The Hardware–Software Model: A New Conceptual Framework of Production, R&D, and Growth with AI*

Jakub Growiec

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SGH Warsaw School of Economics, Poland
Department of Quantitative Economics
E-mail: jakub.growiec@sgh.waw.pl.

Abstract

The article proposes a new conceptual framework for capturing production, R&D, and economic growth in aggregative models which extend their horizon into the digital era. Two key factors of production are considered: *hardware*, including physical labor, traditional physical capital and programmable hardware, and *software*, encompassing human cognitive work, pre-programmed software, and artificial intelligence (AI). Hardware and software are complementary in production whereas their constituent components are mutually substitutable. The framework generalizes, among others, the standard model of production with capital and labor, models with capital–skill complementarity and skill-biased technical change, and unified growth theories embracing also the pre-industrial period. It offers a clear conceptual distinction between mechanization and automation as well as between robotization and the development of AI. It delivers sharp, economically intuitive predictions for long-run growth, the evolution of factor shares, and the direction of technical change.

Keywords: production function, R&D equation, technological progress, complementarity, automation, artificial intelligence.

JEL codes: O30, O40, O41.

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1 Introduction

This paper addresses an important challenge to economic growth theory: to adapt the workhorse models to the realities of the incipient digital era, characterized by gradual automation, explosion of data communication and collection, and rapid advances in AI. But for a few forerunners,[1] growth models developed thus far are either rooted entirely in the industrial era, or focus on even earlier eras. Unified growth theory, in particular, pays specific attention to the period of Industrial Revolution but does not speak to the ongoing Digital Revolution which is – arguably – transforming the world before our eyes in a comparably profound way. Existing models tend to be ill-suited to modeling the supply side of the digital-era economy featuring computer and robot hardware, pre-programmed software and AI algorithms, primarily because they are based on the classical capital–labor dichotomy which is incompatible with a world where information processing, communication and storage, as well as decision making, is increasingly detached from human minds.

The contribution of this conceptual study is to lay out the rudiments of a macroeconomic framework for modeling production, R&D and growth across the human history, including and specially focusing on the digital era. The proposed formalization – the *hardware–software model* – is designed so that it nests the following conventional models as special cases:

(i) a standard treatment of the industrial economy respecting Kaldor's facts,

(ii) a model of capital–skill complementarity and skill-biased technical change,

(iii) a unified growth theory addressing the period of Industrial Revolution,

(iv) a theory of inception and further development of the digital era.

To get there, however, I take a big step back and re-evaluate the key inputs to aggregate production and R&D.

The key premise of the proposed new framework lies with the postulate that valuable output can only be generated through purposefully initiated physical actions. Thus, generating output (either in the material or in the informational form) requires both some physical *action* and some *code*, a set of instructions describing the action. In consequence, the general form of any production function should feature some physical *hardware* X, able to perform the action, and some disembodied *software* S, providing the relevant information. This simple observation has profound consequences. It underscores, for example, that physical capital and human physical labor should be modeled as substitutable inputs, contributing to the *hardware* factor: they are the means by which we perform physical action. Analogously,

[1] For example, Acemoglu and Restrepo (2018); Benzell, Kotlikoff, LaGarda, and Sachs (2015); Berg, Buffie, and Zanna (2018).

human cognitive work, pre-programmed software and AI should also be viewed as substitutes, making up the *software* factor: they are the source of instructions for the performed action. In turn, hardware and software are clearly complementary and indispensable in the process. (By *complementarity*, in practice I will mean their gross complementarity in the sense of elasticity of substitution being below unity.) The model also formalizes the observation that programmable hardware, similarly to the human body, has double duty: as means of performing physical actions and as a container for software – stored information and working algorithms.

Key predictions of the hardware–software model in terms of long-run dynamics and the evolution of factor shares are as follows. First, in the digital era, as production gets increasingly automated, software becomes proportional to hardware because it can be virtually costlessly copied and thus can easily scale up to the level of available programmable (computer, robot, etc.) hardware. Under constant returns to scale and in the absence of further technological revolutions[2], this generates long-run endogenous growth by hardware accumulation alone; in the limit, all production is automated. Second, complementarity and substitutability shape the dynamics of factor shares. The Industrial Revolution had vastly different implications for factor shares than the ongoing Digital Revolution because the former featured replacement of humans with machines in the hardware factor (brawn) whereas the latter pertains to the software factor (brains). The Industrial Revolution (or the process of mechanization) raised demand for human skilled labor; the Digital Revolution (or the process of automation) replaces human skilled labor and raises demand for complementary computer hardware, which eventually becomes the long-run growth bottleneck. Third, all technical change is naturally *software-augmenting.*

This paper is related to a few strands of literature. First, the literature on production function specification and estimation, in particular with capital–skill complementarity, unbalanced growth, as well as investment-specific and skill-biased technical change.[3] Second, the literature preoccupied with accounting for the accumulation of information and communication technologies (ICT) and their broad growth-enhancing role as a general purpose technology.[4] Third, studies focusing on automation and its impacts on productivity, employment, wages and factor shares.[5]

[2]Given the observed pace of growth in computing power and AI capabilities, further technological revolutions are actually quite likely.

[3]Including among others Gordon (1990); Jorgenson (1995); Greenwood, Hercowitz, and Krusell (1997); Hercowitz (1998); Kumar and Russell (2002); Koop, Osiewalski, and Steel (1999, 2000); Krusell, Ohanian, Ríos-Rull, and Violante (2000); Henderson and Russell (2005); Caselli and Coleman (2006); Klump, McAdam, and Willman (2007, 2012); Growiec (2012); Mućk (2017); McAdam and Willman (2018).

[4]Including among others Bresnahan and Trajtenberg (1995); Timmer and van Ark (2005); Jorgenson (2005); Brynjolfsson and McAfee (2014); Gordon (2016); Brynjolfsson, Rock, and Syverson (2017); Nordhaus (2017); Aum, Lee, and Shin (2018).

[5]Including among others Acemoglu and Autor (2011); Autor and Dorn (2013); Graetz and

Fourth, the nascent literature on macroeconomic implications of development of AI and autonomous robots.[6] Last but not least, the voluminous literature on R&D based endogenous growth.[7]

The remainder of the paper is structured as follows. Section 2 defines the factors of production of the hardware–software model. Section 3 discusses the conceptual underpinnings of the aggregate production function. Section 4 tackles the R&D equation. Section 5 discusses the special case of CES functions. Section 6 concludes with a general discussion of the framework, spelling out the key concepts and misconceptions of the digital era, and speculating about the future.

2 The Hardware–Software Model

In any conceivable technological process, valuable output is generated through some physical action. It is a local reduction of entropy, and so it typically does not occur by chance but is purposefully initiated. In other words, producing valuable output requires both some physical *action* and some *code*, a set of instructions describing the action. Hence, I shall posit that the postulated general production function (for whatever the output is) should feature some physical *hardware* X, able to perform the action, and some disembodied *software* S, providing information on what should be done and how. This naturally leads to a general form:

$$Output = \mathcal{F}(X, S), \tag{1}$$

where $\mathcal{F}$ is increasing in both factors and is specified such that hardware X and software S are always mutually complementary. The degree of their complementarity is an open question; the plausible range spans from perfect complementarity (Leontief form) if just one method of producing output exists, to imperfect complementarity if firms are allowed to choose their preferred technology from a technology menu (Jones, 2005; Growiec, 2013, 2018). Intuitively, some degree of substitutability is likely because the same outcome can be generated with more resources (larger X) but less efficient code (smaller S), or vice versa. One natural way to instantiate this assumption is to take a CES specification with an elasticity of substitution $\sigma \in (0, 1)$, cf. Klump, McAdam, and Willman (2007, 2012). The particular CES

Michaels (2015); Acemoglu and Restrepo (2018); Andrews, Criscuolo, and Gal (2016); Arntz, Gregory, and Zierahn (2016); Frey and Osborne (2017); Barkai (2017); Autor, Dorn, Katz, Patterson, and Van Reenen (2017); Jones and Kim (2017); Hemous and Olsen (2018).

[6] Including Yudkowsky (2013); Graetz and Michaels (2015); Sachs, Benzell, and LaGarda (2015); Benzell, Kotlikoff, LaGarda, and Sachs (2015); DeCanio (2016); Acemoglu and Restrepo (2018); Aghion, Jones, and Jones (2017); Berg, Buffie, and Zanna (2018).

[7] Including among others Romer (1990); Jones and Manuelli (1990); Aghion and Howitt (1992); Jones (1995); Acemoglu (2003); Ha and Howitt (2007); Madsen (2008); Bloom, Jones, Van Reenen, and Webb (2017); Kruse-Andersen (2017).

form of the $\mathcal{F}$ function is however not necessary for the results.[8]

It must be noted that such treatment abstracts from raw materials and energy which are being used up in the production process. The specification (1) works as if we assumed that natural resources and energy required for production were given for free and in infinite supply, or at least they were sufficiently cheap and abundant that they never become a bottleneck. Relaxing this simplifying assumption is left for further research.

Hardware X encompasses both physical labor performed by humans (and domesticated animals), as well as physical actions performed by machines. Hence, X encompasses both the services of physical capital K and of unskilled labor L, where the latter variable excludes any know-how or skill of the worker.

Software S, in turn, encompasses all useful instructions which stem from the available information, in particular the practical implementation of state-of-the-art technologies. Hence, it includes the skills and technological knowledge employed in human cognitive work, H, as well as pre-programmed software Ψ, which is essentially a task-specific list of instructions to be performed by the associated programmable hardware.[9] Software Ψ may in particular include artificial intelligence (AI) algorithms, defined as the software which is able to learn from data as well as potentially self-improve and self-replicate. It is implicitly assumed that there are no physical obstacles precluding pre-programmed software from performing (or providing the hardware with instructions to perform) any task available to a human (Yudkowsky, 2013; Dennett, 2017).

Within hardware, I view the agents of physical action as perfectly substitutable. This reflects the idea that whatever it is that performs a given set of actions, if the actions are precisely defined then the outcome should be the same. The same logic applies to software: regardless of whether a set of instructions comes from a human brain or a mechanical information processing unit, if the actual information content of instructions is the same, then the outcome should be the same, too.[10]

[8]For example, Growiec and Mućk (2018) propose a more flexible framework that also allows the modeler to control whether the factors are gross substitutes or gross complements.

[9]Contemporary programmable hardware consists typically of computers, robots, and other devices embodying digital chips. In principle, it does not have be silicon-based, though. In fact the first pieces of non-biological programmable hardware were mechanical devices such as the Jacquard loom using punchcards, first invented in 1804.

[10]An important caveat is that by saying this I exclude complex, multi-step tasks that have not been yet fully automated. For example, if a cognitive task consists of two necessary steps, the first of which can be performed by a computer but the latter (under current technology) only by a human, then pre-programmed software and human cognitive work will turn out complementary at the level of the whole task even though they are perfectly substitutable at the level of the two sub-tasks. This apparent complementarity disappears, however, once the whole task becomes fully automatable. A more detailed treatment of complex tasks within the hardware–software model is an important objective for further research.

This leads to the following formula:

$$Output = \mathcal{F}(X, S) = \mathcal{F}(L + K, H + \Psi). \tag{2}$$

Each of the four identified factors of production has its unique properties (Table 1).

- *Human physical labor* L is rivalrous and given in fixed supply per worker and unit of time, $L = \zeta N$ where $\zeta \in [0, \bar{\zeta}]$ denotes the supply of physical labor per worker in a unit of time, expressed in physical capital units, and N is the total number of workers.

- *Physical capital* K is rivalrous but can be unboundedly accumulated in per-capita terms. Physical capital K may be non-programmable or programmable. The share of programmable (computer, robot, etc.) hardware in total physical capital is denoted by χ (so that $\chi \in [0, 1]$).

- *Human cognitive work* H consists itself of three components, technological knowledge A, skill level h, and the number of workers N, as in $H = AhN$. Technological knowledge A, or the size of the "repository of codes" is non-rivalrous (Romer, 1990) and accumulable. Per-capita skill levels h are rivalrous and bounded above, theoretically by the optimal code for performing a given task, but in practice by a much lower number $\bar{h} > 0$ due to the human inability to rewire our brains in order to perform cognitive tasks more efficiently (Yudkowsky, 2013) as well as human mortality and decreasing returns in education (Growiec, 2010).

- *Pre-programmed software* Ψ also consists of three components, technological knowledge A, "AI skill level" ψ which captures the degree to which pre-programmed software is able to perform the tasks collected in A (and the associated efficiency), and the stock of programmable hardware χK on which the software is run, as in $\Psi = A\psi\chi K$. Technological knowledge A is the same as above.[11] The AI skill level ψ is assumed to be bounded above by the optimal code for performing a given task (e.g., perfect accuracy), though there may be in fact a much lower upper bound (Hanson and Yudkowsky, 2013). Because software can be virtually costlessly copied, it is assumed that it can scale up to the level of all available programmable hardware χK.[12]

It is important to observe that the hardware–software model envisages technological progress (growth in A) as expansion of the "repository of codes", i.e., as the development of new, better instructions allowing to produce higher output with a

[11] If in reality the sets of codes available to humans and AI are different, the discrepancy between the measures of both sets can be captured by the factor ψ relative to h.

[12] Which implies that, in its basic form, the model abstracts from economic and legal constraints on the diffusion of software, such as the protection of intellectual property rights.

Table 1: Factors of Production and R&D

Hardware X	Human physical labor	$L = \zeta N$
	Non-programmable physical capital	$(1-\chi)K$
	Programmable physical capital	χK
Software S	Human cognitive work	$H = AhN$
	Pre-programmed software†	$\Psi = A\psi\chi K$

Note: † includes AI algorithms.

given amount of hardware. These instructions can be applied to the tasks at hand both by humans, deterministic pre-programmed code, and AI. This is intuitive: technological progress may take the form of new abstract ideas, scientific theories, systematically catalogued facts, codes specifying certain actions, or blueprints of physical items; all this is *information* and not actual *objects* or *actions*, and it is precisely this informational character that makes technologies non-rivalrous (Romer, 1990). Thus all technological progress is naturally modeled as *software-augmenting*. In the hardware–software model, in contrast to the standard capital–labor model, there is no room for discussion on the direction of technical change (Acemoglu, 2009) – a property that is highly valuable from a reductionist point of view.

3 The Aggregate Production Function

3.1 Setup

The aggregate production function is a key element of any macroeconomic model, and particularly so of any long-run growth theory. Since the 1950s (Solow, 1956, 1957), it has become commonplace to take capital K and labor L as the key inputs of this function, and value added (or GDP) as its output Y. Furthermore, in the temporal dimension it has become a very frequent, if not standard, practice to assume labor-augmenting (Harrod-neutral) technical change, as in

$$Y = F(K, AL). \tag{3}$$

This specification is naturally a simplification, as any production function is (Temple, 2006); both K and L are obviously amalgamates of heterogeneous components. The question is whether this simplified form is sufficient for capturing the key macroeconomic facts. From the literature[13] it becomes more and more apparent that the standard treatment of inputs as in (3) may have been sufficient to model

[13] Such as Gordon (1990); Greenwood, Hercowitz, and Krusell (1997); Krusell, Ohanian, Ríos-Rull, and Violante (2000); Caselli and Coleman (2006); Growiec (2012); McAdam and Willman (2018).

the classic Kaldor (1961) facts but fails in capturing the new facts, which emphasize irreducible heterogeneity within the K and L factors (Jones and Romer, 2010).

The hardware–software production function proposed in this paper, following directly from equation (2), structures this heterogeneity so that it embraces both the key historical macro facts and the incipient digital production technology using AI. It also provides sharp implications for the distinction between skilled and unskilled labor, "traditional" (non-programmable) physical capital and its programmable (computer, robot, etc.) counterpart, mechanization and automation.

Using the concepts from the previous section, the aggregate production function F can be formalized as:

$$Y = F(X, S) = F(\zeta N + K, A(hN + \psi\chi K)), \tag{4}$$

where Y is aggregate value added (or GDP). The standard replication argument applies to this production function specification, implying constant returns to scale with respect to X and S.

From the laws of thermodynamics, implying in particular that performing physical action requires expediting energy, it is expected that an essential fraction of GDP must consist of material outputs, serving – at the very least – to sustain the hardware (including human bodies) and allow it to work.

On historical evidence I assume that pre-programmed software can be deployed in production processes only if the technology level A is high enough for programmable hardware to exist ($\chi > 0$). Furthermore, I posit that the skill level ψ of deterministic pre-programmed software can be at best equal to ηh, where $\eta \in (0, \bar{\eta})$ captures the fraction of human skills that have been deterministically programmed into machines, multiplied by a positive factor that ensures that human and computer cognitive work is denominated in common units ($\bar{\eta}$ denotes human-level performance). AI, in contrast, has the ability to learn from data and potentially self-improve its architecture; thus with AI it cannot be precluded that $\psi > \eta h$ and even $\psi \gg \bar{\eta} h$ (superhuman performance). In the following I assume that $\bar{\psi} > \bar{\eta}\bar{h}$, allowing for the scenario where AI would eventually achieve superhuman performance.[14] The model embraces the possibility of full automation for two reasons: first, I exclude the case where $\bar{\psi}$ is prohibitively low, and second, I assume that with sufficient computing power χK, all essential cognitive tasks are amenable to automation.[15]

Equations (2) and (4) signify that AI is viewed here just as (improved) computer software and not as a separate production input. This is because machine learning algorithms may allow drastic improvements in the applicability, efficiency,

[14] See the discussion of this assumption in Section 6.

[15] However, in a more general model with complex, multi-step tasks, human cognitive work can become essential for generating output if at least one step of at least one essential task is not automatable. Essentiality implies that there is no way around this particular step as well as no possibility of substituting out the entire task.

and versatility of software, but do not constitute a qualitative change in its function as means of providing instructions to programmable hardware. Hence, the model does not feature a separate AI revolution, and rather sees AI development as a massive boost to the Digital Revolution which already began with the early computer hardware and software. In my view, AI is to the digital era like the development of electricity and internal combustion engines was to the industrial era: a second wave of key breakthroughs, forcefully accelerating the impact of the initial revolutionary technological ideas on the economy and society (Gordon, 2016).

3.2 Economic Growth and Technological Change

Log-differentiating (4) with respect to time, I obtain the following Solow-type decomposition of economic growth:

$$g_Y = \pi_X g_X + \pi_S g_S, \tag{5}$$

where $\pi_X = \frac{\partial Y}{\partial X}\frac{X}{Y}$ is the hardware share of output, and analogously $\pi_S = \frac{\partial Y}{\partial S}\frac{S}{Y}$ is the software share. By Euler's theorem, under constant returns to scale we have $\pi_X + \pi_S = 1$.

Decomposing (4) further, I obtain:

$$g_Y = \pi_X[\pi_L g_N + \pi_K g_K] + \pi_S[\pi_H(g_h + g_N) + \pi_\Psi(g_\psi + g_\chi + g_K)] + \pi_S g_A, \tag{6}$$

where – due to the assumption of perfect substitutability of the constituent components of hardware and software – the shares are simply $\pi_L = \frac{L}{X}$, $\pi_K = \frac{K}{X}$, $\pi_H = \frac{H}{S}$ and $\pi_\Psi = \frac{\Psi}{S}$.

Equation (6) presents formally that there are multiple potential sources of output growth in the hardware–software model. Each of them may have a different asymptotic properties, though:

- Population growth g_N increases the total amounts of both human physical and cognitive work. If there is physical capital or pre-programmed software in the economy, though, this impact is less than proportional to output growth and thus, *ceteris paribus*, growth in output per worker $(g_Y - g_N)$ decreases with population growth.

- Physical capital accumulation g_K affects output growth both directly via the hardware component and indirectly via the pre-programmed software component (if $\pi_\Psi > 0$). It is subject to decreasing returns, but to a decreasing degree, and as $\pi_K \to 1$ and $\pi_\Psi \to 1$, the returns become asymptotically constant, allowing for unbounded output growth (Jones and Manuelli, 1990).

- Growth in human capital per worker g_h and growth in the AI skill level g_ψ can be decisive in the short to medium run, but their impact on growth is by definition transitory and bound to disappear as $h \to \bar{h}$ and $\psi \to \bar{\psi}$.

- Growth in the share of programmable hardware g_χ illustrates the spread of digital technologies across the economy and the potential for automation (which however also requires software). This factor can be important in the short to medium run, but should not play any role over the long run because χ is bounded between zero and one.

- Technological change g_A, understood as the increase in technological knowledge A, the size of the "repository of codes", is conceptually independent of human capital and AI skill accumulation. It adds to output growth with an elasticity equal to the software share and can be potentially unbounded.

At this point, note that while the software-augmenting character of technological change comes out as a very natural implication of the proposed conceptual framework, this regularity stands in stark contrast to the discussions in the literature on the direction of factor-augmenting technical change (e.g. Acemoglu, 2003; Jones, 2005; León-Ledesma, McAdam, and Willman, 2010). This is because the prevalent concepts of aggregate production factors such as capital and labor conflate hardware and software: technical change augments cognitive work but not physical labor, and pre-programmed software but not the hardware on which it is run. The new framework also resolves the conundrum whether technological progress is disembodied or embodied in new investment goods (e.g. Gordon, 1990; Greenwood, Hercowitz, and Krusell, 1997; Hercowitz, 1998): in itself, it is the *disembodied information* that allows for more efficient actions. Nevertheless it may require investment in the complementary hardware in order to deliver the desired effects for output.

3.3 Stages of Economic Development

Let us now trace how the hardware–software model can be used to capture the key properties of production processes across the human history.

Stage 1. Pre-industrial production. In a pre-industrial economy, output was produced primarily in farming. At that stage of development, there was no significant accumulation of productive capital K, and hence output was produced with a technology that could only use human (and animal) physical labor for performing the physical actions. There was also no pre-programmed software Ψ. Setting $K = 0$ in (4) yields the following simple form:

$$Y = F(X, S) = F(\zeta N, AhN) = N \cdot F(\zeta, Ah). \tag{7}$$

Hence, under gross complementarity of hardware and software (actions and instructions), pre-industrial output per worker was bounded above due to the scarcity of hardware.

In fact, this result holds qualitatively also when assuming positive but bounded K, so long as it remains small relative to ζN. Such a model would feature land, a vital but fixed factor of agricultural production.

Stage 2. Industrial production. In an industrial economy, output was produced in farming and industry. Following the Industrial Revolution (≈1800 CE onwards), human (and animal) physical labor was gradually replaced with machines in a process of *mechanization* of production. The physical actions were, however, dependent solely on the instructions based on human cognitive work: there was no programmable hardware and no pre-programmed software yet. This implied a rising demand for human cognitive skills, setting up an upward trend in wages (Galor, 2005) due to the relative scarcity of software. Setting $\chi = 0$ in (4) yields:

$$Y = F(X, S) = F(\zeta N + K, AhN). \tag{8}$$

The limit of full mechanization and skill satiation, $K \to \infty$ and $h \to \bar{h}$, where $\bar{h}$ is the upper limit of human capital (skill) accumulation, implies $Y = F(K, \bar{h}AN)$. Hence, under this specification we obtain – in the limit – the standard balanced growth path result with gross complementarity of inputs and purely "labor-augmenting" (though generally, software-augmenting) technical change (Uzawa, 1961; Acemoglu, 2003). K/N grows at the same rate as technological knowledge A.

Equation (8) naturally embraces the concept of capital-skill complementarity (Krusell, Ohanian, Ríos-Rull, and Violante, 2000; Caselli and Coleman, 2006; McAdam and Willman, 2018): physical capital is complementary to skilled labor H but substitutable with unskilled labor L. It can be also made consistent with the standard capital–labor specification of the production function $Y = F(K, AN)$, but only in the hypothetical case of all human work being of the cognitive type ($\zeta = 0$). Then, capital and labor would be naturally gross complements, as suggested by bulk of the recent empirical literature (Klump, McAdam, and Willman, 2007, 2012; Mućk, 2017).

Stage 3. Digital production. In a digital economy, output is increasingly produced in automated processes. Following the Digital Revolution (≈1980 CE onwards), ongoing *automation* of production is observed: human cognitive skills are gradually replaced by pre-programmed routines which scale with programmable hardware χK and not with (working) population N. Skill-biased technical change gives way to routine-biased technical change (Acemoglu and Autor, 2011; Autor and Dorn, 2013). This is where we are now.

At a later stage of the digital era, however, case-based software is gradually replaced with self-improving AI algorithms, allowing for multiple-fold increases in ψ (Berg, Buffie, and Zanna, 2018). The formula (4) holds in its generality. The limit of full automation implies

$$Y = K \cdot F(1, A\bar{\psi}\bar{\chi}), \tag{9}$$

where $\bar{\psi}$ is the upper limit of AI skill accumulation and $\bar{\chi} \in (0, 1]$ is the limiting share of programmable hardware in all physical capital as $K \to \infty$. This specification implies full automation of the production process in the limit, leaving no jobs in the production sector to be performed by humans.

Equation (9) delivers an AK-type implication: there is long-run endogenous growth is due to the accumulation of (programmable) hardware alone (Jones and Manuelli, 1990; Barro and Sala-i-Martin, 2003). This result is obtained thanks to two forces: (i) software expands proportionally with hardware, (ii) hardware and software are gross complements, and thus in the long run hardware remains the bottleneck of development. Although asymptotically constant, the pace of hardware accumulation (and thus economic growth) may be nevertheless stupefying: global volumes of computational capacity, data storage and data communication exhibit doubling times between 1.5 and 3 years since the 1980s (Hilbert and López, 2011); the costs of a standard computation are declining by 53% per year on average since 1940 (Nordhaus, 2017). What has been bringing growth down in the recent decades, however, was the large share of "traditional" (non-programmable) capital, and – crucially – lack of AI algorithms able to fully tap the available computing power. Neither of these two constraints is guaranteed to persist into the indefinite future.

Note that with Cobb-Douglas technology or unbounded hardware-augmenting technical change, the model would imply explosive growth with unbounded growth rates and a finite-time singularity.

Hypothetical stage 4. Post-digital production. Under high to full automation of production processes, programmable hardware based on silicon chips, χK, will gradually become the bottleneck of further development. This will increase the incentives to invest in R&D directed towards radical innovations holding the promise to eliminate this bottleneck. Such breakthrough technology would have to tap an entirely new source of energy, fundamentally increase energy efficiency, or otherwise massively improve unit productivity of programmable hardware. Among the probable scenarios, one could envision the arrival of disruptive nanotechnology, quantum computing, massively improved solar power, or perhaps something yet unimagined. Extrapolating past trends in information processing and data accumulation and expecting them to feed into R&D productivity (see the next section of this paper), it is conceivable that such new breakthrough technology may arrive quite soon.

Formally, such an episode of "new mechanization" (one may imagine e.g., a "nanobot revolution" or "quantum revolution") may be modelled by introducing an additional component to the hardware amalgamate, as in:

$$X = \zeta N + K + \omega M, \tag{10}$$

where M denotes the new form of hardware, and $\omega \gg 1$ captures its unit productivity relative to K. This form of hardware must be programmable, so that AI could scale with M and avoid becoming a growth bottleneck itself.

Long-run implications include gradual replacement of K-type hardware with M and a permanent acceleration in growth. In fact, this additional acceleration in hardware X accumulation may eventually lead to a new growth regime "with a doubling time measured in days, not years." (Hanson, 2000).

In a world with fully mechanized and automated production, a new form of programmable hardware M and AI that is able to scale with M, in the limit of $K/M \to 0$ the aggregate production function becomes again linear:

$$Y = F(\omega M, A\bar{\psi}M) = M \cdot F(\omega, A\bar{\psi}). \tag{11}$$

This means that despite the new breakthrough and the acceleration, hardware remains the bottleneck (i.e., key factor constraining the pace) of long-run growth.

3.4 Factor Shares

The assumption of gross complementarity of production inputs (as exemplified by CES technology with $\sigma \in (0,1)$) provides a clear-cut implication for factor shares: factor income will be disproportionately directed towards the scarce factor. In consequence, we should expect the following developments, all of which are empirically testable and intuitively explicable.

Stage 1. Pre-industrial production. In a pre-industrial economy, setting aside the (relevant) problem of increasing land scarcity because our analysis abstracts from the role of raw materials in production, we observe an increasing portion of value added being directed to "hardware" (physical labor) at the expense of "software" (human knowledge) as A grows, with the hypothetical limit of zero software share as $A \to \infty$ without an industrial revolution (with a steady $K = 0$).

Stage 2. Industrial production. The first stage of development of an industrial economy features gradual *mechanization* of production: physical capital accumulation systematically decreases the role of human labor in "hardware". Given the substitutability between capital and physical labor, unskilled labor shares go down whereas capital shares go up. Assuming furthermore that (temporarily) the pace of capital accumulation outruns technical change (growth in A), this is however accompanied by increasing output shares accruing to "software" (i.e., human cognitive work) which thus becomes scarce, raising the skilled wage and the skill premium. As the economy tends to a BGP, the hardware share stabilizes around some level $\bar{\pi}_X \in (0,1)$. From then onwards the economy respects Kaldor's facts (Kaldor, 1961).

Stage 3. Digital production. The first stage of development of a digital economy features gradual *automation* of production: pre-programmed software and AI accumulation gradually decreases the role of human skilled labor in software. Given the substitutability of these two factors, skilled labor shares go down whereas preprogrammed software and AI shares go up. This is where we are now.

Next, however, the model predicts that due to ongoing technological progress hardware gradually becomes the bottleneck of the economy. Then the overall software share begins to decline and, in the absence of a next technological revolution, the hardware share tends to a hypothetical limit of unity. Note that over the long run, factor remuneration goes increasingly to the owners of capital goods, in particular programmable hardware. In the limit, none of the remuneration goes to human workers because human skills by then have been fully automated.

Hypothetical stage 4. Post-digital production. Perhaps the functional distribution of income becomes less of an issue in a world where neither hardware nor software requires any human input, but nevertheless one may observe that the episode of "new mechanization" (replacement of K with M in hardware) would incur a dynamic that is partly similar to the one following the Industrial Revolution. Namely, accumulation of M systematically decreases the role of K in hardware, so that the share of K goes down whereas the share of M goes up. Next, if all software is able to scale with M then its share is bound to remain low. A different scenario would follow if software were not able to scale with M – then it would become scarce and its share of output would increase.

4 The R&D Equation

4.1 Setup

Technological change due to purposeful R&D activities is widely acknowledged as a key driver of long-run growth in output per worker in the industrial and early digital era. Due to the non-rivalrous character of technological ideas, they act a source of increasing returns to scale (Romer, 1986, 1990), allowing output to grow even when the use of inputs is constant over time. The exact specification of the R&D process at the macroeconomic level is however subject to dispute. In particular, and perhaps somewhat surprisingly, the existing R&D-based growth literature almost unequivocally assumes that researchers' labor is the only input in the R&D process (Romer, 1990; Jones, 1995, 1999; Ha and Howitt, 2007). Alternatively, a few studies embrace the "lab equipment" specification of the R&D process, conditioning R&D output on overall R&D spending (Rivera-Batiz and Romer, 1991; Bloom, Jones, Van Reenen, and Webb, 2017; Kruse-Andersen, 2017). In reality, however, productivity of the R&D sector depends not just on the labor of researchers but increasingly also on the services of *R&D capital*. Modern R&D capital may range from modest offices at university campuses or computers at researchers' laps to such exquisite machinery as the Very Large Telescope (VLT) and the Large Hadron Collider (LHC).

Consistently with the hardware–software model, I postulate that R&D output is a function of two inputs to the R&D process: hardware X and software S. Hardware includes R&D capital alongside human physical labor. Software encompasses all the sophisticated and ingenuous ideas supplied by scientists and technical personnel, as well as – again – pre-programmed software and AI.

Intuitively, the difference between the production process and the R&D process is that the latter tends to involve relatively less physical action and more sophisticated instructions. R&D is also not bound by the thermodynamical requirement that an essential fraction of its output must be material; in fact probably almost all its output comes in the form of information. Yet, note, even pure *thinking* is in fact computational action carried out in the thinker's brain – so it needs some hardware, too; and the further we go from genuinely abstract, philosophical reflection towards more applied R&D, the more actual physical action is necessary (e.g., laboratory experiments, survey data collection, model building, prototype testing, etc.). In fact, the practicality and complexity of research equipment has undergone systematic, cumulative changes over the centuries. The difference in usefulness of Ptolemy's astrolabe, Galileo's telescope, and the modern Very Large Telescope (VLT) is breathtaking, and so is to think how early statisticians actually computed correlations and ran regressions without relying on computer hardware.

This framework implicitly assumes that there is no qualitative difference between human thought and computer software in digital-era R&D processes. In line with Brynjolfsson and McAfee (2014) I hypothesize that *ideation*, creativity and intuition represent sophisticated pattern recognition. Thus there is no theoretical argument precluding the possibility that R&D will also be subject to gradual automation in the digital era. For one thing, AI algorithms improve at a pace that is of the same order of magnitude as hardware accumulation (which is subject to Moore's law), Grace (2013). Furthermore, in the recent decade we have witnessed a surge in breakthrough results, ranging from autonomous vehicles and simultaneous language interpretation to self-taught superhuman performance at chess and Go, following from the same broad methodology of *deep neural networks* (deep learning), Tegmark (2017).

As of today, AI is already used in e.g., genome sequencing, not to mention web browser engines, which are of enormous help to modern researchers. In the future, the use of AI in R&D may revolutionize it by not just helping answer research questions, but also ask new ones. Brynjolfsson and McAfee (2014) predict that AI technologies will turn out decisive for growth dynamics in the near future by developing "gradually, then suddenly", fueled by their highly scalable character and – potentially – ability to self-improve. Indeed, right before our eyes AI algorithms are getting better and better at pattern recognition based on big data, classification, categorization of various sorts of content, and making adaptive decisions in noisy, variable environments – and they are already much faster than humans at all that.

In line with the above discussion, I postulate that the knowledge accumulation equation should also obey the general equation (2):[16]

$$\dot{A} = \gamma\Phi(X, S) = \gamma\Phi(\zeta N + K, A(hN + \psi\chi K)), \qquad \gamma > 0. \tag{12}$$

It is assumed that Φ is increasing and concave in both factors, X and S. The function Φ should be understood as an idea production function which is active within a certain technological paradigm. Observable technological progress then comes from incremental innovations which, in turn, rely on radical innovations for new research avenues to be opened (Kondratieff, 1935; Olsson, 2000, 2005; Growiec and Schumacher, 2013). Even if there are "fishing out effects" within each technological paradigm (decreasing returns to scale in Φ), opening new paradigms may rejuvenate technological opportunity, allowing the parameter γ to go up.

4.2 R&D Across Stages of Economic Development

Let me now discuss how the overarching hardware–software framework specializes to deal with the realities of consecutive eras of economic development.

Stage 1. Pre-industrial R&D. In a pre-industrial economy, R&D was carried out by individual scholars (and possibly small research teams of their disciples). R&D output was generated essentially from their thought and simple experiments, with little or no aid of R&D capital. Setting $K = 0$ in (12) yields:

$$\dot{A} = \gamma\Phi(X, S) = \gamma\Phi(\zeta N, AhN). \tag{13}$$

Hence, under gross complementarity of hardware and software and assuming constancy of γ, the pool of technological opportunity was gradually depleted and "ideas were getting harder to find" (Olsson, 2005; Bloom, Jones, Van Reenen, and Webb, 2017). The model implies that in the absence of R&D capital, the knowledge increment $\dot{A}$ tends to a positive constant and the rate of technological progress $\dot{A}/A$ – to zero.

Stage 2. R&D of the industrial era. In an industrial economy, R&D output was produced increasingly in laboratories, specialized research units, and corporate R&D divisions. More and more specialized machines were employed in order to advance the state of knowledge. All physical actions were, however, dependent on the instructions provided by skilled scientists and their personnel: there was no

[16] In order to better describe the early millennia of human history, equation (12) could be augmented with knowledge depreciation. As the focus here is on the more recent centuries, after the development of writing and the printing press, which made depreciation of aggregate human knowledge negligible, I set this complication aside.

programmable hardware and no pre-programmed software yet. Transforming (12), the following form is obtained:

$$\dot{A} = \gamma\Phi(X, S) = \gamma\Phi(\zeta N + K, AhN). \tag{14}$$

In the limit of full mechanization and skill satiation, $K \to \infty$ and $h \to \bar{h}$, the model implies that $\dot{A} = \gamma\Phi(K, \bar{h}AN)$, where $\bar{h}$ is the upper limit of human capital (skill) accumulation. Thus again "ideas are getting harder to find" ($\dot{A}/A$ decreases with A).

Yet, if Φ exhibits constant returns to scale then thanks to R&D capital accumulation the economy tends to an asymptotic BGP where K and A grow at the same rate:

$$g_A = \frac{\dot{A}}{A} = \gamma\Phi\left(\frac{K}{A}, hN\right). \tag{15}$$

Hence, in the counterfactual scenario of balanced growth without a Digital Revolution, increases in average skills h and R&D employment N tend to increase the pace of technological progress only up to a point, after which it is pinned by the scarce factor, K/A. R&D is the key source of growth and accumulation of R&D capital is the underlying mechanism that allows to sustain it.

Stage 3. R&D in the digital era. In the early days of the digital era, such as the contemporary times as of 2019, human research skills are increasingly augmented with sophisticated R&D hardware. Moreover, some of the more tedious research tasks already are gradually automated. This process may accelerate fast in the future after AI algorithms become sufficiently advanced to contribute in such non-structured, complex environments as cutting-edge R&D.

In the digital era, equation (12) holds in its general form. The limit of full automation implies:

$$\dot{A} = \gamma\Phi(K, A\bar{\psi}\bar{\chi}K). \tag{16}$$

If Φ exhibits constant returns to scale then again the economy tends to an asymptotic BGP where K and A grow at the same rate:

$$g_A = \frac{\dot{A}}{A} = \gamma\Phi\left(\frac{K}{A}, \psi\chi K\right). \tag{17}$$

Hence, increases in AI skills ψ and the accumulation of programmable hardware χK tend to increase the pace of technological progress but only up to a point, after which it is pinned by the scarce factor, K/A. The hardware–software model implies that it is the accumulation of programmable hardware that would eventually become the unique source of long-run growth of a digital economy. In a world where software is able to scale with hardware, technological progress ceases to be the key contributor to growth – which it remains only as long as the overall supply of software is pinned to the size of the human population.

5 The Role of Automation for Aggregate Production, R&D and Growth: A CES Example

Let me now provide a more detailed treatment of the impact of automation (and particularly AI-driven automation) on production and R&D under the proposed framework. To this end, I will specifically assume that both production functions, F and Φ, take the normalized CES form (Klump, McAdam, and Willman, 2012), whereas capital accumulation follows the standard equation of motion due to Solow (1956). What follows is a two-sector growth model with two interlinked growth engines, factor accumulation and R&D. Neither of them is able to drive long-run growth alone: capital accumulation is not sufficient because of decreasing returns under a fixed level of technology; and R&D is not sufficient because its operations require the deployment of R&D capital (unlike endogenous growth models à la Romer (1990) or Jones (1995) where human cognitive work was the only essential factor in R&D).

The model consists of the following equations:

$$
\begin{aligned}
X &= \zeta N + K, && (18)\\
S &= A(hN + \psi\chi K), && (19)\\
Y &= Y_0\left(\pi_0\left(\frac{u_X X}{u_{X0}X_0}\right)^{\xi} + (1-\pi_0)\left(\frac{u_S S}{u_{S0}S_0}\right)^{\xi}\right)^{\frac{1}{\xi}}, && (20)\\
\dot{A} &= \dot{A}_0\left(\gamma_0\left(\frac{(1-u_X)X}{(1-u_{X0})X_0}\right)^{\mu} + (1-\gamma_0)\left(\frac{(1-u_S)S}{(1-u_{S0})S_0}\right)^{\mu}\right)^{\frac{1}{\mu}}, && (21)\\
\dot{K} &= sY - \delta K, && (22)
\end{aligned}
$$

where $s \in [0,1]$ is the savings rate, $u_X, u_S \in [0,1]$ are the shares of hardware and software, respectively, allocated to the production sector, and $(1-u_X), (1-u_S)$ are the respective shares allocated to R&D. The parameter $\xi < 0$ captures the degree of substitutability between hardware and software in production, and $\mu < 0$ – in R&D. The parameters with subscript 0 are normalization constants.

This framework allows me to provide a comparison of two polar scenarios: (i) without any digital revolution ($\chi = 0$), and (ii) with a digital revolution, eventually leading to full automation. In both scenarios I will assume a constant population size, a prediction that appears realistic over the long run: population projections indeed suggest that global population will plateau within the next century.

Industrial-era economy without automation. In an economy without programmable hardware and pre-programmed software, as the stock of capital tends to infinity and as $h \to \bar{h}$ (growth in the average level of education flattens out), one may approximate $X \approx K$ and $S \approx A\bar{h}N$. Inserting these approximations into the system

(18)–(22) and setting a constant population size $N = N_0$ yields the following system of equations describing the balanced growth path of the economy:

$$\frac{Y}{A} = \frac{Y_0}{A_0}\left(\pi_0\left(\frac{u_X}{u_{X0}}\frac{K}{A}\frac{A_0}{K_0}\right)^{\xi} + (1-\pi_0)\left(\frac{u_S}{u_{S0}}\frac{\bar{h}}{h_0}\right)^{\xi}\right)^{\frac{1}{\xi}}, \tag{23}$$

$$g = g_0\left(\gamma_0\left(\frac{(1-u_X)}{(1-u_{X0})}\frac{K}{A}\frac{A_0}{K_0}\right)^{\mu} + (1-\gamma_0)\left(\frac{(1-u_S)\bar{h}}{(1-u_{S0})h_0}\right)^{\mu}\right)^{\frac{1}{\mu}}, \tag{24}$$

$$g = s\frac{Y}{K} - \delta, \tag{25}$$

$$\frac{Y}{A} = \frac{Y}{K}\frac{K}{A}. \tag{26}$$

This is a four-equation system in four stationary variables: the growth rate g and the three ratios, Y/A, Y/K and K/A. Additional calculus uncovers that the long-run economic growth rate g depends on the key endogenous variables of the model, s, u_X and u_S. The dependence of g on s is unambiguously positive, whereas growth effects of the latter two variables are ambiguous.

Along the balanced growth path of the industrial economy without automation, the economy respects Kaldor (1961) facts: the "great ratios" (K/Y, C/Y) and factor shares are constant.

Digital-era economy with full automation. As the stock of capital tends to infinity, and $\chi \to \bar{\chi}, \psi \to \bar{\psi}$, one may approximate $X \approx K$ and $S \approx A\bar{\psi}\bar{\chi}K$, underscoring that in the limit of full automation, production and R&D get totally decoupled from the employed human population.[17] Inserting these approximations into the system (18)–(22) and letting $A \to \infty$ yields the following asymptotic balanced growth path of this economy:

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = s\pi_0^{\frac{1}{\xi}}\left(\frac{u_X}{u_{X0}}\frac{Y_0}{K_0}\right) - \delta, \tag{27}$$

$$\frac{Y}{K} = \pi_0^{\frac{1}{\xi}}\frac{u_X}{u_{X0}}\frac{Y_0}{K_0}, \tag{28}$$

$$\frac{K}{A} = \gamma_0^{-\frac{1}{\mu}}\left(\frac{1-u_{X0}}{1-u_X}\right)\frac{g}{g_0}. \tag{29}$$

Hence, this scenario leads to an AK-type model of fully endogenous growth (Jones and Manuelli, 1990; Barro and Sala-i-Martin, 2003). The accumulation of programmable hardware becomes the unique engine of growth because it does double duty once software is able to scale up to hardware, $S \propto K$. The endogenous variables positively affecting the long-run growth rate are (i) the savings rate s, and (ii) the share of hardware in production u_X. The allocation of software u_S becomes irrelevant in the limit because if software is able to scale with hardware, it is ultimately only

[17] Putting it more harshly, under full mechanization and automation humans become useless, irrelevant for the economy (Harari, 2017).

the hardware that constitutes the crucial growth bottleneck. For the same reason, in the limit it does not pay to allocate any more hardware to R&D: the impact of R&D on growth eventually vanishes.

Along the asymptotic balanced growth path of the digital economy with full automation, the economy respects the Kaldor (1961) fact of constancy of the "great ratios" (K/Y, C/Y), but the software share falls to zero.

6 Discussion

6.1 Key Concepts and Misconceptions of the Digital Era

In the current paper I have carried out some baseline conceptual work, needed by economic growth theory in order to achieve progress in modelling the realities of the digital era. The key contribution of the proposed hardware–software model lies with the formalization of production processes across all eras of economic development, with specific focus on capturing the effects of the Digital Revolution. In particular, it provides a conceptually consistent approach to delineating such key concepts – that are sometimes confused in the literature – as mechanization, automation, ICT, hardware, software, and AI (as well as: robots and robotization).

Viewed through the lens of the hardware–software model:

- *Mechanization* of production consists in replacing human (and animal) physical labor with machines (K in place of L). Large-scale mechanization is observed since the Industrial Revolution. It applies to the mode of action but not the instructions, which have been historically provided by humans, according to their expertise and judgment.

- *Automation* of production consists in replacing humans with pre-programmed software in providing instructions to machines (Ψ in place of H). Automation pertains to cases where a task, previously involving human decisions, is carried out entirely by machines without any human intervention. Automation is observed since information technologies came into use as general purpose technologies (Bresnahan and Trajtenberg, 1995). Routine tasks (both physical and cognitive) are typically the first to be automated (Autor and Dorn, 2013).

Historically mechanization preceded automation. Therefore the automation processes of the digital era tend to affect tasks where no human labor is needed anymore. This ordering is however not obligatory. A fun example of automation without mechanization is when you walk around town blindly following the instructions of your GPS.

- *Information and communication technology* (ICT) is a concept that is orthogonal to the hardware–software dichotomy because ICTs include both programmable hardware χ and pre-programmed software Ψ. They constitute a breakthrough compared to non-programmable machines because they allow to replace humans in providing instructions. Code, once programmed, can be run multiple times, also concurrently on many machines, without the need of any human intervention. Hence ICTs were necessary for initiating automation.

- *Artificial intelligence (AI)* is a special type of pre-programmed software that has the ability to learn from data. In contrast to "traditional" software which consists of a deterministic, fixed set of instructions (e.g., if–then loops), artificial intelligence embraces uncertainty and can improve its performance based on experience and new information. This happens even under a static architecture of the AI – though it is conceivable that AI may modify its own architecture while heading towards self-improvement. Machine learning bears similarity to human learning, but its advantage is that many networked pieces of equipment can pool their data, whereas humans cannot. The development of AI opens new opportunities for speeding up automation because AI allows to substitute humans in non-routine tasks as well (Brynjolfsson, Rock, and Syverson, 2017). According to Agrawal, Gans, and Goldfarb (2017), while computers drastically lowered the costs of computing (arithmetic), AI drastically lowers the costs of *prediction*.

In light of the above discussion, it is a misconception to identify computers and robots with their hardware (Benzell, Kotlikoff, LaGarda, and Sachs, 2015; Berg, Buffie, and Zanna, 2018). Computers, robots, smartphones and other ICTs consist both of their hardware and software. Their hardware can be productive and useful only when provided with appropriate instructions, either from human operators or pre-programmed software.

Another frequent misconception is to automatically associate AI with robots. AI is software that can learn from data. This software may indeed provide instructions to robots, but also to conventional computers, smartphones and other devices which embody digital chips.

It is also rather problematic to identify AI development with automation (Aghion, Jones, and Jones, 2017), because automation may proceed also without AI, as it has been the case for decades e.g. in the auto industry, and AI – especially at initial stages of development – may be complementary to some human skills such as judgment (Agrawal, Gans, and Goldfarb, 2017).

Automation also should not be conflated with mechanization. This is done, for example, in the famous question "will humans go the way of horses?" (Brynjolfsson and McAfee, 2014). The answer is: as far as human physical labor is concerned, humans have long went the way of horses; for cognitive tasks (for which horses are of no use) this has not been the case, at least not yet. By the same token, it is false comfort to say that the history of the Industrial Revolution teaches us that when jobs are destroyed, new ones are bound to emerge. It only teaches us that when physical labor is mechanized, additional workers will be demanded in cognitive occupations; it tells us nothing about cognitive occupations being automated.

6.2 AI and the Future of Production and R&D

The role of AI in future production and R&D processes depends critically on the value of $\bar{\psi}$ relative to $\bar{\eta}\bar{h}$, i.e., the upper limit of AI skills relative to human skills. Full automation will be possible only if AI would achieve superhuman performance across a broad array of tasks and gain sufficient adaptability and versatility to be able to endogenously expand the breadth of its expertise. Otherwise, there will be a point at which automation must stop.

This caveat, in turn, depends on the answers to two following questions. First, is *ideation* a sophisticated incarnation of pattern recognition or a qualitatively different feature? Can AI be creative, imaginative and insightful in the way humans can be? Preliminary evidence suggests that even some of the contemporary AI algorithms can indeed be perceived as creative, e.g., in devising innovative strategies in chess and Go (DeepMind's AlphaZero, Silver, Hubert, Schrittwieser, et al., 2018), drawing artistic pictures (Schmidhuber, 2009a), or composing music (Amper Music, IBM's Watson Beat, Google's Magenta, AIVA). Arguments have also been formulated that the lines between creativity, insight and complexity are rather arbitrary, further favoring the case of a high $\bar{\psi}$ (Dennett, 2017; Tegmark, 2017).

Second, how high are the *returns to cognitive reinvestment* in machine intelligence? (Yudkowsky, 2013). How efficient will the future AI be in re-designing itself and its environment in order to improve its cognitive capacity? Humans are in this regard limited by our inability to rewire our brains, and so we circumvent this limitation by increasingly relying on external memory, data collection equipment, and computational power. We also increasingly pool our resources by working in ever larger research teams whose members have increasingly specialized sets of skills. As our knowledge set is growing but our brains are not, interdisciplinary "Renaissance Men" are long gone (Jones, 2009). Unfortunately, speed and accuracy of our interpersonal communication are far from perfect, and thus we may be missing plenty of interdisciplinary insights. AI algorithms running on fast computers, in contrast, communicate extremely fast and without error. They also by far surpass us in terms of speed and serial depth of computation (Hanson and Yudkowsky, 2013). In contrast to humans, machine intelligence is also (so far, theoretically) able to recursively rewrite its code provided that it is able to prove that the rewrite is beneficial (Schmidhuber, 2009b). All this points to a rather high $\bar{\psi}$ and motivates the baseline parametrization used in the current paper. Nevertheless, thus far AI is markedly lagging behind the human brain in terms of versatility and adaptivity. If this is resolved, we may observe a rapid buildup of AI skills, and even an uncontrolled intelligence explosion (Hanson and Yudkowsky, 2013; Bostrom, 2014). If not, then perhaps $\bar{\psi}$ is low and full automation is not possible.

6.3 Singularity?

The notion of an uncontrolled intelligence explosion naturally leads to the question whether we are approaching a technological *singularity* (Kurzweil, 2005; Nordhaus, 2017; Aghion, Jones, and Jones, 2017). The answer depends on the precise definition of singularity, though. On the one hand, the hardware–software model embraces the possibility of a singularity in the sense of an "AI takeover", when human cognitive work is no longer required for production. At that moment, AI becomes better than humans in *everything*, including inventing new tasks (Acemoglu and Restrepo, 2018) and building AI. With a sufficiently high degree of substitutability between human cognitive work and pre-programmed software, this may well appear in finite time.[18] On the other hand, however, the model does not allow for a singularity understood as a vertical asymptote in the level of GDP, i.e., arbitrarily high production in finite time, which – given that a non-degenerate fraction of output must be material to sustain the hardware – would be inconsistent with the laws of thermodynamics.

In sum, the hardware–software model expects a growth acceleration in the future, but it also expects that its fruits will not necessarily benefit the humankind.

* * *

Future work on the hardware–software model should forge a link between the proposed conceptual framework and general-equilibrium modelling of economic growth. It is important to identify the equilibrium forces determining the extent of automation and to quantify the timing at which AI development becomes critical for economic growth. One could also review alternative scenarios, such as the one where R&D could be carried out without R&D capital or where AI software does not scale proportionally to hardware. Another promising line of work would be to analyze complex tasks within the hardware–software model in order to quantify the extent to which human cognitive work and AI can be complementary on the run-up to full automation.

[18] This has tremendous philosophical, political and even *existential* implications (see e.g. Hanson and Yudkowsky, 2013; Bostrom, 2014; Harari, 2017).

References

ACEMOGLU, D. (2003): "Labor-and Capital-Augmenting Technical Change," *Journal of the European Economic Association*, 1, 1–37.

——— (2009): *Introduction to Modern Economic Growth*. Princeton University Press.

ACEMOGLU, D., AND D. AUTOR (2011): "Skills, Tasks and Technologies: Implications for Employment and Earnings," in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, vol. 4, chap. 12, pp. 1043–1171. Elsevier.

ACEMOGLU, D., AND P. RESTREPO (2018): "The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares and Employment," *American Economic Review*, 108, 1488–1542.

AGHION, P., AND P. HOWITT (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60, 323–351.

AGHION, P., B. F. JONES, AND C. I. JONES (2017): "Artificial Intelligence and Economic Growth," Working paper.

AGRAWAL, A., J. S. GANS, AND A. GOLDFARB (2017): "What to Expect From Artificial Intelligence," *MIT Sloan Management Review*, Spring 2017.

ANDREWS, D., C. CRISCUOLO, AND P. N. GAL (2016): "The Global Productivity Slowdown, Technology Divergence and Public Policy: A Firm Level Perspective," Working party no. 1 on macroeconomic and structural policy analysis, OECD.

ARNTZ, M., T. GREGORY, AND U. ZIERAHN (2016): "The Risk of Automation for Jobs in OECD Countries: A Comparative Analysis," OECD Social, Employment and Migration Working Paper No 189, OECD Publishing, Paris.

AUM, S., S. Y. LEE, AND Y. SHIN (2018): "Computerizing Industries and Routinizing Jobs: Explaining Trends in Aggregate Productivity," *Journal of Monetary Economics*, 97, 1–21.

AUTOR, D., D. DORN, L. F. KATZ, C. PATTERSON, AND J. VAN REENEN (2017): "The Fall of the Labor Share and the Rise of Superstar Firms," Working Paper No. 23396, NBER.

AUTOR, D. H., AND D. DORN (2013): "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market," *American Economic Review*, 103(5), 1553–97.

Barkai, S. (2017): "Declining Labor and Capital Shares," Job market paper, University of Chicago.

Barro, R. J., and X. X. Sala-i-Martin (2003): *Economic Growth*. MIT Press.

Benzell, S. G., L. J. Kotlikoff, G. LaGarda, and J. D. Sachs (2015): "Robots Are Us: Some Economics of Human Replacement," Working Paper No. 20941, NBER.

Berg, A., E. F. Buffie, and L.-F. Zanna (2018): "Should We Fear the Robot Revolution? (The Correct Answer is Yes)," *Journal of Monetary Economics*, 97, 117–148.

Bloom, N., C. I. Jones, J. Van Reenen, and M. Webb (2017): "Are Ideas Getting Harder to Find?," Working paper, Stanford University.

Bostrom, N. (2014): *Superintelligence: Paths, Dangers, Strategies*. Oxford University Press.

Bresnahan, T. F., and M. Trajtenberg (1995): "General Purpose Technologies: Engines of Growth?," *Journal of Econometrics*, 65, 83–108.

Brynjolfsson, E., and A. McAfee (2014): *The Second Machine Age: Work, Progress, and Prosperity in a Time of Brilliant Technologies*. W.W. Norton & Co.

Brynjolfsson, E., D. Rock, and C. Syverson (2017): "Artificial Intelligence and the Modern Productivity Paradox: A Clash of Expectations and Statistics," Chapter No. 14007, NBER.

Caselli, F., and W. J. Coleman (2006): "The World Technology Frontier," *American Economic Review*, 96, 499–522.

DeCanio, S. J. (2016): "Robots and Humans – Complements or Substitutes?," *Journal of Macroeconomics*, 49, 280–291.

Dennett, D. (2017): *From Bacteria to Bach and Back: The Evolution of Minds*. W. W. Norton & Company.

Frey, C. B., and M. Osborne (2017): "The Future of Employment: How Susceptible Are Jobs to Computerisation?," *Technological Forecasting and Social Change*, 114, 254–280.

Galor, O. (2005): "From Stagnation to Growth: Unified Growth Theory," in *Handbook of Economic Growth*, ed. by P. Aghion, and S. N. Durlauf, pp. 171–293. North-Holland.

Gordon, R. J. (1990): *The Measurement of Durable Goods Prices*. University of Chicago Press.

Gordon, R. J. (2016): *The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War*. Princeton University Press.

Grace, K. (2013): "Algorithmic Progress in Six Domains," Technical report 2013-3, Berkeley, CA: Machine Intelligence Research Institute.

Graetz, G., and G. Michaels (2015): "Robots at Work," Discussion paper, Uppsala University and London School of Economics.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997): "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 87, 342–362.

Growiec, J. (2010): "Human Capital, Aggregation, and Growth," *Macroeconomic Dynamics*, 14, 189–211.

——— (2012): "The World Technology Frontier: What Can We Learn from the US States?," *Oxford Bulletin of Economics and Statistics*, 74, 777–807.

——— (2013): "A Microfoundation for Normalized CES Production Functions with Factor-Augmenting Technical Change," *Journal of Economic Dynamics and Control*, 37, 2336–2350.

Growiec, J. (2018): "Factor-Specific Technology Choice," *Journal of Mathematical Economics*, 77, 1–14.

Growiec, J., and J. Mućk (2018): "Isoelastic Elasticity of Substitution Production Functions," *Macroeconomic Dynamics*, p. forthcoming.

Growiec, J., and I. Schumacher (2013): "Technological Opportunity, Long-Run Growth, and Convergence," *Oxford Economic Papers*, 65, 323–351.

Ha, J., and P. Howitt (2007): "Accounting for Trends in Productivity and R&D: A Schumpeterian Critique of Semi-Endogenous Growth Theory," *Journal of Money, Credit and Banking*, 39(4), 733–774.

Hanson, R. (2000): "Long-Term Growth as a Sequence of Exponential Modes," Working paper, George Mason University.

Hanson, R., and E. Yudkowsky (2013): *The Hanson-Yudkowsky AI-Foom Debate*. Machine Intelligence Research Institute.

Harari, Y. N. (2017): *Homo Deus: A Brief History of Tomorrow*. Vintage.

Hemous, D., and M. Olsen (2018): "The Rise of the Machines: Automation, Horizontal Innovation and Income Inequality," Working paper, University of Zurich.

Henderson, D. J., and R. R. Russell (2005): "Human Capital and Convergence: A Production–Frontier Approach," *International Economic Review*, 46, 1167–1205.

Hercowitz, Z. (1998): "The 'Embodiment' Controversy: A Review Essay," *Journal of Monetary Economics*, 41, 217–224.

Hilbert, M., and P. López (2011): "The World's Technological Capacity to Store, Communicate, and Compute Information," *Science*, 332, 60–65.

Jones, B. F. (2009): "The Burden of Knowledge and the "Death of the Renaissance Man": Is Innovation Getting Harder? ," *Review of Economic Studies*, 76, 283–317.

Jones, C. I. (1995): "R&D-Based Models of Economic Growth," *Journal of Political Economy*, 103, 759–84.

——— (1999): "Growth: With or Without Scale Effects?," *American Economic Review*, 89(2), 139–144.

——— (2005): "The Shape of Production Functions and the Direction of Technical Change," *Quarterly Journal of Economics*, 120, 517–549.

Jones, C. I., and J. Kim (2017): "A Schumpeterian Model of Top Income Inequality," *Journal of Political Economy*, p. forthcoming.

Jones, C. I., and P. M. Romer (2010): "The New Kaldor Facts: Ideas, Institutions, Population, and Human Capital," *American Economic Journal: Macroeconomics*, 2, 224–245.

Jones, L. E., and R. E. Manuelli (1990): "A Convex Model of Equilibrium Growth: Theory and Policy Implications," *Journal of Political Economy*, 98, 1008–1038.

Jorgenson, D. W. (1995): *Productivity. Volume 1: Postwar U.S. Economic Growth*. MIT Press.

——— (2005): "Accounting for Growth in the Information Age," in *Handbook of Economic Growth*, ed. by P. Aghion, and S. Durlauf. North-Holland.

Kaldor, N. (1961): "Capital Accumulation and Economic Growth," in *The Theory of Capital*, ed. by L. A. Lutz, and D. C. Hague, pp. 177–222. Palgrave Macmillan.

Klump, R., P. McAdam, and A. Willman (2007): "Factor Substitution and Factor Augmenting Technical Progress in the US," *Review of Economics and Statistics*, 89, 183–192.

——— (2012): "Normalization in CES Production Functions: Theory and Empirics," *Journal of Economic Surveys*, 26, 769–799.

Kondratieff, N. D. (1935): "The Long Waves in Economic Life," *Review of Economics and Statistics*, 17, 105–115.

Koop, G., J. Osiewalski, and M. F. J. Steel (1999): "The Components of Output Growth: A Stochastic Frontier Analysis," *Oxford Bulletin of Economics and Statistics*, 61, 455–487.

——— (2000): "Measuring the Sources of Output Growth in a Panel of Countries," *Journal of Business and Economic Statistics*, 18, 284–299.

Kruse-Andersen, P. K. (2017): "Testing R&D-Based Endogenous Growth Models," Working paper, University of Copenhagen.

Krusell, P., L. E. Ohanian, J.-V. Ríos-Rull, and G. L. Violante (2000): "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 68, 1029–1054.

Kumar, S., and R. R. Russell (2002): "Technological Change, Technological Catch-up, and Capital Deepening: Relative Contributions to Growth and Convergence," *American Economic Review*, 92, 527–548.

Kurzweil, R. (2005): *The Singularity is Near*. New York: Penguin.

León-Ledesma, M. A., P. McAdam, and A. Willman (2010): "Identifying the Elasticity of Substitution with Biased Technical Change," *American Economic Review*, 100, 1330–1357.

Madsen, J. (2008): "Semi-endogenous Versus Schumpeterian Growth Models: Testing the Knowledge Production Function Using International Data," *Journal of Economic Growth*, 13, 1–26.

McAdam, P., and A. Willman (2018): "Unraveling the Skill Premium," *Macroeconomic Dynamics*, 22, 33–62.

Mućk, J. (2017): "Elasticity of Substitution Between Labor and Capital: Robust Evidence from Developed Economies," Working paper no. 271, Narodowy Bank Polski.

Nordhaus, W. D. (2017): "Are We Approaching an Economic Singularity? Information Technology and the Future of Economic Growth," Working paper, Cowles Foundation, Yale University.

Olsson, O. (2000): "Knowledge as a Set in Idea Space: An Epistemological View on Growth," *Journal of Economic Growth*, 5, 253–275.

——— (2005): "Technological Opportunity and Growth," *Journal of Economic Growth*, 10, 31–53.

Rivera-Batiz, L. A., and P. M. Romer (1991): "Economic Integration and Endogenous Growth," *The Quarterly Journal of Economics*, 106(2), 531–555.

Romer, P. M. (1986): "Increasing Returns and Long-run Growth," *Journal of Political Economy*, 94(5), 1002–1037.

——— (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98, S71–S102.

Sachs, J. D., S. G. Benzell, and G. LaGarda (2015): "Robots: Curse or Blessing? A Basic Framework," Working Paper Np. 21091, NBER.

Schmidhuber, J. (2009a): "Driven by Compression Progress: A Simple Principle Explains Essential Aspects of Subjective Beauty, Novelty, Surprise, Interestingness, Attention, Curiosity, Creativity, Art, Science, Music, Jokes," in *Anticipatory Behavior in Adaptive Learning Systems, from Sensorimotor to Higher-level Cognitive Capabilities*, ed. by G. Pezzulo, M. V. Butz, O. Sigaud, and G. Baldassarre. Springer, LNAI.

——— (2009b): "Ultimate Cognition à la Gödel," *Cognitive Computation*, 1, 177–193.

Silver, D., T. Hubert, J. Schrittwieser, et al. (2018): "A General Reinforcement Learning Algorithm That Masters Chess, Shogi, and Go Through Self-Play," *Science*, 362, 1140–1144.

Solow, R. M. (1956): "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70, 65–94.

——— (1957): "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics*, 39, 312–320.

Tegmark, M. (2017): *Life 3.0: Being Human in the Age of Artificial Intelligence.* New York: Knopf.

Temple, J. (2006): "Aggregate Production Functions and Growth Economics," *International Review of Applied Economics*, 20, 301–317.

Timmer, M. P., and B. van Ark (2005): "IT in the European Union: A driver of productivity divergence?," *Oxford Economic Papers*, 57(4), 693–716.

Uzawa, H. (1961): "Neutral Inventions and the Stability of Growth Equilibrium," *Review of Economic Studies*, 28, 117–124.

Yudkowsky, E. (2013): "Intelligence Explosion Microeconomics," Technical report 2013-1, Machine Intelligence Research Institute.