Household Heterogeneity and the Value of Government Spending Multiplier: an Analytical Characterization[1]

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[1] The views presented in this paper are those of the author, and should not be attributed to Narodowy Bank Polski.

Introduction 1

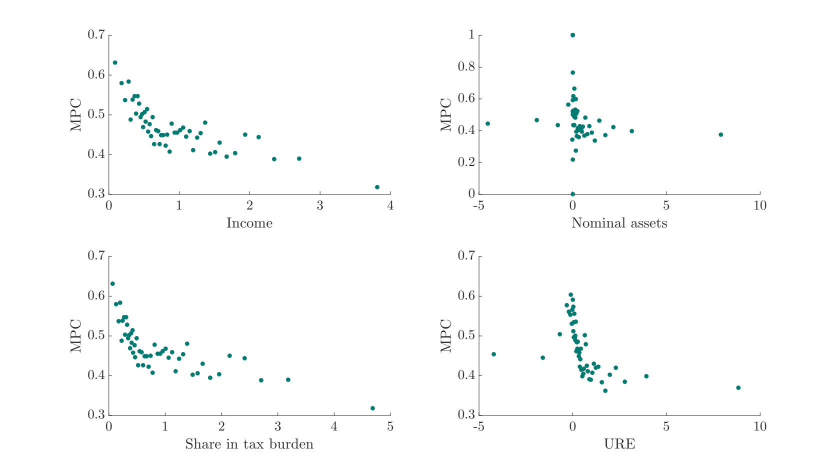
- Last decade: renaissance in fiscal research (see Ramey (2019))
- Central issue: additional output generated by 1$ of government expenditures
- Woodford (2011): discussion based on old-fashioned models (Keynesian cross in the ISLM model)

$$\frac{dY}{dG} = \frac{1}{1 - MPC} = 1 + MPC + MPC \cdot MPC + \ldots$$

Introduction 2

- Voluminous empirical literature: individual characteristics crucial for consumption behavior
- Important works: Carroll et al. (2014), Jappelli and Pistaferri (2014), Kaplan and Violante (2014), Krueger et al. (2016)
- Keynesian cross logic: consumption pass-through essential for the multiplier's value
- Key issue: distribution of MPC across households of different characteristics

Introduction 3: cross-correlations in SHIW 2016



Introduction 4

- Accounting for the cross-sectional consumption patterns: prerequisite for better multiplier estimates
- Standard tool: Bewley-Huggett-Aiyagari model (BHA)
- Quantitative works: Challe and Ragot (2011), Navarro and Ferriere (2016), Hagedorn et al. (2017), Brinca et al. (2017)
- What are the exact determinants of the multiplier when households are unequal?
  - Paper-and-pencil solutions are insightful
  - Problem: BHA is inherently complex
  - BHA: limited possibility of obtaining analytical results

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Introduction 5

- Intertemporal Keynesian Cross: Auclert et al. (2018)
- Multiplier is a function of iMPCs and the path of fiscal deficits
- Sufficient-statistics approach
- Assumption: constant-real-rate monetary rule
- Problems:
  - channels operating through prices and interest rates are shut off
  - consumer balance sheets, public debt management: unaffected
  - monetary-fiscal interactions ignored

Outline

- This paper:
  - analytical formula for the multiplier in a heterogeneous agent economy
  - central bank follows a standard Taylor rule
  - formula decomposed into interpretable channels (most of them expressed as sufficient statistics)
  - calibrated model is used to estimate the multiplier and the magnitude of channels
  - 3 alternative scenarios analyzed

Technical Contribution

Restrictive assumptions made to derive analytical results in the Bewley-Huggett-Aiyagari model (BHA)

1. Extreme illiquidity: Krusell et al. (2011), Werning (2015), McKay and Reis (2016), Ravn and Sterk (2016)
2. Constant real interest rates: Auclert et al. (2018), Patterson (2018)
3. Partial equilibrium: Auclert (2017)

This paper: frictional product market assumed to relax 1., 2. and 3.

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Frictional product market

- Arguments for specifying the market for goods in a decentralized manner:
  - product market frictions are ubiquitous (Michaillat and Saez (2015))
  - easy to introduce sticky prices (in comparison to Rotemberg (1982) and Calvo (1983))
  - all GE effects summarized with only one variable
- Frictional product market in the literature:
  - First paper: Diamond (1982)
  - More recent works: Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2015), Kaplan and Menzio (2016), Storesletten et al. (2017)
- Almost inconsequential for fiscal policy transmission mechanism (comparison to NK model by Woodford (2011))

Model: households

$$V(b,z) = \max_{c,v,b'} \left\{ \tilde{u}(c,v) + \beta \mathbb{E}_{z'|z} V\left(b',z'\right) \right\} \tag{1}$$

subject to:

$$c + T(z) + \frac{b'}{1+i} = \frac{b}{\Pi} + z \cdot f$$

$$c = q \cdot v$$

$$b' \geq -\xi$$

Model: government

- Fiscal authority:

$$\int_{B\times Z} T(z)\, d\mu(b,z) + \frac{\bar{B}'}{1+i} = \frac{\bar{B}}{\Pi} + G \tag{2}$$

$$G = q \cdot v_G \tag{3}$$

- Central bank:

$$i = \bar{i} + \phi_Y \cdot \left(\frac{Y-\bar{Y}}{\bar{Y}}\right) + \phi_\Pi \cdot \left(\Pi - \bar{\Pi}\right)$$

Matching and price-setting

- Matching technology (CRS):

$$M\left(\int_{B\times Z} v\left(b,z\right)d\mu\left(b,z\right)+v_G,\int_{B\times Z} zd\mu\left(b,z\right)\right)$$

- Product market tightness

$$x\equiv\frac{\int_{B\times Z} v\left(b,z\right)d\mu\left(b,z\right)+v_G}{\int_{B\times Z} zd\mu\left(b,z\right)} \tag{4}$$

- Price-setting mechanism:

$$\Pi=\Pi(x),\ \Pi'\left(x\right)>0. \tag{5}$$

Consistency conditions and market clearing

- Matching probabilities:

$$f(x) = \frac{M\left(\int_{B\times Z} v(b,z)\, d\mu(b,z) + v_G, \int_{B\times Z} z d\mu(b,z)\right)}{\int_{B\times Z} z d\mu(b,z)} = M(x,1) \tag{6}$$

$$q(x) = \frac{M\left(\int_{B\times Z} v(b,z)\, d\mu(b,z) + v_G, \int_{B\times Z} z d\mu(b,z)\right)}{\int_{B\times Z} v(b,z)\, d\mu(b,z) + v_G} = M\left(1, \frac{1}{x}\right) \tag{7}$$

- Asset market clearing:

$$\bar{B}' = \int_{B\times Z} b'(b,z)\, d\mu(b,z) \tag{8}$$

- Product market clearing:

$$\int_{B\times Z} c(b,z)\, d\mu(b,z) + G = \underbrace{f(x) \cdot \int_{B\times Z} z d\mu(b,z)}_{\equiv Y(x)} \tag{9}$$

Law of motion for the distribution of agents

- Distribution of agents evolves according to:

$$\mu'\left(\mathscr{B}', z'\right) = \int_{\{b: b'(b,z) \in \mathscr{B}'\} \times Z} \mathbb{P}(z'|z) d\mu(b,z) \tag{10}$$

- Operator Γ is defined as:

$$\mu' = \Gamma(\mu). \tag{11}$$

- Standardization:

$$\int_{B \times Z} z d\mu(b,z) = 1 \tag{12}$$

Stationary equilibrium

Definition

A stationary equilibrium is: positive numbers x, q, f, i, value function V, policy functions c, v, b', distribution μ such that given $\bar{B}$, G, v_G, Π and T:

1. Given f, q, i, Π and T function V solves household's maximization problem 1 and c, v and b' are associated policy functions.

2. Given $\bar{B}$, G, Π, v_G, q and i equation 3 and government budget constraint 2 hold.

3. Consistency conditions 4, 6, 7, price-setting relationship 5 and resource constraints 8, 9 are satisfied.

4. Measure μ is a fixed point of operator Γ defined by 10 and 11.

Expressing GE effects as functions of x and G

- Aggregate output:

$$Y(x) = f(x) \cdot \underbrace{\int_{B \times Z} z d\mu(b, z)}_{=1} = f(x)$$

- Interest rate:

$$i(x) = \bar{i} + \phi_Y \cdot \left(\frac{Y(x) - \bar{Y}}{\bar{Y}} \right) + \phi_\Pi \cdot \left(\Pi(x) - \bar{\Pi} \right)$$

- Search effort:

$$v = \frac{c}{q(x)} \Rightarrow u(c, x) \equiv \tilde{u}\left(c, \frac{c}{q(x)} \right)$$

- Assumption:

$$u_{cx} = 0$$

Expressing GE effects as functions of x and G

- I concentrate on an unexpected fiscal shock G_t that arrives at time t
- Fiscal rule (as the Ricardian equivalence fails):

$$\Lambda : \; G_t \to \left[\left\{ G_s\left(G_t\right) \right\}_{s \geq t}, \left\{ \bar{B}_{s+1}\left(G_t\right) \right\}_{s \geq t} \right]$$

- Share of debt-financed public expenditures in period t:

$$\lambda \equiv \frac{d\bar{B}_{t+1}}{dG_t}$$

- Decomposing the individual tax burden:

$$T\left(z\right) \equiv \tau\left(z\right) \cdot \Theta, \; where \; \int_{B \times Z} \tau\left(z\right) d\mu\left(b, z\right) = 1$$

- Fiscal rule Λ and prices pin down the budget income from taxes for $s \geq t$:

$$\Theta\left(x_s, G_t\right) = \frac{1}{\Pi\left(x_s\right)} \cdot \bar{B}_s\left(G_t\right) - \frac{1}{1 + i\left(x_s\right)} \cdot \bar{B}_{s+1}\left(G_t\right) + G_s\left(G_t\right)$$

Reformulated consumer problem: GE effects depend on x and G

Time-dependent Bellman equation in period t under Λ:

$$V_t^{\Lambda}(b_t, z_t|G_t) = \max_{c_t,\, b_{t+1}} \left\{ u(c_t, x_t) + \beta \mathbb{E}_{z_{t+1}|z_t} V_{t+1}^{\Lambda}(b_{t+1}, z_{t+1}|G_t) \right\}$$

subject to:

$$c_t + \tau(z_t) \cdot \Theta(x_t, G_t) + \frac{b_{t+1}}{1 + i(x_t)} = \frac{b_t}{\Pi(x_t)} + z_t \cdot f(x_t)$$

$$b_{t+1} \geq -\xi$$

Under perfect foresight aggregate resource constraint becomes:

$$\underbrace{\int_{B \times Z} c^{\Lambda}(b_t, z_t|x_t, G_t)\, d\mu_t(b_t, z_t)}_{\equiv C^{\Lambda}(x_t, G_t)} + G_t = Y(x_t).$$

Multiplier: a preliminary formulation

Lemma

Suppose that economy is in stationary equilibrium at the beginning of period t and government follows fiscal rule Λ. Then the value of government spending multiplier in period t is:

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^{\Lambda}}{\partial G_t}}{1 - \frac{\partial C_t^{\Lambda}}{\partial x_t} \cdot \frac{1}{f'(x_t)}}$$

Some additional notation

- Aggregation of variable m over distribution of agents μ:

$$\mathbb{E}_\mu m \equiv \int_{B\times Z} m(b,z)\, d\mu(b,z)$$

- Marginal propensity to consume/save:

$$MPC \equiv \frac{dc}{dy},\ MPS \equiv \frac{1}{1+i}\cdot\frac{db'}{dy},\ \text{where } y \equiv z\cdot f(x) - \tau(z)\cdot\Theta$$

- Unhedged interest rate exposure (like in Auclert (2017)):

$$URE \equiv \frac{b}{\Pi} + z\cdot f - \tau\cdot\Theta - c$$

- Comovement of prices and output resulting from a positive demand shock:

$$\alpha \equiv \frac{\frac{d\Pi}{dx}}{\frac{dY}{dx}}$$

- Strength of central bank's reaction:

$$\Omega \equiv \phi_\Pi \cdot \alpha + \phi_Y$$

Main result: formula for the multiplier in the BHA model

Theorem

Under perfect foresight and fiscal rule Λ we have:

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^\Lambda}{\partial G_t}}{1 - \frac{\partial C_t^\Lambda}{\partial x_t} \cdot \frac{1}{f'(x_t)}}$$

where:

$$\frac{\partial C_t^\Lambda}{\partial x_t} \cdot \frac{1}{f'(x_t)} \equiv \underbrace{-\frac{\Omega}{1+i} \cdot \mathbb{E}_\mu (MPS \cdot c)}_{\text{Intertemporal substitution channel}(-)} + \underbrace{\frac{\Omega}{1+i} \cdot \mathbb{E}_\mu (MPC \cdot URE)}_{\text{Interest rate exposure channel}(-/+)}$$

$$+ \underbrace{\mathbb{E}_\mu (MPC \cdot z)}_{\text{Income channel}(+)} \underbrace{- \left(\frac{\Omega}{(1+i)^2} - \alpha \right) \cdot \bar{B} \cdot \mathbb{E}_\mu (MPC \cdot \tau)}_{\text{Debt service costs channel}(-/+)} \underbrace{- \alpha \cdot \mathbb{E}_\mu (MPC \cdot b)}_{\text{Fisher channel}(-/+)}$$

and:

$$\frac{\partial C_t^\Lambda}{\partial G_t} \equiv \underbrace{-\left(1 - \frac{\lambda}{1+i}\right) \cdot \mathbb{E}_\mu (MPC \cdot \tau)}_{\text{Taxation channel}(-)} + \underbrace{\beta \cdot (1+i) \cdot \mathbb{E}_\mu \left(MPS \cdot \frac{1}{u_{cc}(c)} \cdot \mathbb{V}_{bG}^\Lambda \right)}_{\text{Expectations channel }(-/+)}$$

Some additional notation

- Change in the forward-looking consumer sentiments:

$$\mathbb{V}^{\Lambda}_{bG} \equiv \mathbb{E}_{z_{t+1}|z_t} V^{\Lambda}_{t+1,bG}\left((1+i)\cdot URE_t, z_{t+1}|G_t\right)|_{URE_t=URE,\, G_t=G,\, V^{\Lambda}_{t+1}=V}$$

Special case: identical agents and comparison to Woodford (2011)

- Comparison to the RA case highlights the role of heterogeneity
- A one-time, tax-financed shock is considered
- Several channels cancel out:

$$\underbrace{\alpha \cdot \bar{B} \cdot \mathbb{E}_\mu (MPC \cdot \tau)}_{\text{Debt service costs channel: repayment}} - \underbrace{\alpha \cdot \mathbb{E}_\mu (MPC \cdot b)}_{\text{Fisher channel}} = 0$$

$$\underbrace{\frac{\Omega}{1+i} \cdot \mathbb{E}_\mu (MPC \cdot URE)}_{\text{Interest rate exposure channel}} - \underbrace{\frac{\Omega}{(1+i)^2} \cdot \bar{B} \cdot \mathbb{E}_\mu (MPC \cdot \tau)}_{\text{Debt service costs channel: issuance}} = 0$$

$$\underbrace{\beta \cdot (1+i) \cdot \mathbb{E}_\mu \left(MPS \cdot \frac{1}{u_{cc}(c)} \cdot \mathbb{V}^{\Lambda}_{bG} \right)}_{\text{Expectations channel}} = 0$$

$$\underbrace{\mathbb{E}_\mu (MPC \cdot z)}_{\text{Income channel}} = \underbrace{\left(1 - \frac{\lambda}{1+i} \right) \cdot \mathbb{E}_\mu (MPC \cdot \tau)}_{\text{Taxation channel}}$$

Special case: identical agents and comparison to Woodford (2011)

- Government spending multiplier in the RA case:

$$\frac{dY_t}{dG_t} = \frac{1}{1 + \beta \cdot \frac{1}{\eta_u} \cdot \Omega}$$

- The corresponding expression in Woodford (standard NK model with endogenous labor supply)

$$\frac{dY_t}{dG_t} = \frac{1}{1 + F\left(\beta \cdot \frac{1}{\eta_u} \cdot \Omega\right)}$$

where $F' > 0$

- Identical determinants of $\frac{dY_t}{dG_t}$ in both environments!

Calibration

- Jappelli and Pistaferri (2014): relatively high average level of MPC in the SHIW data (equal to 0.475) can be hardly matched by the Bewley-Huggett-Aiyagari model
- Jappelli and Pistaferri (2014) suggest two solutions:
  - introduce a proportion of rule-of-thumb (hand-to-mouth, HTM) agents with $MPC = 1$
  - decrease discount factor β significantly
- Lowering β generates unrealistically high real interest rates
  - I follow Auclert (2017) and set: β_H to match real interest rate and β_L to match average MPC
- I consider both variants of the model and choose the better one
- A GHH-like utility function:

$$u(c,x) = \frac{1}{1-\sigma} \cdot \left[\left(c - \frac{\kappa}{\phi} \cdot \left(\frac{c}{q(x)} \right)^{\phi} \right)^{1-\sigma} - 1 \right]$$

Calibration: parameters set w/o simulations, both versions of the model

Parameter	Name	Value	Target/Source
f	Probability of selling output	0.763	Capacity utilization
Π	Price index	1	Standardization
ϕ_Y	Parameter of Taylor rule	0.125	Galí (2008)
ϕ_Π	Parameter of Taylor rule	1.5	Galí (2008)
$\bar{i}$	Parameter of Taylor rule	0.02	Fisher equation
α	Demand-driven comovement of Y and Π	0.51	SVAR evidence
$\bar{B}$	Real value of public debt	0.99	Debt to GDP ratio
σ	Risk aversion	1	Condition $u_{cx} = 0$
ϕ	Search effort curvature	1	Condition $u_{cx} = 0$
$\{\tau(z)\}_{z \in Z}$	Shares in total tax burden	not reported	Italian tax system
G	Government purchases	0.28	budget constraint
λ	Stimulus financing rule	$\{0, 1.02\}$	Tax/debt financed dG

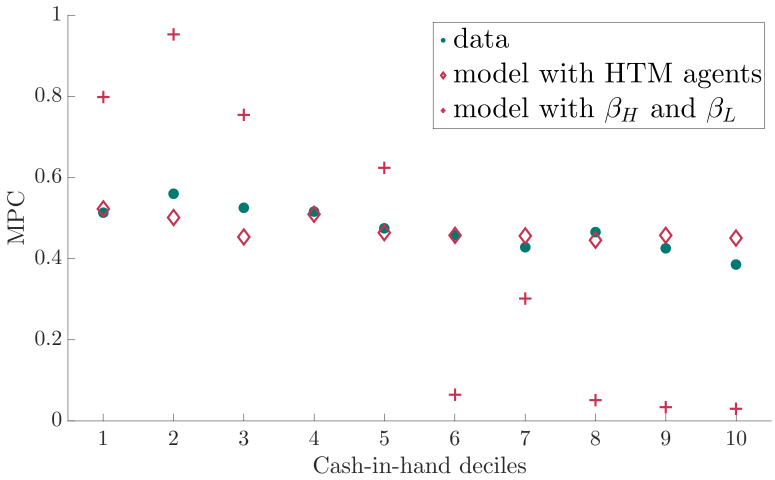
Calibration: parameters set with simulations, model with *HTM* agents

Parameter	Name	Value	Target/Source
β	Discount factor	0.9703	Real interest rate
ξ	Liquidity constraint	-2.2	Ratio of debt to assets
μ_{HTM}	Proportion of HTM agents	0.42	Average MPC
σ_T^2	Variance of transitory shocks	0.05	MPC distribution
σ_P^2	Variance of persistent shocks	0.04	MPC distribution
ρ_P	Autocorrelation of persistent component	0.958	MPC distribution

Calibration: parameters set with simulations, model with heterogeneous β

Parameter	Name	Value	Target/Source
β_H	Discount factor of patient agents	0.9736	Real interest rate
β_L	Discount factor of impatient agents	0.69	Average MPC
ξ	Liquidity constraint	-1.35	Ratio of debt to assets
σ_T^2	Variance of transitory shocks	0.05	MPC distribution
σ_P^2	Variance of persistent shocks	0.04	MPC distribution
ρ_P	Autocorrelation of persistent component	0.958	MPC distribution

Key calibration target: MPC across cash-in-hand deciles



Fiscal Multiplier: decomposition, benchmark scenario, 2 variants of the model

	Model with HTM agents		Model with β_L and β_H	
	Value	Counterfactual $\frac{dY_t}{dG_t}$	Value	Counterfactual $\frac{dY_t}{dG_t}$
Taxation channel	−0.63	1.95	−0.42	0.79
Expectations channel	−0.03	0.76	−0.08	0.50
Intertemporal substitution channel	−0.13	0.94	−0.24	0.54
Interest rate exposure channel	0.56	0.32	−0.50	0.76
Income channel	0.63	0.30	0.43	0.31
Debt service costs channel	−0.22	1.22	−0.14	0.49
Fisher channel	−0.34	2.17	0.29	0.35
MUTLIPLIER: $\frac{dY_t}{dG_t}$	0.69		0.43	

Fiscal Multiplier: decomposition, alternative scenarios, model with HTM agents

Channel\Scenario	Benchmark	More active monetary policy	Debt-financed stimulus
Taxation channel	−0.63	−0.63	0
Expectations channel	−0.03	−0.06	−0.39
Intertemporal substitution channel	−0.13	−0.26	−0.13
Interest rate exposure channel	0.56	1.12	0.56
Income channel	0.63	0.63	0.63
Debt service costs channel	−0.22	−0.75	−0.22
Fisher channel	−0.34	−0.34	−0.34
MUTLIPLIER	0.69	0.52	1.24

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The End

Thank your for your attention!