Household Heterogeneity and the Value of Government Spending Multiplier: an Analytical Characterization

Paweł Kopiec

Narodowy Bank Polski

1The views presented in this paper are those of the author, and should not be attributed to Narodowy Bank Polski.
Introduction 1

- Last decade: renaissance in fiscal research (see Ramey (2019))
- Central issue: additional output generated by 1$ of government expenditures
- Woodford (2011): discussion based on old-fashioned models (Keynesian cross in the ISLM model)

\[
\frac{dY}{dG} = \frac{1}{1 - MPC} = 1 + MPC + MPC \cdot MPC + \ldots
\]
Voluminous empirical literature: individual characteristics crucial for consumption behavior


Keynesian cross logic: consumption pass-through essential for the multiplier’s value

Key issue: distribution of MPC across households of different characteristics
Introduction 3: cross-correlations in SHIW 2016
Introduction 4

- Accounting for the cross-sectional consumption patterns: prerequisite for better multiplier estimates
- Standard tool: Bewley-Huggett-Aiyagari model (BHA)

- What are the exact determinants of the multiplier when households are unequal?
  - Paper-and-pencil solutions are insightful
  - Problem: BHA is inherently complex
  - BHA: limited possibility of obtaining analytical results
Accounting for the cross-sectional consumption patterns: prerequisite for better multiplier estimates

Standard tool: Bewley-Huggett-Aiyagari model (BHA)

Quantitative works: Challe and Ragot (2011), Navarro and Ferriere (2016), Hagedorn et al. (2017), Brinca et al. (2017)

What are the exact determinants of the multiplier when households are unequal?

- Paper-and-pencil solutions are insightful
- Problem: BHA is inherently complex
- BHA: limited possibility of obtaining analytical results
Introduction 5

- Intertemporal Keynesian Cross: Auclert et al. (2018)
- Multiplier is a function of iMPCs and the path of fiscal deficits
- Sufficient-statistics approach
- Assumption: constant-real-rate monetary rule
- Problems:
  - channels operating through prices and interest rates are shut off
  - consumer balance sheets, public debt management: unaffected
  - monetary-fiscal interactions ignored
Outline

- This paper:
  - analytical formula for the multiplier in a heterogeneous agent economy
  - central bank follows a standard Taylor rule
  - formula decomposed into interpretable channels (most of them expressed as sufficient statistics)
  - calibrated model is used to estimate the multiplier and the magnitude of channels
  - 3 alternative scenarios analyzed
Restrictive assumptions made to derive analytical results in the Bewley-Huggett-Aiyagari model (BHA)


This paper: frictional product market assumed to relax 1., 2. and 3.
Restrictive assumptions made to derive analytical results in the Bewley-Huggett-Aiyagari model (BHA)


This paper: frictional product market assumed to relax 1., 2. and 3.
Frictional product market

- Arguments for specifying the market for goods in a decentralized manner:
  - product market frictions are ubiquitous (Michaillat and Saez (2015))
  - easy to introduce sticky prices (in comparison to Rotemberg (1982) and Calvo (1983))
  - all GE effects summarized with only one variable

- Frictional product market in the literature:
  - First paper: Diamond (1982)

- Almost inconsequential for fiscal policy transmission mechanism (comparison to NK model by Woodford (2011))
Model: households

\[ V(b, z) = \max_{c, v, b'} \left\{ \tilde{u}(c, v) + \beta \mathbb{E}_{z'|z} V(b', z') \right\} \]

subject to:

\[ c + T(z) + \frac{b'}{1+i} = \frac{b}{\Pi} + z \cdot f \]

\[ c = q \cdot v \]

\[ b' \geq -\xi \]
Model: government

- Fiscal authority:
  \[
  \int_{B \times Z} T(z) d\mu(b, z) + \frac{\tilde{B}'}{1+i} = \frac{\tilde{B}}{\Pi} + G
  \]
  \[G = q \cdot v_G\] (3)

- Central bank:
  \[
  i = \bar{i} + \phi_Y \cdot \left( \frac{Y - \bar{Y}}{\bar{Y}} \right) + \phi_\Pi \cdot (\Pi - \bar{\Pi})
  \]
Matching and price-setting

- Matching technology (CRS):
  \[
  M \left( \int_{B \times Z} v(b, z) \, d\mu(b, z) + v_G, \int_{B \times Z} zd\mu(b, z) \right)
  \]

- Product market tightness
  \[
  x \equiv \frac{\int_{B \times Z} v(b, z) \, d\mu(b, z) + v_G}{\int_{B \times Z} zd\mu(b, z)}
  \]  (4)

- Price-setting mechanism:
  \[
  \Pi = \Pi(x), \; \Pi'(x) > 0.
  \]  (5)
Consistency conditions and market clearing

- Matching probabilities:

\[
 f(x) = \frac{M \left( \int_{B \times Z} v(b, z) \, d\mu(b, z) + v_G, \int_{B \times Z} zd\mu(b, z) \right)}{\int_{B \times Z} zd\mu(b, z)} = M(x, 1) 
\]

\[
 q(x) = \frac{M \left( \int_{B \times Z} v(b, z) \, d\mu(b, z) + v_G, \int_{B \times Z} zd\mu(b, z) \right)}{\int_{B \times Z} v(b, z) \, d\mu(b, z) + v_G} = M \left( 1, \frac{1}{x} \right) 
\]

- Asset market clearing:

\[
 \bar{B}' = \int_{B \times Z} b' (b, z) \, d\mu(b, z) 
\]

- Product market clearing:

\[
 \int_{B \times Z} c(b, z) \, d\mu(b, z) + G = f(x) \cdot \int_{B \times Z} zd\mu(b, z) \equiv Y(x) 
\]
Law of motion for the distribution of agents

- Distribution of agents evolves according to:

\[
\mu' (B', z') = \int_{\{b: b'(b, z) \in B'\} \times Z} \mathbb{P}(z' | z) d\mu(b, z) \quad (10)
\]

- Operator \( \Gamma \) is defined as:

\[
\mu' = \Gamma (\mu). \quad (11)
\]

- Standardization:

\[
\int_{B \times Z} zd\mu(b, z) = 1 \quad (12)
\]
A stationary equilibrium is: positive numbers $x$, $q$, $f$, $i$, value function $V$, policy functions $c$, $v$, $b'$, distribution $\mu$ such that given $\bar{B}$, $G$, $v_{G}$, $\Pi$ and $T$:

1. Given $f$, $q$, $i$, $\Pi$ and $T$ function $V$ solves household’s maximization problem 1 and $c$, $v$ and $b'$ are associated policy functions.
2. Given $\bar{B}$, $G$, $\Pi$, $v_{G}$, $q$ and $i$ equation 3 and government budget constraint 2 hold.
3. Consistency conditions 4, 6, 7, price-setting relationship 5 and resource constraints 8, 9 are satisfied.
4. Measure $\mu$ is a fixed point of operator $\Gamma$ defined by 10 and 11.
Expressing GE effects as functions of $x$ and $G$

- Aggregate output:

$$Y(x) = f(x) \cdot \int_{B \times Z} zd\mu(b, z) = f(x)$$

- Interest rate:

$$i(x) = \bar{i} + \phi_Y \cdot \left( \frac{Y(x) - \bar{Y}}{\bar{Y}} \right) + \phi_{\pi} \cdot (\Pi(x) - \bar{\Pi})$$

- Search effort:

$$v = \frac{c}{q(x)} \Rightarrow u(c, x) \equiv \tilde{u} \left( c, \frac{c}{q(x)} \right)$$

- Assumption:

$$u_{cx} = 0$$
Expressing GE effects as functions of $x$ and $G$

- I concentrate on an unexpected fiscal shock $G_t$ that arrives at time $t$.
- Fiscal rule (as the Ricardian equivalence fails):
  \[ \Lambda : G_t \rightarrow \left[ \{ G_s(G_t) \}_{s \geq t}, \{ \bar{B}_{s+1}(G_t) \}_{s \geq t} \right] \]
- Share of debt-financed public expenditures in period $t$:
  \[ \lambda \equiv \frac{d \bar{B}_{t+1}}{d G_t} \]
- Decomposing the individual tax burden:
  \[ T(z) \equiv \tau(z) \cdot \Theta, \text{ where } \int_{B \times Z} \tau(z) d\mu(b,z) = 1 \]
- Fiscal rule $\Lambda$ and prices pin down the budget income from taxes for $s \geq t$:
  \[ \Theta(x_s, G_t) = \frac{1}{\Pi(x_s)} \cdot \bar{B}_s(G_t) - \frac{1}{1 + i(x_s)} \cdot \bar{B}_{s+1}(G_t) + G_s(G_t) \]
Reformulated consumer problem: GE effects depend on $x$ and $G$

Time-dependent Bellman equation in period $t$ under $\Lambda$:

$$V^\Lambda_t(b_t, z_t | G_t) = \max_{c_t, b_{t+1}} \left\{ u(c_t, x_t) + \beta \mathbb{E}_{z_{t+1} | z_t} V^\Lambda_{t+1}(b_{t+1}, z_{t+1} | G_t) \right\}$$

subject to:

$$c_t + \tau(z_t) \cdot \Theta(x_t, G_t) + \frac{b_{t+1}}{1 + i(x_t)} = \frac{b_t}{\Pi(x_t)} + z_t \cdot f(x_t)$$

$$b_{t+1} \geq -\xi$$

Under perfect foresight aggregate resource constraint becomes:

$$\int_{B \times Z} c^\Lambda(b_t, z_t | x_t, G_t) \, d\mu_t(b_t, z_t) + G_t = Y(x_t).$$

$$\equiv C^\Lambda(x_t, G_t)$$
Lemma

Suppose that economy is in stationary equilibrium at the beginning of period $t$ and government follows fiscal rule $\Lambda$. Then the value of government spending multiplier in period $t$ is:

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C^\Lambda_t}{\partial G_t}}{1 - \frac{\partial C^\Lambda_t}{\partial x_t} \cdot \frac{1}{f'(x_t)}}$$
Some additional notation

- Aggregation of variable $m$ over distribution of agents $\mu$:

$$E_\mu m \equiv \int_{B \times Z} m(b, z) \, d\mu(b, z)$$

- Marginal propensity to consume/save:

$$MPC \equiv \frac{dc}{dy}, \quad MPS \equiv \frac{1}{1 + i} \cdot \frac{db'}{dy}, \text{ where } y \equiv z \cdot f(x) - \tau(z) \cdot \Theta$$

- Unhedged interest rate exposure (like in Auclert (2017)):

$$URE \equiv \frac{b}{\Pi} + z \cdot f - \tau \cdot \Theta - c$$

- Comovement of prices and output resulting from a positive demand shock:

$$\alpha \equiv \frac{d\Pi}{dx} \frac{dY}{dX}$$

- Strength of central bank’s reaction:

$$\Omega \equiv \phi_\Pi \cdot \alpha + \phi_Y$$
Main result: formula for the multiplier in the BHA model

\[ \frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t}{\partial G_t}}{1 - \frac{\partial C_t}{\partial x_t} \cdot f'(x_t)} \]

where:

\[ \frac{\partial C_t}{\partial x_t} \cdot \frac{1}{f'(x_t)} \equiv -\frac{\Omega}{1+i} \cdot \mathbb{E}_\mu (MPS \cdot c) + \frac{\Omega}{1+i} \cdot \mathbb{E}_\mu (MPC \cdot URE) \]

\[ \text{Intertemporal substitution channel} (-) \]

\[ + \mathbb{E}_\mu (MPC \cdot z) - \left( \frac{\Omega}{(1+i)^2} - \alpha \right) \cdot \bar{B} \cdot \mathbb{E}_\mu (MPC \cdot \tau) - \alpha \cdot \mathbb{E}_\mu (MPC \cdot b) \]

\[ \text{Interest rate exposure channel} (-/+). \]

\[ + \mathbb{E}_\mu (MPC \cdot z) - \left( \frac{\Omega}{(1+i)^2} - \alpha \right) \cdot \bar{B} \cdot \mathbb{E}_\mu (MPC \cdot \tau) - \alpha \cdot \mathbb{E}_\mu (MPC \cdot b) \]

\[ \text{Income channel} (+) \]

\[ + \mathbb{E}_\mu (MPC \cdot z) - \left( \frac{\Omega}{(1+i)^2} - \alpha \right) \cdot \bar{B} \cdot \mathbb{E}_\mu (MPC \cdot \tau) - \alpha \cdot \mathbb{E}_\mu (MPC \cdot b) \]

\[ \text{Debt service costs channel} (-/+). \]

\[ + \mathbb{E}_\mu (MPC \cdot z) - \left( \frac{\Omega}{(1+i)^2} - \alpha \right) \cdot \bar{B} \cdot \mathbb{E}_\mu (MPC \cdot \tau) - \alpha \cdot \mathbb{E}_\mu (MPC \cdot b) \]

\[ \text{Fisher channel} (-/+). \]

and:

\[ \frac{\partial C_t}{\partial G_t} \equiv -\left( 1 - \frac{\lambda}{1+i} \right) \cdot \mathbb{E}_\mu (MPC \cdot \tau) + \beta \cdot (1+i) \cdot \mathbb{E}_\mu \left( MPS \cdot \frac{1}{u_{cc}(c)} \cdot \nu^\Lambda bG \right) \]

\[ \text{Taxation channel} (-) \]

\[ + \beta \cdot (1+i) \cdot \mathbb{E}_\mu \left( MPS \cdot \frac{1}{u_{cc}(c)} \cdot \nu^\Lambda bG \right) \]

\[ \text{Expectations channel} (-/+). \]
Some additional notation

- Change in the forward-looking consumer sentiments:

$$V_{bG}^\Lambda \equiv E_{z_{t+1}|z_t} V_{t+1,bG}^\Lambda ((1+i) \cdot URE_t, z_{t+1}|G_t) \mid_{URE_t = URE, G_t = G, V_{t+1}^\Lambda = V}$$
Special case: identical agents and comparison to Woodford (2011)

- Comparison to the RA case highlights the role of heterogeneity
- A one-time, tax-financed shock is considered
- Several channels cancel out:

\[
\alpha \cdot \bar{B} \cdot E_\mu (MPC \cdot \tau) - \alpha \cdot E_\mu (MPC \cdot b) = 0
\]

Debt service costs channel: repayment

\[
\frac{\Omega}{1+i} \cdot E_\mu (MPC \cdot URE) - \frac{\Omega}{(1+i)^2} \cdot \bar{B} \cdot E_\mu (MPC \cdot \tau) = 0
\]

Interest rate exposure channel

\[
\beta \cdot (1 + i) \cdot E_\mu \left( MPS \cdot \frac{1}{u_{cc}(c)} \cdot V^\Lambda_{bG} \right) = 0
\]

Expectations channel

\[
E_\mu (MPC \cdot z) = \left( 1 - \frac{\lambda}{1+i} \right) \cdot E_\mu (MPC \cdot \tau)
\]

Income channel

\[
E_\mu (MPC \cdot \tau) = \left( 1 - \frac{\lambda}{1+i} \right) \cdot E_\mu (MPC \cdot \tau)
\]

Taxation channel
Special case: identical agents and comparison to Woodford (2011)

- Government spending multiplier in the RA case:

\[
\frac{dY_t}{dG_t} = \frac{1}{1 + \beta \cdot \frac{1}{\eta_u} \cdot \Omega}
\]

- The corresponding expression in Woodford (standard NK model with endogenous labor supply)

\[
\frac{dY_t}{dG_t} = \frac{1}{1 + F \left( \beta \cdot \frac{1}{\eta_u} \cdot \Omega \right)}
\]

where \( F' > 0 \)

- Identical determinants of \( \frac{dY_t}{dG_t} \) in both environments!
Calibration

- Jappelli and Pistaferri (2014): relatively high average level of MPC in the SHIW data (equal to 0.475) can be hardly matched by the Bewley-Huggett-Aiyagari model
- Jappelli and Pistaferri (2014) suggest two solutions:
  - introduce a proportion of rule-of-thumb (hand-to-mouth, HTM) agents with \( MPC = 1 \)
  - decrease discount factor \( \beta \) significantly
- Lowering \( \beta \) generates unrealistically high real interest rates
  - I follow Auclert (2017) and set: \( \beta_H \) to match real interest rate and \( \beta_L \) to match average MPC
- I consider both variants of the model and choose the better one
- A GHH-like utility function:
  \[
  u(c, x) = \frac{1}{1-\sigma} \cdot \left[ \left( c - \frac{\kappa}{\phi} \cdot \left( \frac{c}{q(x)} \right)^{\phi} \right)^{1-\sigma} - 1 \right]
  \]
Calibration: parameters set w/o simulations, both versions of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>Probability of selling output</td>
<td>0.763</td>
<td>Capacity utilization</td>
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<td>$\Pi$</td>
<td>Price index</td>
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<td>$\phi_Y$</td>
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<td>Galí (2008)</td>
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<td>$\bar{i}$</td>
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<td>$\alpha$</td>
<td>Demand-driven comovement of $Y$ and $\Pi$</td>
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<td>SVAR evidence</td>
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<td>$\bar{B}$</td>
<td>Real value of public debt</td>
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<td>Debt to GDP ratio</td>
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<td>$\sigma$</td>
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<td>Condition $u_{cx} = 0$</td>
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<tr>
<td>$\phi$</td>
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<td>Italian tax system</td>
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<td>Stimulus financing rule</td>
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Calibration: parameters set with simulations, model with *HTM* agents

<table>
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<th>Parameter</th>
<th>Name</th>
<th>Value</th>
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<td>$\mu_{\text{HTM}}$</td>
<td>Proportion of HTM agents</td>
<td>0.42</td>
<td>Average MPC</td>
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<td>$\sigma_T^2$</td>
<td>Variance of transitory shocks</td>
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<td>MPC distribution</td>
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<td>$\sigma_P^2$</td>
<td>Variance of persistent shocks</td>
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<td>MPC distribution</td>
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<td>$\rho_P$</td>
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<td>MPC distribution</td>
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Calibration: parameters set with simulations, model with heterogeneous $\beta$

<table>
<thead>
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<th>Parameter</th>
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Key calibration target: MPC across cash-in-hand deciles
## Fiscal Multiplier: decomposition, benchmark scenario, 2 variants of the model

<table>
<thead>
<tr>
<th>Channel</th>
<th>Model with HTM agents</th>
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<th>Model with $\beta_L$ and $\beta_H$</th>
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<tr>
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<td>Value</td>
<td>Counterfactual $\frac{dY_t}{dG_t}$</td>
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Fiscal Multiplier: decomposition, alternative scenarios, model with HTM agents

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<th>Channel\Scenario</th>
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References


The End

Thank you for your attention!