Sources of U.S. Wealth Inequality in the Past, Present, and Future

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Macroeconomics and inequality

inequality has become a major part of macroeconomics

- significant recent public interest in inequality
- significant recent academic interest in how inequality affects macroeconomic aggregates:
  - affects average MPC, and hence fiscal and monetary multipliers
  - affects the distribution of MPCs, opens up for state dependence
  - affects distribution of marginal propensities to work too
- idea here: evaluate our new workhorse of macro and inequality—the Bewley/Huggett/Aiyagari model—from the perspective of U.S. data on wealth inequality
Evolution of top wealth inequality in the U.S.

Overview: objective

- examine the workhorse model quantitatively: can it match the data?
  - its average shape
  - its evolution over time
- in particular, study the role of a number of wealth-inequality determinants: marginal tax rates, preferences, earnings, and portfolio returns—all varying across households and over time
- we tie all of the parameters to micro data; does the benchmark framework do an adequate job?
Overview: findings

- average shape:
  - yes
  - due to portfolio heterogeneity, very small (or no) role for preference heterogeneity

- dynamic evolution:
  - yes, except for very, very top
  - lower tax progressivity plays key role for cumulative
  - portfolio heterogeneity and asset prices key for swings
  - earnings variance plays little role

- predictions for future: slow but significant further widening of inequality
Quantitative incomplete-markets model

- extended Aiyagari 1994 framework (optimal growth model, idiosyncratic wage shocks, precautionary saving):
  - log labor income as sum of persistent and transitory component; adjusted at the top to match the observed Pareto tail in labor income
  - transitory component incorporates zero earnings state
  - heterogeneous returns: reduced-form portfolio choice, returns increasing in wealth and have i.i.d. idiosyncratic component
  - stochastic discount factor follows AR(1) process (Krusell-Smith 1998 extended)
  - progressive taxation: use data on federal effective tax rates for 11 income brackets (Piketty & Saez 2007)
  - parsimonious modeling of social safety net: 60% of tax revenues rebated as lump-sum transfers

- time-varying taxes, labor income process, and excess returns
- finding: saving rates (key consumer choice) very robust and unresponsive to all drivers
Return heterogeneity

- total return given asset holdings $a_t$ is
  $$r_t + r^X_t(a_t) + \sigma^X(a_t) \eta_t$$

- $r_t$ is endogenous
- $r^X_t(\cdot)$ and $\sigma^X(\cdot)$ are exogenous excess return schedules (mean and st.dev.), taken from the data
- $\eta_t$ is an i.i.d. standard normal shock
- rationalize as reduced form of portfolio choice model
The consumer’s problem

\[ V_t(x_t, p_t, \beta_t) = \max_{a_{t+1} \geq a} \{ u(x_t - a_{t+1}) + \beta_t \mathbb{E} [V_{t+1}(x_{t+1}, p_{t+1}, \beta_{t+1})|p_t, \beta_t] \} \]

subject to:

\[ x_{t+1} = a_{t+1} + y_{t+1}^{\text{ord}} - \tau_{t+1}^{\text{ord}}(y_{t+1}^{\text{ord}}) + (1 - \tau_{t+1}^{\text{cg}})y_{t+1}^{\text{cg}} + T_{t+1} \]

\[ y_{t+1}^{\text{ord}} = (r_{t+1} + r_{t+1}^{X}(a_{t+1}))a_{t+1} + w_{t+1}l_{t+1}(p_{t+1}, \nu_{t+1}) \]

\[ y_{t+1}^{\text{cg}} = \sigma^{X}(a_{t+1})\eta_{t+1}a_{t+1} \]

\( x_t \) cash on hand

\( l_{t+1}(\cdot, \cdot) \) efficiency units of labor, moves over time

— \( p_t \) persistent component of earnings process

— \( \nu_{t+1} \) transitory earnings shock

\( \tau_{t+1}^{\text{ord}}(\cdot) \) progressive tax on ordinary income, moves over time

\( \tau_{t}^{\text{cg}} \) flat capital gains tax, moves over time

\( T_t \) lump-sum transfer
Whence wealth inequality?

- a dynasty model with complete markets, identical (standard) preferences and returns: generates no long-run wealth inequality beyond initial conditions $\Rightarrow$ inadequate model of wealth inequality
- incomplete markets added: has predictions, i.e., generates unique distribution in steady state
- Aiyagari (1994) delivers far too little wealth inequality: Gini of wealth becomes that of earnings (in data: $\gg$)
- the literature has struggled with this (no clear consensus)
  - finite lives/OG?
  - preference heterogeneity
  - returns increase with wealth, entrepreneurs
  - different earnings processes
- here:
  - no “tricks”: just feed in micro observations, works well
  - portfolio heterogeneity important but next step is to explain it!
Nontrivial mechanisms at top of the distribution

- In the data, both earnings and wealth distribution have Pareto shapes at the top
  - Again, wealth has a fatter tail (lower Pareto coefficient)
- We calibrate earnings as in Aiyagari but add Pareto distribution at the top—calibrated to data
  - This generates Pareto in wealth but with same coefficient $\Rightarrow$ too thin a tail
- However: stochastic returns or $\beta$s generate a Pareto tail in the wealth distribution endogenously!
  - Follows from random growth theory (Kesten 1973, see also Gabaix 2009)
  - Mechanism has been employed by Benhabib, Bisin and Zhu 2015, Nirei & Aoki 2015, Piketty & Zucman 2015
Calibration strategy

1. calibrate earnings process, tax rates, return process, social safety net to observables

2. choose randomness in discount factor residually so as to replicate the wealth distribution in the initial steady state (1967)
Calibration: return process

\[ r^X_t(a_t) = \sum_{c \in C} w_c(a_t) \left( \bar{r}_{c,t} + \tilde{r}^X_c(a_t) \right) \]

\[ \left( \sigma^X(a_t) \right)^2 = \sum_{c \in C} \left( w_c(a_t) \tilde{\sigma}_c^X(a_t) \right)^2 \]

- asset classes \( C \): risk-free, public equity, private equity, housing
- \( \bar{r}_{c,t} \): aggregate return on asset class \( c \) (U.S. data), time-varying
- fixed over time, based on Swedish administrative data from Bach, Calvet, Sodini (2016):
  - \( w_c(\cdot) \): portfolio weights
  - \( \tilde{r}_c^X(\cdot) \): within asset class return heterogeneity
  - \( \tilde{\sigma}_c^X(\cdot) \): asset \( c \) idiosyncratic return standard deviation
Excess return schedule

Excess Return Schedule

- Mean excess return
- Standard deviation
- St. dev. (priv.equ. re-scaled)
Calibration: stochastic-$\beta$ and other

Single-$\beta$ model:
- no $\beta$-heterogeneity
- returns as measured in the data

Stochastic-$\beta$ model (benchmark):
- $\beta_t$ follows AR(1) process with: $\mu = 0.94$, $\rho = 0.992$, $\sigma = 0.0006$
- in cross-section, standard deviation $= 0.005$; over 50 years, mean reversion is $1/3$
- re-scale standard deviation of private equity return by 0.52

Other:
- CRRA $= 1.5$
- zero earnings with prob 0.075
- borrowing constraint at level of annual transfer
- Cobb-Douglas with capital’s share $= 0.36$; $\delta = 0.048$
### Results, I: steady state (1967)

<table>
<thead>
<tr>
<th></th>
<th>Top 10%</th>
<th>Top 1%</th>
<th>Top 0.1%</th>
<th>Top 0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data*</td>
<td>70.8%</td>
<td>27.8%</td>
<td>9.4%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Single-(\beta) Model</td>
<td>66.6%</td>
<td>23.7%</td>
<td>11.2%</td>
<td>7.2%</td>
</tr>
<tr>
<td>Benchmark Model</td>
<td>73.8%</td>
<td>27.4%</td>
<td>8.4%</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Bottom 50%</th>
<th>Fraction (a &lt; 0)</th>
</tr>
</thead>
<tbody>
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<td>Data*</td>
<td>4.0%</td>
<td>8.0%</td>
</tr>
<tr>
<td>Single-(\beta) Model</td>
<td>3.5%</td>
<td>7.3%</td>
</tr>
<tr>
<td>Benchmark Model</td>
<td>3.0%</td>
<td>6.6%</td>
</tr>
</tbody>
</table>

(*top wealth shares (capitalization): Saez & Zucman 2016; bottom 50% share (SCF): Kennickell 2011)

- model also matches wealth distribution well on its entire domain
## Contribution of different channels

<table>
<thead>
<tr>
<th></th>
<th>top 10%</th>
<th>top 1%</th>
<th>top 0.1%</th>
<th>top 0.01%</th>
<th>Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$-heterogeneity</td>
<td>8.8%</td>
<td>7.7%</td>
<td>3.8%</td>
<td>2.0%</td>
<td>0.050</td>
</tr>
<tr>
<td>earnings heterogeneity</td>
<td>-27.5%</td>
<td>-17.8%</td>
<td>-9.5%</td>
<td>-6.4%</td>
<td>-0.173</td>
</tr>
<tr>
<td>persistent</td>
<td>-5.0%</td>
<td>-7.5%</td>
<td>-4.2%</td>
<td>-2.9%</td>
<td>0.009</td>
</tr>
<tr>
<td>transitory</td>
<td>-11.6%</td>
<td>-4.3%</td>
<td>-1.7%</td>
<td>-0.9%</td>
<td>-0.109</td>
</tr>
<tr>
<td>tax progressivity</td>
<td>-21.3%</td>
<td>-61.8%</td>
<td>-71.2%</td>
<td>-67.1%</td>
<td>-0.148</td>
</tr>
<tr>
<td>return heterogeneity</td>
<td>29.5%</td>
<td>18.4%</td>
<td>6.6%</td>
<td>2.8%</td>
<td>0.192</td>
</tr>
<tr>
<td>mean differences</td>
<td>25.8%</td>
<td>16.7%</td>
<td>6.0%</td>
<td>2.6%</td>
<td>0.174</td>
</tr>
<tr>
<td>return risk</td>
<td>0.7%</td>
<td>2.2%</td>
<td>3.3%</td>
<td>2.5%</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Interpretation: e.g. when removing $\beta$-heterogeneity from the benchmark model, the top 10% share decreases from 73.8% to 65.0% (in general equilibrium). Thus, $\beta$-heterogeneity contributes +8.8 percentage points to the top 10% wealth share.
Observed change 1: decrease in tax progressivity

- federal effective tax rates (Piketty & Saez 2007): income, payroll, corporate and estate taxes
Observed change 2: increase in labor income risk

Observed change 3: increase in top labor income shares

- adjust standard AR(1) in idiosyncratic productivity by imposing a Pareto tail for the top 10% earners: calibrated tail coefficient decreases from 2.8 to 1.9 (Piketty & Saez 2003, updated series in 2011)
Observed change 4: valuation effects

Aggregate Excess Returns (10Y moving averages)

- housing
- public equity
- private equity
Results, II: historical evolution

- Top 10% wealth share
- Top 1% wealth share
- Top 0.1% wealth share
- Top 0.01% wealth share
Summary of transitional dynamics

- model captures the salient features of the evolution of the U.S. wealth distribution
- assumptions that we found are not critical:
  - perfect foresight (details)
  - robust to CES production function with elasticity $> 1$ (details)
- shortcomings:
  - explosion of wealth concentration at the extreme top (0.01%) as measured by Saez & Zucman 2016 not explained well
Main channels

- what fraction of the cumulative increase in the top wealth shares do the four channels account for? (graphs)

<table>
<thead>
<tr>
<th></th>
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<th>Top 1%</th>
<th>Top 0.1%</th>
<th>Top 0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxes</td>
<td>1.57</td>
<td>1.15</td>
<td>0.72</td>
<td>0.36</td>
</tr>
<tr>
<td>Top Earnings Inequality</td>
<td>0.44</td>
<td>0.14</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td>Earnings Risk</td>
<td>-0.84</td>
<td>-0.21</td>
<td>-0.09</td>
<td>-0.05</td>
</tr>
<tr>
<td>Return Premia</td>
<td>-0.58</td>
<td>-0.28</td>
<td>-0.13</td>
<td>-0.08</td>
</tr>
<tr>
<td>Combined</td>
<td>0.71</td>
<td>0.66</td>
<td>0.54</td>
<td>0.29</td>
</tr>
</tbody>
</table>

- larger earnings risk can induce higher precautionary savings
  - especially among the less wealthy, reducing tendency of heterogeneous discount factors to drive apart the wealth distribution (Becker 1980)
  - interest rate falls, thereby increasing the Pareto tail coefficient (i.e., decreasing top wealth inequality)
Decomposition of transitional dynamics

**top 10% wealth share**

- Blue line: full model
- Red dashed line: taxes
- Yellow line: earnings risk
- Purple line: top earnings
- Green line: excess returns

**top 1% wealth share**

**top 0.1% wealth share**

**top 0.01% wealth share**
Capital in the 21st century?

- Top 10% wealth share
- Top 1% wealth share
- Top 0.1% wealth share
- Top 0.01% wealth share

Model vs data (SZ)
Capital in the 24th century???
Conclusion: (surprising) success, challenging new questions

- main findings, steady state:
  - need for $\beta$ heterogeneity almost gone in new calibration
  - key reason: (realistic) return heterogeneity

- main findings, historical evolution:
  - declining tax progressivity most powerful force for generating increases in wealth inequality
  - asset-price movements account very well for short-run dynamics (U-shape in wealth inequality)
  - speed of changes at the very top hard to match (if you believe in Saez & Zucman data)

- remaining questions from perspective of this paper:
  - missing rise at top: increased idiosyncratic return volatility, shift toward private equity?
  - why are portfolios heterogeneous (both across and within wealth levels), what drives returns?

- implications for macro: concomitant evolution of mpcs.
dziękuję bardzo
Trends in wealth inequality: recent literature

- **models of Pareto tails**: Piketty and Zucman 2015, Benhabib, Bisin, and Luo 2015, Nirei and Aoki 2015.
Equilibrium: capital market clearing

need to find two equilibrium objects \((K_t, r_t)\) for market clearing:

1. aggregate capital (as usual)

   \[
   K_t = \int a_t d\Gamma(a_t)
   \]

2. aggregate capital income (redundant if \(r_t^X(\cdot) = 0\))

   \[
   (MPK(K_t) - \delta)K_t = \int \left( r_t + r_t^X(a_t) \right) a_t d\Gamma(a_t)
   \]
Multiplicative shocks and Pareto tails

- linear savings rules as wealth grows large (Bewley 1977; Carroll 2012; Benhabib et al. 2015): \( \lim_{x \to \infty} s(x, \beta) = \bar{s}_\beta x \).
- asset accumulation for large \( x \):

\[
\begin{align*}
a_{t+1} &= s(x_t, \beta) \\
    &= s(a_t + y_t - T(y_t), \beta) \\
    &\approx \bar{s}_\beta a_t (1 + (1 - \tau_{\text{max}})r) + \bar{s}_\beta (1 - \tau_{\text{max}})e_t \\
    &\equiv \hat{s} a_t + z_t,
\end{align*}
\]

where \( e_t \) is earnings.

- \( \beta \) and/or \( r \) random \( \to \hat{s} \) is random.

- with reflecting barrier (borrowing constraint) and/or random earnings, the invariant distribution for wealth has a Pareto tail with coefficient \( \zeta \) solving: \( \mathbb{E}[\hat{s}^\zeta] = 1 \).
Stochastic-\(\beta\) yields stochastic, linear savings decisions

![Graph showing marginal propensity to save vs log(k)]

- high beta, high earnings
- high beta, low earnings
- low beta, high earnings
- low beta, low earnings

Marginal propensity to save:
- 0.85
- 0.9
- 0.95
- 1
Gives rise to a Pareto tail in the wealth distribution
## Cumulative change in top wealth shares

<table>
<thead>
<tr>
<th></th>
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<th>Top 1%</th>
<th>Top 0.1%</th>
<th>Top 0.01%</th>
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</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967</td>
<td>70.8</td>
<td>27.8</td>
<td>9.4</td>
<td>3.1</td>
</tr>
<tr>
<td>2012</td>
<td>77.2</td>
<td>41.8</td>
<td>22.0</td>
<td>11.2</td>
</tr>
<tr>
<td>Relative Δ</td>
<td>9.0%</td>
<td>50.4%</td>
<td>134.0%</td>
<td>261.3%</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967</td>
<td>73.8</td>
<td>27.4</td>
<td>8.4</td>
<td>3.2</td>
</tr>
<tr>
<td>2012</td>
<td>78.5</td>
<td>36.5</td>
<td>14.4</td>
<td>5.6</td>
</tr>
<tr>
<td>Relative Δ</td>
<td>6.4%</td>
<td>33.2%</td>
<td>72.2%</td>
<td>75.4%</td>
</tr>
<tr>
<td><strong>Fraction explained</strong></td>
<td>70.8%</td>
<td>65.9%</td>
<td>53.8%</td>
<td>28.9%</td>
</tr>
</tbody>
</table>

Wealth shares in %.

... when compared to SCF data

<table>
<thead>
<tr>
<th></th>
<th>Top 10%</th>
<th>Top 1%</th>
<th>Top 0.1%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>67.1</td>
<td>30.1</td>
<td>10.8</td>
</tr>
<tr>
<td>2013</td>
<td>75.3</td>
<td>35.8</td>
<td>13.5</td>
</tr>
<tr>
<td>Relative Δ</td>
<td>12.2%</td>
<td>19.1%</td>
<td>25.4%</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>69.3</td>
<td>24.5</td>
<td>7.4</td>
</tr>
<tr>
<td>2013</td>
<td>78.9</td>
<td>37.1</td>
<td>14.8</td>
</tr>
<tr>
<td>Relative Δ</td>
<td>13.7%</td>
<td>51.5%</td>
<td>100.3%</td>
</tr>
<tr>
<td><strong>Fraction Explained</strong></td>
<td>112.5%</td>
<td>270.1%</td>
<td>394.5%</td>
</tr>
</tbody>
</table>

Wealth shares in %.
Data: SCF, as reported by Saez & Zucman 2016.
Other parts of the distribution

<table>
<thead>
<tr>
<th></th>
<th>Bottom 50%</th>
<th>personal wealth ( \frac{Y}{Y} )</th>
<th>nat’l wealth ( \frac{Y}{Y} )</th>
<th>( K ) ( \frac{Y}{Y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967</td>
<td>4.0%</td>
<td>3.6</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>1.1%</td>
<td>4.1</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>Relative ( \Delta )</td>
<td>−73%</td>
<td>14%</td>
<td>14%</td>
<td></td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967</td>
<td>3.0%</td>
<td></td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>1.4%</td>
<td></td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td>Relative ( \Delta )</td>
<td>−53%</td>
<td></td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td><strong>Fraction explained</strong></td>
<td>74%</td>
<td></td>
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Bottom 50% Data: SCF, as reported by Kennickell 2011.
Excess return schedule details

Aggregate Excess Returns in 1967 steady state (over risk-free rate):

- public equity 0.067
- private equity 0.129
- housing 0.037 (incl. imputed rent)

<table>
<thead>
<tr>
<th></th>
<th>P0-P40</th>
<th>P40-P50</th>
<th>P50-P60</th>
<th>P60-P70</th>
<th>P70-P80</th>
<th>P80-P90</th>
<th>P90-P95</th>
<th>P95-P97.5</th>
<th>P97.5-P99</th>
<th>P99-P99.5</th>
<th>P99.5-P99.9</th>
<th>P99.9-P99.99</th>
<th>Top 0.01%</th>
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<tr>
<td>fixed portfolio weights</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cash</td>
<td>0.722</td>
<td>0.412</td>
<td>0.248</td>
<td>0.182</td>
<td>0.156</td>
<td>0.134</td>
<td>0.115</td>
<td>0.102</td>
<td>0.090</td>
<td>0.079</td>
<td>0.071</td>
<td>0.051</td>
<td>0.029</td>
</tr>
<tr>
<td>housing</td>
<td>0.162</td>
<td>0.394</td>
<td>0.580</td>
<td>0.662</td>
<td>0.678</td>
<td>0.674</td>
<td>0.658</td>
<td>0.626</td>
<td>0.572</td>
<td>0.482</td>
<td>0.363</td>
<td>0.253</td>
<td>0.155</td>
</tr>
<tr>
<td>public equity</td>
<td>0.113</td>
<td>0.189</td>
<td>0.165</td>
<td>0.147</td>
<td>0.153</td>
<td>0.170</td>
<td>0.189</td>
<td>0.207</td>
<td>0.219</td>
<td>0.232</td>
<td>0.230</td>
<td>0.185</td>
<td>0.179</td>
</tr>
<tr>
<td>private equity</td>
<td>0.002</td>
<td>0.005</td>
<td>0.007</td>
<td>0.009</td>
<td>0.013</td>
<td>0.021</td>
<td>0.038</td>
<td>0.065</td>
<td>0.118</td>
<td>0.207</td>
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excess return schedule in 1967

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<th>P60-P70</th>
<th>P70-P80</th>
<th>P80-P90</th>
<th>P90-P95</th>
<th>P95-P97.5</th>
<th>P97.5-P99</th>
<th>P99-P99.5</th>
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<td>0.041</td>
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<td>st. dev. (priv.equ. re-scaled)</td>
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Housing details

- financial return on housing as sum of capital gains term and rental income
- we set capital gains term to zero in steady states (in long run 0-0.5% real price growth)
- over transition, use growth in aggregate house price index (Case-Shiller)
- rental income set to 5.33% (average for U.S. from Jorda, Knoll, Kuvshinov, Schularick, Tayler ”Rate of Return on Everything”)

Public and private equity

Public Equity
▶ U.S. stock market return

Private Equity
▶ Kartashova (AER, 2014) documents private equity premium over stock market
▶ aggregate time series for U.S. starting in 1960
### Capital in the 21st century?

<table>
<thead>
<tr>
<th>Year</th>
<th>Top 10%</th>
<th>Top 1%</th>
<th>Top 0.1%</th>
<th>Top 0.01%</th>
<th>Bottom 50%</th>
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<tr>
<td>1967</td>
<td>73.8</td>
<td>27.4</td>
<td>8.4</td>
<td>3.2</td>
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<tr>
<td>2017</td>
<td>80.0</td>
<td>39.2</td>
<td>16.2</td>
<td>6.5</td>
<td>1.2</td>
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<tr>
<td>2100</td>
<td>89.1</td>
<td>61.6</td>
<td>35.2</td>
<td>17.0</td>
<td>0.3</td>
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</table>

Model predictions for 21st century. Wealth shares in %.

- long-run effects of decrease in tax progressivity
Perfect foresight vs. myopic transition; CES

**capital - net output ratio**

- **benchmark**
- **CES**
- **myopic**

**bottom 50% share**
Tax changes: changes in savings behavior vs. resources

- **Top 10% Wealth Share**
  - Full equilibrium
  - New s(.), fix tax
  - Fix s(.), new tax

- **Top 1% Wealth Share**

- **Top 0.1% Wealth Share**

- **Top 0.01% Wealth Share**
Only changes in earnings risk I
Only changes in earnings risk II

- capital - net output ratio
- bottom 50% share

- model (capital)
- data (national wealth)
- data (private wealth)
- model
- data (SCF)
Only changes in top earnings shares I
Only changes in top earnings shares II
Only changes in taxes I

**Top 10% Wealth Share**
- Model
- Data (SZ)
- Data (SCF)

**Top 1% Wealth Share**

**Top 0.1% Wealth Share**

**Top 0.01% Wealth Share**
Only changes in return premia I

**top 10% wealth share**

![Graph showing the top 10% wealth share from 1970 to 2010 with different data sources and models.]

**top 1% wealth share**

![Graph showing the top 1% wealth share from 1970 to 2010 with different data sources and models.]

**top 0.1% wealth share**

![Graph showing the top 0.1% wealth share from 1970 to 2010 with different data sources and models.]

**top 0.01% wealth share**

![Graph showing the top 0.01% wealth share from 1970 to 2010 with different data sources and models.]
Only changes in return premia II

**capital - net output ratio**

- model (capital)
- data (national wealth)
- data (private wealth)

**bottom 50% share**

- model
- data (SCF)
Dynamics in single-$\beta$ model I

**top 10% wealth share**

- **model**
- **data (SZ)**
- **data (SCF)**

**top 1% wealth share**

**top 0.1% wealth share**

**top 0.01% wealth share**
Dynamics in single-\(\beta\) model II

**capital - net output ratio**

- model (capital)
- data (national wealth)
- data (private wealth)

**bottom 50% share**

- model
- data (SCF)
Inflation I

**top 10% wealth share**

- benchmark
- benchmark + inflation

**top 1% wealth share**

**top 0.1% wealth share**

**top 0.01% wealth share**