A NON-PARAMETRIC MODEL-BASED APPROACH TO RISK ANALYSIS OF MACROECONOMIC FORECASTS

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OUTLINE OF PRESENTATION

- Motivation & This paper
- Key idea and ingredients of model-based judgmentally-adjusted risk analysis
- Short review of the literature (Bank of England, Sveriges Riksbank, Banco de Portugal, Bundesbank)
- A simulation-based non-parametric approach (with discrete and continuous skewing functions)
    - Focus on the skewing mechanism
- An example: Risks surrounding the January 2009 Banca d'Italia's projections
- Remarks on empirical results

MOTIVATION

- Growing tendency, at least in the CB community, to provide to the public not only an assessment of the central projection, but also an appraisal of the main risks surrounding that projection: their source, their direction, their intensity

- Recent crisis: obviously, no one could predict exactly when and where the crisis would have started; but the main risks to the world economy which played a role in the genesis and the unfolding of the crisis had long been identified and played a prominent role in CBs' risk analyses

- Those analyses went unheard (or at any rate have been forgotten). Lack of effective means of communication?

- Effective, catchy, easy way to communicate risk assessment? → Variants of the Bank of England's "fan charts" (BoE; mid-1990s)

THIS PAPER

- Various variants of BoE's approach available, overcoming limitations of original proposal
- All based on combining judgmental assessment on risks with the empirical properties of the model underlying the forecasts
- Most approaches (BoE, Sveriges Riksbank, Banco de Portugal) are parametric, with few exceptions (Bundesbank)
- This paper: Drawing on previous work by Altissimo and Siviero (1997), we put forward a viable approach to model-simulation-based non-parametric construction of fan charts
- Empirical results offer a number of interesting insights (e.g., on why parametric methods would be inappropriate)
- Eventually, the approach might be adopted to appraise uncertainty on the occasion of the publication of the forecasts (for the time being: symmetric fan charts, July 2009 Economic Bulletin)

KEY IDEAS AND INGREDIENTS, 1/4

Original idea was developed at the Boe in the mid 1990s, with the following basic ingredients (retained in all subsequent refinements):

- **Aim**: communicate uncertainty and risks surrounding a given point forecast by means of the distribution of possible outcomes (rather than, say, array of alternative scenarios)
- **Inputs**: (i) judgmental assessment about the prevailing direction of risks (and possibly their relative intensity *vis-à-vis* the past) ***and*** (ii) the model structure as well as measures of past forecast performance
- **Outputs**: density distribution for GDP and inflation ("***fan charts***")

Combining both inputs is arguably the key idea behind the success of the Boe'S approach: (i) without (ii) would look like arbitrary, *ad-hoc* appraisal of risks; (ii) without (i) would look mechanical and largely uninteresting

Remark - What is left out of Inputs: Model uncertainty; Parameter uncertainty; Uncertainty about initial conditions

KEY IDEAS AND INGREDIENTS, 2/4

Let us consider a generic (linear) dynamic macroeconometric model:

$$\Gamma y_t = B y_{t-1} + A x_t + e_t$$

The forecast error in period *t* may be written as:

$$y_t - \hat{y}_t = \Pi_1 (x_t - x_t^*) + \Pi_0 \Pi_1 (x_{t-1} - x_{t-1}^*) + \ldots + \Pi_0^{t-\tau-1} \Pi_1 (x_{\tau+1} - x_{\tau+1}^*)$$
$$+ \varepsilon_t + \Pi_0 \varepsilon_{t-1} + \ldots + \Pi_0^{t-\tau-1} \varepsilon_{\tau+1}$$

(where the reduced-form coefficients are defined in the usual way)

KEY IDEAS AND INGREDIENTS, 3/4

Hence, the forecast error (for the whole vector of endogenous variables) combines:

- Impact of contemporaneous projection errors for exogenous variables
- Impact of previous projection errors for exogenous variables
- Impact of contemporaneous “genuine” forecast errors
- Impact of previous “genuine” forecast errors

(NB: dynamic dimension often overlooked)

➢ Impose distribution on projection errors for exogenous variables and on “genuine” forecast errors (from now on: “inputs”) and obtain distribution for endogenous variables of interest (from now on: “outputs”); distribution is shaped by subjective assessment of risks

KEY IDEAS AND INGREDIENTS, 4/4

Other common features:

- Only a subset of inputs (taking on board the impact of all inputs to standard macroeconometric models would be prohibitive)
- Re-scaling thus needed to impose that the size of uncertainty delivered by those inputs be in line with typical uncertainty of the forecasts
- Possibility to adjust not only direction but also intensity of risks (by tampering with their variance)
- Interpret baseline point forecast as *mode* of the distribution of output and assume that result of risk analysis coincides with *mean* of the distribution
- In almost all cases, assumed distribution for inputs is parametric

PREVIOUS LITERATURE, 1/2

- Bank of England (Britton, Cunningham and Whitley (1997); Britton, Fisher and Whitley (1998)):
  - assume two-piece normal (*tpn*) for all inputs
  - assume that resulting outputs are also *tpn*
- Sveriges Riksbank (Blix and Sellin (1998) and (2000):
  - similar to BoE's
- Banco de Portugal (Novo and Pinheiro (2003); Pinheiro and Esteves (2009)):
  - starting point: *tpn* is not a closed family of functions wrt linearity
  - skewed generalised normal (*sgn*) is closed (under some assumptions; limits to the degree of skewness: everything does not go)
  - joint distribution in latest version

PREVIOUS LITERATURE, 2/2

Non parametric approaches (less restrictive):

- Banca d'Italia (Altissimo and Siviero (1997)):
  - skewed binomial distribution used to draw randomly from inputs projection errors

- Deutsche Bundesbank (Knüppel and Tödter (2006)):
  - wide variety of approaches

Not only are non parametric approaches less restrictive; they also offer interesting insights (see empirical results)

THE APPROACH, 1/5: KEY FEATURES

- **Non parametric**: the forecast distribution is generated by stochastic simulations of the Banca d'Italia's Quarterly Macroeconometric Model (BIQM)
- **Bootstrapping procedure**: extract draws (10,000 found to be enough) from historical projection errors of inputs (about 15 of them considered), changing skewness and volatility so as to take subjective appraisal of risk on board
- **Dynamics**: fully retained (visible impact on results)
- **Representation**: from 2nd to 9th decile of distribution (unlike original fan charts)

THE APPROACH, 2/5: SOURCES OF RISK

Included:

- Assumed developments in exogenous variables
  - World demand, competitors' prices, exchange rates
- Error terms of behavioural equations
  - Stochastic components of main GDP sub-aggregates, main equations in wage/price block

Excluded:

- Model uncertainty
- Parameter uncertainty
- Uncertainty about initial conditions

THE APPROACH, 3/5: ASSESSMENT OF RISK

- Assessments collected on direction (numerical assessment of probability of an outcome below the one assumed in the baseline, for each input) and size (numerical assessment on the perceived degree of uncertainty relative to average historical uncertainty) of risk (in BoE: MPC)

- Risks are not independent $\Rightarrow$ projection errors for inputs are first orthogonalized; correlation very limited in history (interpretation/ communication issue?)

THE APPROACH, 4/5: SKEWING MECHANISM

Problem: how to extract draws consistently with subjective beliefs?

- Let $f(z)$ be the empirical distribution of a specific source of risk

- Following Ferreira and Steel (2006), we use a probability density function $p(u)$, with $0 \leq u \leq 1$, as a weight function to define a skewed probability density function $s(z)$ corresponding to the original symmetric one $f(z)$:

$$s(z) = f(z)\,p(F(z))$$

More details below

THE APPROACH, 5/5: PRODUCING THE OUTPUT

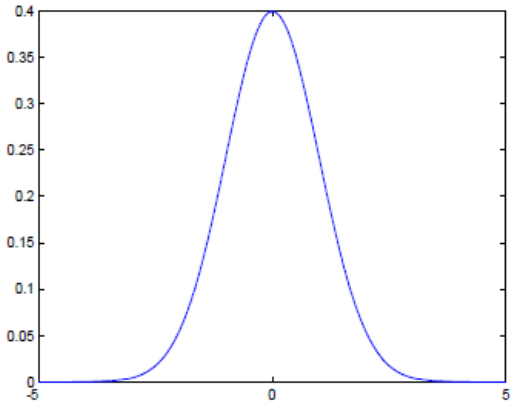
- Extract 10,000 draws for all sources of risk
- Simulate the model 10,000 times (simulation code is fully vectorized $\rightarrow$ high efficiency; 10,000 simulations of BIQM's 700 equations require about 5 minutes)
- Rescaling (as in other approaches; details here)
- Compute distribution (deciles) and moments of all endogenous variables

FOCUS ON THE SKEWING MECHANISM, 1/2

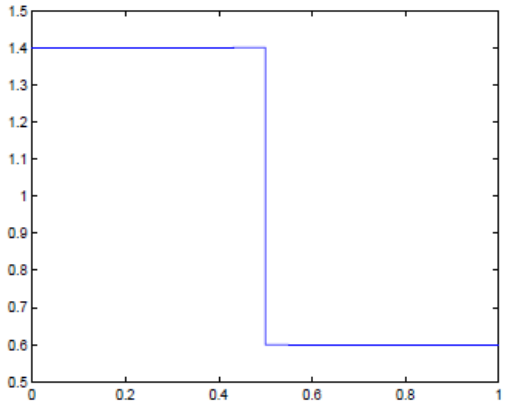
With discrete skewing mechanism

$$s(z) = f(z)\,p(F(z))$$

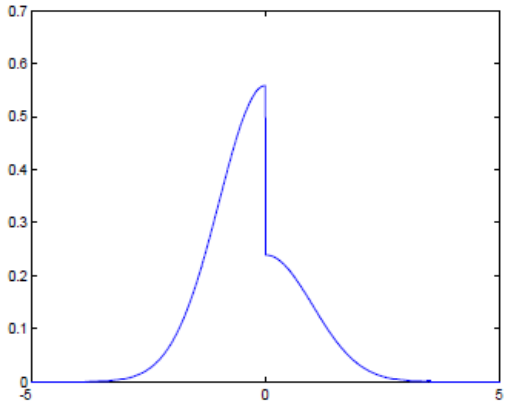
Binomial *p*, 70% probability of outcome below zero:



original *f*



skewing *p*



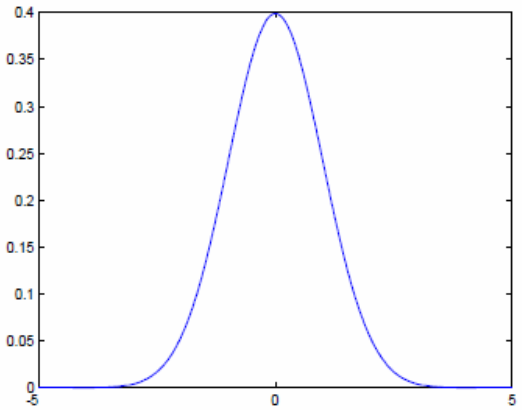
resulting *s*

FOCUS ON THE SKEWING MECHANISM, 1/2

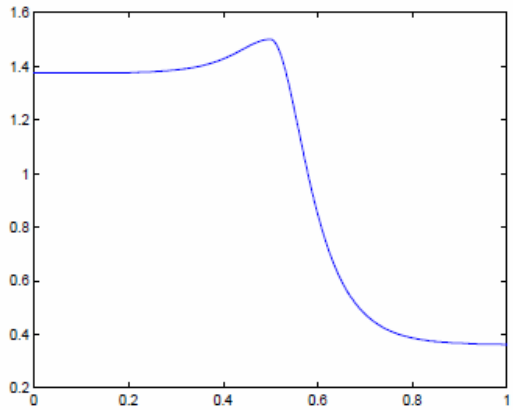
With continuous skewing mechanism

$$s(z) = f(z)\,p(F(z))$$

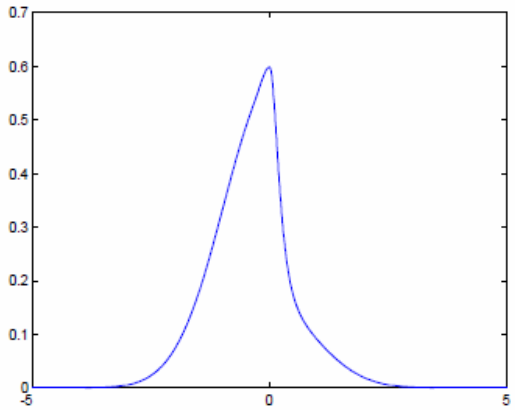
Smooth *p* (hyperbolic family), 70% probability of outcome below zero:



original *f*



skewing *p*



resulting *s*

This is the one we use

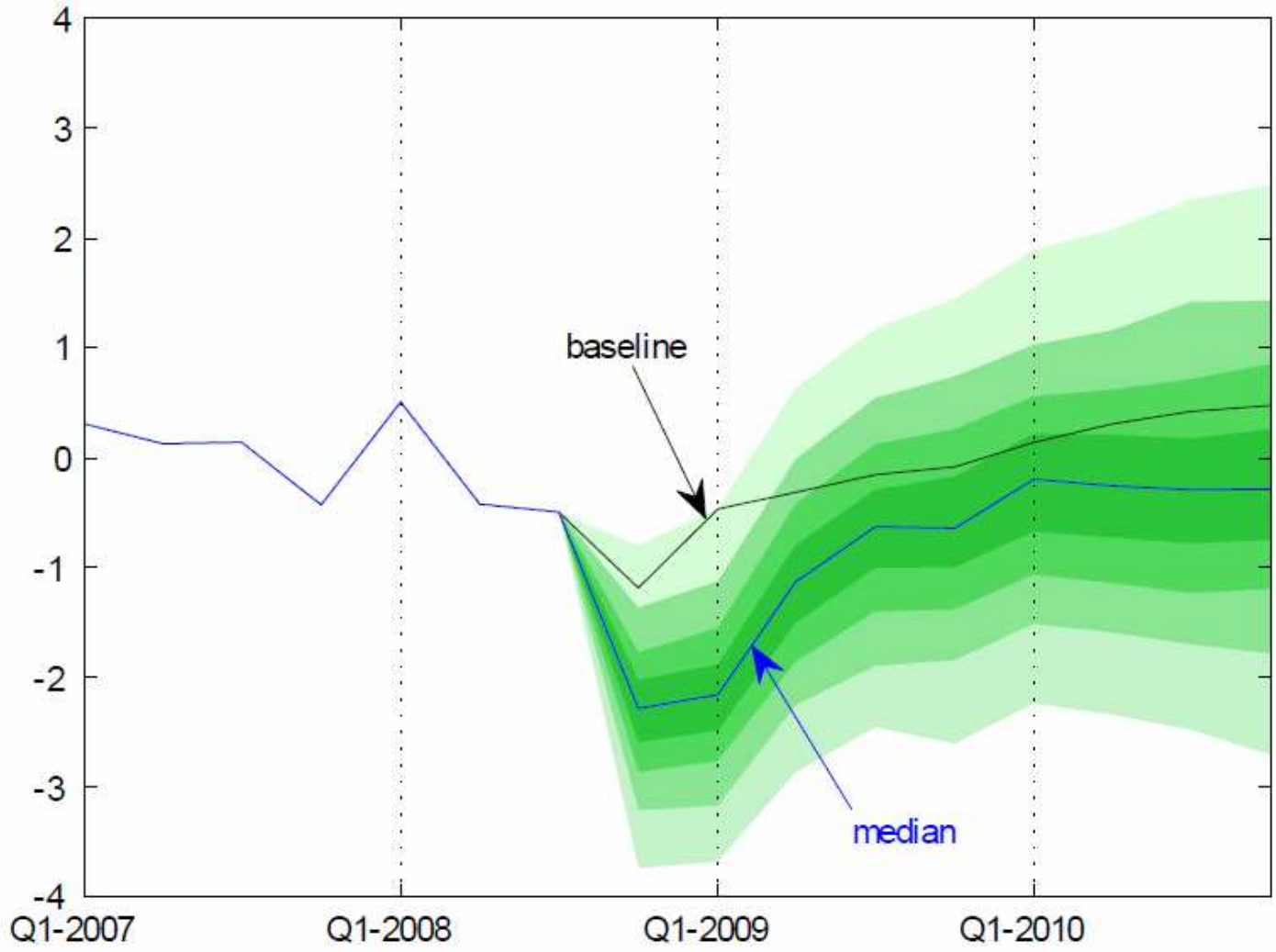
AN EXAMPLE, 1/4

Inputs:

	2008 4 2009 1		2009 2 2009 3		2009 4 2010 4	
Variable	Uncertainty adjustment factor	Pr(shock<0)	Uncertainty adjustment factor	Pr(shock<0)	Uncertainty adjustment factor	Pr(shock<0)
Domanda mondiale	10.0	0.90	1.0	0.70	1.0	0.80
Prezzi manufatti esteri (pesi export)	1.0	0.80	1.0	0.10	1.0	0.30
Prezzi manufatti esteri (pesi import)	1.0	0.60	1.0	0.20	1.0	0.40
Tasso di cambio dollaro-euro	1.0	0.80	1.0	0.20	1.0	0.30
Prezzi delle materie prime	1.0	0.50	1.0	0.50	1.0	0.30
Prezzi dei beni intermedi agricoli	1.0	0.50	1.0	0.40	1.0	0.50
Brent	1.0	0.55	1.0	0.50	1.0	0.57
Consumo economico	2.0	0.70	1.0	0.40	1.0	0.30
Consumi durevoli	2.0	0.80	1.0	0.20	1.0	0.70
Investimenti in macchinari	3.0	0.80	1.0	0.40	1.0	0.40
Importazioni beni non-farm non energy	1.0	0.50	1.0	0.60	1.0	0.50
Esportazioni beni non-farm non energy	5.0	0.90	1.0	0.50	1.0	0.65
Occupazione settore privato	1.0	0.90	1.0	0.50	1.0	0.60
Deflatore valore aggiunto settore privato	1.0	0.50	1.0	0.35	1.0	0.35
Deflatore dei consumi dei beni non durevoli	1.0	0.50	1.0	0.40	1.0	0.40
Aspettative d'inflazione	1.0	0.80	1.0	0.30	1.0	0.50
Retribuzioni settore privato	1.0	0.70	1.0	0.30	1.0	0.30

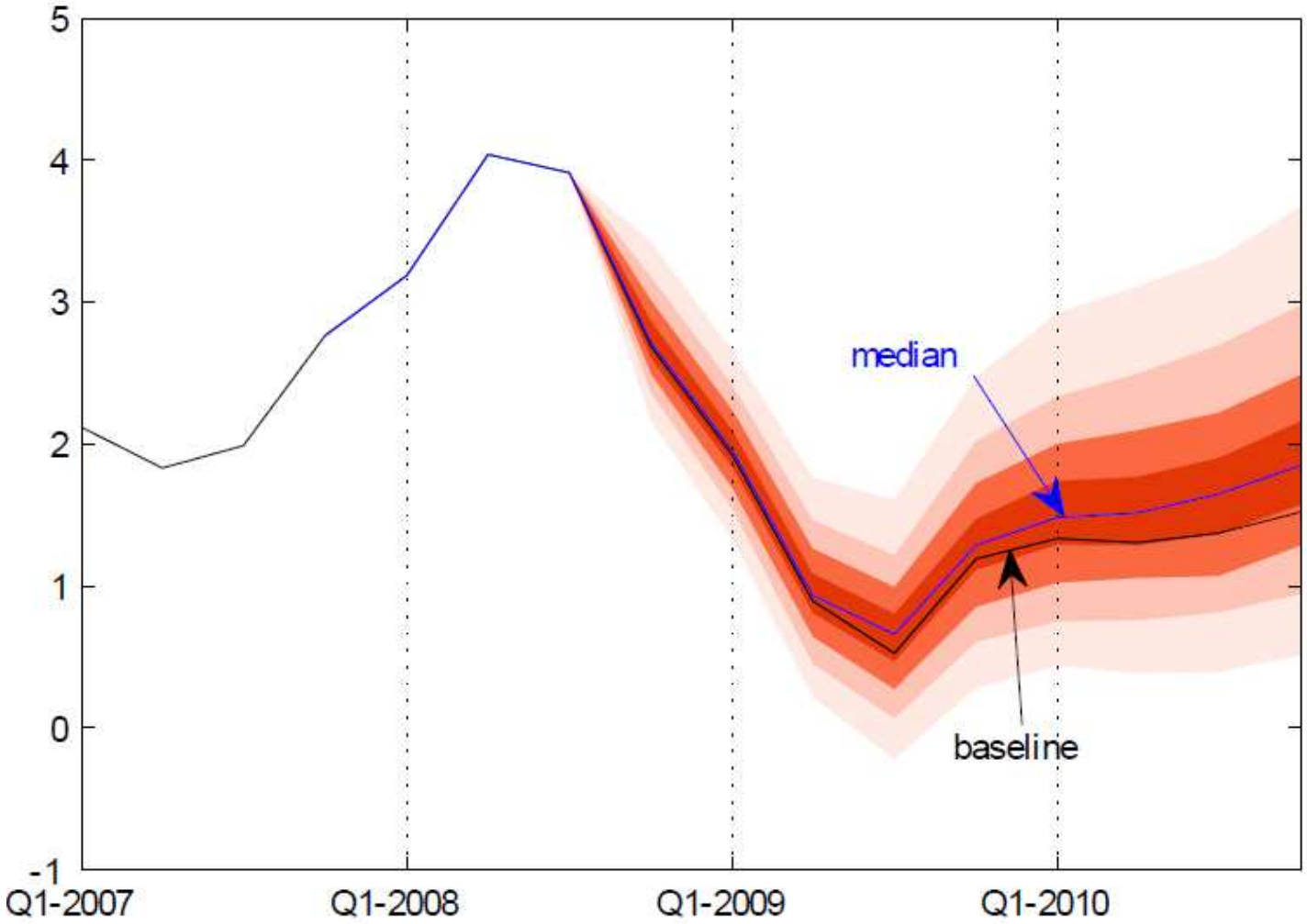
AN EXAMPLE, 2/4

Output for GDP growth rate



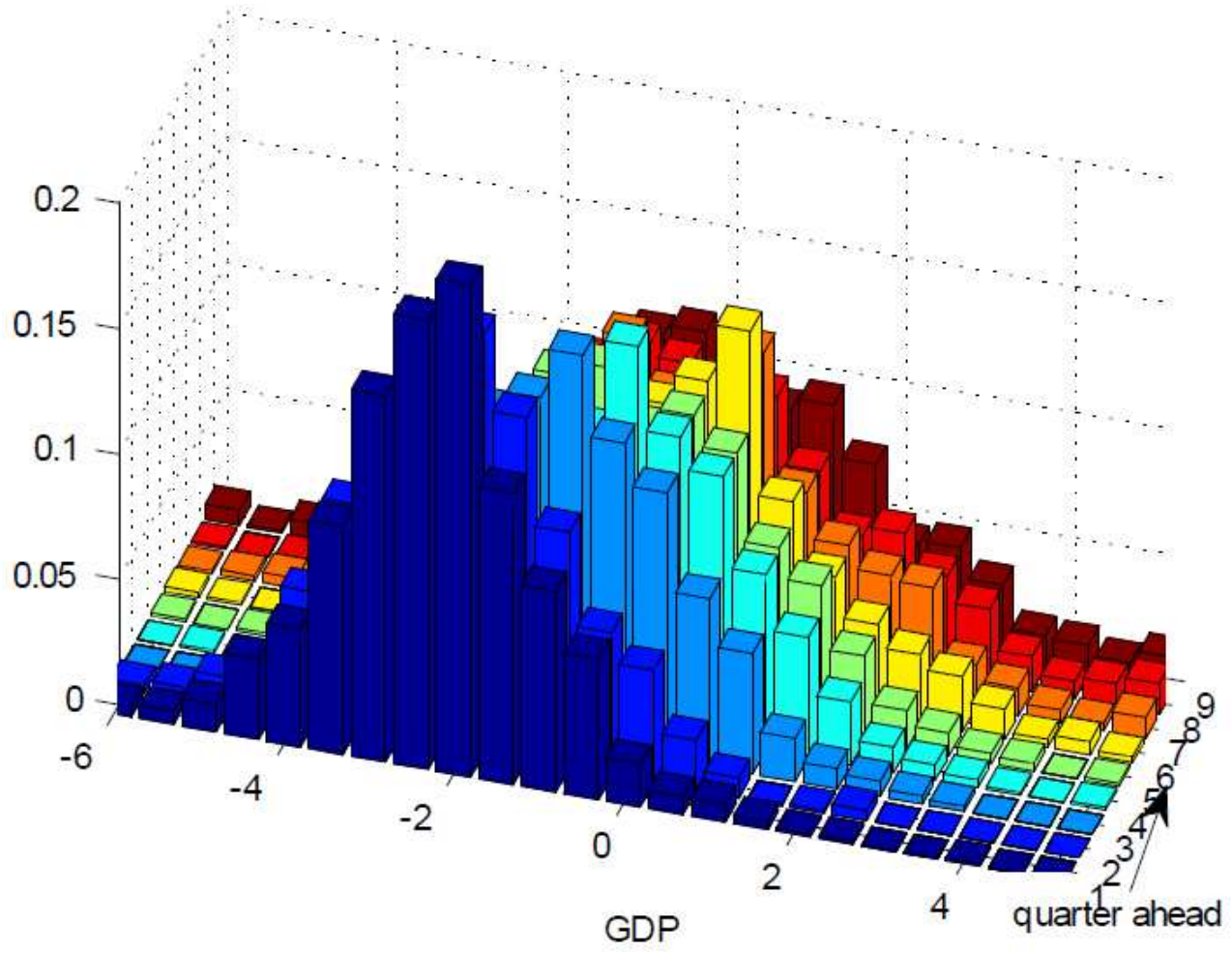
AN EXAMPLE, 3/4

Output for inflation



AN EXAMPLE, 4/4

Non trifling asymmetry in early periods vanishes as the forecast horizon lengthens



REMARKS ON EMPIRICAL RESULTS

- Asymmetry vanishes rather quickly
- Even with just one, extremely asymmetric source of risk, asymmetry vanishes quickly
- With parametric approaches, this is not usually the case
- Whatever asymmetry one assumes, it tends to spread rather evenly at the far end of the forecast horizon (IFT)

SUMMING UP

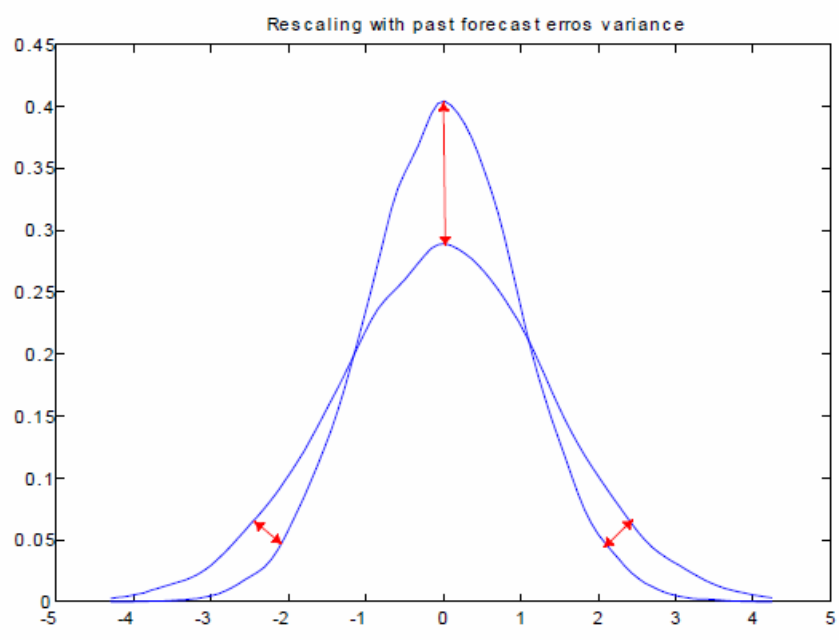
- Appraisal of risk no longer an optional feature: effective means to communicate risks is a “must have”
- BoE (mid-1990s) introduced a way to produce risk assessments which are **both** (i) model-consistent **and** (ii) judgmentally adjusted
- A number of subsequent refinements (Sveriges Riksbank, Banco de Portugal)
- This paper: propose a non-parametric, stochastic-simulation-based approach (similar to Altissimo and Siviero (1997) and Knüppel and Tödter (2006)), with continuous-function skewing mechanism
- In the empirical applications, asymmetry of outputs tends to vanish rather quickly, whatever degree of asymmetry one assumes for inputs

Thank you for your attention

Extras

RESCALING

- Relatively large, but still limited, number of risk factors (some factors are ignored altogether)
- Resulting degree of risk may not match historical uncertainty
- ➢ Rescaling factor (computed once and for all for symmetric uncertainty)



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