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An application of the Choleskized multivariate distributions for constructing inflation 'fan charts'

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The prologue

Two main approaches to building fan charts

Analytical: Parameters are analytically derived from a dynamic model (see e.g. Kemp, 1991, 1999, Chaudhuri *et al.*, 2008, Cogley *et al.*, 2004).

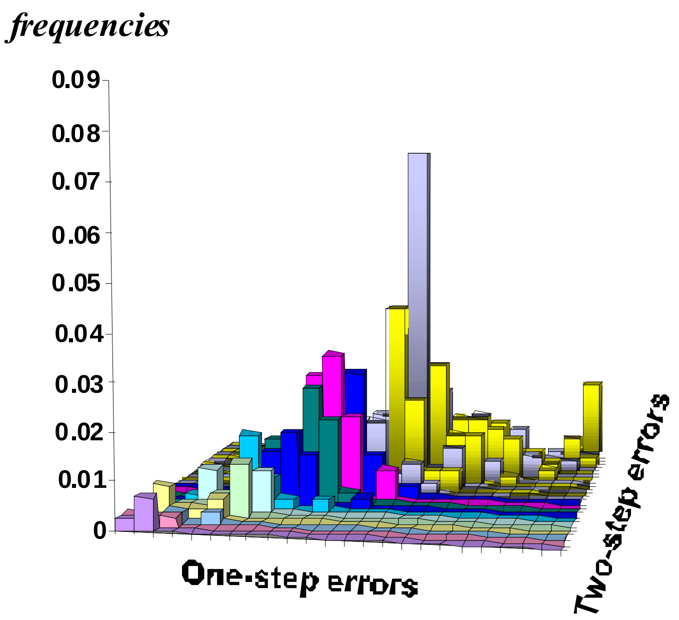
Pragmatic: Parameters are calibrated or set with the use of pooled forecasts, experts opinions, past forecasts errors etc. (see Wallis, 1999, and Giordani and Söderlind, 2003).

Basic approach used here: pragmatic, with parameters (and functional approximations) derived on the basis of past forecasts errors of point (deterministic) forecasts.

General features of real-life inflation forecast errors distributions:

- They have ‘heavy tails’ (due to episodes of hyperinflation and rapid disinflation)
- Usually the simpler forecasting technique is, the heavier the tails are
- They are skewed (not necessarily to the right)
- Their distributions are dependant between different forecast horizons

Joint distribution of one and two step ahead errors of *BARMA* forecast, OECD countries



In case of y/y inflation forecasting errors *must* be dependent, as there is the overlapping events effect here.

How to model forecast errors?

Some convenient skewed and heavy-tailed distributions

These are *stable distributions* after 'tilting' (tempering the tails)

Definition 1. **(Rosiński, 2007)*:* A random variable X in $\mathbb{R}^d$ is said to be a tempered stable if the corresponding measure M in R^d in polar coordinates has a form**

$$M_0(dr,du) = \alpha r^{-\alpha-1} q(r,u) dr d\sigma(u) ,$$

where $q:(0,+\infty)\times S^{d-1} \to (0,+\infty)$ is a Borel function such that $q(.,u)$ is completely monotone for each $u \in S^{d-1}$, $q(0+,u)=1$ and $q(\infty,u)=0$.

One-dimensional case:

$$q(r,u) = \exp(-\theta \cdot r)\ ,\ \theta > 0$$

θ **controls tempering**

Definition 2: Choleskized multivariate stable and tempered stable distributions

Choleskized distribution of X **is a distribution of multivariate random variable** $Z = CX$ **where** C **is a Cholesky upper triangular matrix, such that** $C'C = \Sigma$**, where** Σ **is a positively defined matrix.**

Further X **is called the 'initial' vector.**

What’s good about Choleskized distributions?

- Very easy to apply (simulate)
- Give interesting (interpretable) results (see later)

What is bad about Choleskized distributions?

Nobody knows (so far) what is their characteristic function and moments (except of some special cases)

How do the stable and tempered stable distributions fit to the OECD forecasts errors?

Better than normal and skewed-normal (see the paper for details)

Tempered stable are better for more sophisticated forecasts and stable for naïve forecasts.

One particular application: the analysis of runs and turning points

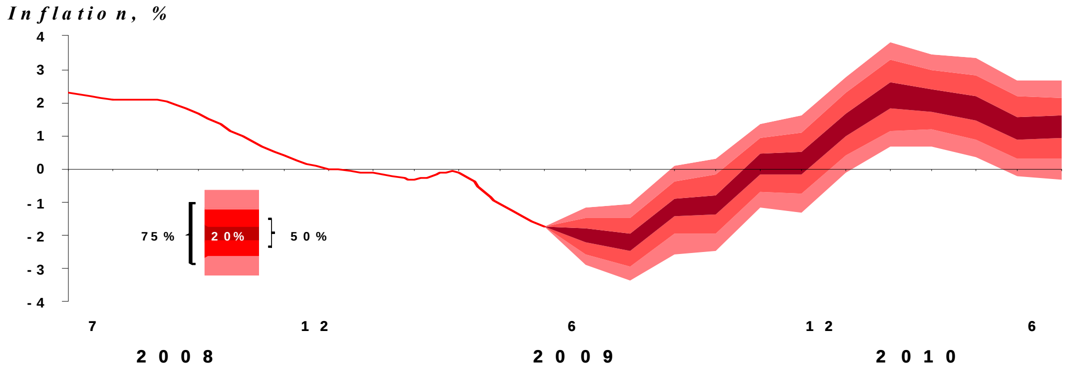
The case of Japan: y/y monthly data on inflation

Two types of forecasts:

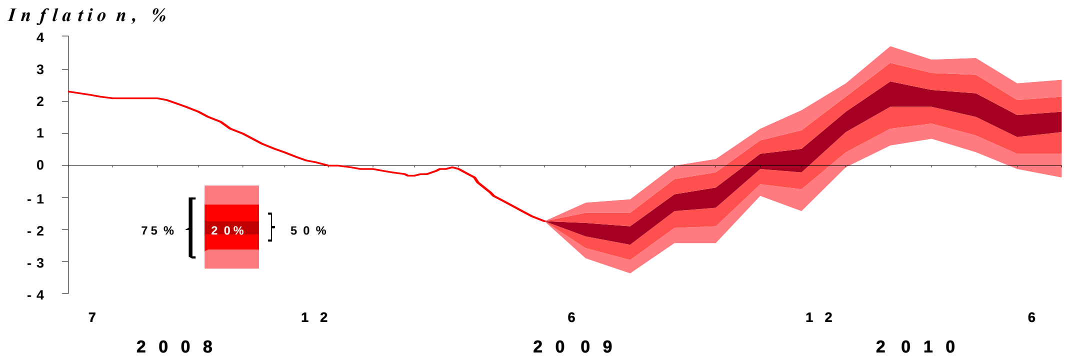
1. **Using dependent (Choleskized) 12-dimensional tempered stable distributions**
2. **Using independent 12-dimensional tempered stable distributions**

It is expected that there will be a turning point in Japanese inflation dynamics at the end of 2009

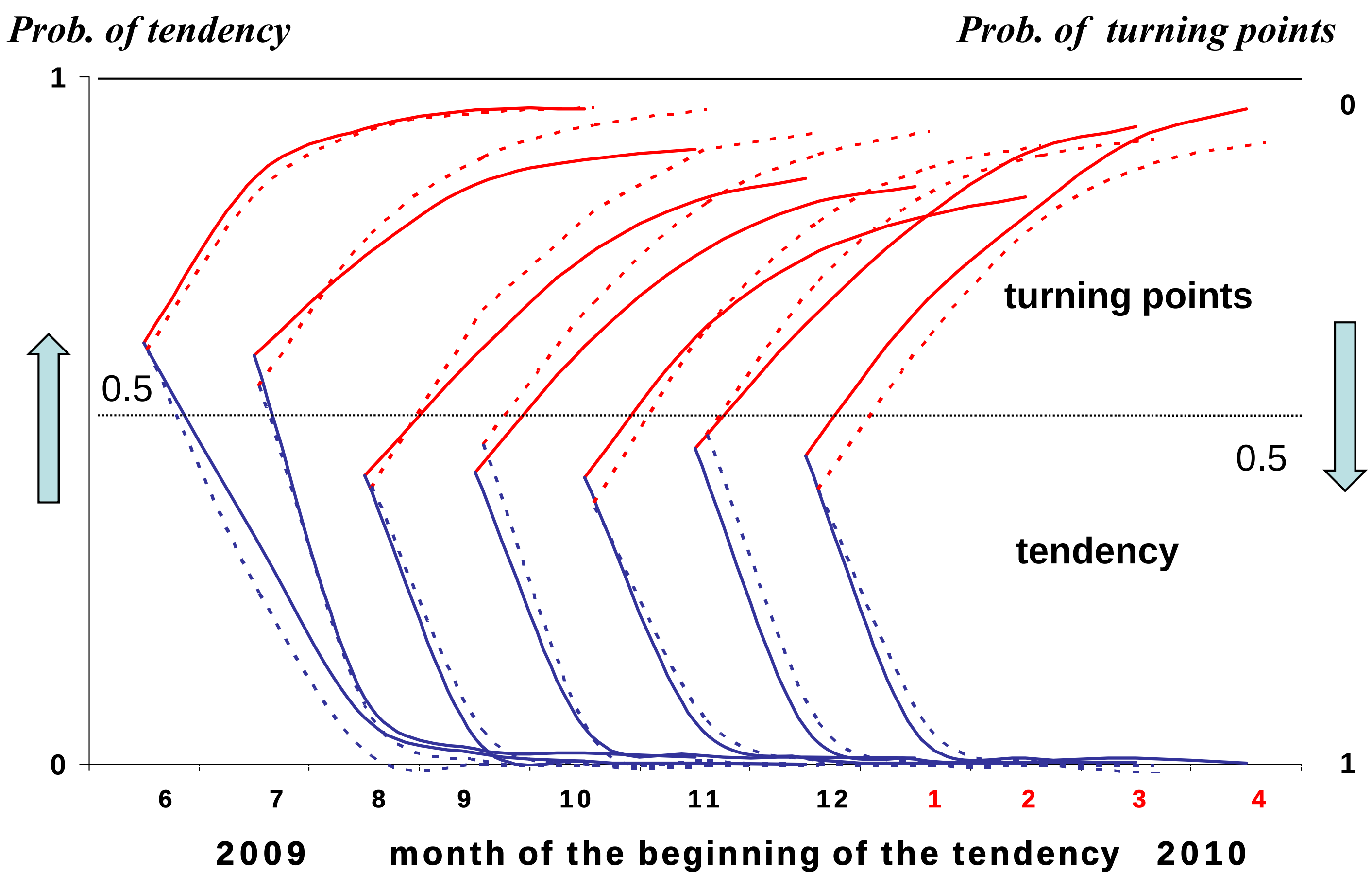
Fan chart for Japan, dependent distributions



Fan chart for Japan, independent distributions



The Dinosaur for Japan



Interpretation of the Dinosaur

r_t : the event that, in time t, a continuation of the current tendency appears, that is, $\text{sgn}(\Delta\pi_{t+1}) = \text{sgn}(\Delta\pi_t)$

s_t :the event that, in time t, the tendency reversion, that is $\text{sgn}(\Delta\pi_{t+1}) \neq \text{sgn}(\Delta\pi_t)$

Points on the red line: $\Pr\left(\bigcap_{i=1}^{k} r_{t+i} \mid s_t\right)$ for $k = 1,2,3,4$.

Points on the blue line $\Pr\left(\bigcap_{i=1}^{k} r_{t+i} \mid r_t\right)$ for $k = 1,2,3,4$.

Expected durations of current run continuation and after the turning point

Month	Current run		After turning point	
	Dep. forecast	Indep. forecast	Dep. forecast	Indep. forecast
Jun 2009	1.58	1.33	1.72	1.37
Jul 2009	1.62	1.13	1.75	1.87
Aug 2009	1.70	1.18	1.74	1.94
Sep 2009	1.66	1.11	1.77	1.99
Oct 2009	1.67	1.27	1.82	1.82
Nov 2009	1.61	1.13	1.73	1.81
Dec 2009	1.63	1.97	1.76	1.63

Conclusions

1. **Fan charts built under the assumption of symmetric or skewed normality might not lead to precise results, as normal and skewed normal distributions do not approximate the empirical distributions of forecast errors well.**
2. **Better fit is usually obtained by the α-stable and tempered stable distributions.**
3. **It is important to incorporate dependence into the multivariate distributions which are used for constructing the fan charts.**
   a. **Real life inflation forecast errors are usually dependent,**
   b. **It is easy to compute probabilities of turning points appearing at a specific time in future, expected lengths of the increasing and decreasing runs of inflation, *etc*.**

4. **The Choleskized multivariate distributions seems to be of a particular use here, as it is computationally straightforward and intuitively simple.**

Drawbacks and limitations

1. **Not all parameters of the Choleskized distributions can be easily estimated, especially if the sample size is limited.**

2. **Little is known of the properties of the Choleskized stable and tempered stable distributions.**