Time Varying Structural Vector Autoregressions: Some New Perspective

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Introduction

Motivation and Main Findings

- No identification results for TVP-SVARs (in particular Primiceri (2005))
- Some evidence that Primiceri's TVP-SVAR setup may be nonidentified (Lubik et al. (2014), Yamamura (2017))
- Identification necessary to settle the issue of sources of variability in the data (coefficients vs. volatilities)
- I failed to demonstrate nonidentifaction of Primiceri's model
- ... But I came up with the alternative TVP-SVAR, for which I gave sufficient conditions for global identification
- These suggest the following:
  - TV contemporaneous relation matrix is identifed without any restrictions (i.e. you don't need Choleski scheme, actually you don't need any other one!)
  - In contrast, you should severely restrict the covariance structure for TV coefficients on lagged data

Primiceri's setup

$$y_t = c_t + x_t B_t + A_t^{-1} D_t \varepsilon_t$$

where $\varepsilon_t \sim N(0, \mathrm{I}_m)$; $x_t = \mathrm{I}_m \otimes (y'_{t-1} ... y'_{t-p})$; A_t is lower diagonal with 1's on the diagonal; $D_t = diag(\sigma_{1,t}, ..., \sigma_{m,t})$ and

$$\begin{bmatrix} c_t \\ B_t \end{bmatrix} = \begin{bmatrix} c_{t-1} \\ B_{t-1} \end{bmatrix} + \begin{bmatrix} \omega_t^c \\ \omega_t^B \end{bmatrix}; \quad \begin{bmatrix} \omega_t^c \\ \omega_t^B \end{bmatrix} \sim N\left(0, \begin{bmatrix} \Omega_c & \Omega_{cB} \\ \Omega'_{cB} & \Omega_B \end{bmatrix}\right)$$

In addition, let α_t denote all free elements in A_t (stacked in a column vector) and $\sigma_t = (\sigma_{1,t}, ..., \sigma_{m,t})'$, then

$$\alpha_t = \alpha_{t-1} + \zeta_t; \quad \zeta_t \sim N(0, S) \qquad \log\sigma_t = \log\sigma_{t-1} + \eta_t; \quad \eta_t \sim N(0, W)$$

CAVEAT: Choleski scheme for $A_t^{-1} D_t$ has nothing to do with identification. Identification is about whether the parameters i.e. $\Omega_c, \Omega_{cB}, \Omega_B, S, W$ are identified. That is whether we can distinguish between all sources of variabiliity for all possible data

My setup and contribution

My TVP-SVAR:

$$y_t = c_t + x_t B_t + \Psi_t \varepsilon_t; \quad \varepsilon_t \sim N(0, \mathrm{I}_m)$$

$$\begin{bmatrix} c_t \\ B_t \end{bmatrix} = \begin{bmatrix} c_{t-1} \\ B_{t-1} \end{bmatrix} + \begin{bmatrix} \omega_t^c \\ \omega_t^B \end{bmatrix}; \quad \begin{bmatrix} \omega_t^c \\ \omega_t^B \end{bmatrix} \sim N\left(0, \begin{bmatrix} \Omega_c & 0 \\ 0 & \Omega_B \end{bmatrix}\right)$$

$$vec(\Psi_t) = vec(\Psi_{t-1}) + u_t; \quad u_t \sim N\left(0, \begin{bmatrix} \Sigma_{11} & 0 & \dots & 0 \\ 0 & \Sigma_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Sigma_{mm} \end{bmatrix}\right)$$

NOTE: The pattern of Ψ_t is unrestricted !

all shocks are mutually independent, $c_0 \sim N(\underline{c}_0, \underline{V}_{c_0})$, $B_0 \sim N(\underline{B}_0, \underline{V}_{B_0})$ and

$$vec(\Psi_0) \sim N\left(vec(\underline{\Psi}_0), \begin{bmatrix} (\underline{V}_{\Psi_0})_{11} & 0 & \dots & 0 \\ 0 & (\underline{V}_{\Psi_0})_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & (\underline{V}_{\Psi_0})_{mm} \end{bmatrix}\right) \quad \text{and } \underline{\Psi}_0 \text{ is nonsingular}$$

Let us denote the model parameters as $\theta = (\Omega_c, \Omega_B, \Sigma_{11}, \dots, \Sigma_{mm}) \in \Theta$, initial observations as $y^0 = (y_0, \dots, y_{-p+1})$ and let $p(y_1, \dots, y_T | y^0, \theta)$ be the pdf of my TVP-SVAR (latent processes are integrated out, though it depends on hyperparameters)

Key results:

DEFINITION: My TVP-SVAR is globally identified at $\theta \in \Theta$ iff $p(y_1, \dots, y_T | y^0, \theta) = p(y_1, \dots, y_T | y^0, \bar{\theta})$ for all $y_1, \dots, y_T \in \mathbb{R}^{m \times T}$ implies $\theta = \bar{\theta}$

THEOREM 1:

a) Let $m = 2$. Then my TVP-SVAR is globally identified at almost all Σ_{11}, Σ_{22};

b) Let $m \geq 3$. Denote the i-th row of $\underline{\Psi}_0^{-1}$ as l_i. My TVP-SVAR is globally identified at almost all $\Sigma_{11}, \Sigma_{22}, \dots, \Sigma_{mm}$ provided that

$$l_1(\Sigma_{22} + (\underline{V}_{\Psi_0})_{22})l'_1 > l_1(\Sigma_{33} + (\underline{V}_{\Psi_0})_{33})l'_1 > \dots > l_1(\Sigma_{mm} + (\underline{V}_{\Psi_0})_{mm})l'_1$$
$$l_2(\Sigma_{11} + (\underline{V}_{\Psi_0})_{11})l'_2 > l_2(\Sigma_{33} + (\underline{V}_{\Psi_0})_{33})l'_2 > \dots > l_2(\Sigma_{mm} + (\underline{V}_{\Psi_0})_{mm})l'_2$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$l_m(\Sigma_{11} + (\underline{V}_{\Psi_0})_{11})l'_m > l_m(\Sigma_{22} + (\underline{V}_{\Psi_0})_{22})l'_m > \dots > l_m(\Sigma_{m-1m-1} + (\underline{V}_{\Psi_0})_{m-1m-1})l'_m$$

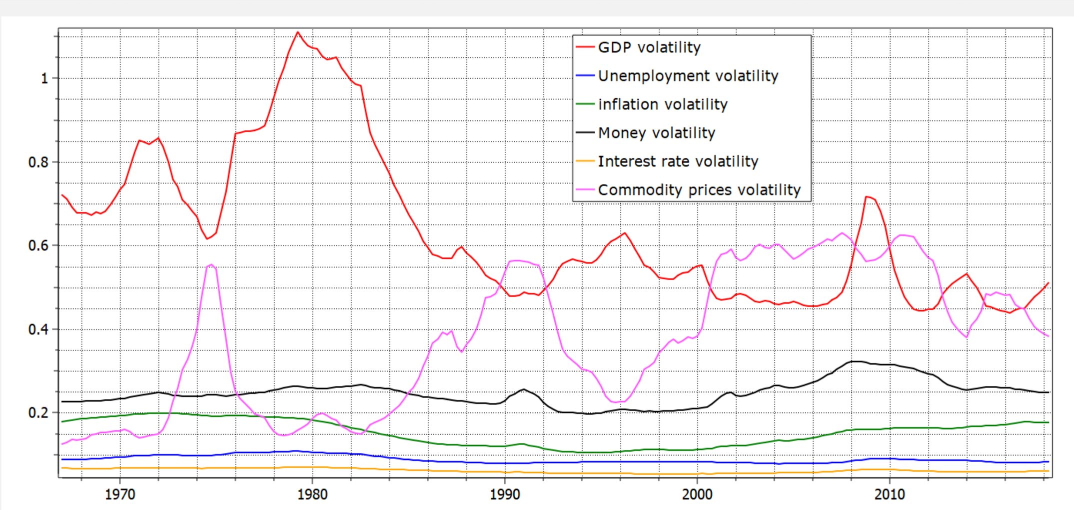
THEOREM 2: *Assume the number of lags is 2. Assume that Ω_B is diagonal (but Ω_c is just positive definite). Then my TVP-SVAR is globally identified at almost all Ω_c, Ω_B (for almost all intitial observations and hyperparameters)*

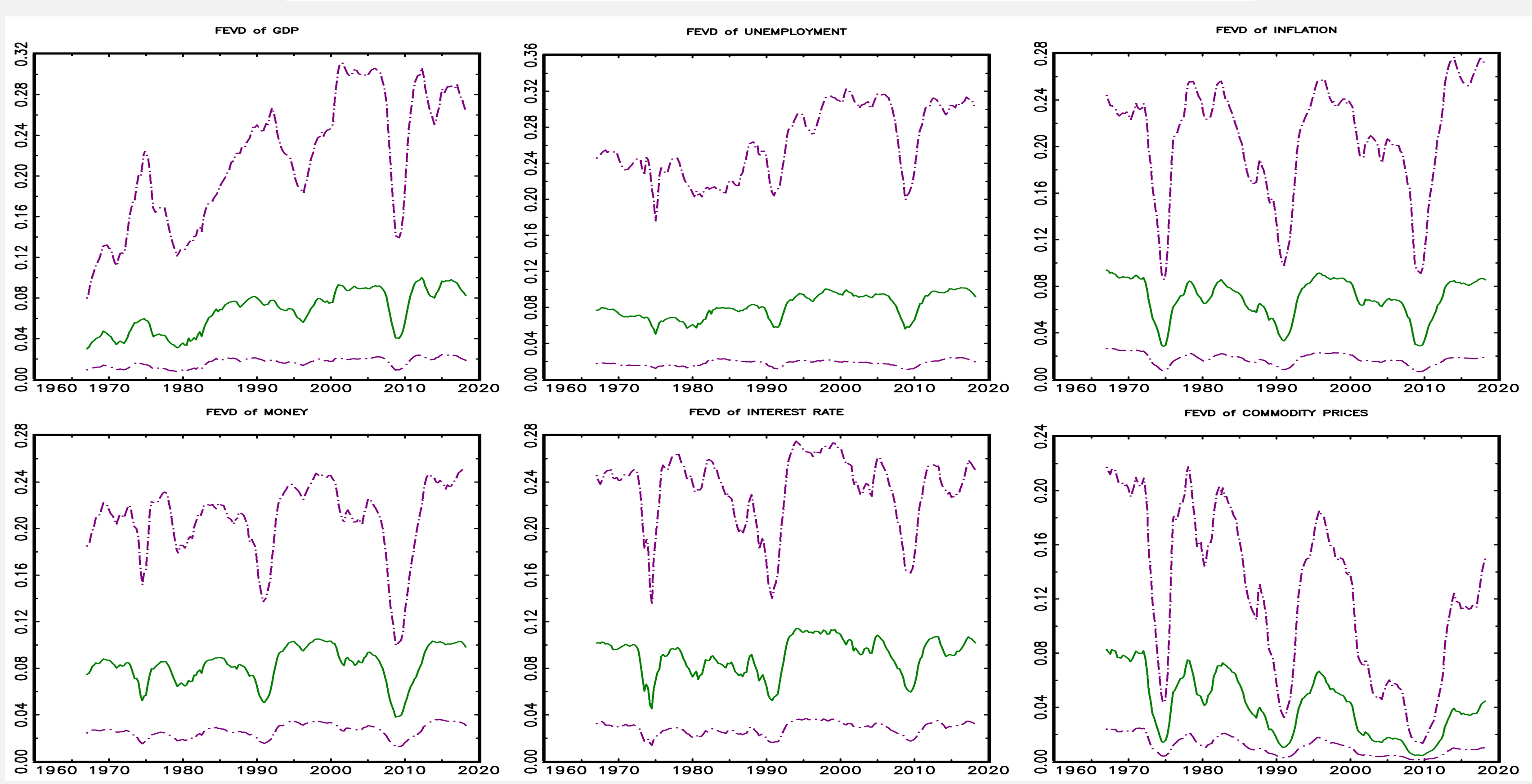
Econometric contribution:

Very efficient Bayesian sampling. In contrast to Primiceri (2005), I managed to provide "pure" Gibbs sampling. That is all Gibbs steps use exact sampling from the full conditional posterior (i.e. no Metropolis-Hastings within Gibbs sampling)

Empirical illustration: U.S. and 6 variables

Real GDP, unemployment rate, GDP deflator, M2 money, federal funds rate, commodity prices, 7 years training sample, effective sample 1967:Q1-2018:Q2





IRFs, monetary policy shock normalized to 25 bp: