Optimal investment taxes and efficient market provision of liquidity in the Diamond-Dybvig model*

Borys Grochulski† Yuzhe Zhang‡

PRELIMINARY DRAFT
May 5, 2014

Abstract

We study taxation of assets as an alternative to regulation of intermediaries in a Diamond-Dybvig economy with private ex post retrade and with private ex ante investment. The possibility of private investment imposes an additional constraint on the social planner beyond resource feasibility and incentive compatibility. With this constraint, the simple asset market equilibrium is efficient. If access to private investment is costly, i.e., the private return on the illiquid investment is smaller than the publicly observable return on illiquid investment, then an asset market equilibrium is efficient with investment taxes. An optimal tax system consists of a proportional subsidy to liquid investment and a proportional tax on illiquid investment. In either case, efficient levels of liquidity are attained in a market equilibrium without intermediation.

1 Introduction

The recent financial crisis sparked a renewed interest in studying financial intermediation and optimal regulation of intermediaries. The Diamond and Dybvig (1983) model is one of the main workhorses in the theory of intermediation and banking. In the Diamond-Dybvig (DD) model, intermediaries are useful because intermediated equilibrium can provide better liquidity insurance than competitive asset market equilibrium in which agents hold assets directly. In this paper, we are interested in taxation as an alternative to intermediation in the DD model. We seek to characterize a tax system implementing the optimal amount of liquidity insurance thus enabling markets to do as well as intermediation.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

†Federal Reserve Bank of Richmond, borys.grochulski@rich.frb.org.

‡Texas A&M University, yuzhe-zhang@econmail.tamu.edu.

To this end, in addition to private ex post retrade markets studied in Farhi et al. (2009), we explicitly include in the DD economy the possibility of private investment ex ante. In that economy, we study optimal allocations and compare them with asset market equilibria with investment taxes. We find an optimal tax system, i.e., one under which the asset market equilibrium with direct asset holding and no intermediation is efficient. In an optimal tax system, proportional taxes are levied on investment ex ante. In particular, the illiquid investment is taxed and the liquid investment is subsidized.

Private investment is modeled as follows. Ex ante, agents have the option to invest privately, i.e., not subject to taxation, in the liquid and the illiquid technology. We interpret private investment as transferring initial assets to another jurisdiction (another "island") and investing there. We allow for this activity to be costly, which in the model is represented a lower return on the illiquid asset if invested on the other island.

The possibility of private investment shrinks the set of feasible allocations in the planning problem. If the cost of private investment is zero, i.e., agents earn the same rate of return on the other island as at home, then the set of feasible allocations is a singleton: the only feasible allocation is the competitive asset market equilibrium allocation without taxes. This result is essentially already in Jacklin (1987).

If the cost of investing on the other island is positive, i.e., the return on private investment is lower there than the return on (publicly observable and taxable) investment at home, then the set of feasible allocations is larger than a singleton. We characterize the optimal allocation as a function of the level of return on the illiquid investment available on the other island. If this return is lower that a threshold, the ex ante private investment constraint does not bind and the first-best allocation is attainable as in Diamond and Dybvig (1983) and Farhi et al. (2009). Otherwise, the ex ante private investment constraint binds and the optimal allocation provides less liquidity insurance than the first best.

We show that all these allocations can be implemented with in an asset market equilibrium with investment taxes. An optimal tax system consists of a proportional subsidy to the liquid investment and a proportional tax on the illiquid investment. These taxes describe an investment wedge consistent with optimal provision of liquidity in the DD model. The optimal investment wedge becomes larger as the private return on the illiquid asset becomes smaller and the agents' value of the private investment option decreases.

Our analysis shows that investment taxes and subsidies can be an effective tool encouraging the provision of liquidity in the DD environment. These taxes thus can serve as an alternative to regulated intermediation. Farhi et al. (2009) show how in the presence of private ex post retrade markets the first-best allocation can be implemented with intermediation subject to minimum liquidity requirements. The tax implementation we consider can achieve the same outcome but has the advantage of not requiring balance sheet information. In particular, a unit of illiquid asset generates a proportional tax liability for the originator regardless of the composition of the originator's balance sheet, which makes it particularly easy to implement in practice. This form of liquidity regulation can be applied to banks as well as to non-bank originators of illiquid assets.

In our analysis, we follow Farhi et al. (2009) in expressing the planning problem in terms of the interest rate R in the private retrade market and the level of income I that agents take into this market. We show how the private investment constraint can be reduced to an additional constraint on the set of pairs (R, I) feasible in the planning problem.

This paper is related to two literatures. The first one studies optimal provision of liquidity insurance in DD economies: Diamond and Dybvig (1983), Jacklin (1987), Allen and Gale (2004), Farhi et al. (2009). The contribution of this paper to this literature is in characterization of optimal allocations of liquidity in the presence of private retrade markets ex post and private investment ex ante. The second literature is the optimal taxation literature in the spirit of Mirrlees (1971) (see Kocherlakota (2010) for a recent review of this literature). The contribution of this paper is to characterize optimal asset origination taxes in the DD environment.

In this paper, we assume full commitment on the side of the government. An interesting extension could be to consider government commitment frictions as in Bisin and Rampini (2006).

2 A Diamond-Dybvig economy with private markets and private investment

The model of DD is as follows. There is a continuum of ex ante identical agents. Each with endowment e. There are three dates: $t = 0, 1, 2$. At $t = 0$, agents can invest in two technologies (assets). The liquid, short-term asset pays at $t = 1$ a payoff of 1 per unit invested at date zero (let's call it the cash asset). The illiquid, long-term asset pays nothing at $t = 1$ and $\hat{R} > 1$ at $t = 2$ per unit invested at $t = 0$. Agents do not consume at $t = 0$. Their preferences over consumption at dates 1 and 2 are represented by the following utility function:

$$(1 - \theta)u(c_1) + \theta \rho u(c_1 + c_2),$$

where $\theta \in \{0, 1\}$ is the idiosyncratic shock and $0 < \rho < 1$. Let $\Pr\{\theta = 0\} = \pi > 0$. Note that if $\theta = 0$, the agent becomes at $t = 1$ extremely impatient: he only values consumption at

$t = 1$. If $\theta = 1$, the agent is indifferent with respect to the timing of consumption between date 1 and 2. Note that the marginal utility of consumption of the patient type is low, as $\rho < 1$. In addition, we follow DD assuming that relative risk aversion is at least 1, $-cu''(c)/u'(c) \geq 1$ for all c, and that $\rho\hat{R} > 1$. These assumptions imply a motive for interim-date redistribution to the impatient agent type.

An allocation c consists of $\{c_1(0), c_2(0), c_1(1), c_2(1)\}$, where $c_t(\theta)$ denotes date-t consumption for an agent with shock θ. Associated with any such allocation are initial asset investments $s_0 \geq 0$ in the short, liquid asset and $x_0 \geq 0$ in the illiquid asset.

An allocation is resource-feasible if

$$\pi\left(c_1(0) + \frac{c_2(0)}{\hat{R}}\right) + (1-\pi)\left(c_1(1) + \frac{c_2(1)}{\hat{R}}\right) \leq e. \tag{1}$$

Note that asset initial investments (s_0, x_0) associated with allocation $c = \{c_1(0), c_2(0), c_1(1), c_2(1)\}$ are

$$s_0 = \pi c_1(0) + (1-\pi)c_1(1)$$

and

$$x_0 = \left(\pi c_2(0) + (1-\pi)c_2(1)\right)/\hat{R},$$

so a more direct way of writing the feasibility constraints in this model could be to write the above two and $s_0 + x_0 \leq e$.

We follow DD in assuming that realizations of θ are private information. In addition, we follow FGT in assuming that individual consumption $c_t(\theta)$ is private and that agents have access to a hidden retrade market. More precisely, agents have private/unfettered access to a perfectly competitive and hidden market for 1-period IOUs at $t = 1$.

Comment: One can think of this market not necessarily as hidden but rather as unfettered. The government cannot intervene in this market. The reason why can but does not have to be private information.

Specifically, given an allocation $c = \{c_1(0), c_2(0), c_1(1), c_2(1)\}$ and a gross interest rate R in the hidden market, the agent of type θ solves

$$\begin{aligned}
&\max_{\tilde{\theta},\tilde{c}_1,\tilde{c}_2,\tilde{s}_1} U(\tilde{c}_1, \tilde{c}_2; \theta) \\
&s.t. \\
&\tilde{c}_1 + \tilde{s}_1 \leq c_1(\tilde{\theta}) \\
&\tilde{c}_2 \leq R\tilde{s}_1 + c_2(\tilde{\theta})
\end{aligned}$$

where $U(c_1, c_2; 0) = u(c_1)$ and $U(c_1, c_2; 1) = \rho u(c_1 + c_2)$. Denote this value by $\tilde{V}(c, R; \theta)$. Note that this value represents the agent's best strategy with respect to reporting θ as well as saving/borrowing in the private market. The IC constraints are

$$U(c_1(\theta), c_2(\theta); \theta) \geq \tilde{V}(c, R; \theta) \tag{2}$$

for both θ.

In addition to the IC constraint, we will impose the following ex ante participation constraint. We assume that agents have private access to both the liquid and illiquid investment technology at $t = 0$. Specifically, we assume that agents can transfer their initial endowment e to another jurisdiction/island and invest over there. To reflect potential costs of transferring resources to another jurisdiction, we allow for the rate of return on the illiquid technology to be lower on the other island. We denote this rate of return by R^o and assume $R^o \leq \hat{R}$. To avoid trivial cases, we assume $R^o > 1$. Importantly, if agents invest their resources in the other island, they can still access the private retrade market at $t = 1$ on their "home" island.

Formally, we add the following ex ante constraint

$$\pi U(c_1(0), c_2(0); 0) + (1 - \pi) U(c_1(1), c_2(1); 1) \geq \tilde{V}_0(R^o, R), \tag{3}$$

where

$$\begin{aligned}
\tilde{V}_0(R^o, R) \quad = \quad & \max_{\substack{\tilde{s}_0, \tilde{x}_0, \\ \tilde{s}_1(\theta), \\ \tilde{c}_1(\theta), \tilde{c}_2(\theta)}} \pi u(\tilde{c}_1(0)) + (1 - \pi) \rho u(\tilde{c}_1(1) + \tilde{c}_2(1)) \\
s.t. \quad & \tilde{s}_0 + \tilde{x}_0 \leq e, \\
& \tilde{c}_1(\theta) + \tilde{s}_1(\theta) \leq \tilde{s}_0, \ \theta = 0, 1, \\
& \tilde{c}_2(\theta) \leq R^o \tilde{x}_0 + R \tilde{s}_1(\theta), \ \theta = 0, 1.
\end{aligned}$$

In the above, $\tilde{s}_1(\theta)$ denotes the amount invested in IOUs in the private retrade market at date 1. For short, we will refer to the constraint (3) as constraint P.

The planning problem is to find allocation c that maximizes ex ante expected utility of the representative agent

$$\pi U(c_1(0), c_2(0); 0) + (1 - \pi) U(c_1(1), c_2(1); 1) \tag{4}$$

subject to the RF constraint (1), the IC constraint (2), and the private invetment constraint (3).

3 The first-best optimum

Without the private investment constraint (3), the optimal allocation is the first-best optimum characterized by Diamond and Dybvig. That is, in the planning problem without (3), the IC constraint (2) does not bind. The first-best optimum is characterized as follows. It is clear that it is without loss of generality to take $c_2(0) = c_1(1) = 0$. (Below, we will often omit the agent type in denoting consumption, as c_1 means consumption of the impatient type and c_2 of the patient type.) The objective function (4) becomes

$$\pi u(c_1) + (1-\pi)\rho u(c_2).$$

Disregard the IC constraint and maximize the objective subject to just the resource constraint $\pi c_1 + (1-\pi)c_2/\hat{R} \leq e$. The FOCs imply

$$u'(c_1) = \rho \hat{R} u'(c_2).$$

The assumption about RRA greater or equal 1 and $\rho\hat{R} > 1$ imply that at the solution $c_1 > e$ and $c_2 < e\hat{R}$. Also, $c_1 < c_2$. At the first-best allocation, to be denoted by $c^* = \{c_1^*(0), c_2^*(0), c_1^*(1), c_2^*(1)\}$, we thus have $c_2^*(0) = c_1^*(1) = 0$ and

$$e < c_1^*(0) < c_2^*(1) < e\hat{R}. \tag{5}$$

To see that this first-best optimal allocation does satisfy the IC constraints note that interest rate R in the retrade market associated with the first-best allocation, i.e., the shadow interstest rate at first best, to be denoted by R^*, is $R^* = \frac{c_2^*}{c_1^*} > 1$. At this rate, the present value of each type's consumption allocation is the same:

$$c_1^* = \frac{c_2^*}{R^*}.$$

A patient agent could claim c_1^*, save it in the private market earning net interest $R^* - 1 > 0$, and consume $R^* c_1^*$ at $t = 2$. But doing so would give him final consumption equal to c_2^*. An impatient agent could claim c_2^*, borrow against it in the private market, and consume at $t = 1$. But doing so would give him c_2^*/R^*, which is the same as c_1^*.

However, the first-best optimum does not satisfy the private investment constraint (3) with $R^o = \hat{R}$. We have $R^* = \frac{c_2^*}{c_1^*} \leqslant \rho\hat{R} < \hat{R}$ (with log preferences $R^* = \rho\hat{R}$, see Thm 1 in FGT). With that interest rate, the outside value $\tilde{V}_0(\hat{R}, R^*)$ is strictly larger than the on-eqm value

$\pi u(c_1^*) + (1-\pi)\,\rho u(c_2^*)$. The trades that support $\tilde{V}_0(\hat{R}, R^*)$ are:

$$\begin{aligned} \tilde{s}_0 &= 0,\ \tilde{x}_0 = e, \\ \tilde{s}_1(1) &= 0,\ \tilde{c}_1(1) = 0,\ \tilde{c}_2(1) = \hat{R}e > c_2^*, \\ \tilde{s}_1(0) &= -\frac{\hat{R}}{R^*}e,\ \tilde{c}_1(0) = \frac{\hat{R}}{R^*}e > c_1^*,\ \tilde{c}_2(0) = \hat{R}\tilde{x}_0 + R^*\tilde{s}_1(1) = 0. \end{aligned}$$

These trades thus reproduce the "all long" deviation strategy described in Jacklin (1987).

4 Competitive asset market equilibrium

Diamond and Dybvig start out their analysis by considering the natural model of trade in their environment: agents invest directly in the two assets at date 0. At date 1, after they find out their preference shock (i.e., their type), they trade. Due to the corner preferences, the structure of trade is simple. The impatient agents sell their holdings x_0 of the illiquid asset to the patient agents. The patient agents pay for that asset with the cash they hold, i.e., the payoff from their initial liquid investment s_0. The impatient agents consume their cash and the cash they get from the patient agents in return for the illiquid asset. DD show that the outcome of this trade provides a final consumption allocation

$$c_1 = e, \quad c_2 = Re, \tag{6}$$

which is inefficient relative to the first-best allocation c^*.

Theorem 1 *(Diamond and Dybvig) A unique equilibrium of the competitive asset market economy exists. In equilibrium, $s_0 = \pi e$, $x_0 = (1-\pi)e\hat{R}$, $c_1 = e$, $c_2 = e\hat{R}$, and the date-1 cash price of the illiquid asset is $p = 1$.*

Proof Proof in the Appendix. ■

Naturally, the competitive asset market equilibrium allocation is incentive compatible. The associated gross interest rate in the private retrade market is given by $\frac{c_2}{c_1} = \hat{R}$. We also note that this allocation satisfies the ex ante private investment constraint if the outside rate of return on the illiquid asset is $R^o = \hat{R}$.

5 Transformed planning problem

We follow Farhi et al. (2009) in transforming the planning problem into an equivalent formulation in which the planner's choice variables are the interest rate in the private retrade market,

R, and income which agents bring into this market, I. Farhi et al. (2009) study the DD environment with RF and IC constraints only, i.e., without the ex ante private investment constraint. They show that these constraints can be expressed in terms of R and I only. We show that the private investment constraint, as well, can be expressed in terms of R and I.

Following FGT, we define $V(I, R; \theta)$ as the value an agent of type θ can get with income I if the gross interest rate he faces in the private IOU market is R. Thus,

$$\begin{aligned} V(I,R;\theta) &= \max_{c_1,c_2\geq 0} U(c_1,c_2;\theta) \\ & \quad s.t. \\ & \quad c_1 + \frac{c_2}{R} \leq I \end{aligned}$$

with the associated consumption demands $c_t(I, R; \theta)$ for $t = 1, 2$ and $\theta = 0, 1$. With these indirect utility functions, the ex ante social welfare function is

$$\pi V(I,R;0) + (1-\pi)\, V(I,R;1). \tag{7}$$

The IC constraint manifests itself with the fact that income I does not depend on (reported) type θ. The RF constraint is

$$\pi \left(c_1(I,R;0) + \frac{c_2(I,R;0)}{\hat{R}} \right) + (1-\pi) \left(c_1(I,R;1) + \frac{c_2(I,R;1)}{\hat{R}} \right) \leq e. \tag{8}$$

The ex ante private investment constraint is

$$\pi V(I,R;0) + (1-\pi)\, V(I,R;1) \geq \tilde{V}_0(R^o, R). \tag{9}$$

A key simplification of this constraint comes from the following lemma.

Lemma 1 *For any* $1 < R^o \leq \hat{R}$

$$\tilde{V}_0(R^o,R) = \pi V\left(\tilde{I},R;0\right) + (1-\pi)\, V\left(\tilde{I},R;1\right), \tag{10}$$

where

$$\tilde{I} = \frac{R^o e}{\min\{R^o, R\}}.$$

Proof Proof in the Appendix. ■

In (10), $\tilde{I}$ represents the level of income at the onset of the private IOU market that the agent can guarantee himself by investing privately, i.e., by transferring his initial wealth e to the other island and investing on his own. With this, the ex ante private investment constraint in the planning problem can be simply expressed as $I \geq \tilde{I}$, or

$$I \geq \frac{R^o e}{\min\{R^o, R\}}. \tag{11}$$

In sum, the planning problem is to choose I and R so as to maximize (7) subject to the RF constraint (8) and the private investment constraint (11).

6 Optimal allocation with $R^o = \hat{R}$

As noted earlier, the first-best optimal allocation (c_1^*, c_2^*) violates the ex ante private investment constraint if $R^o = \hat{R}$. With (11), this can be easily seen as

$$I^* = c_1^* < c_1^* \frac{\hat{R}e}{c_2^*} = \frac{\hat{R}e}{R^*} = \frac{\hat{R}e}{\min\{\hat{R}, R^*\}}.$$

The asset market equilibrium $c_1 = e$, $c_2 = e\hat{R}$ corresponds to $I = e$ and $R = \hat{R}$. Note that this allocation satisfies (11):

$$I = e = \frac{\hat{R}e}{\hat{R}} = \frac{\hat{R}e}{\min\{\hat{R}, R\}}.$$

Proposition 1 *(Jacklin) If $R^o = \hat{R}$, then the competitive asset market equilibrium allocation $c_1 = e$, $c_2 = e\hat{R}$ is the only feasible allocation in the social planning problem. Thus, the competitive asset market equilibrium is efficient.*

Proof Proof in the Appendix. ■

The top blue solid line in Figure 1 represents the ex ante private investment constraint with $R^o = \hat{R}$. For all $R < \hat{R}$, this line is downward-slopping with $I = \frac{\hat{R}e}{\min\{\hat{R},R\}} = \frac{\hat{R}e}{R}$. For all $R > \hat{R}$, this line is horizontal with $I = \frac{\hat{R}e}{\min\{\hat{R},R\}} = e$. All allocations (R, I) on and above this line satisfy the private investment constraint (11). The black line is the planner's RF constraint (8). All allocations (R, I) on and below this line are resource feasible. It is clear in Figure 1 that there is only one point (R, I) weakly below the RF constraint and above the private investment constraint. That point is $(R, I) = (\hat{R}, e)$, which represents the competitive asset market equilibrium allocation $c_1 = I = e$, $c_2 = \hat{R}I = \hat{R}e$.

7 Optimal allocations with $1 < R^o < \hat{R}$

Define

$$\bar{R}^o := \frac{\hat{R}}{\pi \frac{\hat{R}}{R^*} + (1-\pi)}$$

and note that $R^* < \bar{R}^o < \hat{R}$. (The left inequality is easy to see when we write $\bar{R}^o$ as $R^* \frac{\hat{R}}{\pi \hat{R} + (1-\pi) R^*}$.)

Lemma 2 *For any $1 < R^o \leq \hat{R}$, the ex ante investment constraint binds iff $R^o > \bar{R}^o$.*

Proof Proof in the Appendix. ■

This lemma says that if the outside investment yields a return sufficiently lower than the return on investment on the home island, in particular when $R^o \leq \bar{R}^o$, then the private investment constraint does not bind and the first-best allocation remains feasible (and thus is optimal).

The bottom blue solid line in Figure 1 represents the ex ante private investment constraint with $R^o = \bar{R}^o$. It intersects the black RF constraint line at the first-best optimum point (R^*, I^*). For $R^o < \bar{R}^o$, the blue line moves further to the left keeping the first-best optimum feasible.

We saw in the previous section (Proposition 1) than when $R^o = \hat{R}$, then the competitive asset market equilibrium allocation (6) is the only feasible allocation, which makes it optimal. Next, we find optimal allocations in other cases with binding ex ante investment constraint, i.e., the cases with $\bar{R}^o < R^o < \hat{R}$. We will denote the optimal allocation (R, I) with the private investment return R^o as (R^{**}, I^{**}) indexed by R^o for $R^o \in [\bar{R}^o, \hat{R}]$. The corresponding optimal consumption allocations are $c_1^{**} = I^{**}$ and $c_2^{**} = R^{**} I^{**}$.

Proposition 2 *For any $R^o \in [\bar{R}^o, \hat{R}]$, the optimal allocation is*

$$\begin{aligned} R^{**} &= \frac{\pi}{\frac{1}{R^o} - \frac{1-\pi}{\hat{R}}}, \\ I^{**} &= \frac{e}{\pi}\left(1 - (1-\pi)\frac{\hat{R}}{R^o}\right). \end{aligned}$$

Proof Proof in the Appendix. ■

Note that if $R^o = \bar{R}^o$, then $R^{**} = R^*$ and $I^{**} = I^*$ thus replicating the first-best optimum (5). If $R^o = \hat{R}$, then $R^{**} = \hat{R}$ and $I^{**} = e$ thus replicating the competitive asset market equilibrium allocation (6). For intermediate values of R^o, the optimal allocation (R^{**}, I^{**}) is between these two points. That is, $e < I^{**} < I^*$ and $\hat{R} > R^{**} > R^*$. The dashed blue line in Figure 1 shows the P constraint for an intermediate value of $R^o \in (\bar{R}^o, \hat{R})$.

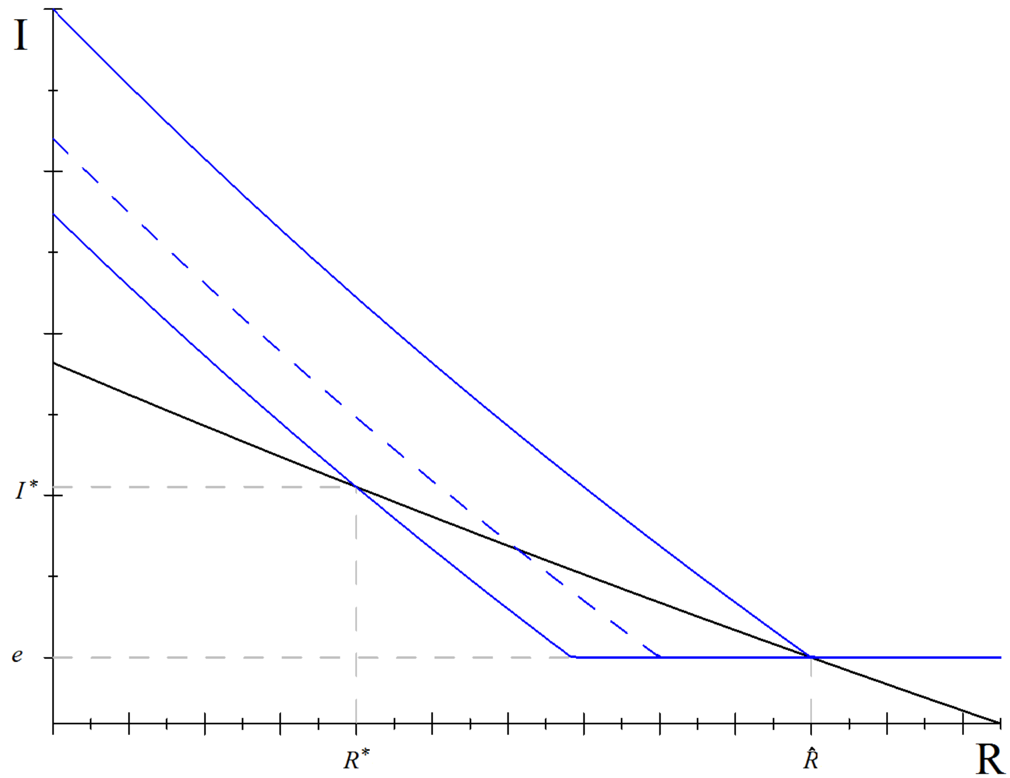

Figure 1: Optimal allocations for various values of R^o.

The intuition behind this result is as follows. Since ex post redistribution to the impatient type is desirable (relative to (6)), the planner wants to increase I and decrease R. The agent's value of the option to invest privately ex ante and borrow in the private IOU market ex post should she turn out impatient increases as the cost of borrowing in the private market, R, decreases. Thus, the planner decreases R from $\hat{R}$ toward R^* as much as possible without triggering private investment ex ante.

Note that ex ante social welfare attained by the planner is decreasing in R^o as higher R^o means tighter ex ante private investment constraint P.

8 Optimal investment taxes

Having characterized the optima for all $1 < R^o \leq \hat{R}$, we now discuss implementation of these allocations as equilibria of the competitive asset market, where agents hold and trade assets directly, i.e., without intermediation.

We saw already that if $R^o = \hat{R}$, the optimum coincides with the competitive asset market equilibrium allocation (6). If $1 < R^o < \hat{R}$, however, this equilibrium allocation is inefficient. It is easy to extend the analysis in FGT to show that Prescott-Townsend equilibria are also inefficient due to the so-called pecuniary externality. FGT use intermediation with government regulation (in particular, with minimum liquidity requirements on the intermediaries) to implement the first-best allocation (c_1^*, c_2^*). Next, we show how this allocation, as well as all other allocations (R^{**}, I^{**}) that are optimal with a given $R^o \in (\bar{R}^o, \hat{R})$, can be implemented in competitive asset market equilibrium with taxes.

8.1 Tax on illiquid asset and subsidy to liquid asset

For each optimal allocation of consumption (c_1^{**}, c_2^{**}) , let's compute the shadow price of the illiquid asset at date 1 (i.e., its price implied by the optimal allocation). To make the allocation (c_1^{**}, c_2^{**}) feasible, the initial investments must be $s_0 = \pi c_1^{**}$ and $x_0 = (1-\pi)c_2^{**}/\hat{R}$. At $t = 1$, the implicit trade is as follows: $(1-\pi)s_0 = (1-\pi)\pi c_1^{**}$ units of cash are exchanged for $\pi x_0 = \pi(1-\pi)c_2^{**}/R$ units of the illiquid asset. So the implicit price per unit of asset is

$$\begin{aligned} p^{**} &= \frac{(1-\pi)\pi c_1^{**}}{\pi(1-\pi)c_2^{**}/\hat{R}} \\ &= \hat{R}\frac{c_1^{**}}{c_2^{**}} \\ &= \frac{\hat{R}}{R^{**}} \\ &< \hat{R} \end{aligned}$$

At $t = 0$, as we saw in Theorem 1, the agent's individual preferences over investment in s_0 and x_0 have linear indifference curves with slope $-\frac{1}{p}$. To support the optimal investment point $(s_0, x_0) = (\pi c_1^{**}, (1-\pi)c_2^{**}/\hat{R})$, this point must be feasible and the slope of the budget line must be $-\frac{1}{p^{**}}$.

To achieve this, suppose then the government imposes a proportional tax τ^x on illiquid investment and hands out a proportional subsidy τ^s to the liquid asset origination at $t = 0$. With these, the budget constraint for the agent at $t = 0$ take the form of

$$(1-\tau^s)s_0 + (1+\tau^x)x_0 \le e.$$

The government announces the tax rates before agents decide whether to invest publicly (subject to taxes) or privately (i.e., with return $R^o \le \hat{R}$ on the illiquid investment). The government is committed to the preannounced rates. In equilibrium, all agents will invest publicly but the option to invest privately constrains the government's choice of taxes. Once the decision to invest at home subject to taxes is made, agents invest and pay taxes at $t = 0$, and trade assets at $t = 1$ as the competitive asset market equilibrium of Theorem 1.

Theorem 2 *For any $R^o \in [\bar{R}^o, \hat{R}]$, if*

$$
\begin{aligned}
1 + \tau^x &= \frac{e}{\pi \frac{c_1^{**}}{p^{**}} + (1-\pi)\frac{c_2^{**}}{\hat{R}}}, \\
1 - \tau^s &= \frac{e}{\pi c_1^{**} + (1-\pi)\frac{c_2^{**} p^{**}}{\hat{R}}},
\end{aligned}
$$

then the competitive asset market equilibrium exists and the equilibrium consumption allocation is the optimum (c_1^{**}, c_2^{**}).

Proof Proof in the Appendix. ■

Note that Theorem 1 and Proposition 1 amount to a special case of Theorem 2 with $R^o = \hat{R}$: the tax-free competitive asset market equilibrium allocation (6) is optimal, i.e., taxes τ^s, τ^x are both zero in this case. In the special case with $R^o = \bar{R}^o$, Theorem 2 provides a tax-based implementation of the first-best optimal allocation (5).

8.2 Other tax systems

The tax system (τ^s, τ^x) is not unique. Geometrically, what tax instruments must accomplish is a rotation of the budget line in the investment space (s_0, x_0) at $t = 0$ so as to match the slope of the (linear) indifference curve, namely $-\frac{1}{p^{**}}$, while keeping the optimal investment point $(s_0, x_0) = (\pi c_1^{**}, (1-\pi)c_2^{**}/\hat{R})$ feasible. Other ways of doing it can involve a lump-sum transfer or tax T given to/imposed on all agents who invest domestically. For example, a proportional tax $\tau^x = \hat{R}\frac{c_1^{**}}{c_2^{**}} - 1 > 0$ on illiquid investment and a lump-sum transfer $T = (1-\pi)(c_1^{**} - \hat{R}c_2^{**}) > 0$ for anyone who does not choose to invest on the other island could be used. Alternatively, a proportional subsidy τ^s to the liquid investment and a lump-sum tax T could be used to accomplish the same goal.

9 Conclusion

TO BE COMPLETED

Appendix

Proof of Theorem 1

Let's start with the date-1 market in which the illiquid asset is sold for cash, given that all agents hold the same s_0 and x_0, which they had chosen at $t = 0$. Let p be the cash price of an unit of x at $t = 1$.

The impatient types solve

$$\begin{aligned} &\max \; u(c_1) \\ &s.t. \; c_1 \leq s_0 + px_0 \end{aligned}$$

so their supply of the illiquid asset to the market is perfectly inelastic: each impatient agent supplies x_0 units at any price. The market supply is thus πx_0 as there are π impatient agents at $t = 1$.

The patient types solve

$$\begin{aligned} &\max \; \rho u(c_1 + c_2) \\ &s.t. \; c_1 + pn \leq s_0 \\ &c_2 \leq (x_0 + n)\hat{R} \end{aligned}$$

where $n \geq -x_0$ is net demand at date 1. Their demand has a jump. If $p > \hat{R}$, then the patient agents want to sell the asset and consume immediately (both their cash s and the sale proceeds px), so $n(p) = -x$ for all $p > \hat{R}$. If $p < \hat{R}$, the patient agents want to hold onto the long-term assets they already have, and spend their cash s buying the asset in the market. Thus individual net demand of each patient agent is $n(p) = \frac{s}{p}$ for all $p < \hat{R}$. Aggregate net demand from the patient agent is then $(1-\pi)\frac{s}{p}$. At price $p = \hat{R}$, the individual demand is anything between $\frac{s}{\hat{R}}$ and $-x$ as a patient agent is indifferent between selling now and consuming or buying some more asset and consuming later. But this will not matter as the whole action will be in the region where $p < \hat{R}$.

The equilibrium price, $\hat{p}$, given s_0 and x_0 (assuming s_0 and x_0 are held uniformly by all agents), is thus

$$\hat{p} = \min\left\{\frac{1-\pi}{\pi}\frac{s_0}{x_0}, \hat{R}\right\} \tag{12}$$

with $\hat{p} = \hat{R}$ if $\pi x_0 \leq (1-\pi)\frac{s_0}{\hat{R}}$ and $\hat{p}$ solving $\pi x_0 = (1-\pi)\frac{s_0}{p}$ if $\pi x_0 > (1-\pi)\frac{s_0}{\hat{R}}$, which means $\hat{p} = \frac{1-\pi}{\pi}\frac{s_0}{x_0}$ in this case.

The continuation values attained at $t = 1$ by the two types are therefore

$$V(s_0, x_0; 0) = u(s + \hat{p}x_0)$$

for the low type and

$$V(s_0, x_0; 1) = \begin{cases} \rho u(s_0 + \hat{p}x_0) & \text{if} \quad \hat{p} = \hat{R}, \\ \rho u((\frac{s_0}{\hat{p}} + x_0)R) & \text{if} \quad \hat{p} < \hat{R}. \end{cases}$$

where $\hat{p}$ is as above (assuming everyone holds the same s_0 and x_0).

Consider now the agents' investment problem at $t = 0$. They maximize the expected value

$$\pi V(s_0, x_0; 0) + (1 - \pi)V(s_0, x_0; 1)$$

subject to $s_0 + x_0 \leq e$ taking the expectation of the price $\hat{p}$ at $t = 1$ as given.

If $\hat{p} = \hat{R}$, then the expected value ex ante is

$$\begin{aligned} \pi V(s_0, x_0; 0) + (1 - \pi)V(s_0, x_0; 1) &= \pi u(s + \hat{p}x) + (1 - \pi)\rho u(s + \hat{p}x) \\ &= \pi u(s_0 + \hat{R}x_0)\left(1 + (1 - \pi)\rho\right). \end{aligned}$$

The indifference curves between s_0 (on the horizontal axis) and x_0 are linear with slope $-\hat{R}^{-1}$, which is flatter that the budget line with the slope of -1. So the agent wants to go all-long: $x_0 = e$, $s_0 = 0$. But then $\pi x_0 \leq (1 - \pi)\frac{s_0}{\hat{R}}$ is violated, and so with $x_0 = e$ the price $\hat{p} = \hat{R}$ is not consistent with eqm in the market for the illiquid asset at $t = 1$. So there cannot be an equilibrium with $\hat{p} = \hat{R}$.

With $\hat{p} < \hat{R}$, the expected value is

$$\pi V(s_0, x_0; 0) + (1 - \pi)V(s_0, x_0; 1) = \pi u(s + \hat{p}x_0) + (1 - \pi)\rho u((\frac{s_0}{\hat{p}} + x_0)\hat{R}).$$

The slope of the agent's indifference curve between s and x is

$$\begin{aligned} \frac{dx_0}{ds_0} &= -\frac{\pi u'(s_0 + \hat{p}x_0) + (1 - \pi)\rho u'((\frac{s_0}{\hat{p}} + x_0)\hat{R})\frac{\hat{R}}{\hat{p}}}{\pi u'(s_0 + \hat{p}x_0)\hat{p} + (1 - \pi)\rho u((\frac{s_0}{\hat{p}} + x_0)\hat{R})\hat{R}} \\ &= -\frac{1}{\hat{p}}\frac{\pi u'(s_0 + \hat{p}x_0)\hat{p} + (1 - \pi)\rho u'((\frac{s_0}{\hat{p}} + x_0)\hat{R})\hat{R}}{\pi u'(s_0 + \hat{p}x_0)\hat{p} + (1 - \pi)\rho u((\frac{s_0}{\hat{p}} + x_0)\hat{R})\hat{R}} \\ &= -\frac{1}{\hat{p}} \end{aligned}$$

The slope of the agent's budget line is -1. Thus, in equilibrium, it must be $\hat{p} = 1$. This value is consistent with $\hat{p} < \hat{R}$. What is the solution to the agent's problem at $t = 0$? The agent maximizes

$$\begin{aligned} \pi V(s_0, x_0; 0) + (1-\pi)V(s_0, x_0; 1) &= \pi u(s_0 + \hat{p}x_0) + (1-\pi)\rho u((\frac{s}{\hat{p}} + x_0)\hat{R}) \\ &= \pi u(s_0 + x_0) + (1-\pi)\rho u((s_0 + x_0)\hat{R}) \end{aligned}$$

s.t., $s_0 + x_0 \leq e$. So the agent is indifferent between all investment choices as long as $s_0 + x_0 = e$. For any such choice his expected value is

$$\pi u(e) + (1-\pi)\rho u(e\hat{R})$$

Equilibrium will exist at $t = 1$ if and only if the date-0 choices are consistent with (1). I.e., $1 = \hat{p} = \min\left\{\frac{1-\pi}{\pi}\frac{s_0}{x_0}, \hat{R}\right\} = \frac{1-\pi}{\pi}\frac{s_0}{x_0}$. So for the the eqm to exist we must have simultaneously

$$\begin{aligned} \frac{1-\pi}{\pi}\frac{s_0}{x_0} &= 1 \\ s_0 + x_0 &= e \end{aligned}$$

This system has a unique solution: $s_0 = \pi e$, $x_0 = (1-\pi)e$.

Thus, this model has a unique equilibrium with equilibrium consumptions $\hat{c}_1(0) = e$, $\hat{c}_2(1) = e\hat{R}$. And this is not the optimal allocation. In particular, $\hat{c}_1$ is too low, i.e., there is not enough "liquidity" available at $t = 1$.

Proof of Lemma 1

(NEEDS CLEANING UP) We work backwards from $t = 1$. If $\theta = 0$, then the agent wants to borrow in the private IOU market against the return he will have on his initial investment $\tilde{x}_0$ in the illiquid asset, regardless of the interest rate R. Since $c_2 = 0$, we have $\tilde{s}_1(0) = -\frac{R^o}{R}\tilde{x}_0$ and thus $c_1 = \tilde{s}_0 + \frac{R^o}{R}\tilde{x}_0$. The value the agent gets is $u(\tilde{s}_0 + \frac{R^o}{R}\tilde{x}_0)$.

If $\theta = 1$, then what the agent wants to do depends on how the market return R compares to his intertemporal rate of substitution, which is 1. If $R > 1$, the agent wants to consume at $t = 2$ and not at $t = 1$, so he sets $c_1 = 0$ and buys IOUs $\tilde{s}_1(1) = \tilde{s}_0$. He then has $c_2 = R^o\tilde{x}_0 + R\tilde{s}_0$. The value he gets in this case is $\rho u(R^o\tilde{x}_0 + R\tilde{s}_0)$. If $R < 1$, the agent wants to consume at $t = 1$ and so, just like the impatient type, he borrows in the private IOU market against the return on his initial investment $\tilde{x}_0$. Since $c_2 = 0$, we have $\tilde{s}_1(0) = -\frac{R^o}{R}\tilde{x}_0$ and thus $c_1 = \tilde{s}_0 + \frac{R^o}{R}\tilde{x}_0$. The value this agent gets in this case is $\rho u(\tilde{s}_0 + \frac{R^o}{R}\tilde{x}_0)$. If $R = 1$, the agent is indifferent. The value he gets is $\rho u(R^o\tilde{x}_0 + \tilde{s}_0)$ doesn't matter what he would do.

We now put the behavior of the impatient types together with the behavior of the patient types to determine the optimal ex ante investments $\tilde{s}_0, \tilde{x}_0$ behind the opt-out value $\tilde{V}_0(R^o, R)$.

If $R \leq 1$, that ex ante expected value of opting out, $\tilde{V}_0(R^o, R)$, is

$$\pi u(\tilde{s}_0 + \frac{R^o}{R}\tilde{x}_0) + (1-\pi)\,\rho u(\tilde{s}_0 + \frac{R^o}{R}\tilde{x}_0).$$

Note $R \leq 1$ implies $\frac{R^o}{R} \geq R^o > 1$, so that value is maximized (subject to the budget constraint $\tilde{s}_0 + \tilde{x}_0 \leq 1$) by setting $\tilde{s}_0 = 0$ and $\tilde{x}_0 = e$. This is intuitive. Since c_1 is cheap in the hidden market relative to c_2, it is always good to maximize the payoff at $t = 2$ by investing all-long, borrow against it at $t = 1$ and consume at $t = 1$, regardless of θ. With $\tilde{s}_0 = 0$ and $\tilde{x}_0 = e$, we can write $\tilde{V}_0(R^o, R)$ as

$$\pi u(\frac{R^o}{R}e) + (1-\pi)\,\rho u(\frac{R^o}{R}e).$$

Now let's switch to analyzing $\pi V(\frac{R^o e}{\min\{R^o,R\}}, R; 0) + (1-\pi)\,V(\frac{R^o e}{\min\{R^o,R\}}, R; 1)$ with $R \leq 1$. We have $\frac{R^o e}{\min\{R^o,R\}} = \frac{R^o e}{R}$. Given income $I = \frac{R^o e}{\min\{R^o,R\}} = \frac{R^o e}{R}$ and interest rate $R \leq 1$, the ex post values are $V(I, R; 0) = u(\frac{R^o}{R}e)$ and $V(I, R; 1) = \rho u(\frac{R^o}{R}e)$ by the same argument (both types want to consume when consumption is cheap). Thus (10) does hold when $R \leq 1$.

If $R > 1$, the ex ante expected value of opting out is

$$\begin{aligned}
& \pi u(\tilde{s}_0 + \frac{R^o}{R}\tilde{x}_0) + (1-\pi)\,\rho u(R^o\tilde{x}_0 + R\tilde{s}_0) \\
= \quad & \pi u(e - \tilde{x}_0 + \frac{R^o}{R}\tilde{x}_0) + (1-\pi)\,\rho u(R^o\tilde{x}_0 + eR - R\tilde{x}_0) \\
= \quad & \pi u(e + \frac{1}{R}(R^o - R)\tilde{x}_0) + (1-\pi)\,\rho u(eR + (R^o - R)\tilde{x}_0)
\end{aligned}$$

which shows that the optimal investment in the long term asset depends on the sign of $R^o - R$. If $R < R^o$, then optimal $\tilde{x}_0 = e$ (the same as in the case of $R \leq 1$ we saw in the previous paragraph). If $R > R^o$, then it is optimal to choose $\tilde{x}_0 = 0$. If $R = R^o$, then any choice of $0 \leq \tilde{x}_0 \leq e$ is optimal for an opting-out agent. The value that agent obtains therefore is

$$\begin{array}{cl}
\pi u(e + \frac{1}{R}(R^o - R)e) + (1-\pi)\,\rho u(eR + (R^o - R)e) & \\
= \pi u(\frac{R^o}{R}e) + (1-\pi)\,\rho u(R^o e) & \text{if} \quad 1 < R < R^o, \\
\pi u(e) + (1-\pi)\,\rho u(R^o e) & \text{if} \qquad R = R^o, \\
\pi u(e) + (1-\pi)\,\rho u(eR) & \text{if} \qquad R > R^o,
\end{array}$$

which we can write as

$$\begin{array}{ccc}
\pi u(\frac{R^o}{R}e) + (1-\pi)\,\rho u(R^o e) & \text{if} & 1 < R < R^o \\
\pi u(e) + (1-\pi)\,\rho u(eR) & \text{if} & R \geq R^o
\end{array} \tag{13}$$

Now let's switch and analyze $\pi V(\frac{R^o e}{\min\{R^o,R\}}, R; 0) + (1-\pi)\, V(\frac{R^o e}{\min\{R^o,R\}}, R; 1)$ with $R > 1$.

For any $R > 1$, consumption demand is $c_1(0) = I$ and $c_1(1) = 0$, $c_2(1) = RI$ because R is greater than the patient's type MRS of 1. Thus

$$\pi V(\frac{R^o e}{\min\{R^o, R\}}, R; 0) + (1-\pi)\, V(\frac{R^o e}{\min\{R^o, R\}}, R; 1) = \pi u(\frac{R^o e}{\min\{R^o, R\}}) + (1-\pi)\, \rho u(R\frac{R^o e}{\min\{R^o, R\}}).$$

If $1 < R < R^o$, then $I = \frac{R^o e}{\min\{R^o,R\}} = \frac{R^o e}{R}$ and the above value is $\pi u(\frac{R^o e}{R}) + (1-\pi)\, \rho u(R\frac{R^o e}{R})$, which is the same as the top line in (13). If $R \geq R^o$, then $I = \frac{R^o e}{\min\{R^o,R\}} = \frac{R^o e}{R^o} = e$ and the above value is $\pi u(e) + (1-\pi)\, \rho u(eR)$, which is the same as the bottom line in (13). Thus (10) does hold at all $R > 1$. This completes the proof of (10).

Proof of Proposition 1

The planner chooses I and R subject to (8) and (11). In order to evaluate the resource feasibility constraint, we need to first calculate agents' consumption demands given I and R. But these demands are easy to compute because the impatient type always wants to consume early and the patient type wants to consume late when $R > 1$ and early if $R \leq 1$.

Let's start with $R \leq 1$. Given any I, both types want to consume early, so their consumption demands are $c_1(0) = I$ and $c_1(1) = I$. The resource feasibility constraint reduces to $\pi c_1(0) + (1-\pi) c_1(1) \leq e$ or $I \leq e$. But the participation constraint (11) is $I \geq \frac{\hat{R} e}{R} > e$ as $\min\{\hat{R}, R\} = R < \hat{R}$. These two constraints can't be simultaneously satisfied, so there are no feasible allocations with $R \leq 1$.

Now consider $R > 1$. Given some I, the agents' demands are $c_1(0) = I$ and $c_1(1) = 0$, $c_2(1) = RI$ (the patient agent wants to consume at $t = 2$ because his intertemporal MRS is 1 and $R > 1$). Thus, resource feasibility requires

$$\pi I + (1-\pi)\frac{RI}{\hat{R}} \leq e$$

or

$$I \leq \frac{\hat{R} e}{\pi \hat{R} + (1-\pi) R}. \tag{14}$$

Writing this constraint together with the agents' ex ante participation constraint (11) we have:

$$\frac{\hat{R} e}{\min\{\hat{R}, R\}} \leq I \leq \frac{\hat{R} e}{\pi \hat{R} + (1-\pi) R} \tag{15}$$

Now note that for all R

$$\min\{\hat{R}, R\} \leq \pi\hat{R} + (1-\pi)R$$

with equality only if $R = \hat{R}$. Thus,

$$\frac{\hat{R}e}{\min\{\hat{R}, R\}} \geq \frac{\hat{R}e}{\pi\hat{R} + (1-\pi)R}$$

with strict inequality for all R different than $R = \hat{R}$. Thus, the only R for which the two inequalities in (15) can be simultaneously satisfied is $R = \hat{R}$. At this R, the upper and lower bounds on I in (15) are both equal to e (the feasibility constraint is $I \leq e$ and the participation is $I \geq e$), so the only feasible level of income associated with $R = \hat{R}$ is $I = e$.

Proof of Lemma 2

This lemma establishes a threshold R^o at which the outside option constraint begins to bind.

Clearly, if $R^o \leq R^*$, then the outside option constraint does not bind.

With outside return R^o, the income $\tilde{I}$ the agent can get himself outside of the system is $\tilde{I} = \frac{R^o e}{\min\{R, R^o\}}$. Taking $R = R^*$, we have $\tilde{I} = \frac{R^o e}{\min\{R^*, R^o\}} = \frac{R^o e}{R^o} = e$. At the first-best optimum ψ^*, the agent gets the return R^* in the hidden IOU market and implicit income $I^* = \frac{\hat{R}e}{\pi\hat{R}+(1-\pi)R^*} > \frac{\hat{R}e}{\hat{R}} = e$ (FGT, Thm1). Thus, no agent would step out of the system to get a smaller income $\tilde{I} < I^*$ (and the same rate of return in the IOU market). That threshold is actually higher (see below).

So the relevant range for R^o to consider is $R^* < R^o < \hat{R}$.

For any R^o in this range, there is no reason for the planner to select $R > R^o$ (this is a conjecture). Intuitively, the planner wants to reduce R from $\hat{R}$ to R^* (as much as feasible given R^o). If $R = R^o$, then the ex ante participation constraint will be satisfied (see below). Thus, there is no need to make R higher.

Thus, we can look at $R^* < R^o < \hat{R}$ and $R \leq R^o$. For these values, the outside income is always $\tilde{I} = \frac{R^o e}{\min\{R, R^o\}} = \frac{R^o e}{R}$. Thus, the quitting constraint the planner faces is $I \geq \frac{R^o e}{R}$. Together with the resource feasibility constraint, they are

$$\frac{R^o e}{R} \leq I \leq \frac{\hat{R}e}{\pi\hat{R} + (1-\pi)R}$$

Define $\bar{R}^o$ as

$$\bar{R}^o = R^* \frac{\hat{R}}{\pi\hat{R} + (1-\pi)R^*} = \frac{\hat{R}}{\pi\frac{\hat{R}}{R^*} + (1-\pi)}$$

for $R^o \leq \bar{R}^o$, the outside value constraint will not bind, and for $R^o > \bar{R}^o$ it will.

Note that $R^* < \bar{R}^o < \hat{R}$.

Proof of Proposition 2

The planner's preferences (representing ex ante social welfare) are convex, and the unconstrained maximum is at point (R^*, I^*). The constrained solutions are given by the intersection of the RF line $I = \frac{\hat{R}e}{\pi\hat{R}+(1-\pi)R}$ and the downward-sloping part of the private investment constraint $I = \frac{R^o e}{R}$. Solving these two, we get the optimum (R^{**}, I^{**}).

As mentioned, these taxes must be such that the optimal investment point $s_0 = \pi c_1^*$ and $x_0 = (1-\pi)c_2^{**}/\hat{R}$ is feasible, and the slope of this line is $-\frac{1}{p^{**}}$. This can be accomplished with the two numbers τ^s and τ^x that the planner can choose. In particular, setting the slope of the segment connecting the optimal investment point $(s_0, x_0) = (\pi c_1^{**}, (1-\pi)c_2^{**}/\hat{R})$ with each axis equal $-\frac{1}{p^{**}}$ we get

$$\begin{aligned}
\frac{\frac{e}{1+\tau^x} - (1-\pi)\frac{c_2^{**}}{\hat{R}}}{-\pi c_1^{**}} &= -\frac{1}{p^{**}}, \\
\frac{(1-\pi)\frac{c_2^{**}}{\hat{R}}}{\pi c_1^{**} - \frac{e}{1-\tau^s}} &= -\frac{1}{p^{**}}.
\end{aligned}$$

From here we get

$$\begin{aligned}
1+\tau^x &= \frac{e}{\pi\frac{c_1^{**}}{p^{**}} + (1-\pi)\frac{c_2^{**}}{\hat{R}}} > 1, \\
1-\tau^s &= \frac{e}{\pi c_1^{**} + (1-\pi)\frac{c_2^{**}p^{**}}{\hat{R}}} < 1.
\end{aligned}$$

With these taxes, we have

$$(1-\tau^s)s_0 + (1+\tau^x)x_0 = \frac{e}{\pi c_1^{**} + (1-\pi)\frac{c_2^{**}p^{**}}{\hat{R}}}(s_0 + p^{**}x_0),$$

so the budget line is given as

$$s_0 + p^{**}x_0 = \pi c_1^{**} + (1-\pi)\frac{c_2^{**}p^{**}}{\hat{R}}.$$

To verify individual optimality, substitute the budget constraint to the indirect utility function and obtain

$$\begin{aligned}\pi V(s_0, x_0; 0) + (1-\pi)V(s_0, x_0; 1) &= \pi u(s_0 + p^{**}x_0) + (1-\pi)\rho u((\frac{s_0}{p^{**}} + x_0)\hat{R}) \\ &= \pi u\left(\pi c_1^{**} + (1-\pi)\frac{c_2^{**}p^{**}}{\hat{R}}\right) + (1-\pi)\rho u\left(\left(\pi c_1^{**} + (1-\pi)\frac{c_2^{**}p^{**}}{\hat{R}}\right)\frac{\hat{R}}{p^{**}}\right)\end{aligned}$$

which is independent of (s_0, x_0), i.e., any choice of (s_0, x_0) on the budget line is individually optimal. In particular, the optimal investment point $(s_0, x_0) = (\pi c_1^{**}, (1-\pi)c_2^{**}/\hat{R})$ is optimal. It is easy to verify that the government breaks even, i.e., the government's budget constraint is met.

References

Allen, F. and D. Gale (2004). Financial intermediaries and markets. *Econometrica 72*(4), 1023–1061.

Bisin, A. and A. A. Rampini (2006). Markets as beneficial constraints on the government. *Journal of Public Economics 90*(45), 601–629.

Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy 91*(3), 401–419.

Farhi, E., M. Golosov, and A. Tsyvinski (2009). A theory of liquidity and regulation of financial intermediation. *The Review of Economic Studies 76*(3), 973–992.

Jacklin, C. J. (1987). Demand deposits, trading restrictions, and risk sharing. In E. C. Prescott and N. Wallace (Eds.), *Contractual Arrangements for Intertemporal Trade*, pp. 26–47. Minneapolis, MN: University of Minnesota Press.

Kocherlakota, N. (2010). *The New Dynamic Public Finance.* Princeton, NJ: Princeton University Press.

Mirrlees, J. (1971). An exploration in the theory of optimum income taxation. *The Review of Economic Studies 61*, 681–700.