EX-POST INFLATION FORECAST UNCERTAINTY AND SKEW NORMAL DISTRIBUTION: 'BACK FROM THE FUTURE' APPROACH

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The aim:

To look for footprints of economic policy in measures of macroeconomic uncertainty.

Quick vocabulary:

***Ex-post* forecast uncertainty** explains by how much forecasters err.

***Ex-ante* forecast uncertainty** expresses current lack of knowledge regarding the future (SPF's, *etc*.).

It is often claimed that (see e.g. Clements, 2014), these two should, at the level of population, coincide. The difference is due to overconfidence or underconfidence of the forecasters making *ex-ante* predictions.

Hypotheses and main findings, in brief:

- We claim that the *ex-post* and *ex-ante* uncertainties can differ at the population level, that is, they can differ even if the forecasters are confidence-neutral.
- They can differ because in macroeconomics the *ex-ante* predictions might affect outcomes (unlike the weather predictions).
- As the research tool, we use a new statistical distribution fitted to *ex-post* forecast errors which parameters are interpretable as characteristics of an economic policy.
- We check that this distribution fits best to *ex-post* forecast inflation uncertainties for most of 38 analysed countries.
- We compute measures of *ex-post* and (*quasi*) *ex-ante* uncertainty for inflation in these countries and evaluate effects of monetary policy.

Ex-post and *ex-ante* uncertainty

Ex-post net forecast uncertainty: root mean square error, $RMSE_U$, of $U_{t,h}$:

$$U_{t,h} = \frac{Z_t - \mu_{t|t-h}}{\sigma_{t|t-h}} \sigma_{t,h} \sim D_P(\mu_{t,h}, \sigma^2_{t,h}). \qquad (1)$$

Ex-ante uncertainty: st. dev. of $U_{t|t-h}$:

$$U_{t|t-h} = \frac{Z_{t|t-h} - \mu_{t|t-h}}{\sigma_{t|t-h}} \sigma_{A,t|t-h} \sim D_A(0, \sigma^2_{A,t|t-h}). \qquad (2)$$

see Clements (2014), Lahiri, Peng and Sheng (2014).

The reason why $U_{t,h}$ is not equal to $U_{t|t-h}$ (and $RMSE_U \neq \sigma_{A,t|t-h}$) is that there might be some economic policy action undertaken on the basis of knowledge of $U_{t|t-h}$ which affects $U_{t,h}$ (e.g. action by a policy body undertaken on the basis of information included in $U_{t|t-h}$).

Distribution of *ex-post* inflation uncertainties

- The forecasting model:

$$\pi_t = \hat{\pi}_{t|t-h} + e_{t|t-h} \quad , \tag{3}$$

$$Var(e_{t|t-h} \,|\, \Omega_{t-h}) = \sigma^2_{t,h} \;, \qquad \hat{\pi}_{t|t-h} \text{ - } \textit{baseline forecast.}$$

$u_{t|t-h}$ is *GARCH*-adjusted $e_{t|t-h}$ (U-uncertainties).

- We propose that:

$$U_{t|t-h} \equiv U = X + \alpha \cdot Y \cdot I_{Y>\bar{m}} + \beta \cdot Y \cdot I_{Y<\bar{k}} \,, \qquad (X,Y) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right) \tag{4}$$

$\bar{m} \in \mathbb{R}$, $\bar{k} \in \mathbb{R}$, $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$, $-1 < \rho < 1$ and $I_{\{\bullet\}}$ is an indicator of set $\{\bullet\}$

The distribution of U is called the *Weighed Skew Normal*, WSN, distribution.

Weighted skew normal distribution: interpretation

$$U = X + \alpha \cdot Y \cdot I_{Y>\bar{m}} + \beta \cdot Y \cdot I_{Y<\bar{k}} \quad , \qquad (X,Y) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

X : containing two elements:

1) *ontological uncertainty*, related to the purely random (unpredictable in mean) nature of future inflation;
2) *epistemic uncertainty*, related to fragmentary and incomplete knowledge of the lead forecaster.

The epistemic element in *X* can be predicable by experts, whose predictions are not, in general, observable by researchers, but are known to the policy makers. We call them the *private experts*.

Y : represents decisions of the private experts. As their decisions are heterogeneous (each expert has access to different private information), this heterogeneity also creates uncertainty (*uncertainty by disagreement*).

Weighted skew normal distribution: interpretation (*cont.*)

$$U = X + \alpha \cdot Y \cdot I_{Y>\bar{m}} + \beta \cdot Y \cdot I_{Y<\bar{k}} \quad , \qquad (X,Y) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

We assume that $\mathrm{var}(X) = \mathrm{var}(Y) = \sigma^2$.

ρ: correlation coefficient between X and Y. It shows to what extent forecasts represented by Y are ‘educated’, or accurate. If, either X is totally unpredictable (that is, if the quasi-uncertainty becomes fully ontological) or, if the private experts are ignorant, then $\rho = 0$.

$\bar{m}$, $\bar{k}$: respectively ‘upper’ and ‘lower’ thresholds for decisions based on Y.

α, β: actual outcomes of these decisions (policy parameters).

Rational behaviour of the policymakers and forecasters implies that

$\alpha \le 0$, $\beta \le 0$, $\bar{m} \ge 0$, $\bar{k} \le 0$ and $0 < \rho < 1$.

Distribution of U

$$U = X + \alpha \cdot Y \cdot I_{Y>\bar{m}} + \beta \cdot Y \cdot I_{Y<\bar{k}} \quad , \qquad (X,Y) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix}\right)$$

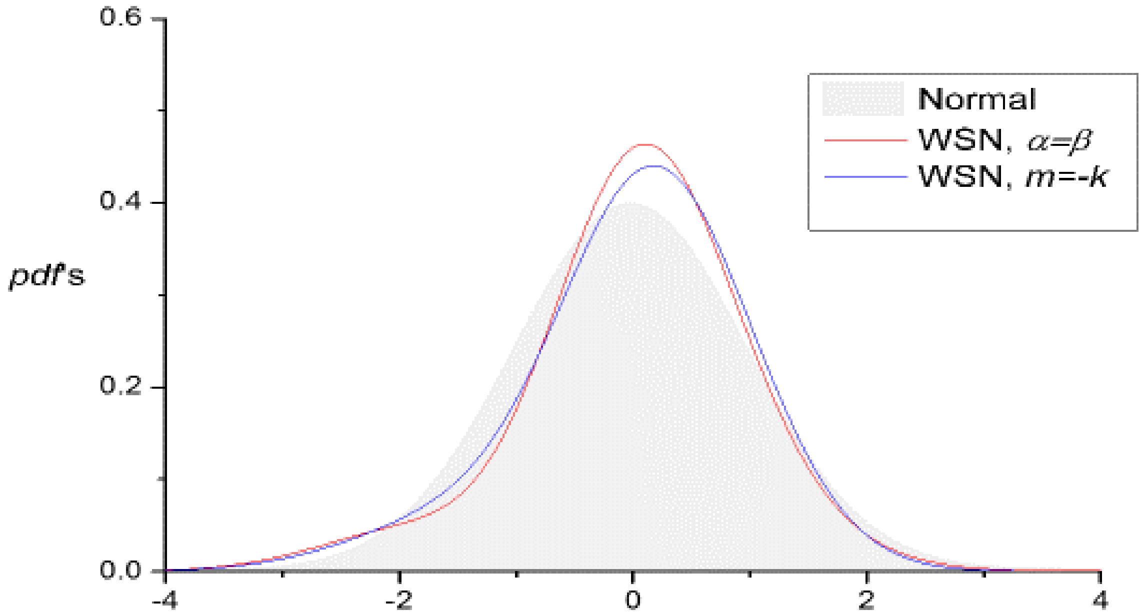
$U \sim \mathrm{WSN}_\sigma\left(\alpha, \beta, \bar{m}, \bar{k}, \rho\right)$: *weighted skew-normal* distribution.

Some properties:

- It is normal only $\alpha = \beta = 0$.
- It is symmetric only if $\alpha = \beta = 0$ or $m = -k$ and $\alpha = \beta$.

A few plots of WSN (mean=0, variance=1, skewness=-0.5)

$$U = X + \alpha \cdot Y \cdot I_{Y > \bar{m}} + \beta \cdot Y \cdot I_{Y < \bar{k}}$$



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---WSN: $\alpha = \beta = -1.64; \quad m = 1.40; k = -0.24; \quad \rho = 0.75$

---WSN: $\alpha = -1.77; \beta = -0.96; \; m = -k = 0.96; \quad \rho = 0.75$

How to recover *ex-ante* uncertainty?

If there was no effective policy action, both *ex-post* and *ex-ante* distributions would have been identical. The difference occurs because there was some epistemic element left in X and retrieved from it by the private experts.

The epistemic uncertainty can partially be extracted from U as:

$$V = U - E(X \mid Y) = U - \rho Y \quad .$$

Further in the text we refer to V as V-*uncertainties*, $V \sim \mathrm{WSN}$.

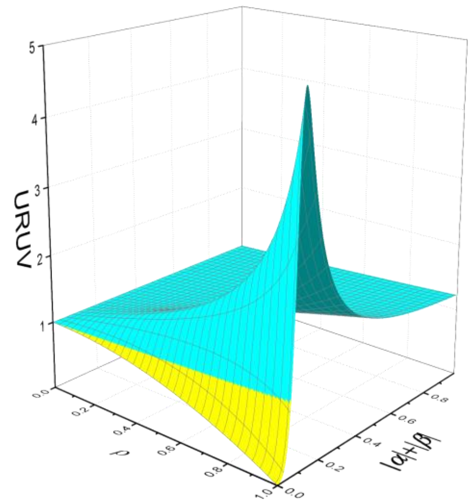
Measure of V-uncertainty, σ_V, can be regarded an analogous to *ex-ante* uncertainty, that is $\sigma_{A,t|t-h} \simeq \sigma_V$.

The concept of URUV (*uncertainty ratio of U and V*):

$$\mathrm{URUV} = \frac{\sigma_V^2}{RMSE_U^2}$$

Deviation of URUV from unity represents:

- effect of the epistemic element on the uncertainty (through ρ)
- effect of monetary policy



The concept of $\mathrm{URUV}_{\max}$:

$\mathrm{URUV}_{\max}(\rho)$ - maximum of URUV for a given ρ.

The ratio of $\dfrac{\mathrm{URUV}}{\mathrm{URUV}_{\max}}$ tells about possible room for policy improvement.

Empirical results

Data:

- Data on CPI (headline) inflation, π_t, for 38 countries: 32 OECD countries, 5 BRICS countries and Indonesia. All series end in January or February 2013.

Recovery of U-uncertainties: point *pseudo ex ante* forecast errors (see e.g. Stock and Watson, 2007) from recursively updated *GARCH-ARIMA* model, scaled by conditional standard deviations.

- With the use of these scaled *pseudo ex-ante* forecast errors, parameters α, β and σ of the WSN distribution are estimated (for fixed $m = \bar{m} / \sigma = -\bar{k} / \sigma = 1$ and $\rho = 0.75$).

Estimation

Simulated minimum distance estimation, *SMDE* (see Charemza *et. al.*, 2012; Dominicy and Veredas, 2013):

$$\hat{\theta}_n^{SMDE} = \arg\min_{\theta\in\Theta} \left\{ \xi \left\{ HD(d_n \,,\, f_{r,\theta} \right\}_{r=1}^{R} \right\} \quad ; \quad \theta = \{\alpha, \beta, \sigma\} \in \Theta \subset \mathbb{R}^3$$

where:

$HD(d_n \,,\, f_\theta) = 2\sum_{i=1}^{m}[d_n(i)^{1/2} - f_\theta(i)^{1/2}]^2$ Hellinger distance measure,

n	sample size,
$d_n(i)$	empirical frequency of data falling into the i^{th} interval,
$f_{r,\theta}$	Monte Carlo approximation of the theoretical probabilities, f_θ,
R	number of replications (drawings) from WSN with given values of $\{\alpha, \beta, \sigma\}$,
ξ	aggregation operator for the noisy criterion function.

Forecast uncertainty measures

h	Theoretical uncertainty	normalised $RMSE_U$	normalised σ_V	URUV	$\frac{URUV}{URUV_{max}}$
U.S.					
3	0.11	0.40	0.43	2.14	0.95
6	0.28	0.49	0.52	2.10	0.93
9	0.50	0.60	0.63	2.03	0.90
12	**0.75**	**0.71**	**0.73**	1.94	0.86
24	1.00	1.00	1.00	1.85	0.82
Poland					
3	0.11	0.38	0.36	1.21	0.53
6	0.28	0.52	0.50	1.22	0.54
9	0.50	0.63	0.59	1.12	0.50
12	**0.75**	**0.70**	**0.69**	1.23	0.54
24	1.00	1.00	1.00	1.29	0.57

Similarities and differences to Clements' (2014) results for U.S.

1 Clements' *ex-ante* uncertainty is greater than *ex-post* uncertainty and similarly, in most cases, $\sigma_V > RMSE_U$ (that is URUV>1).

2 For shorter horizons Clements' *ex-ante* measures, being greater than the corresponding *ex-post* measure, indicate short-horizon underconfidence.

3 In our approach the fact that *pseudo ex-ante* uncertainty, σ_V, is greater than the *ex-post* uncertainty, $RMSE_U$, is interpreted as a positive effect of the monetary policy undertaken on the basis of information obtained by the private experts rather than underconfidence.

Other comments (comparison of Poland and U.S.):

1. For both U.S. and Poland and for 12-months ahead forecasts, the normalized uncertainties are smaller than the theoretical ones. It presumably indicates good quality of forecasts and policy for longer horizons.
2. For U.S., the effects of monetary policy on uncertainty are stronger than for Poland. However for U.S., URUV is gradually decreasing with the increase in forecast horizon. For Poland it is not changing much. It indicates a stronger long-run effect of the Polish monetary policy, which is not 'loosing footprints' for longer horizons.
3. For Poland, the ratio of URUV to $\mathrm{URUV}_{\max}$ is lower than for U.S. It suggests more potential 'room for improvement' in Polish monetary policy than for the U.S., as the U.S. policy is closer to its optimum.

Conclusions

- The weighed skew normal distribution proposed in this paper works well for tracing footprints of economic policy in inflation forecast uncertainties.

- U-uncertainty measure might be of interest to economic agents who do not have an influence on economic policy and who do not really care of what is epistemic and what is not.

- V-uncertainty measure might be used as an alternative (complement or substitute) to the *ex-ante* uncertainty measures derived from surveys of forecasts. It could be of interest to the policy makers, who does not want the picture of uncertainty be blurred by their own decisions.

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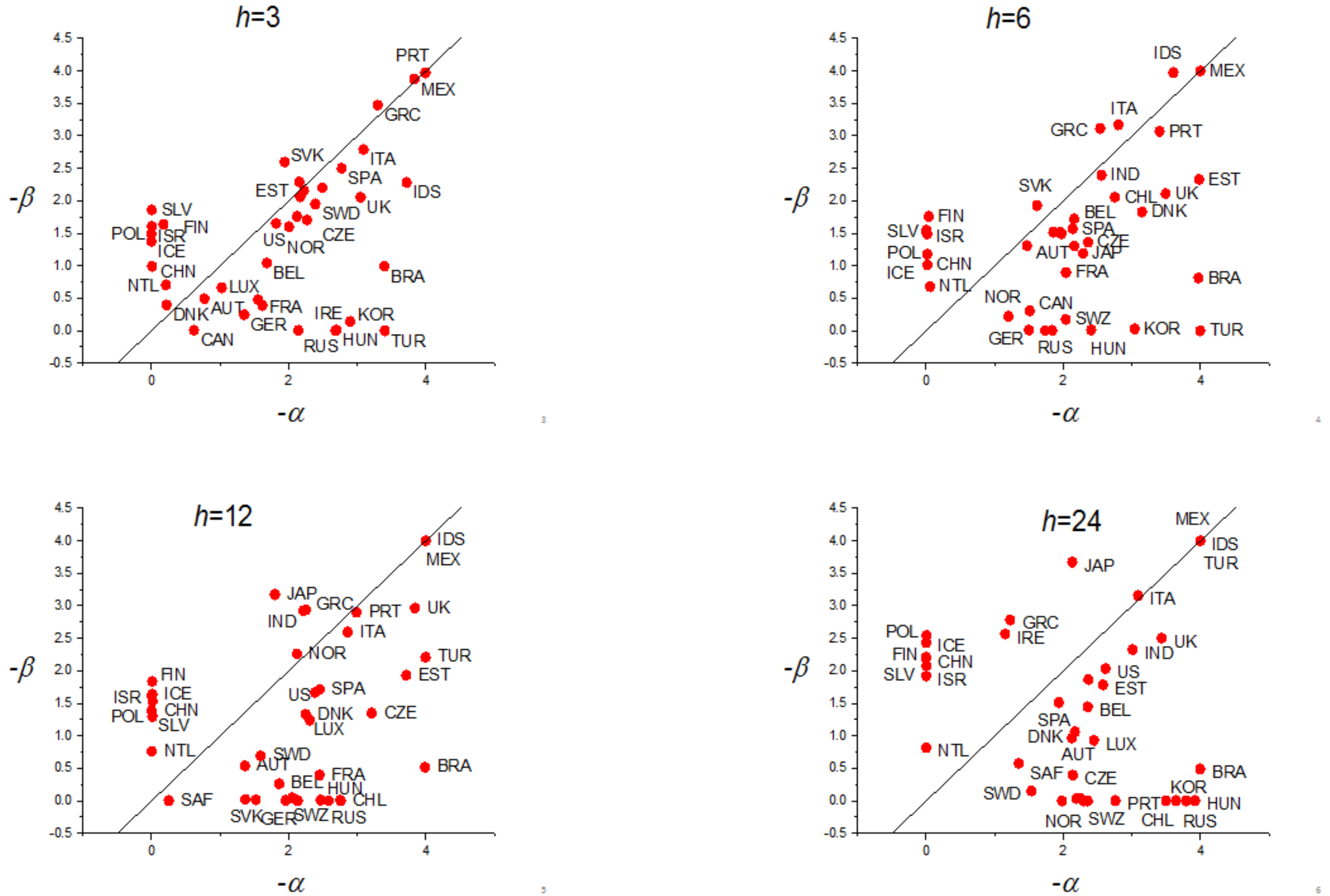
Additional slides

Estimated α and β parameters



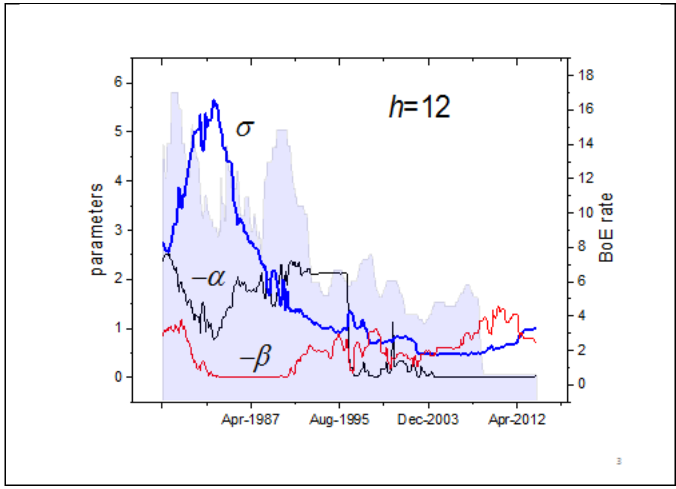
General interpretation:

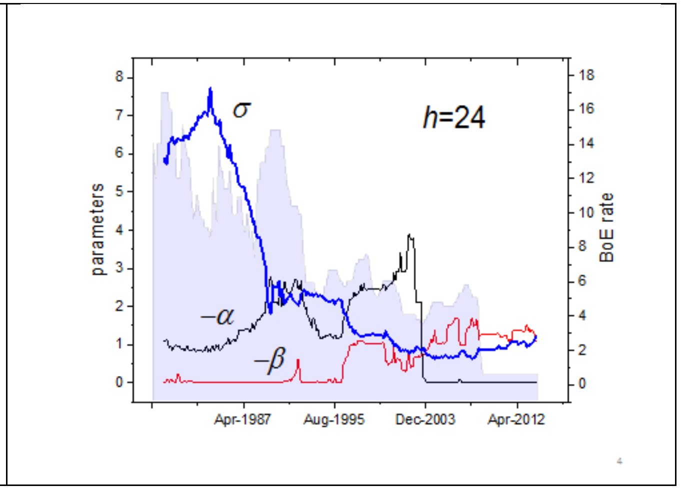
(i) The non-leading large Euro countries that were strongly affected by the economic crisis of 2008, Greece, Italy and Portugal, are predominantly high up the 45 degrees line, showing strong imprints of the decisions of the European Central Bank. Effects are close to being symmetric, with Greece gradually moving, with the increase in forecast horizon, into the output-stimulating zone, with Portugal moving the other way.

(ii) For longer horizons, Japan is among the countries with the highest $-\beta$, which is markedly greater than its $-\alpha$. This might reflect the preference to output-stimulating and anti-deflationary policy.

(iii) Turkey is among the countries with the highest $-\alpha$'s. With $-\beta$ close to zero, this might indicate stronger preference to anti-inflationary, rather than output-stimulating, reaction to forecasts signals. However, with the increase in forecast horizon, $-\beta$ increases, bringing Turkey, for the 2-years horizon, close to the situation of more balanced reaction.

Recursive estimates of $-\alpha$, $-\beta$ and σ for UK.





(i) For one-year uncertainty, $-\alpha$ has been higher than $-\beta$ until December 1996, after which $-\beta$ started to rise and $-\alpha$ first rapidly fell, and then stabilize at a low level (that is, close to zero). It coincides with - stimulating policy of BoE,

(ii) A similar switch between the strength of α and β can be observed for the 2-years uncertainty in August 2003.

Estimation of $\sigma_{A,t|t-h}$

Usually from surveys of forecasts like SPF (see e.g. Giordani and Söderlind, 2003, Wallis, 2004, Lahiri and Sheng, 2010, Clements, 2014, Lahiri, Peng and Sheng, 2014).

Pros:

- ✓ Truly *ex-ante* measure.
- ✓ Based on transparent and intuitive assumptions.

Cons:

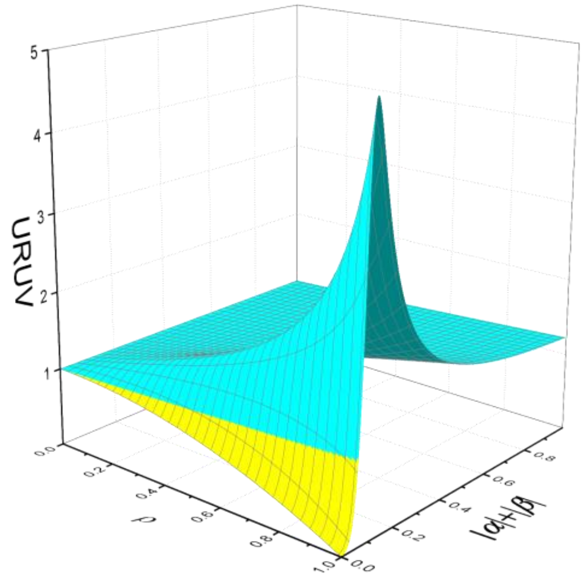
- ✓ Bias of *ex-ante* uncertainty due to homogeneity of information.
- ✓ Possible psychological bias of probabilistic and interval forecasts (Clements, 2014, Hansson, Juslin & Winman, 2008, Soll & Klayman, 2004).

Estimation of $RMSE_U$

(1) By computing root mean square errors of a set of point forecast errors after correcting for variance forecastability.

(2) From surveys of forecasts, by computing differences between the realisation and either the aggregated (e.g. median) forecasts, or individual forecasts. (Lahiri, Peng and Sheng, 2014).

Relation between URUV, ρ and $|\alpha|+|\beta|$



If the private experts are successful in explaining the epistemic element in X, but the monetary decisions are of a weak strength, variance of U increases in relation to the variance of V. In order to achieve a reduction in uncertainty, coordination between the forecasters and decision-makers is needed (policy should not be too strong or too weak).