

Fiscal Risk in a Monetary Union*

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Abstract

We present a dynamic and quantitative model of a fiscal solvency crisis in a monetary union. Diverse fiscal policies, which are subject to fiscal limits and stochastic shocks, can threaten a monetary union. The fiscal limits arise due to distortionary taxation and political will. Stochastic shocks are random and could push a fiscally sound policy towards its limit. In equilibrium agents refuse to lend along a path which violates the fiscal limits, creating a fiscal solvency crisis. The dynamics leading to the crisis depend on the policy response to restore lending. We focus on two responses, default and policy switching. We simulate our model to quantify the probability of a fiscal solvency crisis in the European Monetary Union with fiscal variables at end of 2009 values. Our model predicts the Greek crisis which occurred and warns of an Italian one.

Key Words: European Monetary Union, Fiscal Theory of the Price Level, Policy Switching, Default, Financial Crisis

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1 Introduction

The Greek debt-swap in March 2012 and the turmoil in government bond markets for other Euro zone countries has highlighted the importance of fiscal risk in a monetary union. Monetary union eliminates a government's ability to unilaterally reduce government debt with inflation, raising the possibility of fiscal insolvency. Sims (1997) argues that passive fiscal policy is not sufficient to eliminate risk since no country can unconditionally commit to raise the expected present value of government surpluses to equal debt obligations. We develop a dynamic and quantitative model in which stochastic shocks can send a government with passive fiscal policy to a position into which agents refuse to lend, thereby creating a fiscal solvency crisis. The government must respond to the crisis, and we consider two responses: default, which we define as a reduction in the value of outstanding government debt, and policy-switching, whereby the fiscal authority switches to active policy and the monetary to passive. We use this model to understand the crises in Euro zone countries which began in 2009.

Government debt has risks due to the interaction of stochastic shocks and fiscal limits. Stochastic fiscal shocks are random, created by natural disaster, political change, or government response to economic events as with the 2008 financial crisis. Fiscal limits are endogenous to a country's economic and political system and have no relation to the limits in the Stability and Growth Pact (SGP). We model these internal fiscal limits as an upper bound on the present value of future primary surpluses relative to GDP that a government can generate. Under the assumption that no country is willing to accumulate

an unlimited quantity of another's government debt, country-by-country intertemporal budget constraints must hold.¹ The fiscal limit on the present value of future primary surpluses, combined with the intertemporal government budget constraint, implies a limit on each country's debt/GDP.

World-wide increases in government debt/GDP ratios have sparked interest in a new literature on fiscal limits. Davig, Leeper, and Walker (2010, 2011), Davig and Leeper (2011), and Cochrane (2011) have explored the implications of fiscal limits for inflation. Bi, Leeper and Leith (2010) examine the tradeoffs between sustainability and stabilization near fiscal limits. Bi (2011) considers the effects of fiscal limits on risk premia. Sims (1997) argues that stochastic fiscal shocks can lead to a violation of fiscal limits. Daniel (2010) explores how fiscal limits can interact with stochastic fiscal policy to create an exchange rate crisis. Our paper considers the implications of fiscal limits for a fiscal solvency crisis in a monetary union.

A fiscal solvency crisis with a policy response of default yields an alternative model of sovereign default, where default is due to fiscal limits, and not to the willingness to repay as in Eaton and Gersovitz (1981), Eaton and Fernandez (1995), and Arellano (2008). Bi, Leeper, and Leith (2010) model default as a policy to reduce debt once it reaches a fiscal limit, and Schabert (2010) models the joint effects of inflation and default on debt.² In our model, default is the policy response to a crisis in which agents refuse to lend and the crisis occurs before debt reaches its fiscal limit. The magnitude of capital loss on government debt is determined endogenously to restore fiscal solvency and is never

¹ Sims (1997), Woodford (1997), Daniel (2001), and Bergin (2000) argue that no country, acting to maximize utility of its own agents, would commit to a policy of permanently taxing its own agents to purchase foreign bonds necessary to meet expenses of other countries.

² Uribe (2006) shows that a surprise default can be used to end a hyperinflation.

hundred percent, consistent with actual experience. The Greek debt-swap in March 2012 satisfies our definition of default. When the policy response to a fiscal solvency crisis is default, the crisis has no effect on the common price level in the monetary union.³ However, we show that after default, markets remain turbulent with high interest rates and additional defaults.

When the policy response to the fiscal solvency crisis is switching, the post-crisis equilibrium is characterized by the Fiscal Theory of the Price Level (FTPL). Pressure on the European Central Bank (ECB) to buy sovereign debt in the later part of 2011 could indicate pressure to resolve the crisis with policy switching. Leeper (1991) demonstrates that the monetary authority maintains control of the price level when fiscal policy is passive and monetary policy is active, but loses control with the FTPL policy combination of active fiscal policy and passive monetary policy. Bergin (2000) considers an FTPL equilibrium in a monetary union. He finds that active fiscal policy in a single country creates price instability for the entire monetary union, not just for the crisis country.⁴ Sims (1997) presents a dynamic model in which the probability of policy switching is stochastic and increasing in government debt. Davig and Leeper (2011) and Davig, Leeper, and Walker (2010, 2011) present dynamic policy-switching models in which switching occurs either exogenously, or becomes more likely once some variable crosses a threshold.

We draw on models developed by these authors. The policy-switching model presented here can be viewed as modeling the dynamic scenario which could characterize the transit of a monetary union from an equilibrium, in which the monetary authority initially has

³ Sims (1997, 1999) notes that inflation in a monetary union can be avoided if the crisis government could commit to default.

⁴ Woodford (1997), Leith and Wren-Lewis (2006), and Cooper, Kempf, and Peled (2010) also discuss the implications of fiscal policy for price instability in a monetary union.

full control over prices, to Bergin's (2000) FTPL equilibrium, in which it loses control. Or alternatively, it modifies the dynamic models of Sims (1997), Davig and Leeper (2011), and Davig, Leeper, and Walker (2010, 2011) to allow policy switching to be an endogenous response to a crisis in which agents refuse to lend. In our model, policy-switching is the reaction to a crisis whose timing is market-determined, as in a generation-one currency crisis model (Krugman 1979, Flood and Garber 1984). The sudden stop in lending by risk-neutral agents replaces the speculative attack of the generation-one model.⁵ We show that crises with market-determined timing develop suddenly and occur before government debt actually reaches its fiscal limit.

Additionally, we show that monetary policy can be designed to retain price level stability when there is a positive probability of policy-switching or default. Davig and Leeper (2011) and Davig, Leeper, and Walker (2010, 2011) demonstrate that the monetary authority can lose control of the price level when there is risk of policy-switching. Bi, Leeper, and Leith (2010) obtain the same result when there is risk of default.

The final contribution of our paper is quantitative. We simulate our model to estimate the probability of a crisis in the EMU using the 2009 values for government debt and the primary surplus. Other papers provide estimates of fiscal risk, based on VAR models of debt, but this risk is that of debt relative to GDP reaching an upper bound (Garcia and Rigobon 2005), or beginning to grow (Tanner and Samake 2008) over a particular horizon. Neither of these events need cause a crisis, and both measures miss the non-linear acceleration of debt in the neighborhood of a crisis due to expectations. Based on 2009 values, our model implies a positive probability of a crisis for Greece under both policy

⁵ Daniel (2010) presents a switching model with market determined crisis timing.

responses to the crisis and indicates some risk for other countries, particularly Italy.

This paper is organized as follows. The second section contains a simple open-economy macroeconomic model. The third section considers dynamics leading to a crisis under alternative responses to the crisis. The fourth section contains simulations of fiscal risk, and the fifth concludes.

2 Model

2.1 Overview

In this section, we set up a simple model of a monetary union, which we use to address fiscal solvency risk. We extend the model in Daniel (2010) to allow default, a monetary union, and a more general fiscal rule. In the initial equilibrium, monetary policy is active and fiscal policy is passive. The model contains four key assumptions. First, international creditors lend to a country only when they expect to receive the market rate of return. Second, government budgets are subject to fiscal limits. Third, fiscal policy follows a rule which is subject to stochastic shocks. Together the fiscal limit and stochastic shocks imply risk on government debt, reflecting the concern by the EMU founders and the reality that a government's commitment to raise taxes to finance expenditures cannot be totally unconditional. Fourth, we assume that a solvent sovereign always repays.

2.2 Credit Markets

We assume that the monetary union consists of N countries. The $j = 1, 2, \dots, N$ countries are small enough that they cannot affect the world price level or world interest rate. There is a single good in the world, implying that equilibrium in goods markets requires the law

of one price. Normalizing the world price level at unity and assuming no world inflation implies that the equilibrium price level in the monetary union is the exchange rate.

We assume that member countries issue bonds denominated in the common currency. The **first key assumption** is that international creditors are willing to buy and sell country j 's government bonds as long as its interest rate, i_{jt} , satisfies interest rate parity. Interest rate parity is implied by the Euler equations for a representative world agent when the covariance of the country- j interest rate with world-agent consumption is zero, or when the world agent is risk neutral. Under the additional assumptions that the world interest rate (i) is constant, interest rate parity can be expressed as

$$\frac{1}{1 + i_{jt}} = \left(\frac{1}{1 + i} \right) E_t \left[\frac{P_t}{P_{t+1}} \delta_{jt+1} \right], \quad j = 1, 2, \dots, N, \quad (1)$$

where E_t denotes the expectation conditional on time t information, P_t denotes the common price level in the monetary union, and δ_{jt+1} is the fraction of the value of the j 'th country's bond that will be repaid in period $t + 1$. For agents to be willing to lend to country j , its interest rate must be allowed to rise above the world interest rate when there is some possibility of a crisis which will be resolved with either default ($E_t \delta_{jt+1} < 1$) or inflation ($E_t \frac{P_t}{P_{t+1}} < 1$).

2.3 Monetary Policy

We assume that initially the common monetary authority follows an active policy, targeting inflation with a Taylor rule. Davig, Leeper, and Walker (2010, 2011) and Bi, Leeper, and Leith (2010) demonstrate that the monetary authority can lose the ability to control the price level when it sets the interest rate on an asset with risk of inflation or default. Since the monetary authority can choose which interest rate to set, we assume that it

chooses the interest rate that gives it the best chance of controlling prices in the event of a crisis. We assume that it sets an interest rate, i_t , which is indexed for expectations of inflation or default, and is given by

$$(1 + i_t) = (1 + i_{jt}) E_t \left[\frac{P_t}{P_{t+1}} \delta_{jt+1} \right].$$

When country j experiences a positive probability of inflation ($E_t \frac{P_t}{P_{t+1}} < 1$) or default ($E_t \delta_{jt+1} < 1$), the monetary authority makes no changes in the interest rate it sets, i_t , allowing country's j interest rate, i_{jt} , to rise and accommodate expectations of inflation or default.⁶ When there is no possibility of inflation, this assumption is equivalent to having the monetary authority set interest rates on bonds which are free of default risk. When there is no possibility of default, this policy requires that the monetary authority set the interest rate on an asset indexed for expected inflation.

However, a policy of fixing i_t would leave the current price level indeterminate, since a fixed value for i_t , together with equation (1), is consistent with any value for the current price level, P_t . Therefore, we assume that the monetary authority sets this interest rate according to a Taylor Rule. We follow Davig and Leeper (2011) and write a Taylor Rule in the inverse of the interest rate, yielding

$$\frac{1}{1 + i_t} = \frac{1}{1 + i} + \kappa \left(\frac{P_{t-1}}{P_t} - 1 \right) \quad \kappa > 0, \quad (2)$$

where we set the inflation target to zero. Combining equations (1) and (2) demonstrates that the monetary authority keeps prices fixed as long as it follows this Taylor Rule and fiscal policy remains passive. The monetary authority rules out sunspot equilibria by promising to react to them in way which eliminates them.⁷

⁶ We demonstrate uniqueness of $E_t \left[\frac{P_t}{P_{t+1}} \delta_{jt+1} \right]$, with one exception, when we solve the model.

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2.4 Fiscal Policy

2.4.1 Government Flow Budget Constraint

Assuming that a fraction, η_j , of the monetary union's money supply, M_t , is supported by country j 's domestic bonds, a member country's nominal flow government budget constraint is given by

$$B_{jt} + \eta_j M_t = \delta_{jt} [(1 + i_{jt-1}) B_{jt-1} + \eta_j M_{t-1}] + G_{jt} - \tau_{jt} P_t Y_{jt},$$

where B_{jt} is nominal government bonds held by the public, G_{jt} is nominal government expenditures, Y_{jt} is real output and τ_{jt} is the tax rate on nominal output of country j .⁸ Letting small letters denote values relative to output, the values of debt relative to output and the primary surplus relative to output for country j can be expressed respectively as

$$b_{jt} = \frac{1}{P_t Y_{jt}} \left(B_{jt} + \frac{1}{1 + i_{jt}} \eta_j M_t \right),$$

$$s_{jt} = \frac{1}{P_t Y_{jt}} \left(\tau_{jt} P_t Y_{jt} - G_{jt} + \left(\frac{i_{jt}}{1 + i_{jt}} \right) \eta_j M_t \right).$$

The government's flow budget constraint can be expressed in terms of debt and the primary surplus relative to output as

$$b_{jt} = \left(\frac{\delta_{jt}}{1 + \pi_t} \right) \left(\frac{1 + i_{jt-1}}{1 + g_j} \right) b_{jt-1} - s_{jt}, \quad (3)$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation rate, and $g_j = \frac{Y_{jt}}{Y_{j,t-1}} - 1$ is the non-stochastic output growth rate for the j 'th country.⁹

If P_t rises, the monetary authority does not accommodate by increasing the money supply. Instead, it promises to let i_t rise according to the Taylor Rule. However, an increase in the interest rate is incompatible with interest rate parity. Therefore, the price level returns to its initial equilibrium value, ruling out sunspots.

⁸ This specification assumes that a default on government bonds would be applied equally to all independent of whether the bonds are held by the monetary authority or by the public.

⁹ We assume growth is non-stochastic to simplify the analysis. We could analyze the implications of stochastic growth using a linearized model, but we reserve this for future work.

To solve the model, we express current debt as a linear function of lagged debt and the current primary surplus by isolating the term containing the capital loss on government debt.¹⁰ Define γ_{jt} as capital loss on debt due to inflation or default as

$$\gamma_{jt} = \left(1 - \frac{\delta_{jt}}{1 + \pi_t}\right) \left(\frac{1 + i_{jt-1}}{1 + g_j}\right) b_{jt-1}.$$

If there is no inflation ($\pi_t = 0$) and no default ($\delta_{jt} = 1$), then there is no capital loss ($\gamma_{jt} = 0$). But if $\pi_t > 0$ or $\delta_{jt} < 1$, then $\gamma_{jt} > 0$. Unanticipated capital loss can be expressed as

$$\gamma_{jt} - E_{t-1}\gamma_{jt} = -\left(\frac{\delta_{jt}}{1 + \pi_t}\right) \left(\frac{1 + i_{jt-1}}{1 + g_j}\right) b_{jt-1} + E_{t-1} \left(\frac{\delta_{jt}}{1 + \pi_t}\right) \left(\frac{1 + i_{jt-1}}{1 + g_j}\right) b_{jt-1}. \quad (4)$$

Note that interest rate parity, equation (1), implies

$$(1 + i) = (1 + i_{jt-1}) E_{t-1} \left(\frac{\delta_{jt}}{1 + \pi_t}\right). \quad (5)$$

Multiplying both sides of equation (5) by $\frac{b_{jt-1}}{1 + g_j}$ and recognizing that $\frac{b_{jt-1}}{1 + g_j}$ is known at time $t - 1$ yields

$$\left(\frac{1 + i}{1 + g_j}\right) b_{jt-1} = E_{t-1} \left(\frac{1 + i_{jt-1}}{1 + g_j}\right) \left(\frac{\delta_{jt}}{1 + \pi_t}\right) b_{jt-1}. \quad (6)$$

Subtracting equation (6) from equation (4) yields

$$\gamma_{jt} - E_{t-1}\gamma_{jt} - \left(\frac{1 + i}{1 + g_j}\right) b_{jt-1} = -\left(\frac{\delta_{jt}}{1 + \pi_t}\right) \left(\frac{1 + i_{jt-1}}{1 + g_j}\right) b_{jt-1}.$$

Using this expression to substitute into equation (3), the evolution of debt relative to output can be expressed as

$$b_{jt} = (1 + r_j) b_{jt-1} - s_{jt} - (\gamma_{jt} - E_{t-1}\gamma_{jt}). \quad (7)$$

¹⁰This is not a linearization, but a way to rearrange the model so that it is linear in a specific way.

where $1 + r_j = \left(\frac{1+i}{1+g_j}\right)$ is the gross growth-adjusted interest rate and $(\gamma_{jt} - E_{t-1}\gamma_{jt})$ is the unanticipated capital loss on government debt. Under our assumption of non-stochastic output growth, r_j is time invariant, yielding a linear equation for debt. Expectations of capital loss $(E_{t-1}\gamma_{jt})$ raise the interest rate, and when the capital loss does not occur, debt accumulates in response to the higher interest rate.

In a closed economy, optimization by the representative agent yields the government's intertemporal budget constraint as a transversality condition (Woodford 1994). However, in a world with open economies, agent transversality conditions require satisfaction of a government budget constraint aggregated over all countries in the world.¹¹ Bergin (2000) uses this implication to argue that in a monetary union, equilibrium requires satisfaction of the aggregate monetary union budget constraint, not individual country budget constraints. It is possible to have an equilibrium in which one country's debt grows at a rate faster than the interest rate forever, as long as the aggregate of the remaining monetary union debt shrinks fast enough such that the transversality condition holds for aggregate monetary-union debt.

Sims (1997), Woodford (1997), Bergin (2000), and Daniel (2001), argue that no country, acting to maximize utility of its own agents, would commit to a policy of taxing its own agents to accumulate government debt indefinitely to finance expenses chosen by member governments. Sims (1997) argues that even in a non-stochastic world, in which each government could commit to a fixed present-value surplus, the coordination problems would be enormous. These problems are magnified in a stochastic world. Additionally,

¹¹Dupor (2000) uses this result to argue that price levels and exchange rates are indeterminate in a world with active fiscal policy.

agreements between EMU members explicitly state that member country governments are not responsible for the debts of others. These arguments do not rule out discretionary gifts in response to stochastic shocks or agreements made in advance for specific fiscal transfers among members.¹² However, they rule out unlimited purchases of another's growing debt, Woodford's (1997) "blank check". If no member government will allow its present-value debt/GDP to become negative in the limit, then no member government can have positive present-value debt/GDP in the limit. Under the assumption of no blank checks, the government's intertemporal budget constraint must hold in equilibrium country by country.

Using equation (7), country j 's intertemporal government budget constraint (IBC) is given by

$$\lim_{T \rightarrow \infty} E_t b_{jt+T} \left(\frac{1}{1+r_j} \right)^T = (1+r_j) b_{jt-1} - (\gamma_{jt} - E_{t-1} \gamma_{jt}) - E_t \sum_{k=0}^{\infty} s_{jt+k} \left(\frac{1}{1+r_j} \right)^k = 0. \quad (8)$$

Note that only unanticipated capital loss reduces the value of debt, contributing to government revenue. Anticipated capital loss creates an increase in the interest rate, and when actual loss equals the anticipated loss, there is no addition to government revenue.¹³

¹²A government's decision to raise taxes to provide a discretionary gift to another country's government does not violate its intertemporal budget constraint. The present-discounted value of its government debt remains zero in the limit.

¹³Cochrane (2001, 2011) introduces long-term bonds, allowing the maturity structure of the debt to determine the timing of inflation. Following a shock which requires an equilibrium capital loss on debt, he demonstrates that the necessary capital loss can be achieved with current inflation, which raises the current price level, and/or with future inflation (anticipated following the shock) which reduces the price of long-term bonds. Both the increase in the current price level, due to current inflation, and the decrease in the price of long term bonds, in response to an increase in expected future inflation, create capital loss on outstanding government debt.

2.4.2 Fiscal Limits

The **second key assumption** is that governments face fiscal limits. This assumption follows a growing literature with papers by Bi (2011), Bi, Leeper and Leith (2010), Cochrane (2011), Davig and Leeper (2011), Davig, Leeper and Walker (2010, 2011), and Sims (1997). The fiscal limits are in part due to the top of the Laffer curve. Since taxes are distortionary, there is an upper bound to the tax revenue that can be raised. There is also a limit below which transfers and spending on public goods cannot be reduced.

Following Bi (2011), we assume that there is an upper bound on the expected present value of the future primary surplus relative to output (s_t) that a government can raise. This upper bound is given by

$$E_t \sum_{k=0}^{\infty} s_{jt+k} \left(\frac{1}{1+r_j} \right)^k \leq \frac{(1+r_j)\bar{\varphi}_j}{r_j}, \quad (9)$$

where $\bar{\varphi}_j$ has the interpretation of the value of the primary surplus at the upper bound if the primary surpluses were constant. Together equations (8) and (9) imply an upper bound on debt relative to GDP for each country, given by¹⁴

$$b_{jt} \leq \frac{\bar{\varphi}_j}{r_j}. \quad (10)$$

However, Bi (2011) and Bi, Leeper and Leith (2010) recognize that a government might not have the political will to raise surpluses to their upper bound. Public demonstrations and even riots against austerity programs are evidence of the existence of the difficulties in the political process of attaining these upper bounds. Bi, Leeper and Leith (2010),

¹⁴Substitute the upper bound into the government budget constraint, equation (8), to yield $(1+r_j)b_{jt-1} - (\gamma_{jt} - E_{t-1}\gamma_{jt}) = E_t \sum_{k=0}^{\infty} s_{jt+k} \left(\frac{1}{1+r_j} \right)^k \leq \frac{(1+r_j)\bar{\varphi}_j}{r_j}$. Use the equation for the government's flow budget constraint, equation (7), to write this equation as $b_{jt} + s_{jt} \leq \frac{(1+r_j)\bar{\varphi}_j}{r_j}$. Letting the surplus take on its largest value of $\bar{\varphi}_j$ yields the expression in the text.

model the fiscal limit as a stochastic fraction of the actual upper bound, and motivate this with assumptions of either changing political leaders, with different propensities to raise surpluses, or leaders who will implement the upper bound surplus only with some probability. We model the fiscal limit, $\hat{\varphi}_j$, as a deterministic fraction ($\rho_j < 1$) of its actual upper bound, where that fraction is determined by political will, yielding

$$b_{jt} \leq \frac{\hat{\varphi}_j}{r_j} = \frac{\rho_j \bar{\varphi}_j}{r_j}. \quad (11)$$

Stronger political will, represented by a larger value for ρ_j , implies a higher fiscal limit.

Fiscal limits have implications for fiscal solvency. Solvency requires that debt and surpluses are expected to take on values satisfying the government's intertemporal budget constraint without exceeding the country's fiscal limit on debt.

2.4.3 Surplus Rule

We assume that the fiscal authority is able to anchor beliefs about future fiscal policy by committing to a surplus rule, under which fiscal limits are expected to be satisfied, at least initially. Since the model is specified in terms of debt and the primary surplus relative to output, we refer to these variables simply as debt and the surplus. The surplus rule must yield a long-run equilibrium which respects the fiscal limit. Therefore, the response of the surplus to debt must be large enough to make the system in debt and the surplus globally stable, ruling out an explosive equilibrium.

The **third key assumption** is that the government follows a surplus rule which is subject to bounded, zero-mean, stochastic shocks, ν_{jt} . Stochastic shocks represent both truly unanticipated fiscal shocks, as with a war, natural disaster, political change, or the 2008 financial crisis, as well as fiscal policy responses to shocks to the economy. Together,

fiscal limits and stochastic shocks imply risk to government debt.

We specify a surplus rule, in which the surplus (s_{jt}) responds to its own lag (s_{jt-1}) and a linear combination of the target value for the long-run primary surplus (φ_j) and debt service ($r_j b_{jt-1}$) at the growth-adjusted interest rate.¹⁵ The surplus rule for country j is given by

$$s_{jt} = (1 - \alpha_j) s_{jt-1} + \alpha_j [(1 - \lambda_j) \varphi_j + \lambda_j r_j b_{jt-1}] + \nu_{jt}, \quad (12)$$

$$\frac{r_j}{1 + r_j} < \alpha_j < 1, \quad \lambda_j > 1, \quad 0 < \varphi_j \leq \hat{\varphi}_j \leq \bar{\varphi}_j,$$

where $(1 - \alpha_j)$ measures persistence in the surplus, reflecting the desire to smooth the effect of shocks over time, consistent with empirical evidence. The parameter λ_j determines the responsiveness of the surplus to debt service. The restrictions are imposed to yield a stationary long-run equilibrium which respects the fiscal limit, $\hat{\varphi}_j$.

The parameters α_j , λ_j , and φ_j are policy choices. We show below that the risk of entering regions where the fiscal limits might bind is affected by these parameter choices. Governments understand this risk, and the parameters they choose reflect their risk tolerance, determined in part, by the cost of a crisis. Empirically, countries do choose surplus rules with risk, and the SGP limits on debt and deficits reflect policy-maker concerns that at least some EMU countries might choose relatively risky rules.

Equations (7) and (12) are the dynamic equations for debt and the surplus. Letting θ_j represent eigenvalues, which are assumed to be real and distinct, the characteristic equation for each country is given by

$$(1 - \alpha_j)(1 + r_j) - \theta_j [2 + r_j(1 - \alpha_j \lambda_j) - \alpha_j] + \theta_j^2 = 0. \quad (13)$$

¹⁵We are assuming that the fiscal authority can adjust the primary surplus to offset the effect of capital gains or losses on seigniorage revenue $\left(\frac{i_{jt}}{1+i_{jt}} \frac{\eta_j M_t}{P_t Y_{jt}}\right)$, allowing the surplus to follow the rule.

Both eigenvalues are less than one under the restriction that $\lambda_j > 1$, ensuring that the model given by equations (7) and (12) is globally stable. Given initial values for debt and the surplus, both are expected to reach long-run equilibrium values determined by the target surplus. This fiscal policy is passive since the government's intertemporal budget constraint is satisfied for any initial values of debt and the surplus.

The **fourth key assumption** is that a government, which is able to borrow to follow its surplus rule, equation (12), honors its debt. Assuming that governments honor debt, even when it is not in their strategic interests, follows Bi, Leith, and Leeper (2010) and Schabert (2010), where default is used to restore fiscal solvency.

2.5 Dynamics

Consider the dynamic behavior of debt and the surplus in a monetary union in which each country is committed to passive fiscal policy. Solutions for equations (7) and (12) are given in the appendix. It is useful to represent the dynamics of the debt-surplus system using country phase diagrams. We construct the phase diagram for each country by subtracting the lagged value of the surplus from equation (12) and the lagged value of debt from equation (7) to yield:

$$\Delta s_{jt} = s_{jt} - s_{jt-1} = -\alpha_j s_{jt-1} + \alpha_j (1 - \lambda_j) \varphi_j + \alpha_j \lambda_j r_j b_{jt-1} + \nu_{jt}, \quad (14)$$

$$\Delta b_{jt} = b_{jt} - b_{jt-1} = (1 - \alpha_j \lambda_j) r_j b_{jt-1} - (1 - \alpha_j) s_{jt-1} - \alpha_j (1 - \lambda_j) \varphi_j - \nu_{jt} - \gamma_{jt} + E_{t-1} \gamma_{jt}. \quad (15)$$

The phase diagram under passive fiscal policy, with $\nu_{jt} = \gamma_{jt} = E_{t-1} \gamma_{jt} = 0$, is given in Figure 1. Debt service ($r_j b_j$) is on the vertical axis and the surplus (s_j) is on the horizontal axis. The upper bound on debt service is given by $\bar{\varphi}_j$ on the vertical axis, and

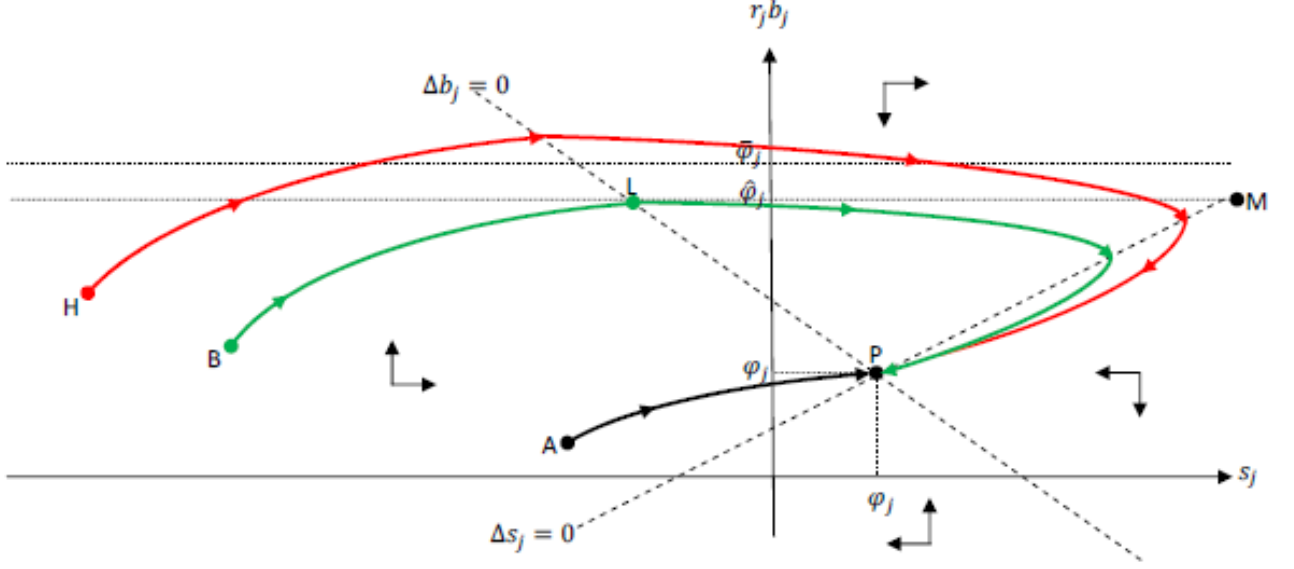


Figure 1: Passive Fiscal Policy

the fiscal limit by $\hat{\varphi}_j$. The $\Delta s_j = 0$ and $\Delta b_j = 0$ schedules intersect at point P with $s_{jt} = \varphi_j = r_j b_{jt}$, representing the long-run equilibrium of the globally stable system. If initial debt and the surplus are at point A, then the system is expected to travel along AP, eventually reaching the long-run equilibrium point P.

Taking time $t - 1$ expectations of equations (14) and (15) with $\gamma_{jt} = E_{t-1}\gamma_{jt} = 0$, yields the slope of a passive adjustment path as

$$\frac{r_j (E_{t-1}b_{jt} - b_{jt-1})}{E_{t-1}s_{jt} - s_{jt-1}} = r_j \left[\frac{r_j b_{jt-1} - s_{jt-1}}{\alpha_j (\lambda_j r_j b_{jt-1} - s_{jt-1} + (1 - \lambda_j) \varphi_j) + E_{t-1}\nu_{jt}} - 1 \right]. \quad (16)$$

Note that in the range for which the slope of the path is positive, a positive expected future fiscal shock ($E_{t-1}\nu_{jt}$) reduces the one-period-ahead slope of the adjustment path, such that debt is expected to attain lower values in its approach to a long-run equilibrium.

Figure 1 is drawn with expected future fiscal shocks equal to zero.

Initial fiscal policy is described by the passive surplus rule. We define current fiscal policy as sustainable if it is expected to meet solvency requirements, satisfying the govern-

ment intertemporal budget constraint without exceeding the fiscal limit in expectations. Therefore, when the debt and the surplus begin at point A, the passive fiscal policy rule is sustainable. Over time, actual fiscal shocks (ν_{jt}) could move the system away from its initial passive adjustment path, labelled AP, possibly to point H along the adjustment path HP. HP is not an equilibrium path because it requires that debt be expected to pass through points where it exceeds its fiscal limit, violating solvency requirements. Restoration of equilibrium requires a policy-response or expectations of a response to restore solvency.¹⁶ We consider two responses, default and policy-switching.

3 Fiscal Solvency Crises

Definition 1 *Equilibrium*: *Given constant values for the world interest rate and world price level, a monetary policy rule (equation 2), a surplus rule (equation 12), a fiscal limit on debt (equation 11), and a policy-response in the event of a fiscal crisis for each country, an equilibrium is a set of time series processes for each country's primary surplus, debt, and capital loss on debt, $\{b_{jt}, s_{jt}, \gamma_{jt}\}_{t=0}^{\infty}$, such that each government's flow and intertemporal budget constraints (equations 7 and 8) hold, expectations are rational, the debt for each country does not exceed its fiscal limit, and world agents expect to receive the return on government bonds determined by interest rate parity (equation 1) and are therefore willing to lend freely.*

A crisis occurs once fiscal shocks have increased debt in one country sufficiently that agents refuse to lend. Lending ends when agents expect that current fiscal policy cannot generate the revenue necessary to compensate for expectations of future capital loss on debt. Restoration of lending, and, equivalently of equilibrium, requires enactment of the planned policy response to restore solvency. We consider two possible responses.¹⁷ In the first, the fiscal authority reduces the value of debt through default and thereafter retains

¹⁶Current policy can be unsustainable, but the government can be solvent due to expectations of a future change in policy. Solvency is an equilibrium requirement. Sustainability is not.

¹⁷The policy response considered by Kumhof (2004) of raising taxes is not likely to be available near the fiscal limit.

its passive surplus rule. The monetary authority maintains its active rule. In the second, the fiscal authority switches from passive policy to active, with the monetary authority switching to a passive policy of interest rate pegging. We assume that agents know the policy response to the crisis,¹⁸ and that the response is immediate.¹⁹

3.1 Default

We consider the case in which a single member of the monetary union faces a fiscal solvency crisis. We assume that, when a government cannot borrow to continue following the passive surplus rule, it uses default to reduce debt by the minimum amount consistent with solvency. This assumption implies that the magnitude of default is never equal to one hundred percent. The monetary authority maintains its active Taylor Rule, equation (2), assuring that inflation meets its target throughout the crisis and beyond.

We define a boundary locus for debt service at risk-free interest rates ($r_j b_j$). We assume that the system begins with debt service and the surplus in a region below the boundary locus, and show that in equilibrium debt service cannot travel above this boundary.

Definition 2 *Conditional on the expectation that a fiscal crisis will be resolved with default, a **boundary locus** for debt service ($r_j b_{jt}$) is the piecewise-continuous path, given by the upward sloping portion of the adjustment path leading to the fiscal limit on debt service ($\hat{\varphi}_j$) and continuing with $r_j b_j = \hat{\varphi}_j$ thereafter.*

Figure 1 shows the boundary locus for debt service as BLM. Debt service reaches its

¹⁸Davig, Leeper, and Walker (2011) show that uncertainty regarding how a shock will ultimately be financed can affect dynamic behavior. Cooper, Kempf, and Peled (2010) show how alternative policy responses can represent multiple equilibria based on agents' beliefs about the policy response.

¹⁹In real world crises, agents do not actually know the policy response, there could be multiple responses, and the response is delayed while officials determine the response. The delay involves a period in which the crisis government cannot access private capital markets. Therefore, our model considers a more orderly way of dealing with crises and considers the implications of different official responses.

fiscal limit $\hat{\varphi}_j$ at point L, and the value of the surplus at point L is given by

$$s_j^L = \frac{(1 - \alpha_j \lambda_j) \hat{\varphi}_j - \alpha_j (1 - \lambda_j) \varphi_j}{1 - \alpha_j}.$$

The boundary locus represents the maximum value of debt service, consistent with fiscal solvency, for each value of the surplus. For values of the surplus below s_j^L , any value of debt service above the boundary locus would put debt on a path which violates fiscal limits and is therefore inconsistent with solvency. For values of the surplus larger than s_j^L , debt service at its fiscal limit of $\hat{\varphi}_j$ is consistent with solvency because the adjustment dynamics imply that future debt falls. Therefore, in a crisis in which agents refuse to lend, the magnitude of default is determined to reduce debt service to the boundary locus.

Given this policy response, the expectation of one-period-ahead capital loss on government debt is determined by the expectation of the distance between debt along the boundary locus and the actual value of debt (b_{jt}). We approximate the value for debt along the upward-sloping portion of the boundary locus, which we label \hat{b}_{jt} , by taking a piecewise linear approximation of this path about s_{jt-1} and \hat{b}_{jt-1} , using equations (14), (15), and (16), to yield²⁰

$$\hat{b}_{jt} = \hat{b}_{jt-1} + \left(\hat{\beta}_{jt-1} - 1 \right) (s_{jt} - s_{jt-1}), \quad (17)$$

where $r_j \left(\hat{\beta}_{jt-1} - 1 \right)$ is the slope of the boundary locus BL,

$$\hat{\beta}_{jt-1} = \frac{r_j \hat{b}_{jt-1} - s_{jt-1}}{\alpha_j \left(\lambda_j r_j \hat{b}_{jt-1} - s_{jt-1} + (1 - \lambda_j) \varphi_j \right)}, \quad (18)$$

²⁰The path for \hat{b}_{jt} is given by the adjustment path in equation (17) for which b_{jt} has a maximum at $\frac{\hat{\varphi}_j}{r_j}$ and has $\gamma_{jt} = E_{t-1} \gamma_{jt} = 0$. We need its value for any given value for s_{jt} . Therefore, we approximate its value at time t using its $t - 1$ value (\hat{b}_{jt-1}) together with its $t - 1$ slope and the change in the actual surplus ($s_{jt} - s_{jt-1}$).

and $s_{jt} - s_{jt-1}$ is given by equation (14). The denominator in (18) represents the change in the surplus along the boundary locus and is always positive. Note that $\hat{\beta}_{jt-1} > 1$ when debt is rising along BL, and $\hat{\beta}_{jt-1} = 1$ once debt reaches its maximum level at point L.

We compute the distance between debt along the boundary locus and its actual value, which we label Ω_{jt} , by subtracting equation (7) from \hat{b}_{jt} , using equation (17), or from $\frac{\hat{\varphi}_j}{r_j}$, depending on the value for s_{jt-1} , to yield

$$\Omega_{jt} = \begin{cases} \hat{\mu}_{jt-1} (\hat{b}_{jt-1} - b_{jt-1}) + \hat{\beta}_{jt-1} \nu_{jt} + \gamma_{jt} - E_{t-1} \gamma_{jt} & s_{jt-1} \leq s_j^L \\ \frac{\hat{\varphi}_j}{r_j} + (1 - \alpha_j) s_{jt-1} - (1 + r_j - \alpha_j \lambda_j r_j) b_{jt-1} + \alpha_j (1 - \lambda_j) \varphi_j + \nu_{jt} + \gamma_{jt} - E_{t-1} \gamma_{jt} & s_{jt-1} \geq s_j^L \end{cases}$$

where

$$\hat{\mu}_{jt-1} = 1 + \frac{r_j (1 - \lambda_j) (\varphi_j - s_{jt-1})}{(\lambda_j r_j \hat{b}_{jt-1} - s_{jt-1} + (1 - \lambda_j) \varphi_j)} > 0.$$

The sign for $\hat{\mu}_{jt-1}$ reflects the fact that adjustment paths, conditional on different initial values, do not cross. Note that we are approximating the current distance based on the most recent value for the slope of the boundary locus.²¹ Defining x_{jt-1} as the state variable determining the distance, according to

$$x_{jt-1} = \begin{cases} \hat{b}_{jt-1} - b_{jt-1} & s_{jt-1} \leq s_j^L \\ \frac{\hat{\varphi}_j}{r_j} + (1 - \alpha_j) s_{jt-1} - [1 + r_j (1 - \alpha_j \lambda_j)] b_{jt-1} + \alpha_j (1 - \lambda_j) \varphi_j & s_{jt-1} \geq s_j^L \end{cases}$$

we can write a general expression for the distance as

$$\Omega_{jt} = \mu_{jt-1} x_{jt-1} + \beta_{jt-1} \nu_{jt} + \gamma_{jt} - E_{t-1} \gamma_{jt}, \quad (19)$$

where

$$\mu_{jt-1} = \begin{cases} \hat{\mu}_{jt-1} & s_{jt-1} \leq s_j^L \\ 1 & s_{jt-1} \geq s_j^L \end{cases}$$

²¹This error should be small, given that the boundary locus is very flat in the neighborhood of s_j^L .

and

$$\beta_{jt-1} = \begin{cases} \hat{\beta}_{jt-1} & s_{jt-1} \leq s_j^L \\ 1 & s_{jt-1} \geq s_j^L \end{cases}$$

We define a shadow value of default, analogous to the shadow value of the exchange rate in generation-one currency-crisis models (Flood and Garber 1984). The shadow value of default represents the reduction in the value of debt needed for the economy to reach the boundary locus. The shadow value can be positive or negative.

Definition 3 *The **shadow value** of capital loss on debt due to default at time t , $\tilde{\gamma}_{jt}$, is defined as the value of γ_{jt} for which $\Omega_{jt} = 0$.*

Setting Ω_{jt} to zero in equation (19) yields

$$\tilde{\gamma}_{jt} = E_{t-1}\gamma_{jt} - (\mu_{jt-1}x_{jt-1} + \beta_{jt-1}\nu_{jt}). \quad (20)$$

Substituting into equation (19) yields an expression for Ω_{jt} as

$$\Omega_{jt} = \gamma_{jt} - \tilde{\gamma}_{jt}, \quad (21)$$

implying that when default does not occur ($\gamma_{jt} = 0$), the distance and the negative of the shadow value are equivalent.

We **assume that agents expect default with** $\gamma_{jt} = \tilde{\gamma}_{jt}$ **if** $\tilde{\gamma}_{jt} > 0$. We prove that this assumption is consistent with a rational expectations equilibrium below in Proposition 2. Under this assumption, when default occurs at time t , the actual value of capital loss due to default is given by

$$\gamma_{jt} = \max\{\tilde{\gamma}_{jt}, 0\} = \max\{E_{t-1}\gamma_{jt} - (\mu_{jt-1}x_{jt-1} + \beta_{jt-1}\nu_{jt}), 0\}, \quad (22)$$

where, in the second equality, we have used equation (20) to substitute for $\tilde{\gamma}_{jt}$.

To solve for the magnitude of default, γ_{jt} , we must first solve for the expectations of default, $E_{t-1}\gamma_{jt}$.

Proposition 1 *Given a surplus rule with plans for default when the government cannot borrow, an equilibrium solution for the magnitude of expected default ($E_{t-1}\gamma_{jt}$) exists if and only if $x_{jt-1} \geq 0$.*

Corollary 1 *When $x_{jt-1} > 0$, the probability of a crisis in period t is less than one, and when $x_{jt-1} = 0$, the probability of a crisis in period t is one.*

All proofs are contained in the appendix. Intuitively, the Proposition and Corollary imply that creditors can be compensated for expectations of default only if there are some values of the fiscal shock (ν_{jt}) for which there would be no default next period. If default next period would occur for all values of the fiscal shock, then there are no values for actual and rationally expected one-period-ahead default, which both restore fiscal solvency and provide the market rate of return to international creditors.

We can use the phase diagram in Figure 1 to understand expectations of default and the probability of a crisis. When the system is substantially below the boundary locus BLM ($x_{jt-1} > 0$), such that no fiscal shock could send it over, the probability of a crisis next period is zero, implying that rational expectations of default are zero. The system is governed by the arrows of motion toward long-run equilibrium values. If fiscal shocks send the system toward the boundary locus, x_{jt-1} falls, but initially remains positive. The probability of a crisis becomes positive and agents begin to expect default. The associated default-risk premium on debt increases the interest rate, causing debt to increase more rapidly than shown along illustrated adjustment paths.

In regions for which $x_{jt-1} > 0$, expectations are well-defined, as shown in the appendix, and cannot be self-fulfilling. This is because the magnitude of default is determined to

restore fiscal solvency. To demonstrate, consider an arbitrary increase in $E_{t-1}\gamma_{jt}$, large enough to raise the probability of default to unity. Taking expectations of equation (22) with the probability of default equal to unity, yields

$$E_{t-1}\gamma_{jt} = E_{t-1}\gamma_{jt} - \mu_{jt-1}x_{jt-1}, \quad (23)$$

an impossibility since $x_{jt-1} > 0$. Therefore, expectations of default cannot arbitrarily increase sufficiently to be self-fulfilling when $x_{jt-1} > 0$. An increase in expectations too small to generate default would also not be self-fulfilling, since no default would occur.

Once debt has risen so much that it lies on the boundary locus ($x_{jt-1} = 0$), the probability of a crisis next period is unity, and expectations of default are so high that default next period could be avoided only for the most favorable fiscal shock.²² Using equation (22) to solve for the magnitude of default once $x_{jt-1} = 0$ yields

$$\gamma_{jt} = E_{t-1}\gamma_{jt} - \beta_{jt-1}\nu_{jt} \geq 0.$$

The sign restriction is necessary since default must be greater than or equal to zero for any realization of ν_{jt} , including its upper bound value of $\bar{\nu}$. This yields $E_{t-1}\gamma_{jt} \geq \beta_{jt-1}\nu_{jt}$ for all values of ν_{jt} , implying

$$E_{t-1}\gamma_{jt} \geq \beta_{jt-1}\bar{\nu}.$$

Therefore, when debt is on the boundary locus, there are multiple equilibria, in which actual and expected default must be positive and can be arbitrarily large. To verify, take the $t - 1$ expectation of equation (22), when the probability of default is unity, to yield an identity in the expectation.

²²The probability of the most favorable shock in a continuous distribution is zero.

A value of $x_{jt-1} < 0$ would imply that debt is above the boundary locus even with the most favorable fiscal shock, such that the probability of default is unity. Taking the expectation of equation (22), when the probability of default is unity, yields equation (23), an impossibility with $x_{jt-1} < 0$. Rationally anticipated default cannot restore fiscal solvency because actual default cannot equal itself plus a gap. Therefore, in equilibrium, the dynamics must bound the system away from positions for which $x_{jt-1} < 0$. This criterion determines crisis timing.

Proposition 2 *There is no equilibrium without default in period t if $\tilde{\gamma}_{jt} > 0$. Default given by $\gamma_{jt} = \tilde{\gamma}_{jt}$, restores equilibrium.*

The proof in the appendix shows that if $\tilde{\gamma}_{jt} > 0$ in the region of the upward sloping boundary locus and there is no default, then $x_{jt} < 0$, violating conditions for an equilibrium. In the flat region of the boundary locus, if $\tilde{\gamma}_{jt} > 0$ without default, then debt would be above its fiscal limit, a position into which agents refuse to lend. Therefore, when $\tilde{\gamma}_{jt} > 0$, agents refuse to lend, and the country defaults in magnitude sufficient to place the system on the boundary locus, thereby restoring fiscal solvency. Proposition 2 validates agents' assumption that the government will default whenever $\tilde{\gamma}_{jt} > 0$.

Corollary 2 *Equilibrium after default requires additional default each period until debt falls below the boundary locus on its approach to the long-run equilibrium.*

Default places the system on the boundary locus, implying that the expectations of default are large enough that default occurs for any fiscal shock. Interest rates are high and additional default is necessary. Therefore, following the crisis, markets remain turbulent for some time. This pattern does eventually end once the dynamics increase the surplus, moving the economy toward the long-run equilibrium below the boundary locus.

Corollary 3 *A government can reduce the probability of a crisis by strengthening political will with an increase in ρ_j .*

A larger value for ρ_j increases $\hat{\varphi}_j$ from equation (11). This raises the boundary locus, implying that the distance between debt along the boundary locus and any initial value of debt is greater. The greater is this distance, the lower is the probability of a crisis. Political will can be important in avoiding a crisis.

Crises generally occur with debt below its upper bound for three reasons. First, agents refuse to lend into positions for debt above the boundary locus, implying that crises occur with debt below the boundary locus. Second, for values of the surplus below s_j^L , the boundary locus is below the fiscal limit. Third, the fiscal limit could be below the upper bound due to less than perfect political will to implement large surpluses.

To summarize, default creates wealth transfers away from the crisis country's creditors toward the crisis government and yields market turbulence after the initial default. However, the monetary union can avoid price-level instability at the cost of allowing member-country default. The active Taylor rule is able to keep inflation on target. The ability to control the price level as the crisis approaches is sensitive to the assumption that the monetary authority targets the interest rate on bonds which have no probability of default. Bi, Leeper, and Leith (2010) show that when the monetary authority targets the interest rate on bonds in the crisis country, anticipations of default can create inflation in the monetary union.

3.2 Monetary and Fiscal Policy Switching

The second possibility we consider is that a government, which cannot borrow to continue its passive fiscal policy, switches to active fiscal policy with the common monetary authority switching to passive policy. Sims (1997), Davig and Leeper (2011), and Davig, Leeper, and Walker (2010, 2011) consider policy-switching models. Ours differs primarily in two respects. First, the timing of the policy switch is determined by the market's failure to lend under current policy, and not as a government decision, reached either stochastically or once debt attains some limit. Second, the monetary authority sets an interest rate, i_t , indexed for expectations of inflation created by the anticipated policy switch.

Before analyzing the switching model, it is necessary to understand equilibrium in a monetary union with one active fiscal policy country, $N - 1$ passive fiscal policy countries, and a passive monetary authority which pegs the interest rate.

3.2.1 Active Fiscal Policy in One Country and Passive in the Others

Consider a monetary union in which fiscal policy is active in the j 'th country and passive in all others. Passive monetary policy sets the interest rate to be consistent with its inflation target of zero, implying that $E_{t-1}\gamma_{jt} = 0$. The active fiscal authority replaces the term $(1 - \lambda_j)\varphi_j + \lambda_j r_j b_{jt-1}$ in the surplus rule, equation (12), with a revised target for the long-run surplus which satisfies the fiscal limit.

Definition 4 *The revised **target surplus** after switching equals the long-run equilibrium value of the surplus, conditional on crisis-period values for debt and the surplus, if the long-run value for the surplus is less than the fiscal limit. Otherwise, the revised target surplus is the fiscal limit, $\hat{\varphi}_j$.*

For a target surplus equal to the fiscal limit, $\hat{\varphi}_j$, the active fiscal rule becomes

$$s_{jt} = (1 - \alpha_j) s_{jt-1} + \alpha_j \hat{\varphi}_j + \nu_{jt}. \quad (24)$$

Substituting into equation (7) yields the evolution of debt under active fiscal policy with target surplus $\hat{\varphi}_j$ as

$$b_{jt} = (1 + r_j) b_{jt-1} - (1 - \alpha_j) s_{jt-1} - \alpha_j \hat{\varphi}_j - \nu_{jt} - \gamma_{jt} + E_{t-1} \gamma_{jt}. \quad (25)$$

The eigenvalues of the characteristic equation for the active fiscal system, given by equations (24) and (25) are $1 + r_j$ and $1 - \alpha_j$. This is a saddlepath-stable system, in which the government's IBC is not satisfied for positions off the saddlepath, and therefore is not satisfied for any initial value of debt, and hence for any price level. Therefore, the equilibrium value for γ_{jt} is determined to set the coefficient on the explosive root to zero. This is an FTPL policy regime. Since all other fiscal policies are passive, there is only one unstable root in the system of N countries.

The time paths for the surplus and debt in the active-fiscal-policy country are given in the appendix. These equations can be used to express the saddlepath relationship between debt and the surplus as

$$\hat{b}_{jt}^{sp} = \left(\frac{1 - \alpha_j}{\alpha_j + r_j} \right) s_{jt} + \frac{\alpha_j (1 + r_j) \hat{\varphi}_j}{(\alpha_j + r_j) r_j}. \quad (26)$$

We construct the phase diagram for an active-fiscal-policy country by subtracting lagged values of the surplus from equation (24) and lagged values of debt from equation (25) to yield

$$\Delta s_{jt} = s_{jt} - s_{jt-1} = -\alpha_j s_{jt-1} + \alpha_j \hat{\varphi}_j + \nu_{jt}, \quad (27)$$

$$\Delta b_{jt} = b_{jt} - b_{jt-1} = r_j b_{jt-1} - (1 - \alpha_j) s_{jt-1} - \alpha_j \hat{\varphi}_j - \gamma_{jt} + E_{t-1} \gamma_{jt} - \nu_{jt}. \quad (28)$$

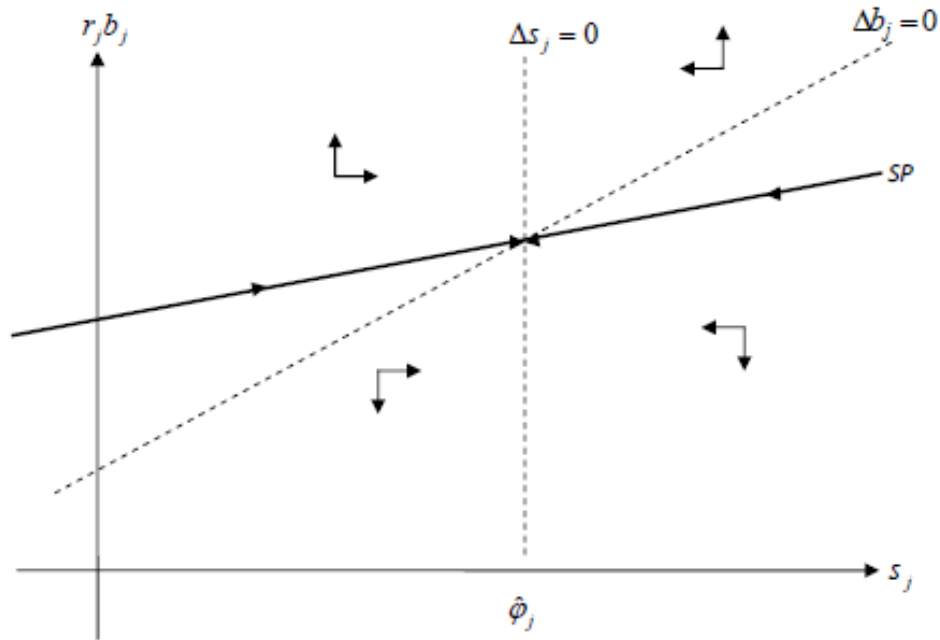


Figure 2: Active Fiscal Policy

The phase diagram under active fiscal policy and with $\nu_{jt} = \gamma_{jt} = E_{t-1}\gamma_{jt} = 0$ is given in Figure 2. The saddlepath has a positive slope and is labeled SP.

Fiscal shocks, ν_{jt} , move the system away from the saddlepath. There must be one jumping variable to assure that the system is on the saddlepath. Price level jumps create jumps in γ_{jt} . From equation (25), b_{jt} jumps with each jump in γ_{jt} , allowing the system for the active-fiscal-policy country to remain on the saddlepath. The monetary authority loses control over the actual price level because price must respond to offset fiscal shocks. However, it retains control over expected and average inflation. Capital gains and losses on government debt are symmetric, implying that expectations of gains and losses are zero in the active fiscal policy, passive monetary policy regime.²³

²³Cochrane (2001) shows that the introduction of long-term government bonds allows the monetary authority to trade off some of the jump in the price level for post-crisis expected inflation.

Since prices are equal in all monetary-union countries, fiscal shocks in the j' th country affect the price level in all the other $N - 1$ countries in the monetary union, as in Bergin (2000). Since debt levels in these countries differ, values for γ_{ht} , $h \in (1, N - 1)$, do not equal γ_{jt} , but the signs of γ_{jt} and γ_{ht} are identical. This creates windfall capital gains and losses on outstanding debt in member countries, to which they passively respond according to equation (12). Additionally, since price-level jumps are determined to assure intertemporal budget balance for the active-fiscal-policy country, the degree of union price-level instability is not related to the size of the active-fiscal-policy country. A crisis in a small country could create just as much instability as a crisis in a large country.

3.2.2 Active Fiscal Policy in Two Countries

Consider a monetary union with active fiscal policy in two countries and passive fiscal policy in $N - 2$. Passive fiscal policy assures intertemporal budget balance for the $N - 2$ countries. Although there are N values for γ_{jt} , there is only a single independent one. The value for the independent γ_{jt} is determined such that the expected present-value of surpluses for the two active-fiscal-policy countries equals the sum of their initial debt (Bergin 2000). Under this criterion, one country could have rising debt, and therefore a positive value for present-value debt in the limit, while the other could have falling debt and a negative present-value limit.

However, we have argued that the negative limit would not be tolerated since, in Woodford's (1997) words, this country would be writing a "blank check" to finance expenditures by the other country. The negative-limit country would optimally switch back to passive fiscal policy, allowing surpluses to fall and avoiding the resource transfer. Therefore, there

will be at most a single active fiscal policy country in a monetary union.

3.2.3 Fiscal Crisis Resolved with Policy Switching

We consider the case in which a single member of the monetary union faces a fiscal solvency crisis. We assume that **when country j cannot borrow to continue its passive fiscal rule, the fiscal authority switches to an active fiscal policy with a new fiscal target, and the monetary authority switches to a passive policy of pegging the interest rate at a value consistent with an inflation target of zero.** Prior to the date of policy switching, the monetary authority follows the Taylor Rule in equation (2), allowing the identical interest rates, i_{jt} for all j countries, to rise in anticipation of the inflation which is expected with policy switching. This Taylor Rule keeps the price level fixed prior to the crisis date. In addition to requiring policy switching when a country cannot borrow, we allow the policy switching to occur stochastically at any time, in contrast to the default model.²⁴

We solve the model by specifying a boundary locus, an algorithm for agent expectations, and a shadow value for capital loss on government debt, as before.

Definition 5 *Conditional on the expectation that a fiscal crisis will be resolved with policy switching accompanied by a revised target surplus, a **boundary locus** for debt service ($r_j b_j$) is defined as the piecewise-continuous path, given by the saddlepath leading to $\hat{\varphi}_j$ for $s_j \leq \hat{\varphi}_j$ and by $r_j b_j = \hat{\varphi}_j$ for $s_j > \hat{\varphi}_j$.*

Figure 3 superimposes the saddlepath for an active policy system with a target surplus equal to $\hat{\varphi}_j$ on the passive policy system for country j . The boundary locus for debt service is CKM.

²⁴Given the definition of the post-switching target surplus, an unanticipated policy switch with $\tilde{\gamma}_{jt} \leq 0$ does not cause a price-level jump. After the policy switch, the system travels along the saddlepath, conditional on current values of debt and the surplus, to a lower long-run value for the surplus than $\hat{\varphi}_j$.

analysis, but changes little substantively since few crises occur in this region. To focus on intuition, we relegate analysis in the neighborhood of the kink to the appendix.

Equation (29) shows that for $\nu_{jt} = \gamma_{jt} = E_{t-1}\gamma_{jt} = 0$, a positive value for x_{jt-1} implies that b_{jt} is below the boundary locus. However, fiscal shocks (ν_{jt}), expectations of inflation ($E_{t-1}\gamma_{jt}$), and inflation (γ_{jt}) can all affect the position of b_{jt} relative to the boundary locus.

We define a shadow value of capital loss on government debt due to inflation ($\tilde{\gamma}_{jt}$) as in Definition 3. Setting $\Omega_{jt} = 0$, from equation (29), and solving yields

$$\tilde{\gamma}_{jt} = E_{t-1}\gamma_{jt} - \frac{\alpha_j(1+r_j)}{\alpha_j+r_j} \left(x_{jt-1} + \frac{\nu_{jt}}{\alpha_j} \right). \quad (32)$$

Substituting into equation (31) yields

$$x_{jt} = \frac{(r_j + \alpha_j)}{\alpha_j} (\gamma_{jt} - \tilde{\gamma}_{jt}) - (\hat{\varphi}_j - \varphi_j) - \lambda_j (\varphi_j - r_j b_{jt}). \quad (33)$$

Note that, in contrast to the default case, in the absence of actual capital loss, x_{jt} is not determined by the shadow value alone. This happens because the adjustment path under the original passive fiscal policy crosses the boundary locus CKM.

Assume that agents expect policy switching with $\gamma_{jt} = \tilde{\gamma}_{jt}$ if $\tilde{\gamma}_{jt} > 0$. We prove that this assumption is consistent with a rational expectations equilibrium below.²⁵ If we redefine $\mu_{jt-1} = \mu_j = \frac{\alpha_j(1+r_j)}{\alpha_j+r_j}$ and $\beta_{jt-1} = \beta_j = \frac{(1+r_j)}{\alpha_j+r_j}$, and let $\tilde{\gamma}_{jt}$ be given by equation (32), then inflation in the crisis period is given by equation (22). Using these redefinitions, Proposition 1 applies directly to the switching case if we replace "plans for default" with "plans for switching." But Proposition 2, determining crisis timing, changes because the

²⁵In contrast to the default case, under switching, a crisis could occur with $\tilde{\gamma}_t < 0$, as we show below.

adjustment path under passive fiscal policy intersects the post-crisis adjustment path under active fiscal policy.

Proposition 3 *There is no equilibrium without policy-switching in period t if $x_{jt} < 0$ or if $\tilde{\gamma}_{jt} > 0$. Policy switching restores equilibrium.*

The proof is in the appendix. Consider the intuition behind crisis dynamics when the crisis will be resolved with policy-switching. A crisis occurs when the government can no longer borrow to continue with the passive fiscal rule. Assume that debt at time $t-1$ is at point H along path HP in Figure 3. The dynamics imply that the distance between debt along the boundary locus CK and the value of debt along the path HP eventually becomes negative. Since this is inconsistent with equilibrium, HP cannot be an equilibrium path. However, the expectation of a future regime switch makes point H feasible because the expectation raises the expected present-value surplus to equal the value of outstanding debt.

In the neighborhood of the boundary locus CK, the market begins to anticipate inflation. This anticipation forces the interest rate to increase. Once agents anticipate inflation, the system approaches the boundary locus CK at a faster rate than implied by the adjustment path HP, as shown in Figure 3 by the arrow from point H.

A crisis occurs when agents refuse to lend, and there are three ways in which this can happen. As the system travels along a passive adjustment path near the boundary locus CK, a negative fiscal shock ($\nu_{jt} < 0$) could send it over such that $x_{jt} < 0$ and $\tilde{\gamma}_{jt} > 0$. The government's response is policy switching. This implies a policy switch with a price level jump to bring the system to the boundary locus. The policy switch restores lending since creditors can expect the market rate of return on debt, and the system is expected

to travel along the boundary locus CK toward the long-run equilibrium at point K.²⁶

Alternatively, the system could be below the boundary locus CK in the region where the slope of the passive adjustment path becomes negative. A negative fiscal shock could send the system over the boundary locus CK such that $\tilde{\gamma}_{jt} > 0$, but the dynamics could imply that $x_{jt} > 0$. If agents refuse to lend, policy-switching occurs and $\gamma_{jt} = \tilde{\gamma}_{jt} > 0$ sets $\Omega_{jt} = 0$, as expected. If agents lend and the government does not switch, then equilibrium expectations of capital loss exist since $x_{jt} > 0$, and the policy can continue another period. However, if agents lend and the government switches, then a price level jump sets $\gamma_{jt} = \tilde{\gamma}_{jt} > 0$, yielding an instantaneous capital loss on debt. Since the government could choose to switch, generating an infinitely negative rate of return, agents refuse to lend in the region for which $\tilde{\gamma}_{jt} > 0$, precipitating a crisis. Policy switching with a price level jump to yield a capital loss of $\tilde{\gamma}_{jt}$ restores lending.

Finally, the system could be below the boundary locus, such that $\tilde{\gamma}_{jt} < 0$, in the region for which the slope of the passive adjustment path exceeds the slope of the boundary locus CK. In this region, the dynamics of the surplus and debt under passive policy could imply that debt next period, in the absence of regime switch, would travel above the boundary locus CK ($x_{jt} < 0$). Agents would not lend into this position since no rationally expected value for future inflation could place the system on the saddlepath. Policy switching with no change in the price level allows debt and the surplus to move along a saddlepath below CK, implying a long-run surplus below $\hat{\varphi}_j$. Therefore, if policy switching occurs with $\tilde{\gamma}_{jt} < 0$, no price level jump occurs, and fiscal authorities choose a lower target surplus,

²⁶Since the probability of inflation is less than one, when a shock occurs requiring inflation, its magnitude is greater than expected allowing debt to jump downwards.

determined by the long-run value of the debt and surplus, conditional on their current values and the new policy. Policy-switching with no price-level jump restores lending.

As the crisis approaches, the monetary authority keeps the price level fixed with its active Taylor Rule, equation (2). This contrasts with Davig and Leeper (2011) and Davig, Leeper and Walker (2010, 2011) who assume that the monetary authority sets the unindexed nominal interest rate in the run-up to the crisis. With their specification, as a crisis approaches, the expectation of a price-level jump raises both expected and actual inflation. Our assumption lets the monetary authority accommodate the effect of expected inflation on the interest rate, eliminating the actual inflation.

Equilibrium after policy switching is characterized by the FTPL. The monetary authority pegs the interest rate. The price level experiences both positive and negative jumps, offsetting fiscal shocks in the country with active fiscal policy, to keep the system on the saddlepath. On average the jumps are zero, implying that expected inflation and, therefore $E_{t-1}\gamma_{jt}$, are at the monetary authority's target value of zero.

Given that all interest rates in the monetary union are identical when crises will be resolved with policy switching, all countries experience an increase in interest rates when a single country enters a region where policy switching becomes possible. Therefore, once $E_{t-1}\gamma_{jt} > 0$, $E_{t-1}\gamma_{ht} > 0$ for all $h \in (1, N - 1)$ countries. Values for expected capital loss on debt in each country are not identical because values of outstanding debt differ. The increase in interest rates acts like a shock to each country's debt, to which they subsequently adjust surpluses according to passive fiscal policy, equation (12). After policy-switching, all $N - 1$ countries retain their passive fiscal policies, adjusting surpluses to price level jumps required by fiscal shocks for the single country with active fiscal policy.

The present values of surpluses for these $N - 1$ countries with passive fiscal policy are always zero, leaving the price level jump free to assure that the present value of surpluses for the active fiscal policy country is always zero.

We outline the implications of the model for the monetary union if we allow multiple countries to have crises simultaneously. The magnitude of the price level increase is determined such that policy switching occurs in only a single country, the one which requires the largest price level increase to restore solvency. The fiscal policy switch in the single country, together with the monetary policy switch and the associated price level jump, reduces the value of outstanding debt in the remaining countries sufficiently that agents are willing to continue to lend under the original passive fiscal policies. Fiscal policy switching in all crisis countries, together with a price level jump, determined to assure intertemporal budget balance for the aggregate of the crisis countries, would not resolve the crisis. Some countries would be left with debt so high that lending is not restored, while others would be on a path whereby the expected present value of debt in the limit is negative.

4 Simulations of Crisis Risk

In this section, we quantify the risk of fiscal crises, created by a failure of private creditors to lend, for five EMU countries. Explicitly, we ask what are the risks of fiscal solvency crises, both in normal times and following a series of negative shocks like those created by the Great Recession and financial crisis? We consider how a country could modify the parameters of its surplus rule to reduce risk, and we compare the risks under the alternative policy responses of default and policy switching.

Given parameter values for the N surplus rules, the distribution of ν_{jt} , and the method of crisis resolution, the system can be solved numerically and simulated to quantify the risk of one country in the N -country monetary union encountering a crisis over a given period of time. For the simulations, we use estimates for the parameters of the surplus rule from Daniel and Shiamptanis (2011). They provide group-mean panel estimates of parameters for the surplus rule in real terms using cointegration and error-correction models for ten EMU countries with annual data over the 1970-2006 period. Since we have group-mean panel values, not country-specific values, we remove the j subscripts from the parameters. The baseline parameter values for the simulations are reported in Table 1.²⁷

Table 1 : Baseline Parameters

	i	α	λ	g	φ
parameters	0.0422	0.5214	3.1750	0.0262	0.0039
standard errors	0.0061	0.0708	0.2355	0.0027	0.0009

These parameters imply a growth-adjusted interest rate given by $r = \frac{i-g}{1+g} = 0.0156$. With these parameter values, the slope of the BL segment of the boundary locus for default is very flat and point F in Figure 3 is out of range. Under the assumption that fiscal shocks have a normal distribution with mean zero, the panel estimate of their standard error is 1.83% of GDP. We let the upper bound on the fiscal shocks, $\bar{\nu}$, be 5.49% of GDP, which corresponds to three standard deviations. We set the fiscal limit on debt to 141% of GDP, larger than any of these countries has experienced within the sample.²⁸ Additionally, we

²⁷The variables in the paper of Daniel and Shiamptanis (2011) are in real terms, whereas the variables in this paper are expressed as a fraction of output. This implies the following mapping between the parameters of the two papers: $\alpha = 1 - \frac{\beta_1}{1+g}$, $\lambda = \frac{\beta_3}{\alpha(i-g)}$, and $\varphi = \frac{-\beta_2}{(\beta_1 + \beta_3/r - 1 - g)}$. To obtain the group-mean panel estimate of the long-run value of output growth g we estimated the following equation $\log(Y_{jt}) = c_j + gt + \varepsilon_{jt}$, where c_j denotes the country specific fixed effects and t denotes time.

²⁸

show that this value for the fiscal limit is consistent with market stability in normal times when debt is low and surpluses are relatively high, and with market volatility when debt rises and surpluses fall.

To estimate the probability of a fiscal crisis, we use 1,000 replications of a ten-year simulation under the two fiscal responses. In each simulation, values of debt/GDP, b_{jt-1} , and the primary surplus/GDP, s_{jt-1} are used to set the initial value for the state variable determining the distance, x_{jt-1} . Based on x_{jt-1} , the critical value for the shock, ν_{jt}^* , defined as the largest value which would create a crisis, and the expectation for capital loss, $E_{t-1}\gamma_{jt}$, are calculated. The dynamic system then receives a fiscal shock, ν_{jt} , from the truncated normal distribution and the value for capital loss, γ_{jt} , is computed. If $\gamma_{jt} = 0$, then next period's surplus and debt are updated using equations (7) and (12), which are then used to update x_{jt} . The process is repeated for ten years. If during the ten-year simulation we have a value of $\gamma_{jt} > 0$ or $x_{jt} < 0$, then there is a crisis and the simulation ends. The probability of a crisis over ten-years is the number of crises divided by 1000, the number of replications.

In normal times, creditors are willing to lend to sovereign governments without risk premia on interest rates. A correctly parameterized model should show the risk of crises to be very low. We proxy normal times as those when countries adhere to the SGP rules, and simulate the model with values for initial debt and the primary surplus equal to the upper bound of the SGP limits. We set debt at 60% of GDP and the primary surplus at -2.06%, which implies an actual surplus of -3% of GDP. Under the baseline parameter

We set the maximum value of the surplus, $\hat{\varphi}$, at 2.2% of GDP which implies a fiscal limit for debt, $\frac{\hat{\varphi}}{r}$, of 141%. The largest debt/GDP within the sample, preceding the recent worldwide recession and financial crisis, was 140% for Belgium in 1993.

values, fiscal policy is safe with no crises over ten years in the 1,000 replications.

Countries which have experienced a negative series of fiscal shocks have higher debt and lower surpluses than SGP requires.²⁹ The model implies that these countries have higher risk, and we can use the simulations to quantify this risk. In 2009, Belgium, France and Germany had modest deviations from the SGP limits. We repeat the simulations, using 2009 values.³⁰ Under baseline parameters, these three countries were perfectly safe over the ten-year horizon.

Next we consider whether high-debt countries like Greece and Italy, which had severely violated the SGP rules by 2009, faced risk over the next ten years. The countries had similar values for debt/GDP levels, 131.6% for Greece and 127.8% for Italy, but Greece had a much lower 2009 primary surplus/GDP, -10.5%, compared with Italy's -1.0%. Under baseline parameter values, the probability of a crisis for Greece was 3.4% and 58.3% under default and switching respectively, and for Italy was zero under both policy responses. Our model implies that one important reason for the difference in risk between Italy and Greece is the large difference between their primary surpluses. Had Greece been able to raise its primary surplus/GDP to Italy's -1.0%, then the probability of a crisis would have fallen to 0.1% under both policy responses.

The OECD is projecting that 2011 values for debt will be higher for many European countries. Therefore, we consider how high debt would have to be for Belgium, France, Germany, and Italy to begin experiencing risk under baseline parameter values, and how crisis probability changes for Greece as its debt increases. Figure 4 plots the probability

²⁹Our data source for values of debt/GDP and primary surpluses/GDP is OECD Economic Outlook July 2011.

³⁰The 2009 values for debt/GDP and primary surplus/GDP for Belgium were 100.5% and -2.5%, for France were 89.3% and -5.3%, and for Germany were 76.4% and -0.8%.

of a fiscal crisis as a function of debt/GDP at baseline parameter values, with the primary surplus at its 2009 value. Under baseline parameter values, the crisis probability becomes positive for Belgium, France, Germany and Italy once debt exceeds 128%, 126%, 131% and 131% of GDP, respectively. The OECD forecasts 2011 debt for Belgium, France and Germany to be well below these levels³¹, implying no risk of a crisis for the countries with moderate SGP deviations. However, the 2011 debt forecast for Italy is 129.7% of GDP, just shy of the debt/GDP level which implies some risk. If realized debt turns out to be a tiny bit larger than forecast, then Italy will have risk. The 2011 debt OECD forecast for Greece is 157.2%. At this level of debt, the risk of a fiscal crisis for Greece rises to 100% under both policy responses. Therefore, our model implies that the Greek crisis would have occurred prior to the end of 2011, consistent with events.³²

Additionally, the diagrams illustrate two interesting implications of the model. First, crises develop suddenly. There are debt levels with very high crisis probability which are similar to debt levels with low crisis probability. In this critical neighborhood for debt, a small increase in debt can propel a country from safe status to potential crisis status. This is because expectations of capital loss rise as the economy approaches the boundary locus. Higher expected capital loss increases the interest rate and increases the rate at which debt accumulates. The reverse is also true. A country which manages to get its debt below the critical level can reattain safe status. Second, switching is always riskier than default. This is because the boundary locus with default is above the boundary locus with switching. This implies that if the ECB could solidify its position against switching,

³¹2011 debt OECD forecasts for Belgium, France, and Germany are 100.7%, 97.3% and 87.3% of GDP, respectively.

³²Greece lost access to the private market for debt in mid-2010.

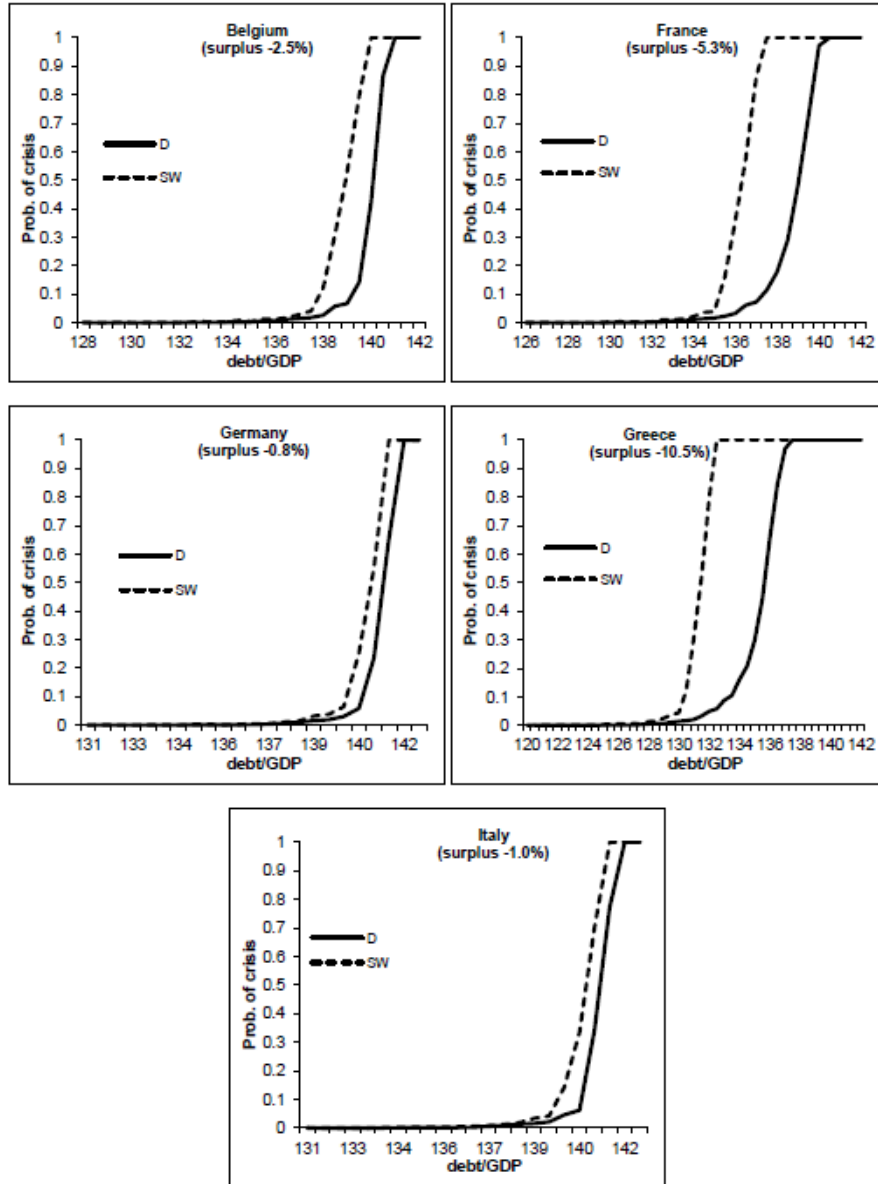


Figure 4: The probability of a crisis as a function of debt/GDP

it could reduce the probability of crisis.

Next, we consider sensitivity analysis. Individual-country parameter values can differ from the group-mean panel values. We change parameter values one at a time by one standard deviation in the risky direction.³³ For a country with debt and the surplus at the SGP limits, the probability of a fiscal crisis is zero under all sensitivity analyses designed to increase risk. Therefore, countries with debt and primary surplus at the SGP limits are perfectly safe over the ten year horizon. However, all five EMU countries have deviated from SGP limits.

We perform the same sensitivity analysis using the 2009 values for debt and the primary surplus. We find that Germany was perfectly safe under all one-standard-deviation parameter changes. Belgium and France were safe under all sensitivity scenarios except for the one-standard-deviation increase in the real interest rate, whereas Italy was not safe under the one-standard-deviation increase in the real interest rate or the reduction in the output growth rate. The increase in risk is moderate for France, but dramatic for Belgium and Italy.³⁴ Since risk appears so sensitive to the real interest rate, we consider how high the real interest rate (i) would have to be for Italy, Belgium, France and Germany to have positive risk. Figure 5 plots crisis probability as a function of i for all five countries. Italy becomes risky with a value for $i = 4.26\%$ (4 basis points higher than the baseline), Belgium for a value of $i = 4.54\%$ (32 basis points higher than the baseline), France for a value of $i = 4.69\%$ (47 basis points higher than the baseline), and Germany for $r = 4.99\%$

³³Experiments included raising i to 0.0483, raising φ to 0.0048, reducing λ to 2.9395, reducing α to 0.4406, and reducing g to 0.0235.

³⁴The one-standard-deviation increase in the real interest rate raises the probability of a crisis for Italy to 100% under both policy responses, for Belgium to 51.1% and 100%, and for France to 0.7% and 1.5%, under default and switching, respectively. The one-standard-deviation reduction in the output growth rate raises the probability of a crisis only for Italy to 100% under both policy responses.

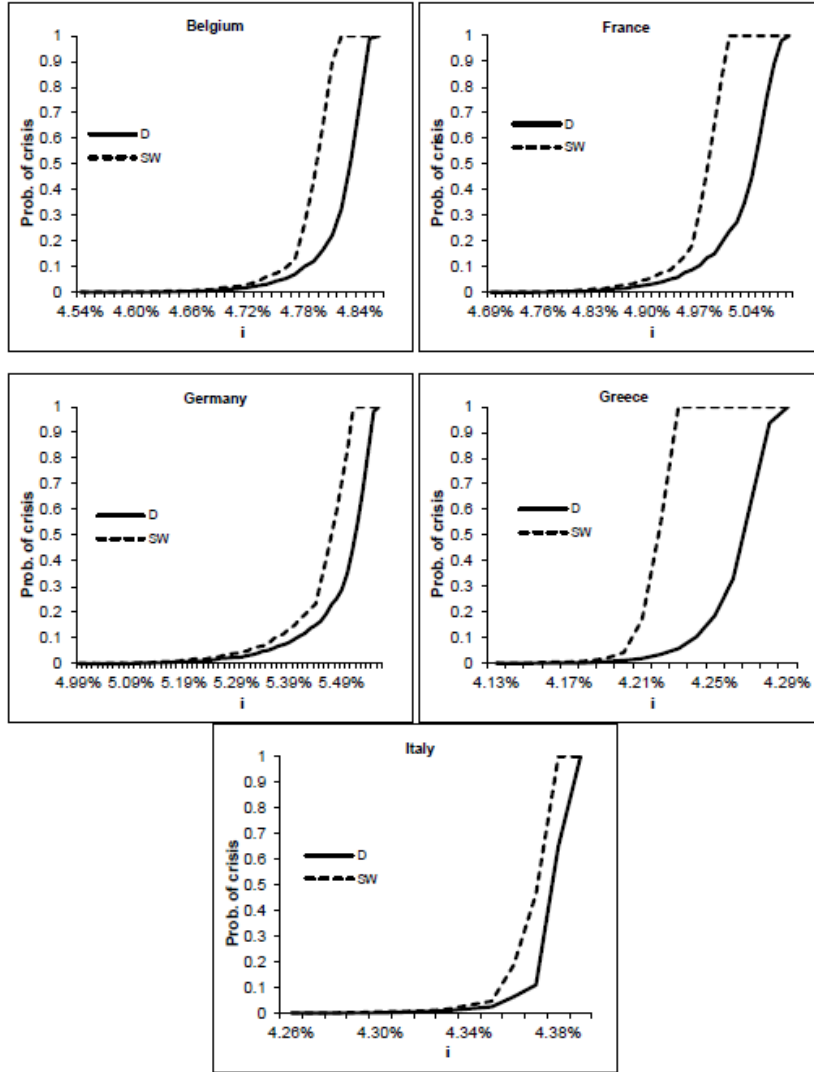


Figure 5: The probability of a crisis as a function of real interest rate

(77 basis points higher). Figure 5 reveals that risk of a crisis is increasing at an increasing rate in the real interest rate. Even though Italy is safe at baseline parameter estimates, a 12 basis-point increase in i , well within the 61 basis-point standard deviation, raises risk to 46.4% and 11.0% under switching and default, respectively.

Next, we investigate how risk changes due to a change in policy parameters since these are choice variables for the government. We use Greece to illustrate since Greece had risk

under 2009 baseline parameter values. Figure 6 plots crisis probability as a function of the policy parameters α , λ and φ within plus and minus one standard deviation of their means. These results show that modest changes in λ and φ could not have reduced risk for Greece close to zero. However, a modest increase in α , which implies reduced surplus persistence, could have. When α increases by one standard deviation to 0.5923, the probability of a fiscal crisis for Greece falls to 0.2% and 0.7%, under default and switching respectively. The Greek primary surplus in 2009 was so low that its persistence was a significant factor in risk.

Another possible policy parameter that could have reduced risk for Greece is political will. Therefore, we determine how high the fiscal limit had to have been to eliminate risk for Greece. Under baseline parameters and 2009 values, Figure 7 shows that crisis probability becomes zero once the fiscal limit increases to 148%. Given that private agents were not willing to lend to Greece in mid-2010 at lower levels of debt, our results imply that the fiscal limit is not this high.

We summarize our results on crisis probability for the five EMU countries as follows. Germany appears perfectly safe under baseline parameter values and under all scenarios designed to increase risk, including letting debt rise to higher forecast levels and letting parameters take on riskier values. At the other extreme, Greece exhibits risk at baseline parameters with 2009 values of the debt and surplus, and that risk rises to 100% with debt rising to the 2011 value forecast by OECD, indicating that a Greek crisis would have occurred. For 2009 values, France has small risk with a real interest rate increase of one standard deviation, while Belgium exhibits a dramatic increase in risk with the same interest rate change. The 2009 risk for Italy rises dramatically if either the real interest

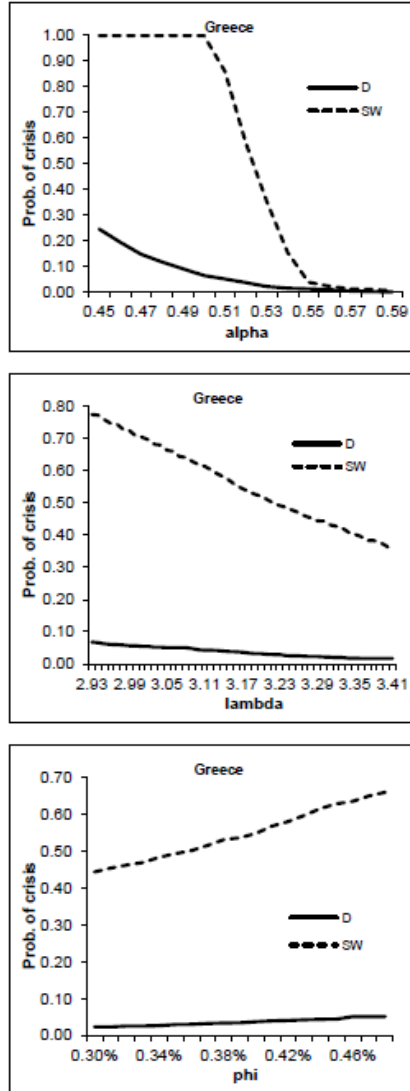


Figure 6: The probability of crisis as a function of alpha, lambda and phi

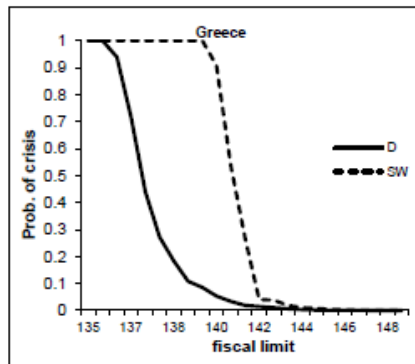


Figure 7: The probability of crisis as a function of fiscal limit

rate increases or the output growth rate decreases by a small amount. Additionally, the 2011 forecast for Italian debt is near the critical level yielding risk. These results seem to justify late-2011 market concern over Italian debt.

These simulations of fiscal risk are conditional on fiscal policy following the rule estimated for the panel over the period 1970-2006. The sensitivity analyses allow countries to differ in their parameter values by a single standard deviation. The simulations ignore the likely correlation of fiscal shocks across countries combined with the fact that a fiscal crisis in one country can affect the interest premium in another under policy-switching. This implies that risk is actually higher, and future research is needed to address this.

5 Conclusions

We present a dynamic and quantitative model of a fiscal solvency crisis in a monetary union. Government debt has risks due to stochastic shocks and fiscal limits. Under these assumptions, passive fiscal policy is not sufficient to assure that a government will always be able to borrow from the private market to carry out its passive surplus rule. Even when a government is following passive fiscal policy with a long-run value for debt well below its fiscal limit, shocks could send debt and the surplus toward a path which violates the fiscal limit. Agents would not lend along such a path because they cannot expect to receive the market rate of return, creating a crisis. The crisis requires a policy response because the government cannot continue its passive surplus rule when it cannot borrow. The dynamics in the run-up and aftermath of a crisis depend on the policy response.

We consider two responses: maintenance of the passive fiscal policy combined with default to reduce the magnitude of outstanding debt, and policy switching whereby fiscal

policy becomes active and monetary policy passive. We summarize characteristics of a fiscal solvency crisis as follows. The timing of the crisis is determined by the market, as in a generation-one currency crisis model. Once agents refuse to lend, policy-makers must respond to restore lending. Crises occur prior to debt reaching its upper bound. This is due to the upward-sloping boundary locus and to the fact that agents could believe that policy-makers have limited political will to tolerate debt at its feasible upper bound, yielding a lower fiscal limit. The equilibrium magnitude of capital loss on debt is endogenously determined to restore fiscal solvency for either policy response. This implies that when the policy response is default, its magnitude is never one hundred percent, in contrast to standard models of sovereign default.

Additionally, we use simulations to quantify the probability of a fiscal solvency crisis in the EMU over the next ten years. Our results illustrate that the probability of a crisis rises at an increasing rate in either debt or the interest rate. Crises develop quickly as interest rates rise in anticipation of the crisis, accelerating the growth rate of debt. Once debt reaches a critical range, a small change in debt creates a large change in crisis probability. The probability of a crisis under policy switching is always higher, implying that the ECB could reduce the probability of a fiscal crisis by solidifying its position against policy switching. We also show that the monetary authority can design policy such that it avoids inflation in the presence of positive crisis risk by targeting an interest rate which has no risk of capital loss.

Finally, our simulation results indicate that a country operating at the upper bound of the SGP rules is perfectly safe under the baseline parameter values. Additionally, countries like Belgium, France and Germany with modest violations, were also perfectly

safe under the baseline parameter values. However, simulations using 2009 data show that Greece was not safe under the baseline parameter values, and that Italy has risk for either a small increase in the real interest rate or a higher level of debt relative to GDP. Therefore, our model predicts the Greek crisis which occurred and warns of an Italian one, as yet unresolved.

6 Appendix: Default

6.1 Solutions

When fiscal policy is passive and monetary policy active, the time paths for each country's surplus and debt relative to output are

$$\begin{aligned}
s_{jt} = & \varphi_j + \frac{(\theta_{2j} - 1 + \alpha_j) \theta_{1j}^t}{(1 - \alpha_j) (\theta_{1j} - \theta_{2j})} \left\{ (\alpha_j - 1) (s_{j0} - \varphi_j) + (\theta_{1j} - 1 + \alpha_j) \left(b_{j0} - \frac{\varphi_j}{r_j} \right) \right. \\
& \left. + \sum_{k=1}^t \theta_{1j}^{-k} [-\theta_{1j} \nu_{jk} - (\theta_{1j} - 1 + \alpha_j) (\gamma_{jk} - E_{k-1} \gamma_{jk})] \right\} \\
& + \frac{(\theta_{1j} - 1 + \alpha_j) \theta_{2j}^t}{(1 - \alpha) (\theta_1 - \theta_2)} \left\{ (1 - \alpha_j) (s_{j0} - \varphi_j) - (\theta_{2j} - 1 + \alpha_j) \left(b_{j0} - \frac{\varphi_j}{r_j} \right) \right. \\
& \left. + \sum_{k=1}^t \theta_{2j}^{-k} [\theta_{2j} \nu_{jk} + (\theta_{2j} - 1 + \alpha_j) (\gamma_{jk} - E_{k-1} \gamma_{jk})] \right\} \quad (34)
\end{aligned}$$

$$\begin{aligned}
b_{jt} = & \frac{\varphi_j}{r_j} + \frac{\theta_{1j}^t}{\theta_{1j} - \theta_{2j}} \left\{ (\alpha_j - 1) (s_{j0} - \varphi_j) + (\theta_{1j} - 1 + \alpha_j) \left(b_{j0} - \frac{\varphi_j}{r_j} \right) \right. \\
& \left. + \sum_{k=1}^t \theta_{1j}^{-k} [-\theta_{1j} \nu_{jk} - (\theta_{1j} - 1 + \alpha_j) (\gamma_{jk} - E_{k-1} \gamma_{jk})] \right\} \\
& + \frac{\theta_{2j}^t}{\theta_{1j} - \theta_{2j}} \left\{ (1 - \alpha_j) (s_{j0} - \varphi_j) - (\theta_{2j} - 1 + \alpha_j) \left(b_{j0} - \frac{\varphi_j}{r_j} \right) \right. \\
& \left. + \sum_{k=1}^t \theta_{2j}^{-k} [\theta_{2j} \nu_{jk} + (\theta_{2j} - 1 + \alpha_j) (\gamma_{jk} - E_{k-1} \gamma_{jk})] \right\} \quad (35)
\end{aligned}$$

where $\theta_{1j} \leq 1$ and $\theta_{2j} < 1$ are the eigenvalues of the characteristic equation (13). When the country is far from a crisis, $\gamma_{jt} = E_{t-1} \gamma_{jt} = 0$. The values for γ_{jt} and its expectations

in the neighborhood of a crisis are endogenized as part of the model's full solution.

6.2 Proofs

6.2.1 Proof of Proposition 1

We prove that there is no value for $E_{t-1}\gamma_{jt}$ when $x_{jt-1} < 0$. Define $f(\nu_{jt})$ as a bounded, symmetric, mean-zero distribution for ν_{jt} , with bounds $\pm \bar{\nu}$. Define ν_{jt}^* as a critical value for ν_{jt} such that

$$\begin{aligned}\gamma_{jt} &> 0 \text{ for } \nu_{jt} < \nu_{jt}^*, \\ \gamma_{jt} &= 0 \text{ for } \nu_{jt} \geq \nu_{jt}^*.\end{aligned}$$

When such a critical value exists, taking the expectation of equation (22) yields

$$E_{t-1}\gamma_{jt} = \int_{-\bar{\nu}}^{\nu_{jt}^*} \gamma_{jt} f(\nu_{jt}) d\nu_{jt} = \int_{-\bar{\nu}}^{\nu_{jt}^*} [E_{t-1}\gamma_{jt} - \mu_{jt-1}x_{jt-1} - \beta_{jt-1}\nu_{jt}] f(\nu_{jt}) d\nu_{jt}. \quad (36)$$

Defining $F(\nu_{jt}^*)$ as the cumulative at ν_{jt}^* and collecting terms on the expectation yields

$$[1 - F(\nu_{jt}^*)] E_{t-1}\gamma_{jt} = -\mu_{jt-1}x_{jt-1}F(\nu_{jt}^*) - \beta_{jt-1} \int_{-\bar{\nu}}^{\nu_{jt}^*} \nu_{jt} f(\nu_{jt}) d\nu_{jt}. \quad (37)$$

Substituting into equation (22) yields an implicit expression for γ_t as

$$[1 - F(\nu_{jt}^*)] \gamma_{jt} = \max \left\{ 0, - \left[\mu_{jt-1}x_{jt-1} + \beta_{jt-1} \int_{-\bar{\nu}}^{\nu_{jt}^*} \nu_{jt} f(\nu_{jt}) d\nu_{jt} + \beta_{jt-1} [1 - F(\nu_{jt}^*)] \nu_{jt} \right] \right\}, \quad (38)$$

where $F(\nu_{jt}^*)$ has the interpretation as the probability of crisis.

To determine the probability of crisis, $F(\nu_{jt}^*)$, and the expectations of default, $E_{t-1}\gamma_{jt}$, first solve for ν_{jt}^* . Define $\chi_{jt} = \int_{-\bar{\nu}}^{\nu_{jt}^*} \nu_{jt} f(\nu_{jt}) d\nu_{jt} + [1 - F(\nu_{jt}^*)] \nu_{jt}^*$. A solution for ν_{jt}^* exists iff there exists a value for ν_{jt}^* , satisfying $-\bar{\nu} \leq \nu_{jt}^* \leq \bar{\nu}$, which sets $\mu_{jt-1}x_{jt-1} + \beta_{jt-1}\chi_{jt} = 0$ such that $\gamma_{jt} = 0$ in equation (38).

Given that $\beta_{jt-1} \geq 1$ and $\mu_{jt-1} > 0$, the proof must show that $\chi_{jt} \leq 0$ for all possible values for ν_{jt}^* . Let ν_{jt}^* take on its smallest possible value of $-\bar{\nu}$. Then $\chi_{jt} = -\bar{\nu} < 0$. The derivative of χ_{jt} with respect to ν_{jt}^* is given by $1 - F(\nu_{jt}^*)$. For $\nu_{jt}^* < \bar{\nu}$, the derivative is positive. Therefore, as ν_{jt}^* rises, χ_{jt} rises monotonically. Once ν_{jt}^* takes on its largest possible value, given by $\bar{\nu}$, $1 - F(\bar{\nu}) = 0$, and χ_{jt} takes on its maximum value of zero. Therefore, $\chi_{jt} \leq 0$ for all feasible values of ν_{jt}^* . Since $\chi_{jt} \leq 0$, a necessary and sufficient condition for $\mu_{jt-1}x_{jt-1} + \beta_{jt-1}\chi_{jt} = 0$ is $x_{jt-1} \geq 0$.

When $x_{jt-1} \geq 0$, a solution for ν_{jt}^* exists, and the expectations of default are given by the solution of equation (37). When $x_{jt-1} < 0$, there is no equilibrium interest which can compensate the lender for expectations of future default and restore fiscal sustainability, such that there is no equilibrium without default.

6.2.2 Proof of Corollary 1

When $x_{jt-1} > 0$, $\chi_{jt} < 0$, requiring $\nu_{jt}^* < \bar{\nu}$. Therefore, the probability of a crisis, given by $F(\nu_{jt}^*)$, is less than one. When $x_{jt-1} = 0$, ν_{jt}^* must set $\chi_{jt} = 0$, implying that $\nu_{jt}^* = \bar{\nu}$. Therefore, the probability of a crisis, given by $F(\bar{\nu})$, is one.

6.2.3 Proof of Proposition 2

Consider $s_{jt-1} \leq s_{jL}$. Equilibrium in period t requires $x_{jt} \geq 0$. This is because Proposition 1 shows that there can be no rational expectations value for $E_t\gamma_{jt+1}$ when $x_{jt} < 0$ under the initial policy mix without default. Therefore, if $x_{jt} < 0$, there is no equilibrium unless the country defaults. Using equations (19) and (20), yields

$$x_{jt} = \mu_{jt-1}x_{jt-1} + \beta_{jt-1}\nu_{jt} - E_{t-1}\gamma_{jt} + \gamma_{jt} = \gamma_{jt} - \tilde{\gamma}_{jt}.$$

Therefore, when $\tilde{\gamma}_{jt} > 0$, $x_{jt} < 0$ unless the country defaults. A positive shadow rate triggers default. Default with $\gamma_{jt} = \tilde{\gamma}_{jt}$ sets $x_{jt} = 0$, restoring equilibrium by Proposition 1.

Consider $s_{jt-1} > s_{jL}$. $\Omega_t = \frac{\hat{\varphi}_j}{r_j} - b_{jt} = \gamma_{jt} - \tilde{\gamma}_{jt}$. Therefore, if $\tilde{\gamma}_{jt} > 0$ and $\gamma_{jt} = 0$, then $b_{jt} > \frac{\hat{\varphi}_j}{r_j}$, an impossibility. Default with $\gamma_{jt} = \tilde{\gamma}_{jt}$ sets $x_{jt} = 0$, restoring equilibrium by Proposition 1.

6.2.4 Proof of Corollary 2

A default in period t , which brings the system to the boundary locus, implies that $x_{jt} = 0$. When $x_{jt} = 0$, the probability of a crisis in period $t + 1$ is unity by Corollary 1, and Proposition 1 yields $E_t \gamma_{jt+1} \geq \beta_{jt} \bar{\nu}$. Given a realization for ν_{jt+1} , default occurs in the magnitude to set $x_{jt+1} = 0$. The pattern persists until the dynamics imply that debt falls below BLM.

6.2.5 Proof of Corollary 3

The position of the boundary locus is increasing in $\hat{\varphi}_j$.

7 Appendix: Switching

7.1 Solutions

The time paths for each country's surplus and debt relative to output are

$$\begin{aligned}
 s_{jt} &= \hat{\varphi}_j + (1 - \alpha_j)^t \left[s_{j0} - \hat{\varphi}_j + \sum_{k=1}^t (1 - \alpha_j)^{-k} \nu_{jk} \right], \\
 b_{jt} &= \frac{\hat{\varphi}_j}{r_j} + (1 - \alpha_j)^t \left(\frac{1 - \alpha_j}{r_j + \alpha_j} \right) \left[s_{j0} - \hat{\varphi}_j + \sum_{k=1}^t (1 - \alpha_j)^{-k} \nu_{jk} \right].
 \end{aligned} \tag{39}$$

The requirement that the coefficient on the explosive root be zero implies

$$b_{j0} - \left(\frac{1 - \alpha_j}{\alpha_j + r_j} \right) s_{j0} + \sum_{k=1}^t (1 + r_j)^{-k} \left[E_{k-1} \gamma_{jk} - \gamma_{jk} - \frac{1 + r_j}{\alpha_j + r_j} \nu_{jk} \right].$$

7.2 Proof of Proposition 3

Assume first that the system is far enough from the kink in the boundary locus such that it is not relevant.

Proposition 1 with redefinitions shows that there is no equilibrium rational expectations value for $E_t \gamma_{jt+1}$ when $x_{jt} < 0$, and there is an equilibrium with $x_{jt} \geq 0$. Therefore, if $x_{jt} < 0$, then there is no equilibrium in the absence of policy switching.

Policy switching restores equilibrium by setting $\Omega_{jt} = 0$. There are three ways in which this can happen, depending on the value for $\tilde{\gamma}_{jt}$. When $\tilde{\gamma}_{jt} > 0$ and $x_{jt} < 0$, a price level jump setting $\gamma_{jt} = \tilde{\gamma}_{jt}$, assures $\Omega_{jt} = 0$, placing the system on the saddlepath.

However, from equation (31), it is possible for $x_{jt} < 0$, when $\tilde{\gamma}_{jt} \leq 0$ since $-(\hat{\varphi}_j - \varphi_j) - \lambda_j (\varphi_j - r_j b_t)$ can be negative. In this event, policy switching entails choosing a lower target surplus than $\hat{\varphi}_j$, in order to place the system on a lower saddlepath without a price level change. The lower target surplus reduces the distance between debt along the new lower saddlepath and its current value to zero, reducing Ω_{jt} to zero.

Since $-(\hat{\varphi}_j - \varphi_j) - \lambda_j (\varphi_j - r_j b_t)$ can also be positive, it is possible for $\tilde{\gamma}_{jt} > 0$ and $x_{jt} > 0$. If this occurs when $r_j b_{jt} < \hat{\varphi}_j$, such that debt is below its fiscal limit, then agents could choose to lend, since there would be an equilibrium with well-defined expectations of inflation. However, if agents lend and switching with a price level jump $\gamma_{jt} = \tilde{\gamma}_{jt}$ occurs, they would experience an instantaneous capital loss to set debt on the boundary locus. Therefore, agents refuse to lend and switching with $\gamma_{jt} = \tilde{\gamma}_{jt}$ occurs.

Now consider modifications necessary in the neighborhood of the kink. The distance between the value of debt along the boundary locus and its current value is given by

$$\Omega_{jt} = \begin{cases} \hat{b}_{jt}^{sp} - b_{jt} = \frac{1-\alpha_j}{\alpha_j+r_j} (s_{jt} - \hat{\varphi}_j) + \frac{\hat{\varphi}_j}{r_j} - b_{jt} & \text{for } s_{jt} \leq \hat{\varphi}_j \\ \hat{b}_{jt}^{sp} - b_{jt} = \frac{\hat{\varphi}_j}{r_j} - b_{jt} & \text{for } s_{jt} \geq \hat{\varphi}_j \end{cases}$$

Define

$$y_{jt-1} = (1 - \alpha_j) s_{jt-1} - r_j (1 - \alpha_j \lambda_j) b_{jt-1} + \alpha_j (1 - \lambda_j) \varphi_j,$$

$$w_{jt-1} = \frac{\hat{\varphi}_j}{r_j} - b_{jt-1}.$$

Using these definitions

$$\Omega_{jt} = \begin{cases} \frac{1+r_j}{\alpha_j+r_j} (y_{jt-1} + \alpha_j w_{jt-1} + \nu_{jt}) - E_{t-1} \gamma_{jt} + \gamma_{jt} & \text{for } s_{jt} \leq \hat{\varphi}_j \\ (y_{jt-1} + w_{jt-1} + \nu_{jt}) - E_{t-1} \gamma_{jt} + \gamma_{jt} & \text{for } s_{jt} \geq \hat{\varphi}_j \end{cases}$$

Define ν_t^c as the critical value of the fiscal shock for which the two distances are equal, and equivalently for which $s_t = \hat{\varphi}$. Equating the two values for the distance yields

$$\frac{1+r_j}{\alpha_j+r_j} (y_{jt-1} + \alpha_j w_{jt-1}) - (y_{jt-1} + w_{jt-1}) = \frac{\alpha_j - 1}{\alpha_j + r_j} \nu_{jt}^c.$$

When $\nu_{jt} \leq \nu_{jt}^c$, then $s_{jt} \leq \hat{\varphi}_j$ and when $\nu_{jt} \geq \nu_{jt}^c$, then $s_{jt} \geq \hat{\varphi}_j$.

The capital loss on debt is given by

$$\gamma_{jt} = \begin{cases} \max \left\{ \left(-\frac{1+r_j}{\alpha_j+r_j} (y_{jt-1} + \alpha_j w_{jt-1} + \nu_{jt}) + E_{t-1} \gamma_{jt} \right), 0 \right\} & \text{for } \nu_{jt} \leq \nu_{jt}^c \\ \max \left\{ \left(-(y_{jt-1} + w_{jt-1} + \nu_{jt}) + E_{t-1} \gamma_{jt} \right), 0 \right\} & \text{for } \nu_{jt} \geq \nu_{jt}^c \end{cases} \quad (40)$$

Letting ν_t^* be the critical value below which capital loss occurs and above which it does not, as in Proposition 1, and taking the expectation of the capital loss yields

$$E_{t-1} \gamma_{jt} = \int_{-\bar{\nu}}^{\min\{\nu_{jt}^c, \nu_{jt}^*\}} \left(E_{t-1} \gamma_{jt} - \frac{1+r_j}{\alpha_j+r_j} (y_{jt-1} + \alpha_j w_{jt-1} + \nu_{jt}) \right) f(\nu_{jt}) d\nu_{jt}$$

$$+ \int_{\min\{\nu_{jt}^c, \nu_{jt}^*\}}^{\nu_{jt}^*} \left(E_{t-1} \gamma_{jt} - (y_{jt-1} + w_{jt-1} + \nu_{jt}) \right) f(\nu_{jt}) d\nu_{jt},$$

which implies

$$\begin{aligned} [1 - F(\nu_{jt}^*)] E_{t-1} \gamma_{jt} &= - \int_{-\bar{\nu}}^{\min\{\nu_{jt}^c, \nu_{jt}^*\}} \frac{1 + r_j}{\alpha_j + r_j} (y_{jt-1} + \alpha_j w_{jt-1} + \nu_{jt}) f(\nu_{jt}) d\nu_{jt} \\ &\quad - \int_{\min\{\nu_{jt}^c, \nu_{jt}^*\}}^{\nu_{jt}^*} (y_{jt-1} + w_{jt-1} + \nu_{jt}) f(\nu_{jt}) d\nu_{jt}. \end{aligned}$$

Substituting into equation (40) and letting ξ_{jt} be an indicator function, which equals zero

when $\nu_{jt} \leq \nu_{jt}^c$ and unity otherwise, yields an implicit expression for γ_{jt}

$$\begin{aligned} [1 - F(\nu_{jt}^*)] \gamma_{jt} &= - \frac{1 + r_j}{\alpha_j + r_j} \left[(y_{jt-1} + \alpha_j w_{jt-1}) + \nu_{jt} (1 - F(\nu_{jt}^*)) + \int_{-\bar{\nu}}^{\nu_{jt}^*} \nu_{jt} f(\nu_{jt}) d\nu_{jt} \right] \\ &\quad + \xi_{jt} \frac{1 - \alpha_j}{\alpha_j + r_j} \left[\nu_{jt} (1 - F(\nu_{jt}^*)) - \nu_{jt}^c (1 - F(\nu_{jt}^c)) + \int_{\nu_{jt}^c}^{\nu_{jt}^*} \nu_{jt} f(\nu_{jt}) d\nu_{jt} \right]. \end{aligned}$$

To determine the probability of a crisis, $F(\nu_{jt}^*)$, and expectations of debt devaluation, $E_{t-1} \gamma_{jt}$, first solve for ν_{jt}^* . When $\nu_{jt}^* \leq \nu_{jt}^c$, this can be done as in Proposition 1 by solving for the value of ν_{jt}^* which satisfies $(y_{jt-1} + \alpha_j w_{jt-1}) + \nu_{jt}^* (1 - F(\nu_{jt}^*)) + \int_{-\bar{\nu}}^{\nu_{jt}^*} \nu_{jt} f(\nu_{jt}) d\nu_{jt} = 0$. Results are identical to those in Proposition 1.

If not, then the indicator function takes on the value of unity, and the extra term must be included. The term, $\nu_{jt}^* (1 - F(\nu_{jt}^*)) - \nu_{jt}^c (1 - F(\nu_{jt}^c)) + \int_{\nu_{jt}^c}^{\nu_{jt}^*} \nu_{jt} f(\nu_{jt}) d\nu_{jt} \geq 0$ for $\nu_{jt}^* \geq \nu_{jt}^c$. To prove, set $\nu_{jt}^* = \nu_{jt}^c$, and note the term is zero. The derivative with respect to ν_{jt}^* equals $(1 - F(\nu_{jt}^*)) > 0$ for $\nu_{jt}^* < \bar{\nu}$. Therefore, the value of $[1 - F(\nu_{jt}^*)] \gamma_{jt}$ is larger for any value for $\nu_{jt}^* > \nu_{jt}^c$, implying that γ_{jt} will be positive for more values of ν_{jt} . This implies a larger ν_{jt}^* .

If there is no value of $\nu_{jt}^* \leq \bar{\nu}$, then no one would have lent into this period, and the crisis would occur in period $t - 1$.

References

- [1] Arellano, Christina, (2008) "Default Risk and Income Fluctuations in Emerging Economies." *American Economic Review* 98(3), 690-712.
- [2] Bergin, Paul, (2000) "Fiscal Solvency and Price Level Determination in a Monetary Union." *Journal of Monetary Economics* 45, 37-53.
- [3] Bi, Huixin, (2011) "Sovereign Risk Premia, Fiscal Limits, and Fiscal Policy." *European Economic Review*, forthcoming.
- [4] Bi, Huixin, Eric Leeper and Campbell Leith, (2010) "Stability versus Sustainability: Macroeconomic Policy Tradeoff", working paper.
- [5] Cochrane, John, (2001) "Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level." *Econometrica* 69(1), 69-116.
- [6] Cochrane, John, (2011) "Understanding Fiscal and Monetary Policy in the Great Recession: Some Unpleasant Fiscal Arithmetic." *European Economic Review*, 55, 2-30.
- [7] Cooper, Russell, Hubert Kempf, and Dan Peled, (2010) "Regional Debt in Monetary Unions: Is it Inflationary?" *European Economic Review*, 54(3), 345 -358.
- [8] Daniel, Betty C., (2001) "The Fiscal Theory of the Price Level in an Open Economy." *Journal of Monetary Economics* 48, 293-308.
- [9] Daniel, Betty C., (2010) "Exchange Rate Crises and Fiscal Solvency," *Journal of Money, Credit, and Banking*, 1109-1135.
- [10] Daniel, Betty C. and Christos Shiamptanis, (2011) "Pushing the Limit? Fiscal Policy in the European Monetary Union," working paper, University at Albany.
- [11] Davig, Troy, Eric Leeper, and Todd B. Walker, (2010) "Unfunded Liabilities and Uncertain Fiscal Financing." *Journal Monetary Economics* 57(5), 600-619.
- [12] Davig, Troy, Eric Leeper, and Todd B. Walker, (2011) "Inflation and the Fiscal Limit." *European Economic Review* 55, 31-47
- [13] Davig, Troy and Eric Leeper, (2011) "Temporarily Unstable Government Debt and Inflation," working paper.
- [14] Dupor, B. "Exchange Rates and the Fiscal Theory of the Price Level." *Journal of Monetary Economics* 45 (2000), 613-630.
- [15] Eaton, Jonathan and Mark Gersovitz, (1981) "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *The Review of Economic Studies* 48(2), 289-309.
- [16] Eaton, Jonathan and Raquel Fernandez, (1995) "Sovereign Debt." In Gene M. Grossman and Kenneth Rogoff, eds., *Handbook of International Economics*, vol 3. Amsterdam: North Holland.
- [17] Flood, R. P. and P. M. Garber, (1984) "Collapsing Exchange Rate Regimes: Some Linear Examples," *Journal of International Economics* 17, 1-13.

- [18] Garcia, Marcio and Roberto Rigobon, (2005) "A Risk Management Approach to Emerging Market's Sovereign Debt Sustainability with an Application to Brazilian Data" in Giavazzi, Francesco, Ilan Goldfajn, and Santiago Herrera (eds.) *Inflation Targeting, Debt, and the Brazilian Experience: 1999 to 2003*. Cambridge, MA: MIT Press.
- [19] Krugman, Paul, (1979) "A Model of Balance of Payments Crises." *Journal of Money, Credit and Banking*, 11, 311-325.
- [20] Kumhof, Michael, (2004) "Fiscal Crisis Resolution: Taxation vs. Inflation." Festschrift in Honor of Guillermo A. Calvo.
- [21] Leeper, Eric, (1991) "Equilibria under 'Active' and 'Passive' Monetary Policies. *Journal of Monetary Economics* 27(1), 129-147.
- [22] Leith, Campbell and Simon Wren-Lewis, (2006) "Compatibility between Monetary and Fiscal Policy under EMU." *European Economic Review* 50, 1529-56.
- [23] Schabert, Andreas (2010). "Monetary Policy Under a Fiscal Theory of Sovereign Default." *Journal of Economic Theory*, 145 (2), 860-868.
- [24] Sims, Christopher A. (1997) "Fiscal Foundations of Price Stability in Open Economies." Working Paper, Yale University.
- [25] Sims, Christopher A. (1999) "The Precarious Fiscal Foundations of EMU, Working Paper.
- [26] Tanner, Evan and Issouf Samake. (2008) "Probabilistic Sustainability of Public Debt: A Vector Autoregression Approach for Brazil, Mexico, and Turkey." *International Monetary Fund Staff Papers* 55(1), 149-182.
- [27] Uribe, Martin, (2006) "A Fiscal Theory of Sovereign Risk." *Journal of Monetary Economics* 53, 1857-1875.
- [28] Woodford, Michael, (1994) "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy." *Economic Theory*, 43(3), 345-380.
- [29] Woodford, Michael, (1997) "Control of the Public Debt: A Requirement for Price Stability?" in *The Debt Burden and Monetary Policy*, ed. by G. Calvo and M. King. London: Macmillan, 117-154.