The Hardware–Software Model
A New Conceptual Framework of Production, R&D, and Growth with AI

Jakub Growiec

SGH Warsaw School of Economics
Narodowy Bank Polski

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Objective

I address a challenge to economic growth theory:

- to adapt growth models to the realities of the digital era (ICT, automation, AI);
- to lay out the rudiments of a macroeconomic framework for modeling production and R&D across the entire human history, including and specially focusing on the digital era.

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The existing conceptual framework:

- aggregate output is produced from capital and labor, $Y = F(A, K, L)$;
- R&D is a function of R&D labor, $\dot{A} = \Phi(L_A, A)$;
- only the industrial era (e.g., R&D-based endogenous growth), or industrial+agricultural era (unified growth theory).

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I propose a new conceptual framework: the hardware–software model.

Outline of the Hardware–Software Model

In any conceivable technological process, valuable output is generated through purposefully initiated physical action:

1. the physical action is a local reduction of entropy and requires expending **energy**
2. the set of instructions, or code, is disembodied **information**

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1. the physical action is a local reduction of entropy and requires expending **energy**
2. the set of instructions, or code, is disembodied **information**
3. the postulated general production function is

$$Output = \mathcal{F}(X, S), \tag{1}$$

where X – hardware, S – software. The function $\mathcal{F}$ is increasing in both factors. Both X and S are essential and mutually complementary ($\sigma < 1$).

What's in Hardware and Software

$$Output = \mathcal{F}(X, S) = \mathcal{F}(L + K, H + \Psi). \qquad (2)$$

Hardware X	Human physical labor	$L = \zeta N$
	Non-programmable physical capital	$(1 - \chi)K$
	Programmable physical capital	χK
Software S	Human cognitive work	$H = AhN$
	Pre-programmed software (including AI algorithms)	$\Psi = A\psi\chi K$

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Within hardware, agents of physical action are substitutable(*): whatever performs a given set of actions, if the actions are precisely defined then the outcome should be the same.

The same logic applies to software(*): regardless of whether a set of instructions comes from a human brain or a digital information processing unit, if the information is the same, then the outcome should be the same, too.

(*) beware of complex, multi-step processes

Hardware

$$Output = \mathcal{F}(X, S) = \mathcal{F}(\zeta N + K, H + \Psi). \tag{3}$$

Components of hardware:

- Human physical labor L is rivalrous and given in fixed supply per worker and unit of time, $L = \zeta N$.
- Physical capital K is rivalrous but can be unboundedly accumulated in per-capita terms. Physical capital K may be non-programmable or programmable. The share of programmable (computer or robot) hardware in total physical capital is denoted by χ (so that $\chi \in [0, 1]$).

Software

$$Output = \mathcal{F}(X, S) = \mathcal{F}(\zeta N + K, AhN + A\psi\chi K). \tag{4}$$

Components of software:

- Human cognitive work H consists of technological knowledge A, skill level h, and the number of workers N, as in $H = AhN$. Technological knowledge A, or the size of the "repository of codes" is non-rivalrous (Romer, 1990) and accumulable. Per-capita skill levels h are rivalrous and bounded above.
- Pre-programmed software Ψ consists of technological knowledge A, "AI skill level" ψ, and the stock of programmable hardware χK on which the software is run, as in $\Psi = A\psi\chi K$. The AI skill level ψ is bounded above. Software can be virtually costlessly copied and thus can scale up to the level of all available programmable hardware χK.

Technological Progress

$$Output = \mathcal{F}(X, S) = \mathcal{F}(\zeta N + K, A(hN + \psi\chi K)). \quad (5)$$

Technological progress (growth in A):

- expands the "repository of codes",
- consists in development of new, better instructions allowing to produce higher output with a given amount of hardware,
- examples: abstract ideas, scientific theories, systematically catalogued facts, codes specifying certain actions, or blueprints of physical items.

New technologies are information and not actual *objects* or *actions*. It is precisely this informational character that makes technologies non-rivalrous (Romer, 1990).

These instructions can be applied to any task at hand both by humans, deterministic pre-programmed code, and AI. Thus, all technological progress is naturally modeled as software-augmenting.

Related Literature (1)

1. **Production function specification and estimation**, in particular with capital–skill complementarity, unbalanced growth, as well as investment-specific and skill-biased technical change (Gordon, 1990; Jorgenson, 1995; Greenwood et al., 1997; Hercowitz, 1998; Kumar and Russell, 2002; Koop et al., 1999, 2000; Krusell et al., 2000; Henderson and Russell, 2005; Caselli and Coleman, 2006; Klump et al., 2007, 2012; Growiec, 2012; Mućk, 2017; McAdam and Willman, 2018);
2. **Accounting for the accumulation of information and communication technologies (ICT)** and their broad growth-enhancing role as a general purpose technology (Bresnahan and Trajtenberg, 1995; Timmer and van Ark, 2005; Jorgenson, 2005; Brynjolfsson and McAfee, 2014; Gordon, 2016; Brynjolfsson et al., 2017; Aum et al., 2018);
3. **Automation** and its impacts on productivity, employment, wages and factor shares (Acemoglu and Autor, 2011; Autor and Dorn, 2013; Graetz and Michaels, 2015; Acemoglu and Restrepo, 2018; Andrews et al., 2016; Arntz et al., 2016; Frey and Osborne, 2017; Barkai, 2017; Autor et al., 2017; Jones and Kim, 2017);

Related Literature (2)

4. Macroeconomic implications of development of **AI and autonomous robots** (Yudkowsky, 2013; Graetz and Michaels, 2015; Sachs et al., 2015; Benzell et al., 2015; DeCanio, 2016; Acemoglu and Restrepo, 2018; Aghion et al., 2017; Berg et al., 2018);
5. **R&D-based endogenous growth** (Romer, 1990; Jones and Manuelli, 1990; Aghion and Howitt, 1992; Jones, 1995; Ha and Howitt, 2007; Madsen, 2008; Bloom et al., 2017; Kruse-Andersen, 2017).

Generality of the New Framework

$$Output = \mathcal{F}(X, S) = \mathcal{F}(\zeta N + K, A(hN + \psi\chi K)). \quad (6)$$

The proposed hardware–software model encompasses as special cases:

- a standard treatment of the industrial economy (respecting Kaldor's facts),
- a model of capital–skill complementarity and skill-biased technical change,
- a unified growth theory addressing the period of Industrial Revolution,
- a theory of inception and further development of the digital era.

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Output:

- GDP or value added, Y,
- technological change, $\dot{A}$.

Key Concepts for the Digital Era

- Mechanization (hardware);
- Automation (software);
- ICT adoption (programmable hardware);
- Robotization (specific type of programmable hardware + software);
- AI (pre-programmed software that is able to learn from data).

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- Industrial revolution ⇒ mechanization (without automation),
- Digital revolution ⇒ automation (following the earlier mechanization).

The Hardware–Software Model and the Human History

- Hardware revolution – arrival of the *homo sapiens* (ca. 200 000 BP)
- Software revolution – cognitive revolution (ca. 70 000 BP)
- Hardware revolution – agricultural revolution (ca. 10 000 BP)
- Software revolution – scientific revolution (ca. 1550 CE)
- Hardware revolution – industrial revolution (ca. 1800 CE)
- Software revolution – digital revolution (ca. 1980 CE)
- Hardware revolution – *coming up next?*

Hardware revolution = revolution in energy
Software revolution = revolution in learning

The Production Function

The aggregate production function F:

$$Y = F(X, S) = F(\zeta N + K, A(hN + \psi\chi K)), \tag{7}$$

where Y is aggregate value added (or GDP).

Replication argument $\Rightarrow$ constant returns to scale with respect to $X = \zeta N + K$ and $S/A = hN + \psi\chi K$ (excluding A).

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From the **laws of thermodynamics** (i.a., performing physical action requires expending energy) it is expected that an essential fraction of GDP must consist of material outputs.

Growth Accounting

Log-differentiating (7) with respect to time, we obtain the following Solow-type decomposition of economic growth:

$$g_Y = \pi_X g_X + \pi_S g_S, \tag{8}$$

or:

$$g_Y = \pi_X[\pi_L g_N + \pi_K g_K] + \pi_S[\pi_H(g_h + g_N) + \pi_\Psi(g_\psi + g_\chi + g_K)] + \pi_S g_A. \tag{9}$$

Each source of output growth has different asymptotic properties.

Stages of Economic Development (1)

Pre-industrial production ($K = \tilde{K} \approx 0$, $\chi = 0$):

$$Y = F(X, S) = F(\zeta N + \tilde{K}, AhN) \approx N \cdot F(\zeta, Ah). \qquad (10)$$

Output per worker is bounded above due to scarcity of hardware.

Stages of Economic Development (2)

Industrial production ($\chi = 0$):

$$Y = F(X, S) = F(\zeta N + K, AhN). \tag{11}$$

Equation (11) naturally captures the concept of capital-skill complementarity (Krusell et al., 2000; McAdam and Willman, 2018): physical capital is complementary to skilled labor H but substitutable with unskilled labor L.

The limit of full mechanization and skill satiation, $K \to \infty$ and $h \to \bar{h}$, implies the standard balanced growth path result (Uzawa, 1961; Acemoglu, 2003) with

$$Y = F(K, \bar{h}AN). \tag{12}$$

Stages of Economic Development (3)

Digital production:

$$Y = F(X, S) = F(\zeta N + K, A(hN + \psi\chi K)). \tag{13}$$

The limit of full automation ($N/K \to 0$) implies:

$$Y = K \cdot F(1, A\bar{\psi}\bar{\chi}). \tag{14}$$

Equation (14) delivers an AK-type implication. Long-run endogenous growth is due to the accumulation of (programmable) hardware alone (Jones and Manuelli, 1990; Barro and Sala-i-Martin, 2003):

- software expands proportionally with hardware,
- hardware and software are gross complements, and thus in the long run hardware remains the bottleneck of development.

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Note: the pace of hardware accumulation (and thus economic growth) may be nevertheless stupefying.
Note #2: the full automation limit rests on the assumption that $\bar{\psi}$ **is high (AI is potentially very capable)**.

Factor Shares

Gross complementarity ($\sigma < 1$): factor income will be disproportionately directed towards the scarce factor.

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4. **Digital production (1).** Automation: substitution within S. Towards $A\psi\chi K$ (scarce pre-programmed software, including AI).

   ...

   here human work becomes irrelevant

   ...
5. **Digital production (2).** Increasing hardware demand by AI. Towards χK (scarce programmable hardware).

The R&D Equation

Existing R&D-based growth literature often assumes that researchers' labor is the only input in the R&D process (Romer, 1990; Jones, 1995, 1999; Ha and Howitt, 2007). Alternatively, a few studies embrace the "lab equipment" specification of the R&D process, conditioning R&D output on overall R&D spending (Rivera-Batiz and Romer, 1991; Bloom et al., 2017; Kruse-Andersen, 2017).

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In reality, however, productivity of the R&D sector depends increasingly on the services of R&D capital.

I postulate the **knowledge accumulation equation** of form:

$$\dot{A} = \Phi(X, S) = \Phi(\zeta N + K, A(hN + \psi\chi K)). \tag{15}$$

Analogous analysis follows. (The only difference: $\dot{A}$ is information.)

R&D Across Stages of Economic Development (1)

Pre-industrial R&D ($K = \tilde{K} \approx 0$, $\chi = 0$):

$$\dot{A} = \Phi(X, S) = \Phi(\zeta N + \tilde{K}, AhN) \approx \Phi(\zeta N, AhN). \tag{16}$$

Ideas are getting harder to find (Olsson, 2005; Bloom et al., 2017).

R&D Across Stages of Economic Development (2)

Industrial R&D ($\chi = 0$):

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In the limit of full mechanization and skill satiation, $K \to \infty$ and $h \to \bar{h}$, it is obtained that

$$\dot{A} = \Phi(K, \bar{h}AN). \tag{18}$$

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Yet, if Φ exhibits constant returns to scale then thanks to R&D capital accumulation the economy tends to an asymptotic BGP where K and A grow at the same rate:

$$g_A = \frac{\dot{A}}{A} = \Phi\left(\frac{K}{A}, hN\right). \tag{19}$$

R&D Across Stages of Economic Development (3)

Digital R&D:

$$\dot{A} = \Phi(X, S) = \Phi(\zeta N + K, A(hN + \psi\chi K)). \tag{20}$$

Human research skills are intensively augmented with R&D hardware. Some routine research tasks are gradually automated. This process may accelerate fast in the future with AI.

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The limit of full automation implies:

$$\dot{A} = \Phi(K, A\bar{\psi}\bar{\chi}K). \tag{21}$$

If Φ exhibits constant returns to scale then again the economy tends to an asymptotic BGP where K and A grow at the same rate:

$$g_A = \frac{\dot{A}}{A} = \Phi\left(\frac{K}{A}, \bar{\psi}\bar{\chi}K\right). \tag{22}$$

Open Questions

AI and the future of production and R&D

- **Limits of AI capability.** Are ideation, innovation, creativity only sophisticated incarnations of pattern recognition, or something more?
- **Returns to cognitive reinvestment.** How efficient will future AI be in re-designing itself and its environment to improve its cognitive capacity?

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Singularity?

- **“AI takeover”.** Will human skills be always required for production and R&D? What would the AI do once it becomes superintelligent?

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Thank you for your attention.

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