔC	81Q4-04Q4	0.16	0.38	0.50
ΔI	81Q4-04Q4	0.39	1.12	1.27
Inflation	69Q1-04Q4	0.02	0.10	0.10
Interest-Rate*	89Q1-04Q4	-0.02	0.11	0.13

* Interest rate FE refers to the federal funds futures data.

Table: SPF vs DSGE Model: One step ahead median forecast error mean

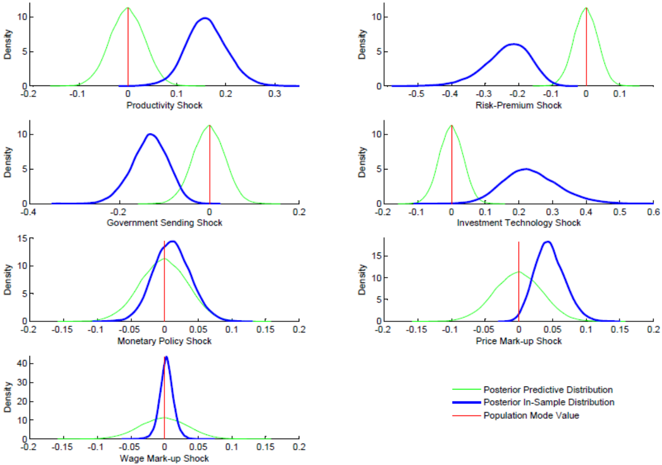

Figure: Mean value for the structural shocks estimated on at posterior mode

Variable	DSGE Model (population)	DSGE Model (observed data)
productivity	0.00	0.16
risk premium	0.00	-0.25
govt. spend.	0.00	-0.12
investment	0.00	0.27
mon. policy	0.00	0.01
price mark-up	0.00	0.05
wage mark-up	0.00	0.00

Mean value for shock processes

Table: Mean of the updated structural shocks estimated at posterior mode

Conclusion

- As an example, I showed that Smets-Wouters DSGE model is highly overidentified
- In order to improve the overall fit of the model to the observed sample, the model ends up being incorrectly specified in other key dimensions
- Argued it is important to not get tied up in a horse race for lower RMSPE when we know that all horses are badly misspecified i.e. running in the wrong direction
- Provided a set of Bayesian tools to carefully study and analyze the structure of one step ahead forecast errors.
  - ease of story telling is an important reason for using DSGE models
  - I filled gaps in the story telling for monetary policy analysis