Credibility For Sale*

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Abstract

We develop a sovereign debt model with official and private creditors where the probability of default depends on both the level and the composition of liabilities. Higher exposure to official lenders improves incentives to repay but carries extra costs, due to overcharging or reduced ex-post flexibility. The model implies that official lending to sovereigns takes place in times of debt distress; carries a favorable rate; and can *displace* private funding even under pari passu provisions. Moreover, in the presence of long-term debt overhang, the availability of official funds increases the probability of default on existing debt although default does not trigger exclusion from private credit markets. These findings help shed light on joint default and debt composition choices such as those observed during the recent sovereign debt crisis in Europe.

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1 Introduction

The recent sovereign debt crisis in the Eurozone exhibited a rich set of patterns: A heavily indebted country, Greece, defaulted on its debt while at the same time receiving substantial official funds (from European sources as well as the IMF) carrying a low interest rate. Greece's default decision was encouraged by its official creditors. Other countries, namely Ireland, Portugal and Spain, did not default but the composition of their new loans (including commitments by the Eurogroup to Spain) shifted significantly

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towards official sources. That is, in spite of pari passu provisions on new private loans, official funds crowded out private sources of financing. Yet other countries including Italy opted to abstain from official funding all together and relied on more expensive private loans.

To account for these phenomena, we propose a simple extension of the standard sovereign debt model (Eaton and Gersovitz, 1981) that addresses default decisions, the choice of composition of sovereign debt between private and official sources and the interaction between these two decisions. We introduce two plausible and realistic features (motivated below): A class of official creditors who possess superior enforcement power because they can impose more severe sanctions in the case of default than private creditors;[1] and a pari passu provision that prevents ex-post discrimination against private lenders. With these extensions, the standard model provides important insights on the phenomena described above.

A sovereign borrower that lacks commitment only repays her debt when the cost of default is sufficiently large. This limits the amount the country can borrow. In order to procure more funds ex ante, the sovereign may want to structure its debt in a way that increases default sanctions ex post. Exposure to official lenders accomplishes this and under pari passu, it strengthens the sovereign's credibility both vis-a-vis official and private creditors. But borrowing from official lenders also carries costs. On the one hand, official funds may be more expensive than private funds on a risk-adjusted basis.[2] On the other hand, official lending may limit ex-post flexibility. For more severe default sanctions imply that debt will be repaid in some states of the world where the sovereign would have defaulted if the debt had been issued to private creditors instead.[3]

The model has the following implications: First, countries typically find it optimal to borrow from private sources when financing needs are low. When these needs are high, sovereigns favor stronger reliance on official sources of funding because the benefits of credibility outweigh the costs associated with those funds. The resulting variation in the debt composition does not arise because of variation in risk-adjusted interest rates—these are constant over the debt cycle. It rather reflects the fact that the sovereign's cost of borrowing includes additional components, namely expected social losses from default and lack of consumption smoothing, and that the magnitude of these components varies with total borrowing needs.[4]

[1] That the identity—official vs. private—of the creditor can make a difference for sovereign risk assessments appears rather undisputed. IMF lending is a case in point. But the identity of the creditor also seems to matter in private loan transactions. According to a widely shared presumption and anecdotal evidence the incentive to repay loans to Mafia is much stronger than the incentive to repay other creditors, due to Mafia's more extensive set of enforcement tools.

[2] For example, official rates may contain a surcharge over private lending rates to cover administrative and other costs, as is the case with IMF loans. Or the rate on official loans may contain a "penalty" component as was the case during the Eurozone debt crisis.

[3] See Zame (1993) for a discussion of the insurance benefits of *implicitly* state contingent debt. Under incomplete markets, a country may trade a higher interest rate on its debt for the option to declare default in states where debt repayment would have been very costly.

[4] In a benchmark case without mark-down on official funds (no rent extraction by official lenders) official and private loans have identical risk and return characteristics. Nevertheless, the government is not indifferent to the debt composition because this composition affects the risk and return characteristics

Second, large official loans may go hand in hand with interest rates that look inexpensive in the sense that private debt of the same amount would result in much higher interest rates. This is a direct consequence of strengthened credibility. Whether the interest rate reduction is large depends on factors that determine the bargaining positions of official lenders and the sovereign.

Third, when the hedging benefits of private—and thus riskier—loans are negligible, debt quotas tend to be higher under official funding. However, default risk tends to be equal or lower when funds are borrowed from official sources, as intended, unless the credibility gains are small and creditors are willing to subsidize the loans.

Fourth, while pari passu provisions succeed in strengthening credibility vis-a-vis both official and private lenders they may fail to induce private investors to continue providing significant, fresh funds. That is, official financing may still crowd out private funding. Crowding out is less likely when the credibility gains of official intervention or the price elasticity of debt are large.

Finally, the preferred debt composition depends on the current default decision and, if the sovereign opts against default, on the stock of outstanding long-term debt (among other factors). The default decision, in turn, depends on the stock of debt overhang and on the sources of refinancing available to the borrowing country. With long-term debt overhang, the availability of official credit makes current default more likely in spite of the fact that default does not trigger exclusion from private credit markets. Official lenders may not only tolerate default on private long-term debt in this case but actually encourage it because they can earn rents by refinancing the country.[5] If high costs discourage current default, however, larger debt overhang makes it more likely that a country shies away from official funds.

These model implications square well with patterns observed during the recent debt crisis in the Eurozone. The case of Greece (with a high debt overhang) conforms with the prediction that in debt distress periods, default is more likely when the debt overhang is large and that when it occurs, it is accompanied by a switch from private towards official sources of funding. The cases of Italy, Ireland, Portugal and Spain (in decreasing order of debt overhang) are broadly consistent with the model implication that absent a default, the share of official funds in fresh borrowing depends negatively on the stock of outstanding long-term debt.[6] The fact that relatively low interest rates have been negotiated for official loans suggests strong bargaining power on the part of the borrowing countries.[7]

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of both assets.

[5]That is, the sovereign and his official would be creditors may gang up against existing, private holders of sovereign debt.

[6]IMF (2012, p. 17) contains debt quota data. While Spain has not yet received official funding the Eurogroup committed to provide up to 100 billion Euro. There exist other explanations for why a debt distressed country shuns official funds, for example that the provision of such funds may be accompanied by demands for politically costly reforms. Our analysis offers a complementary explanation that does not rely on political economy considerations.

[7]According to observers, the IMF is repositioning itself in favor of a more "aggressive" stance vis-a-vis holders of outstanding private debt, "seeking earlier and deeper haircuts of government creditors" (Financial Times, June 5, 2013, Creditors Likely Losers From IMF Sovereign Rethink). If true, such

The first key new element in our model relative to the standard sovereign debt setup concerns the existence of differential enforcement powers across groups of creditors. Such differential powers may arise for various reasons. One reason is that the credit relationship is part of a broader arrangement between the borrower and the lender, as it is typically the case from participation in the same club. Consider for instance the relationship between Greece and the other members of the Eurozone. A Greek default on official loans from those countries could trigger retaliation and lower Greece's benefits from club membership in the European Monetary Union (EMU) or even the European Union (EU): Structural fund payments and other transfers might be cut; Greece might be forced to leave the Eurozone; official lenders might be tempted to adopt policies that are less favorable to Greek interests; support for Greek foreign policy positions might wither; and so on.

As the ongoing crisis constitutes the first instance in which certain members of the Eurozone have borrowed large amounts from other members, and since no default against official funds has occurred we cannot yet know whether official lenders would be in a position to inflict sanctions of the type described above and if they were, whether they would actually choose to do so.[8] But what matters for the behavior of agents in our model—and hence for the properties of equilibrium—is the *perception* of the existence and likely use of such sanctioning powers, rather than the use itself.[9] In our view, the public debate in Europe and statements by policy makers provide ample evidence for a widely shared belief that superior sanctioning powers do exist and official lenders would be willing to use them.

For example, in Germany which provides most of the official financing, the statements of politicians, the debates in parliament and the public reaction all conjure the impression that the loans are perceived to face a low probability of default. In fact, such a perception was a sina qua non for large German loan provision at low rates to be politically feasible in the first place, given voters' expressed antipathy to solidarity (transfers) towards Greece. This perception is also founded in the knowledge that a default by Greece on debt held by official creditors amounts to violating EU treaties and breaking national laws, leaving Greece in uncharted and treacherous political territory regarding its future within the EU.[10] Naturally, time consistency is an issue as it would also be costly for Germany to impose sanctions ex post. But repeat business within the club (lending to Portugal, Ireland and Spain is but one example) makes reputational considerations important, and not imposing sanctions following a Greek default could undermine Germany's credibility.[11]

repositioning would seem to be consistent with the model.

[8]Superior power certainly existed during the times when mighty countries would use military force to enforce repayment (for instance, when the British navy bombarded Athens).

[9]Naturally, in a model with asynchronous borrowing and default decisions of multiple borrowers, default by one country could reveal the existence of such powers and affect perceptions in those countries that have not made a default decision yet.

[10]The German government spokesman Steffen Seibert argued that the countries of the Eurozone could not accept a reduction in the value of their loans to Greece because this would contradict European Union treaties as well as national legislation in Germany and other countries that prohibits member countries to assume the debts of other countries (Kathimerini, November 27, 2012).

[11]Steffen Seibert has argued that debt forgiveness would lead to a huge loss of credibility for Germany and could encourage other countries with debt problems to ask for similar treatment (Kathimerini,

Note also that in order to ensure broad political support for enforcement ex post, Germany has required club-wide participation in the official lending operations.

Similar perceptions about the additional, severe cost of Greek default as a consequence of Eurozone loans are also held in Greece: Voters have opted for parties that strongly oppose default and warn about its dire consequences for Greece's membership in EMU and even EU. While the main opposition party advocates default on both private and official loans this position may not reflect the view that official lenders are powerless but rather an underlying desire to actually subject Greece to the enforcement and get the country expelled from EMU.[12] In our view, the evidence thus points to a widely shared belief in both the existence of superior sanctioning powers on the part of official lenders and their willingness to use them.

The second key new element in the model, the pari passu provision, also fits the European experience. Anxious to avoid a crowding out of private funding, official lenders conceded that safeguards should be put in place to impede ex-post discrimination against their private counterparts. Consistent with this intention, the Greek debt exchange in Spring 2012 put private and official lenders (the EFSF) on an equal footing,[13] and more recent discussions about financial support for Spain revealed similar considerations.[14] In another context and looking forward, the New York court case between Argentina and Elliott Management might end up undermining the preferred creditor status of certain lenders and further contribute to sovereign debt arrangements characterized by pari passu.

Related Literature and Outline The literature on the composition of sovereign debt by type of creditor is scant. Boz (2011) reviews the literature on IMF lending, summarizes empirical evidence and presents a quantitative model of a sovereign that may borrow from private lenders and the IMF. In her model private lending is subject to default risk while IMF lending is default risk free. The cost of IMF funds exceeds the risk free rate by an exogenous surcharge. It predicts modest, countercyclical and intermittent IMF lending with complete crowding out of private funding. We think that the last feature makes the model unsuitable for analyzing the recent Euro debt crisis as private and official new lending co-exist and this is a key fact that needs to be explained.[15]

November 27, 2012).

[12]The main opposition party is called the "party of the drachma" in the sense that special interest groups that support it—such as heavily indebted press barons, labor unions in the public sector, professional guilds etc.—are thought to profit from Greece's exit from European institutions.

[13]Zettelmeyer, Trebesch and Gulati (2012, p. 25) report that "Greece and the remaining signatories of the agreement committed to a payment schedule in which the EFSF and bondholders would be repaid pro-rata and on the same day. In the event of a shortfall in payments by Greece, the common paying agent committed to distributing allocating this shortfall pro rata between the EFSF and the bondholders. Hence, the co-financing agreement makes it difficult for Greece to default on its bondholders without also defaulting on the EFSF."

[14]The Wall Street Journal (June 29, 2012, Investors Cheer Europe Deal) reports that Angela Merkel's agreement "to make ESM loans to Spain equal to Spanish bonds in creditors' pecking order was largely a recognition by Germany that this was necessary to protect Spain's ability to sell bonds ...."

[15]Boz's model also has the unsatisfactory property that accessing the IMF lending facilities changes preferences in the sense that it triggers an increase in the sovereign's discount factor.

Bolton and Jeanne (2011) analyze the interaction between multiple sovereigns of different credit quality and the banking system in a financially integrated area. They argue that a country issuing 'safe haven' government debt may derive rents from exploiting its position as monopolistic supplier of this safe asset. In the model proposed here, we also allow for non-competitive rents, but in contrast to Bolton and Jeanne (2011), we consider the possibility that (official) lenders rather than the borrower extract rents. Niepelt (2011) analyzes the composition of sovereign debt across maturities rather than lenders, as considered here, and Diamond and He (2012) analyze the implications of the maturity structure of debt overhang on investment decisions. Finally, Tirole (2012) distinguishes between ex-post bailouts that aim at avoiding collateral damage and ex-ante risk-sharing (for example joint-and-several liability) among sovereigns.

The rest of the paper is organized as follows. The model is set up in section 2. The equilibrium is characterized in section 3. Section 4 contains a series of tractable examples that help develop intuition and illustrate the main results. Section 5 concludes.

2 Model

The economy lasts for two periods, $t = 1, 2$. It is inhabited by a representative taxpayer, a government and foreign investors. Taxpayers neither save nor borrow.[16] They have time- and state-additive preferences over consumption with strictly increasing and concave felicity function $u(\cdot)$ and discount factor $\delta \in (0, 1)$. Welfare of taxpayers in period $t = 1$ is given by

$$\mathbb{E}\left[\sum_{t=1}^{2} \delta^{t-1} u(y_t^p - \tau_t) | s_1, \pi_1\right],$$

where y_t^p denotes exogenous, pre-tax income, τ_t taxes, s_t the state (to be specified below) and π_t the policy choice in period t. We often write $\mathbb{E}_1[\cdot]$ instead of $\mathbb{E}[\cdot|s_1, \pi_1]$.

Foreign investors are risk neutral, require a risk free gross interest rate $\beta^{-1} > 1$ and hold all government debt (since taxpayers do not save).[17] To guarantee positive debt positions, we assume $\delta \ll \beta$ as is standard in the sovereign debt literature.[18] Foreign investors are composed of private and official lenders. Private lenders are competitive. Official lenders—we refer to them as "the enforcer"—may coordinate amongst themselves and behave non-competitively vis-a-vis the borrowing country. Either as a consequence of this, or due to differences in the cost of funds across lenders, the interest rate charged by official lenders may differ from that charged by private lenders. The central implications of the model are independent of this feature.

The government maximizes the welfare of taxpayers. In period t, it chooses the repayment rate on maturing debt, r_t, issues zero-coupon, one-period debt, b_{t+1}, of which b_{t+1}^e is

[16] Mankiw (2000) or Matsen, Sveen and Torvik (2005) analyze fiscal policy in economies with "savers" and "spenders."

[17] The assumption that the sets of taxpayers and investors do not "overlap" simplifies the analysis and does not matter for the main results.

[18] For recent examples, see Aguiar and Gopinath (2006) or Arellano (2008).

held by the enforcer and $b_{t+1} - b^e_{t+1}$ by private lenders, and (residually) levies taxes. Without loss of generality, public spending other than debt repayment is normalized to zero. Crucially, the government cannot commit its successors (or future selves). Short-sales are ruled out.

Let b_{02} denote the stock of debt issued to private investors in the past that is due in period 2. Define $\tilde{b}_2 \equiv b_{02}\xi_1 + b_2$ to be the stock of maturing debt in period $t = 2$, where ξ_1 is a variable linked to the default decision in the first period: If default in the first period applies to debt maturing in that period and also to the outstanding long-term debt, then $\xi_1 \equiv r_1$. If, instead, default in the first period does not affect the repayment rate on long-term debt, then $\xi_1 \equiv 1$ and $\tilde{b}_2 = b_{02} + b_2$. While the latter specification is consistent with a strict notion of lack of commitment, the former often seems plausible on the basis of legal and economic grounds and it also generates more closely intertwined default and refinancing choices. We solve the model under either specification. Outstanding long-term debt may be repurchased by the government in period $t = 1$. The short-sale constraints in the first period therefore read $b_2 - b^e_2 \geq -b_{02}\xi_1$ and $b^e_2 \geq 0$ or, more compactly, $\tilde{b}_2 \geq b^e_2 \geq 0$. Let $B(b_{02}\xi_1)$ denote the set of debt ownership structures (b_2, b^e_2) that are consistent with the two short-sale constraints.

A sovereign default—a situation where the repayment rate falls short of unity—triggers income losses for taxpayers (see Eaton and Gersovitz, 1981; Cole and Kehoe, 2000; Aguiar and Gopinath, 2006; Arellano, 2008). More specifically, a default in period t triggers an income loss $L_t \geq 0$ where L_t is the realization of an i.i.d. random variable with cumulative distribution function $F_t(\cdot)$ and associated density function $f_t(\cdot)$, $f_t(L) > 0$ for all $L_t \geq 0$.[19] In the presence of official lending, default triggers additional income losses for the borrowing country (see the discussion in the introduction). These losses are given by $\mathcal{L}(b^e_2)$ with $\mathcal{L}(0) = 0$ and $\mathcal{L}'(b^e_2) \geq 0$ for all $b^e_2 > 0$.[20] Default occurs uniformly across privately and officially held debt (see the discussion on pari passu in the introduction).

The sequence of events in each period is as follows. In the beginning of period t, L_t and y_t become known. The state is given by $s_t = (y_t, L_t, \tilde{b}_t, b^e_t)$. Conditional on s_t, the government chooses policies, $\pi_1 = (r_1, b_2, b^e_2)$ or $\pi_2 = r_2$, taking as given the equilibrium relationship between these choices and bond prices.

Let $q_1(s_1, \pi_1)$ and $p_1(s_1, \pi_1)$ denote the period $t = 1$ state s_1 price of debt issued to private and official lenders, respectively, if the government implements policy π_1. When choosing its policy, the government takes the price functions $q_1(s_1, \cdot)$ and $p_1(s_1, \cdot)$ as given. Letting $\Delta_1(s_1, \pi_1) \equiv q_1(s_1, \pi_1) - p_1(s_1, \pi_1)$ denote the difference between the two prices we define the borrowing country's deficit in period $t = 1$ as

$$d_1(s_1, \pi_1) \equiv b_2 q_1(s_1, \pi_1) - b^e_2 \Delta_1(s_1, \pi_1). \tag{1}$$

The budget constraint of the government is $\tau_1 = b_1 r_1 - d_1(s_1, \pi_1)$. The pre-tax income of taxpayers is $y^p_1 = y_1 - \mathbf{1}_{[r_1<1]} L_1$ and $y^p_2 = y_2 - \mathbf{1}_{[r_2<1]}(L_2 + \mathcal{L}(b^e_2))$ where $\mathbf{1}_{[x]}$ denotes the indicator function for event x. Taxpayers' consumption therefore is given by $c_1 =$

[19] Default costs are sometimes assumed to depend on income, $L_t(y_t)$. Our specification allows for arbitrary correlation between y_t and L_t.

[20] Our results would effectively remain unchanged under different assumptions about the function $\mathcal{L}(\cdot)$. For example, $\mathcal{L}(\cdot)$ could depend on b^e_2/b_2 rather than b^e_2.

$y_1 - b_1 r_1 - \mathbf{1}_{[r_1<1]} L_1 + d_1(s_1, \pi_1)$ in the first period and $c_2 = y_2 - \tilde{b}_2 r_2 - \mathbf{1}_{[r_2<1]}(L_2 + \mathcal{L}(b_2^e))$ in the second period.

Let $G_1(s_1)$ denote the value of the government's program conditional on state s_1 and let $G_1(s_1; \pi_1)$ denote the value conditional on a particular first-period policy choice. We have

$$\begin{aligned} G_1(s_1) &= \max_{r_1 \in [0,1],\ (b_2, b_2^e) \in B(b_{02}\xi_1)} u(y_1 - b_1 r_1 - \mathbf{1}_{[r_1<1]} L_1 + d_1(s_1, \pi_1)) + \delta \mathbb{E}_1 \left[G_2(s_2) \right] \\ &\qquad \text{s.t. } p_1(s_1, \cdot), q_1(s_1, \cdot), \\ G_2(s_2) &= \max_{r_2 \in [0,1]} u(y_2 - \tilde{b}_2 r_2 - \mathbf{1}_{[r_2<1]}(L_2 + \mathcal{L}(b_2^e))). \end{aligned}$$

The government chooses the repayment rate on maturing debt as well as debt issuance in period $t = 1$ in order to maximize the sum of the flow utility from consumption in that period as well as the discounted expected continuation value. The latter represents the maximized flow utility from consumption in period $t = 2$, as reflected by the second value function. Importantly, the default rate in period $t = 2$ is chosen by the government in that period alone, due to the lack of commitment.

An *equilibrium* conditional on the official-funds price function $p_1(s_t, \cdot)$ then consists of value and policy functions in periods $t = 1$ and $t = 2$ and a private-funds price function $q_1(s_t, \cdot)$ such that

i. conditional on s_1 as well as the price functions, the policy choices are optimal for the borrowing country,

$$\pi_t(s_t) \text{ solves } G_t(s_t),\ t = 1, 2;$$

ii. the private-funds price function reflects rational expectations as well as the participation constraint of competitive private lenders (i.e., investors earn the expected, competitive rate of return),

$$q_1(s_1, \pi_1) = \beta\ \mathbb{E}_1 \left[r_2(s_2) \right]. \tag{2}$$

Note that equilibrium is defined conditional on a price function for official funds, $p_1(\cdot, \cdot)$. This allows us to study debt policy under alternative assumptions about the institutional environment in place and the enforcer's cost of funds. Consider for example the case in which the enforcer has negligible bargaining power. In this case, the equilibrium price $p_1(s_1, \pi_1)$ is set so that the enforcer attains no more than his outside option. If exposure to the borrowing country after a default generates some costs $\mathcal{C}(b_2^e)$ (beyond capital losses) to the enforcer then the enforcer's binding participation constraint implies

$$b_2^e p_1(s_1, \pi_1) = \beta b_2^e\ \mathbb{E}_1[r_2(s_2)] - \beta\ \text{Prob}[r_2(s_2) < 1]\ \mathcal{C}(b_2^e). \tag{3}$$

As another example, consider the case where the enforcer has sufficient bargaining power vis-a-vis the borrowing country to negotiate a fixed "mark-down" relative to the price on private markets. The equilibrium price of official funds then equals

$$p_1(s_1, \pi_1) = \kappa\ q_1(s_1, \pi_1),\ 0 < \kappa < 1. \tag{4}$$

In both examples, $p_1(s_1, \pi_1) \leq q_1(s_1, \pi_1)$.[21]

We proceed under the assumption that the government's program is well behaved and gives rise to smooth policy functions. In the examples considered below, we verify that this is indeed the case.

3 Analysis

Choice of Repayment Rate in the Second Period Consider first the government's choice of repayment rate in the last period, r_2. Since the marginal cost of lowering r_2 is zero when $r_2 < 1$, the optimal repayment rate equals either zero or unity. In particular,

$$r_2(s_2) = \begin{cases} 1 & \text{if } L_2 \geq \tilde{b}_2 - \mathcal{L}(b_2^e) \\ 0 & \text{if } L_2 < \tilde{b}_2 - \mathcal{L}(b_2^e) \end{cases} . \tag{5}$$

Condition (5) states that the government chooses to default when the resulting income losses, $L_2 + \mathcal{L}(b_2^e)$, are smaller than the amount of debt coming due.[22] Condition (5) is consistent with the notion that governments tend to default when the political costs—specifically income losses of pivotal pressure groups—are low. Governments also tend to default when economic activity is depressed (Borensztein, Levy Yeyati and Panizza, 2006; Tomz and Wright, 2007). The model is consistent with this fact as well when it is slightly extended to include direct default costs for the government in addition to the income losses for taxpayers. Note that corner solutions for the optimal repayment rate follow under more general assumptions about default costs than those made here.

Equation (5) pins down the expected repayment rate. From (2), the equilibrium price of private funds equals

$$q_1(s_1, \pi_1) = \beta(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e))) \tag{6}$$

and is decreasing in the quantity of debt issued, b_2. If $b_{02} > 0$ and $\xi_1 = r_1$ then the choice of repayment rate in the first period, r_1, also affects the price because it determines $\tilde{b}_2$. We return to this point later, when discussing the equilibrium choice of r_1.

Choice of Debt Issued to Private Lenders Issuing debt to private lenders has two effects on the deficit. On the one hand, it raises funds from the marginal unit of debt, in

[21] It is also possible to think of situations where $\kappa > 1$, for example because official lenders subsidize borrowing in order to account for externalities.

[22] Letting b_2^p denote privately held debt and r_2^p, r_2^e the repayment rates on b_2^p and b_2^e, respectively, our assumption in the main model corresponds to the case where

$$c_2 = y_2 - b_2^e r_2^e - b_2^p r_2^p - \mathbf{1}_{[r_2^e \cdot r_2^p < 1]} L_2 - \mathbf{1}_{[r_2^e \cdot r_2^p < 1]} \mathcal{L}(b_2^e).$$

An alternative specification,

$$c_2 = y_2 - b_2^e r_2^e - b_2^p r_2^p - \mathbf{1}_{[r_2^e \cdot r_2^p < 1]} L_2 - \mathbf{1}_{[r_2^e < 1]} \mathcal{L}(b_2^e),$$

could give rise to selective default against private lenders (as long as $b_2^e < \mathcal{L}(b_2^e)$) but never to selective default against official lenders. Boz (2011) completely rules out default against the enforcer.

proportion to its price. On the other hand, it reduces the funds raised from inframarginal units of private and official lending, by changing the price of these units. This latter effect is a direct consequence of the government's lack of commitment and reflects the endogeneity of subsequent repayment decisions. Formally, from (1) and (6),

$$\begin{aligned}\frac{\partial d_1(s_1,\pi_1)}{\partial b_2} &= q_1(s_1,\pi_1) + b_2\frac{\partial q_1(s_1,\pi_1)}{\partial b_2} - b_2^e\frac{\partial \Delta_1(s_1,\pi_1)}{\partial b_2} \\ &= q_1(s_1,\pi_1) - b_2\beta f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) - b_2^e\frac{\partial \Delta_1(s_1,\pi_1)}{\partial b_2}.\end{aligned}$$

Funding from private sources is maximized at the top of the "debt-Laffer curve" which is reached when the above marginal effect equals zero. A completely myopic government ($\delta = 0$) maximizes the deficit and attains the maximum of the debt-Laffer curve. A non-myopic government ($\delta > 0$), in contrast, does not maximize the deficit because each additional unit of debt strictly reduces the continuation value. In either case, the equilibrium value of b_2 is therefore (weakly) smaller than the value that attains the maximum of the debt-Laffer curve. Moreover, this equilibrium value (weakly) exceeds $b_2^e - b_{02}\xi_1$, due to the short-sale constraint vis-a-vis private investors ($b_2 - b_2^e \geq -b_{02}\xi_1$). In the following, we refer to the range of b_2 values defined by the lower bound of $b_2^e - b_{02}\xi_1$ and the upper bound of the maximizer of the debt-Laffer curve as the "relevant range" for b_2.

Let λ and μ denote the multipliers associated with the short-sale constraints $b_2^e \geq 0$ and $b_2 \geq b_2^e - b_{02}\xi_1$, respectively. The effect of a marginal increase in debt issued to private lenders on the government's objective is given by

$$\frac{\partial G_1(s_1;\pi_1)}{\partial b_2} = u'(c_1)\frac{\partial d_1(s_1,\pi_1)}{\partial b_2} + \delta\frac{\partial \mathbb{E}_1[G_2(s_2)]}{\partial b_2} + \mu$$

which can be expressed as[23]

$$\begin{aligned}&(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))(\beta u'(c_1) - \delta\mathbb{E}_1[u'(y_2 - \tilde{b}_2)]) \\ &\qquad - u'(c_1)\left(b_2\beta f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) + b_2^e\frac{\partial \Delta_1(s_1,\pi_1)}{\partial b_2}\right) + \mu. \qquad (7)\end{aligned}$$

The first part of this marginal effect represents the consumption smoothing benefit from the marginal unit of debt. It differs from the corresponding expression in the case without default risk because the price of debt equals $\beta(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))$ rather than β and because debt repayment occurs with probability $(1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))$ rather than

[23] We use the fact that

$$\begin{aligned}&\frac{\partial \mathbb{E}_1[G_2(s_2)]}{\partial b_2} = \\ &\quad \frac{\partial}{\partial b_2}\int_0^{\tilde{b}_2 - \mathcal{L}(b_2^e)} \mathbb{E}_1[u(y_2 - L_2 - \mathcal{L}(b_2^e))|L_2]\mathrm{d}F_2(L_2) + \frac{\partial}{\partial b_2}\int_{\tilde{b}_2 - \mathcal{L}(b_2^e)}^{\infty} \mathbb{E}_1[u(y_2 - \tilde{b}_2)]\mathrm{d}F_2(L_2) \\ &= \mathbb{E}_1[u(y_2 - \tilde{b}_2)]f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) - \mathbb{E}_1[u(y_2 - \tilde{b}_2)]f_2(\tilde{b}_2 - \mathcal{L}(b_2^e)) - (1 - F_2(\tilde{b}_2 - \mathcal{L}(b_2^e)))\mathbb{E}_1[u'(y_2 - \tilde{b}_2)].\end{aligned}$$

always.[24] The marginal rate of substitution between current and future consumption and thus, the profile of output, as well as the relative price between current and future consumption determine the strength of the consumption smoothing benefit.

The second part of the marginal effect arises because the repayment probability depends on the quantity issued: Each extra unit of debt issued lowers the price of all inframarginal units or, equivalently, raises the interest rate on them. This increase in the interest rate—which would be absent in a model with commitment—makes first period consumption more expensive. As a consequence, the equilibrium amount of debt issued (conditional on b_2^e) tends to be smaller than that under commitment. The second part also reflects the fact that issuance of b_2 might affect the price difference $\Delta_1(s_1, \pi_1)$. The final part of the marginal effect, the multiplier μ, is strictly positive if the short-sale constraint $b_2 \geq b_2^e - b_{02}\xi_1$ is binding, and equals zero otherwise.

It may seem surprising that the negative welfare effect associated with the reduction of funds raised from inframarginal units of debt (the second part discussed above) is not balanced by a positive welfare effect from the reduced repayment probability of these inframarginal units in the future. In fact, this effect is present. However, it does not appear in (7) because it is equal in absolute value to a third welfare effect of opposite sign, reflecting the increased risk of future social losses in the wake of default.[25] It is these social losses that are at the source of the reduced incentive (relative to the commitment case) for the government to issue debt. Niepelt (2011) contains a detailed discussion in the context of a model with multiple maturities.

Choice of Debt Issued to Official Lenders Issuing debt to official lenders while holding total debt constant (that is, substituting official for private debt) affects the deficit threefold. First, by raising the output losses of the borrowing country in case of future default, it reduces default risk and increases the price of debt. This has a positive effect on the deficit. Second, it reduces the deficit at the margin by the amount $\Delta_1(s_1, \pi_1)$ if private creditors purchase debt at a higher price than official lenders. Finally, it may change the price discount applied on the inframarginal units of debt issued to the enforcer. Formally, from (1) and (6),

$$\frac{\partial d_1(s_1, \pi_1)}{\partial b_2^e} = b_2 \beta f_2(\tilde{b}_2 - \mathcal{L}(b_2^e))\mathcal{L}'(b_2^e) - \Delta_1(s_1, \pi_1) - b_2^e \frac{\partial \Delta_1(s_1, \pi_1)}{\partial b_2^e}.$$

The effect of substituting official for private funds on the government's objective is given by

$$\frac{\partial G_1(s_1; \pi_1)}{\partial b_2^e} = u'(c_1)\frac{\partial d_1(s_1, \pi_1)}{\partial b_2^e} + \delta \frac{\partial \mathbb{E}_1[G_2(s_2)]}{\partial b_2^e} + \lambda - \mu$$

[24] With risk free debt, the marginal effect would reduce to $\beta u'(c_1) - \delta \mathbb{E}_1[u'(y_2 - \tilde{b}_2)]$.

[25] Higher debt issuance increases subsequent default risk and thus, the risk of future output losses in the wake of default. The corresponding first-order welfare effects that operate through the continuation value are zero. This is a consequence of an envelope condition—the subsequent government is indifferent at the margin between bearing the costs of debt repayment on the one hand or income losses in the wake of default on the other (see footnote 23).

where the multipliers reflect the two short-sale constraints. This can be expressed as[26]

$$\mathcal{L}'(b_2^e)\left(u'(c_1)\beta f_2(\tilde{b}_2-\mathcal{L}(b_2^e))b_2-\delta\mathbb{E}_1\left[\int_0^{\tilde{b}_2-\mathcal{L}(b_2^e)}u'(y_2-L_2-\mathcal{L}(b_2^e))\mathrm{d}F_2(L_2)\right]\right)$$
$$-u'(c_1)\left(\Delta_1(s_1,\pi_1)+b_2^e\frac{\partial\Delta_1(s_1,\pi_1)}{\partial b_2^e}\right)+\lambda-\mu. \tag{8}$$

The first part of this marginal effect reflects the benefit of stronger credibility on the one hand and the cost of reduced flexibility on the other. A larger share of official debt generates stronger repayment incentives and hence lower default risk; this raises $q_1(s_1,\pi_1)$ and the deficit, and it allows the country to consume more in the first period. But the larger share of official debt also inflicts additional income losses in case default nevertheless occurs subsequently which happens for low realizations of L_2. These income losses hurt particularly if marginal utility is high in default states. Conditional on the distribution F_2, the cost of reduced flexibility therefore is a more important concern when income y_2 negatively correlates with L_2 such that default states are states with low consumption. The second part of the marginal effect reflects the price difference between private and official loans, as well as the fact that changing the debt composition may affect this price difference.

Choice of Repayment Rate in the First Period The trade-off governing the choice of r_1 differs depending on whether $\xi_1=1$ or $\xi_1=r_1$. Consider first the latter case. When $\xi_1=r_1$ (and $b_{02}>0$), then the trade-off governing the choice or r_1 is a dynamic one because default does not only wipe out maturing debt but also long-term debt overhang. This reduces the probability of default in the second period and raises the price $q_1(s_1,\pi_1)$. For $r_1<1$, the net marginal benefit of reducing the repayment rate further is positive and as a consequence, the optimal repayment rate equals either zero or unity. Letting $G_1(s_1;r_1=\rho)$ denote the value of the government's program conditional on state s_1 and repayment rate $r_1=\rho$, the equilibrium choice thus satisfies

$$r_1(s_1)=\begin{cases}1 & \text{if } G_1(s_1;r_1=1)\geq G_1(s_1;r_1=0)\\ 0 & \text{if } G_1(s_1;r_1=1)<G_1(s_1;r_1=0)\end{cases}. \tag{9}$$

We discuss the interdependence between long-term debt overhang and the default decision (9) in more detail below.

If $\xi_1=1$, in contrast, default wipes out maturing debt, b_1, but not outstanding long-term debt, b_{02}. The choice of r_1 therefore does not affect the price of debt paid by private lenders, $q_1(s_1,\pi_1)$. If the same holds true for the price paid by official lenders, $p_1(s_1,\pi_1)$, (and thus, the price difference $\Delta_1(s_1,\pi_1)$) then the deficit is independent of r_1 as well and the repayment decision in the first period parallels the one in the second period, namely

$$r_1(s_1)=\begin{cases}1 & \text{if } L_1\geq b_1\\ 0 & \text{if } L_1<b_1\end{cases}.$$

[26]Note that $\partial\mathbb{E}_1[G_2(s_2)]/\partial b_2^e=-\mathcal{L}'(b_2^e)\mathbb{E}_1\left[\int_0^{\tilde{b}_2-\mathcal{L}(b_2^e)}u'(y_2-L_2-\mathcal{L}(b_2^e))\mathrm{d}F_2(L_2)\right]$ (see footnote 23).

Independence of $p_1(s_1, \cdot)$ and r_1 if $\xi_1 = 1$ may be a reasonable assumption in some environments but not in others. The assumption is satisfied in the particular specifications discussed above (see equations (3) and (4)) where the trade-offs present in the lending relationship between the enforcer and the borrowing country from period $t = 1$ onwards are independent of the repayment rate r_1. But it would not be satisfied if the enforcer's participation constraint held "before" r_1 were chosen. Equation (3) then would be replaced by

$$b_2^e p_1(s_1, \pi_1) = \beta b_2^e \, \mathbb{E}_1[r_2(s_2)] - \beta \, \text{Prob}[r_2(s_2) < 1] \, \mathcal{C}(b_2^e) + b_1^e r_1$$

and the equilibrium price of debt purchased by the enforcer would depend on the repayment rate in the first period. In this case, the enforcer would be indifferent between lowering r_1 by an amount ϵ and increasing p_1 by the amount $\epsilon b_1^e / b_2^e$. Such a combination of changes in r_1 and p_1 could strictly increase the welfare of the borrowing country if $b_1^e < b_1$, that is, if there were another group of investors that could be "burned."[27] Consequently, a default in the first period could be in the joint interest of the borrowing country and the enforcer.[28] We do not pursue this variation of the model here.

Properties of Equilibrium The equilibrium conditions make clear that the quantity of debt issued, the ownership structure, and the default choices depend on factors such as the intensity of the borrowing needs, as manifested in the ratio β/δ and the steepness of the output profile; the distribution function of output losses, $F_2(\cdot)$; preferences; the enforcement technology, $\mathcal{L}(\cdot)$; and the price discount, $\Delta_1(\cdot)$. Since the general model cannot be solved in closed form, the exact contribution of these factors is difficult to identify. Nevertheless, some general conclusions can be drawn, in particular relating to the effects of pari passu on the equilibrium debt ownership structure and to the interaction between this structure and the default decision in the first period.

Conditions (7) and (8) imply that four types of equilibria may emerge. Letting $\mathcal{M}(b_2, b_2^e)$ and $\mathcal{M}^e(b_2, b_2^e)$ denote the marginal effects without multipliers in (7) and (8), respectively, these four types can be summarized as follows:

i. $\mu = \lambda = 0$. b_2, b_2^e interior with $\mathcal{M}(b_2, b_2^e) = \mathcal{M}^e(b_2, b_2^e) = 0$.

ii. $\mu = 0, \lambda > 0$. b_2 interior, $b_2^e = 0$ with $\mathcal{M}(b_2, 0) = 0$, $\mathcal{M}^e(b_2, 0) < 0$.

iii. $\mu > 0, \lambda = 0$. $b_2 = b_2^e > 0$ with $\mathcal{M}(b_2, b_2) + \mathcal{M}^e(b_2, b_2) = 0$.

iv. $\mu > 0, \lambda > 0$. $b_2 = b_2^e = 0$ with $\mathcal{M}(b_2, b_2) + \mathcal{M}^e(b_2, b_2) < 0$.

Suppose for simplicity that the mark-down on official funds is constant, $1 - \Delta_1(s_1, \pi_1)/q_1(s_1, \pi_1) = \kappa$, and normalize (7) and (8) by the direct contribution of a marginal unit of debt to utility

[27] Naturally, a proper specification of the problem would require that such incentives are recognized and priced ex ante.

[28] Broner, Martin and Ventura (2010) argue that secondary markets undermine the ability of a sovereign to discriminate between groups of lenders. The above argument suggests that the borrowing country may collude with lenders rolling over its debt and discriminate against other holders of outstanding debt by choosing r_1 and p_1 appropriately.

in the first period, $u'(c_1)\beta(1-F_2(\tilde{b}_2-\mathcal{L}(b_2^e)))$. This yields

$$\mathcal{M}(b_2,b_2^e) \propto \left(1-\frac{\delta\mathbb{E}_1[u'(c_2^{\rm nd})]}{\beta u'(c_1)}\right)-(b_2-b_2^e(1-\kappa))H_2(\tilde{b}_2-\mathcal{L}(b_2^e)), \tag{10}$$

$$\mathcal{M}^e(b_2,b_2^e) \propto \mathcal{L}'(b_2^e)\times \left((b_2-b_2^e(1-\kappa))H_2(\tilde{b}_2-\mathcal{L}(b_2^e))-\frac{\delta\mathbb{E}_1\left[\int_0^{\tilde{b}_2-\mathcal{L}(b_2^e)}u'(c_2^{\rm d})\mathrm{d}F_2(L_2)\right]}{\beta u'(c_1)(1-F_2(\tilde{b}_2-\mathcal{L}(b_2^e)))}\right) -(1-\kappa). \tag{11}$$

Here, $H_2(\cdot)$ denotes the hazard function, $H_2(\cdot)\equiv f_2(\cdot)/(1-F_2(\cdot))$, and $c_2^{\rm d}$ and $c_2^{\rm nd}$ denotes consumption in default and non-default states, respectively. In (10), the term in parentheses represents the consumption smoothing benefit from the marginal unit of debt and the second term the negative price effect due to higher debt issuance, weighted by the quantity of debt. In (11), the term in parentheses represents the marginal benefit of credibility net of the cost of reduced flexibility, and the last term reflects the reduction in the deficit due to the marginal substitution towards marked-down official funds.

Under standard assumptions, the hazard function is weakly increasing. As a consequence, the debt-Laffer curve is hump shaped and for any given value of b_2^e, a unique value for b_2 solves the condition $\mathcal{M}(b_2,b_2^e)=0$.[29] Moreover, this value for b_2 tends to rise as b_2^e increases.[30] This is a direct consequence of pari passu in the sense that higher b_2^e improves the price elasticity of official *and private* funds.[31] But the model makes clear that this feature may not prevent crowding out of private funds—in contrast to what appears to be intended (see the discussion in the introduction). In fact, such crowding out takes place whenever b_2 increases by less than one-to-one with b_2^e. This is more likely to be the case when the functions $\mathcal{L}(\cdot)$ and $H_2(\cdot)$ are relatively flat.[32]

Pari passu also crucially affects the cost benefit analysis of official funds as represented by $\mathcal{M}^e(b_2,b_2^e)$. For the pari passu restriction to put official and private lenders on an equal footing at the time of repayment implies that strengthened credibility raises the prices of both official *and private* funds (as reflected in the first term of (11)). This gives rise to a complementarity between total and official lending that parallels the complementarity in (10) although it works in the opposite direction: While the complementarity in (11)

[29]The hazard function is closely related to the price elasticity of debt, for $b_2/q_1\partial q_1/\partial b_2=-b_2H_2$. Examples of distribution functions with increasing hazard functions include uniform, normal, exponential, logistic, extreme value, Laplace, power, Weibull, gamma, chi-squared, chi, or beta distributions (see, e.g., Bagnoli and Bergstrom, 2005). Abstracting from official lending, the deficit is given by $\beta(1-F_2)b_2$ and its derivative with respect to b_2 equals $\beta(1-F_2)(1-b_2H_2)$. With a weakly increasing hazard function, this derivative strictly decreases as b_2 increases (at least up to the top of the debt-Laffer curve).

[30]Suppose for simplicity that marginal utility is constant such that an interior b_2 is pinned down by the condition $1-\delta/\beta=(b_2-b_2^e(1-\kappa))H_2(b_{02}+b_2-\mathcal{L}(b_2^e))$. If $\mathcal{L}(\cdot)$ and $H_2(\cdot)$ are increasing and $\kappa\leq 1$ then a rise of b_2^e leads b_2 to increase as well.

[31]Without pari passu, a higher quantity of official loans is likely to increase default risk on private loans, depending on the exact relationship between the then, two default choices and default costs.

[32]For example, if $F(\cdot)$ is exponential such that the hazard function is constant and if marginal utility is constant as well then an increase of b_2^e leaves the equilibrium value of b_2 unaltered. Official lending then fully crowds out private lending.

arises because a higher quantity of total borrowing increases the benefit of strengthened credibility, the complementarity in (10) occurs because a higher quantity of official lending affects the debt-Laffer curve in a favorable way. Higher b_2 therefore increases the marginal benefit of b_2^e while higher b_2^e increases the net benefit of total borrowing, and it is precisely the pari passu restriction that is at the source of these complementarities.

Absent sufficient curvature of $u(\cdot), F(\cdot)$ or $\mathcal{L}(\cdot)$ the complementarities may give rise to corner solutions for the debt composition. This is confirmed by the analysis of the special cases of the model discussed below. Numerical simulations of the model under general functional form assumptions suggest that although interior debt compositions are not unusual, the forces that push the debt composition into a corner in the fully linear case seem to operate strongly also in the general case.

Turning to the interaction between the composition of fresh funds and the default decision in the first period, consider the role of long-term debt overhang, $b_{02}\xi_1 > 0$. For given values of b_2 and b_2^e, long-term debt overhang has two consequences for the marginal effects $\mathcal{M}(b_2, b_2^e)$ and $\mathcal{M}^e(b_2, b_2^e)$. On the one hand, it lowers the price of newly issued debt (and thus, the deficit) and changes the elasticity of the price, as is evident from the fact that the density functions $F_2(\cdot)$ and $f_2(\cdot)$ in (10) and (11) depend on $\tilde{b}_2$. This affects the marginal benefit of both b_2 and b_2^e. On the other hand, long-term debt overhang increases the marginal expected cost of enforcer funds, due to reduced flexibility in the future, as reflected by the term $-\mathcal{L}'(b_2^e)\int_0^{\tilde{b}_2-\mathcal{L}(b_2^e)} u'(c_2^{\mathrm{d}})\mathrm{d}F_2(L_2)$ in (11). Long-term debt overhang therefore may reduce the attractiveness of official relative to private funding, and precisely for this reason, it may encourage default when refinancing from official sources is particularly desirable. We discuss this in more detail in one of the following examples.

4 Examples

In order to characterize equilibrium in closed form and present solutions that shed light on the first-order determinants of default choices and the debt ownership structure, we abstract from all non-essential sources of non-linearity.[33] (At the end of the section, we briefly discuss the consequences of relaxing the linearity assumption.) In particular, we let $u'(c) = 1$, $\mathcal{L}'(b_2^e) = \mathcal{L}'$ with $0 \leq \mathcal{L}' < 1$, and $F_2'(L_2) = f_2$ over the relevant range.[34] This implies (net of some constants)

$$\begin{aligned} G_1(s_1) \;=\; \max_{r_1\in[0,1],\ (b_2,b_2^e)\in B(b_{02}\xi_1)} & -b_1 r_1 - \mathbf{1}_{[r_1<1]}L_1 + \beta(1 - f_2\cdot(\tilde{b}_2 - \mathcal{L}'b_2^e))b_2 - \Delta_1(s_1,\pi_1)b_2^e \\ & - \delta\left\{\int_0^{\tilde{b}_2-\mathcal{L}'b_2^e}(L_2+\mathcal{L}'b_2^e)f_2\mathrm{d}L_2 + (1-f_2\cdot(\tilde{b}_2-\mathcal{L}'b_2^e))\tilde{b}_2\right\}. \end{aligned} \tag{12}$$

We highlight the role played by the intensity of borrowing needs, β/δ, enforcement power, $\mathcal{L}'$, the price discount, $\Delta(\cdot)$, and long-term debt overhang, $b_{02}\xi_1$, by working

[33] This comes at the cost of ignoring the hedging benefits of privately held (and thus, riskier) debt.

[34] The restriction $\mathcal{L}' < 1$ is required for a debt-Laffer curve to exist. Without it, official lending could completely eliminate default risk. The analysis and implications of the case of $\mathcal{L}' \geq 1$ are straightforward. Note that if $\mathcal{L}' \geq 1$ and $\Delta_1(s_1, \pi_1) = 0$, then the country attains the commitment outcome.

through a series of examples. These examples illustrate that the model can account for the issuance of debt to official lenders in periods of debt distress (and no such issuance in other periods) at yields that appear favorable to the borrower compared with the yields that would have to be paid on private markets. They also illustrate the general properties of equilibrium discussed earlier, including the interaction between debt overhang, default and the composition of new borrowing.

Exogenous Price Discount, No Long-Term Debt Overhang Suppose that funds provided by the enforcer carry an exogenous, constant price discount relative to funds obtained from private investors, $p_1(s_1, \pi_1) = \kappa q_1(s_1, \pi_1)$ with $\kappa \leq 1$, and let $b_{02} = 0$. The constant price discount implies $\Delta_1(s_1, \pi_1) \equiv q_1(s_1, \pi_1)(1 - \kappa)$ and the marginal effects defined earlier equal

$$\begin{aligned} \mathcal{M}(b_2, b_2^e) &= (1 - F_2)(\beta - \delta) - \beta f_2(b_2 - b_2^e(1 - \kappa)), & (13) \\ \mathcal{M}^e(b_2, b_2^e) &= \mathcal{L}'(\beta f_2(b_2 - b_2^e(1 - \kappa)) - \delta F_2) - \beta(1 - F_2)(1 - \kappa) & (14) \end{aligned}$$

where $F_2 \equiv f_2 \cdot (b_2 - \mathcal{L}' b_2^e)$ denotes the probability of default. Holding b_2^e constant, G_1 is concave in b_2. The determinant of the Hessian is negative, and the Hessian thus is indefinite.[35] This implies that any interior critical point of (12) constitutes a saddle point such that the equilibrium necessarily is in a corner. We consider the two interesting corner equilibria—one with private debt and the other with official debt—in turn. The uninteresting third corner equilibrium with zero debt is ruled out by assuming that δ/β is sufficiently small.

If sovereign debt is exclusively funded from private sources then $\mathcal{M}(b_2, 0) = 0$. Solving for the equilibrium yields the following values for the debt levels and the government's program in the first period (for $\delta \leq \beta$):[36]

$$b_2^{\mathrm{PR}} = \frac{1}{f_2} \frac{\beta - \delta}{2\beta - \delta}, \quad b_2^{e\,\mathrm{PR}} = 0, \quad G_1^{\mathrm{PR}} = \frac{1}{2f_2} \frac{(\beta - \delta)^2}{2\beta - \delta}.$$

The maximum of the debt-Laffer curve is obtained at the debt level $(2f_2)^{-1}$, the level chosen by a myopic government with $\delta = 0$, and is associated with a default probability of $1/2$.

If instead all debt is funded from official sources then $\mathcal{M}(b_2, b_2) + \mathcal{M}^e(b_2, b_2) = 0$ and the equilibrium values (for $\delta \leq \beta\kappa$) are given by

$$b_2^{\mathrm{OF}} = \frac{1}{f_2} \frac{\beta\kappa - \delta}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}, \quad b_2^{e\,\mathrm{OF}} = b_2^{\mathrm{OF}}, \quad G_1^{\mathrm{OF}} = \frac{1}{2f_2} \frac{(\beta\kappa - \delta)^2}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}.$$

The maximum of the debt-Laffer curve is now obtained at the debt level $(2f_2(1 - \mathcal{L}'))^{-1}$ and is again associated with a default probability of $1/2$. For $\mathcal{L}' > 0$, the debt level attaining the maximum of the debt-Laffer curve is higher in the corner with official than with private debt.

[35] In the special case of $1 - \kappa = \mathcal{L}'$ the determinant is zero. See, for example, Simon and Blume (1994, Theorem 16.1).

[36] Unless otherwise noted, we let $b_1 = L_1 = 0$.

Comparing the outcomes in the two cases, note that $G_1^{\mathrm{OF}} > G_1^{\mathrm{PR}}$ whenever $b_2^{\mathrm{OF}}(\beta\kappa - \delta) > b_2^{\mathrm{PR}}(\beta - \delta)$.[37] Consequently, $G_1^{\mathrm{OF}} > G_1^{\mathrm{PR}}$ implies $b_2^{\mathrm{OF}} > b_2^{\mathrm{PR}}$ and thus, countries that borrow from official sources tend to be more heavily indebted than countries borrowing from private sources. To understand the country's choice of debt instrument consider first the case of $\delta = 0$. We found above that the debt level corresponding to the maximum of the debt-Laffer curve is higher in the corner with official debt. But this does not imply that a myopic government that aims at maximizing the deficit necessarily chooses official over private debt since the former may be lower priced. In fact, comparing G_1^{PR} with G_1^{OF} for $\delta = 0$ reveals that the borrower will opt for official debt if and only if $1 - \kappa < \mathcal{L}'$ that is, if the positive effect of stronger credibility on prices outweighs the mark-down. In the following, we posit that this condition is met so that a myopic government would favor issuing debt to official creditors.

In the range $0 \leq \delta \leq \beta\kappa$ where both b_2^{PR} and b_2^{OF} are positive the criterion for the choice of debt instrument is

$$G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}} = \frac{1}{2f_2}\left[\frac{(\beta\kappa - \delta)^2}{2\beta\kappa - \delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'} - \frac{(\beta-\delta)^2}{2\beta - \delta}\right].$$

For $\kappa = 1$, this expression is positive and official debt is preferred since it generates benefits of credibility at no cost. For $\kappa < 1$, $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ is strictly positive for $\delta = 0$, negative for $\delta = \beta\kappa$ and convex in δ in-between implying that there exists a unique threshold value $\delta^\star$ such that for $\delta \leq \delta^\star$ (high borrowing needs) official funding is preferred while for $\delta > \delta^\star$ (low borrowing needs) private funding is preferred. The model thus predicts that episodes of high borrowing needs (as captured by a low δ/β ratio) are associated with borrowing from official rather than private sources.

Figure 1 presents a numerical example. It plots the difference $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ against δ for $\beta = 0.9$ and $f_2 = 0.1$. The solid curve corresponds to intermediate values of enforcement power ($\mathcal{L}' = 0.25$) and price discount ($\kappa = 0.9$). Holding δ fixed, the difference $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ increases if $\mathcal{L}'$ is raised (dashed curve for $\mathcal{L}' = 0.4$) and decreases if κ is lowered (dotted curve for $\kappa = 0.8$). Stronger enforcement power therefore raises $\delta^\star$ and renders official funding more likely while higher price discounts lower $\delta^\star$ and increase the relative advantage of private funding. Under the maintained assumptions, these intuitive comparative statics results hold for arbitrary parameter combinations.

The default risk $F_2(b_2 - \mathcal{L}(b_2^e))$ along the equilibrium path is given by $f_2 b_2^{\mathrm{PR}} = (\beta - \delta)/(2\beta - \delta)$ if the sovereign opts to borrow from private lenders and $f_2 b_2^{\mathrm{OF}}(1 - \mathcal{L}') = (\beta\kappa - \delta)/(2\beta\kappa - \delta(1-\mathcal{L}'))$ if it borrows from official sources. As long as $\kappa \leq 1$ the default risk under private lending always exceeds the one under official lending. In equilibrium (where the government chooses between the two sources of funding) default risk therefore is weakly lower than if the sovereign only had the option to borrow from private lenders. More favorable funding conditions offered by official lenders (higher values of κ) increase the amount of borrowing from official sources and reduce the difference between the two default rates.

Turning to prices, a given amount of debt, b_2, carries the price $\beta(1 - f_2 \cdot b_2)$ when issued to private lenders and $\kappa\beta(1 - f_2 \cdot b_2(1-\mathcal{L}'))$ when issued to official lenders. A given

[37] This follows from the fact that $G_1^{\mathrm{OF}} = b_2^{\mathrm{OF}}(\beta\kappa - \delta)/2$ and $G_1^{\mathrm{PR}} = b_2^{\mathrm{PR}}(\beta - \delta)/2$.

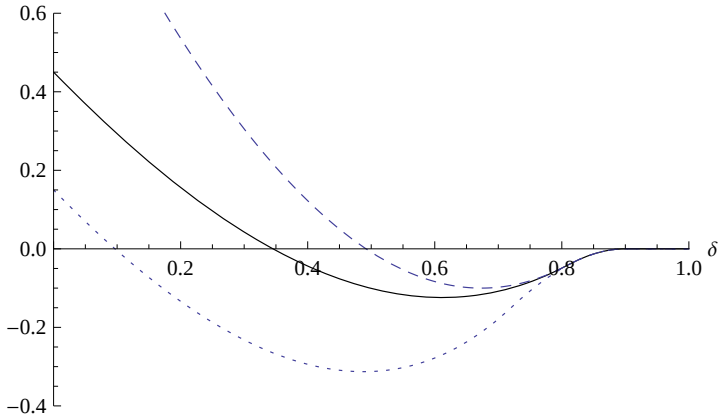

Figure 1: $G_1^{\text{OF}} - G_1^{\text{PR}}$ as function of δ. Higher $\mathcal{L}'$ shifts the curve up (dashed line), lower κ shifts the curve down (dotted line).

amount of debt therefore is cheaper when financed from official sources if and only if

$$\mathcal{L}' \geq \frac{1 - f_2 b_2}{f_2 b_2} \frac{1 - \kappa}{\kappa}.$$

Strong enforcement power, large levels of debt and a small mark-down on official funds (a large value for κ) all contribute to lowering the interest rate on official debt relative to the one on private debt.

Endogenous Price Discount, No Long-Term Debt Overhang Consider next the case where the price discount is determined endogenously as the outcome of bargaining between the sovereign and the enforcer. In the simplest case, all bargaining power lies with the sovereign and default generates a cost $\mathcal{C}(b_2^e)$ to the enforcer (in addition to the capital loss). The binding participation constraint of the enforcer (3) then reads

$$b_2^e p_1(s_1, \pi_1) = b_2^e \beta (1 - F_2) - \beta F_2 \, \mathcal{C}(b_2^e),$$

where, as before, we let $F_2 \equiv f_2 \cdot (b_2 - \mathcal{L}' b_2^e)$. If the cost is linear, $\mathcal{C}'(b_2^e) = \mathcal{C}' \geq 0$, then this participation constraint simplifies to

$$p_1(s_1, \pi_1) = q_1(s_1, \pi_1) - \beta F_2 \mathcal{C}' = \beta (1 - F_2)(1 + \mathcal{C}') - \beta \mathcal{C}'. \tag{15}$$

The properties of the equilibrium in this example are similar to those obtained previously. The equilibrium is in a corner. If sovereign debt is exclusively funded from private sources, the level of debt and the value of the government's program remain unchanged relative to the previous example. But if all debt is funded from official sources then the equilibrium (for $\delta \leq \beta$) is characterized by

$$b_2^{\text{OF}} = \frac{1}{f_2} \frac{\beta - \delta}{2\beta(1 + \mathcal{C}') - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}, \quad G_1^{\text{OF}} = \frac{1}{2 f_2} \frac{(\beta - \delta)^2}{2\beta(1 + \mathcal{C}') - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}.$$

The maximum of the debt-Laffer curve now is at the debt level $(2f_2(1+\mathcal{C}')(1-\mathcal{L}'))^{-1}$, the level chosen by a myopic government, and yields a default probability of $1/2(1+\mathcal{C}')$. Consequently, as long as $(1+\mathcal{C}')(1-\mathcal{L}') \leq 1$ and $\delta = 0$, more debt is issued when the source is official rather than private.

As far as the choice of the debt instrument in the range $0 \leq \delta \leq \beta$ is concerned, the desirability of official relative to private funds is determined by

$$G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}} = \frac{1}{2f_2}(\beta-\delta)^2 \left[\frac{1}{2\beta(1+\mathcal{C}')-\delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'} - \frac{1}{2\beta-\delta}\right] = \frac{\beta-\delta}{2}(b_2^{\mathrm{OF}} - b_2^{\mathrm{PR}})$$

and official funding is preferred if and only if $b_2^{\mathrm{OF}} \geq b_2^{\mathrm{PR}}$.

The difference $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ is positive at $\delta = 0$ if $(1+\mathcal{C}')(1-\mathcal{L}') \leq 1$, attains a zero in the interval $[0,\beta)$ if $\mathcal{L}'(2\mathcal{C}'+\mathcal{L}') < 2\mathcal{C}'$, and always attains a zero at $\delta = \beta$. Hence, if the first two conditions are satisfied, there exists a unique threshold value $\delta^\star$ such that for $\delta \leq \delta^\star$ (high borrowing needs) official funding is preferred while the opposite holds for $\delta > \delta^\star$. The threshold value increases with $\mathcal{L}'$, as in the previous example, and falls with $\mathcal{C}'$. This is intuitive since a higher $\mathcal{C}'$ increases the expected costs (beyond capital losses) that the enforcer bears in case of default; in order to compensate for these expected costs, the enforcer requires a premium relative to the rate charged by private debt buyers. An increase of $\mathcal{C}'$ therefore has the same qualitative effect on $\delta^\star$ as a decrease of κ in the previous example.

As far as the price of funds is concerned, a fixed quantity of debt b_2 carries a higher interest rate when raised from private sources. The price for such debt equals $\beta(1-f_2b_2)$ while the price for the same quantity of debt issued to official creditors equals $\beta(1-f_2b_2(1-\mathcal{L}'))(1+\mathcal{C}') - \beta\mathcal{C}'$ (from (15)) which is larger than $\beta(1-f_2b_2)$ under the first condition described above.

These findings are robust to changing the specification of the cost function $\mathcal{C}(\cdot)$. Suppose, for example, that costs are not proportional but contain a fixed component so that $\mathcal{C}(b_2^e) = c > 0$ if $b_2^e > 0$ and $\mathcal{C}(b_2^e) = 0$ if $b_2^e = 0$. The enforcer's participation constraint (3) satisfied at equality then reads

$$p_1(s_1,\pi_1) = q_1(s_1,\pi_1) - \beta f_2 \cdot (b_2 - \mathcal{L}'b_2^e)c/b_2^e$$

and equilibrium is again at a corner. Under conditions guaranteeing $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}} > 0$ at $\delta = 0$, an increase in c reduces the threshold value $\delta^\star$ at which $G_1^{\mathrm{OF}} = G_1^{\mathrm{PR}}$. That is, a higher fixed cost c has the same qualitative effect on $\delta^\star$ as higher variable costs $\mathcal{C}'$ or a lower κ in the previous examples.[38]

Exogenous Price Discount, Long-Term Debt Overhang Finally, consider the consequences of long-term debt overhang, $b_{02}\xi_1 > 0$, for the choice of debt instrument as well

[38] We have (for $\delta \leq \beta(1-cf_2(1-\mathcal{L}'))$)

$$b_2^{\mathrm{OF}} = \frac{1}{f_2}\frac{\beta(1-cf_2(1-\mathcal{L}'))-\delta}{2\beta-\delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'}, \quad G_1^{\mathrm{OF}} = \frac{1}{2f_2}\frac{(\beta(1-cf_2(1-\mathcal{L}'))-\delta)^2}{2\beta-\delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'}.$$

as the decision to default in the first period. In parallel to the first example, we assume an exogenous and constant mark-down, $p_1(s_1, \pi_1) = \kappa q_1(s_1, \pi_1)$. The marginal effects $\mathcal{M}(b_2, b_2^e)$ and $\mathcal{M}^e(b_2, b_2^e)$ then are unchanged relative to (13) and (14) except that the probability of default, F_2, is given by $f_2 \cdot (\tilde{b}_2 - \mathcal{L}' b_2^e)$ rather than $f_2 \cdot (b_2 - \mathcal{L}' b_2^e)$. We also assume that $f_2 b_{02} \xi_1 < 1$ so that the probability of default is smaller than one and new debt issuance depresses the price of debt.

If sovereign debt is exclusively funded from private sources then the equilibrium level of debt (for $\delta \leq \beta$) is given by

$$b_2^{\mathrm{PR}} = \frac{1}{f_2} \frac{\beta - \delta}{2\beta - \delta} (1 - f_2 b_{02} \xi_1).$$

Less debt—by a factor of $(1 - f_2 b_{02} \xi_1)$—is issued relative to the case without long-term debt overhang. This is due to the fact that outstanding long-term debt renders default more likely, raising the (absolute value of the) price elasticity and reducing the attractiveness of new debt issuance.

When all debt is financed by official sources then the equilibrium debt level (for sufficiently low values for δ) is given by

$$b_2^{\mathrm{OF}} = \frac{1}{f_2} \frac{\beta\kappa - \delta}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'} (1 - f_2 b_{02} \xi_1) - \frac{1}{f_2} \frac{\delta \mathcal{L}' f_2 b_{02} \xi_1}{2\beta\kappa - \delta(1 - \mathcal{L}')} \frac{1}{1 - \mathcal{L}'}.$$

As in the case with private refinancing, outstanding long-term liabilities reduce the incentive to issue new debt because they push the borrowing country closer to the top of the debt-Laffer curve. This effect is reflected in the wedge $(1 - f_2 b_{02} \xi_1)$. But long-term debt overhang carries an additional cost when accompanied by the issuance of official debt because it increases the marginal expected cost of enforcer funds due to reduced future flexibility, as discussed in the context of the general model. This lowers the optimal new debt issuance, as reflected in the right-most term in the expression for b_2^{OF}.

If $\delta = 0$ the costs due to reduced future flexibility are not internalized by the government in the first period. Debt overhang therefore affects the trade-off between official and private funds only through its effect on the price elasticity. In particular, the difference between the two values is given by

$$G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}|_{\delta=0} = -\frac{\beta(1 - \kappa - \mathcal{L}')}{4 f_2 (1 - \mathcal{L}')} (1 - f_2 b_{02} \xi_1)$$

such that official funds are preferred over private funds whenever $1 - \kappa < \mathcal{L}'$ (exactly as without debt overhang). While sufficiently strong credibility gains and a low mark-down lead the government to prefer official over private funds this relative attractiveness decreases with the size of the debt overhang.

When $\delta > 0$ the government internalizes costs due to reduced future flexibility and debt overhang reduces the relative attractiveness of official refinancing for this second reason as well. Figure 2 plots the difference $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ as a function of δ for different values of $b_{02}\xi_1$. The parameter values are as in the example without long-term debt overhang. We also assume that $b_1 = L_1 = 0$. The solid line corresponds to $b_{02}\xi_1 = 0$,

the dashed line to $b_{02}\xi_1 = 1$ and the dotted line to $b_{02}\xi_1 = 3$. The figure shows that official debt becomes less desirable (the threshold value of $\delta^\star$ for the choice of official debt becomes smaller) as outstanding debt increases. Intuitively, higher long-term debt overhang places the economy closer to the top of the debt-Laffer curve where the benefits from higher credibility are outweighed by the costs associated with reduced flexibility in the future and the mark-down.

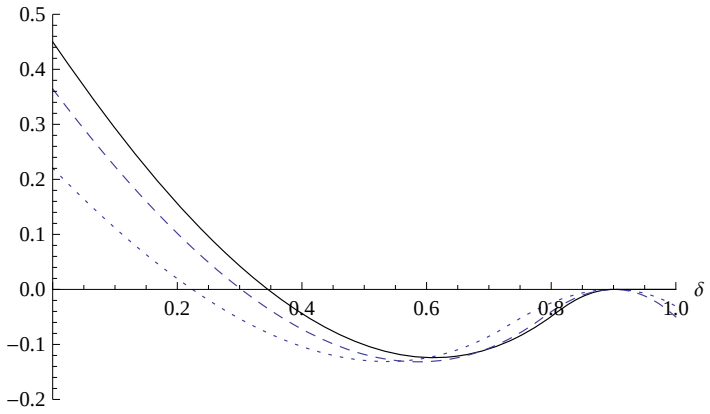

Figure 2: $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ as function of δ. Higher $b_{02}\xi_1$ reduces $\delta^\star$.

Turning to the default decision in the first period, note that this decision is independent of the stock of long-term debt overhang if the government can only default on currently maturing debt ($\xi_1 = 1$). In this case, the threshold value $\hat{L}_1$ at which it becomes optimal to default equals $\hat{L}_1 = b_1$, as without outstanding debt.

If default applies to both maturing and outstanding debt ($\xi_1 = r_1$) then $\hat{L}_1$ exceeds b_1 whenever $b_{02} > 0$ (see (9)) and the sovereign's incentive to default increases with the stock of outstanding long-term debt.[39] As discussed above, this incentive can be particularly strong if refinancing is provided from official sources because in this case default moves the country away from the top of the debt-Laffer curve and also reduces the marginal expected cost of enforcer funds due to reduced flexibility in the future.

Figure 3 illustrates this (ignore the solid curve for now). It displays the threshold values $\hat{L}_1$ as a function of δ and conditional on refinancing from either of the two sources of fresh funds. The default threshold $\hat{L}_1^{\mathrm{PR}}$ applies when new debt must be financed by private investors; and the threshold $\hat{L}_1^{\mathrm{OF}}$ applies when it must be issued to official lenders. Default occurs for realizations of L_1 below the relevant loci. For $b_{02} = 0$, the default thresholds are independent of δ and the two loci coincide and are flat at level b_1. For $b_{02} > 0$, as in the example illustrated in the figure (where $b_1 = 0, b_{02} = 2$), the loci have a non-zero slope because default reduces $b_{02}r_1$ to zero, and the effect of this change on the value of the government's program depends on δ.[40] More to the point, the figure

[39] In two different environments, one with $\xi_1 = r_1$ and the other with $\xi_1 = 1$, the cost of defaulting might differ. We disregard such differences as they are irrelevant for our analysis because we do not compare outcomes across environments.

[40] A reduction of $b_{02}r_1$ affects the price of new debt, the equilibrium quantity of debt issued as well as

shows that for low values of δ, intermediate realizations of L_1 (for instance, $L_1 = 0.9$ for $\delta = 0$) induce the sovereign to default if refinancing is provided from official but not if it is provided from private sources.

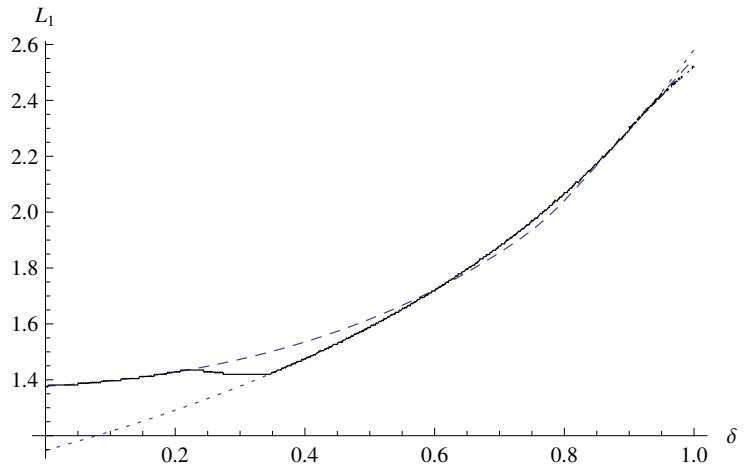

Figure 3: $\hat{L}_1^{\mathrm{PR}}$ (dotted), $\hat{L}_1^{\mathrm{OF}}$ (dashed), $\hat{L}_1$ (solid) as functions of δ.

The solid line in figure 3 represents the equilibrium default threshold $\hat{L}_1$ as a function of δ. It coincides with the default threshold conditional on official refinancing, $\hat{L}_1^{\mathrm{OF}}$, whenever the government *chooses* to borrow from official sources independently of the realization of L_1. This is the case when δ is low such that funding needs are strong. With a low realization of L_1, the country then defaults, long-term debt overhang is wiped out and the benefits of credibility make it optimal to seek official funds. For high realizations of L_1, in contrast, the country does not default and long-term debt overhang persists but the country still prefers official funds because of the low importance (due to the low value of δ) it attaches to future flexibility.

When δ is high the government's choice of refinancing source also is independent of the realization of L_1 but in this case borrowing from private sources is always preferred. The equilibrium default threshold therefore coincides with the default threshold conditional on private refinancing. For intermediate values of δ (in an interval around 0.3), though, default and refinancing decisions interact and the equilibrium default threshold differs from both $\hat{L}_1^{\mathrm{OF}}$ and $\hat{L}_1^{\mathrm{PR}}$. Default occurs less often than when the government can only tap official funds but more often than in the case where the government can only borrow from private sources. Low realizations of L_1 and the ensuing default lead the government to seek official funds with their credibility benefits while conditional on high realizations of L_1 and the ensuing non-default, the presence of honored long-term debt overhang renders official refinancing unattractive. As a consequence, the choices of repayment rate and refinancing source are fully correlated in this intermediate range of δ values.

Figure 4 summarizes how the default decision and the debt ownership structure vary

the amount of long- and short-term debt to be serviced in the future. The price, the quantity, and the weight attached to the future all depend on δ.

with δ and L_1 (ignore the dashed curves for now). The government defaults when L_1 is small and it seeks refinancing from official sources for low values of δ. The solid demarcation lines in the figure identify four resulting policy regions in (δ, L_1)-space. The dashed demarcation lines identify the same policy regions under the assumption of a higher debt overhang ($b_{02} = 3$ instead of $b_{02} = 2$). The increase in the stock of outstanding long-term debt has two effects on the regions. First, it increases the horizontal demarcation line separating the regions with $r_1 = 0$ from those with $r_1 = 1$. Intuitively, a default improves both the price elasticity of fresh funds and the continuation value more strongly if the debt overhang is higher, independently of the source of refinancing. Second, higher debt overhang shifts to the left the upper part of the vertical demarcation line separating the top-left and top-right policy regions ($r_1 = 1$ and $b_2^e > 0$ versus $r_1 = 1$ and $b_2^e = 0$). Intuitively, when default is too costly higher debt overhang reduces the attractiveness of official relative to private refinancing, both because of the higher price elasticity and the higher marginal expected cost of enforcer funds due to reduced future flexibility (see figure 2 and the discussion surrounding it).

Figure 5 illustrates the equilibrium debt issuance in the first period, b_2. New debt issuance is higher after a default (low realizations of L_1) because the latter eliminates long-term debt overhang and thereby improves access to new funding. Default therefore increases consumption both directly and indirectly. With a higher stock of outstanding long-term debt (case not depicted in the figure) new debt issuance falls for high values of L_1 because the government does not default in those states and higher debt overhang renders new funding less attractive. For low values of L_1 the quantity and price of new funds does not change relative to the situation with lower debt overhang because the overhang is wiped out anyway.

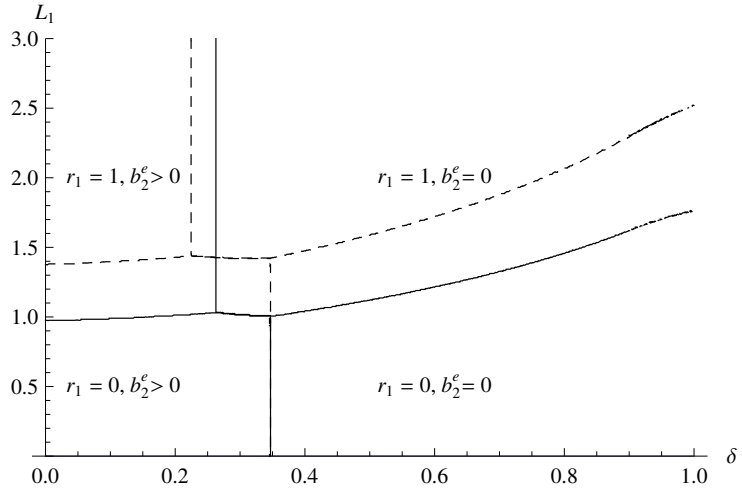

Figure 4: Default and official lending regions for $b_{02} = 2$ (solid) and $b_{02} = 3$ (dashed).

In sum, figures 3–5 illustrate how default and refinancing decisions interact and more specifically, how the availability of official funding may increase default risk on outstanding debt. In fact, figure 4 lends itself to a compact characterization of country experiences

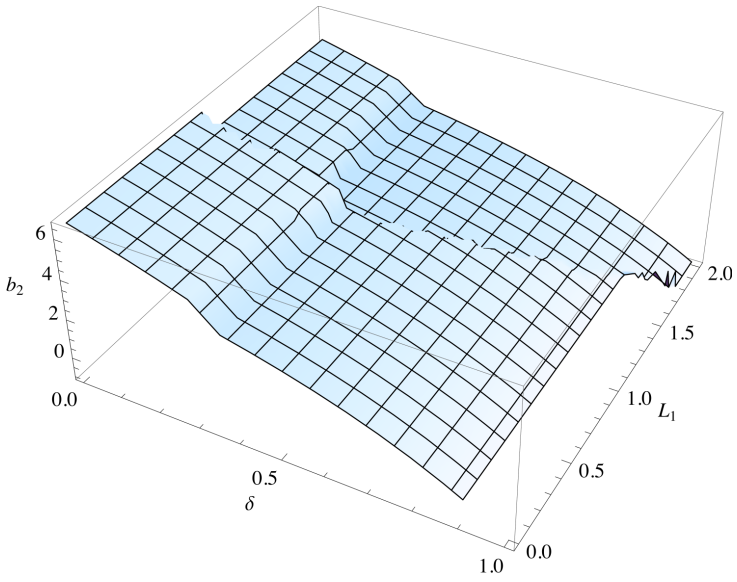

Figure 5: Total borrowing as function of δ and L_1.

during the recent Eurozone sovereign debt crisis. Consider a sudden increase in refinancing needs, indicated by a fall of δ below the lower part of the vertical demarcation line. The model then predicts the following: First, countries that choose to default receive refinancing from official sources. The experience of Greece supports this prediction. Second, countries that choose not to default may seek refinancing from official or private sources, with heavily indebted countries (high debt overhang) less likely to opt for official funds. The experiences of Ireland, Italy and Portugal are consistent with this prediction while the experience of Spain may be considered consistent or not.[41]

Finally, the model has another, somewhat provocative implication. As emphasized several times, the availability of official funding may increase default risk on outstanding debt when refinancing needs are high. Interestingly, it may not only be the borrowing country that favors default in these circumstances. Official creditors may favor it as well since they profit from debt they buy as long as $\kappa < 1$ and, as a consequence, from any increase in the demand for official funds.

Consequences of Relaxing the Linearity Assumptions Relaxing the linearity assumptions may give rise to an interior equilibrium debt composition. For example, maintaining the assumption of linear utility and a uniform distribution function but letting $\mathcal{L}(\cdot)$ be strictly concave rather than linear produces a strictly concave objective function $G_1(s_1)$ if $\kappa < 1$. Depending on parameter values the short-selling constraints may still

[41] At the time when official funds were provided, the debt quotas of Ireland and Portugal (which received refinancing from official sources) were lower than the quota of Italy which did not receive such refinancing. Spain had an even lower debt quota. While it has not actually received official funds the Eurogroup committed to provide up to 100 billion Euro. IMF (2012, p. 17) contains deficit and debt quota data.

bind in this case and induce a corner solution. But if they do not bind, the model predicts an interior equilibrium debt composition, in addition to the interior total quantity of borrowing.

5 Concluding Remarks

During the recent sovereign debt crisis in the Eurozone, patterns pertaining to default decisions and the composition of debt have varied significantly across debt distressed countries. While Greece defaulted and sought refinancing from official sources, Ireland, Portugal and Spain did not default but still drew official funds (or accepted a Eurogroup commitment for conditional support), and Italy neither defaulted nor relied on official loans. We have proposed a simple theory that helps shed light on these patterns.[42]

Our model adds two features to the standard sovereign debt framework: Differential enforcement powers across groups of creditors, and the pari passu provision. These features combine to produce default and debt composition decisions that seem consistent with the empirical evidence, for example default accompanied by "inexpensive" official lending; substitution of official for private debt in times of debt distress, with a tendency towards larger debt quantities when funds are drawn from official sources; crowding out of private debt even under pari passu; and resistance to draw official funds at favorable rates by troubled but non-defaulting countries with large long-term debt obligations. Another prediction of the model, namely that official would be creditors may encourage a sovereign to default on outstanding private debt is unexpected, yet equally plausible under the assumption that these creditors are in a position to extract rents.

An advantage of our model lies in its simplicity. In particular, the setup generates cross-country variation of government choices without incorporating political economy considerations. But this omission also represents an opportunity to expand the model's explanatory power by including such relevant elements. Future research could attempt this fusion. It could also address normative questions, for example with respect to the welfare implications of pari passu provisions or, more generally, the optimal seniority structure of official and private loans.

Finally, note that our analysis is quite general and applies equally well to credit relationships that do not involve sovereign debt as long as classes of creditors differ in terms of the punishment they can inflict on delinquent debtors.

[42]For a recent view related to the one proposed in this paper, see "The eurozone's journey to defaults," *Financial Times*, March 11, 2011.

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