Macroeconomic Implications of Maturity Transformation

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Abstract

In this paper I develop a DSGE model in which the banking sector engages in both monitoring and maturity transformation. Maturity mismatch on the bank balance sheet, namely, long-term assets financed with short-term liabilities, has been widely documented empirically, but has so far been ignored in macroeconomic models. Macroeconomic models typically assume interest rates on loans are reset each period, but in reality many financial contracts are long-term contracts with fixed interest rates. My simulation results show that the incorporation of maturity mismatch alters the transmission channels between the financial and real sectors. Unlike much of the existing literature, in this model the bank net worth functions as a shock absorber and dampens the business cycles. Specifically, the responses of the real sector to shocks are attenuated, while the responses of the financial sector are magnified. These results suggest that by omitting long term loans previous studies overestimate the amplification effects of financial frictions and the effectiveness of monetary policy, and therefore the implications of financial frictions for business cycles and monetary policy should be revisited.

Keywords: maturity mismatch, DSGE, financial frictions, balance sheet channel, transmission mechanism

JEL codes: E22, E32, E44, G21

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1 Introduction

Recent macroeconomics developments have underlined the importance of credit market imperfections in business cycle dynamics. Financial frictions are considered to be one of the prime determinants of macroeconomic fluctuations, and more importantly, the critical propagators and amplifiers of shocks. The frictions in the credit market, which often include the asymmetric information and moral hazard problems in financial contracts, have been widely employed in dynamic general equilibrium models to reproduce the disruption in financial markets and large business cycle fluctuations as observed in the data. Although a wide range of financial factors have been accounted for in the existing studies, such as the financial accelerator channel and the credit supply channel, literature has remained inconclusive on the dynamic effects of financial frictions on the real activities. In this paper I will argue that a financial factor that has been omitted in macroeconomic studies, i.e., maturity mismatch on banks' balance sheets, also plays an influential role in shaping the business cycles through an alternative transmission channel. To complement the previous studies on the macroeconomic consequences of financial frictions, I develop a real business cycle model accounting for the maturity mismatch and assess the quantitative importance of this feature in business cycle dynamics.

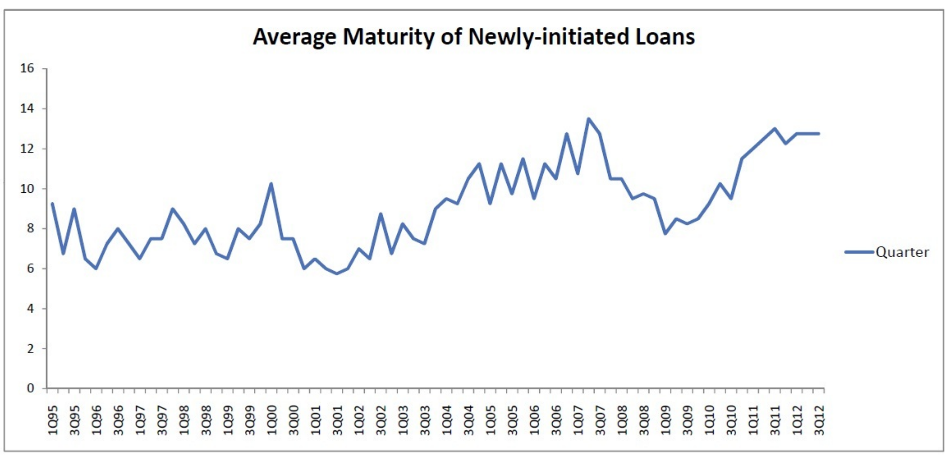


Figure 1: Average Maturity of Newly-initiated Loans

Figure 1 shows the average maturity of newly-initiated loans for corporate purposes[1]. Empirical evidences[2] indicate that the average maturity of total bank assets are four times the maturity of their liabilities. Most macroeconomic models, however, are built under the conjecture that both loans and deposits have a one-period maturity (one period often refers to one quarter). The assumption that all interest rates of loans are reset each period fails to agree with the fact that in practice many loan contracts are fixed rate contracts. Figure 2[3]

[1]Thomson Reuters LPC's DealScan.
[2]Documented by GStüntner (2010)
[3]Flow of Funds and NIPA.

plots the average interest expenses of non-financial corporate sector against the spot interest rates, i.e., the prime rate (loans) and the BAA corporate bond yield (bonds). If all the interest rates on loans were reset each period as DSGE models assume, the adjustments of average interest expenses would track the movements of the prime rate and the BAA yield closely. However, in reality the average borrowing cost, represented by the average interest expenses of the non-financial corporate sector, follows a much smoother pattern. Have misinterpreted the average borrowing cost, macroeconomic models with only one-period loans tend to overestimate the fluctuations of the real sector's borrowing cost. As will be shown in this paper, a model accounting for the sluggish borrowing cost predicts a distinct transmission channel that offsets the effects of financial frictions. Unlike the existing DSGE models which emphasize the role of the financial sector as a shock amplifier, this model also highlights the role of the bank net worth as a shock absorber. To this end, the model in this paper is specifically tailored to capture the feature of maturity mismatch in the banking sector.

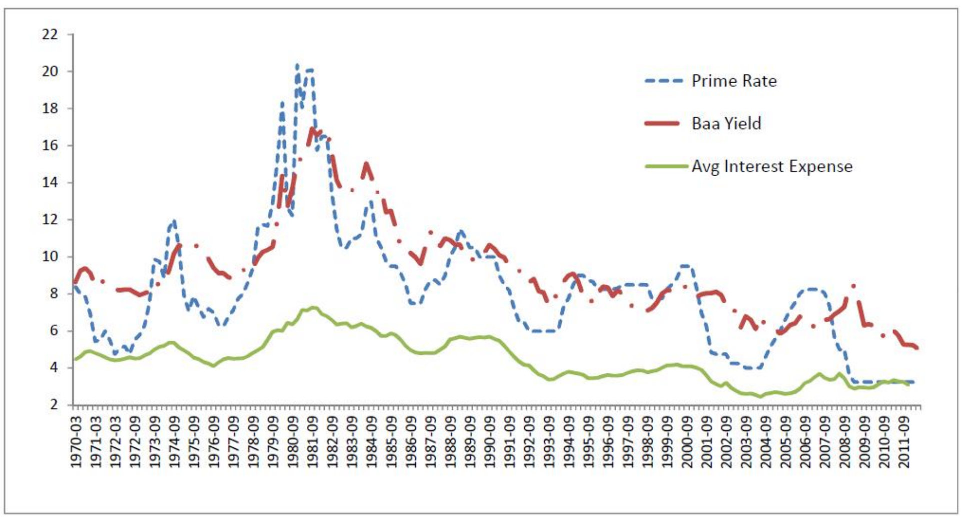


Figure 2: Fixed-rate Loans

Maturity mismatch is desirable because more profitable projects frequently require a longer investment horizon but investors' liquidity needs are uncertain. The bank finances the longer term investment by pooling the demand deposits, thereby providing "transformation" service. As an important feature of the banking sector, maturity mismatch has not obtained enough attention from macroeconomists. In this paper I will show the importance of maturity mismatch on economic dynamics by comparing a full model with maturity mismatch to a baseline model without it. The baseline model shares many similarities with much of the available literature; the full model builds on top of the baseline model with the incorporation of a non-trivial structure of maturity mismatch. The structure of the baseline model is outlined as follows. The credit demand of the real sector is modeled in a very similar manner of Bernanke, Gertler and Gilchrist (1999) (henceforth BGG). The banker and the entrepreneur engage in a financial contract, in which the contractual loan rate is tied to the entrepreneur's financial condition (i.e., the leverage ratio); this is the so-called financial accelerator channel. In addition to this, a (constrained) optimizing banking sector is embedded so as to model the bank lending and pricing behavior. The role of bank net worth is motivated by considering a moral hazard problem between bankers and depositors, borrowed from Gertler and Karadi (2010) (henceforth GK). In the presence of the moral hazard problem the size of the bank balance sheet is endogenously constrained by the level of bank net worth. The bank balance sheet channel and the perceived default risk jointly determine the loan rate and the credit supply. The baseline model outlined features the interplay between the credit demand and the credit supply. As has been widely documented in this strand of literature, credit market frictions affect real activities by magnifying the impact of shocks including monetary policy shocks. For example, Christensen et al. (2007) show that financial frictions boost the response of output after an increase in policy rates by about a third. Bernanke *et al.* (1999) and Gertler and Karadi (2010), among many others, also have similar predictions on the quantitative effect of the credit channel on the transmission of shocks. Although the credit channel theory greatly improves the ability of DSGE models to match empirical evidences, I will argue with the following model that this result is substantially weakened in a setting in which banks are engaging in maturity transformation.

The demand for long-term loans is motivated by the stochastic duration of investment projects. A project faces a constant probability to terminate in each period, and upon termination, the entrepreneur refinances by engaging in a new loan contract. The constant expected loan maturity across entrepreneurs and across time greatly simplifies the aggregation. Given the presence of long-term loans, banks face maturity mismatch on the balance sheets: on the asset side of the balance sheet are multi-period loans with fixed interest rates, while on the liability side there are the demand deposits.[4] The sluggish adjustments of loan rates constitute a transmission channel between the entrepreneurial and bank balance sheets that has been under-explored in literature. To show business cycle implications I simulate the model to various shocks. Impulse response functions suggest that the presence of maturity mismatch on the bank balance sheet attenuates the responses of the real sector in face of shocks at the cost of amplifying the fluctuations of financial variables, such as the bank leverage ratio and retained earnings. Unlike much of the existing literature, the bank net worth in this model acts as a shock absorber and dampens the business cycle fluctuations. The intuition behind these results is simple: the real sector is partially immunized from the fluctuations of interest rates due to fixed-rate loans, while the banking sector is exposed to interest rate risks because of the staggered interest rate adjustments. Therefore bank net worth functions as a buffer and counteracts the amplification effect caused by financial frictions.

[4]It is worth noting that in principle long-term loans don't necessarily imply maturity mismatch; in this paper the underlying assumption of deposits being one-period allows me to use "long-term loans" and "maturity mismatch" interchangeably.

These results suggest that previous studies with exclusively one-period loans overestimate the amplification effect of financial frictions. In reality, in the presence of maturity mismatch the amplification mechanism of financial frictions are largely muted. An exercise comparing the modeled economy with the efficient economy indicates that a certain degree of maturity mismatch on banks' balance sheets help the economy realize its full potential.

This paper is related to the banking literature on maturity mismatch. In fact, maturity mismatch as a core activity of financial intermediaries has been studied extensively in microeconomic literature, beginning with the seminal work of Diamond and Dybvig (1983). Allen and Gale (2007), Rochet and Vives (2004) and Holmstrom and Tirole (1995, 1994), among many others, have contributed to this strand of literature. However, all the models mentioned above are partial equilibrium models with a two or three period horizon, and hence they do not allow for studying the role of maturity mismatch in business cycle fluctuations.

This paper is closely related to the rapidly growing literature on DSGE models with financial frictions. The seminal work of Bernanke Gerlter and Gilchrist (1999) studies the role of the borrower's net worth in a New Keynesian framework. The model generates a countercyclical external finance premium, which is the so-called financial accelerator channel. The banking sector in this model serves as a monitor and diversifies investment risks for the investors. Gertler and Kiyotaki (2010) and Gertler and Karadi (2010) instead focus on the credit supply. They introduce a moral hazard problem between the financial sector and households, and consequently the credit supply is endogenously constrained by the banks' net worth. Rannenberg (2011) incorporates BGG framework into the GK model in order to study the interaction between the credit demand and the credit supply. He finds that with both a financial accelerator and the bank balance sheet channel the model outperforms BGG in terms of matching the data. Meh and Moran (2010) shows that the bank net worth propagates and magnifies the shocks in a model with double moral hazard problems. However, the models mentioned above consider one-period loans exclusively and only identify the amplification channel through which financial frictions magnify shocks. Specifically, Gertler and Karadi (2010) and Meh and Moran (2010) both highlight the role of the bank net worth as a shock amplifier. My model complements the existing studies and and underlines the role of the bank net worth as a buffer and shock attenuator, which casts some doubts on the quantitative results in the available literature.

Andreasen, Ferman and Zabczyk (2012) (henceforth AFZ), to my knowledge, is the first attempt to incorporate the maturity mismatch in a DSGE framework. In their model the demand for long-term loans is motivated by firms' infrequent capital adjustment. The model predicts an attenuating effect on all the aggregate variables such as output and investment, but little implication for the banking sector can be drawn in their model because of the lack of important financial frictions. My model shares many similarities with AFZ, but by further exploiting the credit channel my model arrives at a rather different result and delivers new messages for policymakers. Instead of an ad hoc long-term loan contract like in AFZ, the long-term financial contract between entrepreneurs and bankers in my model is built on credit frictions, taking the default risks of entrepreneurs into account. By allowing for the entrepreneurs' default, my model not only captures the transmission of sticky loan rates on the bank balance sheet, but also the feedback effect driven by the perceived default risk. This feature has been proven to be very important in determining the banks' lending behaviors, and hence the main finds of my model differ from AFZ's in a key respect: AFZ predict a "credit attenuator" effect of maturity mismatch on the economic fluctuations; My model, while providing similar predictions on the fluctuations of real activities, suggests that financial fluctuations are magnified in the presence of maturity mismatch.

The paper is structured as follows: Section 1 is an introduction and motivation; Section 2 discusses the model in detail; Section 3 provides the simulation results and inspects the mechanism; finally Section 4 concludes.

2 An RBC Model with Maturity Mismatch

2.1 Workers

There is a continuum of identical households in the economy. Within each household there are three types of agents: workers, entrepreneurs and bankers. Workers make consumption plans and supply labor. They return their wages to the respective households and deposit funds in competitive banks. Entrepreneurs invest in long-term projects. The acquisition of working capital for the project is partially financed by taking loans from banks. Bankers finance the loans with their own net worth and deposits. Over time the entrepreneur/banker exits the industry and transfers his net worth back to the respective household. A corresponding number of workers randomly become entrepreneurs and bankers, keeping the number of each occupation constant.

There is perfect consumption insurance within a household. Let C_t be the consumption of a representative household, and H_t be worker's labor supply. The worker is risk averse, and he or she derives utility from consumption and disability from labor supply

$$\max E_t \sum_{i=0}^{\infty} \beta^i [\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\phi} H_{t+i}^{1+\phi}]$$

with $0 < \beta < 1$, $0 < h < 1$ and $\chi > 0$ $\phi > 0$. β is the discount factor. To facilitate comparison to many DSGE models I also allow for habit formation h. Demand deposits D_t are the only saving vehicle, which offers a risk-free return R_t in a given period t. W_t is the real wage and Π_t denotes the net transfers from entrepreneurs and bankers to the household. The worker's consumption decision has to satisfy the household's budget constraint,

$$C_t = W_t H_t + R_t D_t - D_{t+1} + \Pi_t$$

Let ϱ_t denote the marginal utility of consumption. The household's optimal choices for consumption and labor supply are given as follows

$$\frac{1}{C_t - hC_{t-1}} - \frac{\beta h}{C_{t+1} - hC_t} = \varrho_t \tag{1}$$

$$\varrho_t W_t = \chi H_t^{\varphi} \tag{2}$$

$$E_t R_t \frac{\rho_{t+1}}{\rho_t} = 1 \tag{3}$$

2.2 Entrepreneurs

Physical Setup Entrepreneurs are in perfect competition, and each entrepreneur invests in a long-term project at a time. Entrepreneurs are risk neutral and aim to maximize the terminal wealth accumulated through retained earnings. The production technology is given by the conventional Cobb-Douglas production function

$$Y_t = A_t K_t^{\epsilon} H_t^{1-\epsilon}$$

where A_t is the total factor productivity. K_t and H_t are the factors of production, physical capital and labor, respectively. The factor market is perfectly competitive, and the real wage is given by the marginal product of labor

$$W_t = (1-\epsilon)\frac{Y_t}{H_t}$$

The acquisition of physical capital is financed by the entrepreneur's net worth as well as external funds from the banking sector. Entrepreneurs have finite business horizons. Specifically, each period a constant fraction $1-\theta^e$ of entrepreneurs retire and exit the industry, implying an expected business horizon of $\frac{1}{1-\theta^e}$. This assumption ensures that the real sector doesn't ultimately accumulate sufficient wealth to stay independent of external financing.

To motivate the demand for long-term loans, in this model the investment project has a stochastic duration. In particular, conditioning on reaching a given period each project faces a constant probability $1-\alpha$ to terminate in that period. The assumption of stochastic duration of the project is intended to capture the fact that projects frequently require multi-period commitments, and therefore there is demand for long-term financial contract in order to secure stable funding. The contract ensures that the interest rate of the loan is fixed throughout the lifetime of the project. Upon project termination, the entrepreneur engages in a new loan contract to finance the next project. An alternative way to introduce fixed interest rate would be to assume that the investment is long-lived while the entrepreneur has stochastic opportunity to refinance. Until the entrepreneur is "tapped by the Calvo fairy" the entrepreneur is locked in the contracted fixed interest rate. The random renewal of the financial contract can be interpreted as a refinancing risk faced by the banking sector. Given that the two assumptions are technically identical, I stick to the first one for its real-life relevance. The stochastic nature of the financial contract implies an average loan maturity of $\frac{1}{1-\alpha}$, and the parameter α governs the degree of maturity mismatch on the bank balance sheet: The greater the α is, the greater the degree of maturity mismatch. Entrepreneurs refinancing at time j are denoted as vintage j. There is an infinite number of vintages in the economy, and the size of each vintage is geometrically declining overtime.

Let Q_t be the price of capital and δ denote the depreciation rate of capital[5]. The *aggregate* return on capital across all entrepreneurs is given by

$$R^k_{t+1} = \frac{\epsilon \frac{Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t} \tag{4}$$

The realized return for an individual project in each period, however, is subject to both aggregate and idiosyncratic risk. Individual entrepreneur i's return is given by $\omega^i_t R^k_t$, with ω^i_t being the idiosyncratic productivity shock. ω^i_t is i.i.d across entrepreneurs and time, following a log-normal distribution with the probability distribution function of $f(\omega)$ and cumulative distribution function of $F(\omega)$. The idiosyncratic shock has a mean value $E\{\omega\} = 1$ and the standard deviation σ. The idiosyncratic risks and the realized capital return are privately observed by the entrepreneur. Bankers need to pay an "auditing" cost to verify the realized return $\omega^i_t R^k_t$.

In the beginning of period t entrepreneur i has available net worth N^i_t. To finance the difference between his expenditures on capital goods and his net worth he takes out loans of L^i_t. The balance sheet of entrepreneur i is given by

$$Q_t K^i_t = N^i_t + L^i_t \tag{5}$$

An entrepreneur takes up a loan at a fixed interest rate throughout the entire life-time of the investment, but the volume of the loan varies in the meantime depending on the optimal capital acquisition in that period. The banker monitors in each period independent of whether the loan matures or not. It is worth noting that unlike auditing, monitoring is costless for bankers. If an entrepreneur's cash flow is not sufficient to cover the debt payment (interests plus the principal), the entrepreneur declares default and exits the industry. Once the project terminates the entrepreneur refinances by taking up a loan at a new interest rate.

[5] Q_t will be derived later

Financial Contract The entrepreneur makes the investment decision, taking the price of capital goods and the expected return to capital as given. After the optimal demand for capital is determined, the capital prices and the returns are endogenized as part of a general equilibrium solution. The financial structure of this model draws extensively from Bernanke, Gertler and Gilchrist (1999), the core of which is a "costly state verification" (CSV) problem in the manner of Townsend (1979). The idiosyncratic productivity shocks are privately observed by the entrepreneur. Bankers must pay a fixed "auditing cost" in order to verify the realized returns. The auditing cost is interpretable as the cost of bankruptcy and equal to a proportion μ of the realized gross payoff to the capital, i.e., $\mu\omega_t^i R_t^k Q_{t-1} K_{t-1}^i$.

The financial contract is characterized by the entrepreneur's leverage ratio $\phi_t^{e,i}$(alternatively, demand for capital K_t^i) and the contractual loan rate $R_t^{L,i}$. It is worth noting that the contractual loan rate $R_t^{L,i}$ is fixed in the contract, in contrast to the state contingent loan rate in BGG. This feature makes the contract more relevant for the real world in that bankers share the default risks with entrepreneurs. Therefore the ex post return on loans earned by banks (adjusted for default cost) generally differs from the contractual loan rate.

It is assume that bankers monitor in each period, and entrepreneurs have to remain solvent in each period regardless of whether the project terminates or not. Each period the entrepreneur has to report to the banker that the entrepreneur has sufficient cash flow to cover debt payment (interests plus the principal), otherwise the banker audits. This is a rather stark assumption [6], but it greatly simplifies the contracting problem and doesn't affect the results qualitatively. The solvency condition is characterized by a threshold value of the idiosyncratic shock $\bar{\omega}_{t+1}^i$: for values of the idiosyncratic shock lower than $\bar{\omega}_{t+1}^i$, the entrepreneur is not able to honor the loan at the contractual loan rate $R_t^{L,i}$

$$\bar{\omega}_{t+1}^i R_{t+1}^k Q_t K_t^i = R_t^{L,i} L_t^i \tag{6}$$

When $\omega^i \geq \bar{\omega}^i$, entrepreneur is able to pay off the loan $R_t^{L,i} L_t^i$ and keeps the difference, equal to $\omega^i R_{t+1}^k Q_{t+1} K_{t+1}^i - R_t^{L,i} L_t^i$. Under the solvency condition, whenever $\omega^i < \bar{\omega}^i$ the entrepreneur declares default and the financial contract terminates automatically. Upon this the banker pays the auditing cost and get to keep the remaining, i.e., $(1-\mu)\omega_{t+1}^i R_{t+1}^k Q_t K_t^i$. A defaulting entrepreneur earns zero profit and exits the industry.

The financial contract is a dynamic version of the contracting problem in BGG. $\Gamma(\bar{\omega}_{t+1}^i)$ denotes the expected gross share of profits going to the banks

$$\Gamma(\bar{\omega}_{t+1}^i) = \bar{\omega}_{t+1}^i \int_{\bar{\omega}_{t+1}^i}^{\infty} f(\omega) d\omega + \int_0^{\bar{\omega}_{t+1}^i} \omega f(\omega) d\omega$$

[6] In principal, given a multi-period contract the lender doesn't monitor each period, and the borrower should be able to accumulate profits and only pay the principle when the loan falls due.

The first term on the right hand side measures the expected return of loans earned from the entrepreneurs who honor the debt at the contractual loan rate. The second term measures the remaining outcome seized by bankers when the entrepreneur declares default. Let $\mu G(\bar{\omega}^i)$ denote the expected auditing cost as a fraction of the total outcome

$$\mu G(\bar{\omega}^i_{t+1}) = \mu \int_0^{\bar{\omega}^i_{t+1}} \omega f(\omega) d\omega$$

The expected net share of profits earned by banks is the gross profits less the deadweight loss $[\Gamma(\bar{\omega}^i_{t+1}) - \mu G(\bar{\omega}^i_{t+1})]$.

The expected share of profits earned by entrepreneurs is therefore given by $1-\Gamma(\bar{\omega}^i_{t+1})$. Entrepreneur's value function Ω_t is the expected discounted profits, taking into account the continuation value of the contract (weighted by the continuation probability α). Entrepreneur's optimization problem is given as following

$$\Omega_t = \max_{\phi^{e,i}_t, E_t\bar{\omega}_{t+1}} E_t\{R^K_{t+1}[1-\Gamma(\bar{\omega}^i_{t+1})]\phi^{e,i}_t N^{e,i}_t + \beta\alpha[1-F(\bar{\omega}^i_{t+1})]\Omega'_{t+1}\}$$

The entrepreneur chooses the optimal leverage ratio $\phi^{e,i}_t$ and the expected default threshold value $E_t\bar{\omega}^i_{t+1}$ to maximize the discounted profits. The entrepreneur may equally choose the contractual loan rate $R^{L,i}_t$ as the alternative to the expected default threshold value. In principle, Ω_t is an infinite sum of profits of all future periods, and Ω'_{t+1} also depends on the choices in period t. To avoid the complications of a fully dynamic contracting problem while preserving the essence of a long-term contract, I simplify the contracting problem by assuming that all non-refinancing entrepreneurs have identical leverage ratio in the beginning of the period. Details of the assumption will be made clear below. Under this assumption Ω'_{t+1} has a slightly different form than Ω_t.[7] The entrepreneur takes into account the fact that the individual leverage ratio will be the aggregate ratio ϕ^e_{t+1} in the beginning of the next period if the financial contract continues to run, and therefore Ω'_{t+1} is a function of aggregate leverage ratio and expected aggregate technology

$$\begin{aligned} \Omega'_{t+1} &= \{R^K_{t+2}[1-\Gamma(\bar{\omega}^i_{t+2})]\phi^e_{t+1}N^{e,i}_{t+1} \\ &+ \beta\alpha[1-F(\bar{\omega}^i_{t+2})]\Omega'_{t+2}\}\} \end{aligned}$$

The entrepreneur's optimal choices are subjected to bankers' participation constraint. Each banker holds an market portfolio of loans, and hence the participation constraint is identical for each banker. Bankers take the expected default risk into account, and the expected return

[7] Appendix.

of loans is given by

$$E_t\{\underbrace{(Q_tK_t - N_t^e)}_{\text{outstanding loan}} R_{t+1}^D\} = E_t\{R_t^K Q_t K_t \quad \underbrace{[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]\}}_{\text{net share of profits earned by banks}} \tag{7}$$

$E_t R_{t+1}^D$ can be interpreted as the expected return that bankers require for a given amount of loans L_t issued. The ex post return R_{t+1}^D is adjusted for default and hence lower than the contractual loan rate R_t^L. $E_t R_{t+1}^D$ is jointly determined by the demand and supply of credit, and the entrepreneur takes it as given when entering the contract.

After solving the optimization problem by choosing the optimal leverage ratio and expected default threshold value, the contracting problem yields the following demand equation

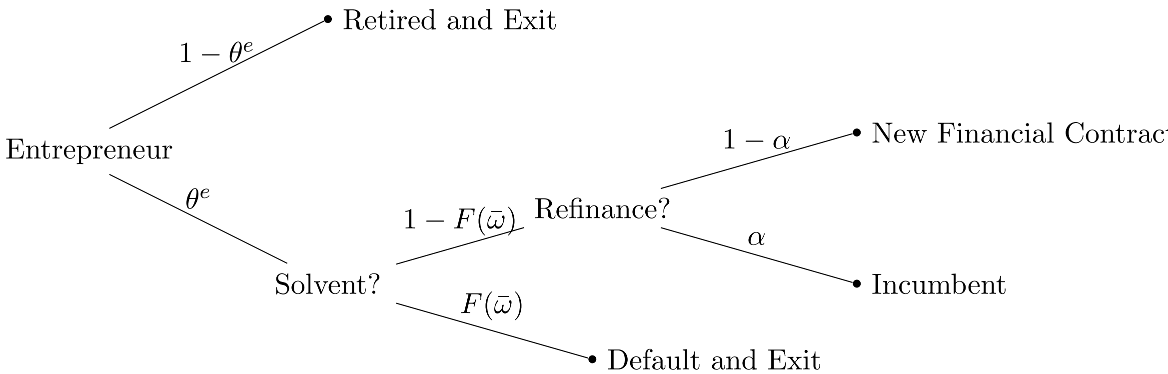
$$E\{R_{t+1}^k\} = s\left(\frac{N_t^{e,i}}{Q_t K_t^i}\right) R_{t+1}^D \tag{8}$$

with $s'(\cdot) < 0$ Details of the contracting problem are derived in the technical appendix. Equation (8) gives a relationship between capital demand and borrowing cost. The proportionality factor $s(\cdot)$ is increasing in maturity parameter α, which reflects the term premium. A rise in $E_t\{\frac{R_{t+1}^k}{R_{t+1}^D}\}$ reduces the expected default probability, therefore the entrepreneur takes on more loan and expand the investment scale. The assumption of constant returns to scale implies that relationship (8) doesn't depend on any entrepreneur-specific factor, which ensures that the leverage ratios are identical within the optimizing entrepreneurs. By the same token, the refinancing entrepreneurs share the same expected default threshold value. Therefore equation (8) can be easily aggregated over refinancing entrepreneurs and expressed in terms of the leverage ratio

$$\tilde{\phi}_t^e = \Psi(E_t\{\frac{R_{t+1}^k}{R_{t+1}^D}\}) \tag{9}$$

with $\Psi(1) = 1, \Psi'(\cdot) > 0$. Equation 9 gives the optimal leverage ratio for the refinancing vintage. To avoid abusing notation let $\tilde{(\cdot)}$ denote the variable associated with the refinancing entrepreneurs, to be distinguished from the aggregate variables.

Aggregation of the Real Activities The following chart summarizes the events that could happen to an entrepreneur with both exogenous and endogenous probabilities



Among the three probabilities that trigger transitions, $1-\theta^e$, $F(\bar{\omega})$ and $1-\alpha$, only the default probability $F(\bar{\omega})$ is endogenous. The default probability of an entrepreneur depends on both the contractual interest rate and the leverage ratio. The former is fixed but the latter varies over time. Specifically, the leverage ratio of a entrepreneur depends on the whole history of idiosyncratic and aggregate shocks, and because of this maintaining tractability of the leverage ratios as well as the default probabilities is very hard. To circumvent this problem I make the following assumption to minimize the degrees of heterogeneity. Before refinancing entrepreneurs obtain new loans, all entrepreneurs engage in an insurance contract, which states that at the end of this period (after the realization of shocks) low-leveraged entrepreneurs transfer net worth to highly-leveraged entrepreneurs up to the point where the leverage ratios across entrepreneurs are equal. This contract works as a partial insurance scheme, since highly-leveraged entrepreneurs face higher default risk than low-leveraged ones and therefore ex ante entrepreneurs have the incentive to share the risk. These transfers reshuffle the net worth across incumbent entrepreneurs while keeping each existing loan to the firm it was initiated to, as well as the associated interest rate. Therefore the insurance contract doesn't imply full risk-sharing in the sense that the interest rates of loans pertaining to the firms stay unchanged and the associated risk for each firm is carried over. By making this simplifying assumption the only heterogeneity across incumbent firms is the fixed interest rate of loans, and the default cost is a reflection of the dispersion of loan rates instead of the leverage ratios. Given that loan rates are vintage-specific, the default probabilities are vintage-specific as well. However, the loan rate of a certain vintage is fixed over time (until refinancing) while the default probability of a certain vintage is a dynamic process. The evolutions of default probabilities will be discussed at the end of this section.

After the transfer of the net worth, the common leverage ratio of incumbent entrepreneurs is determined by the condition

$$\alpha(1-\delta)Q_t K_{t-1} = \alpha\theta^e[1-F(\bar{\omega}_t)]N^e_{e,t}\phi^e_t$$

The left hand side is the capital stock (net of depreciation) held by incumbent entrepreneurs. The leverage ratio of the incumbent entrepreneurs ϕ_t^e is pinned down by their capital holding and net worth. Therefore there are only two prevailing leverage ratios among all firms at a time, ϕ^e pertaining to incumbent entrepreneurs and $\tilde{\phi}^e$ pertaining to refinancing entrepreneurs.

The aggregate entrepreneur net worth N_t^e is comprised of the net worth of existing entrepreneurs, $N_{e,t}^e$ and the net worth brought by newly-entered entrepreneurs $N_{n,t}^e$

$$N_t^e = \theta^e[1 - F(\bar{\omega}_t)]N_{e,t}^e + N_{n,t}^e \tag{10}$$

The evolution of the net worth of the existing entrepreneur in period t,

$$\begin{aligned}
N_{e,t}^e &= (1-\alpha)R_t^k N_{t-1}^e \tilde{\phi}_{t-1}^e [1 - \Gamma(\bar{\omega}_t^{t-1})] \\
&+ \alpha(1-\alpha)R_t^k N_{t-1}^e \phi_{t-1}^e [1 - \Gamma(\bar{\omega}_t^{t-2})] \\
&+ \alpha^2(1-\alpha)R_t^k N_{t-1}^e \phi_{t-1}^e [1 - \Gamma(\bar{\omega}_t^{t-3})] \\
&+ \ldots \\
&+ \alpha^{t-\tau-1}(1-\alpha)R_t^k N_{t-1}^e \phi_{t-1}^e [1 - \Gamma(\bar{\omega}_t^{\tau})] + \ldots
\end{aligned} \tag{11}$$

Incoming entrepreneurs enter the industry with startup funds from their respective households. The amount of startup funds is proportional to the value of the total capital stock, with the implication that the household considers how much an entrepreneur needs to start a business.

$$N_{n,t}^e = \xi^e Q_t K_t \tag{12}$$

The refinancing entrepreneurs' capital demand $\tilde{K}_t$ (as mentioned above, $\tilde{}$ variable refers to the refinancing entrepreneurs) is comprised of the demand of incumbent entrepreneurs and the demand of newly-entered entrepreneurs

$$(1-\alpha)Q_t\tilde{K}_t = \{ \underbrace{(1-\alpha)\theta^e[1 - F(\tilde{\bar{\omega}}_t)]N_{e,t}^e}_{\text{incumbent entrepreneurs whose loans fall due}} + \underbrace{\xi^e Q_t K_t}_{\text{newly-entered entrepreneurs}} \}\tilde{\phi}_t^e$$

$\tilde{\phi}_t^e$ is the common leverage ratio of all the refinancing entrepreneurs. Newly-initiated loans $\tilde{L}_t$ acquired by the refinancing entrepreneurs and the existing loans add up to total outstanding loans

$$L_t = (1-\alpha)\tilde{L}_t + \alpha\theta^e[1 - F(\tilde{\bar{\omega}}_t)]L_{t-1} \tag{13}$$

$\tilde{R}_t^L$ can be interpreted as the spot interest rate on newly-initiated loans, which differs from the aggregate contractual loan rate R^L since the latter consists of all the loan rates fixed in the previous periods. To be specific, the average borrowing cost R^L is a weighted average of the spot rate $\tilde{R}^L_t$ and interest rates of all vintages

$$R^L_t L_t = (1-\alpha)\tilde{R}^L_t \tilde{L}_t + \alpha\theta^e[1 - F(\tilde{\bar{\omega}}_t)]R^L_{t-1}L_{t-1} \tag{14}$$

Let $F(\bar{\omega}^\tau_t)$ be the default rate of vintage τ. The aggregate default rate is the weighted average of default rates across all vintages, expressed in the following way

$$F(\bar{\omega})_t = (1-\alpha)F(\tilde{\bar{\omega}}^{t-1}_t) + \alpha(1-\alpha)F(\bar{\omega}^{t-2}_t) + \alpha^2(1-\alpha)F(\bar{\omega}^{t-3}_t)... \tag{15}$$

Let $\tilde{L}_\tau$ and $R^{D,\tau}_t$ be the outstanding loans and ex post loan rate of vintage τ respectively. The aggregate return of loans R^D can be obtained by adding up returns of all vintages

$$R^D_t L_{t-1} = (1-\alpha)\tilde{R}^{D,t-1}_t \tilde{L}_{t-1} + \alpha(1-\alpha)R^{D,t-2}_t \tilde{L}_{t-2} + \alpha^2(1-\alpha)R^{D,t-3}_t \tilde{L}_{t-3} + ... \tag{16}$$

Equations 31, 16 and 15 contain indefinite terms, which generates an interesting technical challenge since they cannot be calculated without keeping track of the default rates of all vintages. This problem is solved in the following way. Given the assumption of the insurance contract that ensures a common leverage ratio across incumbent entrepreneurs, the default rate of a certain vintage τ at time t ($\tau \leq t-2$) depends on the aggregate technology R^k and balance sheet ϕ, and the vintage-specific loan rate $\tilde{R}^L_\tau$. However, since the only vintage-specific factor, the loan rate, is fixed over time, the evolution of the default rate only depends on the evolutions of the two aggregate variables. In particular, the default threshold value of vintage τ can be expressed in a recursive form

$$\begin{aligned}
& \bar{\omega}^\tau_{t+1} &&= \frac{R^L_\tau(\phi_t - 1)}{R^k_{t+1}\phi_t} \\
\Longrightarrow\quad & \bar{\omega}^\tau_{t+1} &&= \bar{\omega}^\tau_t \frac{R^k_t}{R^k_{t+1}} \frac{\phi_{t-1}}{\phi_{t-1}-1} \frac{\phi_t - 1}{\phi_t} \\
\overset{log-linearized}{\Longrightarrow}\quad & \hat{\bar{\omega}}^\tau_{t+1} &&= \hat{\bar{\omega}}^\tau_t + \hat{R}^k_t - \hat{R}^k_{t+1} + \frac{\hat{\phi}_t - \hat{\phi}_{t-1}}{\bar{\phi}-1}
\end{aligned}$$

The last equation is in log-linearized fashion, with $\hat{x}$ being the percentage deviation, and $\bar{x}$ being the steady state. With $\bar{\omega}^\tau_t$ being in a recursive form, the evolutions, in terms of percentage deviations, of aggregate default probability $F(\bar{\omega}_t)$ and the entrepreneur's share of profits $[1 - \Gamma(\bar{\omega}^{t-1}_t)]$, both of which are linear functions of $\hat{\bar{\omega}}^\tau_t$, can be expressed in recursive form. Therefore the evolutions of aggregate entrepreneur net worth, aggregate default rate and aggregate ex post return of loans can be traced over time in the log-linearized fashion.

2.3 Banking Sector

Bankers are risk neutral, and each one manages a bank. In each period a banker retires with a constant probability $1-\theta^b$. Upon retirement the banker becomes a worker and returns his net worth to the respective family. A corresponding number of workers randomly become bankers to keep the number of each occupation constant. The newly-entered bankers receive startup funds from the families to take over the long-term loans from the retiring bankers. Each bank holds a market portfolio of loans. The banker j aims to maximize his expected terminal wealth, denoted by V_t^j

$$\begin{aligned} V_t^j &= \max \mathbb{E}_t \sum_{\tau} \beta^{\tau}(1-\theta^b)(\theta^b)^{\tau} N_{t+\tau+1}^{b,j} \\ &= \max \mathbb{E}_t \sum_{\tau} \beta^{\tau}(1-\theta^b)(\theta^b)^{\tau}[R_{t+\tau+1}^D L_{t+\tau}^j - R_{t+\tau}(L_{t+\tau}^j - N_{t+\tau}^{b,j})] \end{aligned}$$

$N^{b,j}$ is the net worth of bank j. Alternatively, V_t^j can be expressed in the forward-looking fashion

$$V_t^j = \nu_t L_t^j + \eta_t N_t^{b,j} \tag{17}$$

where the respective shadow value of L_t^j and $N_t^{b,j}$ are given by the following,

$$\nu_t = E_t\{(1-\theta^b)\beta(R_{t+1}^D - R_t) + \beta\theta^b x_{t,t+1}\nu_{t+1}\}$$

$$\eta_t = E_t\{(1-\theta^b)\beta R_t + \beta\theta^b z_{t,t+1}\eta_{t+1}\}$$

with $x_{t,t+1} \equiv L_{t+1}^j / L_t^j$, $z_{t,t+1} \equiv N_{t+1}^{b,j}/N_t^{b,j}$. $x_{t,t+1}$ is the gross growth rate of loans, and $z_{t,t+1}$ is the gross growth rate of bank net worth. ν_t can be interpreted as the expected discounted marginal gain of initiating one more unit of loan, holding net worth constant, while η_t is the expected discounted value of building up another unit of net worth, holding the balance sheet size constant. In a frictionless capital market the banker will build up his assets indefinitely by borrowing from workers until the net return of assets is zero. To place a limit on arbitrage, I follow the setup of GK and introduce a moral hazard problem between bankers and workers, in order to prevent excess credit supply that wipes out the interest spread between loans and deposits. It is assumed that the banker has an incentive to renege on debts (deposits) and divert a fraction λ of his assets (loans) for his own consumption. If this happens the bank is forced into bankruptcy and the workers recover the remaining $(1-\lambda)$ of assets. To prevent the banker from diverting funds, workers refrain from lending excessively to bankers. In particular, the amount of lending will be constrained by

$$V_t^j \geq \lambda L_t^j \tag{18}$$

The left-hand side is the terminal wealth that the banker would lose if he diverts the assets; the right-hand side is his gain from doing so. The incentive constraint can be equivalently expressed in a forward-looking way

$$\nu_t L_t^j + \eta_t N_t^{b,j} \geq \lambda L_t^j \tag{19}$$

The optimal decision for the banker is to borrow up to the point when the gain of diverting assets is exactly balanced by the cost. Equation (19) is binding only if $0 < \nu_t < \lambda$: if $\nu_t > \lambda$, it is not worth diverting the funds since the franchise value of the bank is higher. Under reasonable parameterizations the borrowing constraint is always binding around the steady states. The binding borrowing constraint gives rise to an endogenous bank leverage ratio

$$\begin{aligned} L_t^j &= \frac{\eta_t}{\lambda - \nu_t} N_t^{b,j} \\ &= \phi_t^b N_t^{b,j} \end{aligned}$$

where ϕ_t^b denotes the leverage ratio of bank j. Since ϕ_t^b doesn't depend on any bank-specific factor, the leverage ratio is identical across all banks. The net worth of bank j is evolving according to

$$N_t^{b,j} = \theta^b[(R_t^D - R_{t-1})\phi_{t-1}^b + R_{t-1}]N_{t-1}^{b,j}$$

It follows that the gross growth rates of bank net worth and loans are not bank-specific either

$$z_{t,t+1} \equiv N_{t+1}^{b,j}/N_t^{b,j} = (R_{t+1}^D - R_t)\phi_t^b + R_t \tag{20}$$

$$x_{t,t+1} \equiv L_{t+1}^j/L_t^j = (\phi_{t+1}^b/\phi_t^b)(N_{t+1}^{b,j}/N_t^{b,j}) = (\phi_{t+1}^b/\phi_t^b)z_{t,t+1} \tag{21}$$

Due to the common growth rates and leverage ratios, the evolution of total bank net worth can be aggregated over individual banks

$$N_{e,t}^b = \theta^b[(R_t^D - R_{t-1})\phi_{t-1}^b + R_{t-1}]N_{t-1}^b \tag{22}$$

where $N_{e,t}^b$ on the left hand side denotes the net worth of banks surviving to period $t+1$. The aggregate bank net worth N_t^b consists of the former and the startup funds brought in by incoming bankers

$$N_t^b = N_{e,t}^b + N_{n,t}^b \tag{23}$$

with $N_{n,t}^b$ being the startup funds of incoming bankers, taken from the corresponding families. The amount of startup funds is made sufficient for new banks to take up the nonmaturing loans from retiring bankers. The amount of startup funds is a fraction ξ^b of the total value of loans, with ξ^b calibrated to ensure the takeover of the outstanding long-term loans of retiring banks. An alternative way, as suggested by AFZ, to guarantee the inheritance of old loans is to introduce an insurance agency financed by a proportional tax on banks' profits. In each period the insurance agency endows the new banker with funds out of the tax revenue. The two alternative assumptions result in different calibrated values for λ, θ^b and ξ^b, but the steady state values are not affected by the choice of assumptions. Here I stick to the first assumption for its brevity.

$$N_{n,t}^b = \xi^b L_t \tag{24}$$

Combining equations (22) and (24) yields the law of motion of aggregate bank net worth

$$N_t^b = \theta^b[(R_t^D - R_{t-1})\phi_{t-1}^b + R_{t-1}]N_{t-1}^b + \xi R_t^D L_t \tag{25}$$

Given a common leverage ratio across banks, the aggregate credit supply is obtained by

$$L_t = \phi_t^b N_t^b \tag{26}$$

2.4 Capital Producing Firms

Capital goods producers are competitive and owned by households. Each period they buy capital from entrepreneurs, refurbish depreciated capital and produce new capital. The cost of repairing worn out capital is unity. In the beginning of each period capital producing firms package both the new and repaired capital and re-sell them to entrepreneurs at a unit price Q_t. Following Christiano, Eichenbaum and Evans (2005), I allow for adjustment costs of investment flow $f(\cdot)$. Capital producer's objective is to maximize profits by choosing I_t

$$\max_{I_t} E_t \sum_{\tau}^{\infty} \beta^{\tau} \frac{\varrho_{t+1+\tau}}{\varrho_{t+\tau}} \{Q_{t+\tau} I_{t+\tau} - [1 + f\left(\frac{I_{t+\tau}}{I_{t+\tau-1}}\right)] I_{t+\tau}\}$$

The adjustment technology has the following properties: $f(1) = f'(1) = 0$, and $f''(1) > 0$. These properties are sufficient to justify that there is no adjustment cost in the steady state. The optimal investment decision yields the price of capital

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \frac{\varrho_{t+1}}{\varrho_t}\left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right)$$

2.5 Market Clearing Conditions

The aggregate demand for capital is the sum of the demand by refinancing entrepreneurs and the existing capital stock of ongoing projects,net of depreciation.

$$K_t = (1 - \alpha)\tilde{K}_t + \alpha(1 - \delta)K_{t-1} \tag{27}$$

Table 1: Conventional Parameters

Parameter	Description	Value
β	Household discount factor	0.995
ϵ	Factor share of capital	0.33
φ	Inverse Frisch elasticity	0.27
h	External habit formation	0.7
η^i	Investment adjustment cost	1.5
δ	Depreciation rate	0.025

The economy-wide resource constraint is given by

$$Y_t = C_t + I_t + R_t^k Q_{t-1} K_{t-1} \mu \Gamma(\bar{\omega}_t) + [1 + f(\frac{I_t}{I_{t-1}})] I_t \tag{28}$$

2.6 Macroeconomic Implications

2.6.1 Parameters

The conventional parameter values are set to be consistent with much of the available literature, as reported in Table 2.6.1. Table 2.6.1 reports the parameters specific to financial frictions. The parameters pertaining to the entrepreneurial sector are mostly set to be in line with BGG: the steady state quarterly default rate is 0.0075; the bankruptcy cost μ is 0.12. The standard deviation of the idiosyncratic productivity shock σ is calibrated to achieve the targeted entrepreneurial leverage ratio of 1.8, which matches the average leverage ratio of the nonfinancial corporate business sector in the Flow of Funds. The default rate, the bankruptcy cost and standard deviations jointly pin down the steady state interest spread $R^k - R^D$, which equals to 40 basis points, and the steady state interest spread of $R^D - R$ is set to 20 basis points [8]. θ^e and ξ^e are pinned down by the balance sheet of non-refinancing entrepreneurs and refinancing entrepreneurs respectively. λ, θ^b are calibrated to achieve the targeted bank leverage ratio of 8, consistent with Gertler and Kiyotaki (2012). ξ^b is calibrated to ensure that incoming bankers have sufficient net worth to effectively take up the outstanding long-term contracts. In the following experiments, the parameter α that governs the average maturity of loans are set to various values in order to examine its implications for business cycles.

2.6.2 Steady State Analysis

As shown by Figure 3, the sensitivity of interest spread $R^K - R^D$ to the entrepreneurial leverage ratio ϕ^e, measured by χ, is increasing in the degree of maturity mismatch α. The

[8] Data source: Bankscope

Table 2: Parameters Pertaining to Financial Frictions

Parameter	Description	Value
μ	Bankruptcy cost	0.12
σ^2	Variance of idiosyncratic shocks	0.3
$F(\bar{\omega})$	Steady state default rate	0.75%
θ^e	Survival probability of entrepreneur	0.968
ϕ^e	Firm leverage ratio	1.8
λ	Fraction that banker can divert	0.33
θ^b	Survival probability of banker	0.975
ϕ^b	Bank leverage ratio	8
$R^K - R$	Spread	60 BPS

positive relationship between χ and α reflects the term premium, which implies that the future default risk has been priced into the loan rate. Since χ is one of the measures of the strength of the credit channel, it can be inferred that the greater the degree of maturity mismatch, the greater the magnitude of the credit channel in the transmission of shocks. However, it is worth noting that the steady state values, including the interest spread $R^K - R^D$, don't depend on the value of α. Put differently, the degree of maturity mismatch affects the economy dynamics, but doesn't have an impact on resource allocation in the steady state.

2.6.3 Impulse Responses Analysis

Technology shock The properties of the model are shown by impulse response functions. To illustrate the transmission mechanism of shocks, I start with the analysis on the baseline mode, which is shown by the black dashed line. In the baseline model all loans are single-period ($\alpha = 0$), and the financial contract collapses to the one-period contract as in BGG. The technology shock considered here is characterized by a one percent innovation in technology with a quarterly autoregressive factor of 0.9. On impact, the aggregate return on capital declines and more entrepreneurs fall into bankruptcy. Aggregate entrepreneurial net worth falls sharply due to the lower return and the higher default rate, and the leverage ratio rises. Perceiving persistently low productivity, investment falls and drags down the price of capital. The return on capital shoots up in the second period because the capital price bounces back right after the shock. Due to the surge of capital return soon after the shock, entrepreneurs' profits start to pick up. The default rate stays high persistently and banks require a higher expected return on loans to compensate for the default cost. This is observed in the hike of both the contractual loan rate and the ex post loan rate. The banking sector builds up net worth since the rise in the loan rate is only partially passed to depositors and the interest spread increases. Depositors are less willing to lend to banks, and it can be seen as banks undergo deleveraging. The contraction of the bank balance sheet means a decline in loans initiated. Both higher demand for credit and lower credit supply contribute to the surge of interest spreads. $R^L - R^D$ represents the magnitude of the deadweight loss arising from default, and $R^D - R$ is the banking sector's profit margin. The spread between the return on capital and the deposit rate $R^k - R$ measures the overall financial (in)efficiency.

The macroeconomic implications of maturity mismatch can be analyzed by varying α. Solid blue lines show the scenario when $\alpha = 0.75$, i.e., average loan maturity of four quarters. The degree of maturity mismatch has been set to 4 because empirical evidences suggest that the observed average duration of bank assets is about four times the duration of their liabilities, as documented by Jochen Guentner (2010). In what follows, $\alpha = 0.75$ will be most of the time used to calibrate the economy with maturity mismatch. As shown by the solid blue lines, he responses of output, investment and capital price in an economy with maturity mismatch are milder than in the baseline model represented by the dashed black lines. Since three quarters of the entrepreneurs are locked in the fixed interest rates, they are immune to the surge in contractual loan rate. Therefore the decline in aggregate entrepreneur net worth is relatively muted. On the other hand, the responses of financial sector are more pronounced in the presence of long-term loans. Due to sluggish adjustments of loan rates, bank net worth and leverage ratio reach the peak at a later period in comparison to the baseline model. Interest spread $R^D - R$ rises more sharply, suggesting that banks are exposed to more volatile movements of net interest margin. This explains the amplified fluctuations in bank net worth and leverage ratio. Given that the rise in retained earnings outweighs the deleveraging effect, the contraction in credit supply is relatively moderate, and therefore the declines in total investment and output are mitigated.

Financial shocks Financial shocks have received much attention in recent studies since they are considered to be good candidates for explaining many stylized facts. Financial shocks, in general, are defined as the shocks affecting the ability of borrowing. In this model both entrepreneurs and bankers are borrowers within particular financial relationships, and hence financial shocks may emerge from either the real sector or the financial sector, both of which deserving of attention. Firstly, I consider a one-time negative shock (5%) to the entrepreneurial net worth. The negative wealth shock directly affects entrepreneurs' capital buffer and the cost of borrowing. Although all entrepreneurs' net worth is hit by the negative shock, three quarters of the entrepreneurs are immune to the surge of high borrowing cost due to the fixed interest rates. Default probability is much lower in the scenario with fixed interest rates, and so is the default cost. The banks require a higher expected loan rate to lend out loans in response to the shock. This is shown by a more volatile interest margin $R^D - R$ and the retained earnings for banks. Bank net worth functions as a buffer in the propagation of this shock, and dampens the impact of the shock on the real sector.

Next I consider two financial shocks originating from the financial sector: a negative shock to bank net worth and a reduction of the depositors' confidence in bankers. Given that banker's ability to borrow is constrained by the bank net worth, a 5% negative wealth shock reduces the amount of deposits that banks can take up and hence the amount of loans initiated. Since the shock emerges from the financial sector and affects all banks in the same way, the interest spreads of $R^D - R$ in both baseline model and the model with maturity mismatch track closely, and so do the financial variables. But in the presence of maturity mismatch, the contractual loan rate and the ex post loan rate have milder adjustment paths and entrepreneurs are less exposed to default risk. The attenuation effect on the real sector implied from the previous experiments carries over to this scenario.

A reduction in depositors' confidence directly affects the banking sector's leverage ratio allowed by the depositors. This is done by giving a persistent (autoregressive factor of 0.9) negative shock of one percent to Λ, the fraction of assets that bankers can divert. The confidence shock is transmitted form the liability side of the bank balance sheet to the asset side, opposite to shocks originating from the real sector. To attract deposits the bankers have to offer a higher real interest rate, but again, due to the sticky loan rates the banking sector cannot fully pass the higher borrowing cost to entrepreneurs. The banking sector takes the interest rate risks and partially shields the real sector from fluctuations of interest rates.

In general, the conclusion that the banking sector with maturity mismatch is stabilizing the economy over the business cycles is robust to various shocks considered, independent of where the shocks originate. The introduction of maturity mismatch offsets the accelerator effects caused by financial frictions, and the bank net worth functions as a buffer by dampening business cycle fluctuations.

Optimality of maturity transformation The next question that comes naturally is what degree of maturity mismatch on the bank balance sheet is desirable from a macroeconomic point of view. As an attempt to answer this question, Figure 9 provides a comparison between economies with various degrees of maturity mismatch to its efficient level, i.e., the economy absent of any financial frictions. This exercise is conducted by simulating the modeled economies to a negative technology shock of one percent in size. The dash dotted (red) line represents the IRFs of the simple real business cycle model without any financial frictions, i.e., in the absence of asymmetric information problem and moral hazard problem. The incorporation of the asymmetric information problem between entrepreneurs and banker as well as the moral hazard problem between bankers and depositors greatly amplifies the shocks, as shown by the baseline model (the dash black line). The dotted green line stands for the economy with average loan maturity of ten quarters ($\alpha = 0.9$), which predicts a stronger attenuation effect than $\alpha = 0.75$. But in the scenario of $\alpha = 0.9$ the economy is driven further apart from the efficient level, since the real sector is constrained and doesn't fully respond to the disturbance in an optimal manner. Potentially there exists an optimal degree of maturity mismatch that offsets the amplification effect of financial frictions and helps the economy to realize its full potential. This exercise provides insight into the financial regulation from a macroeconomic perspective. Existing financial regulation policies primarily aim to limit the distress of individual institutions; the macroprudential policies, however, can improve the overall welfare of the economy by designing proper regulations and tools. As this exercise suggests, an optimal targeted degree of maturity mismatch on banks' balance sheets can help the economy realize its full potential.

3 A Model with Nominal Rigidities

Allowing for a role of monetary policy, in this section I extend the model with nominal rigidities. While the major framework of the RBC model is preserved, a few modifications are made on the setup of the real sector and the monetary authority.

3.1 Intermediate Goods Firms

Following the conventional setup of New Keynesian models, the real sector is comprised of intermediate goods firms and retail firms. Entrepreneurs, as presented in the previous section, produce intermediate goods and thus will be labeled as intermediate goods firms in this section. Intermediate goods are eventually sold to retail firms at the relative price of mc_t (in the period t). The real wage and the return on capital therefore follow

$$W_t = (1-\varpi)\frac{Y_t}{H_t}mc_t$$

$$R^k_{t+1} = \frac{mc_{t+1}\varpi\frac{Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t} \tag{29}$$

The remaining setup and the evolution of the real variables are the same as in Section 2.

3.2 Retail Firms

Final output Y_t is a CES (constant elasticity of substitution) composite of a continuum of mass unity of differentiated retail firms.

$$Y_t = [\int_0^1 Y_t^{f\,\frac{\varepsilon-1}{\varepsilon}} df]^{\frac{\varepsilon}{\varepsilon-1}} \tag{30}$$

where ε is the elasticity of substitution, and Y_t^f is the product of retail firm f. Cost minimization implies the standard demand function

$$Y_t^f = (\frac{P_t^f}{P_t})^{-\varepsilon} Y_t \tag{31}$$

where P_t^f is the price of the retail good of firm f. The aggregate price level is thus given by

$$P_t[\int_0^1 P_t^{f\,1-\varepsilon} df]^{1/1-\varepsilon} \tag{32}$$

Retailers re-package intermediate output. It takes one unit of intermediate output to make a unit of retail output. The real marginal cost is thus the relative intermediate output price mc_t. Retailers set nominal prices in a staggered fashion à la Calvo (1983). Each period the retailer is able to freely adjust price with probability $1-\varsigma$, and the optimal price set in period t is denoted as P_t^*. The remaining fraction ς of retailers simply keep their prices constant $P_t^f = P_{t-1}^f$. The retailer's pricing problem is to choose the optimal price P_t^* to maximize the expected profits

$$\underset{P_t^*}{max} E_t \sum_{i=0}^{\infty} \varsigma^i \beta^i \frac{\rho_{t+i}}{\rho_t} (\frac{P_t^*}{P_{t+i}} - mc_{t+i}) Y_{t+i}^f \tag{33}$$

subject to the individual demand function (31). From the law of large number, the evolution of the price level is given by

$$P_t = [(1-\varsigma) P_t^{*\,\frac{1}{1-\varepsilon}} + \varsigma P_{t-1}^{1/1-\varepsilon}]^{1-\varepsilon} \tag{34}$$

3.3 Monetary Policy

Monetary policy is characterized by a simple Taylor rule with interest-rate smoothing. Let $\bar{R}^n$ be the steady state nominal rate, $\bar{Y}$ the steady state level of output and $\bar{\Pi}$ the steady state inflation rate.

$$R_t^n = \rho_{R^n} R_{t-1}^n + (1-\rho_{R^n}) \left[\bar{R}^n + \phi_\Pi \log\left(\frac{\Pi_t}{\bar{\Pi}}\right) + \phi_Y \log(\frac{Y_t}{\bar{Y}}) \right] + \epsilon_t^{R^n} \tag{35}$$

where the smoothing parameter ρ_{R^n} lies between zero and unity, and ϵ_t is an exogenous shock to monetary policy. The relationship between nominal and real interest rates is given by the Fisher equation

$$R^n = R_t \Pi_{t+1} = R_t \frac{P_{t+1}}{P_t} \tag{36}$$

3.4 Parameters

The parameters pertaining to the retail sector and the monetary authority are fairly standard in the New Keynesian literature. The steady state inflation rate $\bar{\Pi}$ is assumed to be 1. The elasticity of substitution ε is set to be 7, and the probability of reoptimization of price $1-\varsigma$ is 0.25. The policy parameters ρ_{R^n}, ϕ_Π and ϕ_Y are given by 0.88, 1.5 and 0.5/4, respectively.

3.5 Implications for Monetary Policy

This section examines the effects of maturity mismatch on the transmission mechanism of monetary policy. Figure 10 and 11 report the impulse responses of aggregate variables to an expansionary monetary policy shock with the size of one percent. The dotted red line represents the standard New Keynesian model without financial frictions, namely, absence of asymmetric information and moral hazard problem. Both the dashed black and the solid blue lines represent the models augmented with financial frictions, with the former being the baseline model with only one-period loans and the latter embedded with maturity mismatch.

The decrease in the nominal interest rate stimulates the investment and leads to an economic boom. The unanticipated rise in inflation following the shock reduces the ex-post real interest rates paid by the intermediate goods firms. The reduction in the real debt repayment increases the entrepreneur's net worth on impact and eases the entrepreneur's borrowing condition. As shown by the differences between the baseline model and the standard New Keynesian model, the presence of financial frictions significantly strengthen the transmission mechanism of monetary policy. The response of output on impact is approximately three times greater in the scenario with financial frictions.

In comparison to the baseline model, the full model with maturity mismatch predicts an attenuation effect on the economic activities. Because a fraction α of entrepreneurs are locked in the fixed-rate contracts, banks' adjustment of lending and pricing of loans doesn't affect this cohort of the entrepreneurs. Put differently, banks cannot fully pass on changes in nominal deposit rates to retail lending rates. This results in the smoother movements of both contractual loan rates and ex post loan rates in the full model, as well as a milder accumulation in the entrepreneurial net worth. By the same token, only a fraction $1-\alpha$ of intermediate goods firms are exposed to the fluctuations in borrowing costs, and therefore the impact on the marginal cost is muted and prices rise by a smaller magnitude. The milder (unanticipated) inflation implies a milder debt-deflation effect on the entrepreneurial net worth, and this also contributes to a mitigated increase in the investment.

In the baseline model the output rises by 2% on impact, which peaks at approximately 3% three quarters after the shock. With the average loan maturity of four quarters, the response of the output on impact is approximately 1.3%, and it peaks at 1.5% two quarters later. The attenuation effect is very persistent throughout the dynamics of the output. The impact of a monetary policy innovation on the output is weakened by roughly one third in the full model. The dampening effect on the inflation dynamics is less persistent, and the response of the inflation is reduced by approximately 1% throughout the first year. These results present a challenge to the quantitative implications of conventional DSGE models, which based on the assumption of one-period loans. Financial frictions do magnify monetary policy shocks, however, the amplification effects can be significantly weakened in the presence of maturity mismatch. These results deliver an important message for policymakers: in order to combat an adverse shock, monetary policy has to act more aggressively than suggested by the existing New Keynesian models.

4 Conclusion

This paper develops a DSGE model in which the banking sector engages in both monitoring and maturity transformation. The banking sector issues long-term loans at fixed interest rates. Banks are exposed to interest rate risks due to the presence of maturity mismatch on banks' balance sheets. This feature gives rise to the amplified fluctuations of financial variables in face of shocks, while the shocks transmitted to the real sector are therefore attenuated. The business cycles implication is that the banking sector is stabilizing the economy by offering stable long-term credit, and the bank balance sheet functions as a shock absorber that shields the real sector from fluctuations in interest rates. These findings suggest that previous studies without the specification of maturity mismatch tend to overestimate the amplification effect caused by financial frictions, and therefore the implications of financial frictions for the business cycles should be revisited.

Maturity mismatch also alters the transmission mechanism of monetary policy because the banking sector cannot fully pass on the change in the interest rate of deposits to the interest rate of loans. This result challenges the previous studies on the qualitative effect of monetary policy and suggests that the monetary policy is less effective than it is documented in DSGE literature. The model also contributes to policy debates on financial regulation issues. In particular, the optimality analysis suggests that there exists an optimal degree of maturity mismatch that can potentially help the economy realize its full potential. This result sheds some light on the design of macroprudential tools.

Extensions on top of this model can be done along many dimensions. One dimension to bring the model closer to reality is to account for the procyclical pattern of loan maturities, as observed in the data[9]. The time-varying loan maturity can be incorporated by solving a fully

[9] See Adrian, Colla and Shin(2012)

dynamic multi-period financial contract. In addition, a welfare analysis can be conducted in search for the optimal degree of maturity mismatch from a macroeconomic perspective.

References

[1] Adrian, T., P. Colla and H. S. Shin, 2012. "Which Financial Frictions? Parsing the Evidence from the Financial Crisis of 2007-9," NBER Chapters, in: *NBER Macroeconomics Annual 2012*, Volume 27 National Bureau of Economic Research, Inc.

[2] Allen, F. and D. Gale, 2009. "Understanding Financial Crises," OUP Catalogue, Oxford University Press, number 9780199251421.

[3] Andreasen, M., M. Ferman and P. Zabczyk, 2012. "The business cycle implications of banks' maturity transformation," *Review of Economic Dynamics*.

[4] Angeloni, I. and E. Faia, 2013. "Capital Regulation and Monetary Policy with Fragile Banks," *Journal of Monetary Economics*, lead article, 60, 3, pp. 3111-382, April 2013.

[5] Bernanke, B. and M. Gertler, 1989. "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, vol. 79(1), pages 14-31, March

[6] Bernanke, B., M. Gertler and S. Gilchrist, 1999. "The financial accelerator in a quantitative business cycle framework," *Handbook of Macroeconomics*, in: J. B. Taylor and M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 21, pages 1341-1393.

[7] Brunnermeier, M. K., T. M. Eisenbach and Y. Sannikov, 2012. "Macroeconomics with Financial Frictions: A Survey," NBER Working Papers 18102, National Bureau of Economic Research, Inc.

[8] Calomiris, C. W. and C. M. Kahn, 1991. "The Role of Demandable Debt in Structuring Optimal Banking Arrangements," *American Economic Review*, vol. 81(3), pages 497-513, June.

[9] Calvo, G. A., 1983. "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, vol. 12(3), pages 383-398, September.

[10] Christiano L., M. Eichenbaum and C. L. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, vol. 113(1), pages 1-45, February.

[11] Christiano L., R. Motto and M. Rostagno, 2010. "Financial factors in economic fluctuations," Working Paper Series 1192, European Central Bank.

[12] Christensen, I. and A. Dib, 2008. "The Financial Accelerator in an Estimated New Keynesian Model," *Review of Economic Dynamics*, vol. 11(1), pages 155-178, January.

[13] Diamond, D. W. and P. Dybvig, 1983. "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, vol. 91(3), pages 401-19, June.

[14] Gerali, A., S. Neri, L. Sessa and F. M. Signoretti, 2010. "Credit and Banking in a DSGE Model of the Euro Area," *Journal of Money, Credit and Banking.*

[15] Gertler, M., 1992. "Financial Capacity and Output Fluctuations in an Economy with Multi-period Financial Relationships," *Review of Economic Studies*, vol. 59(3), pages 455-72, July.

[16] Gertler, M. and P. Karadi, 2011. "A model of unconventional monetary policy," *Journal of Monetary Economics*, vol. 58(1), pages 17-34, January.

[17] Gertler, M. and N. Kiyotaki, 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis," Handbook of Monetary Economics, in: Benjamin M. Friedman and Michael Woodford (ed.), *Handbook of Monetary Economics*, edition 1, volume 3, chapter 11, pages 547-599.

[18] Güntner,J. H. F., 2010. "Borrow Short to Lend Long: A DSGE Model of Maturity Transformation". Mimeo.

[19] Holmstrom, B. and J. Tirole, 1997. "Financial Intermediation, Loanable Funds, and the Real Sector," The *Quarterly Journal of Economics*, vol. 112(3), pages 663-91, August.

[20] Holmstrom, B. and J. Tirole, 1998. "Private and Public Supply of Liquidity," *Journal of Political Economy*, vol. 106(1), pages 1-40, February.

[21] Jermann, U. and V. Quadrini, 2012. "Macroeconomic Effects of Financial Shocks," *American Economic Review*

[22] Kiyotaki, N. and J. Moore, 1997. "Credit Cycles," *Journal of Political Economy*, vol. 105(2), pages 211-48, April.

[23] Meh, C. A. and K. Moran, 2010. "The role of bank capital in the propagation of shocks," *Journal of Economic Dynamics and Control*, vol. 34(3), pages 555-576, March.

[24] Rannenberg, A., 2012. "Asymmetric information in credit markets, bank leverage cycles and macroeconomic dynamics," Working Paper Research 224, National Bank of Belgium.

[25] Rochet, J. and X. Vives, 2004. "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?," *Journal of the European Economic Association*, vol. 2(6), pages 1116-1147, December.

[26] Townsend, R. M., 1979. "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory*, vol. 21(2), pages 265-293, October.

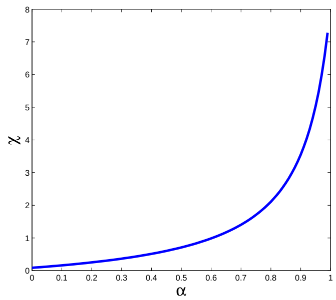


Figure 3: Relationship between χ and α

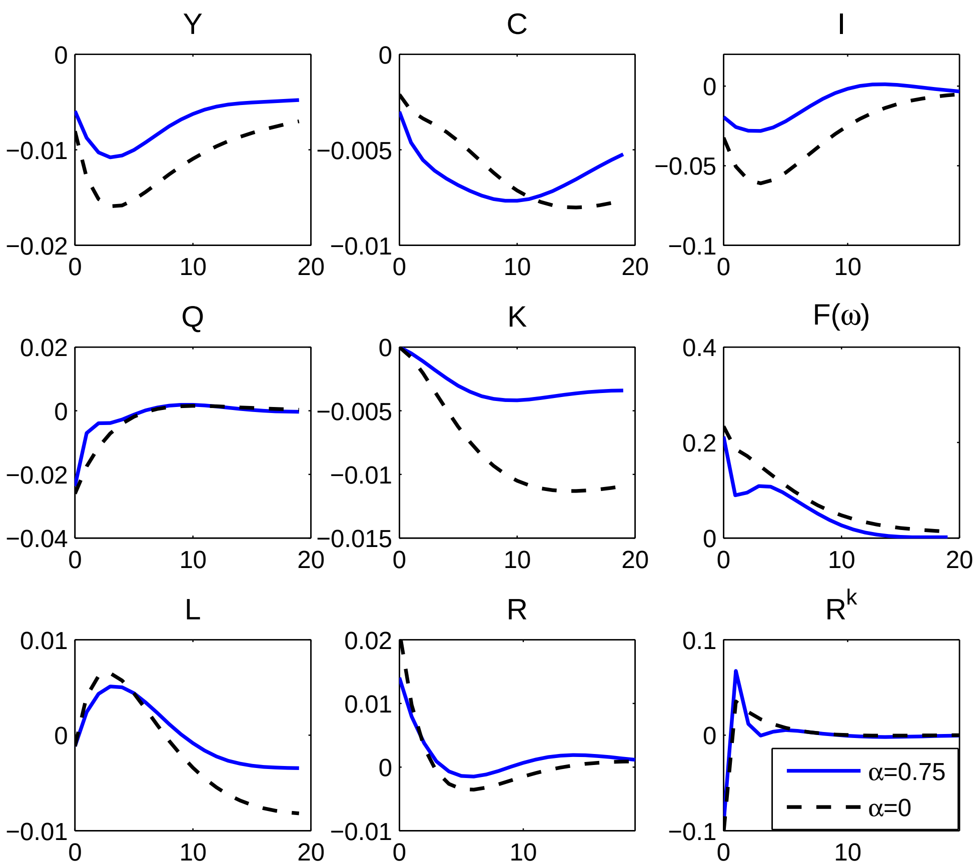


Figure 4: Responses to a 1% technology shock

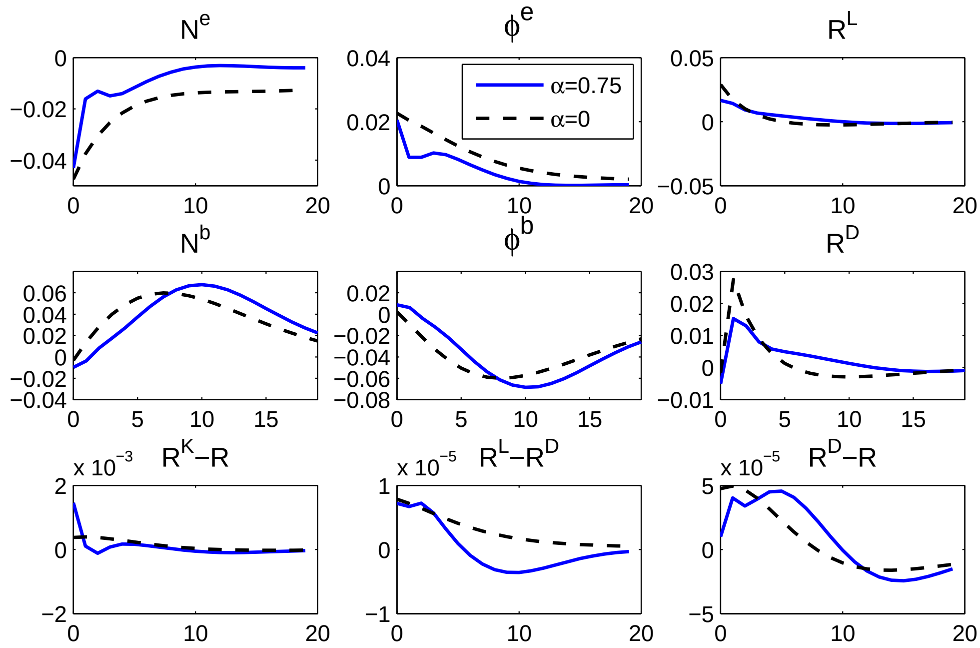


Figure 5: Responses to a 1% technology shock

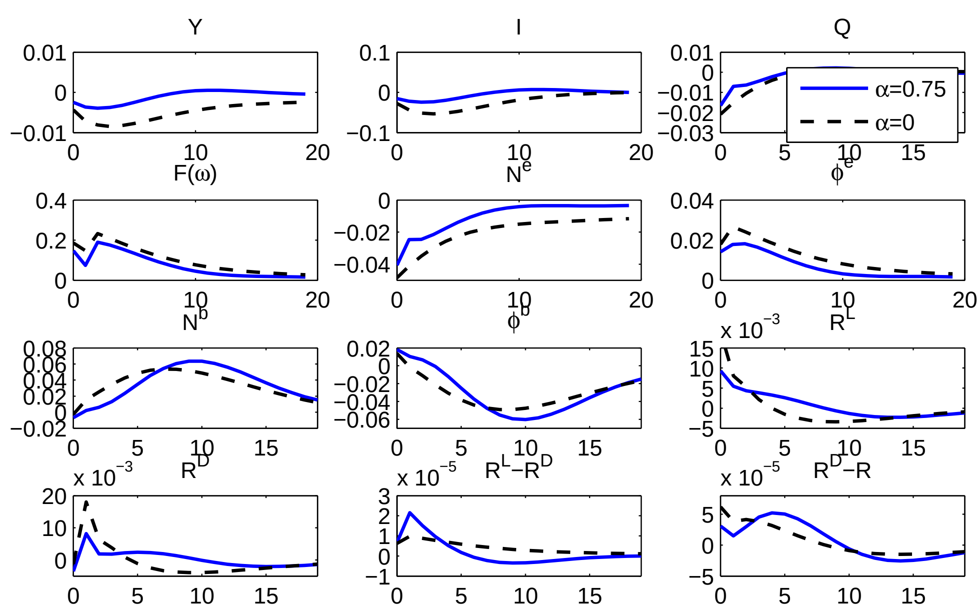


Figure 6: Responses to a 5% entrepreneurial net worth shock

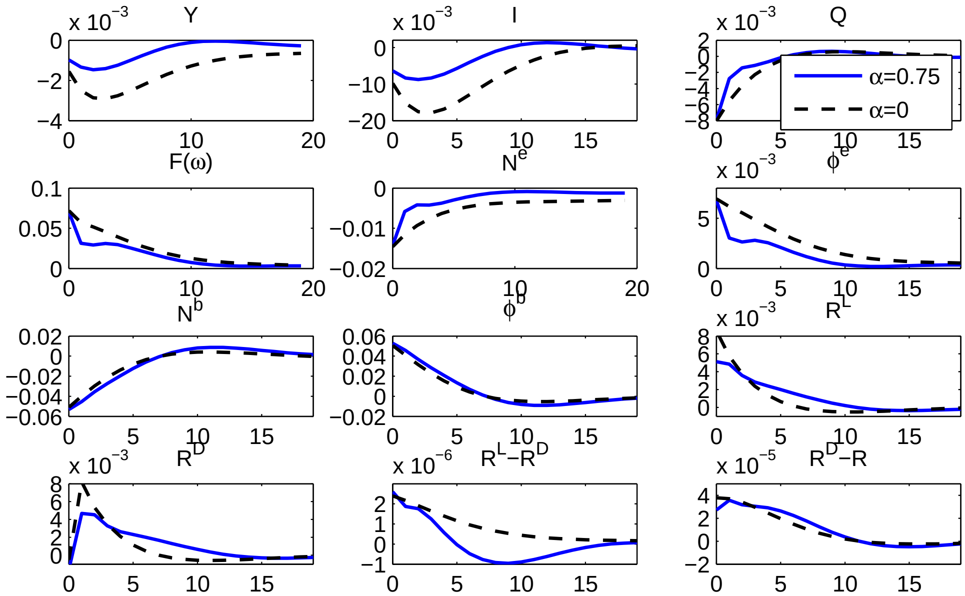


Figure 7: Responses to a 5% bank net worth shock

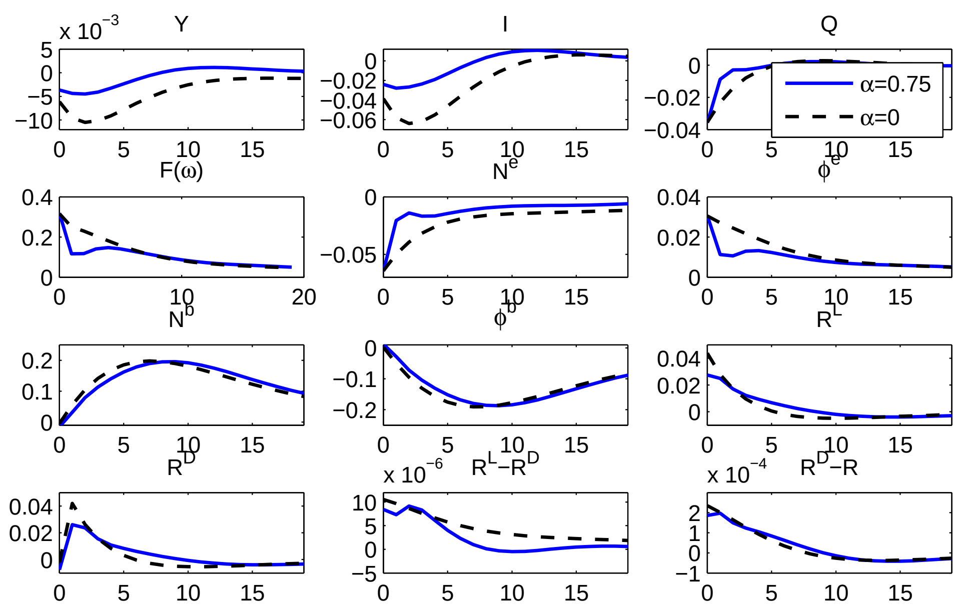


Figure 8: Responses to a 1% shock to depositors' confidence

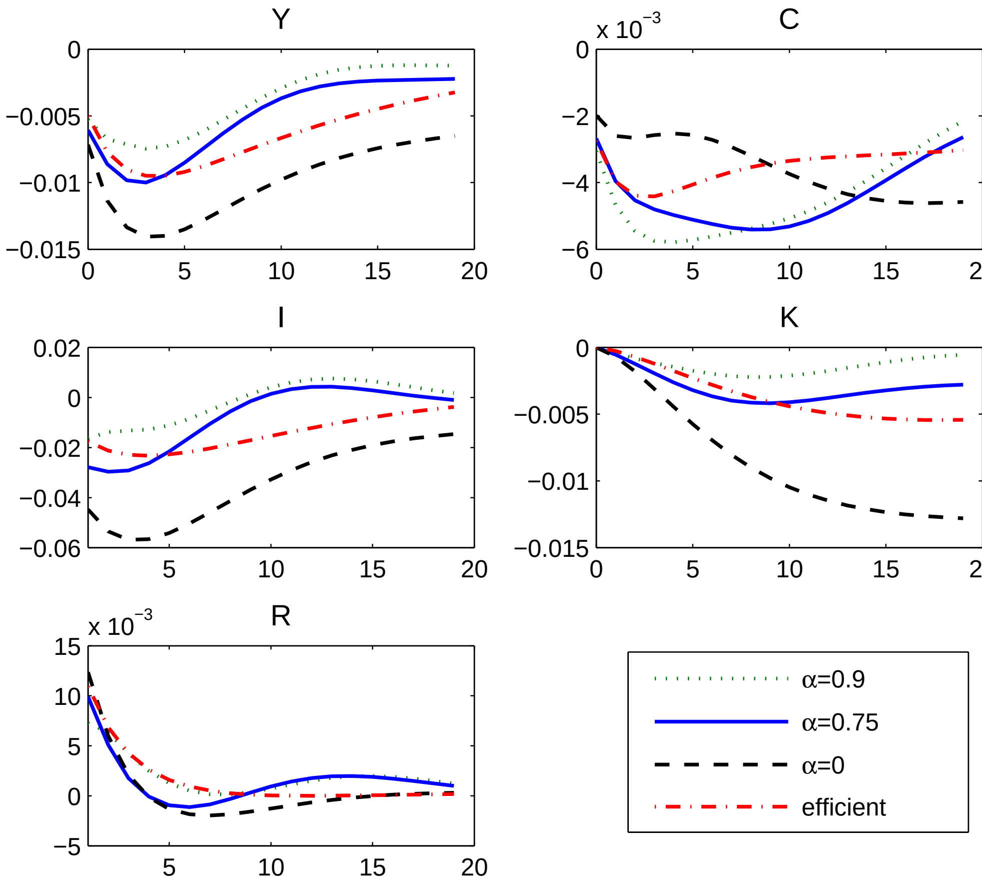


Figure 9: Maturity mismatch and the efficient economy (technology shock)

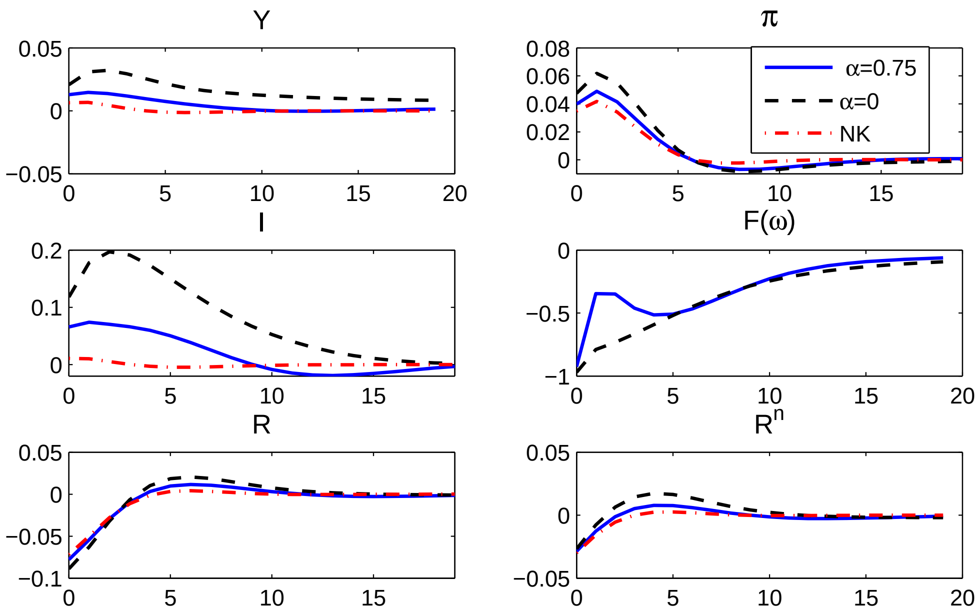


Figure 10: 1% expansionary monetary policy shock

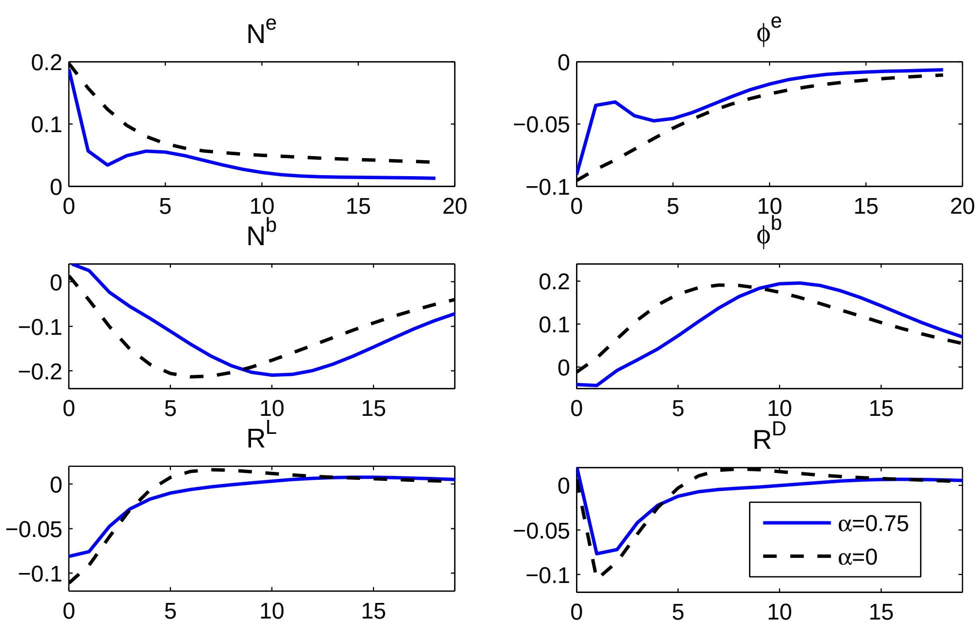


Figure 11: 1% expansionary monetary policy shock