Bailout Uncertainty in a Microfounded General Equilibrium Model of the Financial Sector

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Goals

- Model the exante impact of bailout policy on the volume of leverage, real investments, and the likelihood of a crisis
- Analyze the impact of an expost downward revision of perceptions about the likelihood of bailout on financial markets' liquidity and the real economy (Lehman's collapse)
- Analyze the impact of higher bailout uncertainty on the amplitude of booms and busts
- Analyze the impact of expansionary monetary policy on leverage and risk appetite
- Analyze the impact of attitude towards risk of financial intermediaries on total credit and the likelihood of booms and busts

Outline

- Introduction
- Framework
- Optimizations
- General equilibrium
- Comparative statics
- Conclusions

Chronology of CDS rate around Lehman's collapse (September-October 2008)

Date	Event	CDS Spread
13-14/9		150
15/9	Lehman files for chapter 11	
16-17/9	Paulson suggests TARP to Congress	250
18-19/9		150
22-23/9	Paulson & Bernanke address Congress	450
24-25/9		350
29/9	Congress rejects Tarp proposal	Almost 450
3/10	Amended Tarp approved by Congress	
5-10/10	Aftermath of approval	150

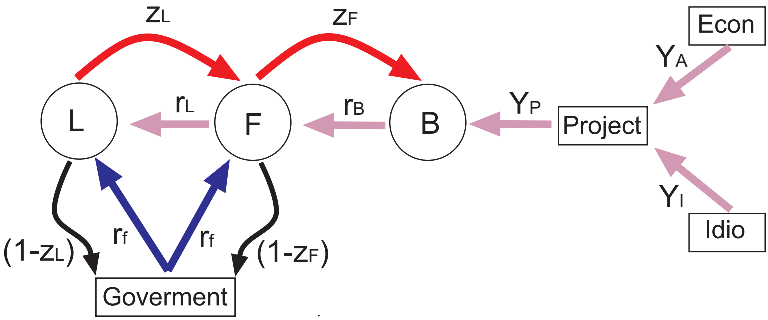
Methods and mechanisms

- Application of Gilboa Schmeidler's conceptualization of increasing uncertainty to Lehman's collapse
- The impact of bailout uncertainty on leverage and the role of duration mismatches
- Extended example of a general equilibrium microfounded model of financial markets aimed at analyzing the interrelationships between financial markets - in essence providing micro-finance foundations to a macro model of the financial sector
- Focus is on the shadow banking system in which long term credit is financed by means of short term liabilities

Recent work

- Meltzer and others, bailout uncertainty
- John Taylor (2009), the impact of low interest rates on the likelihood of a crisis
- Farhi-Tirole (2009), collective moral hazard
- Tirole (2010), illiquidity
- Sieczka, Sornette and Holyst (2010), bankruptcy cascades

Financial flows



Timeline

- 3 periods labeled: $0, 1$ and 2
- All investment decisions are made in period 0
- Once chosen the project size cannot be adjusted
- Return information is revealed partially in period 1 and fully in period 2

Financing - debt maturity

- Only short term loans are available
- Two periods projects are financed by two consecutive one period loans
- If internal resources and credit refinancing does not suffice a default occurs
- In case of default all equity vanishes

Players

There is a large number of each of the following 3 kinds of players:

- **Borrowers (B)** - entrepreneurs, manufacturers, mortgage borrowers
- **Financial intermediaries (F)** - hedge funds, SIVs and conduits
- **Lenders (L)** - pension funds

Mass: M_B, M_F and M_L

Capital: Each individual player (irrespective of type) possesses one unit of equity capital.
For Fs this unit may be composed of capital and exogenous long term debt.

Borrower

- Each borrower invests in a single long term (two periods) investment project
- The project size, x, is selected to maximizes the borrower's expected utility
- The borrower's utility function is

$$u\left(W_B, L_B\right) = \begin{cases} W_B & if \quad W_B \geq 0 \\ -P_B L_B & if \quad W_B < 0, \quad P_B > 0 \end{cases}$$

- When needed $(x > 1)$ B borrows an additional amount, L_B from F at an interest rate $r_{Bt}, \quad t = 1, 2$

Financial intermediary

- Borrows an amount L_F from many lenders at r_L
- In each period F lends a fraction z_F of his resources $(1 + L_F)$ to two borrowers at a rate r_{Bt}, $t = 1, 2$
- The remainder, $1 - z_F$, is invested in a risk free asset whose gross rate, r_f, is set by the central bank
- A F chooses L_F and z_F so as to maximize his expected utility from profits, W_F, in each period
- F possesses a CRRA utility function:

$$u\left(W_F\right) = \frac{W_F^{1-\delta}}{1-\delta}, \quad \delta > 0 \ \ but \ small,$$

and suffers a penalty $P_F L_F$ in case of default

Lender

- Each lender invests a portion z_L of his resources in a fully diversified portfolio of loans to Fs and a portion $1 - z_L$ in the risk free asset
- Each lender lends to a large number of Fs at r_L. As a consequence his risky portfolio is fully diversified
- Lenders risk aversion's is characterized by mean-variance preferences (CARA preferences)

$$u(W_L) = -\frac{1}{\alpha} e^{-\alpha W_L}, \quad \alpha \geq 0$$

Borrower (project) - yield

- Returns on all projects are equally distributed
- A project's net return in each period is the sum of an aggregate and an individual shock

$$\widetilde{R} = \widetilde{R}_A + \widetilde{R}_I$$

- All shocks realize in period 2
- The aggregate (economy-wide shock) $\widetilde{R}_A$, is binomially distributed:

$$\widetilde{R}_A = \begin{cases} y & \text{w.p.} \quad q \\ -y & \text{w.p.} \quad 1-q \end{cases}$$

- The aggregate is revealed in period 2

Borrower (project) - yield

- Three types of projects: lucky, unlucky and regular
- In period 0 all projects are viewed as regular
- Project type is revealed in period 1
- Full project's outcome is revealed in period 2
- For $(1 - \theta_{LB} - \theta_{UB})$ percent of the projects **(regular projects)** the idiosyncratic shock, $\widetilde{R}_I$, is binomially distributed:

$$\widetilde{R_I} = \begin{cases} y & \text{w.p.} \quad q \\ -y & \text{w.p.} \quad 1-q \end{cases}$$

Borrower (project) - yield

- In period 1, for $\theta_{LB} < q$ percents of the projects **(lucky projects)** the idiosyncratic shock, $\widetilde{R}_I$, is binomially distributed:

$$\widetilde{R}_I = \begin{cases} y & \text{w.p.} \quad 1 \\ -y & \text{w.p.} \quad 0 \end{cases}$$

- In period 1, for $\theta_{UB} < 1 - q$ percents of the projects **(unlucky projects)** the idiosyncratic shock, $\widetilde{R}_I$, is binomially distributed:

$$\widetilde{R}_I = \begin{cases} y & \text{w.p.} \quad 0 \\ -y & \text{w.p.} \quad 1 \end{cases}$$

Borrower (project) - yield

- $\widetilde{R}_A$ and $\widetilde{R}_I$ are independent across periods and mutually independent within a period
- The idiosyncratic shock, $\widetilde{R}_I$, is independent across projects
- We assume projects are profitable on average

$$r_B < \mu_B$$

$$\mu_B \equiv E\widetilde{R}$$

$$r_B \equiv (1 + r_{B1})(1 + r_{B2}) - 1$$

Borrower - default

- Can possibly default in either period 1 (illiquidity) or in period 2 (insolvency)
- A borrower who defaults loses his investment project and incurs a penalty $P_B L_B$
- When B defaults the F who lent to him loses the entire debt service, $1 + r_{Bt}, \quad t = 1, 2$
- A borrower defaults in period 1 if he can't refinance the project
- A borrower defaults in period 2 if his cash flow is smaller than the required debt service

Financial requirements

Financial intermediary - yield

The period's 1 income from a portfolio consist of two loans $\widetilde{R}_B$:

	Yield	Probability
The two borrowers are solvent	$(1 + r_{B1})$	$(1 - \theta_{UB})^2 \equiv \gamma_{11}$
One borrower defaults and one is solvent	$\frac{1}{2}(1 + r_{B1})$	$2(1 - \theta_{UB})\theta_{UB} \equiv \gamma_{21}$
Both borrowers default	0	$\theta_{UB}^2 \equiv \gamma_{31}$

Financial intermediary - yield

The corresponding period's 2 probabilities given the information of period 1 are

$$\begin{aligned} q(1-q_1^2)+q_1^2 &\equiv \gamma_{12} \\ 2(1-q)q_1(1-q_1) &\equiv \gamma_{22} \\ (1-q)(1-q_1)^2 &\equiv \gamma_{32} \end{aligned}$$

where

$$q_1 \equiv \frac{q-\theta_{LB}}{1-\theta_{LB}-\theta_{UB}}$$

Financial intermediary - default

- May default in period 1 (in which case he disappears) or in period 2
- Defaults when at least one of his two borrowers defaults
- His creditors (L) lose the fraction of fund, invested in that particular F provided there are no governmental bailouts.

▸ Financial requirements

Governmental bailout policy

- Government may repay the gross debt owed to lenders by defaulting Fs
- In the presence of risk and the absence of uncertainty the perceived probability that the debt service of a defaulting F is paid by government (a bailout) is p
- Likelihood of bailout is independent across Fs
- In case of bailout lenders receives the debt service $1 + r_L$

Lender - yield

Since period's 1 risk is induced only by idiosyncratic factors and a lender is fully diversified his period's 1 return as perceived in period 0 is risk free implying that

$$\widetilde{r}_{L1} = r_{f1}.$$

Lender - yield

Given p, the expected return to a lender on his (fully diversified) portfolio of loans in period 2 as perceived in period 1 is normally distributed with mean

$$E\left(\{\widetilde{r}_{L2}\}\right) = \left[1 - (1-p)\left\{q(1-q_1^2) + q_1^2\right\}\right](1 + r_L)$$

and the variance is a function of q, p, θ_{LB} , θ_{UB} and r_{L1}

Equilibrium's interest rates

Proposition

In a general equilibrium with risk aversion on the part of borrowers, financial intermediaries and lenders, and positive levels of leverage in both the real and the financial sectors, the following inequalities hold

$$r_f \leq r_L < r_B.$$

Summary

Player	Index	Leverage	Mass	Income	Expenses
Borrower	B	L_B	M_B	$\widetilde{R}$	r_B
FI	F	L_F	M_F	$\widetilde{r}_B$	r_L
Lender	L	0	M_L	$\widetilde{r}_L$	0

- $\widetilde{R} = \widetilde{R}_A + \widetilde{R}_I$
- Probability of receiving $R_A = y$ is q
- In period 1 the probability of receiving $R_I = y$ is q, 1 or 0 conditional on the project type.
- Probability of bailout is p
- Probability of receiving r_B is determined by q, θ_{LB} and θ_{UB}
- Probability of receiving r_L is determined by q, p, θ_{LB} and θ_{UB}

Borrower's optimal leverage

- Since Bs payoffs are discrete the probability of default is a step function of L_B
- The optimal leverage is

$$L_B^* = \frac{1}{(1 + r_{B1})(1 + r_{B2}^e) - 1}$$

Financial intermediary's optimization

- F's wealth at the end of each period is

$$\widetilde{W}_F(\widetilde{R}_B, L_F) =$$
$$(1 + L_F)\left[z_F \widetilde{R}_B + (1 - z_F)(1 + r_f)\right] - (1 + r_L)L_F$$

- The distribution of return from two loans, $\widetilde{R}_B$, is

Return	**Probability**	**State**
$1 + r_B$	γ_{1t}	S
$\frac{1}{2}(1 + r_B)$	γ_{2t}	D
0	$1 - \gamma_F^1 - \gamma_{3t}$	D

where $t = 1, 2$.

Financial intermediary's optimization

Proposition

At an optimum with positive leverage F invests all his resources in risky loans to Bs and his/her optimal leverage is

$$L_F^*(r_{Bt}, r_{Lt}) = \left(\frac{\gamma_{1t}}{1-\gamma_{1t}}\right)^{\frac{1}{\delta}} (r_{Bt} - r_{Lt})^{\frac{1-\delta}{\delta}} - \frac{1 + r_{Bt}}{r_{Bt} - r_{Lt}},$$

$t = 0, 1.$

Financial intermediary's optimization

Proposition

(Comparative Statics):

$$\frac{\partial L_F^*}{\partial r_L} < 0, \quad \frac{\partial L_F^*}{\partial \delta} < 0, \quad \frac{\partial L_F^*}{\partial P_F} < 0, \quad \frac{\partial L_F^*}{\partial r_B} > 0, \quad \frac{\partial L_F^*}{\partial \gamma_{1t}} > 0$$

Lender's optimization

Proposition

Conditional on the probability of bailout, p, a lender's period's 1 optimal investment in a fully diversified risky portfolio of loans to Fs satisfies

$$z_L^*(p, r_{L2}, q, q_1) = \frac{E\left(\{\widetilde{r}_L\} \mid p\right) - r_f}{\alpha Var\left(\{\widetilde{r}_L\} \mid p\right)}.$$

Lender's optimization

Proposition

Holding r_{L2} constant a less generous bailout policy (lower p)

1. *reduces the mean return on the portfolio of loans from lenders to financial intermediaries*
2. *raises the covariance between any two loans in the (fully diversified) portfolio, and therefore, systemic risk and the portfolio's variance*
3. *Both changes reinforce each other in inducing a "flight to safety" by lenders*

Lender's optimization

- In the presence of bailout uncertainty and ambiguity aversion a la Gilboa Schmeidler p is replaced by π
- π is the smallest bailout probability within the set of binomial distributions with positive mass

Actual Period 0 equilibrium

Since lenders do no face risk in lending to F's, there is no risk premium in period 0 and r_{L1} is determined exogenously through

$$1 + r_{L1} = \frac{1 + r_{f1}}{1 - (1 - p)\left[1 - (1 - \theta_{UB})^2\right]}$$

Actual Period 0 equilibrium

Actual period's 0 equilibrium conditions in the markets for loans from Ls to Fs and from Fs o Bs are given by

$$
\begin{aligned}
M_F L_F^*(r_{B1}, r_{L1}) &= M_L z_{L1} \\
L_B^*(r_{B1}, r_{B2}^e) &= \frac{M_B}{(1 + r_{B1})(1 + r_{B2}^e) - 1} \\
&= M_F \left\{1 + L_F^*(r_{B1}, r_{L1})\right\}.
\end{aligned}
$$

They determine z_{L1} and r_{B1} as functions of r_{L1} and r_{B2}^e. Note that $L_B^*(r_{B1}, r_{B2}^e)$ also depends on period's 0 expectation of the cost of funds to borrowers, r_{B2}^e, in the subsequent period

Actual Period 0 equilibrium

$$(1-\theta_{UB})^2 M_F L_F^*(r_{B2}^e, r_{L2}^e) = (1+r_{f1}) M_L z_L^*(\pi_0, r_{L2}^e, q, q_1),$$

$$\frac{M_B}{(1+r_{B1})(1+r_{B2}^e)-1} = (1-\theta_{UB})\left\{1 + r_{B1} + (r_{B1}-r_{L1}) L_F^*(r_{B1}, r_{L1}) + L_F^*(r_{B2}^e, r_{L2}^e\right\}$$

The 2 period's 0 actual market clearing conditions and the 2 clearing conditions expected for period 1 jointly determine $r_{B1}, r_{B2}^e, r_{L2}^e$ and z_{L1}. This system is recursive since the last three equations simultaneously determine the first equation three variables leaving the first for the determination of z_{L1}.

Actual period 1 equilibrium

$$(1-\theta_{UB})^2 M_F L_F^*(r_{B2}, r_{L2}) = (1+r_{f1}) M_L z_L^*(\pi_1, r_{L2}, q, q_1),$$

$$\frac{M_B}{(1+r_{B1})(1+r_{B2}^e)-1} = (1-\theta_{UB})\left\{1 + r_{B1} + (r_{B1} - r_{L1}) L_F^*(r_{B1}, r_{L1}) + L_F^*(r_{B2}, r_{L2}\right\}$$

- Those **actual** period's 2 equilibrium conditions determine the **actual** interest rates, r_{B2} and r_{L2} in period 1 for given values of r_{B1}, r_{B2}^e, r_{L1} and π_1
- Importantly $\pi_1 < \pi_0$

An expost (period 1) increase in uncertainty about bailout probability

Proposition

An expost increase in bailout uncertainty (a decrease in π_1 below π_0) about governmental bailout policy leads to

- an increase in the cost of funds, r_{L2} to Fs above what it had been expected to be as of period 0.
- an increase in the cost of funds, $r_{B2,}$ to borrowers above r^e_{B2} reducing the profits expected for period 2 and increasing the fraction of borrowers who default in both periods 1 and 2.
- A decrease in the share of funds invested by lenders with financial intermediaries (flight to safety)
- When the decrease in π is sufficiently large the consequent increase in $r_{B,}$ may induce a total drying up of period's 1 credit to some or all borrowers
- An increase in bailout uncertainty raises the banking spread, $r_{B2} - r_{L2}$

An exante (period 0) decrease in uncertainty about bailout probability

Proposition

Provided F's risk aversion, δ *and the penalty,* P_F*, F incurs when insolvent are sufficiently small, an exante decrease in bailout uncertainty (an increase in* π_0*) about bailout policy leads to:*

- *Higher equilibrium levels of leverage by both borrowers and financial intermediaries*
- *Larger share of lenders' resources invested with financial intermediaries (rush to risk)*
- *Lower cost of funds to both borrowers and financial intermediaries*

The impact of F's risk aversion, δ, on interest rates and the banking spread

Proposition

The higher are the risk aversion parameter, δ, and the default penalty, P_F, of financial intermediaries

- the higher r_B
- the lower r_L
- the higher the banking spread, $r_B - r_L$
- the lower equilibrium leverage

The impact of monetary policy

Proposition

The lower the policy rate, r_f,

- the lower are r_B and r_L
- the higher is leverage and investment activity
- the larger the share of resources channeled by L's to F's

Concluding remarks and open issues

- By limiting leverage exante bailout uncertainty (as well as contractionary monetary policy) reduces leverage and the probability of a future crisis
- Expost bailout uncertainty (as well as contractionary monetary policy) exarcebates the adverse effects of an ongoing crisis
- A "black swan" increase in bailout uncertainty, such as the one that most likely occurred during the first two-three weeks following Lehman's collapse, can explain the sudden evaporation of market liquidity

Concluding remarks and open issues

- The risk aversion parameter and the insolvency penalty of financial intermediaries play important roles in the determination of aggregate leverage, the banking spread and the size of projects undertaken
- The adverse economic effects of bailout uncertainty are larger the lower is the risk aversion of financial intermediaries
- Higher bailout uncertainty is associated with larger systemic risks and wider banking spreads
- The model can be used to identify conditions under which the existence of financial intermediaries leads to more equilibrium leverage in comparison to a structure in which L's lend directly to B's (a direct capital market)
- The model can be used to identify the impact of the policy rate on the banking spread