The Hardware–Software Model
A New Conceptual Framework of Production, R&D, and Growth with AI

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Objective

I address a challenge to economic growth theory:

- to adapt growth models to the realities of the digital era (ICT, automation, AI);
- to lay out the rudiments of a macroeconomic framework for modeling production and R&D across the entire human history, including and specially focusing on the digital era.
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The existing conceptual framework:

- aggregate output is produced from capital and labor, \( Y = F(A, K, L) \);
- R&D is a function of R&D labor, \( \dot{A} = \Phi(L_A, A) \);
- only the industrial era (e.g., R&D-based endogenous growth), or industrial+agricultural era (unified growth theory).
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I propose a new conceptual framework: the hardware–software model.
Outline of the Hardware–Software Model

In any conceivable technological process, valuable output is generated through purposefully initiated physical action:

1. the physical action is a local reduction of entropy and requires expending energy
2. the set of instructions, or code, is disembodied information
Outline of the Hardware–Software Model

In any conceivable technological process, valuable output is generated through purposefully initiated physical action:

1. the **physical action** is a local reduction of entropy and requires expending energy
2. the set of instructions, or code, is disembodied **information**
3. the postulated general production function is

   \[ \text{Output} = \mathcal{F}(X, S), \]  

   \[ (1) \]

where \( X \) – hardware, \( S \) – software. The function \( \mathcal{F} \) is increasing in both factors. Both \( X \) and \( S \) are essential and mutually complementary (\( \sigma < 1 \)).
What’s in Hardware and Software

\[ Output = F(X, S) = F(L + K, H + \psi). \] (2)

<table>
<thead>
<tr>
<th>Hardware ( X )</th>
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<th>( L = \zeta N )</th>
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\[ L = \zeta N \]
\[ (1 - \chi) K \]
\[ \chi K \]
\[ H = AhN \]
\[ \Psi = A\psi\chi K \]

Within hardware, **agents of physical action are substitutable(**): whatever performs a given set of actions, if the actions are precisely defined then the outcome should be the same.

The same logic applies to software(**): regardless of whether a set of instructions comes from a human brain or a digital information processing unit, if the information is the same, then the outcome should be the same, too.

(*) beware of complex, multi-step processes
Output = \mathcal{F}(X, S) = \mathcal{F}(\zeta N + K, H + \Psi). \tag{3}

Components of hardware:

- **Human physical labor** \(L\) is rivalrous and given in fixed supply per worker and unit of time, \(L = \zeta N\).

- **Physical capital** \(K\) is rivalrous but can be unboundedly accumulated in per-capita terms. Physical capital \(K\) may be non-programmable or programmable. The share of programmable (computer or robot) hardware in total physical capital is denoted by \(\chi\) (so that \(\chi \in [0, 1]\)).
Output = \mathcal{F}(X, S) = \mathcal{F}(\zeta N + K, AhN + A\psi\chi K). \tag{4}

Components of software:

- Human cognitive work $H$ consists of technological knowledge $A$, skill level $h$, and the number of workers $N$, as in $H = AhN$. Technological knowledge $A$, or the size of the “repository of codes” is non-rivalrous (Romer, 1990) and accumulable. Per-capita skill levels $h$ are rivalrous and bounded above.

- Pre-programmed software $\Psi$ consists of technological knowledge $A$, “AI skill level” $\psi$, and the stock of programmable hardware $\chi K$ on which the software is run, as in $\Psi = A\psi\chi K$. The AI skill level $\psi$ is bounded above. Software can be virtually costlessly copied and thus can scale up to the level of all available programmable hardware $\chi K$. 

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Technological Progress

\[
Output = F(X, S) = F(\zeta N + K, A(hN + \psi \chi K)).
\]  

Technological progress (growth in \(A\)):
- expands the “repository of codes”,
- consists in development of new, better instructions allowing to produce higher output with a given amount of hardware,
- examples: abstract ideas, scientific theories, systematically catalogued facts, codes specifying certain actions, or blueprints of physical items.

New technologies are information and not actual objects or actions. It is precisely this informational character that makes technologies non-rivalrous (Romer, 1990).

These instructions can be applied to any task at hand both by humans, deterministic pre-programmed code, and AI. Thus, all technological progress is naturally modeled as software-augmenting.
**Related Literature (1)**

1. **Production function specification and estimation**, in particular with capital–skill complementarity, unbalanced growth, as well as investment-specific and skill-biased technical change (Gordon, 1990; Jorgenson, 1995; Greenwood et al., 1997; Hercowitz, 1998; Kumar and Russell, 2002; Koop et al., 1999, 2000; Krusell et al., 2000; Henderson and Russell, 2005; Caselli and Coleman, 2006; Klump et al., 2007, 2012; Growiec, 2012; Mućk, 2017; McAdam and Willman, 2018);

2. **Accounting for the accumulation of information and communication technologies (ICT)** and their broad growth-enhancing role as a general purpose technology (Bresnahan and Trajtenberg, 1995; Timmer and van Ark, 2005; Jorgenson, 2005; Brynjolfsson and McAfee, 2014; Gordon, 2016; Brynjolfsson et al., 2017; Aum et al., 2018);

3. **Automation** and its impacts on productivity, employment, wages and factor shares (Acemoglu and Autor, 2011; Autor and Dorn, 2013; Graetz and Michaels, 2015; Acemoglu and Restrepo, 2018; Andrews et al., 2016; Arntz et al., 2016; Frey and Osborne, 2017; Barkai, 2017; Autor et al., 2017; Jones and Kim, 2017);
4. Macroeconomic implications of development of **AI and autonomous robots** (Yudkowsky, 2013; Graetz and Michaels, 2015; Sachs et al., 2015; Benzell et al., 2015; DeCanio, 2016; Acemoglu and Restrepo, 2018; Aghion et al., 2017; Berg et al., 2018);

Generality of the New Framework

\[ \text{Output} = \mathcal{F}(X, S) = \mathcal{F}(\zeta N + K, A(hN + \psi \chi K)). \] (6)

The proposed hardware–software model encompasses as special cases:
- a standard treatment of the industrial economy (respecting Kaldor’s facts),
- a model of capital–skill complementarity and skill-biased technical change,
- a unified growth theory addressing the period of Industrial Revolution,
- a theory of inception and further development of the digital era.
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**Output:**
- GDP or value added, \( Y \),
- technological change, \( \dot{A} \).
Key Concepts for the Digital Era

- **Mechanization** (hardware);
- **Automation** (software);
- **ICT adoption** (programmable hardware);
- **Robotization** (specific type of programmable hardware + software);
- **AI** (pre-programmed software that is able to learn from data).

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The Hardware–Software Model and the Human History

- **Hardware revolution** – arrival of the *homo sapiens* (ca. 200 000 BP)
- **Software revolution** – cognitive revolution (ca. 70 000 BP)
- **Hardware revolution** – agricultural revolution (ca. 10 000 BP)
- **Software revolution** – scientific revolution (ca. 1550 CE)
- **Hardware revolution** – industrial revolution (ca. 1800 CE)
- **Software revolution** – digital revolution (ca. 1980 CE)
- **Hardware revolution** – *coming up next*?

Hardware revolution = revolution in energy
Software revolution = revolution in learning
The Production Function

The aggregate production function $F$: 

$$ Y = F(X, S) = F(\zeta N + K, A(hN + \psi \chi K)),$$

where $Y$ is aggregate value added (or GDP).

**Replication argument** $\Rightarrow$ constant returns to scale with respect to $X = \zeta N + K$ and $S/A = hN + \psi \chi K$ (excluding $A$).

**Non-rivalry of ideas** (Romer, 1990) $\Rightarrow$ increasing returns to scale with respect to $X$ and $S$ (including $A$).
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From the **laws of thermodynamics** (i.a., performing physical action requires expending energy) it is expected that **an essential fraction of GDP must consist of material outputs**.
Growth Accounting

Log-differentiating (7) with respect to time, we obtain the following Solow-type decomposition of economic growth:

\[ g_Y = \pi_X g_X + \pi_s g_s, \]  

or:

\[ g_Y = \pi_X [\pi_L g_N + \pi_K g_K] + \pi_S [\pi_H (g_h + g_N) + \pi_\psi (g_\psi + g_\chi + g_K)] + \pi_s g_A. \]

Each source of output growth has different asymptotic properties.
Stages of Economic Development (1)

Pre-industrial production \((K = \tilde{K} \approx 0, \chi = 0)\):

\[
Y = F(X, S) = F(\zeta N + \tilde{K}, AhN) \approx N \cdot F(\zeta, Ah). \tag{10}
\]

Output per worker is bounded above due to scarcity of hardware.
Stages of Economic Development (2)

**Industrial production** \((\chi = 0)\): 

\[
Y = F(X, S) = F(\zeta N + K, AhN). 
\]  

Equation (11) naturally captures the concept of **capital-skill complementarity** (Krusell et al., 2000; McAdam and Willman, 2018): physical capital is complementary to skilled labor \(H\) but substitutable with unskilled labor \(L\).

The limit of **full mechanization** and skill satiation, \(K \to \infty\) and \(h \to \bar{h}\), implies the **standard balanced growth path result** (Uzawa, 1961; Acemoglu, 2003) with 

\[
Y = F(K, \bar{h}AN). 
\]  

(12)
Stages of Economic Development (3)

Digital production:

\[ Y = F(X, S) = F(\zeta N + K, A(hN + \psi\chi K)). \] (13)

The limit of full automation \((N/K \to 0)\) implies:

\[ Y = K \cdot F(1, A\bar{\psi}\bar{\chi}). \] (14)

Equation (14) delivers an AK-type implication. Long-run endogenous growth is due to the accumulation of (programmable) hardware alone (Jones and Manuelli, 1990; Barro and Sala-i-Martin, 2003):

- software expands proportionally with hardware,
- hardware and software are gross complements, and thus in the long run hardware remains the bottleneck of development.

Note: the pace of hardware accumulation (and thus economic growth) may be nevertheless stupefying.

Note #2: the full automation limit rests on the assumption that \(\bar{\psi}\) is high (AI is potentially very capable).
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Factor Shares

Gross complementarity ($\sigma < 1$): factor income will be disproportionately directed towards the scarce factor.

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2. **Industrial production (1).** Mechanization: substitution within \(X\). Towards \(K\) (scarce capital).
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4. **Digital production (1).** Automation: substitution within $S$. Towards $A\psi \chi K$ (scarce pre-programmed software, including AI).
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4. **Digital production (1).** *Automation*: substitution within $S$. Towards $A\psi\chi K$ (scarce pre-programmed software, including AI).

... *here human work becomes irrelevant*

...

5. **Digital production (2).** *Increasing hardware demand by AI*. Towards $\chi K$ (scarce programmable hardware).
Existing R&D-based growth literature often assumes that researchers’ labor is the only input in the R&D process (Romer, 1990; Jones, 1995, 1999; Ha and Howitt, 2007). Alternatively, a few studies embrace the “lab equipment” specification of the R&D process, conditioning R&D output on overall R&D spending (Rivera-Batiz and Romer, 1991; Bloom et al., 2017; Kruse-Andersen, 2017).

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I postulate the knowledge accumulation equation of form:

\[
\dot{A} = \Phi(X, S) = \Phi(\zeta N + K, A(\eta N + \psi \chi K)).
\]  

(15)

Analogous analysis follows. (The only difference: \( \dot{A} \) is information.)
Pre-industrial R&D ($K = \tilde{K} \approx 0, \chi = 0$):

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Ideas are getting harder to find (Olsson, 2005; Bloom et al., 2017).
Industrial R&D \((\chi = 0)\):

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In the limit of full mechanization and skill satiation, \(K \to \infty\) and \(h \to \bar{h}\), it is obtained that

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Industrial R&D ($\chi = 0$):

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Yet, if $\Phi$ exhibits constant returns to scale then thanks to R&D capital accumulation the economy tends to an asymptotic BGP where $K$ and $A$ grow at the same rate:

$$g_A = \frac{\dot{A}}{A} = \Phi \left( \frac{K}{A}, hN \right).$$ \hspace{1cm} (19)
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Human research skills are intensively augmented with R&D hardware. Some routine research tasks are gradually automated. This process may accelerate fast in the future with AI.
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The limit of full automation implies:

\[ \dot{A} = \Phi(K, A\bar{\psi} \bar{\chi} K). \]  

(21)

If \( \Phi \) exhibits constant returns to scale then again the economy tends to an asymptotic BGP where \( K \) and \( A \) grow at the same rate:

\[ g_A = \frac{\dot{A}}{A} = \Phi \left( \frac{K}{A}, \bar{\psi} \bar{\chi} K \right). \]  

(22)
Open Questions

AI and the future of production and R&D

- **Limits of AI capability.** Are ideation, innovation, creativity only sophisticated incarnations of pattern recognition, or something more?
- **Returns to cognitive reinvestment.** How efficient will future AI be in re-designing itself and its environment to improve its cognitive capacity?
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- **“AI takeover”.** Will human skills be always required for production and R&D? What would the AI do once it becomes superintelligent?
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Thank you for your attention.


