Fan charts of inflation and GDP growth projections

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1 Introduction

Forecasting inflation and GDP growth is an important aspect of contemporary central banking, especially in direct inflation targeting regime. But each forecast is burdened with uncertainty. Estimating and publishing this uncertainty is important for central banks' communication with the public as it enables to shift the focus from the central path of which probability is zero towards medium-term risks. Also distribution of risks might be relevant if some members of monethary authorities have a non-symmetrical loss function.

Fan charts, which plot the probability density of forecast variable (i.e. an estimate of the probability distribution of possible outcomes at different forecast horizon) have become the most popular method of quantitative representation of forecasts along with their uncertainty.

2 Quantifying the uncertainty

Clements and Hendry (Forecasting Economic Time Series, 1998) distinguish five major sources of uncertainty in large-scale macroeconometric models used for forecasting:

- Future changes in the underlying structure of the economy
- Misspecification of the model
- Inaccuracies in the estimates of the model's parameters (estimation uncertainty)
- Variable mismeasurement:
  - initial condition uncertainty
  - non-modelled variable uncertainty
  - incorrect categorization of some variable as exogenous
  - lack of invariance to policy changes in exogenous variables
- The cumulation of future shocks to the economy

Unfortunately not all of the above-mentioned sources of uncertainty are quantifiable. Moreover model forecasts are usually corrected by experts and these expert adjustments may decrease forecast errors but still in a non-quantifiable way. For these reasons central banks typically use past forecast errors as a catch-all proxy for future uncertainty. This strategy is often complemented with expert adjustments for anticipated changes in uncertainty, e.g. uncertainty in increased for periods of recession and decreased for periods of steady growth.

3 Procedure

Due to problems with quantifying various types of uncertainty in macroeconomic models, past forecast errors seem to be a good catch-all proxy for future uncertainty. Still, uncertainty may vary due to different reasons, e.g. change within a business cycle (being lower in the periods of higher growth and increasing during recessions). So, in the NBP's approach uncertainty consistent with past forecast errors is adjusted for anticipated changes in uncertainty.

Another need for adjustment results from the assumption of fixed reference interest rate under which NBP's projections are prepared. Past forecast errors are based on NBP's forecasts with paths of interest rates taken from market expectations. Assumption of no change of interest rates, i.e. no reaction of monetary authority to any changes in inflation should result in bigger variance/uncertainty of future inflation path[1].

To cope with the above-mentioned problems the following procedure is used:

1. **Determining the distributions of historical forecast errors.** On the basis of past forecast errors from the ECMOD/NECMOD model, the forecast errors variance is estimated for every forecasting horizon. In the case of inflation, account is taken of uncertainty of forecasts starting from the quarter in which the Inflation Report is published. In the case of GDP growth forecasts, due to frequent revisions of national accounts, account is also taken of the uncertainty of past values of the variable (up to 7 quarters preceding the publication of the Report inclusive).

2. **Simulation of paths of exogenous variables (for more details see sections 4 and 5).** On the basis of the NECMOD model's multipliers, set of exogenous variables were selected, whose uncertainty has a prevailing impact on the uncertainty of inflation and GDP forecasts. Experts forecasting exogenous variables, in every forecasting round present the central path (the expected value) and the uncertainty assessment of the forecast of the given variable (whereas the distribution of risks does not have to be symmetric). The simulation procedure of exogenous variables was chosen in such a way that: 1) the expected value of simulated paths of variables conforms to central paths given by experts, 2) the expected value of stochastic disturbances is zero, 3) the autocorrelation of variables observed in the sample is retained and 4) the cross correlation of shocks among particular variables is retained as well. Exogenous variables are simulated in two versions: with historical uncertainty

[1]The same may be not true for GDP growth, as monetary authority stabilising inflation may in the short-term increase the variance of GDP growth.

and with current uncertainty. In the first case the confidence band of simulated exogenous variables are consistent with the average assessment of uncertainty given by experts in the previous forecasting rounds, while in the second - with the assessment of uncertainty in the current forecasting round.

3. **Simulations from the NECMOD model.**[2] The NECMOD model is used to carry out stochastic simulations in two versions. In the first version, the uncertainty of exogenous variables is set at the average level from the previous forecasting rounds and the interest rate is endogenous, determined by the Taylor rule estimated on the basis of historical data. In the second, the uncertainty of exogenous variables is set at the level from the current forecasting round, and the interest rate does not change in the projection horizon. The simulations do not take into consideration other sources of forecast uncertainty (uncertainty related to the error term, estimators, etc.). The goal of the simulation is to assess the impact of changes in the uncertainty assessment of exogenous variables and of the assumption of the exogeneity of the monetary policy on the uncertainty of GDP and inflation forecasts.

4. **Determining the current uncertainty concerning GDP and inflation.** The fan chart based only on past forecast errors from the ECMOD/NECMOD model is an adequate estimate of uncertainty of the **forecast** with the uncertainty of exogenous variables at the level corresponding to the average uncertainty in the previous forecasting rounds. To this fan chart a correction calculated in the previous point is added, by means of which we obtain the uncertainty assessment of the **projection** with the current level of exogenous variables uncertainty.

4 Determining the uncertainty of exogenous variables

Ex ante uncertainty of assumptions underlying projection of inflation and GDP is used in estimation of anticipated change in the overall uncertainty (as compared to past forecast errors). Below we give a more detailed description of series we shock in stochastic simulations.

4.1 Shocked time series

When choosing series to be shocked in stochastic simulations, the following criteria were taken into account:

- The selected variable should have prevailing impact on uncertainty of inflation and/or GDP growth (the property was assessed on the base of model multipliers);
- Uncertainty of the variable should non-negligibly change between forecasting rounds;

[2] Technically, model is solved sequentially, quarter after quarter and solution in each period takes into account model-consistent expectations. In each quarter, agents form their expectations about future paths of all variables, relying on the shocks realized up to this period. More specifically, agents assume that in the future endogenous variables evolve according to their equations and exogenous variables follow the laws of motion for the shock processes. In the next quarter shocks for that period are generated and agents update their expectations.

- Uncertainty of the selected time series should be feasible to measure and forecast.

Based on these criteria the following time series were selected:

- Crude oil price in the world markets
- Natural gas price in the world markets
- Coal price in the world markets
- EUR/USD exchange rate
- 3M interest rates in the euro area
- 3M interest rates in the USA
- Price of food commodities in the world markets
- Weighted GDP of main trading partners
- Weighted potential GDP of main trading partners
- Weighted value-added deflator of main trading partners

It is worth noting that all of the above mentioned variables describe foreign environment. This corresponds with the perception of Polish economy as a small open economy.

4.2 Data sources

Beyond think-thank forecasts and expert opinion, an alternative way to assess uncertainty of interest rates, exchange rates and other prices is to turn to commodity and financial markets. Futures contracts allow market participants to hedge against risk and protect them against losses if the market moves against them, i.e. they eliminate the uncertainty and price risk. In this way, it is possible to infer expected price as well as its risk distribution from futures contracts. Another financial instrument used for the same reason are options, which give right but not obligation to buy/sell a particular asset at a later date at an agreed price. In return for granting the option, the seller collects the premium from the buyer.

4.3 Estimating/evaluating uncertainty of shocked exogenous variables

4.3.1 Commodity price risk

Natural gas, crude oil and coal prices in the international energy exchanges have been rising strongly over the last few years to decline sharply at the end of 2008. Energy prices volatility creates uncertainty for all market participants and has significant impact on the nominal and real side of the economy.

A way to assess expected future commodity prices is to derive them from the current futures price of exchange-traded energy futures contracts. Because there are many different contracts for each resource, the most representative ones for each raw material were chosen.

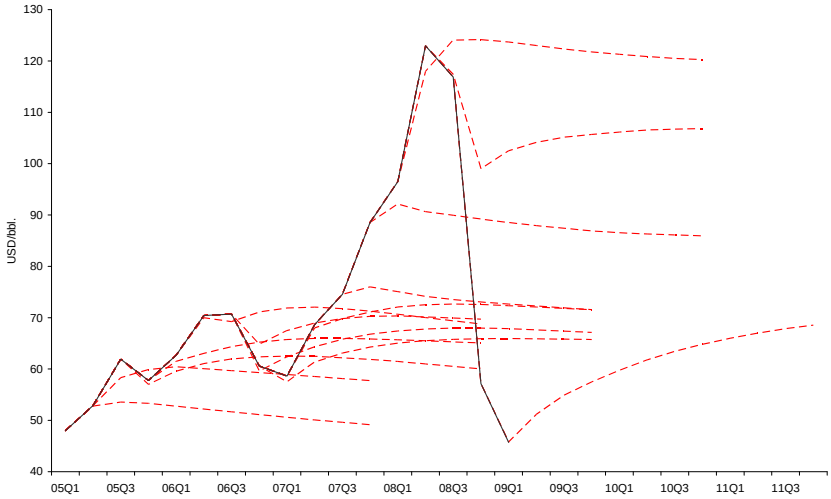
They serve as an international benchmark grade, namely: Brent Crude Oil futures and Natural gas futures (traded at NYMEX) and Richard's Bay coal futures (traded at ICE).

Uncertainty of anticipated energy prices is calculated on the basis of deviations of similarly constructed forecast from the actual (realised) spot prices. In practice, we evaluate the average past error of derivatives market participants in assessing expected energy prices for different durations of contracts. This pure statistical approach can be then modified by including expert judgement. The uncertainty assessment is conducted for each price variable separately.

Future raw material prices are assumed to follow log-normal distribution. The distribution has two desired properties. First, commodity price data tend to be highly skewed. Second, commodity prices have a theoretical minimum value (zero) but no theoretical maximum value.

To assess the risk of the spot prices forecast based on future contract prices the data on contracts traded on the day of the cut-off dates for consecutive forecasting rounds (from May 2005) were collected. Figure 1 displays the futures contracts for oil prices collected on cut-off dates.

Figure 1: Brent oil price with market expectations from futures contracts*.

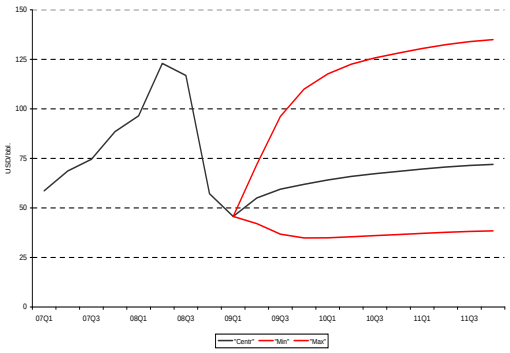


*Information collected on cut-off dates.

Next, for each quarter of projection horizon errors between natural logarithms of adequate future price and realisations were calculated and assumed to follow normal distribution with zero mean and horizon-specific variance obtained with the approach coherent with the one used to calculate variance of the past errors for inflation and GDP.

Experts use statistical description of past future prices based prediction error as a reference point to assess anticipated changes in this uncertainty. They can scale the uncertainty bands in order to indicate e.g. that in the forecasting horizon some markets may be more/less volatile than in the past. Secondly, they can change the balance between upwards and downwards risks for each quarter of their forecast.

Figure 2: Historical uncertainty concerning crude oil price forecasts*.



*Calculation from June 2009 forecasting round.

4.3.2 EUR/USD exchange rate

Above mentioned energy commodities, as well as food commodities in the world markets, are quoted in USD. At the same time, Polish currency behaves much more in line with EUR than with USD. Thus, prices of energy and food commodities quoted in PLN depend heavily on the EUR/USD exchange rate. That was the main rationale for introducing this cross-exchange rate to the risk analysis.

Uncertainty of the EUR/USD exchange rate is based on currency options contracts. Data are extracted from Bloomberg and cover all past cut-off dates and the current one. Unfortunately data are available only for 4-quarter horizon, so for longer horizons trends need to be extrapolated. It was assumed that variance in the later quarters grows linearly, which corresponds with the assumption of white noise process.

4.3.3 3M interest rates in the euro area and in the US

Introduction of foreign short-term interest rates to the risk analysis is important mainly as they affect behaviour of exchange rates. Also, both interest rates - in the euro area and the US are shocked, instead of just one weighted interest rate of main trading partners of Poland. This allows (through cross-correlations of shocks) to catch influence of those interest rates on EUR/USD exchange rate.

Uncertainty of 3M interest rates in the euro area and the US is also (as for EUR/USD exchange rate) based on options contracts and the data also come form Bloomberg. This time data are available for 8-quarter horizon so less extrapolating needs to be done, but the procedure is the same as for EUR/USD exchange rate.

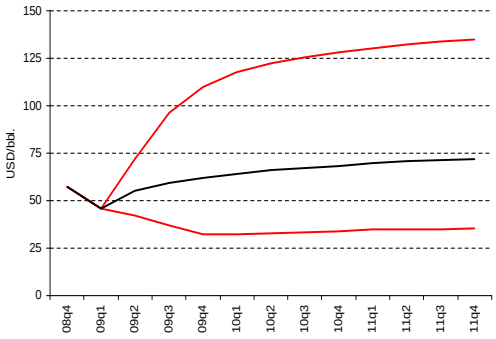
4.3.4 Other variables

Uncertainty of the remaining variables (namely: price of food commodities in the world markets and weighted GDP, potential GDP and value-added deflator of main trading partners) is given by experts. In their assessment of the uncertainty, experts use various available sources of information including their knowledge of the economic processes, sometimes backed by intuition.

5 Simulation of paths of exogenous variables

Experience gathered at the NBP indicates that ascribing 5th and 95th percentiles to respectively minimum and maximum forecast paths of the variable may well serve as an intuitive method of quantifying the uncertainty by experts. Distributions provided by experts can be asymmetrical - see example below for oil price.

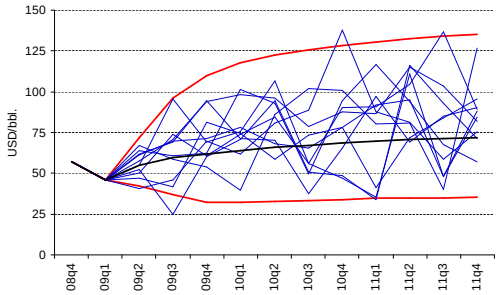
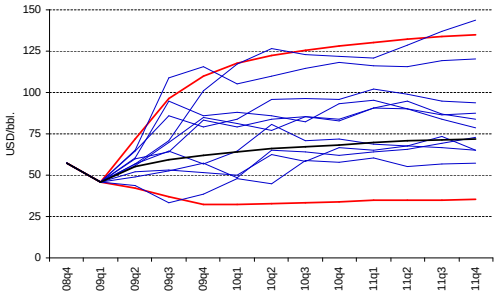
Figure 3: Future uncertatinty concerning crude oil price forecast*.



*Assumption for June 2009 forecasting round.

Importantly, significantly different data generating processes (DGPs) can be consistent with an expert forecast described in terms of central path and minimum and maximum bounds. Graphs below show ten exemplary paths of oil prices with different assumptions about DGPs, but in both cases expected value, 5th and 95th percentile for the distribution are the same.

Table 1: Ten exemplary oil price paths generated with the assumption of:

zero autocorrelation	zero autocorrelation for first differences
	

Intuitively, for each DGP of exogenous variables used in stochastic simulations, resulting uncertainty of inflation and GDP growth (fan charts) would be different. For that reason procedure of simulating exogenous variables needs to have the following properties:

- The expected value of simulated paths of variables conforms to central paths given

by experts.

- The expected value of stochastic disturbances is zero.
- The autocorrelation of variables observed in the sample is retained.
- The cross correlation of shocks among particular variables is retained.

5.1 Data generating process (DGP)

Each variable (where exogenous variables are indexed by i) is assumed to be generated by the process of the form:

$$f(y_{i,t}) = a_{i,t} + b_i \cdot f(y_{i,t-1}) + \xi_{i,t}$$

$$\xi_{i,t} \sim TPN(0, s^1_{i,t}, s^2_{i,t})$$

where: $f(y_t) = y_t$ or $ln(y_t)$, Δy_t, $\frac{\Delta y_t}{y_{t-1}}$.

Transformation $f(.)$ depends on the time series properties of the exogenous variable, i.e. its volatility and stationarity or non-stationarity. Shocks for each variable are correlated ($\mathrm{E}(\xi_{i,t}, \xi_{j,t} \neq 0$) and TPN stands for two-piece normal distribution (also called half-Gaussian distribution).

Therefore, exogenous variables in stochastic simulations are generated by AR(1) process with time-varying constant and shocks following asymmetric distribution with variance changing in time. Parameters describing autocorrelations for each variable (b_i) and values of cross-correlations between shocks for different variables are constant in time and are estimated.

Parameters $a_{i,t}$,$s^1_{i,t}$ and $s^2_{i,t}$ corresponding with equations for each exogenous variable are calculated so that DGPs generating these variables are consistent with the central path and confidence bands given by experts. As three parameters are calculated for each period and also three properties of the distribution are given by experts, calculation of the parameters is unambiguous.

5.2 DGP - Autocorrelations and cross-correlations

Historical time series of exogenous variables are described by the process:

$$f(y_t) = \alpha + \beta \cdot f(y_{t-1}) + \varepsilon_t$$

where y is a vector of variables after appropriate transformations. Parameters of this process estimated jointly with seemingly unrelated regression (SUR) method to allow for cross-correlations between residuals from equations entering the system. Estimator of β_i i.e. b_i , measuring the level of autocorrelation for the series, is used in DGP for the series. Estimators of the constants (α) are not used. Residuals of equations are used to calculate the correlation matrix $\Sigma = \{\rho_{ij}\}$, where ρ_{ij}- empirical Spearman rank correlation between $e_{i,t}$and $e_{j,t}$.

In order to retain those Spearman rank correlations for shocks for each variable, shocks are generated jointly by:

$$\xi_{i,t} = F_{i,t}^{-1}(\Phi(\varepsilon_{i,t}))$$

where:

$F_{i,t}$- cumulative distribution function (CDF) of TPN $(0, s_{i,t}^1, s_{i,t}^2)$,

Φ- CDF of N(0,1),

$e_t \sim N(0, \Sigma)$.

The abovementioned transformation is monotonic and does not change Spearman rank correlations (The correlation between $\varepsilon_{i,t}$and $\varepsilon_{j,t}$ is the same as the one between $\xi_{i,ti,t}$and $\xi_{i,tj,t}$).

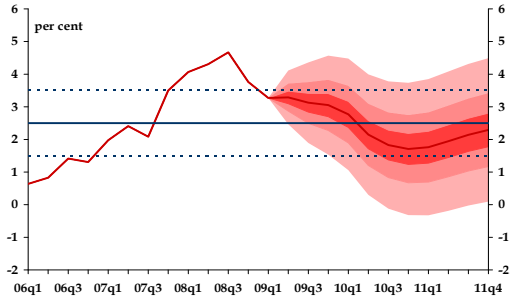
With the transformation $u_{i,t} = \Phi(\varepsilon_{i,t})$ variables from multivariate uniform distribution with retained Spearman rank cross-correlations are produced, and with the transformation $\xi_{i,t}$=$F_{i,t}^{-1}(u_{i,t})$ series from multivariate TPN distribution also with retained Spearman rank cross-correlations.

5.3 Illustrative results

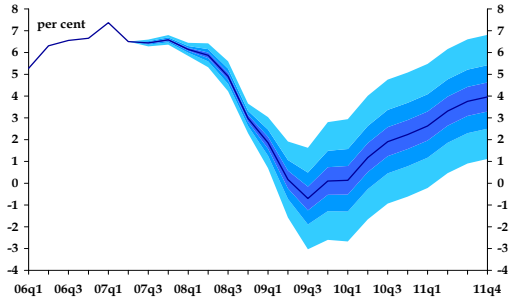
Sample results, basing on the June 2009 forecast round, are shown below. First two figures show uncertainty of the past NBP forecast errors, which are the reference for estimating future uncertainty.

Table 2: Fan charts based on past forecast errors*.

CPI inflation



GDP growth



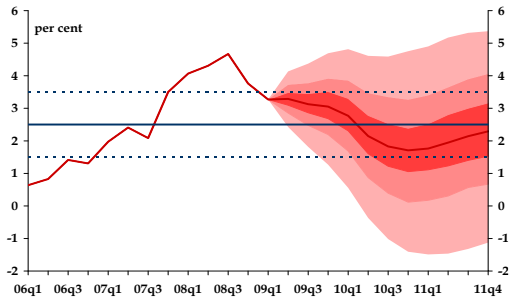
*Calculation up to February 2009 forecast.

Application of the NBP's procedure for future uncertainty evaluation resulted in considerable increase in anticipated uncertainty for CPI inflation projection in the long-term horizon. This should come as no surprise, taking into account that this uncertainty is estimated under the assumption of fixed reference interest rate, i.e. no reaction of monetary policy to inflationary developments. At the same time, anticipated uncertainty for GDP growth was slightly increased in the short term and slightly reduced in the medium term. The increase in the short term may be attributed to higher anticipated uncertainty of external assumptions (effects of the recent turmoil on the financial markets), while the decrease in the medium term is the result of model mechanisms and fixed reference interest rate assumption. Namely, in the face of external shocks monetary policy (which in the model is aimed at stabilising inflation) may slightly destabilise GDP growth in the medium

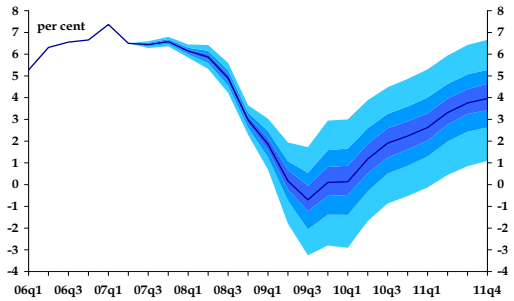
term. Final fan charts for CPI inflation and GDP growth in June 2009 forecasting round are shown below.

Table 3: Final fan charts*.

CPI inflation



GDP growth



*June 2009 forecasting round.

6 Advantages and disadvantages of the NBP's method

Advantages:

- Width of the fan charts is consistent with the expert-adjusted, historical forecast errors from the ECMOD/NECMOD model.
- Fan charts reflect changes in uncertainty between forecasting rounds.
- Fan chart is constructed under the assumption of exogenous monetary policy.
- Revisions of variables (national accounts) are accounted for.
- Method is flexible and can be modified/extended easily.

Disadvantages:

- Changes in "endogenous" uncertainty are not accounted for in the method (e.g. a rise in uncertainty of investment developments).
- Consequences of improvement/deterioration of the model/forecasts and expert adjustments are not accounted for.