Exchange Rate Undershooting: Evidence and Theory

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Abstract

We run local projections to estimate the effect of US monetary policy shocks on the dollar. We find that monetary contractions appreciate the dollar and establish two results. First, the spot exchange rate undershoots: the appreciation is smaller on impact than in the longer run. Second, forward exchange rates also appreciate on impact, but their response is flat across tenors. Next, we develop and estimate a New Keynesian model with information frictions. In the model, investors do not observe the natural rate of interest directly. As a result, they learn only over time whether an interest rate surprise represents a monetary contraction. The model accurately predicts the joint dynamics of spot and forward exchange rates following a monetary contraction.

Keywords: Spot exchange rate, forward exchange rate, monetary policy, information effect, information frictions, UIP puzzle, forward premium puzzle

JEL-Codes: F31, E43

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1 Introduction

Some 40 years ago, Dornbusch (1976) put forward a seminal account of how exchange rates adjust to monetary policy shocks. It goes as follows. In the long run the exchange rate is expected to appreciate in response to a contractionary monetary policy shock. This ensures that purchasing power parity is restored because the shock induces a permanent decline of the domestic price level. In the short run, as domestic interest rates exceed foreign rates, market participants expect the exchange rate to depreciate. This ensures that uncovered interest parity is satisfied. How can expectations of an appreciation in the long run be consistent with expectations of a depreciation in the short run? Only if the exchange rate appreciates more strongly on impact than in the long run, that is, if it overshoots on impact.

Expectations take center stage in Dornbusch’s account and, importantly, are assumed to adjust instantaneously to the shock. By the same token, all relevant information is available and processed on impact. Yet by now there is pervasive evidence that expectations are revised only sluggishly in response to new information (Coibion and Gorodnichenko, 2012, 2015). More specifically, recent work by Nakamura and Steinsson (2018a) illustrates the importance of information frictions for the transmission of monetary policy. Since market participants have incomplete information about the state of the economy, interest rate surprises carry potentially new information about the natural rate of interest. Whenever market participants observe interest rate surprises they are uncertain whether they represent monetary policy shocks or changes in the natural rate.

This matters for exchange rate dynamics because natural rate shocks that signal potential output growth will generally depreciate the exchange rate—unlike monetary policy shocks which appreciate the exchange rate. Information frictions thus dampen the initial response of the exchange rate to a monetary policy shock. Moreover, as market participants learn about the monetary policy shock over time, the exchange rate continues to appreciate. Hence, the exchange rate may undershoot its longer-term level on impact because of information frictions. In the first part of this paper, we provide evidence that the exchange rate indeed undershoots in response to monetary policy shocks. We also show that the impact response of forward exchange rates fails to anticipate the appreciation that is about to take place. In the second part of the paper we develop an open economy model with information frictions. It is able to account for the evidence, both qualitatively and quantitatively.

1 Results of Romer and Romer (2000) support the notion that there are informational asymmetries between the Federal Reserve and private forecasters. They also show that private forecasters adjust their forecasts in response to monetary policy actions. Ellingsen and Söderström (2001) rationalize alternative response patterns of the yield curve to monetary policy shocks through asymmetric information. Melosi (2017) estimates a model with dispersed information. In his model monetary policy shocks have a “signaling effect” as heterogeneous price setters seek to learn about the state of the economy.
In the first part of the paper, we rely on local projections to estimate the effect of monetary policy shocks. For this purpose we use the shock series identified by Romer and Romer (2004) and updated by Coibion et al. (2017). The sample for our baseline specification runs from the post-Bretton Woods period to 2007 and we obtain estimates based on quarterly and monthly observations. We normalize the monetary policy shock such that it raises the federal funds rate on impact by 100 basis points and obtain familiar responses of the federal funds rate, output and prices to the shock. Regarding the exchange rate we obtain two sets of results. They are robust across a number of alternative specifications.

First, we find that the \textit{spot} exchange rate of the dollar appreciates by 1 percent on impact and continues to strengthen over the next 3 to 4 years. Eventually the exchange rate settles on a new level, appreciated by 5 to 10 percent relative to the pre-shock level, depending on the specification. We also find that conditional on the shock, short-term interest rates in the US tend to increase relative to foreign interest rates. Hence, we find that the high interest rate currency appreciates—an instance of the UIP puzzle conditional on monetary policy shocks as in Eichenbaum and Evans (1995).

Second, we find that dollar \textit{forward} exchange rates appreciate on impact by some 1 percent as well. Moreover, the response of forward rates is flat across tenors: the response of the 1-month forward rate is about the same as the response of the 3-months or the 12-months rate. The impact response of forward rates, in other words, fails to signal the appreciation that we find for future spot rates in response to monetary policy shocks. Taken together, our results for spot and forward rates allow us to establish a new result: the forward premium puzzle—documented by Fama (1984) and many others unconditionally—also obtains conditional on monetary policy shocks.

Overall, our empirical analysis supports the notion that the ongoing appreciation of the dollar after impact takes investors by surprise. This informs our model-based analysis which centers on information frictions.\footnote{Following Fama (1984) many contributions seek to rationalize the forward premium puzzle and the UIP puzzle through a time-varying risk premium (Engel, 2014). Yet, Froot and Frankel (1989) reject the notion that time-varying risk accounts for the forward premium puzzle on the basis of survey data. They find, instead, that investors make significant expectational errors. Benigno et al. (2012) develop a New Keynesian model with time-varying risk to study exchange rate determination in general equilibrium. They find that their model can generally account for a negative coefficient in the classic UIP regression. However, their model predicts exchange rate overshooting in response to monetary policy shocks. In our model we thus abstract from a potential role of risk premiums.} Specifically, in the second part of the paper, we rationalize the evidence within a New Keynesian open economy model. In terms of information frictions we follow Nakamura and Steinsson (2018a) and assume that the private sector observes the natural rate of interest imperfectly. As a result, interest rate surprises may reflect a monetary policy shock, a change in the natural rate, or a mixture of both.
Our model differs from the model of Nakamura and Steinsson (2018a) in two respects. First, we consider an open economy rather than a closed economy. Second, we provide a micro-foundation to the setup in Nakamura and Steinsson (2018a), as we model monetary policy shocks and natural rate shocks as distinct and assume that agents process information optimally on the basis of signal extraction. This setup permits us to study the effects of monetary policy shocks and natural rate shocks in isolation—both under full and under incomplete information. Under full information, a monetary contraction triggers an appreciation and exchange rate overshooting, just like in Dornbusch (1976). Following a natural rate shock, on the other hand, the exchange rate depreciates because the natural rate shock signals a rising level of potential output.

Under incomplete information—and this is the key finding of our model-based analysis—the exchange rate can indeed undershoot in response to monetary contractions. Intuitively, as market participants partly mistake monetary policy shocks for a policy response to the natural rate, the impact response of the nominal exchange rate is muted. This is because following natural rate shocks, the exchange rate would depreciate rather than appreciate in the long term. It is only over time that agents learn about the monetary policy shock, and the exchange rate gradually appreciates. Consistent with our account, Coibion et al. (2018) find that estimates of potential output growth are gradually revised downward in response to monetary policy shocks.

We also perform a quantitative analysis and estimate key parameters of the model by matching empirical impulse response functions. Specifically, we follow an indirect inference approach where we interpret the estimated impulse response functions as an “identified moment” (Nakamura and Steinsson, 2018b). We find that the model can explain the empirical impulse response functions of both the spot and of the forward exchange rate very well. The estimated parameters also appear plausible. They imply that market participants attribute about 2/3 of interest rate surprises to natural rate shocks, and only 1/3 to monetary policy shocks. These values are almost identical to the estimate reported by Nakamura and Steinsson (2018a) in a quite different context. Also, the extent of information friction implied by our estimates squares well with the results of Coibion and Gorodnichenko (2012).

Our paper relates to a number of studies on how monetary policy impacts the exchange rate. In a recent paper, Schmitt-Grohé and Uribe (2018) identify temporary and permanent monetary policy shocks separately in an estimated state-space model. In line with our results, they find that the exchange rate undershoots in response to both shocks. Galí (2018) studies how interest rate differentials impact exchange rates. He finds a stronger role of near-term compared to longer-term interest rate differentials. He stresses that conventional full
information models cannot account for this evidence.

In a seminal study, Eichenbaum and Evans (1995) estimate the effects of monetary policy shocks within a vector autoregression (VAR) model and document a pattern that has become known as “delayed overshooting”: in response to a monetary contraction the exchange rate appreciates on impact and depreciates thereafter. However, the depreciation sets in only after a delay of more than two years: relative to the maximum effect, the exchange rate undershoots on impact—like in our analysis.\footnote{Delayed overshooting has been found to be robust across a number of alternative specifications and identification schemes. It has also been documented for the real exchange rate (e.g. Scholl and Uhlig, 2008; Steinsson, 2008; Bouakez and Normandin, 2010). However, the “delayed overshooting puzzle” has not gone unchallenged and some studies have documented overshooting (e.g. Faust and Rogers, 2003; Bjornland, 2009; Forni and Gambetti, 2010). In particular, in recent work Kim et al. (2017) find that delayed overshooting obtains only in a sample that includes the 1980s.} We also find for some specifications a reversal of exchange rate dynamics after the maximum effect—in line with delayed overshooting. However, we find that the reversal is not robust across specifications and, to the extent that we observe it at all, it occurs very late after impact and is quantitatively modest.

A number of explanations have been put forward to account for the pattern in the data that characterizes the exchange rate response to monetary policy shocks. Bacchetta and van Wincoop (2010) develop a model with infrequent portfolio adjustment which is able to predict a hump-shaped adjustment of the exchange rate. Kim et al. (2017) suggest that delayed overshooting obtains in the 1980s because the Volcker disinflation was lacking credibility.\footnote{Kim et al. (2017) find for a post-1980s sample that the exchange rate overshoots almost without delay—at least for some specifications. Yet they focus on the response of the real exchange rate. Instead, we find that undershooting is a robust feature of the response of the nominal exchange rate also as we drop the 1980-period.} This is consistent with the model put forward by Gourinchas and Tornell (2004), in which investors have systematically distorted beliefs about future interest rate differentials. Similarly, Burnside et al. (2011) explain the forward premium puzzle by overconfidence as a result of which investors overreact to information about future inflation. In our model, in contrast, investors are fully rational. They merely lack complete information about the current state of the economy. While in our setup market participants cannot distinguish monetary policy shocks from natural rate shocks, they update their beliefs optimally. And while we show that our estimated model accounts for undershooting, we stress that it is also able to generate either immediate overshooting or delayed overshooting for alternative parameterizations.

The rest of the paper is organized as follows. In the next section we describe our empirical analysis and present results. Section 3 outlines the model economy, explains the estimation procedure and discusses estimation results. Section 4 inspects the mechanism that operates at the heart of the model by contrasting exchange rate dynamics under full and incomplete information. Section 5 assesses the external validity of the model. A final section concludes.
2 Empirical evidence

In this section we establish new evidence on how the exchange rate adjusts to monetary policy shocks—both the spot rate and, for a subsample, also forward exchange rates. We focus on the US and estimate the response of the dollar to US monetary policy shocks. We consider a variety of exchange rate measures. In our baseline, we rely on the narrow nominal effective exchange rate index compiled by the Bank for International Settlement (BIS), which is a trade-weighted index of bilateral exchange rates of the dollar vis-à-vis 15 currencies. We also consider a number of bilateral dollar exchange rates.

In our baseline specification we work with the narratively identified monetary policy shocks due to Romer and Romer (2004). Specifically, we use the times series provided by Coibion et al. (2017) who have updated and extended the original series. Romer and Romer (2004) explain the approach in detail. Here we summarize the main idea. In a first step, Romer and Romer construct a time series for the change in the intended federal funds rate around FOMC meetings on the basis of narrative sources. In a second step these changes are purged of the component that may be caused by the Fed’s assessment of current economic conditions as well as of the economic outlook, as captured by the Fed’s Greenbook. For this purpose Romer and Romer regress the change of the intended federal funds rate on the Greenbook forecasts for inflation, real output growth, and the unemployment rate. The residual of this regression, to which we refer as $u_t$ below, captures non-systematic shifts in policy, that is, monetary policy shocks.

In the following subsection we briefly outline our empirical framework and present results for the spot exchange rate. We find that the dollar undershoots following a monetary policy shock: the impact response of the spot rate is weaker than the response in the longer run. The exchange rate thus continues to appreciate after impact. In order to assess whether this ongoing appreciation is expected by market participants, we study the impact response of forward exchange rates in Section 2.2. Our results suggest that the ongoing appreciation is not anticipated.

2.1 The dynamics of the spot exchange rate

We use local projections to directly estimate the impulse responses to monetary policy shocks (Jordà, 2005). For our baseline specification, we estimate the responses of the nominal exchange rate and key macroeconomic variables based on quarterly observations for the period 1976Q1 until 2007Q3, that is, our sample starts after the Bretton Woods system had been completely abandoned and ends before the financial crisis.

Our empirical specification builds on Coibion et al. (2017). Formally, letting $u_t$ denote a
US monetary policy shock in period $t$, we estimate the following model:

$$x_{t+h} - x_{t-1} = c^{(h)} + \sum_{j=1}^{J} \alpha^{(h)}_{j} (x_{t-j} - x_{t-j-1}) + \sum_{k=0}^{K-1} \beta^{(h)}_{k} u_{t-k} + \varepsilon_{t+h}. \quad (1)$$

In this specification, we estimate the effect on the variable of interest, $x_{t}$, at horizon $h$ relative to the pre-shock level as suggested by Stock and Watson (2018). In this way, we acknowledge that monetary policy shocks may have permanent effects on the variables of interest. This is particularly relevant as we study the dynamic adjustment of the nominal exchange rate which, according to theory, features a unit root unless it is stabilized by monetary policy. Our specification includes $J$ lags of the dependent variable and $K$ lags of the shock. $c^{(h)}$ is a constant for horizon $h$ and $\varepsilon_{t+h}$ is an iid error term with zero mean. We compute heteroscedasticity and autocorrelation consistent standard errors as in Newey and West (1987).

As we estimate the empirical model above, $x_{t}$ is, in turn, the log of US real GDP, the log of the US consumer price index (CPI), and the log of the nominal exchange rate of the US dollar. We also estimate the response of the federal funds rate to the shock. We follow Coibion et al. (2017) and restrict the contemporaneous effect of monetary policy shocks on GDP and the CPI to be equal to zero. We control for past shocks and past values of the endogenous variable during the year preceding the shock. Since we use quarterly observations in the baseline, this yields $J = 4$ and $K = 4$. Our results are robust across alternative specifications for $J$ and $K$, as Figure B.1 in the Appendix illustrates.

Figure 1 shows the impulse responses to a monetary policy shock. It is normalized so that the federal funds rate increases initially by 100 basis points. The solid lines represent the point estimate, while shaded areas indicate 68 percent and 90 percent confidence bands. The horizontal axis measures time in quarters. The vertical axis measures deviations from the pre-shock level, in percentage points for the federal funds rate and in percent for the other variables. The upper-left panel shows that the federal funds rate rises persistently for about 1.5 years. Afterwards it gradually converges back to zero. The upper-right panel shows the response of output which displays a distinct hump-shaped pattern, familiar from earlier work on the monetary transmission mechanism (e.g., Christiano et al., 1999). We observe a maximum effect after about one to two years, when output has declined by approximately 1 percent relative to its pre-shock level. The effect on output ceases to be significant after 2-3 years. The lower-left panel shows the response of the price level. Initially, prices adjust

5Note also that the shocks $u_{t}$ are generated regressors. Pagan (1984) shows that the standard errors on the generated regressors are asymptotically valid under the null hypothesis that the coefficient is zero; see also the discussion in Coibion and Gorodnichenko (2015).

6In this case we exclude the pre-shock level and do not consider differenced lags. All series are obtained from the St. Louis Fed (FRED) except for the exchange rate (BIS).

7In this case, the second sum in equation (1) runs from $k = 1$ to $K$. 

6
sluggishly. We observe a significant decline of prices only after about 1.5 to 2 years, again a familiar finding of earlier studies. However, the price level continues to decline markedly afterwards. Five years after the shock it is reduced by some 3 percent.

Finally, we turn to the response of the nominal exchange rate, shown in the lower-right panel. The exchange rate measures the price of foreign currency in terms of dollars. Hence, a decline of the exchange rate represents an appreciation of the dollar. We observe a significant impact response. The dollar appreciates immediately by approximately 1 percent in response to the shock. Moreover, the appreciation continues over time. It takes the exchange rate more than two years to settle on a new level. At this point it is has gained some 5 percent in value. Relative to this longer-term effect, the impact response is muted: the exchange rate undershoots on impact.

Next, we estimate model (1) also on monthly data, focusing on the response of the nominal
as well as the real effective exchange rate (the latter also taken from the BIS). In line with the baseline specification, we now include 12 lags of the shock and the dependent variable ($J = 12$ and $K = 12$). The upper-left panel of Figure 2 shows the results for a specification where we zoom in on the short-run adjustment: the first 12 months after the shock. Again, we find that the exchange rate undershoots. The effect is stronger still than for the quarterly model in the sense that the impact response is more muted relative to what we observe 12 months after the shock. In the right panel of the same figure we show the response of the real effective exchange rate. It displays a pattern of adjustment to the shock that is very similar to that of the nominal exchange rate—in line with earlier results of Eichenbaum and Evans (1995) and many others and, more generally, in line with the findings by Mussa (1986).

In the bottom row of Figure 2 we still display results for monthly data, but we zoom out

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Figure 2: Impulse response of exchange rate to monetary policy shock: nominal (left) and real (right). Time measured in months against horizontal axis. Solid line corresponds to point estimate. Shaded areas correspond to 68% and 90% confidence intervals, respectively. Time (horizontal axis) is in months. Vertical axis measures deviation from pre-shock level in percent. Sample: 1976M1 to 2007M7.
to a horizon of 60 months to ease the comparison with the baseline estimates shown Figure 1 above. We find that results are very similar, but the effects are somewhat stronger once we estimate the model on monthly observations. The maximum effect of the shock on the nominal exchange rate is about 8 percent after 4 years and similar for the real exchange rate. Note, however, that 5 years after impact, the real appreciation is somewhat weaker than the nominal appreciation. This is consistent with the notion that purchasing power parity holds in the long run and our finding for the longer-term effect of the shock on the price level, shown in Figure 1 above.

Figure 3 shows results for bilateral exchange rates, based on monthly observations as in Figure 2. We consider the response of bilateral dollar exchange rates vis-à-vis major currencies of 8 countries: Canada, Denmark, Germany, Japan, Norway, Sweden, Switzerland, and the UK. We find that results are very similar to the baseline for which we rely on the effective exchange rate. From a qualitative point of view, it is noteworthy that we detect undershooting in each country of the sample. But also quantitatively, we find an appreciation of the dollar against each currency of a magnitude comparable to what we find for the baseline.

In sum, across a range of specifications, the response of the spot exchange rate to a monetary policy shock exhibits the following robust features. First, in response to a monetary contraction the dollar appreciates in the longer run. This is consistent with the notion that monetary shocks have a permanent effect on the exchange rate. Second, while there is appreciation on impact it is smaller than in the longer run; the appreciation continues over time and undershoots on impact. Third, in some instances, there is a reversal of the maximum appreciation at longer horizons, reminiscent of the “delayed overshooting puzzle” (Eichenbaum and Evans, 1995)—see for example the response for Japan in Figure 3. However, in contrast to the earlier contributions, we find no robust evidence for a systematic reversal of exchange rate dynamics, and, to the extent that we observe it at all, the reversal is rather mild and happens very late after impact. In our analysis, we thus focus on undershooting as they key feature characterizing exchange rate dynamics following monetary shocks.

We report additional evidence in the Appendix. First, we explore the robustness of our results across alternative sample periods. In Figure B.2 in the Appendix, we show results for a sample that is longer than in the baseline, as it runs from 1973 and until the end of 2008. We also consider a sample that includes only the post-Volcker period and excludes the financial crisis (1988M01–2007M07). For both sample periods we find results that are broadly in line with our baseline.

Second, we consider an alternative measure of monetary policy shocks. Rather than the Romer-Romer series compiled by Coibion et al. (2017), we use the time series of monetary
Figure 3: Impulse response of bilateral dollar exchange rates. Monthly observations. Specification as in baseline, see Figure 1 for details. Sample: 1976M1 to 2007M7.
policy shocks identified by Jarociński and Karadi (2018). Jarociński and Karadi (2018) rely on high-frequency data around policy announcements to measure interest rate surprises. They then purge these surprises of central bank information shocks in order to isolate monetary policy shocks. In Figure B.2 in the Appendix, we show that the pattern of the exchange rate response to the monetary policy shocks identified by Jarociński and Karadi (2018) is fairly similar to our baseline.

Third, we also test whether we can formally reject the null hypothesis of overshooting for our sample of bilateral US dollar exchange rates. For this purpose, we regress the response of the spot rate differential $s_{t+h} - s_t$ on the monetary policy shock. A positive coefficient would indicate that the exchange rate declines (that is, appreciates) more strongly on impact than in the longer run, that is, the exchange rate overshoots. Table B.1 in the Appendix shows the result. We find that the regression coefficient is negative for all countries at all horizons, and significantly so in many cases, which leads us to reject the hypothesis of exchange-rate overshooting.

Fourth and last, we consider how foreign interest rates respond to US monetary policy shocks. For reasons of data availability we do so for four countries only. In each instance, we estimate regression (1) but consider an interest rate differential, the federal funds rate minus the foreign short-term interest rate, as the dependent variable. Figure B.3 in the Appendix shows the result. Interest rate differentials are generally positive. Put differently, conditional on a US monetary policy shock, dollar interest rates are relatively high for an extended period, not only relative to the pre-shock level but also relative to other currencies.

Taken together our results represent a variant of the UIP puzzle, conditional on monetary policy shocks. Following monetary contractions, the high interest-rate currency continues to appreciate. For our analysis it is of particular interest whether the ongoing appreciation of the exchange rate after impact is expected by market participants. To address this issue we study the impact response of forward exchange rates.

2.2 The response of forward exchange rates

To study the response of forward exchange rates, we rely again on local projections. Specifically, we use a variant of local projection (1):

$$f_{t+h}^h - s_{t+1} = c^{(h)} + \sum_{j=1}^{J} \alpha_j^{(h)} (s_{t+j} - s_{t+j-1}) + \sum_{k=0}^{K-1} \beta_k^{(h)} u_{t-k} + \epsilon_{t+h}.$$  

(2)

In this expression, $f_{t+h}^h$ denotes the forward exchange rate pertaining to a currency exchange in period $t+h$, and $s_t$ is the spot exchange rate in period $t$. Importantly, in this specification we estimate the impact response of the forward exchange rate for various horizons, in each case
relative to the pre-shock level of the spot exchange rate. Hence, our setup mimics specification (1) as closely as possible—we merely replace future spot rates with current forward rates for each horizon $h$ for which data on forward exchange rates are available.

Forward rates are easily available for the UK. Specifically, we obtain from the Bank of England the dollar-pound forward exchange rates for tenors of 1, 3, 6 and 12 months. We estimate model (2) on each time series in turn. Figure 4 shows the results. In the figure, the (blue) solid line reproduces the dollar-pound spot exchange rate for the first 12 months after the shock, shown already in the lower-right panel of Figure 3 above. The response of forward rates is shown by blue diamonds, with bars indicating 90 percent confidence bounds. The 1-month forward rate appreciates by some 1.5 percent. In this regard its response is almost identical to the response of the spot rate upon impact. Note that the response of the spot rate in month 3 is already quite a bit stronger compared to the impact response of the forward rate. This pattern persists as time passes: while the response of forward rates remains basically flat, the spot rate drifts further downwards. Hence, if assessed against the immediate response of forward rates at various horizons, it appears that the dynamics of the
Table 1: Dynamic response of spot exchange rates versus impact response of forward exchange rates, following US monetary policy shocks. Table entries are the percentage change of the exchange rate vis-à-vis the dollar (price of foreign currency in US dollars). Estimation based on models (1) and (2), respectively. ***, **, and * indicate a significant rejection of the null of a zero response at the 1, 5, and 10 percent level, respectively. Sample: 1976M1 to 2007M1, except for Japan (start: 1978M7). Data source: Datastream.

<table>
<thead>
<tr>
<th>Country</th>
<th>Spot $t+1$</th>
<th>Forward $t+1$</th>
<th>Spot $t+3$</th>
<th>Forward $t+3$</th>
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</thead>
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<td>-0.59*</td>
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<td>-1.99**</td>
<td>-4.18***</td>
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<td>-4.26***</td>
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<td>-1.83</td>
<td>-0.61</td>
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<td>-2.82**</td>
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<td>-1.22**</td>
</tr>
</tbody>
</table>

Spot rate are unforeseen by market participants upon impact.

For the other countries, we obtain forward rates for 1 and 3 months from Datastream for our sample period. In Table 1, we report the responses of 1-month and 3-months forward exchange rates of the dollar vis-à-vis the currencies of all eight countries in our sample. We contrast the response of the forward rates to the response of the spot rate for both horizons and find that the response of the spot rate is generally much stronger, in line with the results for the dollar-pound forward rates, shown in Figure 4 above.

Our results are in line with the forward premium puzzle (Fama, 1984). More specifically, we our results illustrate that the puzzle also obtains conditional on US monetary policy shocks. For this purpose, we cast our results reported in Table 1 in the form of the “Fama regressions”: we relate the actual change in the spot exchange rate ($s_{t+h} - s_t$) to the forward premium ($f_{t+h} - s_t$), both conditional on a monetary policy shock in period $t$. Figure 5 displays the results for the 8 countries in our sample, both for $h = 1$ and $h = 3$. The horizontal axis measures the forward premium, while the vertical axis measures the actual change in the spot exchange rate. Note that if both covered and uncovered interest parity hold, the observations should scatter along the 45-degree line. This is not the case. Instead, the relationship is negative (regression coefficient: -2.7), just like in Fama (1984) and many other studies that...

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8 Larger tenors are available only since the mid-1980s. The forward-rate data from Datastream are between the British pound and a foreign currency of our country group. As in Burnside et al. (2010), we obtain dollar-forwards by multiplying the pound-forwards with the dollar-pound spot exchange rate. For Japan, the series starts only in 1978M7.
In sum, we find that—conditional on monetary policy shocks—the forward premium fails to predict the change of future spot exchange rates. This evidence is consistent with the notion that markets fail to anticipate the dynamics of the exchange rate triggered by a monetary contraction.

3 An open-economy model with information frictions

We develop an open economy New Keynesian model with information frictions. In doing so we build on and extend the model by Nakamura and Steinsson (2018a). We estimate the model and show that it can account for the evidence reported in Section 2. In particular, we are interested in two findings regarding the effects of US monetary policy shocks. The first finding is that the spot exchange rate undershoots. The second finding is that the impact response of forward exchange rates is flat across tenors.
3.1 Model

We consider a New Keynesian small open economy model as in Gali and Monacelli (2005). The distinct feature of our model are information frictions. Private agents do not observe potential output growth and, equivalently, the natural rate of interest directly. The central bank sets the policy rate with reference to the natural rate, but subject to errors, that is, monetary policy shocks. These may capture preference shifts of the central bank which induce it to deviate from the natural rate. Alternatively, it may capture measurement errors of the central bank as it, too, observes the natural rate imperfectly. Either way, whenever the central bank adjusts its policy rate, it conveys potentially relevant information about the natural rate to the private sector.

Our analysis accounts for two recent advances in the literature on expectations formation and its links with monetary policy. First, our model features “noisy” information. Coibion and Gorodnichenko (2012, 2015) show that information frictions, or more specifically models with noisy information, are generally able to capture key features of the data on expectations formation. Second, we build on Nakamura and Steinsson (2018a) who show that an interest-rate surprise reflects not necessarily a monetary policy shock, but also carries information about the natural rate. We extend the model of Nakamura and Steinsson (2018a) to an open economy setting. Moreover, we model natural rate shocks as distinct from monetary policy shocks and make the inference problem of agents explicit assuming that, conditional on their information set, market participants form expectations rationally.\(^9\) We thereby provide a micro-foundation to the setup in Nakamura and Steinsson (2018a).

The environment underlying our model is standard. The domestic country is small such that domestic developments have no bearing on the rest of the world. In the domestic economy, monopolistically competitive firms produce a variety of goods which are consumed domestically as well as exported. The law of one price holds at the level of varieties. Prices are set in the currency of the producer and adjusted infrequently due to a Calvo constraint. Goods markets are imperfectly integrated as domestically produced goods account for a non-zero fraction of the final consumption good. The real exchange rate may deviate from purchasing power parity as a result. International financial markets are complete so that there is perfect consumption risk sharing between the rest of the world and the domestic economy.

As shown by Lorenzoni (2009) and others, the non-linear model as well as its first-order

\(^9\)In our setup, agents learn about the underlying shocks, but in doing so are fully aware of the structural model equations. In this regard our approach differs from adaptive learning where agents may be “internally rational”, but are generally unaware of the true stochastic processes (Adam and Marcet, 2011). Gourinchas and Tornell (2004) and Burnside et al. (2011) pursue behavioral approaches to account for exchange rate dynamics.
approximation are not affected by the presence of noisy information. To save space, we thus
delegate the decision problems of households and firms to the Appendix. In what follows
we provide, in turn, a compact exposition of the approximate equilibrium conditions and an
explanation of how expectations are formed in the presence of noisy information.

3.1.1 Approximate equilibrium conditions

We approximate dynamics in the neighborhood of the steady state. The structural parameters
and initial conditions in the domestic economy are the same as in the rest of the world. The
steady state is therefore symmetric. There is no inflation in steady state and international
relative prices are unity. In what follows, we express all variables in logs. Foreign variables
are denoted with a star. They are constant because there are no foreign shocks, and because
they are not affected by developments in the (small) domestic economy.

Inflation dynamics are determined by the New Keynesian Phillips curve:

\[ \pi_t = \beta \tilde{E}_t \pi_{t+1} + \kappa (y_t - y_t^n) + \eta_t, \]

where \( \pi_t \) is inflation of domestically produced goods, \( y_t \) is output, \( y_t^n \) is potential output, and
\( \eta_t \sim N(0, \sigma_{\eta}^2) \) is an exogenous disturbance. In turn, \( 0 < \beta < 1 \) is the time-discount factor
and \( \kappa > 0 \) captures the extent of nominal rigidities.

The fact that expectations are based on an incomplete information set is indicated by a
tilde above the expectations operator. Denoting a generic variable \( x_t \), we define \( \tilde{E}_t (x_t) \equiv E(x_t | I_t) \), where
\( E \) denotes the mathematical expectations operator. The information set \( I_t \) and the resulting properties of \( \tilde{E}_t \) are detailed below.

We assume that potential output growth follows a first-order autoregressive process:

\[ \Delta y_t^n = \rho_y \Delta y_{t-1}^n + \varepsilon_t^y, \quad \varepsilon_t^y \sim N(0, \sigma_y^2), \]

where \( 0 \leq \rho_y < 1 \), such that a positive disturbance \( \varepsilon_t^y > 0 \) sets in motion a gradual increase
of \( y_t^n \) to a permanently higher level. As shown below, this implies that an increase of the
natural rate signals a growing economy.

A second equilibrium condition links output and the real exchange rate

\[ \theta(y_t - y^*) = s_t + p^* - p_t, \]

where \( \theta^{-1} \) denotes the intertemporal elasticity of substitution. Here, \( y^* \) denotes output in the
rest of the world, \( s_t \) denotes the spot exchange rate, defined as the price of foreign currency
expressed in terms of domestic currency (as in our empirical analysis above), \( p_t \) is the price
index of domestically produced goods (such that \( \pi_t = p_t - p_{t-1} \)) and \( p^* \) is the foreign price.
level. The composite term $s_t + p^* - p_t$ determines the country’s terms of trade, which move proportionately with the real exchange rate in our model. Specifically, the real exchange rate is given by

$$q_t = (1 - \omega)(s_t + p^* - p_t),$$

where the degree of openness of the domestic economy is $\omega \in [0, 1]$. A value $\omega < 1$ indicates that the domestic economy is not fully open, or that there is home-bias in consumption. An increase in $s_t$ indicates a nominal depreciation of the domestic currency, whereas an increase in $q_t$ indicates a real depreciation. To obtain equation (4) we combine market clearing for domestically produced goods with the risk-sharing condition implied by complete international financial markets (Backus and Smith, 1993). Equation (4) shows that following a real depreciation, the demand for domestically produced goods increases.

The nominal exchange rate, in turn, is determined via the uncovered interest rate parity (UIP) condition

$$\tilde{E}_t \Delta s_{t+1} = i_t - i^*. \hspace{1cm} (6)$$

Here, $i_t$ is the domestic short-term nominal interest rate, and $i^*$ is the foreign rate. According to this condition, the exchange rate is expected to depreciate whenever domestic interest rates exceed foreign rates. While UIP holds, we stress that the expected depreciation $\tilde{E}_t \Delta s_{t+1}$ is conditional on the (incomplete) information available to investors at time $t$.

Finally, the model is closed by specifying monetary policy. We posit the following interest rate feedback rule

$$i_t = r^n_t + \phi \pi_t + u_t, \hspace{1cm} (7)$$

where $\phi > 1$, in line with the Taylor principle, and where $u_t$ is a monetary policy shock, for which we assume

$$u_t = \rho_u u_{t-1} + \varepsilon^u_t, \hspace{0.5cm} \varepsilon^u_t \sim \mathcal{N}(0, \sigma^2_u).$$

The natural rate, in turn, is defined as the real interest rate that would prevail absent price and information rigidities. In our model this implies

$$r^n_t \equiv (i_t - E_t \pi_{t+1})|_{\kappa=\infty} = \bar{r} + \theta E_t \Delta y^n_{t+1} = \bar{r} + \theta \rho_y \Delta y^n_t, \hspace{1cm} (8)$$

where $\bar{r} = -\log(\beta) > 0$ and where $E_t$ denotes expectations under full information. Notice from equation (8) that, when potential output $y^n_t$ rises, this temporarily raises the natural rate of interest, as this foreshadows a growth path along which potential output approaches a permanently higher level.

---

9To obtain this equation, we set $\kappa = \infty$ in the Phillips curve (3) which yields $y_t = y^n_t$. Second, we combine the equation for the real exchange rate (4) and the UIP condition (6), and replace $y_t = y^n_t$. Finally, we set $i^* = \bar{r} > 0$ because the foreign nominal rate $i^*$ is in steady state throughout.
3.1.2 Information processing

We now detail how market participants process information, given their knowledge of the economy and the variables that they directly observe. Specifically, we assume that households and firms observe the following variables: the policy rate, actual output, the exchange rate, the price level, and inflation. Their information set thus contains the history of the following variables until period \( t \):

\[ \mathcal{I}_t = \{ i_\tau, y_\tau, s_\tau, p_\tau, \pi_\tau; \tau \leq t \}. \]  

(9)

In contrast, we assume that households and firms do not observe the monetary policy shock \( u_t \), potential output \( y^n_t \) (the growth rate of which is tied to the natural rate according to equation (8)) as well as the exogenous disturbance \( \eta_t \). Yet, private agents observe two signals about the unobserved variables.

The first signal stems from the Phillips curve (3). Because private agents observe current inflation and output (and form an expectation about future inflation), they may infer the signal

\[ \xi_{1t} = -\kappa y^n_t + \eta_t. \]  

(10)

The shock \( \eta_t \) thus has the natural interpretation of representing noise in the observation of potential output \( y^n_t \); or it may equally be interpreted as any shift in the Phillips curve that is unrelated with changes in potential output, such as cost push shocks, or short-term financing frictions that affect domestic firms. In the absence of noise in the Phillips curve, that is, if the standard deviation of \( \eta_t \) goes to zero (\( \sigma_\eta = 0 \)), household and firms can observe natural output perfectly.

The second signal stems from the Taylor rule (7). From observing current inflation and the policy rate \( i_t \), agents infer the signal

\[ \xi_{2t} = r^n_t + u_t = \bar{r} + \theta \rho_y \Delta y^n_t + u_t, \]  

(11)

where we use equation (8). By adjusting its policy rate, the central bank thus effectively communicates information about the natural rate to the private sector. However, the signal is not perfectly informative, because of the presence of monetary policy shocks \( u_t \).

Observe that monetary policy shocks play a dual role in our analysis. On the one hand, given our empirical analysis we are interested in the response of the nominal exchange rate following a change in \( u_t \) below. But monetary policy shocks are also key for information frictions to impact macroeconomic dynamics. This is because monetary policy shocks provide a second source of noise in the observation of natural output for households and firms. Indeed, in the absence of monetary policy shocks (\( \sigma_u = 0 \)), while private agents could not infer natural
output from the first signal (10), they could infer it from the second signal (11), and the model
would reduce to one of full information.

At this stage we emphasize that we do not impose any restriction on the importance of the
two shocks \( \eta_t \) and \( u_t \). Instead we will estimate their stochastic processes below, as we bring
the model to the data. If the data favors a full information model, or if the data favors clean
communication of \( r^n_t \) by the central bank to private agents, this will result in a low estimate
of either \( \sigma_\eta \) or \( \sigma_u \). We also reemphasize the difference of our model relative to the model of
Nakamura and Steinsson (2018a). In their model, an interest-rate surprise always represents a
composite disturbance: a simultaneous tightening of monetary policy and a rise of the natural
rate. In our noisy information model instead, the shocks \( u_t \) and \( r^n_t \) are independent sources
of variation.

We now specify how private agents form expectations \( \tilde{E}_t \). From equations (10) and (11)
we have seen that private agents receive two signals about the state of potential output \( y^n_t \)
and the monetary shock \( u_t \). While we assume that private agents do not observe the realized
values of \( u_t, y^n_t \) and \( \eta_t \), we assume that they know their underlying stochastic process (that
is, the parameters \( \rho_y, \rho_u \) and so forth, are common knowledge). Then, the two equations (10)
and (11) form a linear system, inducing rational agents to form expectations in a recursive
manner by using the linear Kalman filter—as in the noisy information models described in
Coibion and Gorodnichenko (2012).

Formally, we write the underlying stochastic process in state-space form:

\[
\begin{pmatrix}
    y^n_t \\
    y^n_{t-1} \\
    u_t
\end{pmatrix}
= F
\begin{pmatrix}
    y^n_{t-1} \\
    y^n_{t-2} \\
    u_{t-1}
\end{pmatrix}
+ K
\begin{pmatrix}
    \xi_t \\\n    \eta_t
\end{pmatrix}
\]

where

\[
\begin{pmatrix}
    \xi_t \\
    \eta_t
\end{pmatrix}
= H
\begin{pmatrix}
    y^n_t \\
    y^n_{t-1} \\
    u_t
\end{pmatrix}
\]

Agents learn using the Kalman filter which yields a recursive formula for expectations \( \tilde{E}_t \)

\[
\tilde{E}_t
\begin{pmatrix}
    y^n_t \\
    y^n_{t-1} \\
    u_t
\end{pmatrix}
= F \tilde{E}_{t-1}
\begin{pmatrix}
    y^n_{t-1} \\
    y^n_{t-2} \\
    u_{t-1}
\end{pmatrix}
+ K_t
\begin{pmatrix}
    \xi_t - HF \tilde{E}_{t-1}
\end{pmatrix}
\]

We compute the Kalman-gain matrix \( K_t \) numerically, assuming, as is standard in the liter-
ature, that the agents’ learning problem has already converged such that matrix \( K_t = K \) is
time-invariant (e.g., Lorenzoni, 2009).
3.2 Estimation

We estimate the model on the basis of an indirect inference approach (Gourieroux et al., 1993; Smith, 2008). Indirect inference estimation relies on finding parameters such that an implied moment of the model matches the same moment that characterizes the data. In our case, we match the impulse response functions following a monetary shock displayed in Figure 1 above. In the language of Nakamura and Steinsson (2018b) we seek to match an “identified moment” rather than an unconditional moment.

This approach comes with several advantages. First, as explained in Nakamura and Steinsson (2018b), it is more robust to misspecification of the structural model because the matching procedure relies only on the part of the model that is needed to generate the particular moment. Second, the approach is also robust to misspecification in the empirical model that is used to generate the moment (the “auxiliary model”), because for indirect inference to work, the auxiliary model need not be correctly specified (Smith, 2008). For the purpose at hand this matters, because the empirical impulse response functions in Figure 1 have been obtained on the basis of identification assumptions which are not generally satisfied in our structural model (see Mertens and Ravn 2011, who are using a similar approach). Finally, indirect inference is identical to maximum likelihood if the auxiliary model is correctly specified.

We fix the behavioral parameters at conventional values and only estimate the shock parameters, which we summarize in vector $\varphi = [\rho_u, \rho_y, \sigma_y, \sigma_\eta]'$. The parameters contained in $\varphi$ are of particular interest, because the relative size of the variances of the shocks determines the extent of information frictions in our model. Notice that the standard deviation of monetary innovations $\sigma_u$ is not included in the vector of parameters to be estimated. This is because the Kalman filter output (12) only depends on variance (signal-to-noise) ratios, but not on the values of variances as such. Therefore, without loss of generality we may fix one of the variances in the estimation. We set $\sigma_u = 0.1$.

For the remaining parameter values we assume conventional values prior to the estimation. We set $\beta = 0.99$, as we assume that a period in the model corresponds to one quarter. Hence, the real interest rate in steady state amounts to one percent per quarter. We use the conventional value for the Taylor coefficient and set $\phi = 1.5$. For the Phillips curve we use a slope of $\kappa = 0.01$, in line with estimates by Gali and Gertler (1999). For the IES we assume $\theta^{-1} = 0.25$. According to Hall (1988) there is no strong evidence for the IES to be different from zero. Other studies have found higher values (e.g., Smets and Wouters, 2007, report a value of about 0.7). For the degree of openness we use $\omega = 0.15$, because imports account for

---

11Output and prices are not predetermined in our structural model, but following Coibion et al. (2017) we assume this to be the case in our empirical model (1). For this reason we cannot match impulse responses directly as, for instance, in Christiano et al. (2005).
roughly 15% of GDP in the US in our sample. Finally, without loss of generality we normalize \( p^* = y^* = 0 \).

For the estimation we proceed as follows. For each parameter draw \( \varphi \) we solve the model numerically and simulate a sequence of 234 observations for the (annualized) nominal interest rate, output, the CPI and the nominal exchange rate.\(^{12}\) We drop the first 100 observations as burn-in period and treat the remaining observations in the same way as the actual time-series data: we run local projections and estimate the impulse response functions for all variables to a monetary policy innovation \( \varepsilon_u^t \). Importantly, at this stage we use the same specification for the local projections as in Figure 1, that is, a lag structure of \( J = 4 \) and \( K = 4 \). We also impose that output and the price level do not respond instantaneously to the shock. We repeat the regression stage 500 times and take the average of the impulse responses to eliminate sample noise. Finally, we compute the (weighted) squared distance of the implied impulse responses to the empirical impulse responses in Figure 1:

\[
\hat{\varphi} = \arg \min_{\varphi} (\hat{\Lambda}^{emp} - \hat{\Lambda}^{sim}(\varphi))' \Sigma^{-1}(\hat{\Lambda}^{emp} - \hat{\Lambda}^{sim}(\varphi)).
\]

Here \( \hat{\Lambda}^{emp} \) are the (vectorized) empirical impulse responses, \( \hat{\Lambda}^{sim} \) are the simulated impulse responses which depend on the parameter draw \( \varphi \), and \( \hat{\varphi} \) is our estimated vector of parameters. The matrix \( \Sigma \) is a diagonal weighting matrix which contains the (point-wise) variances of the empirical impulse response functions. Therefore, our estimator ensures that the model-implied impulse response functions are as close to the empirical ones as possible, in terms of point-wise standard deviations.

Table 2 shows the results. For the natural rate process we estimate an autocorrelation of \( \rho_y = 0.749 \), and the standard deviation of the innovations is \( \sigma_y = 0.063 \). As for the monetary policy shock, we estimate a high autocorrelation of \( \rho_u = 0.96 \).\(^{13}\) Lastly, the standard deviation

\[\text{\textbackslash{}hline}
\text{Parameter} & \rho_y & \rho_u & \sigma_y & \sigma_\eta \\
\hline
\text{Estimate} & 0.749(0.081) & 0.960(0.003) & 0.063(0.011) & 0.007(0.002) \\
\hline
\end{array}
\]

\begin{tabular}{lcc}
\hline
\text{Statistic} & \( K_1^{est}/((\theta \rho_y)/\kappa) \) & \( K_2^{est}/1 \) \\
\hline
\text{Value} & 0.022 & 0.338 \\
\hline
\end{tabular}

Table 2: Parameter estimates (standard errors in parentheses) based on indirect inference procedure and implied noise statistics (see the main text for details).

\(^{12}\)In our model, \( p_t \) is the producer price index whereas the estimation uses the consumer price index (CPI). In our model, the CPI is given by: \( cpi_t = (1-\omega)p_t + \omega s_t \) (see the Appendix for details).

\(^{13}\)In our model, we have abstracted from interest rate smoothing in the Taylor rule. Therefore, the persistence...
of the noise $\eta$ is estimated to be $\sigma_\eta = 0.007$. We also report standard deviations of our parameter estimates in parentheses.\footnote{To compute the standard deviations, we follow Hall et al. (2012) and use}

$$
\Sigma_\varphi = \Lambda_\varphi \left[ \frac{\partial \hat{\Lambda}^{\text{im}}(\varphi)}{\partial \varphi} \Big|_{\varphi = \hat{\varphi}} \right]' \Sigma^{-1} \Sigma_\delta \Sigma^{-1} \left[ \frac{\partial \hat{\Lambda}^{\text{im}}(\varphi)}{\partial \varphi} \Big|_{\varphi = \hat{\varphi}} \right] \Lambda_\varphi,
$$

where $\Lambda_\varphi$ and $\Sigma_\delta$ are defined as

$$
\Lambda_\varphi \equiv \left( \frac{\partial \hat{\Lambda}^{\text{im}}(\varphi)}{\partial \varphi} \Big|_{\varphi = \hat{\varphi}} \right)' \Sigma^{-1} \left[ \frac{\partial \hat{\Lambda}^{\text{im}}(\varphi)}{\partial \varphi} \Big|_{\varphi = \hat{\varphi}} \right]^{-1}, \quad \Sigma_\delta \equiv \Sigma + \frac{1}{500} \sum_{j=1}^{500} \Sigma_j,
$$

where $\Sigma_j$ is the counterpart of matrix $\Sigma$ in the $j$th replication of our model-implied impulse response functions. See also Mertens and Ravn (2011) who perform an identical estimation.
dashed lines) and compares them to our baseline empirical impulse response functions, shown in Figure 1 above. We find that the model is able to replicate the empirical patterns rather well. For example, it is able to generate a hump-shaped response of real GDP. The fact that noisy information models are able to generate humps has already been stressed by Mackowiak and Wiederholt (2015). Importantly, however, the model response tracks the response for the nominal exchange rate particularly well—our key variable of interest. Hence, the estimated model is able to generate undershooting, the distinct feature of the exchange rate dynamics triggered by monetary policy shocks according to our estimates reported in Section 2.

We show in Section 5 that the estimated model also captures successfully other features of the data. Most importantly, we show that the model-implied impulse response of forward exchange rates matches its empirical counterpart as well. Before we discuss this result in detail, however, we build intuition of why the model can successfully account for undershooting. To do so, in the next subsection we compute the extent of information frictions which are implied by our estimates. In Section 4 we discuss in detail why information frictions lead to exchange rate undershooting following a monetary policy shock.

### 3.3 Measuring the extent of information frictions

To set the stage, we note that when there is full information, agents perfectly observe the realization of any random variable $x_t$ in the model: $\hat{E}_t x_t = x_t$. Full information is nested in the model when noise in the Phillips curve (3) is zero: $\sigma_\eta = 0$. By observing the two signals, private agents can perfectly distinguish changes in potential output $y_n^t$ and monetary policy shocks $u_t$.

The last row of the Kalman filter describes the perceived evolution of the monetary policy shock, $\hat{E}_t u_t$. Under full information, because $\hat{E}_t u_t = u_t$ must hold, this equation implicitly defines two numbers $K_{1\text{full}}$ and $K_{2\text{full}}$:

$$
u_t = \rho \nu_{t-1} + \left( K_{1\text{full}} \quad K_{2\text{full}} \right) \begin{pmatrix} -\kappa & 0 \\ \theta \rho_y & -\theta \rho_y \end{pmatrix} \begin{pmatrix} y_n^t - (1 + \rho_y) y_{n,t-1} + \rho_y y_{n,t-2} \\ u_{t} - \rho_{u} u_{t-1} \end{pmatrix}. $$

Rearranging yields $K_{1\text{full}} = (\theta \rho_y) / \kappa$ and $K_{2\text{full}} = 1$. At the opposite end, when the signals are infinitely noisy, the numbers are $K_{1\text{zero}} = K_{2\text{zero}} = 0$ for in this case, agents attach zero weight to new information contained in any of the two signals.\(^{15}\)

---

\(^{15}\)To generate “zero” information in this model, it is not sufficient to set the noise variance to infinity $\sigma_\eta^2 = \infty$. This is because while in this case, the signal which stems from the Phillips curve becomes uninformative (recall equation (3)), agents can still infer about the state of the economy from the signal coming from the Taylor rule (equation (7)). Therefore, to have zero inference for private agents about the monetary policy shock, it is required that also the variance of the natural rate shock is large. Put differently, also $\sigma_{y_n}^2 = \infty$. 

---
This implies that the estimated coefficients $K_{1}^{\text{est}}$ and $K_{2}^{\text{est}}$ must lie in the two intervals $K_{1}^{\text{est}} \in [0, (\theta \rho_{y})/\kappa]$ and $K_{2}^{\text{est}} \in [0, 1]$. We can assess the extent of noisy information implied by our estimates against this range. If the $K^{\text{est}}$ are close to the upper bound of the feasible interval, there is a small degree of noisy information. Conversely, for estimates $K^{\text{est}}$ closer to zero, the degree of information frictions is large.

The last row of Table 2 shows the results. Specifically, we report $K^{\text{est}}/K^{\text{full}}$. By using this normalization, numbers closer to one provide a relative distance to the case of full information. We obtain 0.022 for the first and 0.338 for the second signal, respectively. Recalling that the first signal stems from the Phillips curve whereas the second signal stems from the Taylor rule, we conclude that private agents infer close to nothing about the monetary policy shock from the Phillips curve, and use about one third of the signal coming from the Taylor rule to update their belief about monetary policy shocks—both indicating a high degree of information frictions. Although based on an entirely different approach and data set, our estimates are consistent with the finding of Nakamura and Steinsson (2018a): they find that about two thirds of interest rate surprises are natural-rate innovations, and only one third monetary policy shocks.

We may also compute a composite statistic which merges the two previous statistics into one. As described in Coibion and Gorodnichenko (2012), in the presence of two signals, a composite statistic can be obtained by multiplying the Kalman matrix $K$ with the observation matrix $H$. To evaluate the noise in observing monetary policy shocks, we study the last entry in the last row of the resulting matrix (compare equation (12)). We find that in our model, this composite statistic equals $K_{2}^{\text{est}} \in [0, 1]$, and is thus the same as the weight given to Taylor rule signals described before. Here we reported that $K_{2}^{\text{est}} = 0.338$. Therefore, the overall degree of information processing regarding monetary policy shocks is estimated to be about one third. This number is in line with estimates for the noisy information models in Coibion and Gorodnichenko (2012).

4 Inspecting the mechanism

In this section we zoom in on the transmission mechanism of our model in order to explore how information frictions shape exchange rate dynamics. To set the stage, we first consider the case of full information. This will serve as a benchmark against which we assess the role of information frictions in Section 4.2. Finally, in Section 4.3 we disentangle the different effects which account for the adjustment dynamics in the estimated model.
4.1 A benchmark: the case of full information

As explained above, our model nests the case of full information for $\sigma_\eta = 0$. We study how the economy reacts to a natural rate shock and to a monetary policy shock under full information. Results are shown in Figure 7. In order to solve the model numerically, we use the estimated parameters from Section 3 except that we set $\sigma_\eta = 0$ and that we assume $\rho_y = \rho_u = 0.8$ for the autocorrelation parameters. The figure is meant to provide a qualitative (not so much a quantitative) illustration of how the model works under full information.

Focus first on the natural rate shock. The adjustment dynamics are displayed in the upper row. The shock implies that potential output rises over time until it settles on a permanently higher level. The natural rate rises temporarily, foreshadowing the growth path of potential output. The nominal interest rate rises alongside the natural rate following equation (7). The exchange rate depreciates, both in nominal and in real terms. Observe in particular that the
exchange rate depreciates *permanently* in response to the shock. This is a supply effect: as the supply of domestic goods rises permanently, their price declines on world markets—that is, the exchange rate depreciates.

Next, we focus on the effect of a monetary policy shock (lower row). The central bank tightens interest rates for reasons exogenous to the economy. Inflation declines. The exchange rate appreciates both in nominal and in real terms. In the long run, the real exchange rate converges back to zero, while the nominal exchange rate remains permanently appreciated. The nominal exchange rate appreciates more strongly on impact than in the long run. The model thus predicts exchange rate overshooting—just like in Dornbusch (1976).

Two equations, in particular, govern the nominal exchange rate response. The first is equation (4), repeated here for convenience:

\[ \theta(y_t - y^*) = s_t + p^* - p_t. \]

This equation determines how the nominal exchange rate reacts in the *long run*. In the long run a monetary policy shock is neutral in terms of economic activity. However, because it generates a temporary decline in inflation (recall Figure 7), the domestic price level \( p_t \) declines permanently to a lower level, \( p_\infty < p_{-1} \). This, in turn, implies that the nominal exchange rate must *appreciate in the long run*, even though the monetary contraction is transitory, \( s_\infty < s_{-1} \).

The second equation is the UIP condition (6), also repeated here for convenience. In the case of full information:

\[ i_t - i^* = E_t \Delta s_{t+1}, \]

where we replace expectation operator \( \tilde{E}_t \) with \( E_t \).

A monetary contraction implies a surprise increase of the policy rate at time 0, \( i_0 > i^* \). This is unanticipated in period 0. However, after period 0 all uncertainty is resolved. This implies \( E_t \Delta s_{t+1} = \Delta s_{t+1} \) for all \( t \geq 0 \), because under full information, agents are not making expectational errors. Dornbusch (1976)’s overshooting result follows immediately. \( i_t - i^* > 0 \) implies that \( \Delta s_{t+1} > 0 \), that is, the nominal exchange rate must *depreciate over time*. Because the exchange rate appreciates in the long run, \( s_\infty < s_{-1} \), depreciation in the short run requires that the exchange rate appreciates more strongly on impact \((t = 0)\) than in the long run. We thus have overshooting: \( s_0 < s_\infty < s_{-1} \).

\[ ^{16}\text{Of course, the precise levels of } p_\infty \text{ and } s_\infty \text{ are equilibrium objects, determined by the responses of inflation and the nominal exchange rate in the short run.} \]
4.2 Exchange rate dynamics when information is noisy

We are now in the position to explore the role of information frictions. Once the monetary policy shock occurs, private agents observe an interest rate surprise. However, they are unable to tell whether this represents a rise in the natural rate to which monetary policy responds, or a monetary policy shock. They are thus uncertain whether in the long run, the nominal exchange rate is going to depreciate or to appreciate (recall Figure 7). Because the nominal exchange rate is a forward looking variable, its current response reflects this uncertainty. For example, if agents attach a high probability to the scenario that the interest rate surprise reflects a change in the natural rate, the nominal exchange rate may initially depreciate. As agents are gradually realizing that the interest rate surprise represents a monetary contraction, the exchange rate starts to appreciate over time.

Generally, because of information frictions, our model can thus account for overshooting, undershooting or delayed overshooting—depending on the model parameters which determine how market participants process information.

This intuition can be made precise formally. We repeat equation (6) for convenience

\[ i_t - i^* = \tilde{E}_t \Delta s_{t+1}, \]  

(14)

Under incomplete information, unlike under full information, investors make expectational errors even absent any new fundamental surprises, that is, even after the shock occurred in period zero. In fact, in noisy information models, the expectational error only converges to zero in the long run (Coibion and Gorodnichenko, 2012). Formally, following a shock to \( u_t \) in the initial period, \( \Delta s_{t+1} \neq \tilde{E}_t \Delta s_{t+1} \) for all \( t \geq 0 \). Letting \( v_{t+1} \equiv \Delta s_{t+1} - \tilde{E}_t \Delta s_{t+1} \) denote the expectational error, we may rewrite the UIP condition (14) as

\[ i_t - i^* + v_{t+1} = \Delta s_{t+1}. \]  

(15)

Equation (15) illustrates why our model may predict exchange rate undershooting when information is incomplete. Even though the policy rate \( i_t \) rises, a negative enough expectational error, \( v_{t+1} < 0 \), can imply a nominal appreciation over time even though the domestic currency is the high interest-rate currency. That the expectational error must indeed be negative can again be understood from the exchange rate response in Figure 7. Under incomplete information, agents initially expect the exchange rate to appreciate by less than under full information, from previous arguments. Over time, as they learn about the monetary policy shock, the exchange rate appreciates by more than previously expected. This implies that \( s_{t+1} < \tilde{E}_t \Delta s_{t+1} \), or that \( v_{t+1} < 0 \).

To illustrate the impact of information frictions, we repeat the nominal interest rate and exchange rate response from Figure 7, but we gradually adjust the noise variance \( \sigma_\eta \).
above zero. The result is shown in Figure 8. When $\sigma_\eta = 0$, the nominal exchange rate response is characterized by overshooting, see Figure 7. As we assume larger values for $\sigma_\eta$, the exchange rate appreciates by less on impact. At the same time, the exchange rate continues to appreciate for some periods after impact. For a low level of information frictions, the exchange rate response is thus characterized by *delayed overshooting*. Instead, as information frictions become more severe, the exchange rate response changes from delayed overshooting to undershooting. The left panel of Figure 8 shows the response of the nominal interest rate to the monetary policy shock. It is almost identical across the experiments.

### 4.3 Dissecting the estimated nominal exchange rate response

We now dissect the nominal exchange rate response of the estimated model, shown in Figure 6, as we identify its underlying drivers. First, we split the exchange rate response according to equation (15), making the expectational error $v_{t+1}$ explicit. The result is shown in the upper-left panel of Figure 9.

Under full information, the expectational error would be zero in all periods except in period $t = 0$, as argued above. The lines representing nominal depreciation $\Delta s_{t+1}$ and the interest rate differential $i_t - i^*$ would thus coincide: a positive differential would be accompanied by nominal depreciation over time. Not so under imperfect information. In this case, the expectational error $v_{t+1}$ is negative, which “drags down” the response of the exchange rate relative to the case of full information. In our estimated model, this effect is strong enough to overturn the *sign* of the change of the exchange rate response from positive to negative for
the entire horizon under consideration. Rather than depreciating, the nominal exchange rate appreciates over time despite a positive interest rate differential.

Next, we split the Taylor rule (7) into its individual components, after applying the expectations operator conditional on the information available to investors at time $t$:

$$i_t = \tilde{E}_t r^n_t + \phi \pi_t + \tilde{E}_t u_t,$$

where we use that $\tilde{E}_t i_t = i_t$ and $\tilde{E}_t \pi_t = \pi_t$, because both $i_t$ and $\pi_t$ are observed. As agents observe an interest rate surprise, $i_t$, they partly mistake this to be a policy response to a rise in the natural rate, even though the natural rate has not changed. In fact, according to our estimation, a significant share of the probability weight is initially put on a natural rate disturbance. This is shown in the upper-right panel of Figure 9. Following the monetary contraction, inflation declines and $\tilde{E}_t u_t$ rises persistently, but so does $\tilde{E}_t r^n_t$.

The lower two panels of the same figure contrast the actual response of $u_t$ versus $r^n_t$ with
agents’ expectations regarding their realizations. The monetary policy shock is shown in the lower left panel: $u_t$ rises initially by 0.43 percentage points, then slowly returns to zero due to a high estimated autocorrelation. Yet agents initially underestimate the size of the monetary policy shock: $\tilde{E}_t u_t$ only rises by 0.14 percentage points on impact. In contrast, while $r^n_t$ stays at zero throughout, agents believe that $r^n_t$ has increased for an extended period.

By observing the response of the economy over time, agents update their beliefs and adjust their estimates of the two shocks accordingly. However, it takes more than five years (twenty quarters) until agents revise their estimate of the natural rate to the true value of zero, and about three years (twelve quarters) until agents’ perception and the actual evolution of the monetary policy shock are fairly aligned. We conclude that, for the model to match the empirically observed impulse response functions following a monetary contraction, the required degree of information friction on monetary policy shocks is substantial.

5 External validity

Our analysis shows that a fairly standard open economy model is able to account for the actual exchange rate response to a monetary policy shock once we allow for information frictions. Not only does the model with information frictions predict exchange rate undershooting, it also tracks the empirical response well from a quantitative point of view. In what follows, we subject the model to additional tests in order to assess its external validity: we ask whether the estimated model can account for patterns of the data that have not been targeted in the estimation procedure.

We focus on three features of the data. First, we contrast the prediction of the estimated model for how forward exchange rates respond to a monetary policy shock with the evidence established in Figure 4. Second, we contrast the model-implied nominal exchange rate response following a natural rate shock to an estimated impulse response of the US dollar to a TFP shock. Last, we show that our model is consistent with evidence for the “information effect” of monetary policy shocks, recently put forward by Nakamura and Steinsson (2018a).

5.1 The impact response of forward exchange rates

A central part of our empirical analysis in Section 2 is to study the impact response of forward exchange rates to a monetary contraction. We find, in particular, that forward exchange rates fail to anticipate the appreciation of the spot exchange rate that is about to take place according to our estimates. This observation provides the rationale for our focus on information frictions. We now ask whether the estimated model predicts the impact response of forward exchange rates correctly. Before answering this question, we stress again that the
response of forward rates has not been used as target in the estimation of the model.

In the model, we define the time-$t$ forward exchange rate with tenor $h$ as the time-$t$
conditional expectation of the time $t + h$ spot exchange rate:

$$f_t^h \equiv \tilde{E}_{t+s_{t+h}}.$$ (17)

Hence, we compute forward exchange rates on the basis of the information available to market
participants at date $t$.

Figure 10 compares the model prediction to the empirical response, as reported in Figure
4 above. Recall that the estimates for the response of forward rates are based on monthly
observations and that the time horizon measured along the horizontal axis is 12 months.\textsuperscript{17}
Because we measure time in our model in quarters, we compute the forward rates $f_t^h$ for $h \in \{0, 1, 2, 3, 4\}$. For instance, $h = 0$ corresponds to the spot rate at time $t$, and $h = 4$ to the
12-months-ahead forward rate. The model prediction is shown as the dashed red line in the
figure. Strikingly, it passes almost exactly through the blue diamonds, corresponding to the
point estimates of the impact forward exchange rates obtained empirically. Even though not

\textsuperscript{17} Also, for means of comparison, we include the response of the dollar-pound spot exchange rate in the
figure.
targeted, our model thus matches very well the response of forward exchange rates following monetary contractions.

5.2 The response of the spot rate to supply shocks

For the mechanism that operates at the heart of our model, it is key that natural rate shocks induce a depreciation of the exchange rate in the long run. It is precisely because natural rate shocks depreciate the exchange rate, that the exchange rate undershoots in response to monetary policy shocks under imperfect information.

Against this background, we seek to assess the response of the exchange rate to a natural rate shock empirically. From the perspective of the model, a natural rate shock corresponds to a shock to the growth rate of potential output. For this reason, we estimate the response of the effective dollar exchange rate to a TFP innovation. Specifically, we employ once more our empirical model (1) and project the change of the exchange rate at various horizons on the time series of TFP changes, compiled and adjusted for utilization by Fernald (2014).

Figure 11 shows the result. The solid line corresponds to the point estimate. The grey areas represent, as before, 68% and 90% confidence intervals. We find that the dollar indeed
depreciates in response to TFP shocks, but the effect is not statistically significant for our baseline sample. Once we estimate the model on a longer sample, ranging from 1976Q1 to 2017Q2, we find a significant response of the exchange rate to TFP shocks. We report the result in Figure B.4 in the Appendix.

In Figure 11, we also show the impulse response predicted by the estimated model. More precisely, because the model does not feature TFP explicitly, we consider a natural rate shock instead. Recall that this shock is equivalent to a shock to natural output growth. To make sure that the shock has a magnitude which is comparable to the TFP shock, we rescale the impulse response predicted by the model such that it matches the empirical impulse response function in the last quarter after impact that we consider ($h = 20$).

It turns out that the adjustment path of the exchange rate predicted by the model (red dashed line) aligns very well with the empirical response function. In particular, the model correctly predicts that the exchange rate initially appreciates in response to a supply shock. Intuitively, as the natural rate rises, the central bank raises interest rates. As a result, market participants initially mistake the natural rate shock for a monetary policy shock, or rather, they attach some probability to this possibility. This induces the exchange rate to initially appreciate.

### 5.3 Monetary policy and growth: reassessing the information effect

A striking observation by Nakamura and Steinsson (2018a) is that in response to monetary policy shocks—identified on the basis of high frequency data—survey estimates of expected output growth increase. This finding motivates their analysis of the “information effect” of monetary non-neutrality. In what follows we show that our model also predicts an information effect of monetary policy innovations on output growth expectations.

Specifically, we establish that in our estimated model market participants revise expectations about output growth upwards in response to monetary innovations, $\varepsilon^u_t$. For this purpose we draw a sequence of natural rate and monetary policy innovations and simulate the estimated model. We simulate the model for 12700 periods, discard the first 100 observation, then keep every 100'th observation. This yields a total of 126 observations, that is, as many observations as Romer-Romer shocks that we use in our empirical analysis in Section 2. We then plot the change in one-year-ahead expectations of output growth in these periods against monetary policy innovations $\varepsilon^u_t$.

The left panel of Figure 12 shows the result. We obtain a positive relationship, with a significant positive slope, as in the empirical analysis of Nakamura and Steinsson (2018a). Expected output growth is $\tilde{g}_{yt} = \tilde{E}_t y_{t+4} - y_t$. We plot the change of annualized output growth expectations, $4 \cdot \Delta \tilde{g}_{yt}$, against $\varepsilon^u_t$. 18

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18 Expected output growth is $\tilde{g}_{yt} = \tilde{E}_t y_{t+4} - y_t$. We plot the change of annualized output growth expectations, $4 \cdot \Delta \tilde{g}_{yt}$, against $\varepsilon^u_t$. 33
The underlying reason for this apparent discrepancy are once more the information frictions that operate in our model. Even though monetary policy shocks and natural rate shocks are completely orthogonal in our model, investors cannot tell the shocks apart immediately. And hence they attach some probability to the possibility that an interest rate innovation due to a monetary policy shock is in fact caused by a rise in the natural rate. As a result, they revise their growth expectations upward whenever this probability is sufficiently high. For our estimated model, this turns out to be the case. Perhaps surprisingly, our analysis thus reveals that, for an information effect on expected output growth to arise, it is not required that interest-rate surprises convey actual information about potential output growth. It is only required that they convey potentially new information about the natural rate.\(^{19}\)

Finally, we show that the Romer and Romer shock series, which underlies our empirical analysis in Section 2, also gives rise to an information effect on expected output growth. Recall that the Romer-Romer shocks are by construction orthogonal the information set of the Federal Reserve as captured by the Greenbook. We relate these shocks to quarterly observations from the Survey of Professional Forecasters. The right panel of Figure 12 correlates the quarterly change of one-year-ahead growth expectations with the Romer and Romer shocks.

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\(^{19}\)On the other hand, because we assume expectations to be rational, new information must indeed be revealed by the central bank on average, for otherwise, market participants would not adjust their expectations.
that we use in our baseline. As in Nakamura and Steinsson (2018a) we find a positive association. A regression yields a positive and significant slope coefficient of 0.24. Comparing the two panels of Figure 12, we find once more that the model predictions align well with the evidence.

6 Conclusion

A number of recent contributions have highlighted the importance of information frictions in order to account for the observed sluggishness of expectations and related macroeconomic phenomena (Coibion and Gorodnichenko, 2012, 2015). In this paper, we study how information frictions impact exchange rate dynamics. This is a first order issue in light of Dornbusch’s overshooting hypothesis, which relies on the assumption that expectations adjusts instantaneously following monetary innovations.

In the first part of our analysis we provide new evidence on how the exchange rate responds to monetary policy shocks. We find that the dollar appreciates in response to a contractionary US monetary policy shock, but less strongly on impact than in the longer run: it undershoots. We also find that US monetary policy shocks induce a positive interest rate differential between the US and abroad, even though the dollar continues to appreciate over time. This is an instance of the UIP puzzle conditional on monetary policy shock, as in earlier work by Eichenbaum and Evans (1995). Importantly, we also estimate how forward exchange rates respond to monetary policy shocks. They appreciate like the spot rate on impact, but their response is flat across tenors. Hence, the impact of forward rates does not reflect the adjustment dynamics of future spot rates that is about to take place. This is an instance of the forward premium puzzle conditional on monetary policy shocks.

This evidence is consistent with the notion that information frictions govern the monetary transmission mechanism, in line with earlier work in a closed-economy context (Romer and Romer, 2000; Melosi, 2017; Nakamura and Steinsson, 2018a). In the second part of the paper we confirm this notion as we develop and estimate an open-economy model with information frictions. In the model agents do neither observe monetary policy shocks nor the natural rate of interest directly. Instead, they process information optimally on the basis of a Kalman filter. As a result, they attach some probability to the possibility that an interest rate surprise represents a policy response to the natural rate, rather than a monetary policy shock. We estimate the model and show that it matches the estimated responses of spot and forward exchange rates following a monetary policy shock particularly well.

Overall, our analysis thus underscores the importance of information frictions when accounting for the international repercussions of monetary policy actions.
References


A Appendix: Economic environment

Here we describe the non-linear model in detail, and present details on the log-linearization.

A.1 Non-linear model

Final Good Firms The final consumption good, $C_t$, is a composite of intermediate goods produced by a continuum of monopolistically competitive firms both at home and abroad. We use $j \in [0, 1]$ to index intermediate goods. Final good firms operate under perfect competition and purchase domestically produced intermediate goods, $Y_t(j)$, as well as imported intermediate goods, $Y_{I,t}(j)$. Final good firms minimize expenditure subject to the following aggregation technology

$$C_t = \left[(1 - \omega)\frac{1}{\sigma} \left(\int_0^1 Y_t(j)\frac{1}{\epsilon} \, dj\right)^{-\frac{\epsilon+1}{\epsilon}} + \omega\frac{1}{\sigma} \left(\int_0^1 Y_{I,t}(j)\frac{1}{\epsilon} \, dj\right)^{-\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \quad (A.1)$$

where $\sigma > 0$ is the trade price elasticity. The parameter $\epsilon > 1$ measures the price elasticity across intermediate goods produced within the same country, while $\omega \in (0, 1)$ measures the weight of imports in the production of final consumption goods—a value lower than one corresponds to home bias in consumption.

Expenditure minimization implies the following price indices for domestically produced intermediate goods and imported intermediate goods, respectively,

$$P_t = \left(\int_0^1 P_t(j)\frac{1}{1-\epsilon} \, di\right)^{\frac{1}{1-\epsilon}}, \quad P_{I,t} = \left(\int_0^1 P_{I,t}(j)\frac{1}{1-\epsilon} \, di\right)^{\frac{1}{1-\epsilon}}. \quad (A.2)$$

By the same token, the consumption price index is

$$CPI_t = \left((1 - \omega)P_t^{1-\sigma} + \omega P_{I,t}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \quad (A.3)$$

Regarding the rest of the world (ROW), we assume an isomorphic aggregation technology. Further, the law of one price is assumed to hold at the level of intermediate goods such that

$$P_{I,t} = S_t P_t^*, \quad (A.4)$$

where $S_t$ denotes the nominal exchange rate (the price of foreign currency in terms of domestic currency). $P_t^*$ denotes the price index of imports measured in foreign currency. Because the domestic economy is small, it also corresponds to the foreign consumer price index: $P_t^* = CPI_t^*$. We also define the terms of trade and the real exchange rate as

$$T_{ot,t} = \frac{P_{I,t}^*}{P_t}, \quad Q_t = \frac{CPI_t^* S_t}{CPI_t}. \quad (A.5)$$
While the law of one price holds throughout, deviations from purchasing power parity are possible in the short run, due to home bias in consumption.

**Intermediate Good Firms** Intermediate goods are produced on the basis of the following production function: \( Y_t(j) = A_t H_t(j) \), where \( H_t(j) \) is the amount of labor employed by firm \( j \). The variable \( A_t \) can be interpreted as aggregate TFP. Intermediate good firms operate under imperfect competition. We assume that price setting is constrained exogenously à la Calvo. Each firm has the opportunity to change its price with a given probability \( 1 - \xi \). Given this possibility, a generic firm \( j \) will set \( P_t(j) \) in order to solve

\[
\max E_t \sum_{k=0}^{\infty} \xi^k \rho_{t,t+k} \left( Y^d_{t,t+k}(j) P_t(j) - W_{t+k} H_{t+k}(j) \right),
\]

where \( \rho_{t,t+k} \) denotes the nominal stochastic discount factor and \( Y^d_{t,t+k}(j) \) denotes demand in period \( t + k \), given that prices have been set optimally in period \( t \). Note that expectations have a tilde \( \tilde{E} \) to indicate the presence of incomplete information.

**Households** The domestic economy is inhabited by a representative household that ranks sequences of consumption and labor effort as

\[
\tilde{E}_t \sum_{k=0}^{\infty} \beta^k \left( \frac{C^1_{t+k} - \theta}{1 - \theta} - \frac{H^1_{t+k}}{1 + \phi} \right), \quad \beta \in (0, 1)
\]

The household trades a complete set of state-contingent securities with the rest of the world. Letting \( \Xi_{t+1} \) denote the payoff in units of domestic currency in period \( t + 1 \) of the portfolio held at the end of period \( t \), the budget constraint of the household is given by

\[
CPI_t C_t + \tilde{E}_t \rho_{t,t+1} \Xi_{t+1} = W_t H_t + Y_t + \Xi_t,
\]

where \( Y_t \) are profits of intermediate good firms transferred to households.

**Monetary policy** Domestic monetary policy is specified by an interest rate feedback rule. Defining the one-period interest rate as \( I_t \equiv 1/\tilde{E}_t(\rho_{t,t+1}) \), we posit

\[
I_t = R^n_t \Pi_t U_t, \quad \phi > 1,
\]

where \( \Pi_t = P_t/P_{t-1} \) measures domestic inflation, where \( R^n_t \) is the natural rate and \( U_t \) is a monetary policy shock.
Market clearing: At the level of each intermediate good, supply equals demand of final good firms and the ROW:

\[ Y_t(j) = Y_t^d(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} \left( \frac{P_t}{CPI_t} \right)^{-\sigma} \left( (1 - \omega)C_t + \omega Tot_t^* C_t^* \right), \tag{A.10} \]

where \( C_t^* \) denotes consumption in the ROW. It is convenient to define an index for aggregate domestic output:

\[ Y_t = \left( \int_0^1 Y_t(j)^{(\varepsilon-1)/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)}. \]

Substituting for \( Y_t(j) \) using (A.10) gives the aggregate relationship

\[ Y_t = \left( \frac{P_t}{CPI_t} \right)^{-\sigma} \left( (1 - \omega)C_t + \omega Tot_t^* C_t^* \right). \tag{A.11} \]

A.2 Equilibrium conditions and the linearized model

In the following, we use lower-case letters to denote the log of a variable. Variables in the ROW are assumed to be constant throughout. We linearize around purchasing power parity \( Q = Tot = P/CPI = 1 \) and zero net inflation \( P_t/P_{t-1} = 1 \).

Price indices: The terms of trade, the law of one price, the CPI, CPI inflation and the real exchange rate can be written as

\[
tot_t = p_{t,t} - p_t, \tag{A.12}
\]

\[
p_{t,t} = p^* + s_t, \tag{A.13}
\]

\[
cpi_t = (1 - \omega)p_t + \omega p_{t,t} = p_t + \omega tot_t, \tag{A.14}
\]

\[
\Delta cpi_t = \pi_t + \omega \Delta tot_t, \tag{A.15}
\]

\[
q_t = (1 - \omega)tot_t. \tag{A.16}
\]

Intermediate good firms: The demand for intermediate good \( (j) \) is given by

\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t, \tag{A.17} \]

so that

\[ \int_0^1 Y_t(j) dj = \zeta_t Y_t, \tag{A.18} \]

where \( \zeta_t = \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj \) measures price dispersion. Aggregation gives

\[ \zeta_t Y_t = \int_0^1 H_t(j) dj = H_t. \tag{A.19} \]
A first order approximation is given by \( y_t = h_t \).

The first order condition to the price setting problem is given by

\[
\tilde{E}_t \sum_{k=0}^{\infty} \xi^k p_{t+k} Y_{t,t+k}^d(j) P_t(j) - \frac{\varepsilon}{\varepsilon - 1} W_{t+k} H_{t+k} = 0. \tag{A.20}
\]

Linearizing (A.20) around zero net inflation, one obtains a variant of the New Keynesian Phillips curve (see, e.g., Galí and Monacelli, 2005):

\[
\pi_t = \beta \tilde{E}_t \pi_{t+1} + \lambda(w_t - p_t - a_t), \tag{A.21}
\]

where \( \lambda := (1 - \xi)(1 - \beta \xi) / \xi \). Marginal costs are defined in real terms, deflated with the domestic price index. Equivalently we may write

\[
w_t - p_t - a_t = w_t^r + \omega \text{tot} - a_t, \tag{A.22}
\]

where \( w_t^r = w_t - c_{pi} \) is the real wage deflated with the CPI.

**Households** The log-linear first order conditions are

\[
w_t^r = w_t - c_{pi} = \theta c_t + \varphi h_t, \tag{A.23}
\]

\[
c_t = \tilde{E}_t c_{t+1} - \frac{1}{\theta}(i_t - \tilde{E}_t \Delta c_{pi} t + \rho), \tag{A.24}
\]

where \( \rho = -\log(\beta) > 0 \). Risk sharing implies that consumption is tightly linked to the real exchange rate (see, e.g., Gali and Monacelli, 2005)

\[
\theta(c_t - c^*) = q_t. \tag{A.25}
\]

**Monetary policy** Rewriting the interest rate feedback rule gives

\[
i_t = r^m_t + \phi \pi_t + u_t. \tag{A.26}
\]

**Equilibrium** Linearizing the good market clearing condition (A.11) yields

\[
y_t = (2 - \omega)\sigma \omega \text{tot} + (1 - \omega)c_t + \omega c^*, \tag{A.27}
\]

where we use (A.12)-(A.15).

**Key equations** We show how to obtain equations (3)-(6) from the main text (the New Keynesian Phillips curve, the risk sharing condition and the UIP condition).

Combine good market clearing (A.27), risk sharing (A.25) and the definition of the real exchange rate (A.16) to obtain

\[
y_t = \frac{1}{\theta} \left( 1 + \omega(2 - \omega)(\sigma \theta - 1) \right) \text{tot} + c^*. \tag{A.28}
\]

43
We assume that $\sigma = 1/\theta$ (the Cole and Obstfeld condition), in which case $\varpi = 1$. Rearrange to obtain

$$\text{tot}_t = \theta(y_t - c^*). \quad (A.29)$$

Combine with equations (A.12) and (A.13), and use that $c^* = y^*$ because the domestic country is small, to obtain equation (4) in the main text.

Next, rewrite the Euler equation (A.24)

$$c_t = \tilde{E}_t c_{t+1} - \frac{1}{\theta}(i_t - \tilde{E}_t(\pi_{t+1} + \omega \Delta \text{tot}_{t+1}) - \rho) \quad (A.30)$$

$$= \tilde{E}_t c_{t+1} - \frac{1}{\theta}(i_t - \tilde{E}_t \pi_{t+1} + \omega \theta \tilde{E}_t \Delta y_{t+1} - \rho), \quad (A.31)$$

where we use (A.15) in the first line and (A.29) in the second.

Combine (A.29) with (A.25) and (A.16) to obtain

$$c_t = (1 - \omega)y_t + \omega c^*. \quad (A.32)$$

Use this expression to substitute for consumption in (A.31)

$$y_t = \tilde{E}_t y_{t+1} - \frac{1}{\theta}(i_t - \tilde{E}_t \pi_{t+1} - \rho), \quad (A.33)$$

which is the dynamic IS curve.

Note that a counterpart of equation (A.33) holds in the ROW

$$y^* = y^* - \frac{1}{\theta}(i^* - \rho), \quad (A.34)$$

where we use that $p^*$ is constant to eliminate expected inflation. Thus $i^* = \rho$. Using this and combining the DIS curve (A.33) with equation (4), we obtain the UIP condition (6) from the main text.

Finally, we rewrite the Phillips curve (A.21). We use (A.23), (A.29), (A.32) and production technology $y_t = h_t$ to rewrite marginal cost

$$w_t^r + \omega \text{tot}_t = \theta c_t + \varphi h_t + \omega \text{tot}_t = (\theta + \varphi)y_t. \quad (A.35)$$

Insert this into the Phillips curve (A.21) and define $\kappa \equiv \lambda(\theta + \varphi)$ to obtain

$$\pi_t = \beta \tilde{E}_t \pi_{t+1} + \kappa y_t - \lambda a_t. \quad (A.36)$$

To obtain the Phillips curve (3) from the main text, we assume that $a_t$ is composed of two components. Specifically we assume that

$$-\lambda a_t = -\kappa y_t^h + \eta_t. \quad (A.37)$$

As in Lorenzoni (2009), we assume that $a_t$ is perfectly observed, but not its individual components (see the main text).
B  Appendix: Additional evidence

Figure B.1: Impulse response of the nominal exchange rate for alternative model specifications. $J$ and $K$ refer to number of lags of the dependent variable and the shock included in local projection (1), respectively. Solid line corresponds to point estimate. Shaded areas correspond to 68% and 90% confidence intervals, respectively. Time (horizontal axis) is in quarters. Vertical axis measures deviation from pre-shock level in percent. Sample: 1976Q1 to 2007Q3.
Figure B.2: Impulse response of the nominal exchange rate to two kinds of monetary policy shocks: Upper panels use Romer, Romer (RR) shocks. Lower panels use Jarociński, Karadi (JK) shocks. Shaded areas correspond to 68% and 90% confidence intervals, respectively. Time (horizontal axis) is in months.
Testing for overshooting: $s_{t+h} - s_t = \beta h u_t + \text{lags} + \varepsilon_{t+h}$

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<th></th>
<th>$h = 12$</th>
<th>$h = 24$</th>
<th>$h = 36$</th>
<th>$h = 48$</th>
<th>$h = 60$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-0.29</td>
<td>-2.59**</td>
<td>-4.43**</td>
<td>-6.41**</td>
<td>-5.87**</td>
</tr>
<tr>
<td>Denmark</td>
<td>-2.73</td>
<td>-8.42*</td>
<td>-10.21</td>
<td>-9.23</td>
<td>-7.09</td>
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<tr>
<td>Germany</td>
<td>-2.77</td>
<td>-8.76**</td>
<td>-10.86*</td>
<td>-10.78*</td>
<td>-9.75</td>
</tr>
<tr>
<td>Sweden</td>
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<td>-8.58*</td>
<td>-10.20</td>
<td>-10.96</td>
<td>-6.94</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-2.18*</td>
<td>-8.57**</td>
<td>-10.07**</td>
<td>-10.75*</td>
<td>-9.49</td>
</tr>
<tr>
<td>UK</td>
<td>-5.19**</td>
<td>-8.02*</td>
<td>-11.08*</td>
<td>-10.01</td>
<td>-4.49</td>
</tr>
</tbody>
</table>

Table B.1: Entries in table report $\hat{\beta}$. $s_t$ is the bilateral exchange rate of the dollar vis-à-vis the currencies of the countries listed in the table. Columns refer to different horizons $h$, measured in months. The null hypothesis of overshooting implies $\beta > 0$. ***, **, and * indicate significant rejection of the null at the 1, 5, and 10 percent level, respectively; Sample: 1976M1 to 2007M7; Data: Datastream.
Figure B.4: Response of exchange rate to supply shock according to data and model. Solid line represents point estimate of local projection of exchange rate on TFP shocks series provided by Fernald (2014). Dashed line is model response to natural rate shock: size of shock set to match the empirical response at $h = 20$. Time (horizontal axis) is in quarters. Sample: 1976Q1–2017Q2