The Dynamics of Sovereign Debt Crises and Bailouts

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First draft: January 6, 2011
This revision: March 5, 2014

Abstract

This paper investigates the scope for a bailout guarantee in a sovereign debt crisis. Building on Arellano (2008), Cole-Kehoe (1996, 2000) and Beetsma-Uhlig (1999), we consider an environment where sovereign defaults arise from negative income shocks, government impatience or a "sunspot"-coordinated buyers strike. We introduce a bailout agency or large investor, and characterize the minimal actuarially fair intervention that guarantees the no-buyers-strike equilibrium, relying on the market to provide residual financing. An intervention is actuarially fair when the intervening agency earns market return in expectation. This intervention by an external facility that guarantees some debt purchase at the "good" equilibrium price reduces, but does not eliminate, default events. Intervention makes it cheaper for governments to borrow, inducing them borrow more until the debt-to-GDP ratio is so high that it cannot be rescued by an actuarially fair intervention.

JEL classification: F34, F41.

Keywords: Default, Bailouts, Self-fulfilling Crises, Endogenous Borrowing Constraints, Long-term Debt.

*The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management. We are thankful to a number of insightful discussions of our paper. We are particularly thankful to Fernando Broner, whose discussion at a CREI conference in December 2011 led to some important changes regarding the bailout analysis in the paper. Address: Francisco Roch, IMF, email: froch@imf.org. Harald Uhlig, Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, IL 60637, U.S.A, email: huhlig@uchicago.edu. The research of Harald Uhlig has been supported by the NSF grant SES-1227280. Harald Uhlig has an ongoing consulting relationship with a Federal Reserve Bank, the Bundesbank and the ECB.

1 Introduction

Warnings and analyses of future sovereign debt crises in Europe and their impact on European monetary policy such as Uhlig (2003) seemed to have received little echo during an episode where such fears were judged unfounded. That has changed. Since 2010, doubts persist on financial markets that a number of countries such as Greece, Portugal, Spain or Italy will be able to repay their sovereign debt. Various bailouts and interventions have been proposed or been executed, with mixed success. Of particular interest is the ECB President Mario Draghi's attempt to restore confidence by pledging to do "whatever it takes" to preserve the eurozone. In the end, the ECB announced the program known as outright monetary transactions (OMT) in September 2012, that meant potentially unlimited purchases of distressed government's bonds. The plan was designed to lower borrowing costs in Spain and Italy, and avoid the dissolution of the monetary union.

One symptom of these developments is the observed pattern of eurozone members sovereign yields since 2010, as shown in Figure 1. Until early 2010, all the yields on 10-year bonds in Europe were co-moving together at relatively low levels. Since then, when irregularities in Greece's public finances were disclosed and default concerns arised, spreads spiked in countries such as Greece, Portugal, Ireland, Spain, and Italy. However, with the ECB President Draghis declaration to do "whatever it takes" and the announcement of the OMT, yields have declined and fears of sovereign defaults have receded in countries such as Portugal, Spain and Italy. One view of this policy is that it served as coordination on a "good equilibrium" in a situation where sovereign debt markets are prone to panics and run. Thus, the behavior of spreads seemed to be more sensitive to changes in market sentiment rather than to actual changes in countries' fundamentals.

This paper is motivated by these developments and seeks to understand the dynamics of sovereign debt crises in a single country, when there is an outside intervention by some bail-out agency. We characterize the minimal actuarially fair intervention that restores the "good" equilibrium of Cole-Kehoe (2000), relying on the market to provide residual financing. "Fair value" here means that the resources provided by the bail-out fund earn the market return in expectation. We believe this is an important benchmark, shedding light on the OMT program of the ECB.

The analysis of the dynamics of a sovereign debt crisis builds on and moderately

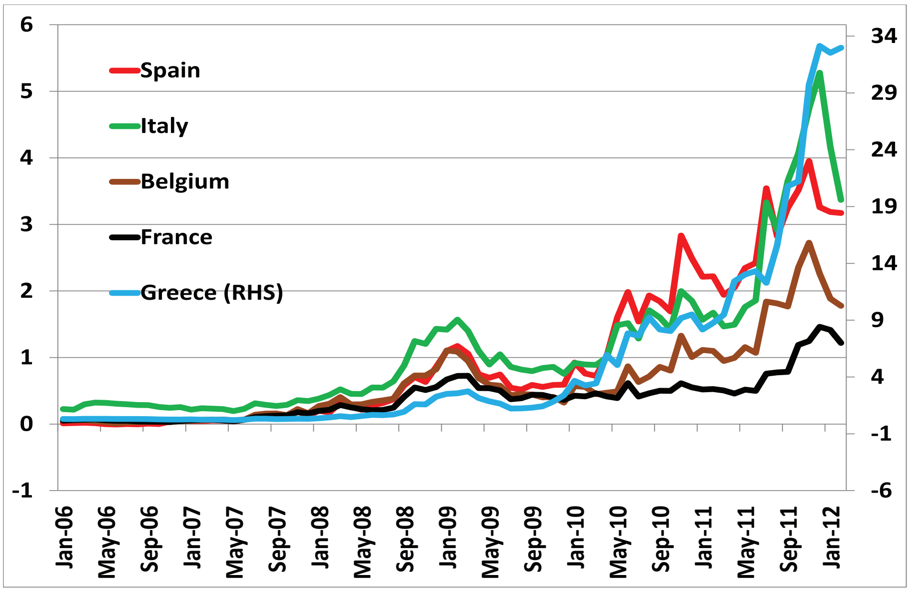

Figure 1: *10yr yield spread to Germany. Source: Bloomberg.*

extends three branches of the literature in particular. First, Arellano (2008) has analyzed the dynamics of sovereign default under fluctuations in income, and shown that defaults are more likely when income is low[1]. Second, Cole and Kehoe (1996,2000) have pointed out that debt crises may be self-fulfilling: the fear of a future default may trigger a current rise in default premia on sovereign debt and thereby raise the probability of a default in the first place. Both theories imply, however, that countries would have a strong incentive to avoid default-triggering scenarios in the first place. We therefore build on the political economy theories of the need for debt contraints in a monetary union of short-sighted fiscal policy makers as in to provide a rationale for a default-prone scenario, see e.g. Beetsma and Uhlig (1999) or Cooper, Kempf and Peled (2010).

We study a dynamic endogenous default model à la Eaton and Gersovitz (1981). This framework is commonly used for quantitative studies of sovereign debt and has been shown to generate a plausible behavior of sovereign debt and spread. The model environment consists of three agents: a single government, international lenders, and a bailout agency. The government finances its consumption with tax receipts and non-contingent long-duration bonds. Tax receipts are exogenous and stochastic. In the model, defaults can occur both from negative income shocks and coordination failures among international investors. If the government defaults on its debt obligations, it then pays an exogenous one-time utility cost of default[2], it is temporarily excluded from debt markets, and it consumes its tax receipts until re-entry into debt markets. The utility cost of default is time-varying and it can be interpreted as an embarrassment of default that changes from government to government. Re-entry into debt markets occurs with some exogenous probability.

We consider a bailout agency, modeled as a particularly large and infinitely lived investor and who is committed to rule out the sunspot-driven defaults of Cole-Kehoe (2000) per debt purchases, even if all other investors do not. We assume that this bailout

[1]That may sound unsurprising, but is actually not trivial. Indeed the recursive contract literature typically implies incentive issues for contract continuation at high rather than low income states, see e.g. Ljungqvist-Sargent (2004).

[2]While including a utility cost opens the door to the free-parameter criticism, it also allows the cost of a crisis to be interpreted broadly. Quantitatively, including a utility cost parameter also allows the model to easily match high debt-totax ratios and default rates, which has been generally hard to achieve in the literature.

agency seeks an actuarially fair return, and characterize the minimal intervention. The bailout agency will not prevent defaults due to fundamental reasons as in Arellano (2008) nor impose additional policy constraints such as conditionality as in e.g. Fink and Scholl (2011).

We find that introducing an actuarially fair bailout agency could effectively serve as a coordination on the "good equilibrium", by issuing debt purchase guarantees and without incurring losses in expectation. However, these guarantees should not be too large. The reason is that this intervention lowers the risk premium for the government, inducing it to borrow more until the debt-to-GDP ratio is so high that it cannot be rescued by an actuarially fair intervention. Thus, while the bailout agency intervention may eliminate multiple equilibria, default events may not be reduced as a result of higher debt levels. However, now defaults would only occur due to fundamental reasons.

Our study is related to the recent literature on quantitative models of sovereign default that extended the approach developed by Eaton and Gersovitz (1981). Different aspects of sovereign debt dynamics and default have been analyzed in these quantitative studies. Aguiar and Gopinath (2006) find that shocks to the trend are important for emerging economies. Moreover, Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) show that long-term debt is essential for accounting for interest rate dynamics in the sovereign default framework. Hatchondo et al. (2012) study the effects of imposing a fiscal rule on debt dynamics and sovereign default risk. Also, Arellano and Ramanarayanan (2012) endogenize the maturity structure and analyze how it varies over the business cycle. Mendoza and Yue (2012) endogenize the output costs of defaulting. Furthermore, Benjamin and Wright (2009) introduce debt renegotiation to explain large delays observed during debt restructuring episodes. However, these studies do not consider defaults driven by a buyers strike and the role of bailouts in eliminating self-fulfilling debt crises.

A few recent papers also analyzed the role of bailouts in models of strategic sovereign default. Boz (2011) introduces a third party that provides subsidized enforceable loans subject to conditionality in order to replicate the procyclical use of market debt but the countercyclical use of IMF loans. Fink and Scholl (2011) also include bailouts and conditionality to reproduce the observed frequency and duration of bailout programs. Juessen and Schabert (2012) include bailout loans at favorable interest rates but conditional to fiscal adjustments, and show that this could not result in lower default rates. However, these studies do not consider self-fulfilling debt crises. Relatedly, Kirsch and Ruhmkorf (2013) incorporate financial assistance to a multiple equilibrium default model. In contrast to our paper, they model bailouts differently: bailout loans are provided at a fixed price schedule, are senior to market debt, and are subject to conditionality. Furthermore, the scope for the bailout is not to resolve the coordination problem completely as in our paper. Uhlig (2013) study the interplay between banks, bank regulation, sovereign default risk and central bank guarantees in a monetary union. He shows that governments in risky countries get to borrow more cheaply, effectively shifting the risk of some of the potential sovereign default losses on the common central bank.

Finally, our paper is also related to the literature on multiple equilibria in models of sovereign default, most notably Cole and Kehoe (1996, 2000), Calvo (1988), Aguiar et al (2013), Conesa and Kehoe (2013), Corsetti and Dedola (2013), and Broner et al (2013). While we share with these papers that crises can be triggered by a buyers strike, we differ in the focus of our analysis. Calvo (1988) shows that there could be multiple equilibria due to the government's inability to commit to its inflation target. Cole and Kehoe (1996, 2000) provide a characterization of the crisis zone and optimal policy in a dynamic stochastic general equilibrium model. Aguiar et al (2013) analyze the effect of inflation credibility in determining the vulnerability to rollover risk. Conesa and Kehoe (2013) show that under certain conditions government may find optimal not to undertake fiscal adjustments, thus "gambling for redemption". Corsetti and Dedola (2013) show that the government's ability to debase debt with inflation does not eliminate self-fulfilling debt crises, when the government lacks credibility. Broner et al (2013) propose a model with creditor discrimination and crowding-out effects to show that an increase in domestic purchases of debt may lead to self-fulfilling crises.

The rest of the article proceeds as follows. Section 2 introduces the model without bailouts. Section 3 introduces and characterizes the bailout agency. Section 4 presents the numerical results. Section 5 concludes.

2 A model of sovereign default dynamics: no bailout agency.

This section closely follows Cole-Kehoe (2000) and Arellano (2008). We assume that there is a single fiscal authority, which finances government consumption $c_t \geq 0$ with tax receipts $y_t \geq 0$ and assets $B_t \in \mathbf{R}$ (with positive values denoting debt), in order to maximize its utility

$$U = \sum_{t=0}^{\infty} \beta^t (u(c_t) - \chi_t \delta_t) \tag{1}$$

where β is the discount factor of the policy maker, $u(\cdot)$ is a strictly increasing, strictly concave and twice differentiable felicity function, χ_t is an exogenous one-time utility cost of default and $\delta_t \in \{0, 1\}$ is the decision to default in period t. We assume that tax receipts y_t are exogenous, while consumption, the level of debt and the default decisions are endogenous and chosen by the government.

In Arellano (2008) as well as Cole and Kehoe (2000), this is the utility of the representative household, y_t is total output and c_t is the consumption of the household, i.e. the fiscal authority is assumed to maximize welfare. The structure assumed here is mathematically the same, and consistent with that interpretation. It is also consistent with our preferred interpretation, where the utility function represents the preferences of the policy maker. For example, given the uncertainty of re-election, a policy maker may discount the future more steeply than would the private sector. Spending may be on groups that are particularly effective in lobbying the government. Finally, y_t should then be viewed as tax receipts, not national income.

A more subtle difference is the cost of a default, modeled here as a one-time utility cost χ_t, while it is modelled as a fractional loss in output in Arellano (2008) with Cole and Kehoe (2000). Note, however, that $c_t = y_t$ in default, and that at least for log-preferences, $u(c_t) = \log(c_t)$, a proportional decline in consumption each period following the default can equivalently be written as a one-time loss in utility. The utility cost formulation provides a free parameter to fine-tune the quantitative implications of the baseline specification of the model: a feature that we exploit in the numerical analysis.

In each period, the government enters with some debt level B_t and the tax receipts y_t as well as some other random variables are realized. Traders on financial markets are assumed to be risk neutral and discount future repayments of debt at some return R, and price new debt B_{t+1} according to some market pricing schedule $q_t(B_{t+1})$. Given the pricing schedule, the government then first makes a decision whether or not to default on its existing debt. If so, it will experience the one-time exogenously given default utility loss χ_t, be excluded from debt markets until re-entry, and simply consume its output, $c_t = y_t$ in this as well as all future periods, while excluded from debt markets. We assume that re-entry to the debt market happens with probability $0 \leq \alpha < 1$, drawn iid each period, and that re-entry starts with a debt level of zero. If the government does not default, it will choose consumption and the new debt level according to the budget constraint

$$c_t + (1-\theta)B_t = y_t + q_t(B_{t+1})(B_{t+1} - \theta B_t) \tag{2}$$

where $0 < \theta \leq 1$ is a parameter, denoting the fraction of debt that currently needs to be repaid. The parameter θ allows to study the effect of altering the maturity structure: the lower θ, the longer the maturity of government debt. The remainder of the debt θB_t will be carried forward, with the government issuing the new debt $B_{t+1} - \theta B_t$.

2.1 State space representation

We shall restrict attention to the following state-space representations of the equilibrium. At the beginning of a period, the aggregate state

$$s = (B, d, z) \tag{3}$$

describes the endogenous level of debt B, the default status d and some exogenous variable $z \in Z$. We assume that z follows a Markov process and that all decisions can be described in terms of the state s. The probability measure describing the transition for z to z' shall be denoted with $\mu(dz' \mid z)$. More specifically, we shall assume that z is given by

$$z = (y, \chi, \zeta) \tag{4}$$

We assume that $y \in [y_L, y_H]$ with $0 < y_L \leq y_H$ either has a strictly positive and continuous density $f(y \mid z_{\text{prev}})$, given the previous Markov state z_{prev}. We assume that $\chi \in \{\chi_L, \chi_H\}$ takes one of two possible values, with $0 = \chi_L \leq \chi_H$. We assume that $\zeta \in [0, 1]$ is uniformly distributed and denotes a "crisis" sunspot. We assume that the three entries in z are independent of each other, given the previous state. For most parts, we shall assume that z is iid, and that therefore the distributions for y and χ also do not depend on z_{prev}. For notation, we shall use $y(s)$ to denote the entry y in the state s, etc..

If the government does not default ($\delta = 0$), the period-per-period budget constraint is

$$c + (1 - \theta)B(s) = y(s) + q(B'; s)(B' - \theta B(s)) \tag{5}$$

where B' is the new debt level chosen by the government and where $q(B'; s)$ is the pricing function for the new debt B'.

If the government defaults ($\delta = 1$), the budget constraint is

$$c = y(s) \tag{6}$$

We assume that the government will be excluded from debt markets until it is given the possibility for re-entry. We assume that re-entry to the debt market happens with probability $0 \leq \alpha < 1$, drawn iid each period[3], and that re-entry starts with a debt level of zero. In that case, "good standing" $d = 0$ in the state s will be turned to "bad standing" or "in default" $d = 1$ in the state s' following a default, and that $d = 1$ is followed by $d = 1$ with probability $1 - \alpha$ and with $d = 0$ with probability α. There is no other role for d. The default decision of the government is endogenous and (assumed to be) a function of the state s, $\delta = \delta(s)$.

We can now provide a recursive formulation of the decision problem for the government. The value function in the default state and after the initial default utility loss is given by

$$v_D(z) = u(y(z)) + \beta(1 - \alpha)E\left[v_D(z') \mid z\right] + \alpha E\left[v_N D(s' = (0, 0, z')) \mid z\right] \tag{7}$$

Given the debt pricing schedule $q(B; s)$, the value from not defaulting is

$$\begin{aligned} v_{ND}(s) = \max_{c,B'} \{ & u(c) + \beta E\left[v(s') \mid z\right] \mid \\ & c + (1 - \theta)B(s) = y(s) + q(B'; s)(B' - \theta B(s)) \\ & s' = (B', d(s), z') \} \end{aligned}$$

[3]Technically, assume that re-entry happens if $\zeta \leq \alpha$, in order to achieve dependence on the state z.

The overall value function is given by

$$v(s) = \max_{\delta \in \{0,1\}} (1-\delta) v_{ND}(s) + \delta(v_D(z(s)) - \chi(s)) \tag{8}$$

Given parameters, a law of motion for z, an **equilibrium** is defined as measurable mappings $q(B'; s)$ in B' and s as well as $c(s), \delta(s)$ and $B'(s)$ in s, such that

1. Given the pricing function $q(B'; s)$, the government maximizes its utility with the choices $c(s), \delta(s)$ and $B'(s)$, subject to the budget constraint (5) and subject to the exclusion from financial markets for all periods, following a default.

2. The market pricing function $q(B'; s)$ is consistent with risk-neutral pricing of government debt and discounting at the risk free return R.

2.2 Debt pricing

Given a level of debt B and "good standing" $d = 0$, let

$$D(B) = \{z \mid \delta(s) = 1 \text{ for } s = (B, 0, z)\} \tag{9}$$

be the default set, and let

$$A(B) = \{z \mid \delta(s) = 0 \text{ for } s = (B, 0, z)\} \tag{10}$$

be the set of all z, such that the government will not default and instead, continue to honor its debt obligations: both are (restricted to be) a measurable set, according to our equilibrium definition. The disjoint union of $D(B)$ and $A(B)$ is the entire set Z. Define the market price for debt, in case of no current default, i.e.

$$\bar{q}(B'; s) = \frac{1}{R} \int_{z' \in A(B)} (1 - \theta + \theta q(B(s' = (B', 0, z')))) \, \mu(dz' \mid z) \tag{11}$$

Here and below, we use the notation $B(s' = (B', 0, z'))$ to denote the new debt level $B(s')$, given the new state $s' = (B', 0, z')$. Due to risk neutral discounting, this is the market price of debt, if there is no default "today". Define the probability of a continuation **next period** per

$$P(B'; s) = \text{Prob}(z' \in A(B') \mid s) = E\left[1_{\delta(s')=0} \mid s\right] \tag{12}$$

If $\theta = 0$, i.e., if all debt has the maturity of one period only, then

$$\bar{q}(B'; s) = \frac{1}{R} P(B'; s) \tag{13}$$

We need to check, whether there could be a default "today". We shall impose the following assumption.

Assumption A. 1 *Given a state s, either $q(B'; s) = \bar{q}(B'; s)$ for all B' or $q(B'; s) = 0$ for all B'.*

This assumption rules out equilibria, where, say, the market expects a current default, if the government tries to finance some future debt level B', but not for others[4].

We now turn to analyzing the possibility for a self-fulfilling expectation of a default. Define the value of not defaulting, if the market prices are consistent with current debt repayment,

$$\begin{aligned}\bar{v}_{ND}(s) = \max_{c,B'} \{ & u(c) + \beta E\left[v(s') \mid z\right] \mid \\ & c + (1-\theta)B(s) = y(s) + \bar{q}(B'; s)(B' - \theta B(s)) \\ & s' = (B', d(s), z')\}\end{aligned}$$

where it should be noted that the continuation value function is as before, i.e. given by (8). Define the value of not defaulting, if the market prices are consistent with a current default,

$$\begin{aligned}\underline{v}_{ND}(s) = \max_{c,B'} \{ & u(c) + \beta E\left[v(s') \mid z\right] \mid \\ & c + (1-\theta)B(s) = y(s) \\ & s' = (B', d(s), z')\}\end{aligned}$$

With that, define two bounds for the current debt levels B, see also figure 19. Above the upper bound $B \geq \bar{B}(z)$, the government finds it optimal to default today, even if the market was willing to finance future debt in the absence of a default now, i.e. even if $q(B'; s) = \bar{q}(B'; s)$. Above the lower bound $B \geq \underline{B}(z)$, the government finds it optimal

[4]Cole and Kehoe (2000) finesse this issue with more within-period detail, having the government first sell new debt at some pricing schedule, before taking the default decision.

to default, if the market thinks it will do so and therefore is unwilling to finance further debt, $q(B';s) = 0$. I.e., let

$$\bar{B}(z) = \inf\{B \mid \bar{v}_{ND}(s = (B, 0, z)) \leq v_D(z(s)) - \chi(s = (B, 0, z))\} \tag{14}$$

as well as

$$\underline{B}(z) = \inf\{B \mid \underline{v}_{ND}(s = (B, 0, z)) \leq v_D(z(s)) - \chi(s = (B, 0, z))\} \tag{15}$$

Whether or not there will be a default at some debt level B between these bounds will be governed by the sunspot random variable ζ. As in Cole-Kehoe (2000), we assume that the probability of a default in this range is some exogenously given probability π.

Assumption A. 2 *For some parameter $\pi \in [0, 1]$, and all s with $\underline{B}(z) \leq B(s) \leq \bar{B}(z)$, we have $q(B';s) = \bar{q}(B';s)$, if $\zeta(s) \geq \pi$ and $q(B';s) = 0$, if $\zeta < \pi$.*

The equilibrium will therefore look as follows (up to breaking indifference at the boundary points):

1. If $B > \bar{B}(z)$, the government will default now and not be able to sell any debt. The market price for new debt will be zero.
2. If $\underline{B}(z) \leq B \leq \bar{B}(z)$, the government will
   (a) default with probability π (more precisely, for $\zeta(z) < \pi$), and the market price for new debt will be zero,
   (b) continue with probability $1 - \pi$ (more precisely, for $\zeta(z) \geq \pi$), and the market price for new debt will be $\bar{q}(B';s)$.
3. If $B < \underline{B}(z)$, the government will not default, and the market price for debt will be given by $\bar{q}(B';s)$.

Following Cole and Kehoe (2000), we shall use the term "crisis zone" for the maximal range for new debt, for which there might be a "sunspot" default next period, i.e. for

$$B' \in \mathcal{B} = \left[\min \underline{B}(z), \max \bar{B}(z)\right]$$

Note that safe debt will be priced at q^* satisfying

$$q^* = \frac{1}{R}(1 - \theta + \theta q^*)$$

and is therefore given by

$$q^* = \frac{1-\theta}{R-\theta} \tag{16}$$

Conversely, given some price q, one can infer the implicit equivalent safe rate

$$R(q) = \theta + \frac{1-\theta}{q} \tag{17}$$

To denote the dependence of the equilibrium on the sunspot parameter π or the dependence on the debt duration parameter θ, we shall use them as superscripts, if needed. Some analysis for the no-bailout case and some insights into the stationary distribution of debt and their dependence on the discount factor are in appendix A.

3 Bailouts

We now introduce the possibility for a bailout per a large and infinitely lived, risk neutral outside investor. More precisely, we envision a facility with sufficiently deep pockets (backed by, say, governments other than the one under consideration here), which aims at ensuring the selection of the "good" equilibrium, while earning the market rate of return in expectation on its bond holdings. I.e., we imagine that this bailout facility insists on actuarially fair pricing. It may well be that actual policy interventions amount to a subsidy or perhaps even a penalty. We view the actuarially fair "restoration-of-the-good-equilibrium" as an important benchmark. It might be interesting to consider other mechanisms, which are not actuarially fair, as well, and we do so in the appendix B. An alternative is to examine the conditionality of such bailouts, combining help with insistence on fiscal discipline, see Fink-Scholl (2011).

If the bailout facility buys the entire debt, then the solution is easy in principle. It should calculate the $\pi = 0$-equilibrium described above, price debt accordingly, and let the country choose the debt level it wants, given this pricing schedule. Since the bailout facility is always there, also in the future, to guarantee the "good" equilibrium, the pricing is actuarially fair.

There is generally no need to buy the entire debt, however, in order to assure the $\pi = 0$ equilibrium. We therefore assume a minimal bailout facility. I.e., we characterize the minimal level of debt $B'_a(s)$ such a facility needs to guarantee buying at the $\pi = 0$ equilibrium price, so that markets must coordinate on this equilibrium. We assume that the facility buys at the $\pi = 0$ equilibrium price, even if the rest of the market does not buy at all: this is only relevant "off-equilibrium". It is important in this construction, that the debt held by the facility is treated the same as the debt held by market participants. The country is indifferent between purchasing this debt from the facility or from the market, and so is the market. The guarantee just needs to be there, in the (now hypothetical) case that the market coordinates on the default outcome.

To characterize the minimal guarantee level $B'_a(s)$, we need to re-examine and slightly modify the value function of the government. We need an assumption about the continuation in the case that the market does not buy, and whether the buyers' strike persists or not. In order to truly characterize the minimal intervention, we make the "optimistic" assumption that a potential buyer's strike only lasts for one period, i.e., given the presence of the large investor, the continuation value following a no-default today shall be given by the value function valid for the $\pi = 0$ equilibrium. Given the policy $B'_a(s)$, define the no default value under assistance (and current buyers' strike, except for the large investor) as

$$\begin{aligned} \underline{v}_{ND;a}(s) = \max_{c,B'} \{ & u(c) + \beta E \left[v^{(\pi=0)}(s') \mid z \right] \mid \\ & c + (1-\theta)B(s) = y(s) + q^{(\pi=0)}(B'; s)(B' - \theta B(s)) \\ & B' \leq B'_a(s) \\ & s' = (B', d(s), z') \} \end{aligned} \tag{18}$$

Note the second constraint, encapsulating the limit of the assistance. Let $\epsilon > 0$ be a parameter and small number to break indifference. Given $q^{(\pi=0)}$ and $v^{(\pi=0)}$, one can therefore solve for $B'_a(s)$ "state by state" such that

$$\underline{v}_{ND;a}(s = (B, 0, z)) = v_D(z(s)) - \chi(s = (B, 0, z)) + \epsilon \text{ for all } 0 \leq B \leq \bar{B}(z) \tag{19}$$

where $\bar{B}(z)$ is the maximum level of current debt consistent with no default in the $\pi = 0$ equilibrium. For $B > \bar{B}(z)$, define $B'_a(s) = 0$, but do note, that $q(B'; s) = 0$ for any $B' > 0$ per definition of $\bar{B}(z)$. In other words, the facility could also provide the (meaningless) guarantee of willing to buy any positive level of debt $B'_a(s)$ at a zero price.

Proposition 1 *Suppose $B'_a(s)$ satisfies (19). Then, $\underline{B}(z) = \bar{B}(z)$, i.e, there will not be a default, unless debt exceeds $\bar{B}(z)$.*

Proof: *Suppose that $\underline{B}(z) \neq \bar{B}(z)$. Then, (14) and (15) imply that $\underline{B}(z) < \bar{B}(z)$. It follows that for every $B \in (\underline{B}(z), \bar{B}(z))$, $\bar{v}_{ND}(s = (B, 0, z)) > v_D(z(s)) - \chi(s = (B, 0, z))$ and $\underline{v}_{ND;a}(s = (B, 0, z)) < v_D(z(s)) - \chi(s = (B, 0, z))$. However, if $B'_a(s)$ satisfies (19), then $\underline{v}_{ND;a}(s = (B, 0, z)) > v_D(z(s)) - \chi(s = (B, 0, z))$ for all $0 \leq B \leq \bar{B}(z)$, which is a contradiction.* •

In the iid case and with a constant embarrassment utility costs $\chi > 0$ of defaulting, a bit more can be said. In that case, some constant value $\beta\tilde{v}_D$

$$\beta E[v_D(z')] \equiv \beta\tilde{v}_D$$

is the continuation value from defaulting. Likewise, when receiving the full guarantee $B'_a(s)$, the continuation value of not defaulting is $\beta\tilde{v}_{ND}(B'_a(s))$, given by

$$\beta E[v(B'_a(s), 0, z')] = \beta\tilde{v}_{ND}(B'_a(s))$$

Criterion (19) becomes

$$\begin{aligned} & u(y(s)) - u\left(y(s) + q^{(\pi=0)}(B'_a(s); s)\,(B'_a(s) - \theta B(s)) - (1-\theta)B(s)\right) \\ = \;\; & \beta\tilde{v}_{ND}(B'_a(s)) - \beta\tilde{v}_D + \chi - \epsilon \end{aligned} \tag{20}$$

comparing the current utility gain from defaulting to the utility continuation loss from defaulting, including the embarrassment cost χ.

Proposition 2 *In the iid and constant-χ case, we have*

1. *For two states s_1, s_2, if $B(s_1) > B(s_2)$, then $B'_a(s_1) \geq B'_a(s_2)$.*
2. *If $B(s) > 0$ and the default set is nonempty, then*

$$q^{(\pi=0)}(B'_a(s); s)\,(B'_a(s) - \theta B(s)) < (1-\theta)B(s)$$

3. *For two states s_1, s_2, if $y(s_1) > y(s_2)$, then $B'_a(s_1) \leq B'_a(s_2)$.*

4. *For two states s_1, s_2, if $\chi(s_1) > \chi(s_2)$, then $B'_a(s_1) \leq B'_a(s_2)$.*

Proof:

1. *Suppose, to get a contradiction, that $B'_a(s_1) < B'_a(s_2)$. Denote the consumption level associated to $(B(s_1), B'_a(s_1))$, $(B(s_2), B'_a(s_2))$, and $(B(s_2), B'_a(s_1))$ by c_1, c_2, and $\tilde{c}_2$ respectively. Criterion (19) becomes*

$$u(c_2) + \beta\tilde{v}_{ND}(B'_a(s_2)) = v_D(z(s)) - \chi + \epsilon$$

Then, by definition of B'_a, we have

$$u(c_2) + \beta\tilde{v}_{ND}(B'_a(s_2)) > u(\tilde{c}_2) + \beta\tilde{v}_{ND}(B'_a(s_1))$$

Given that $\tilde{c}_2 > c_1$, we have

$$u(\tilde{c}_2) + \beta\tilde{v}_{ND}(B'_a(s_1)) > u(c_1) + \beta\tilde{v}_{ND}(B'_a(s_1))$$

But, by definition of B'_a, we have

$$u(c_1) + \beta\tilde{v}_{ND}(B'_a(s_1)) = v_D(z(s)) - \chi + \epsilon$$

which is a contradiction.

2. *From proposition 2 in Arellano (2008) it follows that there is no contract available $\{q^{(\pi=0)}(B'; s), B'\}$ such that $q^{(\pi=0)}(B'; s)(B' - \theta B(s)) - (1-\theta)B(s) > 0$. The definition of our minimal guarantee implies that $B'_a(s) \leq B'$. Thus, the contract $\{q^{(\pi=0)}(B'_a(s); s), B'_a(s)\}$ is available to the economy and it must be the case that $q^{(\pi=0)}(B'_a(s); s)\,(B'_a(s) - \theta B(s)) < (1-\theta)B(s)$.*

3. *Suppose, to get a contradiction, that $B'_a(s_1) > B'_a(s_2)$. Denote the consumption level associated to $(y(s_1), B'_a(s_1))$, $(y(s_2), B'_a(s_2))$, $(y(s_2), B'_a(s_1))$, and $(y(s_1), B'_a(s_2))$ by c_1, c_2, $\tilde{c}_2$, and $\tilde{c}_1$ respectively. By definition of B'_a, $B'_a(s_1) > B'_a(s_2)$ implies*

$$u(\tilde{c}_1) + \beta\tilde{v}_{ND}(B'_a(s_2)) < u(c_1) + \beta\tilde{v}_{ND}(B'_a(s_1)) = u(y(s_1)) + \beta\tilde{v}_D - \chi + \epsilon$$
$$u(\tilde{c}_2) + \beta\tilde{v}_{ND}(B'_a(s_1)) > u(c_2) + \beta\tilde{v}_{ND}(B'_a(s_2)) = u(y(s_2)) + \beta\tilde{v}_D - \chi + \epsilon$$

Also, by concavity of the utility function and part 2 of this proposition, we have

$$u(y(s_2)) - u(\tilde{c}_2) > u(y(s_1)) - u(\tilde{c}_1) = \beta(\tilde{v}_{ND}(B'_a(s_1)) - \tilde{v}_D) + \chi - \epsilon$$

This implies that $u(y(s_2)) + \beta\tilde{v}_D - \chi + \epsilon > u(\tilde{c}_2) + \beta\tilde{v}_{ND}(B'_a(s_1))$, which is a contradiction.

4. *This follows from criterion (19).*

•

Government's risk aversion	σ	1/2
Interest rate	r	3.0%
Income autocorrelation coefficient	ρ	0.945
Standard deviation of innovations	σ_ϵ	3.4%
Mean log income	μ	(-1/2)σ_ϵ^2
Exclusion	α	0.2
Maturity structure	θ	0.8
Discount factor	β	0.4
Cost	χ_L	0
Cost	χ_H	0.5
SFC sunspot probability	π	0.05
Income grid	$y_1, \ldots, y_{20}$	$[0.73, \ldots, 1.37]$
debt grid	$B_1, \ldots, B_{1000}$	

Table 1: Parameter values for the calibration. One period is one year.

4 A numerical example

This section presents the results of a numerical exercise, where the model is solved using value function iteration. First we discuss the functional forms and parametrization, and then we give the results.

The government's within period utility function has the CRRA form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

We assume that the income process is a log-normal autoregressive process with unconditional mean μ

$$\log(y_{t+1}) = (1 - \rho)\mu + \rho \log(y_t) + \varepsilon_{t+1}$$

with $E(\varepsilon) = 0, E(\varepsilon^2) = \sigma_\varepsilon^2$.

A period in the model refers to a year. Table 1 summarizes the key parameters used in this exercise. Additionally, as transition matrix between the two χ-states, we choose

$$\begin{bmatrix} 0 & 1 \\ 0.04 & 0.96 \end{bmatrix}$$

Both the value for χ_H as well as the transition probability from χ_H to χ_L was chosen after some experimentation to hit two target properties. First, we aimed at a debt-to-tax

	Target	$\theta = 0.8$
Debt/Tax ratio	2 .. 3	2.4
Default rate	5% .. 8%	6.6%

Table 2: *Targets and numerical results for the debt/tax ratio and the default rate*

	Buyers present	Buyers' strike
χ_L	38%	2%
χ_H	12%	48%

Table 3: *The structure of defaults.*

ratio somewhere between two and three. Second, we aimed at default rates between 5 and 8 percent. While it tends to be hard to hit these numerical targets with, say, the assumption that the only penalty to default is higher consumption variability, it is comparatively easy to do it here, with these two additional free parameters, see table 2.

Table 3 shows the "anatomy" of defaults. One can see that 12 percent of the defaults happen due to fundamental problems, even with a "responsible" χ_H government and despite buyers willing to buy the bonds in principle. However, nearly half of all defaults occur due to a buyers' strike: it is these occurrences which the bailout agency shall help to avoid.

Figure 2 shows the resulting crisis zones. The intervals in the figure denote the pairs of income and debt levels for which the government would only default in the case of a buyers' strike. For any debt level to the left of the interval, the government always repays independently of whether there is a buyers strike or not. Similarly, for debt levels to the right of the interval, the government will always find it optimal to default. Figure 3 shows the debt purchase assistance policy by the bailout agency. Over a fairly narrow range, the guaranteed purchases quickly rise until they reach 100%. At that point, the risk and incentive of a default due to fundamental reasons tomorrow is so large, that the failure to sell a small fraction of the new debt will be enough to trigger a default. If the current debt is even higher, the fundamental debt price collapses all the way to zero, and so does the bailout guarantee. The country will not be willing to repay or will be unable to repay in the future, and purchasing debt at any positive price will result

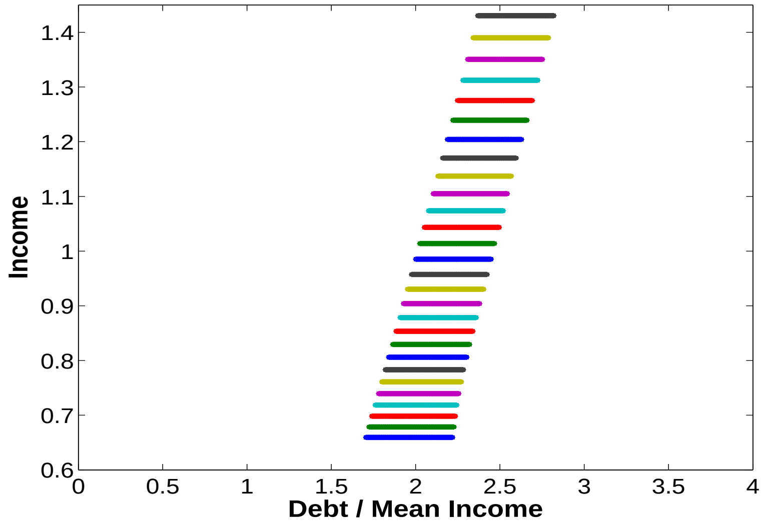

Figure 2: *Crisis zones*

in expected losses. Thus, the bailout guarantee is only positive for pairs of income and debt levels in the crises zones, shown in Figure 2. Figure 4 shows the dependence of this policy on income. With currently higher income, it may well be worth guaranteeing debt purchases, that would lead to default at lower income levels. In other words, the bailout agency should rather support the country during a boom than a recession. This result may be counterintuitive from a policy perspective, but surely makes sense from the perspective of a risk-neutral investor.

Table 4 shows the impact of varying the maturity of debt. As the maturity of debt is increased, the threat from a buyers strike in any given period declines, as an ever smaller fraction of the debt needs to be rolled over. As a result, the incentive to maintain higher debt levels rises, and not much changes with the default rates, as the overall result, while the length of the crisis zones shrink. These results are graphically represented in figures 5,6 and 7. The corresponding shift in the debt purchase assistance policy is shown in 8.

Table 5 shows that the change in the sunspot probability π for a buyers strike has only a modest impact on the overall default probability, while the debt level increases. With the fear of a default due to buyer's strike gone, debt becomes more attractive. Indeed, as table 6 shows, the default probability mass now shifts from the "buyer strike" scenario to

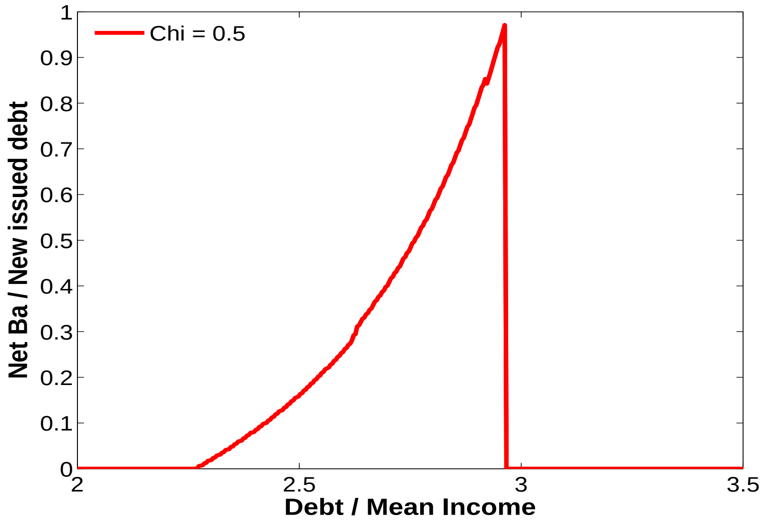

Figure 3: *Debt purchase assistance policy by the bailout agency.*

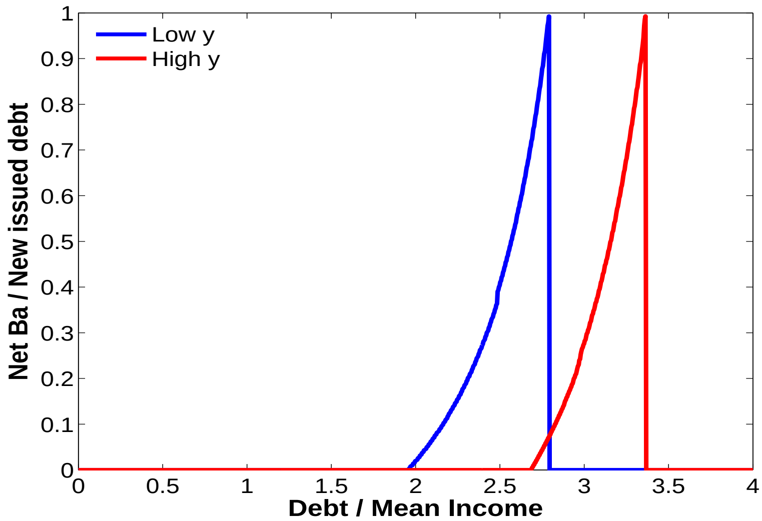

Figure 4: Income and debt purchase assistance

Targets:

	Target	$\theta = 0.9$	$\theta = \mathbf{0.8}$	$\theta = 0.5$	$\theta = 0$
Debt/Tax ratio	2 .. 3	3.3	**2.4**	1.8	1.6
Default rate	5% .. 8%	6.6%	**6.6%**	6.2%	6.2%

Defaults: $\theta = 0.9$:

	Buyers present	Buyers' strike
χ_L	38%	2%
χ_H	16%	44%

Defaults: $\theta = 0$:

	Buyers present	Buyers' strike
χ_L	42%	2%
χ_H	2%	54%

Table 4: *Variations in maturity and their impact on defaults. $\theta = 0$ is one-period debt, whereas $\theta = 0.9$ is essentially 10-period debt.*

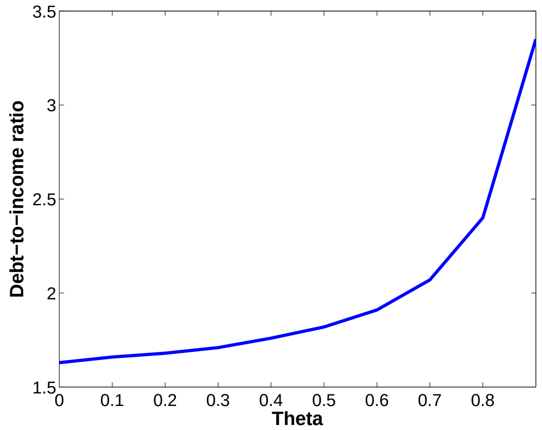

Figure 5: *Debt and θ*

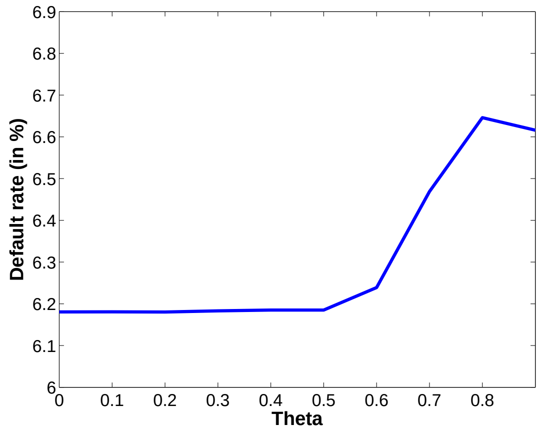

Figure 6: *Default and θ*

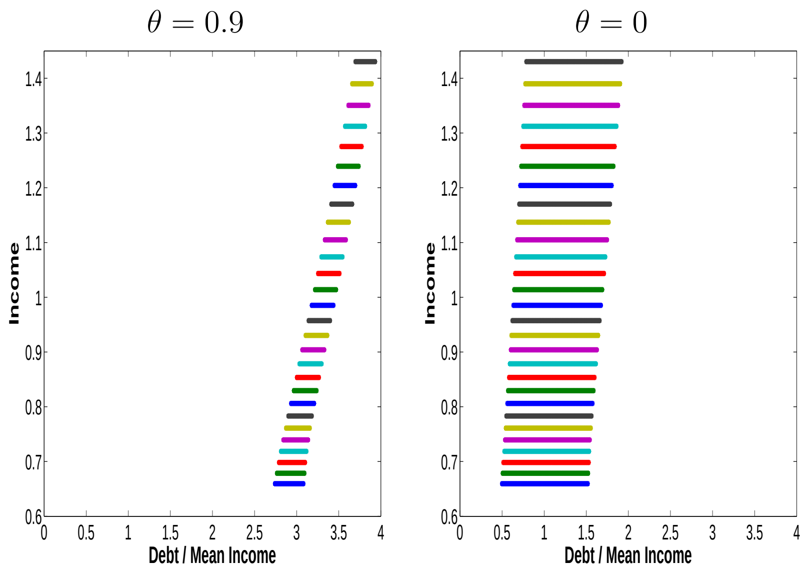

Figure 7: *Maturity and Crisis Zones*

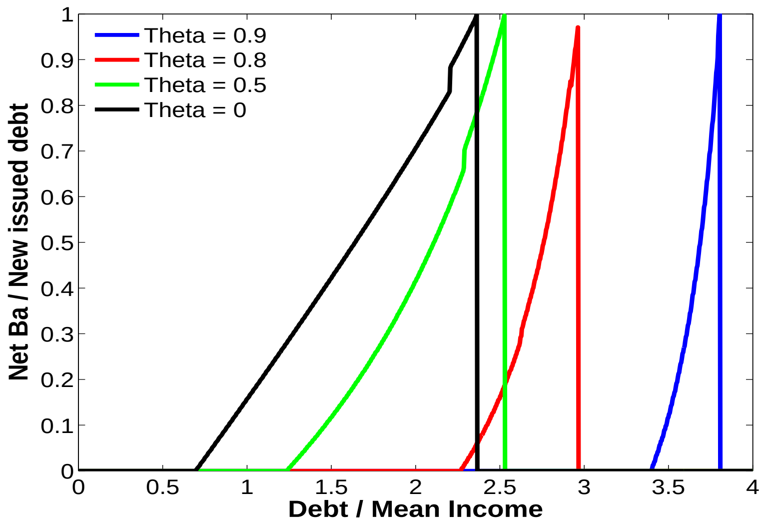

Figure 8: *Maturity and debt purchase assistance*

	Target	$\pi = 0.2$	$\pi = 0.1$	$\pi = \mathbf{0.05}$	$\pi = 0$
Debt/Tax ratio	2 .. 3	1.8	2.1	**2.4**	2.9
Default rate	5% .. 8%	5%	8%	**6.6%**	4%

Table 5: *Sunspot probabilities and debt levels*

the default due to fundamental reasons. Graphical representations of these relationships are in figures 9 and 10. There is a conundrum for the bailout agency here. As that agency is successful in reducing the sunspot default probability from, say, 20 percent to zero percent, the overall default rates only decline modestly from 5% to 4%. In some ways, the problem gets postponed: the government gets a bit more time to accumulate more debt. As far as default rates are then concerned after this transition, not much will have changed.

Figure 11 shows the pricing function for debt at our benchmark value for θ, while 12 shows the pricing function for the somewhat more intuitive case of $\theta = 0$, i.e. one-period debt. Indeed, debt prices rise and thus yields decline, as the bailout agency assures the $\pi = 0$ equilibrium through its purchase guarantees. The resulting debt buildup is rather fast, as figure 13 shows. Figures 14, 15 and 16 show how the stationary debt distribution is shifted to the right, inducing the higher occurrences of defaults due to fundamental

Defaults for $\pi = 0.1$: total prob = 8%:		
	Buyers present	Buyers' strike
χ_L	27%	3%
χ_H	8%	62%
Defaults for $\pi = 0.05$ (Benchmark): total prob = 6.6%:		
	Buyers present	Buyers' strike
χ_L	38%	2%
χ_H	12%	48%
Defaults for $\pi = 0$:total prob = 4%:		
	Buyers present	Buyers' strike
χ_L	81%	0%
χ_H	19%	0%

Table 6: *Sunspot probabilities and default details*

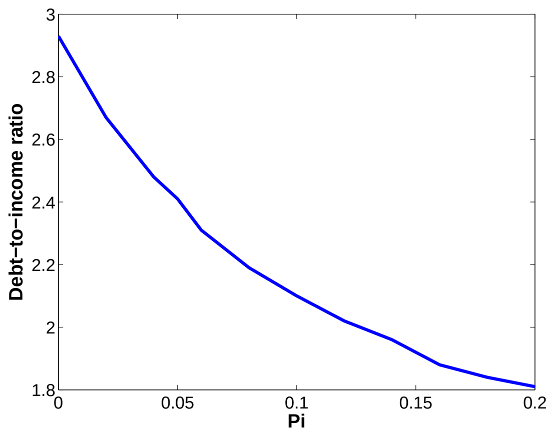

Figure 9: *Debt and π*

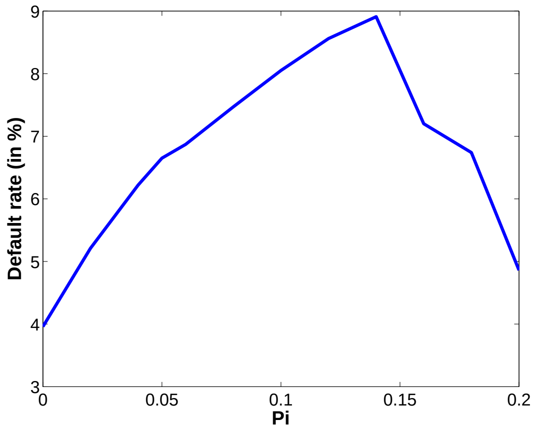

Figure 10: *Default and* π

reasons. A graphical representation of the decision rules underlying the increased debt accumulation under debt purchase assistance is shown in figure 17: the decision rule shifts upwards, indicating a larger willingness of the government to incur debt.

5 Conclusions

We have analyzed the dynamics of sovereign debt defaults, drawing on insights from three literatures, particularly Arellano (2008), Cole-Kehoe (2000) and Beetsma-Uhlig (1999). More precisely, we have analyzed the dynamics of sovereign debt, when politicians discount the future considerably more than private markets and when there are possibilities for both a "sunspot-"driven default as well as a default driven by worsening of economic conditions or weakening of the resolve to continue with repaying the country debt.

We have shown how this can lead to a scenario, where the country perches itself in a precarious position, with the possibility of defaults imminent. We characterized the minimal actuarially fair intervention that restores the "good" equilibrium of Cole-Kehoe, relying on the market to provide residual financing.

Three messages and conclusions emerge. First, an actuarially fair bailout agency may be able to restore the "fundamentals-only" equilibrium, by issuing debt purchase guarantees and without incurring losses in expectation. Second, these guarantees need to go far enough, but not too far. Defaults due to fundamental reasons still lurk around the

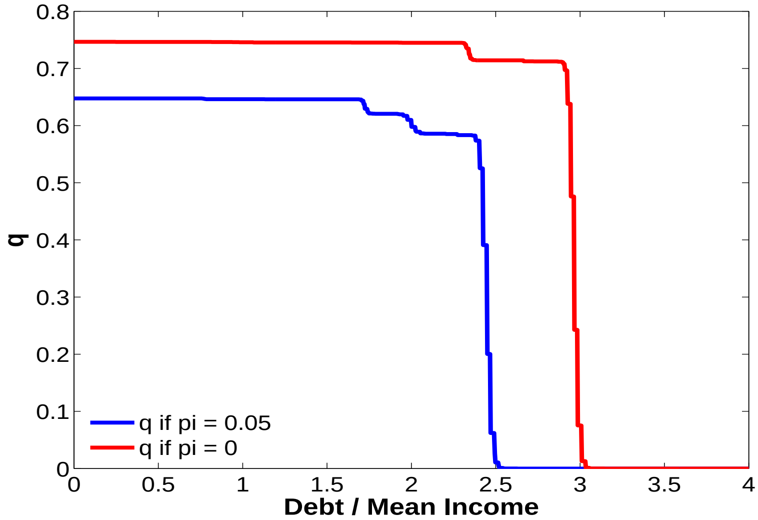

Figure 11: *Debt pricing function,* $\pi = 0.05$ *vs* $\pi = 0$.

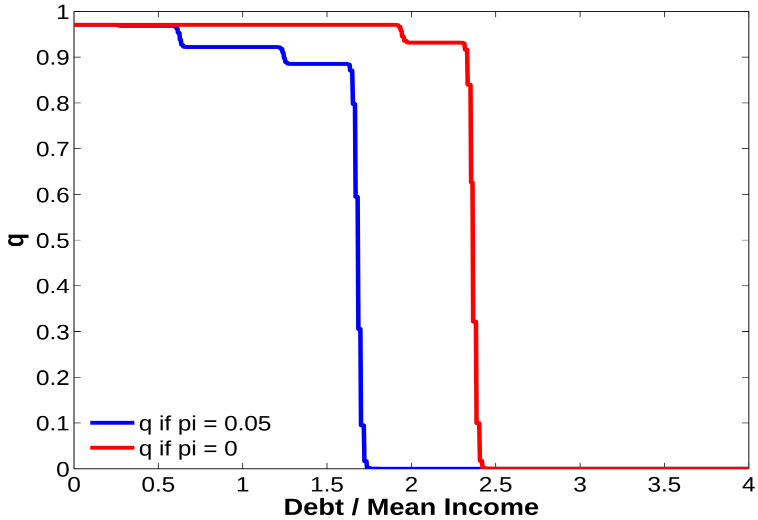

Figure 12: *Debt pricing function,* $\pi = 0.05$ *vs* $\pi = 0$*, when* $\theta = 0$.

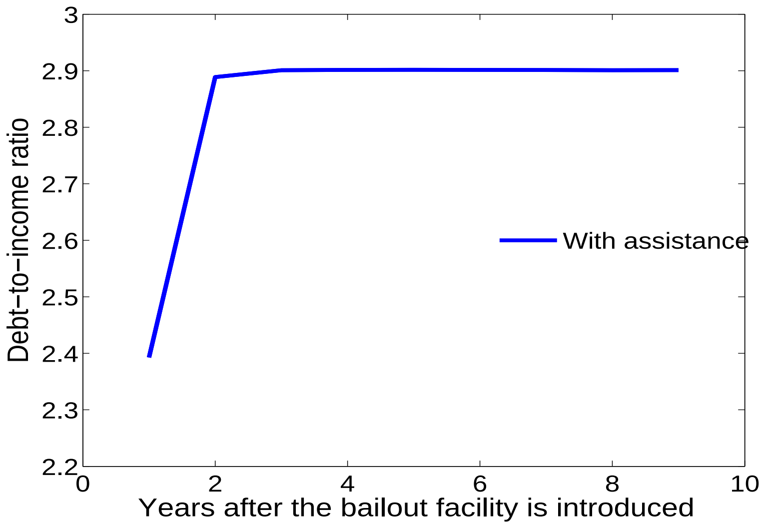

Figure 13: *Debt dynamics after the assistance facility is introduced. Starting point:* $\pi = 0.05$*, mean income, mean debt/gdp ratio.*

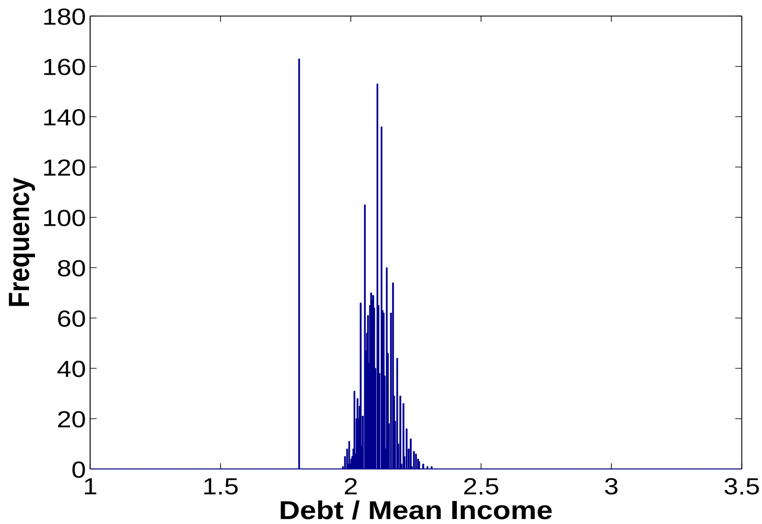

Figure 14: *Debt Distribution with sunspots:* $\pi = 0.1$

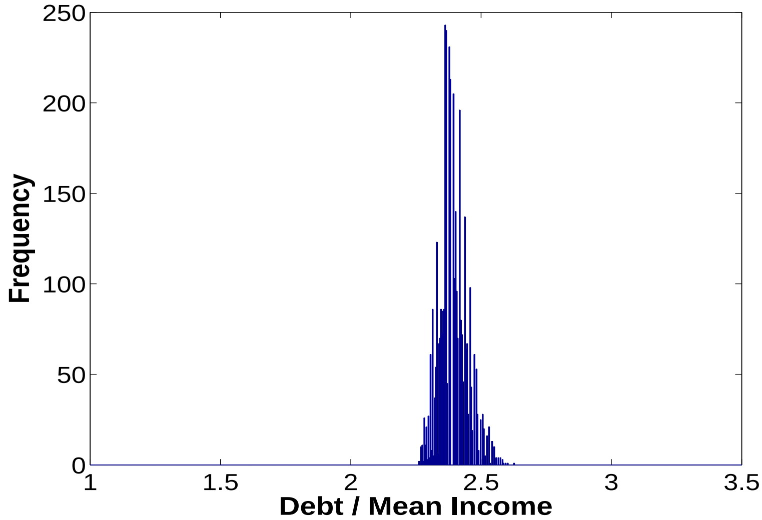

Figure 15: *Debt Distribution with sunspots:* $\pi = 0.05$

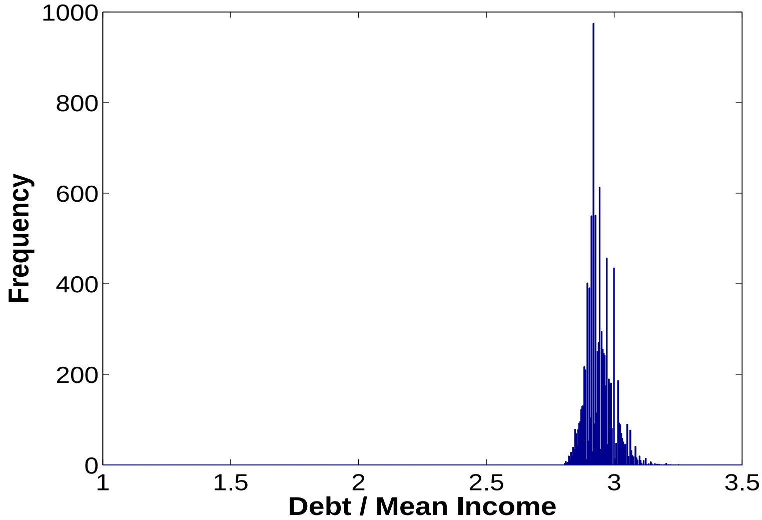

Figure 16: *Debt Distribution without sunspots or with debt purchase assistance:* $\pi = 0$

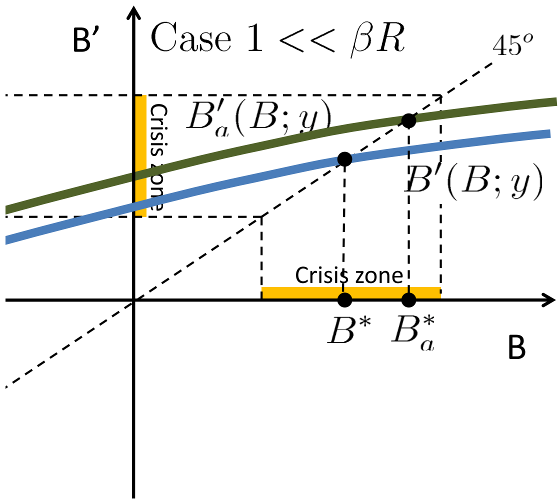

Figure 17: *Stationary debt dynamics, permanent assistance*

corner, and excessive debt purchase guarantees would then invariably lead to losses for the bailout agency. Third, the overall default rates may not change much, as the higher guarantees and the lower yields mean that the current government can relax a bit in its efforts to repay its debt level and incur more deficits instead. The resulting higher debt levels in the future will then make future defaults inevitable on occasions, but this time due to fundamental reasons rather than buyers' strike.

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A No bailouts: analysis

In this section, we exclude assisted debt issuance, i.e. we assume that $q_a(B'; s) \equiv 0$. We therefore furthermore assume, that the bailout sunspot $\psi(s)$ is "irrelevant", i.e. all functions are independent of ψ: it may not be necessary to assume so, but it seems unnecessary to consider it. We finally shall assume that z is iid.

The following results are essentially in Arellano (2008) and state that default incentives increase with higher debt.

Proposition 3 *Suppose z is iid and that all functions are independent of ψ. If default is optimal for $s^{(1)} = (B^{(1)}, 0, z)$, then default is optimal for $s^{(2)} = (B^{(2)}, 0, z)$, whenever $B^{(2)} > B^{(1)}$.*

Proof: *If $s^{(1)} \in D\left(B^{(1)}\right)$ then*

$$u\left(y\right) + \beta E\left[v_d\left(z'\right)\right] - \chi > u\left(y\left(z\right) + q\left(B'; s^{(1)}\right)\left(B' - \theta B\left(s^{(1)}\right)\right) - \left(1-\theta\right) B\left(s^{(1)}\right)\right).$$

Given that

$$q\left(B'; s^{(2)}\right)\left(B' - \theta B\left(s^{(2)}\right)\right) - \left(1-\theta\right) B\left(s^{(2)}\right) < q\left(B'; s^{(1)}\right)\left(B' - \theta B\left(s^{(1)}\right)\right) - \left(1-\theta\right) B\left(s^{(1)}\right)$$

for all B', then

$$u\left(y\left(z\right) + q\left(B'; s^{(1)}\right)\left(B' - \theta B\left(s^{(1)}\right)\right) - \left(1-\theta\right) B\left(s^{(1)}\right)\right) >$$
$$u\left(y\left(z\right) + q\left(B'; s^{(2)}\right)\left(B' - \theta B\left(s^{(2)}\right)\right) - \left(1-\theta\right) B\left(s^{(2)}\right)\right)$$

Then, it follows that

$$u\left(y\right) + \beta E\left[v_d\left(z'\right)\right] - \chi > u\left(y\left(z\right) + q\left(B'; s^{(2)}\right)\left(B' - \theta B\left(s^{(2)}\right)\right) - \left(1-\theta\right) B\left(s^{(2)}\right)\right).$$

Hence, default is optimal for $s^{(2)}$. •

The next proposition states that lower tax receipts y increases default incentives.

Proposition 4 *Suppose z is iid and that all functions are independent of ψ. Default incentives are stronger, the lower are tax receipts. I.e., for all $y^{(1)} \leq y^{(2)}$, if $z^{(2)} = (y^{(2)}, \chi, \zeta, \psi) \in D(B)$, then so is $z^{(1)} = (y^{(1)}, \chi, \zeta, \psi) \in D(B)$.*

Proof: *Let $B^{(1)}$ be the optimal choice for $z^{(1)}$ and $B^{(2)}$ the optimal choice for $z^{(2)}$. If*

$$u\left(y_2 + q\left(B^{(2)}; s^{(2)}\right)\left(B^{(2)} - \theta B\left(s^{(2)}\right)\right) - (1-\theta) B\left(s^{(2)}\right)\right) + \beta E\left[v\left(s'\right)\right]$$
$$-\left\{u\left(y_1 + q\left(B^{(1)}; s^{(1)}\right)\left(B^{(1)} - \theta B\left(s^{(1)}\right)\right) - (1-\theta) B\left(s^{(1)}\right)\right) + \beta E\left[v\left(s'\right)\right]\right\} >$$
$$u\left(y_2\right) + \beta E\left[v_D\left(z'\right)\right] - \left\{u\left(y_1\right) + \beta E\left[v_D\left(z'\right)\right]\right\}$$

then, $z^{(2)} \in D(B)$ implies $z^{(1)} \in D(B)$.

Besides,

$$u\left(y_2 + q\left(B^{(2)}; s^{(2)}\right)\left(B^{(2)} - \theta B\left(s^{(2)}\right)\right) - (1-\theta) B\left(s^{(2)}\right)\right) + \beta E\left[v\left(s'\right)\right] >$$

$$u\left(y_2 + q\left(B^{(1)}; s^{(2)}\right)\left(B^{(1)} - \theta B\left(s^{(2)}\right)\right) - (1-\theta) B\left(s^{(2)}\right)\right) + \beta E\left[v\left(s'\right)\right]$$

Thus, if

$$u\left(y_2 + q\left(B^{(1)}; s^{(2)}\right)\left(B^{(1)} - \theta B\left(s^{(2)}\right)\right) - (1-\theta) B\left(s^{(2)}\right)\right)$$

$$-u\left(y_1 + q\left(B^{(1)}; s^{(1)}\right)\left(B^{(1)} - \theta B\left(s^{(1)}\right)\right) - (1-\theta) B\left(s^{(1)}\right)\right) > u\left(y_2\right) - u\left(y_1\right)$$

then, $z^{(2)} \in D(B)$ implies $z^{(1)} \in D(B)$.

Given that utility is increasing and strictly concave; and that $z^{(2)} \in D(B) \implies q(B'; s)(B' - \theta B(s)) - (1-\theta) B(s) < 0$, the last condition holds implying that $z^{(1)} \in D(B)$. •

This is the non-trivial insight and proposition 3 in Arellano (2008) and follows similarly from the concavity of $u(\cdot)$. A graphical representation is in figure 18. In that figure, a pricing function $q(B'; s)$ is taken as given. We are typicallyk considering two pricing functions in particular. Due to the possibility of a sunspot, the pricing function may be $q = \bar{q}_m(B'; s)$ or $q \equiv 0$. The latter results in a larger default set in the latter case. A graphical representation is in figure 19.

By comparison to proposition 4, the next proposition states that less "shame" χ of defaulting results in higher incentives to default.

Proposition 5 *Suppose z is iid and that all functions are independent of ψ. Default incentives are stronger, the lower is the utility penalty from defaulting. I.e., for all $\chi^{(1)} \leq \chi^{(2)}$, if $z^{(2)} = (y, \chi^{(2)}, \zeta, \psi) \in D(B)$, then so is $z^{(1)} = (y, \chi^{(1)}, \zeta, \psi) \in D(B)$.*

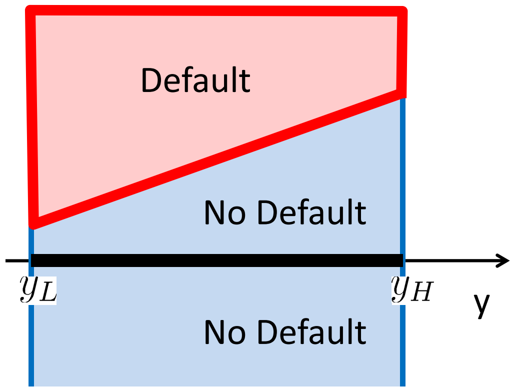

Figure 18: *Relationship between debt, income and the default decision, at a given pricing function* $q(B'; s)$

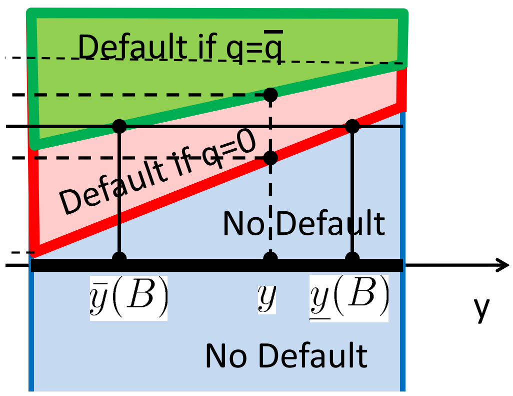

Figure 19: *Relationship between debt, income and the default decision, for the two pricing functions* $q = \bar{q}_m(B'; s)$ *and* $q \equiv 0$

Proof: *If* $z^{(2)} \in D(B)$ *by definition*

$$u(y) + \beta E[v_D(z')] - \chi_2 > u(y + q(B';s)(B' - \theta B(s)) - (1-\theta)B(s)) + \beta E[v(s')]$$

Given that $\chi^{(1)} \leq \chi^{(2)}$

$$u(y) + \beta E[v_D(z')] - \chi_1 > u(y) + \beta E[v_D(z')] - \chi_2$$

Thus

$$u(y) + \beta E[v_D(z')] - \chi_1 > u(y + q(B';s)(B' - \theta B(s)) - (1-\theta)B(s)) + \beta E[v(s')]$$

implying that $z^{(1)} \in D(B)$ *.* •

With these results, we can derive the dependence of the pricing function on the debt level.

Proposition 6 *Suppose that* $q_a(B';s) \equiv 0$*, i.e. no bailouts. Then* $q(B';s)$ *is decreasing in the debt level* B'*. If* y *and/or* χ *is random with a strictly positive and continuous density, then* $q(B';s)$ *is continuous in* B' *with a nonpositive derivative in* B'*, except for finitely many points.*

Proof: *See Chatterjee and Eyigungor (2012), Proposition 3.* •

A graphical representation of the pricing function $q = \bar{q}_m(B';s)$ is in figure 20 for the case of $\theta = 0$, i.e. one-period bonds. If the next period debt level is below the lowest level, at which a default could possibly be expected, $B' \leq \min \underline{B}(z)$, then the debt is safe and will be discounted at R. As B' increases beyond this level, there will be some states of nature in the future, for which a default may occur: these defaults become gradually more likely with increases in B', as one can infer from figure 19. Once the debt level is so high, that a default must surely occur tomorrow, then the current price level must be zero as well. The pricing function depends on the sunspot default probability tomorrow in a subtle way, as figure 21 shows. With a zero probability of a "sunspot" default, the debt B' needs to exceed $\min \bar{B}(z)$ in order for the price $\bar{q}_m(B';s)$ to decline. Indeed, $\bar{B}(z)$ itself depends on π and should intuitively rise, as π falls (since q is shifting upwards): this is indicated by the shift also of $\max \bar{B}(z)$ in that figure.

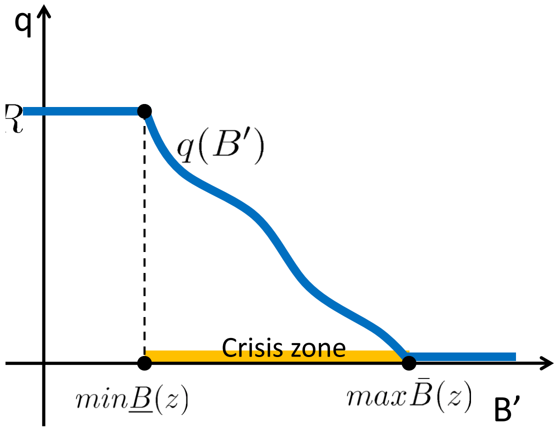

Figure 20: *The market price $q(B') = \bar{q}_m(B';s)$ as a function of future debt B'.*

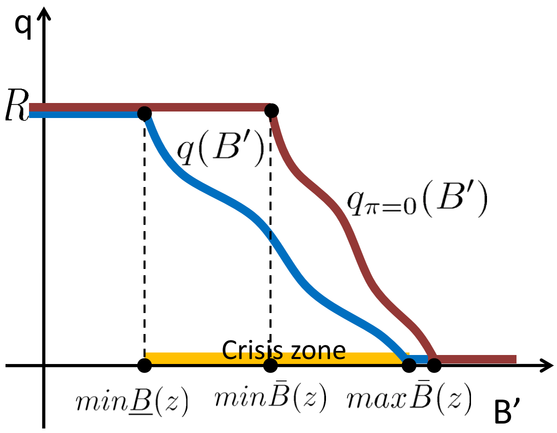

Figure 21: *The market price $q(B') = \bar{q}_m(B';s)$ for nonzero "sunspot" default probability π as well as for $\pi = 0$.*

It is useful to analyze the ensuing debt dynamics. The question is now, how large B' is, compared to the debt level B leading into this scenario. Consider the case where $\beta R = 1$. If income is literally constant, then consumption should be constant and the debt level should likewise remain constant, **except** that the country can also avoid the cost of default altogether by "saving itself" out of the crisis zone, as shown in Cole and Kehoe (2000).

Indeed, with a modest degree of income variation and for $\beta R = 1$, the country will choose to distance itself over time from the default zone as far as possible, saving for precautionary motives. The ensuing dynamics is shown in figure 22. If $\beta R < 1$, but close to 1, then the asset accumulation will not "run away", but still, the country will choose to accumulate large amounts of assets, as shown in figure 23. As a result, a sovereign debt crisis is highly unlikely. Here, it is therefore important to appeal to the political economy literature on sovereign debt accumulation, as in the literature cited in the introduction. If the government discounts the future sufficiently highly, i.e. if βR is considerably smaller than unity, then the country will possibly perch itself at a precarious point with an amount of debt in the crisis zone, as shown in figure 24. Indeed, reintroducing the income fluctuations in this picture results in a stationary distribution for the debt level, under suitable assumptions, as shown in figure 25.

B Other bailout mechanisms

Let us now consider the possibility for a bailouts, which may not necessarily be actuarially fair, as an extension of the discussion in the main body of the paper, and as these may be important for certain policy discussions. We shall focus on a few benchmark cases and explore their implications. First, suppose that, for a single period, debt can be sold at some fixed "assisted" price $0 < q_a < 1/R$ to some outside facility, provided the total amount B' of debt does not exceed some upper limit $\bar{B}_a$. This is a bailout and a stylized version of the one-time rescue for Greece or a one-time intervention by the European Financial Stability Facility. The resulting situation is shown in figure 26. The green line denotes the market price for existing debt sold to private lenders, while the blue line denotes the line, at which debt can be sold to the outside facility. The new debt level $B'_a(s)$ now exceeds the old debt level. Essentially, given the bailout, there is no longer

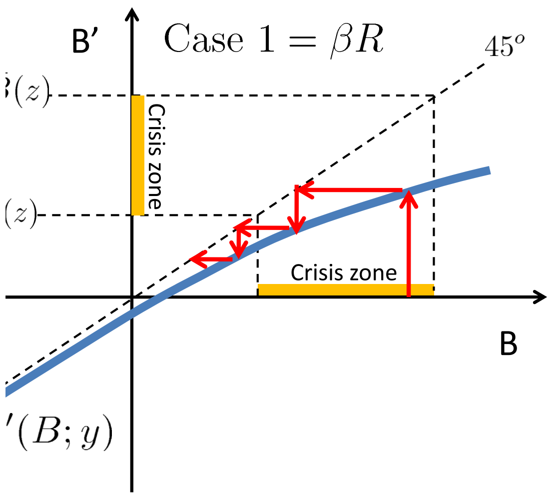

Figure 22: *The debt dynamics for small income fluctuations and $\beta R = 1$.*

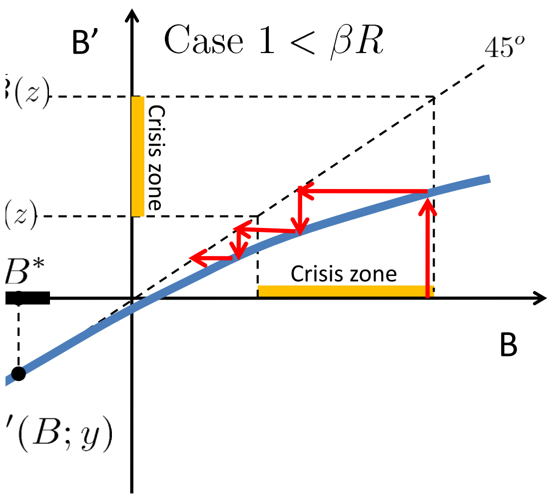

Figure 23: *The debt dynamics for small income fluctuations and βR below, but near 1.*

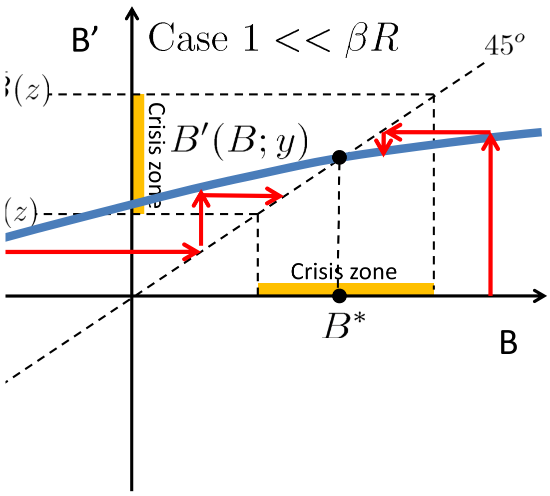

Figure 24: *The debt dynamics for small income fluctuations and βR far below 1.*

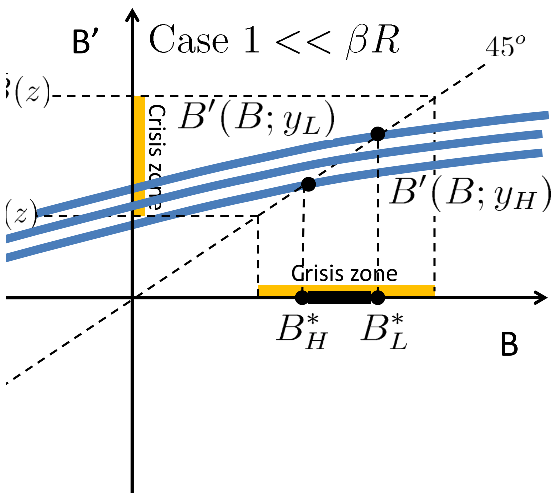

Figure 25: *The stationary debt dynamics for small income fluctuations and βR far below 1.*

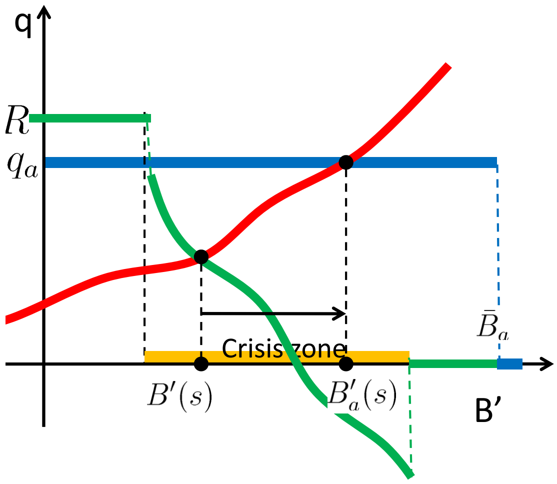

Figure 26: *The choice of the debt level in case of a one-time assistance or bailout.*

quite the same pressure for the government of the country to cut back on government spending, due to the impending financial crisis. Indeed, we have seen how the attempts of government cut backs in Greece and Portugal have run into fierce local resistance: a luxury, that certainly would not have been there, if these countries needed to keep borrowing on private markets only and wished to avoid a default. As this is a one-time bailout, the resulting debt dynamics is given by figure 24, starting towards the right end, and indicated with the red arrow there (indeed, that arrow only applies in this situation: without the bailout, there would have been an assured default at that debt level outside the crisis zone).

It may be more interesting to consider a permanent version of this facility: all future borrowing by the country at hand can be done at some fixed price $0 < q_a < 1/R$, provided the total amount B' of debt does not exceed some upper limit $\bar{B}_a$. In that case, the pricing is given by figure 27. The existence of the borrowing guarantee now removes the doubt of private lenders that the country will be able to borrow tomorrow. As a result, the country debt becomes safe and will be discounted at the usual safe rate R. The mere promise of the permanent facility results in a markedly reduced market interest on the country debt, provided the promised facility is fully credible.

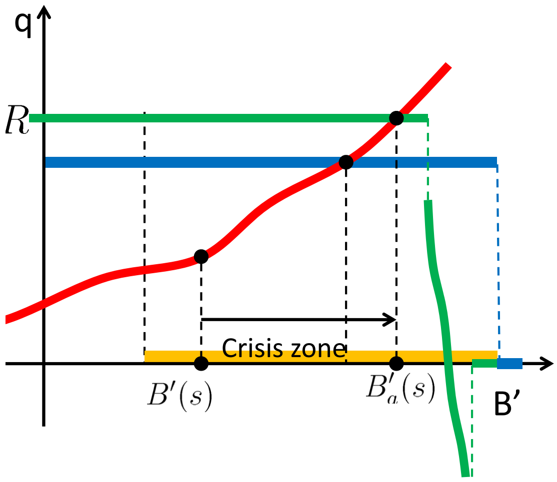

Figure 27: *The choice of the debt level in case of a permanent assistance or bailout.*

This may appear to be a wonderful solution. This is so only at first blush, however. Note that the borrowing increases from $B'(s)$ to $B'_a(s)$. Indeed, the country will once again find its perch in the crisis zone of probabilistic default: this time, however, triggered by the debt limit imposed by the facility[5]. The country will borrow privately at the safe return R, until it gets near the imposed debt limit. At that point, credibility on private credit markets collapses as a default is now viewed as likely, the country will borrow one last time, but this time from the facility at the reduced price, and will default in the next period. The proof is by contradiction: if it would not default in the next period (or if such a default would be very unlikely), then it would borrow privately, rather than at the "penalty rate" from the facility. The ensuing debt dynamics is shown in figure 28.

Both scenarios are in conflict with the observation, however, that yields on, say, Greece, Portugese and Irish debt are high and continue to be high, i.e. that there continue to be default fears by private markets. While it is conceivable, that we are simply in that "terminal" period described in the previous scenario, an alternative view here is that the bailout is probabilistic. This can be modelled in analogy to the default sunspot above. I.e., assume some bailout probability $0 < \omega < 1$. If the "bailout sunspot"

[5]Without a debt limit, the country will choose to run a Ponzi scheme, borrowing forever more without ever repaying.

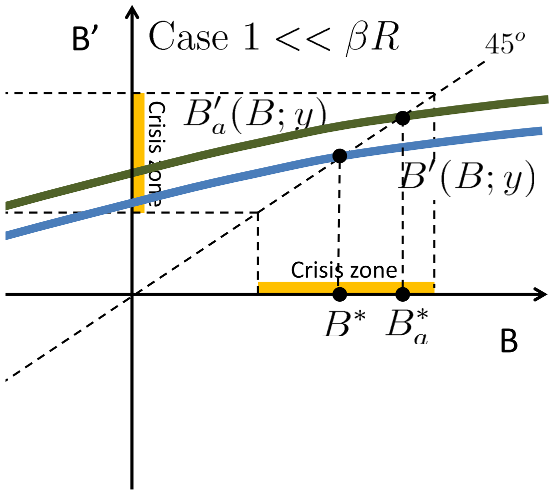

Figure 28: *The stationary debt dynamics for small income fluctuations and a permanent bailout facility.*

ψ is below ω, $\psi < \omega$, then the country can borrow at the price $0 < q_a < 1/R$ from the outside facility, provided the total amount B' of debt does not exceed some upper limit $\bar{B}_a$. If the "bailout sunspot" ψ exceeds ω, $\psi \geq \omega$, then the country must rely on private markets alone.

This will have the effect shown in figure 29. The level of debt at which a country will now prefer a default in those periods when no borrowing from the facility is possible, has increased compared to the "no bailout ever" scenario, as the country can hope for the option of borrowing from that facility in the future. Therefore, the crisis zone shifts to the right. The debt dynamics is shown in figure 30. Essentially, this is now a shifted version of the debt dynamics without that facility: rather than repaying the debt, the country shifts to higher debt levels, and the probability of a default is essentially the same as it was before. This takes a bit of time, of course. The facility therefore provides a temporary, but not a permanent resolution of the fiscal crisis. The debt is once again traded at a premium, as before, except that the probabilistic bailout means that these higher premium will be afforded at a higher debt level, than without that facility, while avoiding the default.

In essence, these scenarios show that the bailout facility only postpones the day of

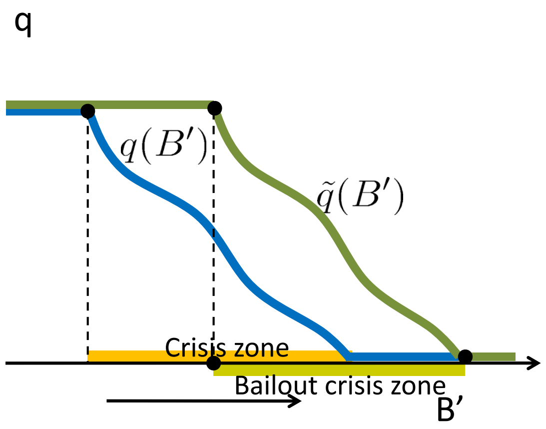

Figure 29: *Comparing the no-bailout private market pricing function $q(B')$ with the pricing function $\tilde{q}(B')$ in case of probabilistic bailouts.*

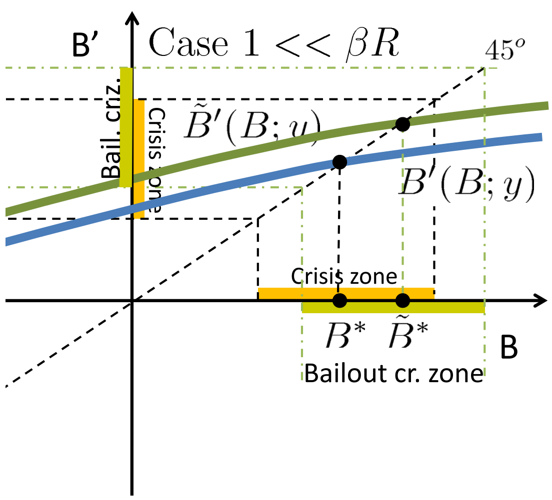

Figure 30: *The stationary debt dynamics for small income fluctuations and probabilistic bailout facility.*

reckoning. It provides temporary relieve to the country in its desire to maintain a high level of government consumption, but leaves the default situation in a very similar and precarious situation as before, once the initial relief is “used up”.