**Introduction**

**Motivation and Main Findings**

- No identification results for TVP-SVARs (in particular Primiceri (2005))
- Some evidence that Primiceri’s TVP-SVAR setup may be nonidentified (Lubik et al., 2014; Yamamura (2017))
- Identification necessary to settle the issue of sources of variability in the data (coefficients vs. volatilities)
- I failed to demonstrate nonidentification of Primiceri’s model

... But I came up with the alternative TVP-SVAR, for which I gave sufficient conditions for global identification

These suggest the following:

- TV contemporaneous relation matrix is identified without any restrictions (i.e. you don’t need Choleski scheme, actually you don’t need any other one!)
- In contrast, you should severely restrict the covariance structure for TV coefficients on lagged data

**My setup and contribution**

**My TV-SVAR:**

\[
y_t = c_t + x_t \beta_t + \Psi \xi_t; \quad \xi_t \sim N(0, I)
\]

\[
\begin{bmatrix}
\alpha^\gamma

\alpha^\beta

\alpha^\delta
\end{bmatrix}
\sim N\left(0, \begin{bmatrix} \Omega & 0 & 0 \\
0 & \Omega & 0 \\
0 & 0 & \Omega \end{bmatrix}\right)
\]

\[
\vec(\Psi_t) = \vec(\Psi_{t-1}) + u_t; \quad u_t \sim N\left(0, \begin{bmatrix} \Sigma_1 & 0 & \ldots & 0 \\
0 & \Sigma_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \Sigma_m \end{bmatrix}\right)
\]

**NOTE:** The pattern of \( \Psi \) is unrestricted!

All shocks are mutually independent, \( c_t \sim N(0, \Sigma_0) \), \( \beta_t \sim N(0, \Sigma_{\beta}) \) and

\[
\vec(\Psi_0) \sim N\left(\begin{bmatrix} \vec(\Psi_{0,11}) & 0 & \ldots & 0 \\
0 & \vec(\Psi_{0,22}) & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \vec(\Psi_{0,mn}) \end{bmatrix}\right)
\]

\( \Psi_0 \) is nonsingular

Let us denote the model parameters as \( \theta = (\Omega, \Sigma_{\beta}, \Sigma_0, \ldots, \Sigma_m) \in \Theta \), initial observations as \( y^0 = (y_{0,t}, \ldots, y_{0,T}) \) and let \( p(y_{t}, \ldots, y_{T}|y^0, \theta) \) be the pdf of my TV-SVAR (latent processes are integrated out, though it depends on hyperparameters)

**Empirical illustration:** U.S. and 6 variables

Real GDP, unemployment rate, GDP deflator, M2 money, federal funds rate, commodity prices, 7 years training sample, effective sample 1967-Q1-2018:Q2

**Primiceri’s setup**

\[
y_t = c_t + x_t \beta_t + A_t^{-1} \Omega \xi_t
\]

where \( \xi_t \sim N(0, I_6) \); \( z_t = \Pi_0 \otimes (y_{t-1, \ldots, y_{t-6}}) \); \( A_t \) is lower diagonal with 1’s on the diagonal; \( \Omega_t = \text{diag}(\sigma_{1,t}, \ldots, \sigma_{m,t}) \) and

\[
\begin{bmatrix}
\epsilon_t \\
\beta_t
\end{bmatrix}
= \begin{bmatrix} \epsilon_{t-1} \\
\beta_{t-1}
\end{bmatrix} + \begin{bmatrix} \alpha^\gamma \\
\alpha^\beta
\end{bmatrix} \sim N\left(0, \begin{bmatrix} \Omega & 0 \\
0 & \Omega_{\beta} \end{bmatrix}\right)
\]

In addition, let \( a_t \) denote all free elements in \( A_t \) (stacked in a column vector) and \( \alpha_t = (\alpha_{1,t}, \ldots, \alpha_{m,t}) \), then

\[
a_t = a_{t-1} + \epsilon_t; \quad \epsilon_t \sim N(0, \Sigma_{\epsilon}) \quad \log(\alpha_t) = \log(\alpha_{t-1}) + \eta_t; \quad \eta_t \sim N(0, W)
\]

**Caveat:** Choleski scheme for \( A_t^{-1} \Omega_t \) has nothing to do with identification. Identification is about whether the parameters i.e. \( \Omega, \Sigma_{\beta}, \Sigma_0, S, W \) are identified.

That is whether we can distinguish between all sources of variability for all possible data

**Key results:**

**DEFINITION:** My TV-SVAR is globally identified at \( \hat{\theta} \in \Theta \) if \( p(y_{t}, \ldots, y_{T}|y^0, \hat{\theta}) = p(y_{t}, \ldots, y_{T}|y^0, \theta) \) for all \( y_{t}, \ldots, y_{T} \in \mathbb{R}^{m+1} \) implies \( \theta = \hat{\theta} \)

**THEOREM 1:**

a) Let \( m = 2 \). Then my TV-SVAR is globally identified at almost all \( \Sigma_1, \Sigma_2 \);

b) Let \( m \geq 3 \). Denote the \( i \)-th row of \( \Psi_1 \) as \( l_i \). My TV-SVAR is globally identified at almost all \( \Sigma_1, \Sigma_2, \ldots, \Sigma_m \) provided that

\[
\begin{align*}
&l_1(\Sigma_2 + (\Psi_{0,22})_{22}) > l_1(\Sigma_3 + (\Psi_{0,33})_{33}) > \ldots > l_1(\Sigma_m + (\Psi_{0,mn})_{mn}) \\
&l_2(\Sigma_1 + (\Psi_{0,11})_{11}) > l_2(\Sigma_3 + (\Psi_{0,33})_{33}) > \ldots > l_2(\Sigma_m + (\Psi_{0,mn})_{mn}) \\
&l_m(\Sigma_1 + (\Psi_{0,11})_{11}) > l_m(\Sigma_2 + (\Psi_{0,22})_{22}) > \ldots > l_m(\Sigma_{m-1} + (\Psi_{0,mn})_{mn-1})_{mn-1}
\end{align*}
\]

**THEOREM 2:** Assume the number of lags is 2. Assume that \( \Omega_0 \) is diagonal (but \( \Omega_0 \) is just positive definite). Then my TV-SVAR is globally identified at almost all \( \Omega_0, \Omega_0 \) (for almost all initial observations and hyperparameters)

**Econometric contribution:**

Very efficient Bayesian sampling. In contrast to Primiceri (2005), I managed to provide “pure” Gibbs sampling. That is all Gibbs steps use exact sampling from the full conditional posterior (i.e. no Metropolis-Hastings within Gibbs sampling)

**IRFs, monetary policy shock normalized to 25 bp:**