Continent debt and performance pricing in an optimal capital structure model*

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FRB Richmond

8th Summer Workshop
Narodowy Bank Polski
June 2019

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

Motivation: financial reorganization of firms

- In the U.S., corporate financial reorganization (Chapter 11) is much more common than liquidation (Chapter 7).
  - In a large sample of filings from 1995-2001, Bris et al (2006) find that 80% of cases were Chapter 11 reorganizations.
  - In Copustat data 1980-2014, Corbea and D'Erazmo (2017) find that 80% of bankruptcy exists were Chapter 11.
- Chapter 7 and 11 filers have different capital structures:
  - e.g., Corbea and D'Erazmo (2017) show that
    - Chapter 7 filers have lower income/assets,
    - Chapter 11 filers have higher leverage (debt/assets).
- Chapter 7 and 11 filers have different recovery rates and security values
  - documented in, e.g., Acharya et al (2007).
- The choice of Chapter 7 and/or 11 bankruptcy is endogenous.
- Models needed to study positive and normative questions on reorganization vs liquidation.

Our model: optimal contract with agency frictions

- We build a model on the trade-off between agency costs and monitoring cost.
  - in the spirit of Jensen (1986) static analysis
  - we add monitoring to DeMarzo and Sannikov (2006) dynamic model
- Monitoring trade-off:
  - when monitored, no information frictions $\rightarrow$ no risk of liquidation
  - but monitoring entails direct costs (legal, operational, etc.)
- Key assumption: monitoring cannot be applied instantaneously, but only after a spell of search (of an uncertain duration)
  - filing preparation delay, search for debtor-in-possession financing
  - reorganization time, search for new agent needed to re-emerge
  - thus, monitoring and agency are states, not actions
  - search for a transition between agency and monitoring is an action
- Capital structure implementation with:
  - equity, long-term debt, revolving credit line (as in DS)
  - plus: contingent debt
  - plus: performance pricing on the credit line

Some related models

- Papers in the "structural" tradition of Merton and Leland, where debt is issued for its tax advantage:
  - Antill and Grenadier (2018) study leverage and debt pricing allowing for Chapter 11 and 7.
  - Manso et al. (2010) introduce performance-sensitive debt as a screening device for high-growth firms.
- Corbae and D'Erasmo (2017) estimate a GE model with Chapter 11 and 7, and evaluate a "fresh start" policy reform.
- Piskorski and Westerfield (2016) add monitoring to DS as a signal of diversion, which affects performance-based compensation but remains off equilibrium.
- Tchistyi (2016) shows that performance-pricing is a part of an optimal capital structure with correlated cashflows, as the incentive to divert increases when liquidation is nearer.

Outline

- Baseline model of DeMarzo and Sannikov.
- Extension with monitoring.
- Capital structure with contingent debt and performance pricing.
- Market values of securities.
- Comparative statics.
- Recap and next steps

Baseline model: DeMarzo-Sannikov

- A dynamic agency friction
  $\rightarrow$ liquidation becomes necessary (despite being ex-post inefficient)
- A firm's cumulative cashflow up to date t, Y_t, follows

$$dY_t = \mu dt + \sigma dZ_t,$$

  $\mu, \sigma > 0$ and Z_t is standard Brownian motion on $(\Omega, \mathcal{F}, P)$.
- A manager with limited liability is hired to run the firm
  - cumulative compensation process I_t is non-decreasing,
  - manager can divert cashflow to private use:
    \$1 diverted $\rightarrow$ private benefit of \$$\lambda$, where $0 < \lambda \leq 1$.
- Firm liquidation value: $L \geq 0$.
- Manager outside option value: $R \geq 0$.
- Contract: (τ, I_t), where
  - τ is the firm liquidation date (a stopping time)
    $\rightarrow$ the manager is dismissed with outside option R.

Baseline model: payoffs

- Agent chooses a reporting process $d\hat{Y}_t \leq dY_t$ to maximize

$$\mathbb{E}\left[\int_0^\tau e^{-\gamma t}\left(dI_t + \lambda(dY_t - d\hat{Y}_t)\right) + e^{-\gamma\tau} R\right]$$

  where $\gamma > 0$ is the time preference rate.

- Under an IC contract, $d\hat{Y}_t = dY_t$ at all t.
- The firm's payoff is

$$\mathbb{E}\left[\int_0^\tau e^{-rt}(\mu dt - dI_t) + e^{-r\tau} L\right].$$

- The agent is impatient: $\gamma > r > 0$.

Baseline model: recursive representation

- State variable: the agent's continuation value: W_t.
- Representation:

$$dW_t = \gamma W_t dt - dI_t + \beta_t (d\hat{Y}_t - \mu dt),$$

  where β_t is the sensitivity to the reported cash flow.
- Firm's profit function with just liquidation: $b_L(W_t)$.
- The contract is IC if $\beta_t \geq \lambda$ at all t.
- With $b_L(W_t)$ concave, $\beta_t = \lambda$.
- State variable dynamics under IC:

$$dW_t = \gamma W_t dt - dI_t + \lambda \sigma dZ_t.$$

Baseline model: HJB and boundary conditions

- The option to pay the agent implies

$$b'_L(W) \geq -1 \text{ for all } W.$$

- Agent payment threshold:

  W^1 defined as the lowest W such that $b'_L(W) = -1$.

- The HJB equation for b_L:

$$rb_L(W) = \mu + \gamma W b'_L(W) + \frac{1}{2}\lambda^2\sigma^2 b''_L(W).$$

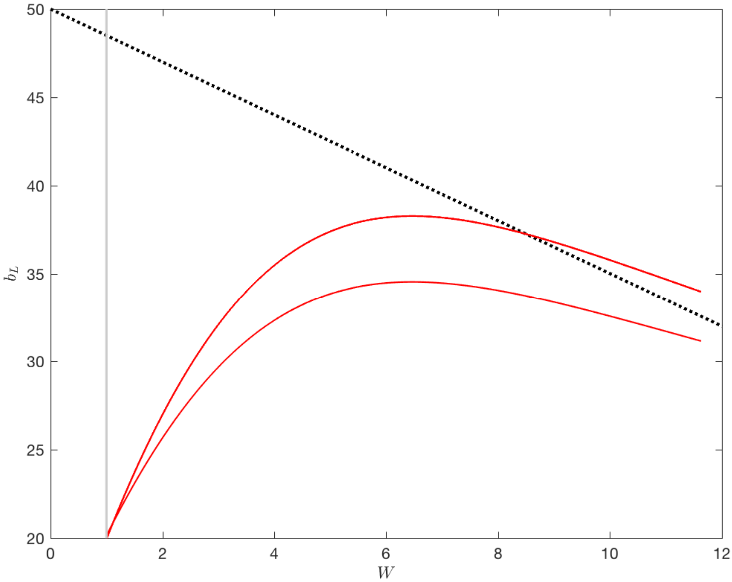
- Agent payment boundary:

$$rb = \mu - \gamma W.$$

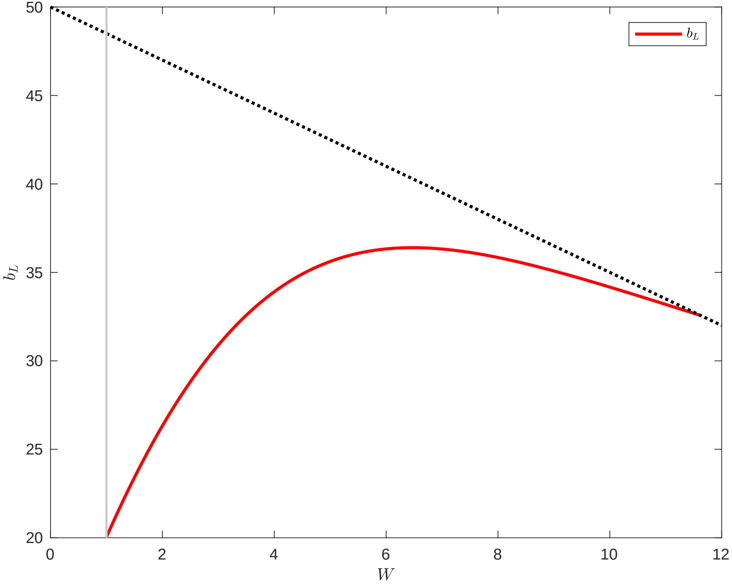
- Boundary conditions for b_L

$$rb_L(W^1) = \mu - \gamma W^1, \;\; b_L(W^1) = -1 \quad \text{and} \quad b_L(R) = L, \;\; b'_L(R) = ?$$

Forward shooting



The profit curve with just liquidation



Model with monitoring: agency stage

- A firm's cumulative cashflow up to date t, Y_t, follows

$$dY_t = \mu dt + \sigma dZ_t,$$

$\mu, \sigma > 0$ and Z_t is standard Brownian motion on $(\Omega, \mathcal{F}, P)$.

- A manager with limited liability is hired to run the firm
  - cumulative compensation process I_t is non-decreasing,
  - manager can divert cashflow to private use:
    \$1 diverted $\rightarrow$ private benefit of \$$\lambda$, where $0 < \lambda \leq 1$.
- The firm can pay the flow cost $\kappa \geq 0$ to search for a transition to the monitoring stage:
  - when κ is being paid, transition to monitoring arrives with Poisson intensity $\rho > 0$,
  - if transition arrives, the manager is forced out, possibly with severance S_t.
- Contract: (τ, I_t, s_t, S_t), where
  - τ is the manager's dismissal date,
  - $s_t \in \{0, 1\}$ is the indicator of searching.

Payoffs: agency stage

- Dismissal of the agent can be due to liquidation or transition to monitoring:

$$\tau = \min\{\tau_M, \tau_L\}.$$

- Agent chooses a reporting process $d\hat{Y}_t \leq dY_t$ to maximize

$$\mathbb{E}\left[\int_0^\tau e^{-\gamma t}\left(dI_t + \lambda(dY_t - d\hat{Y}_t)\right) + 1_{\tau=\tau_L} e^{-\gamma\tau} R + 1_{\tau=\tau_M} e^{-r\tau}(R + S_t)\right]$$

where $\gamma > 0$ is the time preference rate, $R > 0$ is outside option.

- Under an IC contract, $d\hat{Y}_t = dY_t$ at all t.
- The firm's payoff is

$$\mathbb{E}\left[\int_0^\tau e^{-rt}((\mu - s_t\kappa)dt - dI_t) + 1_{\tau=\tau_L} e^{-r\tau} L + 1_{\tau=\tau_M} e^{-r\tau_M}(M - S_t)\right]$$

where M is the value of the firm entering the monitoring state.

Monitoring stage

- Firm run by expert/trustee $\rightarrow$ no agency frictions.
- Flow cost of monitoring and search for exit: $\kappa_B dt$.
- Firm finds a new agent and transitions back to the agency stage with Poisson intensity $\phi > 0$.
- Firm value in monitoring:

$$M = \mathbb{E}\left[\int_0^{\hat{\tau}} e^{-rt}(\mu - \kappa_B)dt + e^{-r\hat{\tau}} b_0\right],$$

  where $\hat{\tau}$ is the time of exit from monitoring, r is the discount rate, and b_0 is the value of starting out in agency with a new agent. Integrating:

$$(r + \phi)M = \mu - \kappa_B + \phi b_0.$$

- Assumption: $\kappa_B \geq \mu$.
  Implication: $M < b_0$, i.e., monitoring is a costly state.

Recursive representation in the agency state

- State variable: the agent's continuation value W_t.
- Representation:

$$dW_t = \gamma W_t dt - dI_t + \beta_t(d\hat{Y}_t - \mu dt) + s_t \Delta_t (dN_t - \rho dt),$$

  where β_t is the sensitivity to the reported cash flow,
  Δ_t is the sensitivity to the switch to the monitoring state,
  N_t is a Poisson process with arrival rate $\rho > 0$.

- Firm's profit function: $b(W_t)$.
- The contract is IC if $\beta_t \geq \lambda$ at all t. With $b(W_t)$ concave, $\beta_t = \lambda$.

State variable dynamics and HJB

- At all $t < \tau$ we have $dN_t = 0$ and $\Delta_t = R + S_t - W_t$, so

$$dW_t = \gamma W_t dt - dI_t + \lambda\sigma dZ_t + s_t(R + S_t - W_t)(0 - \rho dt),$$

  i.e.,

$$dW_t = \big((\gamma + s_t\rho)W_t - s_t\rho(R + S_t)\big)dt - dI_t + \lambda\sigma dZ_t.$$

- The new terms are due to the risk of jump at dismissal.
  - changes to the slope and the level of the drift term
- The HJB equation for the firm's value b is

$$\begin{aligned} rb(W) = \quad \max_{s,S} \quad & \mu - s\kappa + \big((\gamma + s\rho)W - s\rho(R + S)\big)b'(W) \\ & + \frac{1}{2}\lambda^2\sigma^2 b''(W) + s\rho(M - S - b(W)). \end{aligned}$$

Condition for search to be used

- Recall $b_L(W)$ as the value of the firm when liquidation is the only option, as in DS:

$$rb_L(W) = \mu + \gamma W b_L'(W) + \frac{1}{2}\lambda^2\sigma^2 b_L''(W).$$

- Let $b_{L,0} = \max_W b_L(W)$.
- Let M_L be the value in the monitoring state if a transition to agency is followed by liquidation:

$$M_L = \frac{\mu - \kappa_B}{r + \phi} + \frac{\phi}{r + \phi} b_{L,0}.$$

- Searching for a transition to monitoring will be chosen at least in some states iff

$$-\kappa + \rho(M_L - L) > 0.$$

Optimal severance and region of search

LEMMA 1

Optimal severance is $S_t = 0$ at all t.

- Optimal S maximizes $-s\rho S b'(W) - s\rho S = (-b'(W) - 1)s\rho S$, but $b'(W) > -1$ at all $W < W^1$.

LEMMA 2

The region of search for monitoring is an interval $(R, \tilde{W}]$, where $\tilde{W} < W_0 := \operatorname{argmax} b(W)$.

- The gain from searching, $-\kappa + \rho(W - R)b'(W) + \rho(M - b(W))$, is monotone in W.
- The search costs, κ and κ_B, imply no search at $W > W_0$.
- The contract starts out in the no-search region.

Verification

Theorem

The unique solution b of the HJB equation

$$\begin{aligned} rb(W) &= \mu - \kappa 1_{W \le \tilde{W}} + \big(\gamma W + 1_{W \le \tilde{W}} \rho (W - R)\big) b'(W) \\ &+ \frac{1}{2}\lambda^2 \sigma^2 b''(W) + 1_{W \le \tilde{W}} \rho (M - b(W)), \end{aligned}$$

where

$$rM = \frac{r}{r+\phi}(\mu - \kappa_B) + \frac{\phi}{r+\phi} r b_0,$$

with boundary conditions

$b'(W^1) = -1$ *at a point* W^1 *such that* $rb(W^1) + \gamma W^1 = \mu$,

$b(R) = L$

is the true value function for the firm under the optimal contract.

Summary of optimal contract dynamics

- Starts out at W_0 with drift $\gamma W_t > 0$ and volatility $\lambda > 0$.
- Compensation is paid when W_t hits the payment threshold W^1, so $W_t \leq W^1$ at all t.
- When W_t drops below the threshold $\tilde{W} < W^1$, the firm searches for a transition from agency to monitoring:
  - the drift of W_t jumps to $\gamma W_t + \rho(W_t - R) > \gamma W_t$,
  - the firm pays the search cost κdt.
- The firm is liquidated if W_t hits the agent's outside option value $R < \tilde{W}$.
- If a transition to monitoring arrives before W_t hits R:
  - the agent is dismissed with severance $S_t = 0$,
  - firm pays monitoring costs $\kappa_B dt$ and searches for a new agent.
- Reorganization and liquidation are both possible exits from agency.

Computation of the solution

$$b(W) = \max\{b_{NS}(W), b_S(W)\},$$

where we define

$$rb_{NS}(W) = \mu + \gamma W b'_{NS}(W) + \frac{1}{2}\lambda^2\sigma^2 b''_{NS}(W),$$

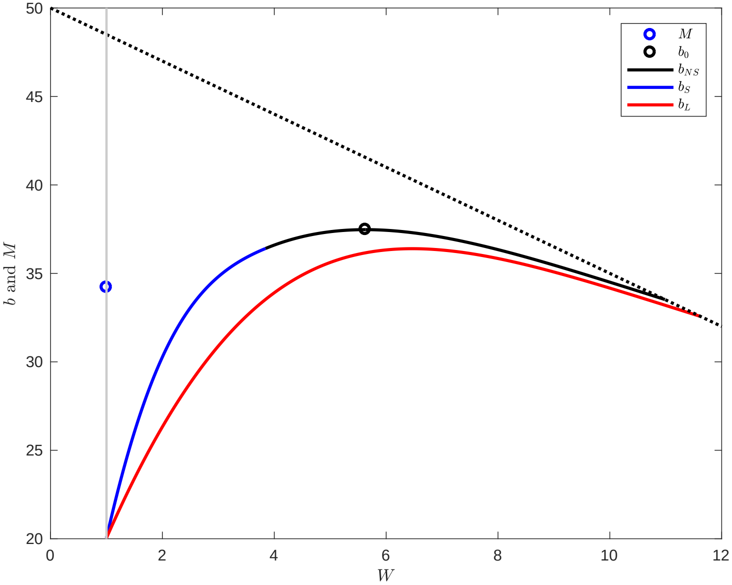
$$rb_S(W) = \mu - \kappa + ((\gamma+\rho)W - \rho R)b'_S(W) + \frac{1}{2}\lambda^2\sigma^2 b''_S(W) + \rho(M - b_S(W)).$$

- Backward-shooting from the payoff boundary point W^1.
- Splicing point $\tilde{W}$ found by smooth pasting and super-contact:

$$\kappa = \rho(W - R)b'_{NS}(W) + \rho(M - b_{NS}(W)).$$

- M known from just b_{NS}: model solvable in a single pass.

Solution with liquidation or monitoring



Capital structure implementation

- The owners set up the following capital structure:
  - credit line with balance B_t, limit C^L, and interest rate $i(B_t)$,
  - long-term consol debt with coupon payment flow x_d at all t,
  - contingent long-term debt with coupon payment x_{cd} suspended when $B_t > \tilde{C}^L$,
  - equity
- Manager's compensation: share λ of equity.
- The manager controls the cashflow, makes payments to debt-holders, and chooses a dividend policy to maximize her own payoff.
  - Cumulative dividend payment process: Div_t.
- Credit line balance dynamics:

$$dB_t = i(B_t)B_t dt + x(B_t)dt + dDiv_t - d\hat{Y}_t$$

Capital structure implementation

Proposition

Let $C^L = \frac{W^1 - R}{\lambda}$ *and* $\tilde{C}^L = \frac{W^1 - \tilde{W}}{\lambda}$*, where* $\tilde{W}$ *and* W^1 *are determined in the optimal contract. Set:*

$$i(B_t) = \begin{cases} \gamma & \text{if} \quad B_t < \tilde{C}^L, \\ \gamma + \rho & \text{if} \quad B_t \geq \tilde{C}^L, \end{cases}$$

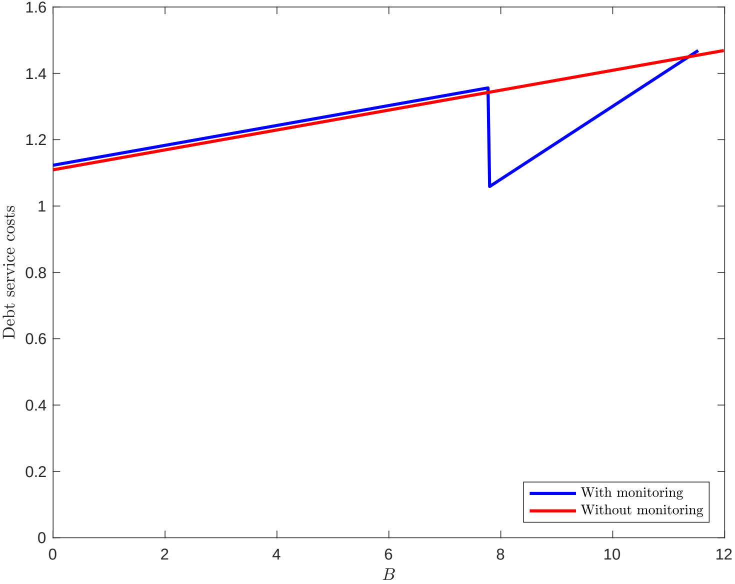
$$x_d = \mu - \frac{\gamma}{\lambda} W^1 - \frac{\rho}{\lambda}(W^1 - R),$$

and

$$x_{cd} = \frac{\rho}{\lambda}(W^1 - R).$$

Then the dividend process $Div_t = \frac{I_t}{\lambda}$*, the deposit process* $\hat{Y}_t = Y_t$*, and the balance process* $B_t = \frac{W^1 - W_t}{\lambda}$*, solve the agent's optimization problem.*

Debt service costs: relief in distress



Market value of securities

$$V_e(B_t) = \mathbb{E}_t \left[\int_0^\tau e^{-rt} dDiv_t + e^{-r\tau} \left(1_{\tau=\tau_L} F_{L,e} + 1_{\tau=\tau_M} F_{M,e} \right) \right],$$

$$V_d(B_t) = \mathbb{E}_t \left[\int_0^\tau e^{-rt} x_d dt + e^{-r\tau} \left(1_{\tau=\tau_L} F_{L,d} + 1_{\tau=\tau_M} F_{M,d} \right) \right],$$

$$V_{cd}(B_t) = \mathbb{E}_t \left[\int_0^\tau e^{-rt} 1_{B_t \le \tilde{C}^L} x_{cd} dt + e^{-r\tau} \left(1_{\tau=\tau_L} F_{L,cd} + 1_{\tau=\tau_M} F_{M,cd} \right) \right],$$

$$\begin{aligned} V_{cl}(B_t) = \mathbb{E}_t \Bigg[& \int_0^\tau e^{-rt} d(Y_t - Div_t) \\ & - \int_0^\tau e^{-rt} (x_d + 1_{B_t \le \tilde{C}^L} x_{cd} + 1_{B_t > \tilde{C}^L} \kappa) dt \\ & + e^{-r\tau} \left(1_{\tau=\tau_L} F_{L,cl} + 1_{\tau=\tau_M} F_{M,cl} \right) \Bigg], \end{aligned}$$

where the constants $F_{L,sec}$ and $F_{M,sec}$ follow some seniority rule.

A tool: Feynman-Kac formula

LEMMA 3

Let W_t follow the equilibrium law of motion for the manager's continuation value until a stopping time $\tau = \min\{\tau_L, \tau_M\}$.
Let g be a function defined on $[R, W^1]$. Let k, F_L, F_M be constant.
Then the same function G defined on $[R, W^1]$ solves both

$$G(W_0) = \mathbb{E}\Big[\int_0^\tau e^{-rt} g(W_t)dt - k\int_0^\tau e^{-rt} dI_t \\ + e^{-r\tau}\left(1_{\tau=\tau_L} F_L + 1_{\tau=\tau_M} F_M\right)\Big]$$

and

$$rG(W) = g(W) + \Big(\gamma W + 1_{W \le \tilde{W}} \rho(W - R)\Big) G'(W) \\ + \frac{1}{2}\lambda^2\sigma^2 G''(W) + 1_{W \le \tilde{W}} \rho(F_M - G(W))$$

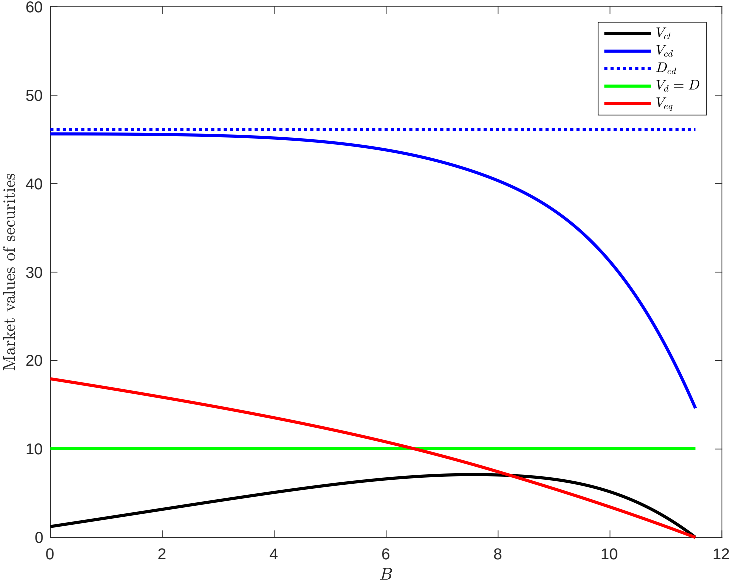
with boundary conditions $G(R) = F_L$ and $G'(W^1) = -k$.

Value of securities: a seniority rule

- Example seniority rule:
  1. long-term debt,
  2. contingent debt,
  3. revolving credit line,
  4. equity.

- Flow and terminal payoffs:
  1. $F_{L,d} = \min\{D, L\}$,
     $F_{M,d} = \min\{D, M\}$,
  2. $F_{L,cd} = \min\{D_{cd}, L - F_{L,d}\}$,
     $F_{M,cd} = \min\{D_{cd}, M - F_{M,d}\}$,
  3. $F_{L,cl} = \min\{B_\tau, L - F_{L,d} - F_{L,cd}\}$,
     $F_{M,cl} = \min\{B_\tau, M - F_{M,d} - F_{M,cd}\}$,
  4. nothing.

Value of securities



Comparative statics

Table: Comparative statics for the capital structure.

	dC^L	dD	dD_{cd}	dW_0	db_0
$d\rho$	$-$	$+$	$+$	$-$	$+$
$d\kappa$	$+$	$-$	$-$	$+$	$-$
$d\phi$	$-$	$+$	$+$	$-$	$+$
$d\kappa_B$	$+$	$-$	$-$	$+$	$-$
dL	$-$	$+$	$+$	$-$	$+$
dR	$-$	$-$	$-$	$+$	$-$
$d\gamma$	$-$	$\pm$	$\pm$	$-$	$-$
$d\sigma^2$	$+$	$-$	$-$	$\pm$	$-$

Recap and next steps

- We extend the DS model of optimal capital structure to allow for financial restructuring as an exit, in addition to liquidation.
- We assume transitions to and out of restructuring are slow.
  - a definition of financial distress
- In an optimal contract, in distress, the manager's faces a risk of dismissal and, correspondingly, a higher drift in her continuation value.
- An optimal capital structure has contingent debt and performance pricing.
- Next steps:
  - testing the perditions,
  - relaxing the assumption of creditor commitment,
  - GE effects.

Extra slide: Corbea-D'Erasmo Table 1

Table 1: Balance Sheet and Corporate Bankruptcies 1980 to 2014

Moment						
Frequency of Exit (%)	1.10					
Fraction of Exit by Chapter 7 (%)	19.83					
Frequency of (All) Bankruptcy (%)	0.96					
Fraction of Chapter 11 Bankruptcy (%)	79.15					
	Non-Bankrupt		Chapter 11		Chapter 7	
	Avg.	Median	Avg.	Median	Avg.	Median
Capital (millions 1983$)	953.18	35.61	408.78*,***	70.05	88.02**	24.58
Cash (millions 1983$)	125.77	9.87	52.84*,***	5.78	14.70**	3.74
Assets (millions 1983$)	1371.17	95.59	503.79*,***	97.49	139.16**	53.57
Op. Income (EBITDA) / Assets (%)	5.49	10.90	-8.34*	-1.18	-12.36	-5.34
Net Debt / Assets (%)	9.11	11.30	29.61*,***	25.25	21.80**	20.28
Total Debt / Assets (%)	28.31	24.45	41.99*,***	36.81	39.74**	34.12
Frac. Firms with Negative Net Debt (%)	36.07	-	21.88*	-	29.30**	-
Secured Debt / Total Debt (%)	43.90	40.77	47.63*	43.91	49.67**	48.59
Interest Coverage (EBITDA/Interest)	14.01	4.89	-0.22*	-0.22	-6.42**	-0.32
Equity Issuance / Assets (%)	4.70	0.06	2.84*	0.01	2.64**	0.01
Fraction Firms Issuing Equity (%)	22.04	-	13.14*	-	15.61**	-
Net Investment / Assets (%)	1.16	0.34	-2.94*	-3.09	-2.24**	-2.30
Dividend / Assets (%)	3.49	2.03	1.80*	0.87	2.31**	1.19
Z-score	3.74	3.20	-1.36*,***	-0.05	-1.42**	0.14
DD Prob. of Default (%)	2.13	0.01	3.60*	1.24	3.71**	1.07

Note: See Appendix A1 for a detailed definition of variables and the construction of bankruptcy and exit indicators. Medians (average) reported in the table correspond to the time series average of the cross-sectional median (mean) obtained for every year in our sample. Test for differences in means at 10% level of significance: * denotes Chapter 11 different from non-bankrupt, ** denotes Chapter 7 different from Non-bankrupt, *** denotes Chapter 11 different from Chapter 7. DD, distance to default, EBITDA, earnings before interest, tax, depreciation and authorization