

Central bank liquidity provision, risk-taking and economic efficiency

Ulrich Bindseil, Juliusz Jablecki*

Abstract

After the Lehman default, but also during the euro area sovereign debt crisis, central banks have tended to extend the ability of banks to take recourse to central bank credit operations through changes of the collateral framework (e.g. Markets Committee, 2013 – in consistence with previous narratives, such as Bagehot, 1873). We provide a simple four-sector model of the economy in which we illustrate the relevant trade-offs, derive optimal central bank collateral policies, and show why in a financial crisis, in which liquidity shocks become more erratic and the total costs of defaults increase, central banks may want to allow for greater potential recourse of banks to central bank credit. The model also illustrates that the credit riskiness of counterparties and issuers is endogenous to the central bank's credit policies and related risk control framework. Finally, the model allows identifying the circumstances under which the counterintuitive case arises in which a relaxation of the central bank collateral policy may reduce its expected losses.

Keywords: central bank, risk-taking, collateral policy, economic efficiency

JEL classification codes: E58, G32

1 Introduction

It is well known ever since the 19th century experience of the Bank of England, as documented in Bagehot (1873) or King (1936), that in financial crises central banks should become lenders of last resort to the economy, while taking into account financial risk management and moral hazard concerns. In this paper we propose a simple model that integrates the issue of central bank lender of last resort policies and financial risk management. The model is driven by both liquidity and solvency shocks hitting financial institutions, and by how the two are correlated. The model allows to derive optimal collateral eligibility and haircut levels for

*UB: European Central Bank, DG Market Operations, email:ulrich.bindseil@ecb.int; JJ: Economic Institute, National Bank of Poland and Faculty of Economic Sciences, Warsaw University, email: juliusz.jablecki@nbp.pl. We wish to thank Benoit Coeuré, Benjamin Sahel, as well as colleagues from the Risk Management Office at the ECB, in particular Carlos Bernadell, Alessandro Calza, Fabian Eser, Fernando Gonzalez, Thomas Kostka, Andreas Manzanares, Fernando Monar and Stephan Sauer for useful comments. Our thanks also go to participants of the ECB seminar on 7 December 2012, in particular: Philippine Cour-Thiemann, Diego Palenzuela, Philippe de Rougemont, Edgar Vogel and Bernhard Winkler, as well as participants of the Fields Quantitative Finance Seminar on 27 February 2013 organized by the Fields Institute, Toronto. All remaining mistakes are ours. The views in this paper do not necessarily reflect the views of the respective central banks.

central banks. We thereby integrate also two normally remote strands of central banking literature, namely on financial risk management and on the management of liquidity crises. Consider briefly each of those strands.

Modern literature on central bank risk management has developed in parallel to the rise of credit and market risk exposures stemming from the lengthening of central banks' balance sheets over the last 15 years.¹ Bernadell, Cardon, Coche, Diebold, and Manganelli (2004) is the first volume dedicated entirely to central bank financial risk management, but focuses only on investment operations and foreign exchange reserves. The risk management solutions proposed therein seem to be largely applicable to any institutional investor. Bindseil, Gonzalez, and Tabakis (2009) covers both central bank investment and policy operations, and aims at elaborating on what makes the central bank special in terms of optimal management of its financial risks. Bindseil (2009) notes first a number of specificities of any public investor, and then a number of specificities of central banks. Part II of the volume deals specifically with policy operations and risk measurement and management for collateralized credit operations of central banks undertaken in the context of monetary policy operations.

The theoretical literature on central bank financial crisis management has so far focused mainly on providing rigorous rationale for the lender of last resort function of central banks as developed by Bagehot (1873)(see also Goodhart, 1999; Freixas, Giannini, Hoggarth, and Soussa, 2000). For example, Diamond and Dybvig (1983), Diamond and Rajan (2001), as well as Rochet and Vives (2004) demonstrate the welfare enhancing effect of some form of liquidity insurance – as a backstop against potential coordination failures (bank runs) and contagion, following essentially from maturity and liquidity transformation inherent in banking operations.² Given that it is normally difficult to distinguish solvent from insolvent banks on a real-time basis (Goodhart, 1999), the question arises whether a lender of last resort is still efficient in such conditions, as it will probably lead to keeping some insolvent institutions afloat. Freixas, Rochet, and Parigi (2004) address this explicitly by introducing a model with both liquidity and solvency shocks that are indistinguishable for the central bank which faces the problem that an insolvent bank may pose as an illiquid one and “gamble for resurrection”, investing the loan in the continuation of economically wasteful projects. It is shown that when it is costly to screen sound firms and solvent banks cannot be easily detected – as would be the case especially in a financial crisis – it is optimal for the central bank to offer emergency liquidity assistance to banks, however at a higher rate (lower than the market) and against collateral, which

¹The length of central bank balance sheets, and hence financial exposures, has increased dramatically for two reasons. First, according to IMF data, emerging and developing economies have increased strongly their foreign exchange reserves, namely from USD 660 billion in 2000 to USD 6,797 billion at end 2011 (i.e. more than a tenfold increase). Second, since 2007, central banks in industrialized countries have increased their balance sheet length in the context of measures taken to combat the financial crisis. From end 2006 to end 2011, the length of the balance sheets of the Fed, Bank of England, and the Eurosystem increased by 233%, 240%, and 138%, respectively (in absolute terms: USD 2,049 billion, GBP 205 billion, EUR 1,585 billion).

²More recently, Holmström and Tirole (2011) have introduced the concept of “inside” and “outside” liquidity to address more broadly the issue of how the economy at large can cope with liquidity shocks. They show that whenever liquidity cannot be endogenously generated within the corporate sector, outside liquidity – e.g. central bank money – needs to be provided.

should serve to deter misuse of its facilities and protect against excessive risk-taking (see also Freixas and Parigi, 2008, for a review of results on lender of last resort and bank closure policy). Finally, Chapman, Chiu, and Molico (2011), study explicitly the effects of central bank collateral policy in the presence of liquidity shocks, credit market imperfections and asset price uncertainty, albeit not necessarily in a crisis. They make two observations relevant to the present paper: (i) that there is a trade-off between relaxing the liquidity constraints of agents, and increasing potential inflation risk and distorting the portfolio choices of agents; and (ii) that a typical risk-management approach to setting the haircuts on collateral is not appropriate for a central bank. Yet, although the authors recognize the systemic impact of the central bank’s collateral framework, they do not look at it in the particular context of crisis management policies and related central bank risk taking – a focal point of this paper.

We extend the available literature by offering a stylized model capturing the effects of liberality in central bank liquidity provision (as specified through its collateral policy) on both central bank risk-taking and economic efficiency. The model provides what is to our knowledge the first formal backing of some of the key statements of Bagehot (1873). Bagehot was well aware of the higher risk-taking associated with enhanced liquidity provision in a crisis, but argued that it was not only necessary to safeguard financial stability but also minimize the central bank’s own financial risks. Such measures, he believed, would be the only way to prevent a financial meltdown and any accompanying massive losses for the central bank.

The model we propose is based on a comprehensive system of financial accounts, similar to the one in Bindseil and Winkler (2013), and follows Freixas, Rochet, and Parigi (2004) in allowing for solvency and liquidity shocks, which are correlated with each other and a priori indistinguishable from the central bank’s perspective. It foresees two time periods and the possibility of bank and corporate default triggered by illiquidity and causing damage in the form of real asset value. Our analysis of how asset value shocks pass through the system of accounts is also inspired by that of Gray, Merton, and Bodie (2007), Gray and Malone (2008) and Castrén and Kavonius (2009), who study the interconnections and shock transmission channels between the risk-adjusted balance sheets of various sectors in the economy using Merton’s (1974) structural credit risk model. Although we explicitly reflect the seniority of companies’ liabilities structure, as in Gray, Merton, and Bodie (2007), we do not introduce the pricing of credit risk, as we assume the values of assets and liabilities are recorded at book values and fair values are established only at the end of period 2. Our approach allows us to explicitly address the central bank’s problem of finding the right balance between the costs of default and the preservation of non-viable economic projects, and show that central bank and general economic efficiency considerations need not necessarily be aligned. Thus, we go beyond Chapman, Chiu, and Molico (2011) by stressing that central bank’s risk management is different from that of granular players not only because it may “affect portfolio choices of other agents,” but because for the central bank, unlike for

other agents, loosening the collateral framework might be fully consistent with protecting the balance sheet. Our paper differs also from Freixas, Rochet, and Parigi (2004) in that we assume more realistically that liquidity and solvency shocks are correlated and that both the secured and the unsecured money markets have broken down completely (which reflects more closely the situation directly after the collapse of Lehman Brothers and experiences from the euro area debt crisis). Most importantly, we explicitly model central bank risk-taking as a major concern that may be relevant for the decisions taken by the central bank and for economic efficiency, while Freixas et al. do not consider this aspect. Finally, Freixas et al. focus on the pricing of emergency central bank credit as a means to discourage moral hazard, while in our view, in the case of a liquidity crisis, the availability of credit (not its price) is the overriding issue, and therefore constraining central bank lending to the right extent seems to be the more relevant parameter to address moral hazard.

The rest of the paper proceeds as follows. In Section 2 we review the particularities of central bank risk management and provide some historical illustration. In Section 3 we propose a formal model that captures in a stylized way the recurrent themes in the debates surrounding central bank financial crisis management. It is shown how one crucial central bank risk control parameter, namely haircuts on central bank collateral, influences both central bank risk-taking and economic efficiency in a way that depends on economic circumstances. Section 4 presents the results of simulation of various parameter changes on optimal haircuts, and Section 5 draws conclusions.

2 Central bank lending and risk-taking in a financial crisis

In what follows, we argue that central bank lending and risk management policies in a crisis are special for a number of reasons, and that ignoring these particularities may lead to sub-optimal central bank decisions both from the point of view of general economic efficiency and risk management.

2.1 The particularities of central bank risk management

Extended liquidity provision by central banks during a crisis comes at the cost of larger exposures compared with normal times. The increase in financial risk is driven by three main factors.

First, probabilities of default of central bank counterparties and issuers of debt instruments used as collateral increase during a crisis.³ Moreover, correlation risks between central bank counterparties and collateral credit quality increase during a financial crisis, because common risk factors (instead of idiosyncratic

³As illustrated e.g. by Standard&Poor's (2009), investment grade debtors (i.e. at least BBB-rated debtors) experience no default at all in good years (e.g. in 1992–1994, 1996, 2004, 2006, 2007 not even one single BBB rated debtor defaulted). In contrast, during bad years even higher rated companies default. For instance, in 2008 the default frequency for AA- and A-rated debtors was both 0.38%.

ones) become predominant. Therefore, the likelihood of the worst case scenario for repo operations, that of a simultaneous default of both the counterparty and the collateral issuer, increases significantly.

Second, central bank lending shifts towards stressed counterparties. During financial crises, stressed banks lose market access and experience funding gaps which are often addressed through increased recourse to central bank credit – a phenomenon that might be called *relative central bank intermediation in the money market*. Hence, central bank lending becomes more concentrated on weaker counterparties which implies that the asset side of its balance sheet becomes, on average, more risky and moreover less diversified. At some stage, central banks may also take over the role of intermediary of the financial system in an *absolute* sense (*absolute central bank intermediation*).

Finally, the central bank's balance sheet may also lengthen due to a flight of households out of bank deposits into banknotes (as it happened in the US and many other countries in the 1930s). This would happen if households were generally worried about the safety of their deposits.

The question now arises as to why exactly should central banks be ready to accept higher risks. We distinguish three main reasons for the central bank to act as the lender of last resort in a financial crisis and to provide elastic credit, even though this leads to higher and more concentrated exposures as argued above.⁴

Negative externalities

Negative social externalities of funding liquidity stress and default due to illiquidity include the following forms: (i) fire-sale spirals induced by liquidity problems of individual banks (Brunnermeier, Crocket, Goodhart, Persaud, and Shin, 2009); (ii) the generalized drying up of funding and market liquidity in the financial system as a consequence of hoarding, driven by the experience that claims, including collateral, can get stuck in a default (relevant after the Lehman default). Due to the systemic escalation inherent in most liquidity crises many entities may find themselves to be (temporarily) illiquid even though they would in principle be solvent (i.e. if they survived the liquidity crisis without liquidity-induced solvency damages); (iii) an eventual credit crunch that affects negatively the real economy and hence households. Central banks as public agents should care about negative externality and should try to prevent them or address them for the sake of reducing deviations from the social optimum. Of course, this recognition does not imply that there are no drawbacks of a too supportive liquidity approach which may create moral hazard, support businesses that should be stopped as they generate social losses, or prevent the necessary price adjustments in markets for certain assets. The negative aspects of too supportive liquidity will also be captured in the model presented

⁴Of course, this recognition does not imply that there are no drawbacks of a too supportive liquidity approach which may create moral hazard, support businesses that should be stopped as they generate social losses, or prevent the necessary price adjustments in markets for certain assets. In this sense, a too supportive central bank attitude can contribute to reduce the efficiency of the price system and the economy at large.

in this paper.

Monopoly to issue legal tender

Central banks have been endowed with the monopoly and freedom to issue the legal tender, thus – unlike leveraged financial institutions – they are not threatened by illiquidity in their own currency. It seems only natural that, in case of a liquidity crisis when all agents attach a high price to liquidity, the central bank remains more willing than others to hold (as collateral or outright) assets which are less liquid. The fact that bank and corporate defaults are costly in themselves even without externalities, as they destroy organizational capital and normally block the efficient use of the underlying resources at least for a while, should also be seen in this context. If a bank or a corporate are threatened by illiquidity (and associated default) in a financial crisis, and if in the case of default the (presumably positive) organizational capital would be destroyed, then saving this capital is part of the “rent” that can be achieved through cooperation between the liquidity-stressed economic agent and the one that has unlimited liquidity. It is important to note that preventing costs of default in this sense through central bank liquidity does not invoke negative externalities, market failures and the public nature of the central bank.⁵ In the model presented below, the cost of corporate default will be one crucial parameter for the optimal degree of elasticity of central bank credit provision. We will not model the negative externalities of default explicitly (although we could), but will simply assume that all costs of defaults (direct and externality-linked) can be captured in one parameter. The model will also allow for positive effects of default – namely to stop corporates/banks with low performance to continue operating in view of the likely persistence in the future of their low performance (which may be viewed as a basic form of moral hazard).

Ability to mitigate risk through haircuts

Haircuts are a powerful risk mitigation tool if credit risk is asymmetric and the cash investor (repo lender) is of very high credit standing. The power of haircuts is limited if cash taker and cash lender are equally credit risky since although haircuts protect the buyer, they expose the seller to unsecured credit risk which increases with the haircut level (Ewerhart and Tapking, 2008).

2.2 Some historical illustration

The trade-off between central bank liquidity provision and risk-taking, and the related experience of central banks was already extensively discussed in the 19th century (e.g. Bagehot, 1873; King, 1936). As the Bank of England’s Jeremiah Harman explained in 1832 regarding the crisis of 1825: “We lent it (money) by every

⁵Empirical estimates of default costs in the corporate finance literature vary between 10% and 44%. See e.g. Glover (2011) and Davydenko, Strebulaev, and Zhao (2012).

possible means and in modes we had never adopted before *consistent with the safety of the bank*. Seeing the dreadful state in which the public were, we rendered every assistance in our power” (quoted in Bagehot, op. cit., emphasis added). Bagehot also emphasized the importance of central bank liquidity provision, “(...) in time of panic it (the Bank of England) must advance freely and vigorously to the public”. Hence, while Bagehot was well aware of the associated higher risk-taking of the central bank, he considered enhanced liquidity provision to be the only possibility to safeguard financial stability. Furthermore, he argued that such measures would be necessary *to minimize the central bank’s eventual own financial risks*:

“(M)aking no loans as we have seen will ruin it (Bank of England); making large loans and stopping, as we have also seen, will ruin it. The only safe plan for the Bank (of England) is the brave plan, to lend in a panic on every kind of current security, or every sort on which money is ordinarily and usually lent. This policy may not save the Bank; but if it do not, nothing will save it.”

What Bagehot suggests is that, in specific cases, a tightening (loosening) of the collateral framework of the central bank could lead to an increase (decrease) of long-term expected central bank losses. Indeed, the aim of “loosening” measures should be to contribute to avoid worst-case scenarios by restoring confidence in a confidence crisis with negative feedback loops and multiple equilibria. If funding stress of banks, together with negative macroeconomic factors, lead to a continued credit crunch and economic downward spiral, solvency deteriorates over time and probabilities of default increase, such as to also increase expected losses of the central bank more and more. If restoring confidence through a more forthcoming collateral and risk control framework allows to prevent such a development from materializing, it could well be that it reduces long-term expected financial losses to the central bank (apart from the positive social welfare aspects of such measures).

It is interesting to note that central banks have usually managed to avoid suffering large-scale losses on their credit operations in financial crises. This could be explained first by the fact that central bank credit operations with banks are typically collateralized. The benefit of not being threatened by illiquidity, and hence having time for liquidation, allows the central bank to take its time with asset liquidation and to await an end of the crisis that triggered counterparty default, i.e. to await mean-reversion in collateral values.⁶ As an illustration, neither the Federal Reserve nor the Bank of England, nor the Bank of Japan, although all having been involved heavily in non-standard forms of liquidity provision to stressed entities over the past few years, have so far faced any losses.

In the case of emergency liquidity measures offered by the Federal Reserve System via the Maiden Lane Facilities (the purpose of which was to facilitate the merger of JP Morgan with Bear Stearns and alleviate

⁶Mean reversion will obviously not materialize in case of the issuer’s default or debt restructuring.

capital and liquidity pressures on American International Group), all credits have been repaid in full with a net gain for the US public. Also the RMBS and agency bond purchases of the Fed were profitable. The case of the AIG rescue is particularly instructive as it illustrates also the inherent endogeneity of risk with respect to central bank's emergency liquidity assistance. The profitable liquidation of the insurer's troubled assets (funded by the Fed and placed with special purpose vehicles called Maiden Lane Facilities) was possible largely due to the general recovery in asset prices stimulated by a combination of low interest rates, extensive liquidity provision and support for credit and mortgage markets. As explained by one Treasury official (quoted by the Financial Times): "We bought at the bottom of the market because we made it the bottom of the market. . . The bottom of the market would have been much deeper if there had been a fire sale of AIG's assets. We pulled back the markets from the brink and Maiden Lanes II and III were a big part of it."⁷

In the case of the Eurosystem, following the default of Lehman Brothers, the Eurosystem was left with some 33 highly complex securities that the investment bank had pledged as collateral securing claims with a total value of EUR 8.5 bn. The process of liquidating collateral took more than four years and brought EUR 7.4 bn, leaving the Bundesbank (in charge of resolving pledged securities) with a residual claim of EUR 1.9 bn, including interest. The Bundesbank is now a creditor in the Lehman Brothers bankruptcy proceedings with a nominal guaranteed claim of USD 3.5 billion, which is expected to be recovered in full.⁸

A word of caution is needed, however. First, the fact that in some recent episodes central banks did not make losses does not imply that the opposite experience could not easily materialize. Moreover, some central banks seem to have solved the problem of large expected losses on their exposures through inflation, achieved by maintaining too low interest rates for a considerable period of time. The most famous example is the Reichsbank in the period 1914 to 1923. When after the loss of World War I large reparation payments were imposed on Germany, it was clear that the Reich was insolvent unless its domestic debt would continue to be inflated away, which did indeed happen. Remarkably, when in 1924 the mark was stabilized again, neither the Reich had defaulted, nor did the Reichsbank have to realize any losses on its claims on the Reich. However, costs to society were huge, as the society had eventually to carry both the costs of the war, and the damages inflation and hyperinflation inflicted on the efficiency of the economy (and on social cohesion).

⁷Henry Sender, "AIG: An improbable profit", The Financial Times, October 22, 2012. We are grateful to Witold Grostal for pointing out this news story to us.

⁸Deutsche Bundesbank, Conclusion of resolution of Lehman collateral, Press notice, 2013-02-20.

3 A model of central bank lending and risk management with real asset value shocks

In this section we present a model to illustrate the problem of risk endogeneity with respect to the central bank’s collateral framework. The model builds on the financial accounts representation introduced in Bindseil and Winkler (2013). It innovates, also relative to previous papers using such models, by capturing asset value shocks, solvency and insolvency, default events and restructuring, and economic efficiency in a well-defined sense. The model contains both asset value (“solvency”) shocks which drive concerns regarding economic performance, and liquidity shocks that may lead to default. Default may occur despite an economic entity being solvent, and poor economic efficiency of banks or corporates may remain unnoticed for an extensive period of time as they continue to receive funding of one or the other kind.

The model is motivated by the recognition that a financial crisis almost always originates from an asset value shock (Kindleberger and Aliber, 2005).⁹ Performance problems do not lead directly to default as it is assumed that they are discovered only with a significant time lag, reflecting the difficulties in valuing non-liquid assets and more generally the opaqueness of banks’ balance-sheets as it could also relate to their significant off-balance-sheet activities or difficult-to-value derivatives transactions. However, liquidity problems are correlated with low quality of loan portfolios as investors receive noisy signals on asset values and tend to withdraw funding on the basis of these signals. The model captures such features in the form of a closed system of financial accounts. Needless to say, the exposition is highly stylized, but aims at capturing one key element of the central bank role in liquidity crises. The model assumes that: (i) the relevant interbank markets have broken down; and (ii) capital market access and deposit flows are uncertain and volatile. This assumption reflects recent experience in the post-Lehman period and the worst phases of the euro area sovereign debt crisis, as well as previous experience from the 1930s (see e.g. Bindseil and Winkler, 2013 on Germany in 1931) or from the 19th century (King, 1936).

3.1 Model setup

At the outset, households are endowed with real assets E (equity). They exchange parts of these real assets in corporate equity P , bank equity Q , banknotes B and bank deposits D (assumed to be divided equally between Bank 1 and Bank 2). Corporates finance their projects by bank loans (equal to $D + B + Q$) and the equity endowment from households (P). The financial sector, consisting of banks and the central bank, is the intermediary between households and corporates (apart from equity stakes in corporates). First, it offers deposits D to households and invests them in loans offered to corporates. Second, the banking sector

⁹In the interest of parsimony, we model explicitly only idiosyncratic asset value shocks, but the extension to systematic shocks is straightforward, although does not alter the results. See on this Bindseil and Jablecki (2013).

Table 1: Financial accounts in the model

Households/Investors			
Assets		Liabilities	
Real assets	$E - D - B - Q - P$	Equity	E
Deposits Bank 1	$D/2$		
Deposits Bank 2	$D/2$		
Bank equity	Q		
Corporate equity	P		
Banknotes	B		

Corporate 1			
Assets		Liabilities	
Real assets	$(D + B + P + Q)/2$	Loans from Bank 1	$(D + B + Q)/2$
		Equity	$P/2$

Corporate 2			
Assets		Liabilities	
Real assets	$(D + B + P + Q)/2$	Loans from Bank 2	$(D + B + Q)/2$
		Equity	$P/2$

Bank 1			
Assets		Liabilities	
Loans to Corporate 1	$(D + B + Q)/2$	Households' deposits	$D/2$
		CB borrowing	$B/2$
		Equity	$Q/2$

Bank 2			
Assets		Liabilities	
Loans to Corporate 2	$(D + B + Q)/2$	Households' deposits	$D/2$
		CB borrowing	$B/2$
		Equity	$Q/2$

Central bank			
Assets		Liabilities	
Credit operations	B	Banknotes	B

is still an intermediary to the operation between the households and the central bank with respect to the issuance of banknotes B . Banks use banknotes to purchase real assets from households, which they sell on to corporates who finance them through a loan from the bank. Thus, total funding, and hence total assets held by banks amount to $B + D + Q$. The resulting financial structure of the economy is presented in Table 1.

When a bank defaults, this has some assumed direct costs. In the model, these costs materialize in the following concrete way: if the bank defaults, also the corporate that the bank was lending to defaults as the bank is no longer able to roll over its credit, and other banks cannot take over quickly enough because they cannot easily assess the quality and solvency of the enterprise.¹⁰ When the corporate defaults, there

¹⁰This assumption is not supposed to reflect the empirically estimated default correlations which tend to be of the order of 1%-5% Moody's (2008a). Rather, it is meant to provide a clear way of including economic costs of default in the model and capturing that these costs ultimately materialize in the real sector by affecting the amount of real resources in the economy. Note that stochasticity could be introduced in a straightforward way by setting default correlation parameter between banks

is economic damage because its management is changed, assets have to be sold to new owners (possibly at distressed prices), changes to the assets have to be made to make them fit into new companies, there is a period of legal uncertainty and associated inertia, etc.¹¹

If, thanks to the central bank’s elastic liquidity provision, defaults of illiquid institutions are prevented, then this may be good since it ensures uninterrupted operation of business projects. However, it can also be bad since banks and corporates may default for good reasons – investors may have withdrawn funding as they receive (noisy) signals on solvency problems relating to bad management. In that case, preventing illiquidity through central bank credit may allow fundamentally unsound projects to continue longer than necessary, and to continue wasting social wealth. It may sometimes be better for the society to discontinue a project through default and go through the one-off cost of reorganization, but then to allow again for a more productive use of the freed up resources.

The central bank in the model is assumed to have no particular information on solvency of banks and corporates, i.e. it does not even receive noisy signals, such as investors do. The central bank, however, can aim at providing liquidity to banks in a way that achieves the optimum with regard to minimizing the expected costs across two possible errors:

- Error 1: letting a bank default for liquidity reasons although it was viable in the sense that there was no reason to expect that it would produce losses in the future;
- Error 2: preventing, through extensive liquidity provision, the default of a bank which is not sound and expected to generate future substantial losses if it is not wound down.

In the model, the parameter of the central bank to achieve the optimum balance between the two errors is the haircut it imposes on collateral.¹² The optimum haircut will depend i.a. on the information content of liquidity shocks with regard to individual banks’ solvency/efficiency problems. If this information content is high, then more conservative haircuts should be optimal, compared to the case of a low information content.

Concretely, we capture the issue of optimal central bank liquidity provision in a two-period model with the following sequence of events.

Period 1:

1. Asset value (“solvency”) shocks materialize, which are modeled as zero-mean random variables:

$$\eta_1 - \text{value shock affecting the assets held by Corporate 1;}$$

and corporates $\rho < 1$. Such an assumption would however complicate the exposition without adding much explanatory value or altering the fundamental conclusions drawn below.

¹¹See also Calvo (1998) pp. 41, 52 for a discussion of the costs of default.

¹²In practice, changes in the restrictiveness of the collateral framework can be brought about also by changes in eligibility. In the model, it has been assumed for the sake of simplicity that there is only one type of asset, and hence there is no scope to differentiate across different asset types in terms of eligibility. It may be an interesting model extension to differentiate across different asset types.

η_2 – value shock affecting the assets held by Corporate 2;

2. Idiosyncratic liquidity shock materializes, defined as $k = \theta + \beta(\eta_1 - \eta_2)$, representing the shift of deposits out of Bank 2 into Bank 1; k is correlated with asset value shocks, reflecting the intuition that liquidity shocks can have information content on debtors' economic performance and solvency.
3. Funding liquidity shocks force banks to adjust their borrowing from the central bank (i.e. Bank 1 adjusts its recourse to central bank credit by k , and Bank 2 by $-k$). The banks pre-deposit all their assets with the central bank as collateral. Recourse to the CB cannot exceed available collateral after haircuts. The haircut level is h , so that the potential borrowing from the central bank is limited to $\frac{1}{2}(1 - h)(B + D + Q)$.
4. If a bank hits its central bank collateral constraint, it defaults. This has two implications. First, the corporate defaults as it depended on the bank for financing (a "credit crunch" occurs). This is assumed to cause a damage to corporate asset value of x . On that basis, the values of the corporate liabilities can be established (assuming the juniority of equity relative to debt). Second, bankruptcy proceedings are initiated and banks' solvency is evaluated, whereby the value of remaining bank assets is divided between the creditors – the central bank and the households. First, the central bank will liquidate its collateral (in fact, by assumption, all assets of the bank), and the remaining asset value is then divided pari passu between the central bank (as far as it still has claims after the liquidation of collateral) and the household.

Period 2:

1. Banks and corporates that have not defaulted continue to exist, and it is assumed that the idiosyncratic real asset shock of period 1 repeats itself precisely. This reflects the assumed persistence of economic performance. Corporates that default are subject to a new draw of the idiosyncratic shock $\tilde{\eta}_{1,2}$ which reflects the fact that they have received a new management and have been re-organized.
2. Economic efficiency and central bank losses are evaluated.

Economic efficiency is understood in the context of the performance of the corporate sector and expressed in terms of the expected change in the stock of real assets in the economy over the two periods. The change in the stock of real assets is defined as the sum of asset value shocks in period 1, costs of default (if any) and asset value shocks in period 2. The latter are equal to a new draw of asset value shocks (in case of default) or period 1 shocks (in case no default occurred). Formally:

$$\Delta_E = \sum_{i=1,2} \{ \eta_i - \mathbb{1}_{\{fail,i\}}x + \mathbb{1}_{\{fail,i\}}\tilde{\eta}_i + (1 - \mathbb{1}_{\{fail,i\}})\eta_i \} \quad (1)$$

where $\mathbb{1}_{\{fail,i\}}$ is equal to 1 if default of Bank i occurs ($i = 1, 2$) and 0 otherwise.

Central bank losses arise from the cascading of asset value shocks and defaults through the respective balance sheets (calculation of the cascade is explained in detail in Bindseil and Jablecki, 2013). We will compare economic efficiency $\mathbb{E}(\Delta)$ (henceforth we drop the subscript E) with the riskiness of the central bank balance sheet, expressed in terms of the expected losses on the collateral portfolio. Since expected loss on an exposure is defined as the product of a counterparty's probability of default (PD) and the loss given default, changes in the central bank's expected losses will have a clear interpretation in terms of changes in counterparties' PD levels, thus reflecting also risk endogeneity. In this setup, the objective of the central bank is to find the optimum level of haircuts that maximizes efficiency and minimizes central bank losses.

Economic efficiency (in the sense of expected change of the stock of real assets over the two periods) is characterized analytically as follows:

Proposition 1. *Let the change in the stock of real assets Δ be defined as in (1), $N(\cdot)$ denote the cumulative standard normal distribution function and $A = \frac{1}{2}(D + B + Q)$. Furthermore, set $\sigma_{Y_1}^2 = \frac{1}{\beta^2}(\sigma_\theta^2 + \beta^2\sigma_{\eta_2}^2)$, $\sigma_{Y_2}^2 = \frac{1}{\beta^2}(\sigma_\theta^2 + \beta^2\sigma_{\eta_1}^2)$ and $\sigma_k^2 = \sigma_\theta^2 + \beta^2\sigma_{\eta_1}^2 + \beta^2\sigma_{\eta_2}^2$ (using the notation introduced above). Then, if $\beta \neq 0$ and $\sigma_{\eta_i} \neq 0$ ($i = 1, 2$), the expected value of Δ is given by:*

$$\mathbb{E}(\Delta) = \sum_{i=1,2} \frac{\sigma_{\eta_i}}{\sqrt{2\pi \left(\frac{\sigma_{Y_i}^2}{\sigma_{\eta_i}^2} + 1 \right)}} \exp \left(\frac{-(-A(1-h) + \frac{1}{2}B)^2}{2\beta^2\sigma_{\eta_i}^2 \left(\frac{\sigma_{Y_i}^2}{\sigma_{\eta_i}^2} + 1 \right)} \right) - 2xN \left(\frac{-A(1-h) + \frac{1}{2}B}{\sigma_k} \right) \quad (2)$$

Proof. See Annex. □

Economic efficiency $\mathbb{E}(\Delta)$ formulas for cases when $\beta = 0$ or $\sigma_{\eta_i} = 0$, $\sigma_{\eta_j} \neq 0$ and $\beta \neq 0$ are derived by straightforward modifications of (2). Note that $\mathbb{E}(\Delta)$ will be driven by the relation between costs of default and the positive expected value of reoccurring asset value shocks. Intuitively, if a bank survives period 1 without being forced to default, it is more likely that it has funded sound projects, and the repetition of such business outcomes in period 2 is obviously associated with increased economic efficiency. It follows from Proposition 1 that the first-order derivative of $\mathbb{E}(\Delta)$ with respect to h can also be obtained in closed form and $\mathbb{E}(\Delta)$ can be both a monotonous and non-monotonous function of h , depending on the interplay of the various parameters describing the state of the financial system (e.g. costs of default, volatilities of idiosyncratic and systemic liquidity and asset value shocks etc.). For example, in a setting with no asset value shocks ($\sigma_{\eta_1} = \sigma_{\eta_2} = 0$) and non-zero costs of default, $\mathbb{E}(\Delta)$ will be decreasing, indicating a preference for a loose collateral framework.

4 The impact of financial sector parameters on optimal haircuts

In this section we consider a number of parameter sets that will illustrate some of the key results of our model. Table 2 shows the parameterization of the various cases considered, whereby the remaining parameters are fixed at: $E = 100$, $B = 20$, $D = 27$, $P = 2$, $Q = 1$, $\sigma_\theta = 1$. We consider four specifications: baseline (I), varying volatility of asset value shocks (II), changing information content of liquidity shocks with respect to solvency shocks (III) and increasing cost of default (IV). Each specification features a number of sub-cases that allow to see how robust is the functional relationship between haircuts and central bank risk-taking and economic efficiency in different environments. Since the central bank loss function is not available in closed form, we derive its distribution using Monte Carlo simulation. Each time the simulations entail 5,000 draws of a “state of the world”, which gives basis for the calculation of the distribution of central bank losses and the calculation of the expected loss as a function of the haircut level, which increases in steps of 0.5% from 0% to 58%.¹³

Table 2: Model specifications considered

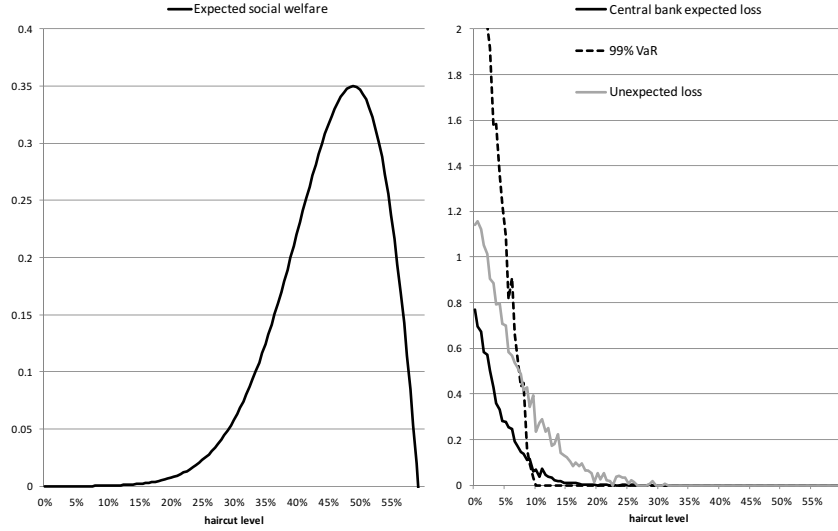
	Specifications			
	I	II	III	IV
$\sigma_{\eta_{1,2}}$	2	0/2/4	2	2
β	1	1	0.1/0.2/1	1
x	1	1	1	0/1/15/25

We start with a baseline specification (I) featuring both key drivers of efficiency, namely firm-specific asset value shocks and costs of default (Figure 1). We also present three measures of central bank risk-taking: the expected loss, the unexpected loss (standard deviation of the loss distribution) and the 99% Value at Risk (i.e. the 99th percentile of the loss distribution). Since all risk curves have similar shapes, for ease of presentation, and in view of the considerations above, the remaining specifications will feature only the expected loss.

Consider first the shape of the efficiency function (left-hand panel). Initially, efficiency increases with the haircut level. However, as default frequency increases, costs of default kick in and begin to weigh on economic efficiency. Ultimately, this generates a hump-shaped economic efficiency curve with a local maximum for $h = 0.48$. Turning to the central bank, Figure 1 (right-hand panel) shows that all risk measures decrease monotonously with the degree of restrictiveness of the haircut policy, and as a result the risk management problem of the central bank can be approached in a similar way as that of a typical granular player with little systemic impact. Consequently, by $h = 0.48$ the balance sheet of the central bank is already fully protected, as all risk measures approach zero. We will see below that such alignment of central bank’s risk management

¹³The upper bound for haircut levels has to be such that banks can at least finance their structural liquidity deficit, stemming from society’s demand for banknotes, i.e. such that $B < (B + D + Q)(1 - h)$. Substituting for B , D and Q , we immediately get $h < 0.5833$.

Figure 1: Economic efficiency and central bank risk-taking in Specification I (here and below see Table 2 for details of the specification)



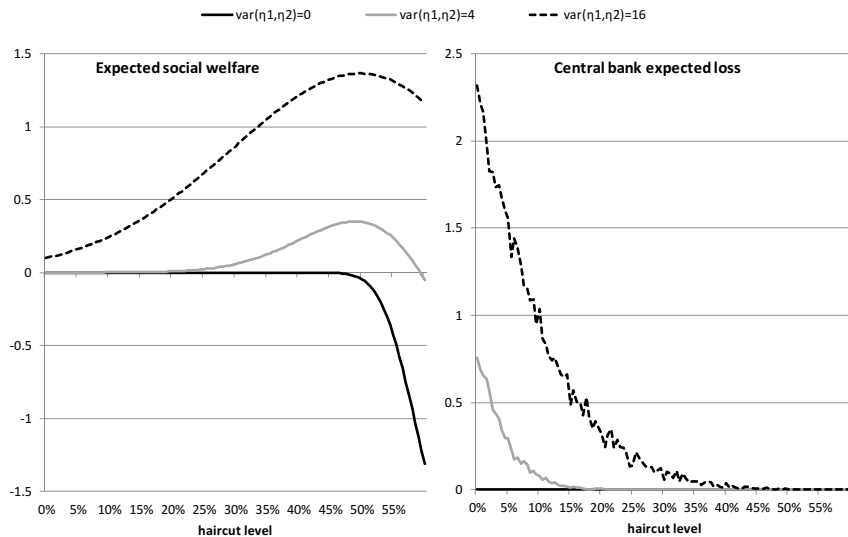
actions with economic efficiency is not universally true.

To verify the robustness of Specification I, we now investigate what happens to efficiency and central bank losses when the volatility of idiosyncratic asset value shocks first drops to zero and subsequently rises to four (Figure 2). When there are no firm-specific asset value shocks, $\mathbb{E}(\Delta) = -2N(24h - 14)$ which is negative and falls further with the restrictiveness of the central bank collateral policy (Figure 2, left-hand panel). To see why this is so, observe that in such case economic efficiency is driven fully by the adverse effects of default and restructuring. The latter are initially very low as banks default very infrequently (e.g. for $h = 0.50$ bank's PD is still below 10% vs. almost 40% in the baseline specification). As haircuts increase, defaults become more prevalent, and the cost of going through reorganization weighs on economic efficiency. When $\sigma_{\eta_i} = 4$, some 30% of solvency shocks will be such as to wipe out households' entire equity stake in corporates. On the other hand, with higher volatility, also sizeable positive shocks are increasingly likely. Thus, increasing the volatility of solvency shocks, while keeping the cost of default constant, produces greater economic efficiency and at the same time higher expected social losses, with the optimal haircut level virtually unchanged. Turning to the central bank (Figure 2, right-hand panel), higher volatility of asset value shocks clearly produces higher expected losses, both on account of higher PDs and more prevalent higher adverse shocks to corporate assets (when $\sigma_{\eta_1} = \sigma_{\eta_2} = 0$ central bank is not expected to suffer any losses).

Specification III allows to analyze the impact of information content of liquidity shocks on economic efficiency. As β decreases from 1 to 0.2 to 0.1 ($\sigma_{\theta}^2 = 1$), the correlation between η_1 and the deposit shift shock k decreases from 0.67 to 0.34 and 0.19, respectively.¹⁴ At the same time, falling β depresses σ_k^2 , which

¹⁴Indeed, with $\sigma_{\eta_1}^2 = \sigma_{\eta_2}^2$, $\text{corr}(\eta_i, k) = \pm\beta\sigma_{\eta_i}(\sigma_{\theta}^2 + 2\beta^2\sigma_{\eta_i}^2)^{-1/2}$ for $i = 1, 2$.

Figure 2: Economic efficiency and central bank losses in Specification II



drops from 9 (with $\beta = 1$) to a mere 1.08 (with $\beta = 0.1$). Such a change also means that banks are initially less frequently forced to default and restructure, as the probability of default stays below 10% until the haircut level of about 50%. Beyond that point, defaults intensify, causing costly restructuring. Ultimately, the initially low PDs overshadow the decreased correlation of liquidity and asset value shocks, and as a result economic efficiency is markedly depressed, while the optimal haircut level is virtually unchanged (Figure 3, left-hand panel).

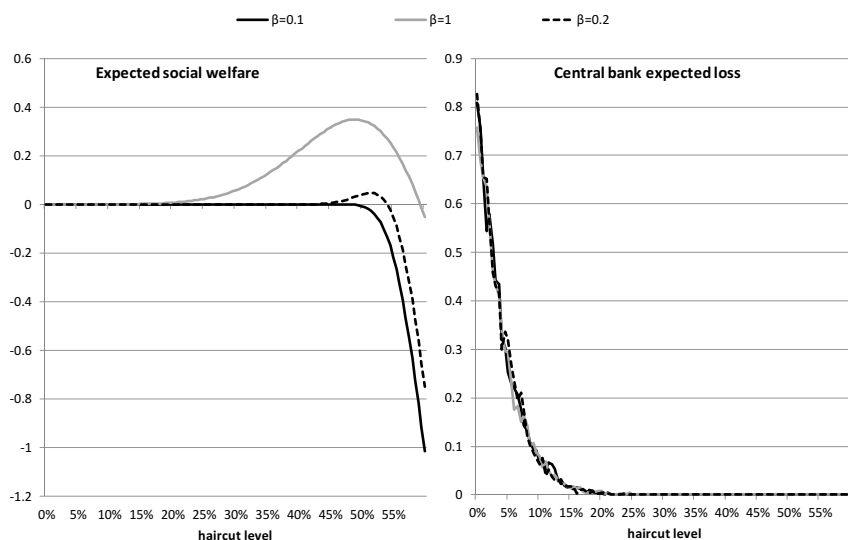
Interestingly, the central bank’s expected loss curve is largely unaffected in this case, despite the changes in banks’ PDs discussed above (Figure 3, right-hand panel). This is due to the fact that the key loss driver – i.e. volatility of asset value shocks – remains unchanged and the cost of default ($x = 1$) can still be absorbed by corporate and bank equity. Thus, in III the risk management conclusion would be to keep the haircut unchanged, even though economic efficiency concerns (e.g. for $\beta = 0.1$) would suggest loosening the collateral framework.

Finally, in Specificaion IV we investigate the impact of the cost of default on both economic efficiency and central bank’s losses. We run the simulations for four different levels of the cost of default which increases in steps from 0 to 1 (4% of asset value), to 15 (60% of asset value) and 25 (100% of asset value).¹⁵

For $x = 0$, $\mathbb{E}(\Delta)$ is monotonously increasing and reaches the maximum for the highest haircut level consistent with accommodating society’s demand for banknotes (Figure 4). Intuitively, more restrictive haircuts ensure that more unsound business projects can be filtered out and costlessly wound down with a net gain for the society. As x increases, however, efficiency gains from killing loss-making businesses are

¹⁵In a recent study Moody’s (2008b) reports that the average firm-wide LGD realized at the resolutions of defaults of 19 U.S. firms in 2007 was around 30%, slightly above 25.1% recorded in 2006 and well below the long-term average of 46% since 1987.

Figure 3: Economic efficiency and central bank losses in Specification III

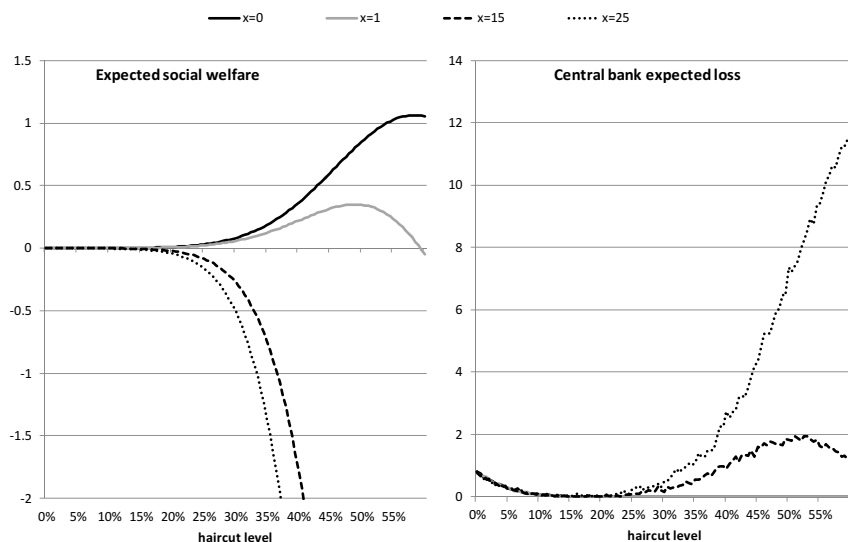


offset by the cost of restructuring and the efficiency curve transforms from a hump-shaped one ($x = 1$) to monotonously decreasing one ($x = 15, 25$) as the negative externalities of default easily outweigh any positive effects of discontinuing malinvestments. Overall, the greater the cost of default, the lower is the optimal haircut from the point of view of economic efficiency.

Turning to the central bank, Figure 4 (right-hand panel) shows that when $x = 0$ expected losses decrease monotonously with the degree of restrictiveness of the haircut policy. Intuitively, the zero cost of default reflects a resilience of the financial system which can always re-organize without disruptions. Therefore, the system can cope even with a very conservative framework without being systemically destabilized. In such an environment, the central bank’s risk management can in fact be approached similarly to the one of a typical granular player on the market, so that increasing haircuts allows it to effectively mitigate credit risk without influencing financial stability. However, when default-related asset value destruction is 60% or higher, the central bank expected loss curve changes from monotonously decreasing to a “U”-shaped one, whereby expected losses fall as the central bank moves from a very liberal framework ($h = 0\%$) to a moderate one ($h = 30\% - 40\%$), but pick up again once that point is breached. As the cost of default increases to 25, in which case reorganization entails almost total destruction of corporate assets (e.g. selling highly sophisticated machinery as metal junk), the losses expected in the most restrictive framework skyrocket. Interestingly however, even at maximum level, central bank losses are about twice lower than losses in real assets and the resulting inefficiency.

Intuitively, these results reflect the fact that when the financial system is not perfectly resilient – as reflected e.g. by the high cost of default – the central bank can no longer be considered to have the same risk management problem as a granular player – the elasticity of its liquidity provision impacts the risk

Figure 4: Economic efficiency and central bank losses in Specification IV



parameters of its counterparties with dramatic consequences for its balance sheet. Excessive risk protection can be self-defeating as it increases default probabilities and expected default related losses for the central bank. The policy conclusion is that, unlike in Specification I, when the cost of default needs to be factored into the analysis, a very restrictive haircut policy may neither be in the interest of society nor of the central bank. Instead, the optimal haircut level – i.e. one that allows striking the right balance between letting a viable bank default for liquidity reasons and preventing the default of a bank which is misallocating resources – is a moderate one.

5 Conclusions

In a financial crisis, central banks play a crucial role as lenders of last resort. But to what extent should they extend credit to banks under funding stress, given that such elastic credit provision might increase their risk-taking and promote moral hazard? More specifically, what are the key trade-offs the central bank needs to consider in limiting the elasticity of its credit provision through collateral eligibility rules and haircuts?

In this paper, we propose a simple model representing the key trade-offs and allowing to derive optimal central bank policies from a risk management and economic efficiency perspective. In particular, the model captures two possible central bank mistakes, namely: (i) letting productive, but temporarily illiquid business projects go bust; and (ii) preserving, through overly elastic liquidity provision, businesses that should default as they are loss-making. While the solvency of banks and corporates was assumed to be unobservable to the central bank prior to default, it is reflected in liquidity shocks, as investors (households) were assumed to receive noisy signals on solvency shocks and the quality of banks' loan portfolios.

The model provides a new formal backing for some of the key ideas of Bagehot (1873), grounded in a comprehensive system of financial accounts, capturing solvency, liquidity, and interaction between the two. The model shows that economic efficiency and central bank risk-taking are in many cases non-monotonous functions of haircuts, and even if the functions are monotonous, they can be either upward- or downward sloping. This means that depending on the haircut level and on economic circumstances, increasing haircuts can either increase or decrease economic efficiency. More surprisingly, in stressed market conditions, characterized by high negative externalities of default, central bank losses can sometimes increase with the level of haircuts. Hence, paradoxically, loosening the collateral framework can be perfectly consistent with protecting the balance sheet of the central bank, as already implied by Bagehot’s dictum that only the “brave” plan of the central bank is the “safe” plan. This is a specific consequence of a more general insight that financial sector risk tends to be endogenous with respect to central bank’s emergency liquidity support. Going beyond model specification, this phenomenon can be illustrated by the following mechanism: if the funding stress of banks, together with other macroeconomic factors, lead to a continued credit crunch and economic downwards spiral affecting collateral values, counterparties’ solvency will deteriorate over time and PDs will increase, eventually increasing also central bank’s risk parameters. To the extent that the central bank’s emergency liquidity operations manage to overcome the negative feedback loops characteristic of a systemic financial turmoil, these actions should then also reduce the central bank’s long-term risk exposure. We believe this reasoning – illustrated formally by our model – goes a long way towards explaining why the major central banks have, over the course of the recent crisis, aimed at increasing the total post-haircut amount of collateral relative to the total balance sheet length of the banking system (Markets Committee, 2013). Indeed, this result is precisely replicated in the model, which shows that under parameter changes that are consistent with a financial crisis, i.e. when costs of default increase and liquidity shocks become more erratic and carry less information on solvency, the central bank should increase the total post-haircut amount of collateral.

Annex

Proof of Proposition 1

Assume first that $\beta \neq 0$ and $\sigma_{\eta_i} \neq 0$ ($i = 1, 2$). Recall that the change in the stock of real assets over the two periods is:

$$\Delta = \sum_{i=1,2} \{ \eta_i - \mathbb{1}_{\{fail,i\}}x + \mathbb{1}_{\{fail,i\}}\tilde{\eta}_i + (1 - \mathbb{1}_{\{fail,i\}})\eta_i \} \quad (3)$$

Since both η_i have zero expected value, the calculation of $\mathbb{E}(\Delta)$, and thus of economic efficiency, reduces to:

$$\mathbb{E}(\Delta) = \mathbb{E} \left(\sum_{i=1,2} \left\{ -\mathbb{1}_{\{fail,i\}}x + \mathbb{1}_{\{fail,i\}}\tilde{\eta}_i - \mathbb{1}_{\{fail,i\}}\eta_i \right\} \right) \quad (4)$$

Observe that

$$\mathbb{1}_{\{fail,i\}} = \begin{cases} 1 & \iff k > A(1-h) - \frac{B}{2} \\ 0 & \iff k \leq A(1-h) - \frac{B}{2} \end{cases}. \quad (5)$$

Since

$$k = \theta + \beta(\eta_1 - \eta_2) \sim N \left(0, \sqrt{\sigma_\theta^2 + \beta^2\sigma_{\eta_1}^2 + \beta^2\sigma_{\eta_2}^2} \right), \quad (6)$$

thus, setting $\sigma_k^2 = \sigma_\theta^2 + \beta^2\sigma_{\eta_1}^2 + \beta^2\sigma_{\eta_2}^2$, we obtain:

$$\mathbb{E} \left(\sum_{i=1,2} \mathbb{1}_{\{fail,i\}}x \right) = 2xN \left(\frac{-A(1-h) + \frac{1}{2}B}{\sigma_k} \right). \quad (7)$$

Note that, by definition, $\tilde{\eta}_i$ and η_i are independent (for $i = 1, 2$), and hence also $\mathbb{1}_{\{fail,i\}}$ and $\tilde{\eta}_i$ must be independent. This implies that:

$$\mathbb{E} \left(\sum_{i=1,2} \mathbb{1}_{\{fail,i\}}\tilde{\eta}_i \right) = 0. \quad (8)$$

To calculate $\sum_i \mathbb{E}(\mathbb{1}_{\{fail,i\}}\eta_i)$ let first $i = 1$. Then (5) can be restated as:

$$\mathbb{1}_{\{fail,1\}} = \begin{cases} 1 & \iff \eta_1 < \frac{1}{\beta} \left(-\theta + \beta\eta_2 - A(1-h) + \frac{B}{2} \right) \\ 0 & \iff \eta_1 \geq \frac{1}{\beta} \left(-\theta + \beta\eta_2 - A(1-h) + \frac{B}{2} \right) \end{cases} \quad (9)$$

Since $\mathbb{1}_{\{fail,1\}}$ and η_1 are not independent, the expectation of their product is a double integral and, by Fubini's theorem, can be calculated using iterated integrals. Thus, assume first that the right-hand-side expression is a constant and denote $Y_1 = \frac{1}{\beta} \left(-\theta + \beta\eta_2 - A(1-h) + \frac{B}{2} \right)$. Then $\mathbb{1}_{\{fail,1\}}\eta_1$ is a normal distribution function truncated to $(-\infty, Y_1)$. Thus,

$$\mathbb{E}(\mathbb{1}_{\{fail,1\}}\eta_1) = \int_{-\infty}^{Y_1} \frac{z}{\sigma_{\eta_1}\sqrt{2\pi}} \exp\left(\frac{-z^2}{2\sigma_{\eta_1}^2}\right) dz. \quad (10)$$

Letting $w = z/\sigma_{\eta_1}$, we immediately obtain

$$\int_{-\infty}^{Y_1} \frac{z}{\sigma_{\eta_1}\sqrt{2\pi}} \exp\left(\frac{-z^2}{2\sigma_{\eta_1}^2}\right) dz = \sigma_{\eta_1} \int_{-\infty}^{\frac{Y_1}{\sigma_{\eta_1}}} \frac{w}{\sqrt{2\pi}} \exp\left(\frac{-w^2}{2}\right) dw = -\sigma_{\eta_1} \phi\left(\frac{Y_1}{\sigma_{\eta_1}}\right), \quad (11)$$

with $\phi(\cdot)$ being the normal PDF, which is the first of the iterated integrals.

Since $Y_1 \sim N((-A(1-h) + \frac{B}{2})/\beta, \sigma_{Y_1})$, it follows that

$$Y_1 = \sigma_{Y_1} U - \frac{-A(1-h) + \frac{B}{2}}{\beta} \quad (12)$$

for $U \sim N(0, 1)$. Thus,

$$\phi\left(\frac{Y_1}{\sigma_{\eta_1}}\right) = \phi\left(\frac{\sigma_{Y_1} U - \frac{-A(1-h) + \frac{B}{2}}{\beta}}{\sigma_{\eta_1}}\right) = \phi(sU + t), \quad (13)$$

for $s = \sigma_{Y_1}/\sigma_{\eta_1}$ and $t = (-A(1-h) + \frac{B}{2})/(\beta\sigma_{\eta_1})$, and we are faced with the calculation of the following integral:

$$\int_{-\infty}^{\infty} \phi(su + t) \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(-\frac{s^2+1}{2}u^2 - stu - \frac{1}{2}t^2\right) du. \quad (14)$$

Since

$$\exp\left(-\frac{s^2+1}{2}u^2 - stu - \frac{1}{2}t^2\right) = \exp\left(-\frac{s^2+1}{2}\left(u + \frac{st}{s^2+1}\right)^2\right) \exp\left(\frac{s^2t^2}{2s^2+2} - \frac{1}{2}t^2\right) \quad (15)$$

we can complete the square and substitute $w = \sqrt{\frac{s^2+1}{2}}\left(u + \frac{st}{s^2+1}\right)$, which leads to:

$$\int_{-\infty}^{\infty} \phi(su + t) \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{s^2+1}} \exp\left(-\frac{t^2}{2(s^2+1)}\right). \quad (16)$$

Substituting for s and t , yields:

$$\mathbb{E}(\mathbb{1}_{\{fail,1\}}\eta_1) = \frac{\sigma_{\eta_i}}{\sqrt{2\pi\left(\frac{\sigma_{Y_i}^2}{\sigma_{\eta_i}^2} + 1\right)}} \exp\left(\frac{-\left(-A(1-h) + \frac{1}{2}B\right)^2}{2\beta^2\sigma_{\eta_i}^2\left(\frac{\sigma_{Y_i}^2}{\sigma_{\eta_i}^2} + 1\right)}\right). \quad (17)$$

The formula for $\sum_i \mathbb{E}(\mathbb{1}_{\{fail,2\}}\eta_2)$ is analogous, and combining (7) and (17) yields the desired formula.

When $\beta = 0$ and volatilities of idiosyncratic solvency shocks are non-zero, then obviously $\mathbb{1}_{\{fail,i\}}$ and η_i are independent, and $\mathbb{E}(\Delta)$ reduces to:

$$\mathbb{E}(\Delta) = -2xN\left(\frac{-A(1-h) + \frac{1}{2}B}{\sqrt{\sigma_\theta^2}}\right). \quad (18)$$

Finally, if $\sigma_{\eta_i} = 0$, then $\mathbb{E}(\mathbb{1}_{\{fail,i\}}\eta_i) = 0$ and the $\mathbb{E}(\Delta)$ will only be driven by $\mathbb{1}_{\{fail,j\}}\eta_j$, as in (17). This completes the proof. \square

References

- BAGEHOT, W. (1873): *Lombard Street: A description of the Money Market*. H. S. King.
- BERNADELL, C., P. CARDON, J. COCHE, F. X. DIEBOLD, AND S. MANGANELLI (eds.) (2004): *Risk management for central bank foreign reserves*. European Central Bank.
- BINDSEIL, U. (2009): “Central banks and other public institutions as financial investors,” in *Risk Management for Central Banks and Other Public Investors*, ed. by U. Bindseil, F. Gonzalez, and E. Tabakis, pp. 3–48. Cambridge University Press.
- BINDSEIL, U., F. GONZALEZ, AND E. TABAKIS (eds.) (2009): *Risk Management for Central Banks and Other Public Investors*. Cambridge University Press.
- BINDSEIL, U., AND J. JABLECKI (2013): “Central bank liquidity provision, risk-taking and economic efficiency,” Working Paper Series 1542/2013, European Central Bank.
- BINDSEIL, U., AND A. WINKLER (2013): “Dual Liquidity Crises-A Financial Accounts Framework,” *Review of International Economics*, 21(1), 151–163.
- BRUNNERMEIER, M., A. CROCKET, C. GOODHART, A. D. PERSAUD, AND H. SHIN (2009): “The Fundamental Principles of Financial Regulation,” Geneva reports on the world economy, International Center for Monetary and Banking Studies.
- CALVO, G. A. (1998): “Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops,” *Journal of Applied Economics*, 0, 35–54.
- CASTRÉN, O., AND I. K. KAVONIUS (2009): “Balance Sheet Interlinkages and Macro-Financial Risk Analysis in the Euro Area,” Working Paper Series 1124, European Central Bank.
- CHAPMAN, J., J. CHIU, AND M. MOLICO (2011): “Central bank haircut policy,” *Annals of Finance*, 7(3), 319–348.
- DAVYDENKO, S. A., I. A. STREBULAEV, AND X. ZHAO (2012): “A Market-Based Study of the Cost of Default,” *The Review of Financial Studies*, 25(10).
- DIAMOND, D. W., AND P. H. DYBVIK (1983): “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy*, 91(3), 401–19.
- DIAMOND, D. W., AND R. G. RAJAN (2001): “Banks and Liquidity,” *American Economic Review*, 91(2), 422–425.
- EWERHART, C., AND J. TAPKING (2008): “Repo markets, counterparty risk and the 2007/2008 liquidity crisis,” Working Paper Series 909, European Central Bank.
- FREIXAS, X., C. GIANNINI, G. HOGGARTH, AND F. SOUSSA (2000): “Lender of Last Resort: What Have We Learned Since Bagehot?,” *Journal of Financial Services Research*, 18(1), 63–84.
- FREIXAS, X., AND B. M. PARIGI (2008): “Lender of Last Resort and Bank Closure Policy,” CESifo Working Paper Series 2286, CESifo Group Munich.
- FREIXAS, X., J.-C. ROCHET, AND B. M. PARIGI (2004): “The Lender of Last Resort: A Twenty-First Century Approach,” *Journal of the European Economic Association*, 2(6), 1085–1115.

- GLOVER, B. (2011): “The Expected Cost of Default,” GSIA Working Papers 2011-E23, Carnegie Mellon University, Tepper School of Business.
- GOODHART, C. A. E. (1999): “Myths about the Lender of Last Resort,” *International Finance*, 2(3), 339–60.
- GRAY, D., AND S. W. MALONE (2008): *Macrofinancial Risk Analysis*. Wiley.
- GRAY, D. F., R. C. MERTON, AND Z. BODIE (2007): “New Framework for Measuring and Managing Macrofinancial Risk and Financial Stability,” NBER Working Papers 13607, National Bureau of Economic Research, Inc.
- HOLMSTRÖM, B., AND J. TIROLE (2011): *Inside and Outside Liquidity*. The MIT Press.
- KINDLEBERGER, C. P., AND R. ALIBER (2005): *Manias, Panics and Crashes: A History of Financial Crises*. Palgrave Macmillan.
- KING, W. T. C. (1936): *History of the London Discount Market*. Frank Cass.
- MARKETS COMMITTEE (2013): “Central bank collateral frameworks and practices,” Report by a Study Group chaired by Guy Debelle, BIS Markets Committee.
- MERTON, R. C. (1974): “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *Journal of Finance*, 29(2), 449–70.
- MOODY’S (2008a): “Asset correlation, realized default correlation, and portfolio credit risk,” Modeling methodology, Moody’s KMV.
- (2008b): “Corporate Default and Recovery Rates, 1920-2007,” Special comment, Moody’s Investors Service.
- ROCHET, J.-C., AND X. VIVES (2004): “Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?,” *Journal of the European Economic Association*, 2(6), 1116–1147.
- STANDARD&POOR’S (2009): “2008 Annual Global Corporate Default Study And Rating Transitions,” Ratings direct.