

The Maastricht Convergence Criteria and Optimal Monetary Policy for the EMU Accession Countries - exercise for Poland¹

Anna Lipińska²

August 2008

¹I would like to thank Kosuke Aoki for his excellent supervision and encouragement. I also thank Evi Pappa, Hugo Rodriguez, Gianluca Benigno and Michael Woodford for helpful comments and suggestions. This version of the paper is based on one of the chapters of my thesis which was published also as ECB Working Paper no 896. The work is not connected in any way to the ECB or Bank of England's research program, and therefore does not necessarily reflect the views of these institutions.

²Bank of England, Monetary Analysis, International Economic Analysis Division, Threadneedle Street, London EC2R 8AH, United Kingdom; e-mail: Anna.Lipinska@bankofengland.co.uk.

Abstract

The EMU accession countries are obliged to fulfill the Maastricht convergence criteria prior to entering the EMU. This paper uses a DSGE model of a two-sector small open economy, to address the following question: How do the Maastricht convergence criteria modify optimal monetary policy in an economy facing domestic and external shocks?

First, we derive the micro founded loss function that represents the objective function of the optimal monetary policy not constrained to satisfy the criteria. We find that the optimal monetary policy should not only target inflation rates in the domestic sectors and aggregate output fluctuations but also domestic and international terms of trade. Second, we show how the loss function changes when the monetary policy is constrained to satisfy the Maastricht criteria. The loss function of such a constrained policy is characterized by additional elements penalizing fluctuations of the CPI inflation rate, the nominal interest rate and the nominal exchange rate around the new targets which are potentially different from the steady state of the unconstrained optimal monetary policy.

Under the parameterization that aims to describe Polish economy, the unconstrained optimal monetary policy violates two criteria: concerning the CPI inflation rate and the nominal interest rate. The constrained optimal policy results in targeting the CPI inflation rate and the nominal interest rate that are 0.6% lower (in annual terms) than the CPI inflation rate and the nominal interest rate in the countries taken as a reference. The welfare costs associated with these constraints need to be offset against credibility gains and other benefits related to the compliance with the Maastricht criteria that are not modelled.

JEL Classification: F41, E52, E58, E61.

Keywords: Optimal monetary policy, Maastricht convergence criteria, EMU accession countries

Streszczenie artykułu

Traktat Akcesyjny do Unii Europejskiej podpisany przez każde z 10 państw Europy Środkowo Wschodniej zobowiązuje te kraje do wstąpienia w najbliższej przyszłości także do Europejskiej Unii Monetarnej (EUM). Oprócz tego, kraje te, aby wstąpić do EUM, muszą spełnić kryteria konwergencji z Maastricht (zgodnie z założeniami Artykułu 109j(1) Traktatu z Maastricht). Zgodnie z kryteriami z Maastricht dany kraj powinien osiągnąć wysoki i trwały stopień stabilności cen, który ilościowo jest odzwierciedlony w niskich stopach inflacji oraz niskich długoterminowych stopach procentowych. Dodatkowo, nominalny kurs walutowy wobec euro w krajach kandydujących do EUM powinien pozostawać w granicach zwykłych marginesów wahań kursów przewidzianych mechanizmem wymiany walut Europejskiego Systemu Walutowego. Kryteria konwergencji, nakładając ograniczenia na zmienne monetarne, wpływają w sposób istotny na kształt polityki monetarnej prowadzonej w krajach kandydujących do EUM. Z drugiej strony, polityka monetarna pełni kluczową rolę w procesie stabilizacji małej gospodarki otwartej, która podlega wstrząsom strukturalnym. Obecnie wiele krajów kandydujących do EUM nie spełnia któregoś z kryteriów z Maastricht.

Celem tego artykułu jest zbadanie w jaki sposób kryteria konwergencji wpływają na kształt optymalnej polityki monetarnej w gospodarce, która podlega wstrząsom strukturalnym o naturze zarówno krajowej jak i zagranicznej. Konstruujemy model DSGE¹ małej gospodarki otwartej, w której ceny mają charakter sztywny. Nasz model zakłada, że polityka makroekonomiczna charakteryzuje się pełną wiarygodnością. Sektor produkcji w opisywanej gospodarce jest złożony z dwóch podsektorów: dóbr handlowych i dóbr niehandlowych. Powyższa struktura produkcji bazuje na wnioskach literatury empirycznej dotyczącej gospodarek krajów kandydujących do EUM. Według tej literatury wstrząsy sektorowej produktywności kształtują w dużym stopniu procesy inflacji i realnego kursu walutowego w krajach kandydujących do EUM (np. Mihaljek i Klau (2004)).

W części teoretycznej charakteryzujemy optymalną politykę monetarną o charakterze ponadczasowym (timeless perspective - Woodford (2003)). Wyprowadzamy wzór na funkcję straty banku centralnego (która wpływa z podstaw mikroekonomicznych modelu) stosując metodologię przybliżenia drugiego rzędu opracowaną przez Rotemberg i Woodford (1997) oraz Benigno i Woodford (2005). Optymalna polityka monetarna (polityka nieograniczona) powinna wyznaczać cele nie tylko wobec stóp inflacji w sektorach krajowych i fluktuacji produkcji zagregowanej, ale także w stosunku do relatywnych cen dóbr krajowych i importowanych (terms of trade).

W związku z tym, że uwzględnienie kryteriów z Maastricht w ich pierwotnej formie jest w naszym modelu raczej złożone, dokonujemy reinterpretacji kryteriów wykorzystując metodologię opracowaną przez Rotemberg i Woodford (1997, 1999) i zastosowaną w problemie ograniczenia zerowego nominalnej stopy procentowej. Metoda ta pozwala na prostą weryfikację, czy dane kryterium jest spełnione na podstawie pierwszego i drugiego momentu zmiennej, wobec której zostało ustalone kryterium. Funkcja straty polityki monetarnej, która jest ograniczona przez kryteria z Maastricht różni się w dwóch aspektach od funkcji straty polityki nieograniczonej. Polityka ograniczona przez kryteria z Maastricht nie tylko ma za cel ograniczenie fluktuacji inflacji, nominalnej

¹stochastyczny, dynamiczny model równowagi ogólnej

stopy procentowej i nominalnego kursu walutowego (komponent stabilizacyjny). Jako czynnik zapobiegawczy stawia sobie nowe cele deterministyczne (które różnią się od stanu ustalonego polityki nieograniczonej) wobec wyżej wymienionych zmiennych (komponent deterministyczny). Oznacza to, że np. bank centralny, prowadząc optymalną politykę ograniczoną przez kryterium inflacyjne, powinien nie tylko prowadzić politykę restrykcyjną w celu ograniczenia fluktuacji inflacji, ale także powinien wyznaczyć cel inflacyjny niższy niż w krajach referencyjnych.

W części empirycznej parametryzujemy nasz model mając na celu zgodność momentów modelu z historycznymi momentami zmiennych makroekonomicznych gospodarki polskiej. Polityka nieograniczona nie spełnia dwóch kryteriów z Maastricht: inflacji i nominalnej stopy procentowej. Polityka ograniczona przez te dwa kryteria spełnia także trzecie kryterium dotyczące nominalnego kursu walutowego. Zarówno komponent stabilizacyjny jak i deterministyczny polityki ograniczonej różni się od polityki nieograniczonej. Polityka ograniczona prowadzi do mniejszej wariacji inflacji, nominalnej stopy procentowej oraz nominalnego kursu walutowego. Poza tym, polityka ograniczona wprowadza tzw. obciążenie deflacyjne, które zakłada cel inflacyjny oraz cel nominalnej stopy procentowej o 0.6% p.a. niższy niż w krajach przyjętych za punkt odniesienia. W rezultacie polityka ograniczona prowadzi do strat dobrobytu społecznego, które wynoszą 0.016% straty konsumpcji w stanie ustalonym względem optymalnej nieograniczonej polityki monetarnej. Należy zaznaczyć, że straty te nie są skorygowane o ewentualne zyski płynące z większej wiarygodności polityki monetarnej ograniczonej przez kryteria.

1 Introduction

The European Union Accession Treaty signed by the Central and Eastern European countries² includes the obligation to participate in the third stage of the economic and monetary union, i.e. the obligation to enter the European Monetary Union (EMU) in the near future.³ In order to enter the EMU these countries are required to satisfy the Maastricht convergence criteria (for details see Appendix B). The criteria are designed to guarantee that prior to joining the European Monetary Union, countries attain a high degree of economic convergence not only in real but also in nominal terms. The countries should achieve a high and durable degree of price stability, which is reflected in low inflation rates and low long-term interest rates. Quantitatively, over a period of one year before the examination the average rate of CPI inflation should not exceed that of the three best performing Member States by more than 1.5% points, while the average nominal long-term interest rate should not exceed by more than 2% that of the best performing Member States in terms of price stability. Additionally, nominal exchange rates of the EMU accession countries versus the euro should stay within normal fluctuation margins (i.e. 15% bound around the central parity) provided for by the Exchange Rate Mechanism of the European Monetary System for at least two years.⁴

By setting constraints on the monetary variables, these criteria affect the way monetary policy should be conducted in the EMU accession countries. Monetary policy plays a crucial role in the stabilization process of an economy exposed to shocks. The obligation to fulfill the Maastricht convergence criteria by the EMU accession countries can restrict the stabilization role of the monetary policy. At the moment, many EMU accession countries do not satisfy some of the Maastricht convergence criteria. Bulgaria, Estonia, Hungary, Latvia, Lithuania, Romania and Slovakia fail to fulfill the CPI inflation rate criterion (see Figure B.3 in Appendix B). Hungary and Romania also violate the nominal interest rate criterion (see Figure B.5 in Appendix B). Finally the nominal exchange rate fluctuations of Polish Zloty, Slovakian Koruna and Romanian Lei versus the euro exceed the band set by the nominal exchange rate criterion (see Figure B.6 in Appendix B).⁵

Keeping this in mind, a natural question arises. How do the Maastricht convergence criteria modify the optimal monetary policy in an economy that faces domestic and external shocks?

To answer this question, we develop a DSGE model of a small open economy with nominal rigidities exposed to both domestic and external shocks. Our model assumes fully credible macroeconomic policies. Moreover this model characterises cyclical behaviour of the economy abstracting from any transitional dynamics that are present in the EMU accession countries.⁶ We take as a point of reference the small open economy model developed by De Paoli (2004). The production structure of the economy is composed of two sectors: a nontraded

²Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia and Slovenia signed the EU Accession Treaty on May 1, 2004. Bulgaria and Romania joined this group on January 1, 2007.

³Slovenia is the first country in this group that joined the European Monetary Union on January 1, 2007. Cyprus and Malta joined the EMU on January 1, 2008.

⁴The Maastricht Treaty also imposes the fiscal criterion, i.e. the sustainability of the government financial position which refers to a government budgetary position without an excessive deficit (Article 104c(6) of the Maastricht Treaty). However, in this paper, we focus on the monetary aspects of the Maastricht convergence criteria.

⁵For the purpose of this exercise we take the bilateral exchange rate from the day of the EU accession to be the central parity.

⁶The phenomenon of nominal price convergence observed in the EMU accession countries results in a positive trend inflation and exchange rate appreciation. This makes the criteria even harder to comply with.

good sector and a traded good sector. The same production structure is present, among others, in the two country model of Liu and Pappa (2005) and the currency area model of Altissimo et al (2005). There are several reasons to impose such a structure in our model. According to the literature, the existence of the nontraded sector helps us to explain international business cycle fluctuations and especially real exchange rate movements (e.g. Benigno and Thoenisen (2003), Corsetti et al. (2003), Stockman and Tesar (1994)). Moreover, the empirical studies regarding the OECD countries find that a major part of the aggregate fluctuations rather have their source in sector-specific than country-wide shocks (e.g. Canzoneri et al. (1999), Marimon and Zilibotti (1998)). Finally we want to match our model with the empirical literature on the EMU accession countries that emphasizes the role of sector productivity shocks in shaping inflation and real exchange rate patterns in these countries (e.g. Mihaljek and Klau (2004)).

In this framework we characterize the optimal monetary policy from a timeless perspective (Woodford (2003)). We derive the micro founded loss function using the second-order approximation methodology developed by Rotemberg and Woodford (1997) and Benigno and Woodford (2005). We find that the optimal monetary policy (unconstrained policy) should not only target inflation rates in the domestic sectors and aggregate output fluctuations, but also domestic and international terms of trade. Since the Maastricht convergence criteria are not easily implementable in our model, we reformulate them using the methodology developed by Rotemberg and Woodford (1997, 1999) for the analysis of the zero bound problem of the nominal interest rate. This method enables us to verify whether a given criterion is satisfied by only computing first and second moments of a variable for which the criterion is set. We focus on the criteria imposed on the CPI inflation rate, the nominal interest rate and the nominal exchange rate as we do not explicitly model the fiscal policy. We present how the loss function changes when the monetary policy is constrained by the Maastricht convergence criteria. Finally, we derive the optimal monetary policy that satisfies all Maastricht convergence criteria (constrained policy).

Under the chosen parameterization (which aims at reflecting the Polish economy)⁷, the unconstrained optimal monetary policy violates the CPI inflation rate and the nominal interest rate criteria. The optimal policy which satisfies these two criteria also guarantees the satisfaction of the nominal exchange criterion. Both the stabilization component and the deterministic component of the constrained policy are different from the unconstrained optimal policy. The constrained policy leads to a smaller variability of the CPI inflation, the nominal interest rate and the nominal exchange rate than under optimal monetary policy. Moreover, this policy is characterized by a deflationary bias which results in targeting a CPI inflation rate and a nominal interest rate that are 0.6% lower (in annual terms) than the CPI inflation rate and the nominal interest rate in the reference countries. As a result, the constrained policy induces additional welfare costs which amount to 85% of the initial deadweight loss associated with the optimal monetary policy. In other words, the permanent percentage shift in the steady state consumption that is lost under the constrained policy with respect to the unconstrained one is equal 0.016%.⁸ These losses need to be offset against the potential benefits from complying with the

⁷Note that the numerical exercise in Lipińska (2008) was performed on the Czech Republic data.

⁸The value is calculated as in Benigno and Lopez-Salido (2003).

criteria which are not analyzed in this paper.

The Maastricht convergence criteria can serve as a commitment technology that improves the credibility of macroeconomic policies in the accession countries (more on this in Ravenna (2005)). Intuitively, the CPI inflation criterion serves to prevent inadequate policies that could lead to higher production costs thus lowering growth. The nominal interest rate restricts the long run inflation expectations to guarantee sustainability of inflation convergence. Finally, the exchange rate criterion aims at preventing possible tensions in the foreign exchange rate market and precludes excessive devaluation before adoption of the euro (more on this in the studies of the National Bank of Hungary (2002), the National Bank of Poland (2004) and the ECB Convergence Report (2006)).

The literature has so far concentrated on two aspects of monetary policy in the EMU accession countries: the appropriate monetary regime in the light of the future accession to the EMU and the ability of alternative monetary regimes to comply with the Maastricht convergence criteria. The first stream of literature represented by, among others, Buitier and Grafe (2003), Coricelli (2002), calls for adopting the peg regime to the euro in these countries, as it enhances credibility of the monetary policy and strengthens the links with the EU and the EMU. Using a DSGE model with nominal rigidities and imperfect credibility, Ravenna (2005) finds that the gain from a credible adoption of the fixed regime towards the euro can outweigh the loss of resignation from the independent monetary policy. Nevertheless, Buitier and Grafe (2003) also claim that an adoption of the fixed regime can seriously endanger the fulfillment of the CPI inflation criterion and therefore call for a change in this criterion. Their reasoning is based on the empirical studies regarding sources of the CPI inflation and real exchange rate developments in the EMU accession countries. A majority of the studies⁹ concentrate on the Balassa–Samuelson effect (Balassa (1964)), which predicts that countries experiencing a higher productivity growth in the traded sector are also characterized by a higher CPI inflation rate and real exchange rate appreciation. Others (e.g. Mihaljek and Klau (2004)) also highlight the role of productivity shocks in the nontraded sector in affecting the CPI inflation rate and the real exchange rate appreciation in the EMU accession countries.¹⁰

The second stream of the literature builds an analysis in the framework of open economy DSGE models. Devereux (2003) and Natalucci and Ravenna (2003) find that the monetary regime characterized by flexible inflation targeting with some weight on exchange rate stability succeeds in fulfilling the Maastricht criteria. Lipińska (2007) shows that probability of a given monetary regime to comply with the Maastricht criteria increases with the degree of openness of an economy and the substitutability of home and foreign goods. The degree of exchange rate pass through determines which of the regimes can comply with the criteria. Low degree of pass through discriminates between regimes: when economy gets more open, variances of the Maastricht variables under the peg and managed float regime diminish while the contrary is true for the CPI targeting regime. If degree of exchange rate pass through is high, then higher openness enables all the regimes to meet

⁹We can list the following empirical studies that analyze CPI inflation and real exchange rate developments in the EMU accession countries: Cipriani (2001), de Broeck and Slok (2001), Egert et al. (2002), Fisher (2002), Halpern and Wyplosz (2001), Coricelli and Jazbec (2001), Arratibel et al. (2002), Mihaljek and Klau (2004).

¹⁰This study goes in line with a recent paper by Altissimo et al (2004) on the sources of inflation differentials in the euro area. The authors find that the nontraded sector (proxied as the service sector) contributes the most to price dispersion among member countries.

the Maastricht criteria. Three other studies are also worth noting: Laxton and Pesenti (2003), Ferreira (2006) and Kolasa (2008). The authors of the first paper study how different interest rate rules perform in stabilizing variability of inflation and output in a small open economy. The second paper focuses on the calculation of the welfare loss that the EMU accession countries will face when they join the EMU. However, contrary to our study, it does not provide the micro founded welfare criterion. Author of the third paper studies the nature of heterogeneity between economies of Poland and the euro area in a two country DSGE model estimated with Bayesian methods.

In contrast to previous studies, our analysis is characterized by the normative approach. We construct the optimal monetary policy for a small open economy and contrast it with the optimal policy that is restricted to satisfy the Maastricht convergence criteria. Therefore, our framework enables us to set guidelines on the way in which monetary policy should be conducted in the EMU accession countries.

The rest of the paper is organized as follows. The next section introduces the model and derives the small open economy dynamics. Section 3 describes derivation of the optimal monetary policy. Section 4 presents the way we reformulate the Maastricht convergence criteria in order to implement them in our framework. Section 5 is dedicated to the derivation of the optimal policy constrained by the Maastricht convergence criteria. Section 6 compares the optimal monetary policy with the optimal monetary policy constrained by the Maastricht convergence criteria under the chosen parameterization of the model. Section 7 concludes.

2 The model

Our modelling framework is based on a one-sector small open economy model of De Paoli (2004) where all goods, i.e. home and foreign ones, are tradable. We extend this model by incorporating two domestic sectors, i.e. a nontraded and a traded sector. Our model is also closely related to the studies of Devereux (2003) and Natalucci and Ravenna (2003). However, we relax an assumption present in their studies regarding perfect competition and homogeneity of goods in the traded sector, which enables us to discuss a role of terms of trade in the stabilization process of a small open economy. In that way our modelling framework is similar to a two-country model with two production sectors of Liu and Pappa (2005).

Following De Paoli (2004), we model a small open economy as the limiting case of a two-country problem, i.e. where the size of the small open economy is set to zero. In the general framework, the model represents two economies of unequal size: a small open home economy and a foreign large economy (which is proxied as the euro area). We consider two highly integrated economies where asset markets are complete. In each of the economies, there are two goods sectors: nontraded goods and traded goods. Moreover, we assume that labour is mobile between sectors in each country and immobile between countries. We assume the existence of home bias in consumption which, in turn, depends on the relative size of the economy and its degree of openness. This assumption enables us to consider a limiting case of the zero size of the home economy and concentrate on the small open economy.

Purchasing power parity (PPP) is violated for two reasons: existence of the nontraded sector and home bias in consumption. Furthermore, in order to study the role of monetary policy in this framework, we introduce monopolistic competition and price rigidities with staggered Calvo contracts in all goods sectors. However, we abstract from any monetary frictions by assuming cashless limiting economies.¹¹ The stochastic environment of the small open economy is characterized by asymmetric productivity shocks originating in both domestic sectors, preference shocks and foreign consumption shocks.

2.1 Households

The world economy consists of a continuum of agents of unit mass: $[0, n)$ belonging to a small country (home) and $[n, 1]$ belonging to the rest of the world, i.e. the euro area (foreign). There are two types of differentiated goods produced in each country: traded and nontraded goods. Home traded goods are indexed on the interval $[0, n)$ and foreign traded goods on the interval $[n, 1]$, respectively. The same applies to nontraded goods. In order to simplify the exposition of the model, we explain in detail only the structure and dynamics of the domestic economy. Thus, from now on, we assume the size of the domestic economy to be zero, i.e. $n \rightarrow 0$.

Households are assumed to live infinitely and behave according to the permanent income hypothesis. They can choose between three types of goods: nontraded, domestic traded and foreign traded goods. C_t^i represents consumption at period t of a consumer i and L_t^i constitutes his labour supply. Each agent i maximizes the following utility function:¹²

$$\max E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} [U(C_t^i, B_t) - V(L_t^i)] \right\}, \quad (2.1)$$

where E_{t_0} denotes the expectation conditional on the information set at date t_0 , β is the intertemporal discount factor and $0 < \beta < 1$, $U(\cdot)$ stands for flows of utility from consumption and $V(\cdot)$ represents flows of disutility from supplying labour.¹³ C is a composite consumption index. We define consumers' preferences over the composite consumption index C_t of traded goods ($C_{T,t}$) (domestically produced and foreign ones) and nontraded goods ($C_{N,t}$):

$$C_t \equiv \left[\mu^{\frac{1}{\phi}} C_{N,t}^{\frac{\phi-1}{\phi}} + (1-\mu)^{\frac{1}{\phi}} C_{T,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad (2.2)$$

where $\phi > 0$ is the elasticity of substitution between traded and nontraded goods and $\mu \in [0, 1]$ is the share of the nontraded goods in overall consumption. Traded good consumption is a composite of the domestically produced traded goods (C_H) and foreign produced traded goods (C_F):

¹¹See Woodford (2003).

¹²In general, we assume U to be twice differentiable, increasing and concave in C_t and V to be twice differentiable, increasing and convex in L_t .

¹³We assume specific functional forms of consumption utility $U(C_t^i)$, and disutility from labour $V(L_t^i)$: $U(C_t^i) \equiv \frac{(C_t^i)^{1-\rho} B_t^\rho}{1-\rho}$, $V(L_t^i) \equiv \varphi_l \frac{(L_t^i)^{1+\eta}}{1+\eta}$ with ρ ($\rho > 0$), the inverse of the intertemporal elasticity of substitution in consumption and η ($\eta \geq 0$), the inverse of labour supply elasticity and B_t , preference shock.

$$C_{T,t} \equiv \left[(1 - \lambda)^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (2.3)$$

where $\theta > 0$ is the elasticity of substitution between home traded and foreign traded goods, and λ is the degree of openness of the small open economy ($\lambda \in [0, 1]$).¹⁴ Finally, C_j (where $j = N, H, F$) are consumption sub-indices of the continuum of differentiated goods:

$$C_{j,t} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \quad (2.4)$$

where $\sigma > 1$ represents elasticity of substitution between differentiated goods in each of the sectors. Based on the above presented preferences, we derive consumption-based price indices expressed in the units of currency of the domestic country:

$$P_t \equiv \left[\mu P_{N,t}^{1-\phi} + (1 - \mu) P_{T,t}^{1-\phi} \right]^{\frac{1}{1-\phi}}, \quad (2.5)$$

$$P_{T,t} \equiv \left[\nu P_{H,t}^{1-\theta} + (1 - \nu) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (2.6)$$

with

$$P_{j,t} \equiv \left[\left(\frac{1}{n} \right) \int_0^n p_t(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}. \quad (2.7)$$

Although we assume the law of one price in the traded sector (i.e. $p(h) = Sp^*(h)$ and $p(f) = Sp^*(f)$ where S is the nominal exchange rate), both the existence of the nontraded goods and the assumed home bias cause deviations from purchasing power parity, i.e. $P \neq SP^*$. The real exchange rate can be defined in the following manner: $RS \equiv \frac{SP^*}{P}$. Moreover, we define the international terms of trade as $T \equiv \frac{P_F}{P_H}$ and the ratio of nontraded to traded goods' prices (domestic terms of trade) as $T^d \equiv \frac{P_N}{P_T}$.

From consumer preferences, we can derive total demand for the generic goods – n (home nontraded ones), h (home traded ones), f (foreign traded ones):

$$y^d(n) = \left[\frac{p(n)}{P_N} \right]^{-\sigma} \left[\frac{P_N}{P} \right]^{-\phi} \mu C, \quad (2.8)$$

$$y^d(h) = \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left[\frac{P_H}{P_T} \right]^{-\theta} (1 - \lambda) C_T + \left[\frac{p^*(h)}{P_H^*} \right]^{-\sigma} \left(\frac{P_H^*}{P_T^*} \right)^{-\theta^*} \lambda C_T^*, \quad (2.9)$$

¹⁴Following de Paoli (2004) and Sutherland (2002), we assume home bias (ν) of the domestic households to be a function of the relative size of the home economy with respect to the foreign one (n) and its degree of openness (λ) such that $(1 - \nu) = (1 - n)\lambda$ where $\lambda \in [0, 1]$. Importantly, the higher is the degree of openness, the smaller is the degree of home bias. Since $n \rightarrow 0$, we obtain that $\nu = 1 - \lambda$.

$$y^d(f) = \left[\frac{p^*(f)}{P_F^*} \right]^{-\sigma} \left[\frac{P_F^*}{P_T^*} \right]^{-\theta} C_T^* \quad (2.10)$$

where variables with an asterisk represent the foreign equivalents of the domestic variables. Importantly, since the domestic economy is a small open economy, demand for foreign traded goods does not depend on domestic demand. However, at the same time, demand for domestic traded goods depends on foreign demand.

Households get disutility from supplying labour to all firms present in each country. Each individual supplies labour to both sectors, i.e. the traded and the nontraded sector:

$$L_t^i = L_t^{i,H} + L_t^{i,N}. \quad (2.11)$$

We assume that consumers have access to a complete set of securities-contingent claims traded internationally. Each household faces the following budget constraint:

$$P_t C_t^i + E_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + TR_t^i + W_{H,t}^i L_{H,t}^i + W_{N,t}^i L_{N,t}^i + \frac{\int_0^n \Pi_{N,t}^i di}{n} + \frac{\int_0^n \Pi_{H,t}^i di}{n}, \quad (2.12)$$

where at date t , D_{t+1} is nominal payoff of the portfolio held at the end of period (t), $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household, $\Pi_{H,t}$ and $\Pi_{N,t}$ are nominal profits from the domestic firms and TR_t^i are nominal lump-sum transfers from the domestic government to household i . Moreover, consumers face no Ponzi game restriction.

The short-term interest rate (R_t) is defined as the price of the portfolio which delivers one unit of currency in each contingency that occurs in the next period:¹⁵

$$R_t^{-1} = E_t \{ Q_{t,t+1} \}. \quad (2.13)$$

The maximization problem of any household consists of maximizing the discounted stream of utility (2.1) subject to the budget constraint (2.12) in order to determine the optimal path of the consumption index, the labour index and contingent claims at all times. The solution to the household decision problem gives a set of first-order conditions.¹⁶ Optimization of the portfolio holdings leads to the following Euler equations for the domestic economy:

$$U_C(C_t, B_t) = \beta E_t \left\{ U_C(C_{t+1}, B_{t+1}) Q_{t,t+1}^{-1} \frac{P_t}{P_{t+1}} \right\}. \quad (2.14)$$

There is a perfect sharing in this setting, meaning that marginal rates of consumption in nominal terms are equalized between countries in all states and at all times.¹⁷ Subsequently, appropriately choosing the

¹⁵Following the literature, we assume one period to be one quarter.

¹⁶We here suppress subscript i as we assume that in equilibrium, all agents are identical. Therefore, we represent optimality conditions for a representative agent.

¹⁷We have to point out here that although the assumption of complete markets conveniently simplifies the model, it neglects a possibility of wealth effects in response to different shocks (Benigno (2001)).

distribution of initial wealth, we obtain the risk sharing condition:

$$\frac{U_C(C_t, B_t)}{U_C(C_t^*, B_t^*)} = v \frac{P_t}{S_t P_t^*} = v R S_t^{-1}, \quad (2.15)$$

where $v > 0$ and depends on the initial wealth distribution. The risk sharing condition implies that the real exchange rate is equal to the marginal rate of substitution between domestic and foreign consumption.

The optimality condition for labour supply in the domestic economy is the following:

$$\frac{W_t^k}{P_t} = \frac{V_L(L_t)}{U_C(C_t, B_t)}, \quad (2.16)$$

where W^k is the nominal wage of the representative consumer in sector k ($k = H, N$).¹⁸ So the real wage is equal to the marginal rate of substitution between labour and consumption.

2.2 Firms

All firms are owned by consumers. Both traded and nontraded sectors are monopolistically competitive. The production function is linear in labour which is the only input. Consequently, its functional form for firm i in sector k ($k = N, H$) is the following:

$$Y_{k,t}(i) = A_t^k L_t^k(i). \quad (2.17)$$

Price is set according to the Calvo (1983) pricing scheme. In each period, a fraction of firms $(1 - \alpha_k)$ decides its price, thus maximizing the future expected profits. The maximization problem of any firm in sector k at time t_0 is given by:

$$\begin{aligned} \max_{P_{k,t_0}(i)} E_{t_0} \sum_{t=t_0}^{\infty} (\alpha_k)^s Q_{t_0,t} [(1 - \tau_k) P_{k,t_0}(i) - MC_t^k(i)] Y_{k,t_0:t}^d(i) \\ \text{subject to } Y_{k,t_0:t}^d(i) = \left(\frac{P_{k,t_0}(i)}{P_{k,t}} \right)^{-\sigma} Y_{k,t}, \end{aligned} \quad (2.18)$$

where $Y_{k,t_0:t}^d(i)$ is demand for the individual good in sector k produced by producer i at time t conditional on keeping the price $P_{k,t_0}(i)$ fixed at the level chosen at time t_0 , $MC_t^k = \frac{W_t^k(i)}{A_t^k}$ is the nominal marginal cost in sector k at time t , and τ_k are revenue taxes in sector k .

Given this setup, the price index in sector k evolves according to the following law of motion:

$$(P_{k,t})^{1-\sigma} = \alpha_k (P_{k,t-1})^{1-\sigma} + (1 - \alpha_k) (P_{k,t_0}(i))^{1-\sigma}. \quad (2.19)$$

¹⁸Notice that wages are equalized between sectors inside each of the economies, due to perfect labour mobility and perfect competition in the labour market.

2.3 Fiscal and monetary policies

The government in the domestic economy is occupied with collecting revenue taxes from firms that are later redistributed to households in the form of lump-sum transfers in such a way that each period, there is a balanced budget:

$$\int_0^n (\tau_N P_{N,t}(i) Y_{N,t}(i) + \tau_H P_{H,t}(i) Y_{H,t}(i)) di = \int_0^n TR_t^j dj. \quad (2.20)$$

A role for the monetary policy arises due to existing nominal and real rigidities in the economy: price stickiness (together with monopolistic competition), home bias and the nontraded good sector, which lead to deviations from PPP. The system is therefore closed by defining appropriate monetary rule for the domestic economy.

2.4 A loglinearized version of the model

This section presents a system of the equilibrium conditions for the small open economy in the loglinear form, which is derived through the first-order approximation around the deterministic steady state with zero inflation defined in Appendix A.1. Here, we characterize the dynamic features of this model where the variables with a hat stand for the log deviations from the steady state. Additionally, the variables with an asterisk represent the foreign equivalents of the domestic variables.

The supply-side of the economy is given by two Phillips curves, one for the nontraded and one for the domestic traded sector, respectively, which are derived from (2.18):

$$\hat{\pi}_{N,t} = k_N(\rho\hat{C}_t + \eta\hat{L}_t - \hat{A}_{N,t} - \rho\hat{B}_t - \hat{p}_{N,t}) + \beta\hat{\pi}_{N,t+1}, \quad (2.21)$$

$$\hat{\pi}_{H,t} = k_H(\rho\hat{C}_t + \eta\hat{L}_t - \hat{A}_{H,t} - \rho\hat{B}_t - \hat{p}_{H,t}) + \beta\hat{\pi}_{H,t+1} \quad (2.22)$$

where $\hat{p}_{N,t} \equiv \ln(\frac{P_{N,t}}{P_t})$, $\hat{p}_{H,t} \equiv \ln(\frac{P_{H,t}}{P_t})$, $\hat{\pi}_{N,t} \equiv \ln(\frac{P_{N,t}}{P_{N,t-1}})$, $\hat{\pi}_{H,t} \equiv \ln(\frac{P_{H,t}}{P_{H,t-1}})$, $k_N \equiv \frac{(1-\alpha_N)(1-\alpha_N\beta)}{\alpha_N}$, $k_H \equiv \frac{(1-\alpha_H)(1-\alpha_H\beta)}{\alpha_H}$ and aggregate labour supply (\hat{L}_t) is defined through the labour market clearing condition ((2.11), (2.17)):

$$\hat{L}_t = \tilde{d}_{Y_N}(\hat{Y}_{N,t} - \hat{A}_{N,t}) + \tilde{d}_{Y_H}(\hat{Y}_{H,t} - \hat{A}_{H,t}), \quad (2.23)$$

where $\tilde{d}_{Y_N} \equiv \frac{\bar{Y}_N}{\bar{Y}_N + \bar{Y}_H}$, $\tilde{d}_{Y_H} \equiv \frac{\bar{Y}_H}{\bar{Y}_N + \bar{Y}_H}$ are ratios evaluated in the steady state.

It is worth underlining that inflation dynamics in both domestic sectors do not only depend on the real marginal costs in a given sector, but also on the relative prices of goods. In particular, a higher relative price of goods in one sector in relation to other goods induces a substitution away effect and leads to deflationary pressures in this sector.

The demand side of the small open economy is represented by the market clearing conditions in both nontraded and domestic traded sectors ((2.8), (2.9)):

$$\widehat{Y}_{N,t} = \widehat{C}_t - \phi \widehat{p}_{N,t}, \quad (2.24)$$

$$\begin{aligned} \widehat{Y}_{H,t} = d_{CH} \widehat{C}_t - \theta \widehat{p}_{H,t} + b(\phi - \theta) d_{CH} \widehat{T}_t^d + (1 - d_{CH}) \theta \widehat{R} \widehat{S}_t + \\ + (1 - d_{CH}) \widehat{C}_t^* + b^*(\phi - \theta)(1 - d_{CH}) \widehat{T}_t^{d*} \end{aligned} \quad (2.25)$$

where $d_{CH} \equiv (1 - \lambda)(1 - \mu) \frac{\overline{C}}{\overline{Y}_H} \overline{p}_H^{-\theta} \overline{p}_T^{\theta - \phi}$, $b \equiv \mu(\overline{p}_N)^{1 - \phi}$, $b^* \equiv \mu^*(\overline{p}_N^*)^{1 - \phi}$ are ratios evaluated in the steady state. Additionally, we define aggregate output as the sum of sector outputs:

$$\widehat{Y}_t = d_{YN}(\widehat{p}_{N,t} + \widehat{Y}_{N,t}) + d_{YH}(\widehat{p}_{H,t} + \widehat{Y}_{H,t}), \quad (2.26)$$

where $d_{YN} \equiv \frac{\overline{p}_N \overline{Y}_N}{\overline{Y}}$ and $d_{YH} \equiv \frac{\overline{p}_H \overline{Y}_H}{\overline{Y}}$ are ratios evaluated in the steady state.

The complete asset market assumption (2.15) gives us the following risk sharing condition:

$$\widehat{C}_t = \widehat{B}_t + \frac{1}{\rho} \widehat{R} \widehat{S}_t + \widehat{C}_t^* - \widehat{B}_t^*. \quad (2.27)$$

From the definition of price indices ((2.5), (2.6)), we obtain the following relations between relative prices, terms of trade, domestic terms of trade and real exchange rate:

$$(a - 1) \widehat{p}_{H,t} = b \widehat{T}_t^d + a \widehat{R} \widehat{S}_t - b^* a \widehat{T}_t^{d*}, \quad (2.28)$$

$$\widehat{p}_{N,t} = (1 - b) \widehat{T}_t^d, \quad (2.29)$$

$$\widehat{p}_{H,t} = -b \widehat{T}_t^d - a \widehat{T}_t, \quad (2.30)$$

where $a \equiv \lambda \left(\frac{RS p_T^*}{\overline{p}_T} \right)^{1 - \theta}$ is the ratio evaluated in the steady state. We also derive the laws of motion for the international terms of trade and the domestic terms of trade from their definitions:

$$\widehat{T}_t = \widehat{\pi}_{F,t} - \widehat{\pi}_{H,t} + \widehat{T}_{t-1}, \quad (2.31)$$

$$\widehat{T}_t^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{T,t} + \widehat{T}_{t-1}^d, \quad (2.32)$$

where $\widehat{\pi}_{T,t} = (1 - a) \widehat{\pi}_{H,t} + a \widehat{\pi}_{F,t}$ and $\widehat{\pi}_{F,t} = \widehat{\pi}_{F,t}^* + (\widehat{S}_t - \widehat{S}_{t-1})$ with $\widehat{\pi}_{T,t} \equiv \ln\left(\frac{P_{T,t}}{P_{T,t-1}}\right)$, $\widehat{\pi}_{F,t} \equiv \ln\left(\frac{P_{F,t}}{P_{F,t-1}}\right)$,

$$\widehat{\pi}_{F,t}^* \equiv \ln\left(\frac{P_{F,t}^*}{P_{F,t-1}^*}\right).$$

Finally, we present equations defining the Maastricht variables: the CPI inflation rate ($\widehat{\pi}_t$), the nominal interest rate (\widehat{R}_t) and the nominal exchange rate (\widehat{S}_t). First, the nominal interest rate can be derived from the loglinearized version of the Euler condition (2.14):

$$\widehat{R}_t = \rho(\widehat{C}_{t+1} - \widehat{B}_{t+1}) - \rho(\widehat{C}_t - \widehat{B}_t) + \widehat{\pi}_{t+1}, \quad (2.33)$$

where $\widehat{\pi}_t \equiv \ln\left(\frac{P_t}{P_{t-1}}\right)$. CPI aggregate inflation is a weighted sum of the sector inflation rates:

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-a)(1-b)\widehat{\pi}_{H,t} + a(1-b)\widehat{\pi}_{F,t}^* + a(1-b)(\widehat{S}_t - \widehat{S}_{t-1}). \quad (2.34)$$

Notice that CPI aggregate inflation does not only depend on the domestic sector inflation rates, but also on the foreign traded inflation rate and changes in the nominal exchange rate. For example, a nominal exchange rate depreciation puts an upward pressure on the CPI inflation rate.

The nominal exchange rate can be derived from the definition of the real exchange rate:

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^* + \widehat{R}S_t - \widehat{R}S_{t-1}. \quad (2.35)$$

The law of motion of the nominal exchange rate depends on the real exchange rate fluctuations and differences in the aggregate inflation rates between the home and the foreign economy. Additionally, by combining the international risk sharing condition (2.27) and Euler conditions for the domestic and foreign economy (2.33), we obtain a relation between the nominal interest rate and the nominal exchange rate:

$$\widehat{S}_t = \widehat{R}_t^* - \widehat{R}_t + \widehat{S}_{t+1}. \quad (2.36)$$

This equation represents a version of the uncovered interest rate parity, which implies that changes in the nominal exchange rate result from differences between the domestic and foreign monetary policy. Let us point out that although very intuitive, this equation does not constitute an independent equilibrium condition.

The system is closed by specifying a monetary rule. In this paper, we derive the optimal monetary policy rule which maximizes welfare of the society subject to the structural equations of the economy. The optimal rule is specified as a rule where the monetary authority stabilizes the target variables in order to minimize the welfare loss of society and provide the most efficient allocation.¹⁹ Apart from the optimal monetary derivation in this framework, we also consider the optimal monetary policy which is additionally constrained by the Maastricht convergence criteria.

Summing up, the dynamics of the small open economy are summarized by the following variables, $\widehat{\pi}_{N,t}$, $\widehat{\pi}_{H,t}$, \widehat{C}_t , \widehat{L}_t , $\widehat{Y}_{H,t}$, $\widehat{Y}_{N,t}$, $\widehat{p}_{N,t}$, $\widehat{p}_{H,t}$, \widehat{Y}_t , $\widehat{R}S_t$, \widehat{T}_t^d , \widehat{T}_t , \widehat{S}_t , $\widehat{\pi}_t$, \widehat{R}_t which are determined by equations (2.21)–(2.35), given the evolution of the stochastic shocks $\widehat{A}_{N,t}$, $\widehat{A}_{H,t}$, \widehat{B}_t and the foreign variables \widehat{C}_t^* , \widehat{T}_t^{d*} , $\widehat{\pi}_t^*$, $\widehat{\pi}_{F,t}^*$.²⁰

¹⁹Giannoni and Woodford (2003) call these type of rules flexible inflation targeting rules.

²⁰For simplicity, we choose to consider only one type of external shocks, foreign consumption shocks (\widehat{C}_t^*). As a result, \widehat{T}_t^{d*} , $\widehat{\pi}_t^*$,

3 The optimal monetary policy

This section characterizes the optimal monetary policy, i.e. the policy maximizing welfare of society subject to the structural equations of an economy. The micro foundations of our model give us a natural welfare measure, i.e. a discounted sum of expected utilities for the agents in the economy (see equation (2.1)).

We use a linear quadratic approach (Rotemberg and Woodford (1997, 1999)) and define the optimal monetary policy problem as a minimization problem of the quadratic loss function subject to the loglinearized structural equations (presented in the previous section). First, we present the welfare measure derived through a second-order Taylor approximation of equation (2.1):

$$W_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z'_v \hat{v}_t - \frac{1}{2} \hat{v}'_t Z_v \hat{v}_t - \hat{v}'_t Z_{\xi} \hat{\xi}_t] + tip + O(3), \quad (3.1)$$

where $\hat{v}'_t = \begin{bmatrix} \hat{C}_t & \hat{Y}_{N,t} & \hat{Y}_{H,t} & \hat{\pi}_{N,t} & \hat{\pi}_{H,t} \end{bmatrix}$; $\hat{\xi}'_t = \begin{bmatrix} \hat{A}_{N,t} & \hat{A}_{H,t} & \hat{B}_t & \hat{C}_t^* \end{bmatrix}$;
 $z'_v = \begin{bmatrix} 1 & -s_{CY_N} & -s_{CY_H} & 0 & 0 \end{bmatrix}$ with $s_{CY_N} \equiv \frac{\bar{\omega}_{Y_N}}{\bar{C}}$ - steady state share of nontraded labour income in domestic consumption, $s_{CY_H} \equiv \frac{\bar{\omega}_{Y_H}}{\bar{C}}$ - steady state share of home traded labour income in domestic consumption and matrices Z_v, Z_{ξ} are defined in Appendix A.4; *tip* stands for *terms independent of policy* and $O(3)$ includes terms that are of a higher order than the second in the deviations of variables from their steady state values.

Notice that the welfare measure (3.1) contains the linear terms in aggregate consumption and sector outputs. These linear terms result from the distortions in the economy. First, monopolistic competition in both domestic sectors leads to inefficient levels of sector outputs and subsequently, an inefficient level of aggregate output. Second, since the domestic economy is open, domestic consumption and aggregate output are not equalized. Importantly, their composition depends on the domestic and international terms of trade. Third, there exists a(n) (international) terms of trade externality (see Corsetti and Pesenti (2001)) according to which monetary policy has an incentive to generate a welfare improving real exchange rate appreciation which leads to a lower disutility from labour without a corresponding decline in the utility of consumption.

The presence of linear terms in the welfare measure (3.1) means that we cannot determine the optimal monetary policy, even up to first order, using the welfare measure subject to the structural equations (2.21)–(2.35) that are only accurate to first order. Following the method proposed by Benigno and Woodford (2005) and Benigno and Benigno (2005), we substitute the linear terms in the approximated welfare function (3.1) by second moments of aggregate output, domestic and international terms of trade using a second-order approximation to the structural equations of the economy.²¹ As a result, we obtain the fully quadratic loss function which can be represented as a function of aggregate output (\hat{Y}_t), domestic and international terms of trade (\hat{T}_t^d, \hat{T}_t) and domestic sector inflation rates ($\hat{\pi}_{H,t}, \hat{\pi}_{N,t}$). Its general expression is given below:

$\hat{\pi}_{F,t}^*$ are assumed to be zero. Moreover, all shocks follow an AR(1) process with normally distributed innovations.

²¹Details of the derivation can be found in Appendix A.4.

$$\begin{aligned}
L_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} & \left[\frac{1}{2} \Phi_Y (\hat{Y}_t - \hat{Y}_t^T)^2 + \frac{1}{2} \Phi_{T^d} (\hat{T}_t^d - \hat{T}_t^{dT})^2 + \right. \\
& \frac{1}{2} \Phi_T (\hat{T}_t - \hat{T}_t^T)^2 + \Phi_{TT^d} \hat{T}_t^d \hat{T}_t + \Phi_{YT^d} \hat{Y}_t \hat{T}_t^d + \Phi_{YT} \hat{Y}_t \hat{T}_t + \\
& \left. + \frac{1}{2} \Phi_{\pi_H} \hat{\pi}_{H,t}^2 + \frac{1}{2} \Phi_{\pi_N} \hat{\pi}_{N,t}^2 \right] + tip + O(3), \quad (3.2)
\end{aligned}$$

where $\hat{Y}_t^T, \hat{T}_t^{dT}, \hat{T}_t^T$ are target variables which are functions of the stochastic shocks and, in general, are different from the flexible price equilibrium processes of aggregate output, domestic terms of trade and international terms of trade.²² The coefficients $\Phi_Y, \Phi_{T^d}, \Phi_T, \Phi_{TT^d}, \Phi_{YT^d}, \Phi_{YT}, \Phi_{\pi_H}, \Phi_{\pi_N}$ are functions of the structural parameters of the model. The term *tip* stands for *terms independent of policy*.

Our loss function can be seen as a generalization of the previous studies encompassing both the closed (Aoki (2001), Benigno (2004), Rotemberg and Woodford (1997)) and open economy frameworks (Gali and Monacelli (2005), De Paoli (2004)).²³ Notice that if we set the size of the nontraded sector to zero and therefore obtain a one-sector small open economy, the loss function becomes identical to the loss function derived by De Paoli (2004).²⁴ In this case, the loss function is a function of the variances of aggregate output, terms of trade²⁵ and home traded inflation. On the other hand, if we set the degree of openness to zero, we obtain the case of a two-sector closed economy which was studied by Aoki (2001) and Benigno (2004). Here, the loss function is a function of the variances of aggregate output, domestic terms of trade, the covariance term between the two and variances of the sector inflation rates. Additionally, our loss function is closely related to the loss function derived for a national policymaker in a two-country model with two sectors of Liu and Pappa (2005). Interestingly, since the monopolistic competition distortion, trade imbalances and also expenditure switching effect are not present in their model, the loss function of a national policymaker depends only on the sector inflation rates and the sector output fluctuations around the flexible price targets.

We characterize the optimal plan under commitment where the policy maker chooses the set of variables $\{\hat{\pi}_{N,t}, \hat{\pi}_{H,t}, \hat{C}_t, \hat{L}_t, \hat{Y}_{H,t}, \hat{Y}_{N,t}, \hat{p}_{N,t}, \hat{p}_{H,t}, \hat{Y}_t, \hat{R}S_t, \hat{T}_t^d, \hat{T}_t, \hat{S}_t, \hat{\pi}_t, \hat{R}_t\}_{t=t_0}^{\infty}$ in order to minimize the loss function (3.2) subject to constraints (2.21)–(2.35), given the initial conditions on this set of variables. The initial conditions (that refer only to period t_0) guarantee the timeless perspective of the problem and make the first-order conditions of the problem time invariant (see Woodford (2003)).

To simplify the exposition of the optimal plan, we reduce the number of variables to the set of five domestic

²²As previously shown in papers by Gali and Monacelli (2005) and De Paoli (2004), in the small open economy framework the target variables will be identical to the flexible price allocations only in some special cases, i.e. an efficient steady state, no markup shocks, no expenditure switching effect (i.e. $\rho\theta = 1$) and no trade imbalances.

²³Rotemberg and Woodford (1998) present a one-sector closed economy model. Aoki (2001) presents a two-sector closed economy model with sticky prices only in one of the sectors. Benigno (2004) analyses the case of a monetary union comprised of two countries, which can be interpreted as a two-sector closed economy with totally segmented labour markets (interpretation as in Woodford (2003)). Gali and Monacelli (2005) and De Paoli (2004) study the case of a one-sector small open economy.

²⁴In our representation, there is a covariance term between terms of trade and aggregate output which can be represented as the weighted sum of the variances of aggregate output and terms of trade.

²⁵In the analysis of De Paoli (2004), it is actually the variance of the real exchange rate. However, it must be kept in mind that in a one-sector small open economy model, terms of trade and real exchange rate are proportional.

variables which determine the loss function (3.2), i.e. $\widehat{Y}_t, \widehat{T}_t^d, \widehat{T}_t, \widehat{\pi}_{N,t}, \widehat{\pi}_{H,t}$. Therefore, we represent the structural equations of the two-sector small open economy (2.21)–(2.35) in terms of these variables. The coefficients are defined in Appendix A.4.2.

The supply side of the economy is represented by two Phillips curves which are derived from equations (2.21) and (2.22) through a substitution of aggregate consumption, aggregate labour and relative prices:

$$\begin{aligned} \widehat{\pi}_{N,t} = k_N(m_{N,Y}\widehat{Y}_t + m_{N,T^d}\widehat{T}_t^d + m_{N,T}\widehat{T}_t + m_{N,A_N}\widehat{A}_{N,t} + \\ + m_{N,A_H}\widehat{A}_{H,t} + m_{N,B}\widehat{B}_t) + \beta\widehat{\pi}_{N,t+1}, \end{aligned} \quad (3.3)$$

$$\begin{aligned} \widehat{\pi}_{H,t} = k_H(m_{H,Y}\widehat{Y}_t + m_{H,T^d}\widehat{T}_t^d + m_{H,T}\widehat{T}_t + m_{H,A_N}\widehat{A}_{N,t} + \\ + m_{H,A_H}\widehat{A}_{H,t} + m_{H,B}\widehat{B}_t) + \beta\widehat{\pi}_{H,t+1}. \end{aligned} \quad (3.4)$$

The equation describing the demand side of the economy is derived from the market clearing conditions ((2.24), (2.25)) and the risk sharing condition (2.27):

$$\widehat{C}_t^* = \widehat{Y}_t + n_{T^d}\widehat{T}_t^d + n_T\widehat{T}_t + n_B\widehat{B}_t, \quad (3.5)$$

where aggregate consumption, relative prices and real exchange rate were substituted out.

The last structural equation represents the law of motion of the domestic and international terms of trade:

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}). \quad (3.6)$$

Finally, the policy maker following the optimal plan under commitment chooses $\{\widehat{Y}_t, \widehat{T}_t^d, \widehat{T}_t, \widehat{\pi}_{H,t}, \widehat{\pi}_{N,t}\}_{t=t_0}^\infty$ in order to minimize the loss function (3.2) subject to the constraints (3.3)–(3.6), given the initial conditions on nonpredetermined variables: $\widehat{Y}_{t_0}, \widehat{T}_{t_0}^d, \widehat{T}_{t_0}, \widehat{\pi}_{H,t_0}, \widehat{\pi}_{N,t_0}$. In accordance with the definition of the optimal plan from a timeless perspective (see Woodford (2003), p.538) the first-order conditions of the problem for all $t \geq t_0$ are the following (where $\gamma_{i,t}$ with $i = 1, 2, 3, 4$ are accordingly the Lagrange multipliers with respect to (3.3)–(3.6)):

- with respect to $\widehat{\pi}_{N,t}$:

$$\Phi_{\pi_N}\widehat{\pi}_{N,t} + \gamma_{1,t} - \gamma_{1,t-1} - \gamma_{4,t} = 0, \quad (3.7)$$

- with respect to $\widehat{\pi}_{H,t}$:

$$\Phi_{\pi_H}\widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} = 0, \quad (3.8)$$

- with respect to \widehat{Y}_t :

$$\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) + \Phi_{YT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{T}_t - k_N m_{N,Y}\gamma_{1,t} - k_H m_{H,Y}\gamma_{2,t} - \gamma_{3,t} = 0, \quad (3.9)$$

- with respect to \widehat{T}_t^d :

$$\begin{aligned} \Phi_{T^d}(\widehat{T}_t^d - \widehat{T}_t^{dT}) + \Phi_{TT^d}\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t - k_N m_{N,T^d}\gamma_{1,t} - k_H m_{H,T^d}\gamma_{2,t} - \\ - n_{T^d}\gamma_{3,t} + \gamma_{4,t} - \beta\gamma_{4,t+1} = 0, \end{aligned} \quad (3.10)$$

- with respect to \widehat{T}_t :

$$\begin{aligned} \Phi_T(\widehat{T}_t - \widehat{T}_t^T) + \Phi_{TT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t - k_N m_{N,T}\gamma_{1,t} - k_H m_{H,T}\gamma_{2,t} - \\ - n_T\gamma_{3,t} + a\gamma_{4,t} - \beta a\gamma_{4,t+1} = 0 \end{aligned} \quad (3.11)$$

Equations (3.7)–(3.11) and constraints (3.3)–(3.6) fully characterize the behavior of the economy under the optimal monetary policy.

4 The Maastricht convergence criteria – a reinterpretation

Including the Maastricht criteria in their original form as additional constraints of the optimal monetary policy requires computationally demanding techniques. In particular, it results in solving the minimization problem of the loss function (3.2) subject to additional nonlinear constraints. On the other hand, the linear quadratic approach has two important advantages that make us decide to reformulate the criteria. First, it provides us with the analytical and intuitive expression for the loss function which can also serve as a welfare measure to rank alternative suboptimal policies. Second, the linear quadratic approach makes it easy to check second-order conditions (which would otherwise be quite difficult) for local optimality of the derived policy.

Therefore, the purpose of this section is to describe the way in which we reformulate the Maastricht criteria in order to implement them as additional constraints faced by the monetary policy in our linear quadratic framework.

First, we summarize the Maastricht criteria (described in Appendix B) by the following inequalities:

- CPI aggregate inflation criterion

$$\pi_t^A - \pi_t^{A,*} \leq B_\pi, \quad (4.1)$$

where $B_\pi = 1.5\%$, π_t^A is annual CPI aggregate inflation in the domestic economy, $\pi_t^{A,*}$ is the average of the annual CPI aggregate inflations in the three lowest inflation countries of the European Union.

- nominal interest rate criterion

$$R_t^L - R_t^{L,A*} \leq C_R \quad (4.2)$$

where $C_R = 2\%$, R_t^L is the annual interest rate for ten-year government bond in the domestic economy, $R_t^{L,A*}$ is the average of the annual interest rates for ten-year government bonds in the three countries of the European Union with the lowest inflation rates.

- nominal exchange rate criterion

$$(1 - D_S)S \leq S_t \leq (1 + D_S)S, \quad (4.3)$$

where $D_S = 15\%$ and S is the central parity between euro and the domestic currency and S_t is the nominal exchange rate.

In order to adjust the criteria to the structure of the model, we assume that the variables $\pi_t^{A,*}$ and $R_t^{L,A*}$, respectively, represent foreign aggregate inflation and the foreign nominal interest rate, i.e. $\hat{\pi}_t^*$, \hat{R}_t^* (which are proxied to be the euro area variables). Here, we implicitly assume that the aggregate inflation rate and the nominal interest rate of the euro area do not differ to any great extent from the average of the three lowest inflation countries of the European Union.²⁶

Second, we impose some simplifying assumptions regarding the criteria to adjust them to the quarterly nature of the model. The CPI inflation rate criterion is stated in annual terms. We decide to reformulate this criterion into the criterion on the quarterly CPI inflation rate with an appropriately changed upper bound, i.e. $B_\pi \equiv ((1.015)^{0.25} - 1)$. Notice that the criterion on the quarterly CPI inflation rate is stricter than the criterion set on the annual CPI inflation rate.²⁷ As far as the nominal interest rate criterion is concerned, we also decide to reformulate it into the criterion on the quarterly nominal interest rate. So, our reformulated criterion with the adjusted upper bound, i.e. $C_R \equiv ((1.02)^{0.25} - 1)$, is stricter than the original criterion.²⁸ Still, to keep the exposition of both criteria simple, we decide to use the reformulated criteria.

Moreover, the nominal exchange rate criterion is stated in terms of the quarterly nominal exchange rate movements. Additionally, we define the central parity of the nominal exchange rate as the steady state value of the nominal exchange rate ($S = \bar{S}$). We are aware that exchange rate criterion is a daily criterion. However strong idiosyncratic movements in exchange rate are rather due to some idiosyncratic events that do not have a fundamental nature. Certainly quarterly exchange rate criterion is less restrictive than the original one. But as long as movements of the exchange rate are driven by structural shocks we approximate closely the original criterion.

In order to implement the already adjusted criteria into the linear quadratic framework, we take advantage of the method proposed by Rotemberg and Woodford (1997, 1999) and Woodford (2003) which is applied to

²⁶We are aware of the CPI inflation rate dispersion among the EMU member countries. Still the framework of the model does not allow us to consider the criteria strictly in their original form.

²⁷This means that it is possible that the original criterion can be still satisfied, even though the quarterly CPI inflation rate violates the reformulated criterion. On the other hand, if the quarterly CPI inflation satisfies the criterion, the original criterion is also satisfied.

²⁸If we assume that the expectations hypothesis holds, an upper bound restriction on the quarterly nominal interest rate implies an upper bound criterion on the ten-year government bond yield. However, the reverse is not true.

the zero bound constraint for the nominal interest rate. The authors propose to approximate the zero bound constraint for the nominal interest rate by restricting the mean of the nominal interest rate to be at least k standard deviations higher than the theoretical lower bound, where k is a sufficiently large number to prevent frequent violation of the original constraint. The main advantage of this alternative constraint over the original one is that it is much less computationally demanding and it only requires computation of the first and second moments of the nominal interest rate, while the original one would require checking whether the nominal interest rate is negative in any state which, in turn, depends on the distribution of the underlying shocks.

Importantly, to further simplify the exposition of the criteria, we assume that the foreign economy is in the steady state, so that foreign CPI inflation and the nominal interest rate ($\widehat{\pi}_t^*$, \widehat{R}_t^*) are zero. We find that this case adequately represents the policy problem in the EMU accession countries as the majority of the shocks in these countries has a domestic nature (see Fidrmuc and Kirhonen (2003)).²⁹

Similarly to Woodford (2003), we redefine the criteria using discounted averages in order to conform with the welfare measure used in our framework. Let us remark that the average value of any variable (x_t) is defined as the discounted sum of the conditional expectations, i.e.:

$$\widehat{m}(x_t) \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t x_t. \quad (4.4)$$

Accordingly, its variance is defined by:

$$\widehat{var}(x_t) \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (x_t - \widehat{m}(x_t))^2. \quad (4.5)$$

Below, we show the reformulated Maastricht convergence criteria.³⁰ Each criterion is presented as a set of two inequalities:

- CPI aggregate inflation criterion:

$$(1 - \beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \geq 0, \quad (4.6)$$

$$(1 - \beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t)^2 \leq K \left((1 - \beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \right)^2; \quad (4.7)$$

- nominal interest rate criterion:

$$(1 - \beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t) \geq 0 \quad (4.8)$$

$$(1 - \beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t)^2 \leq K \left((1 - \beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t) \right)^2 \quad (4.9)$$

²⁹In section 6 we discuss the consequences of relaxing this assumption (e.g. a departure from the steady state of the foreign economy or a suboptimal foreign monetary policy) for the nature of optimal policy constrained by the Maastricht criteria and the associated welfare loss.

³⁰The detailed derivation of the Maastricht convergence criteria can be found in Appendix A.5.

- nominal exchange rate criterion must be decomposed into two systems of the inequalities, i.e. the upper bound and the lower bound:

– upper bound

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S - \widehat{S}_t) \geq 0 \quad (4.10)$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S - \widehat{S}_t)^2 \leq K \left((1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S - \widehat{S}_t) \right)^2 \quad (4.11)$$

– lower bound

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S + \widehat{S}_t) \geq 0 \quad (4.12)$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S + \widehat{S}_t)^2 \leq K \left((1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S + \widehat{S}_t) \right)^2 \quad (4.13)$$

where $K = 1 + k^{-2}$ and $D_S = 15\%$, $B_\pi = (1.015)^{0.25} - 1$, $C_R = (1.02)^{0.25} - 1$, $k = 1.96$.

The first inequality means that the average values of the CPI inflation rate, the nominal interest rate and the nominal exchange rate, respectively, should not exceed the bounds, B_π , C_R and D_S . The second inequality further restrains fluctuations in the Maastricht variables by setting an upper bound on their variances. This upper bound depends on the average values of the Maastricht variables and the bounds, B_π , C_R and D_S . Importantly, it also depends on parameter K which guarantees that the original constraints on the Maastricht variables ((4.1)–(4.3)) are satisfied with a high probability. Under a normality assumption, by setting $K = 1 + 1.96^{-2}$, we obtain that fulfillment of inequalities (4.6)–(4.13) guarantees that each of the original constraints should be met with a probability of 95%.

Summing up, the set of inequalities (4.6)–(4.13) represent the Maastricht convergence criteria in our model.

5 Optimal monetary policy constrained by the Maastricht criteria

This section presents how to construct the loss function of the optimal monetary policy constrained by the Maastricht convergence criteria summarized by inequalities (4.6)–(4.13) (constrained optimal policy). In this respect, we follow Woodford (2003). Specifically, the loss function of the constrained optimal monetary policy is augmented by the new elements which describe fluctuations in CPI aggregate inflation, the nominal interest rate and the nominal exchange rate.

We state the following proposition, which is based on Proposition 6.9 (p. 428) in Woodford (2003):

Proposition 1 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0} \left\{ (1 - \beta) \sum_{t=t_0}^{\infty} \beta^t L_t \right\}, \quad (5.1)$$

subject to (4.6) - (4.13). Let $m_{1,\pi}$, $m_{1,R}$, $m_{1,S}^U$, $m_{1,S}^L$ be the discounted average values of $(B_\pi - \hat{\pi}_t)$, $(C_R - \hat{R}_t)$, $(D_S - \hat{S}_t)$, $(D_S + \hat{S}_t)$ and $m_{2,S}^U$, $m_{2,S}^L$, $m_{2,\pi}$, $m_{2,R}$ be the discounted means of $(B_\pi - \hat{\pi}_t)^2$, $(C_R - \hat{R}_t)^2$, $(D_S - \hat{S}_t)^2$, $(D_S + \hat{S}_t)^2$ associated with the optimal policy. Then, the optimal policy also minimizes a modified discounted loss criterion of the form (5.1) with L_t replaced by:

$$\tilde{L}_t \equiv L_t + \Phi_\pi(\pi^T - \hat{\pi}_t)^2 + \Phi_R(R^T - \hat{R}_t)^2 + \Phi_{S,U}(S^{T,U} - \hat{S}_t)^2 + \Phi_{S,L}(S^{T,L} - \hat{S}_t)^2, \quad (5.2)$$

under constraints represented by the structural equations of an economy. Importantly, $\Phi_\pi \geq 0$, $\Phi_R \geq 0$, $\Phi_{S,U} \geq 0$, $\Phi_{S,L} \geq 0$ and take strictly positive values if and only if the respective constraints (4.7), (4.9), (4.11), (4.13) are binding. Moreover, if the constraints are binding, the corresponding target values π^T , R^T , $S^{T,U}$, $S^{T,L}$ satisfy the following relations:

$$\pi^T = B_\pi - Km_{1,\pi} < 0 \quad (5.3)$$

$$R^T = C_R - Km_{1,R} < 0 \quad (5.4)$$

$$S^{T,U} = D_S - Km_{1,S} < 0 \quad (5.5)$$

$$S^{T,L} = -D_S + Km_{1,S} > 0. \quad (5.6)$$

Proof can be found in Appendix A.6.

In the presence of binding constraints, the optimal monetary policy constrained by the Maastricht convergence criteria do not only lead to smaller variances of the Maastricht variables, it also assigns target values for these variables that are different from the steady state of the optimal monetary policy.

In particular, if the constraints on the nominal interest rate or CPI inflation are binding, the target values for these variables are negative. This means that the constrained optimal monetary policy should target the CPI inflation rate or the nominal interest rate that is actually lower than the foreign CPI inflation or the foreign nominal interest rate, respectively. Therefore, this policy results in a deflationary bias. Finally, the deflationary bias together with a decrease in the nominal interest rate lead to a nominal exchange rate appreciation. Notice that if the upper bound criterion on the nominal exchange rate is binding, the constrained optimal policy is also characterized by a nominal exchange rate appreciation and negative averages of the nominal interest rate and the CPI inflation rate.

6 Numerical exercise

The purpose of this section is twofold, to characterize the optimal monetary policy for the EMU accession countries, given their obligation to satisfy the Maastricht convergence criteria and analyze whether and how it differs from the optimal monetary policy not constrained by the criteria (the unconstrained optimal monetary policy). To this end, in the first step, we present the optimal monetary policy and identify whether such a policy violates any of the Maastricht convergence criteria. Second, based on the results, we construct the

optimal policy that satisfies all the criteria (the constrained optimal monetary policy). Third, we compare both policies by studying their welfare costs and analyzing their response pattern to the shocks.

6.1 Parameterization

In the previous literature on the EMU accession countries (i.e. Laxton and Pesenti (2003), Natalucci and Ravenna (2003) and Lipinska (2008)) model parameters were calibrated to match the moments of the Czech Republic economy. In this exercise we choose to calibrate the model to match the moments of the data that represent Polish economy. As far as the micro parameters are concerned we choose the same values as in Lipińska (2008). We differentiate our calibration from Lipińska (2008) in two respects: we choose parameters of the shocks to match the moments of the Polish economy and also we take different parameters of the Taylor rule in order to account for the different monetary regime of Poland. We focus on matching moments of the key variables: output, consumption and Maastricht variables.

The discount factor, β , equals 0.99, which implies an annual interest rate of around four percent. The coefficient of risk aversion in consumer preferences is set to 2 as in Stockman and Tesar (1995) to get an intertemporal elasticity of substitution equal to 0.5. As far as labour supply elasticity ($\frac{1}{\eta}$) is concerned, the micro data estimates of η consider [3, 20] as a reasonable range. We decide to set η to 3. The elasticity of substitution between nontradable and tradable consumption, ϕ , is set to 0.5 as in Stockman and Tesar (1994) and the elasticity of substitution between home and foreign tradable consumption, θ , is set to 1.5 (as in Chari et al. (2002) and Smets and Wouters (2004)). The elasticity of substitution between differentiated goods, σ , is equal to 10, which together with the revenue tax of 0.12³¹ implies a markup of 1.26.³²

The share of nontradable consumption in the aggregate consumption basket, μ , is assumed to be 0.38, while the share of foreign tradable consumption in the tradable consumption basket, λ , is assumed to be 0.4 (as in Kolasa (2008)). These values correspond to the weights in CPI reported for Poland over the period 1996–2007.³³ As far as the foreign economy is concerned, we set the share of nontradable consumption in the foreign aggregate consumption basket, μ^* , to be 0.6, which is consistent with the value chosen by Benigno and Thoenisen (2003) regarding the structure of euro area consumption.

Following Natalucci and Ravenna (2003), we set the degree of price rigidity in the nontraded sector, α_N , to 0.85. The degree of price rigidity in the traded sector, α_H , is slightly smaller and equals 0.8. These values are somewhat higher than the values reported in the micro and macro studies for the euro area countries.³⁴ Still, Natalucci and Ravenna (2003) justify them by a high share of the government regulated prices in the EMU

³¹This value represents the average share of Taxes less Subsidies in the Gross Domestic Product at 1995 constant prices in Poland for the years 1995-2006 (source: Eurostat).

³²Martins et al. (1996) estimate the average markup for manufacturing sectors at around 1.2 in most OECD countries over the period 1980-1992. Some studies (Morrison (1994), Domowitz et al (1988)) suggest that the plausible estimates range between 1.2 and 1.7.

³³Source: Eurostat.

³⁴Stahl (2004) estimates that the average duration between price adjustment in the manufacturing sector is nine months (which corresponds to the degree of price rigidity: 0.67). On the other hand, Gali et al (2001) and Benigno and Lopez-Salido (2003) estimate the aggregate supply relations for the European countries and find the overall degree of price rigidity for these countries to be 0.78.

accession countries.

All shocks that constitute the stochastic environment of the small open economy follow the AR(1) process. The parameters of the shocks are chosen to match the historical moments of the variables. Similarly to Natalucci and Ravenna (2003) and Laxton and Pesenti (2003), the productivity shocks in both domestic sectors are characterized by a strong persistence parameter equal to 0.85. Standard deviations of the productivity shocks are set to 1.6% (nontraded sector) and 1.8% (traded sector). These values roughly reflect the values chosen by Natalucci and Ravenna (2003), 1.8% (nontraded sector) and 2% (traded sector). Moreover, the productivity shocks are strongly correlated, their correlation coefficient is set to 0.7.³⁵ All other shocks are independent of each other. Parameters defining the preference shock are, 0.4% (standard deviation) and 0.7 (persistence parameter) (as in Laxton and Pesenti (2003)). Parameters of the foreign consumption shock are estimated using quarterly data on aggregate consumption in the euro area over the period 1990-2005 (source: Eurostat). The standard deviation of the foreign consumption shock is equal to 0.23% and its persistence parameter is 0.85.

Following Kolasa (2008), we parametrize the monetary policy rule, i.e. the nominal interest rate follows the rule described by: $\widehat{R}_t = 0.8\widehat{R}_{t-1} + 0.2(2.12\widehat{\pi}_t + 0.258\widehat{Y}_t + 0.03\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $\widehat{\varepsilon}_{R,t}$ is the monetary policy innovation with a standard deviation equal to 0.45%.³⁶ Such a parametrization of the monetary policy rule enables us to closely match the historical moments of the Polish economy.

We summarize all parameters described above in Table A.1 (Structural parameters) and Table A.2 (Stochastic environment) in Appendix A.3. Moreover Table A.3 (Matching the moments) in Appendix A.3 compares the model moments with the historical moments for the Polish economy.

6.2 Unconstrained optimal monetary policy

Now, we characterize the optimal monetary policy under the chosen parameterization. First, we analyze what the main concern of the optimal monetary policy is by studying the coefficients of the loss function given by (3.2).³⁷ In Table 6.1, we present these coefficients.

Φ_{π_N}	Φ_{π_H}	Φ_Y	Φ_{T^d}	Φ_T	Φ_{TT^d}	Φ_{YT^d}	Φ_{YT}
107.14	31.13	2.98	0.11	0.14	-0.03	-0.24	-0.28

Table 6.1: Loss function coefficients under the optimal monetary policy

The highest penalty coefficient is assigned to fluctuations in nontradable sector inflation and home tradable inflation. Therefore, the optimal monetary policy mainly stabilizes domestic inflation. This finding is in line with the literature on core inflation targeting (Aoki (2001)). Apart from that, the optimal monetary policy

³⁵Empirical evidence shows that productivity shocks are highly persistent and positively correlated (see Backus et al (1992)).

³⁶The Taylor rule estimated by Kolasa (2008) does not have nominal exchange rate element. In order to match the variability of nominal exchange rate in Poland we decided to include this element.

³⁷Following Benigno and Woodford (2005), we check whether the second-order conditions of the policy problem are satisfied in order to guarantee that there is no alternative random policy that could improve the welfare of society. This consists in checking whether all eigenvalues of the matrix representing the loss function (3.2) are nonnegative (see Proposition 3, p.23 in Benigno and Woodford (2004)).

faces a trade off between stabilizing the output gap and the sector inflations which is reflected in the positive values of the penalty coefficients assigned to fluctuations in domestic and international terms of trade.

Next, we check whether the optimal monetary policy satisfies the Maastricht convergence criteria. Since the means of all variables under the optimal monetary policy are zero, we can reduce constraints (4.6)–(4.13) to the following set of inequalities:

$$\widehat{\text{var}}(\widehat{\pi}_t) \leq (K - 1)B_\pi^2 \tag{6.1}$$

$$\widehat{\text{var}}(\widehat{R}_t) \leq (K - 1)C_R^2 \tag{6.2}$$

$$\widehat{\text{var}}(\widehat{S}_t) \leq (K - 1)D_S^2, \tag{6.3}$$

where $\widehat{\text{var}}(x_t)$ with $x_t = \widehat{\pi}_t, \widehat{R}_t, \widehat{S}_t$ is defined by (4.5). Notice that these constraints set the upper bounds on the variances of the Maastricht variables. In Table 6.2, we present variances of these variables under optimal monetary policy and the respective upper bounds that represent the right-hand side of equations (6.1)–(6.3). We write that a criterion is violated (satisfied) when the variance of the respective Maastricht variable is higher (smaller) than the upper bound.

	CPI inflation	nominal interest rate	nominal exchange rate
mean (in %)	0	0	0
variance	0.2692	0.3403	13.1943
bound	0.0356	0.0651	58.57
criterion	<i>violated</i>	<i>violated</i>	<i>satisfied</i>
note: Variances and bounds are multiplied by 100^2 (in $(\%)^2$)			

Table 6.2: Moments of the Maastricht variables under the optimal monetary policy

The optimal monetary policy violates two of the Maastricht convergence criteria, the CPI inflation criterion and the nominal interest rate criterion. The nominal exchange rate criterion is satisfied.³⁸ Therefore, the loss function of optimal monetary policy for the EMU accession countries must be augmented by additional terms.

6.3 Constrained optimal policy

Now, we construct the optimal monetary policy that satisfies all Maastricht criteria. First, we augment the loss function of the optimal monetary policy with additional terms reflecting fluctuations of CPI inflation and the nominal interest rate and solve the new policy problem.³⁹ Second, we check whether such a policy also satisfies the nominal exchange rate criterion.

The loss function of the optimal policy that satisfies two additional constraints on CPI inflation and the nominal interest rate is given below:

³⁸Note that currently, Polish economy satisfies the Maastricht criterion regarding nominal interest rate and does not satisfy the criteria regarding CPI inflation and the nominal exchange rate. See Figures B.3 (the CPI inflation criterion), B.5 (the nominal interest rate criterion) and B.6 (the nominal exchange criterion) in Appendix B.

³⁹First-order conditions of this policy are presented in Appendix A.7.

$$\tilde{L}_t = L_t + \frac{1}{2}\Phi_\pi(\pi^T - \hat{\pi}_t)^2 + \frac{1}{2}\Phi_R(R^T - \hat{R}_t)^2, \quad (6.4)$$

where $\Phi_\pi > 0$, $\Phi_R > 0$ and $\pi^T < 0$, $R^T < 0$. Values of the penalty coefficients (Φ_π , Φ_R) and targets (π^T , R^T) can be obtained from the solution to the minimization problem of the original loss function constrained by structural equations (3.3)–(3.6) and also the additional constraints on the CPI inflation rate (4.6)–(4.7) and the nominal interest rate (4.8)–(4.9).⁴⁰ These values are presented in Table 6.3:

Φ_π	Φ_R	π^T (in %)	R^T (in %)
37.06	16.9	-0.1626	-0.173

Table 6.3: The augmented loss function coefficients under the constrained monetary policy

Notice that values of the penalty coefficients on the CPI inflation rate and nominal interest rate fluctuations are of the same magnitude as the penalty coefficients on the domestic inflation rates. The negative target value for the CPI inflation rate means that now, monetary policy targets the CPI inflation rate and the nominal interest rate that in annual terms are 0.6% smaller than their foreign counterparts.

Finally, we check whether this policy also satisfies the nominal exchange rate criterion. In Table 6.4, we present the first and second discounted moments of all Maastricht variables and evaluate whether each of the criteria is satisfied. A criterion is satisfied when the respective set of inequalities that describes this criterion holds. In particular, the CPI inflation criterion is described by the set of inequalities (4.6)–(4.7), the nominal interest criterion is explained by (4.8)–(4.9) and the nominal exchange rate criterion by (4.10)–(4.13).

	CPI inflation	nominal interest rate	nominal exchange rate
mean (in %)	-0.047	-0.0474	-4.6970
variance	0.0453	0.0708	10.1235
criterion	<i>satisfied</i>	<i>satisfied</i>	<i>satisfied</i>
note: Variances are multiplied by 100 ² (in (%) ²)			

Table 6.4: Moments of the Maastricht variables under the constrained optimal policy

Importantly, the nominal exchange rate criterion is satisfied. Not surprisingly, variances of the CPI inflation rate and the nominal interest rate are smaller than under the optimal policy. Notice that the variance of the nominal exchange rate is smaller than the one under the optimal monetary policy. This is due to the fact that the nominal exchange rate changes are, apart from the domestic sector inflation rates, one of the components of the CPI inflation rate (see (2.34)). So the policy that targets domestic inflation rates and the CPI aggregate inflation rate at the same time also decreases the nominal exchange rate variability. Let us remark that the negative targets for the nominal interest rate and the CPI aggregate inflation lead to negative means of all

⁴⁰Special thanks to Michael Woodford for explaining the algorithm to find the parameters of the constrained policy problem (see p. 427–435 in Woodford (2003)).

Maastricht variables. Therefore, a central bank choosing such a policy commits itself to a policy resulting in the average CPI inflation rate and the nominal interest rate being 0.15% smaller in annual terms than their foreign counterparts. Additionally, this policy is characterized by an average nominal exchange rate appreciation of nearly 5%.

Summing up, the optimal monetary policy constrained by additional criteria on the CPI inflation and the nominal interest rate is the policy satisfying all Maastricht convergence criteria.

6.4 Comparison of the constrained and unconstrained optimal policy

Now, we focus on the comparison of the optimal monetary policy and the optimal policy constrained by the convergence criteria. First, we calculate the welfare losses associated with each policy and second, we analyze differences between the policies in their stabilization pattern when responding to the shocks.⁴¹

In Table 6.5, we present the expected discounted welfare losses for both policies:

	UOP	COP
loss	1.8288	3.3889
note: Losses are multiplied by 100^2 (in $(\%)^2$)		

Table 6.5: Welfare losses for the unconstrained and constrained optimal policy

where UOP is the unconstrained optimal policy and COP is the constrained optimal monetary policy.

The obligation to comply with the Maastricht convergence criteria induces additional welfare costs equal to 85% of the optimal monetary policy loss. In order to give more intuition to this result we present it in the metric of permanent shifts in the steady state consumption. Following Benigno and Lopez-Salido (2003) we define index δ_C as:

$$\delta_C \equiv -(1 - \beta) \frac{W_C - W_U}{U_C \bar{C}} \quad (6.5)$$

where W_C is welfare under the unconstrained policy and W_U is welfare under the constrained policy, U_C is the marginal utility of consumption and \bar{C} is the steady state level of consumption. Based on this, δ_C measures the permanent shift in the steady state consumption that is lost under the constrained policy with respect to the unconstrained one. Under our parameterization index δ_C is equal to 0.016%.⁴²

These welfare costs originate from both the stochastic and deterministic component of the constrained policy. The constrained optimal policy reduces variances of the Maastricht variables but at the same time decreases covariance of aggregate output, domestic and international terms of trade with their optimal targets. Moreover in order to satisfy the criteria the constrained policy has to also induce negative targets for the CPI inflation rate and the nominal interest rate. These negative targets result in the negative means of all variables.

⁴¹Note that the losses for both policies are calculated using the formula (3.2).

⁴²Note that Lucas (1988) has evaluated the costs of business cycle to be around 0.05% of permanent shift in steady state consumption. In Lipińska (2008) the corresponding value of the permanent shift in the steady state consumption is 0.022%.

The welfare loss associated with the constrained optimal policy crucially depends on the foreign economy and the way its monetary policy is conducted. In our benchmark case, we assume the foreign economy to be in the steady state. This helps us simplify the exposition of the constrained optimal monetary policy problem. However, by allowing the foreign economy to be hit by stochastic shocks and, moreover, its monetary policy to be suboptimal, we obtain different targets and also penalty coefficients for domestic CPI inflation and the nominal interest rate. It can be shown that in such a situation, the targets on the CPI inflation rate and the nominal interest rate will not only depend on the average values of their foreign counterparts, but also on their fluctuations and covariance of the domestic and foreign processes. However, a deflationary bias feature of the constrained policy is preserved.⁴³

Now, we investigate how the two policies, constrained optimal monetary policy and unconstrained optimal monetary policy, differ when responding to the shocks. First, we analyze which shocks are most important in creating fluctuations of the Maastricht variables. In the table below, we present variance decomposition results for CPI aggregate inflation, the nominal interest rate and the nominal exchange rate.⁴⁴ Since the variance decomposition structure does not change to any considerable extent with the chosen policy, we report results for the constrained policy.

	shocks:			
variables:	A_N	A_H	B	C^*
CPI inflation	90%	4%	2%	3%
nominal interest rate	88%	7%	4%	1%
nominal exchange rate	91%	5%	1%	3%

Table 6.6: Variance decomposition of the Maastricht variables under the constrained monetary policy

Around 90% of the total variability of CPI aggregate inflation, the nominal interest rate and the nominal exchange rate are explained by domestic nontradable productivity shocks. This result is consistent with the literature on the sources of inflation differentials in the euro area (Altissimo et al (2005), Duarte and Wolman (2003), Canzoneri et al. (2005)). We have to stress that the nontraded productivity shocks play such an important role due to a small number of shocks that explain the stochastic environment. Subsequently a portion of variance of the Maastricht variables explained by the nontraded productivity shocks can implicitly reflect other sources of shocks (compare variance decomposition for the euro area economy in the model of Smets and Wouters (2004)). Interestingly, Kolasa (2008) estimates a model much richer than presented here with Bayesian methods and using Polish data and finds that in the long run nontradable productivity shocks account for about 50% of the variability of the Maastricht variables.

Notice that although parameters describing productivity shocks are similar in our setup, each of the productivity shocks has a different impact on the real exchange rate and therefore, on the Maastricht variables. This

⁴³See Proposition 2 and the Discussion in Appendix A.8.

⁴⁴We calculate variance decomposition using discounted variances of the variables.

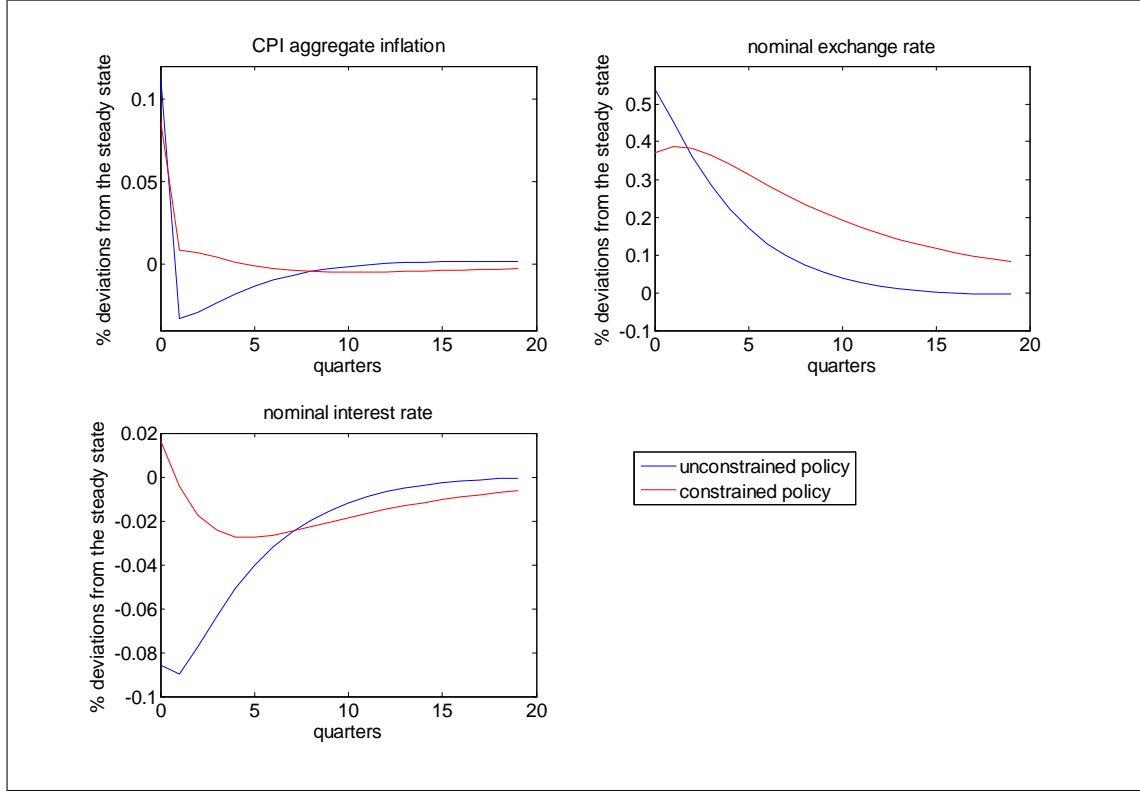


Figure 6.1: Impulse responses of the Maastricht variables to a positive domestic nontradable productivity shock

can easily be understood by analyzing the following equation, which relates the real exchange rate to domestic and international terms of trade (see (2.28), (2.30)):

$$\widehat{RS}_t = b^* \widehat{T}_t^{d^*} - b \widehat{T}_t^d + (1 - a) \widehat{T}_t. \quad (6.6)$$

Both domestic productivity shocks result in real exchange rate depreciation. However, the magnitude of the real exchange rate depreciation differs between the two shocks. Nontradable productivity shocks lead to a decline in the domestic terms of trade and a rise in the international terms of trade. Both changes have a depreciation effect on the real exchange rate. On the other hand, domestic tradable productivity shocks result in a rise of both types of terms of trade. From equation (6.6) we see that increases in both types of terms of trade cancel out and lead to a small change in the real exchange rate. As a result, domestic nontradable productivity shocks lead to a stronger real exchange rate depreciation and therefore, larger changes in the nominal interest rate and the CPI inflation rate.

Having all this in mind, we decide to study the stabilization pattern of both policies in response to domestic nontradable productivity shocks (see Figure 6.1).

Under the unconstrained optimal policy, a positive domestic nontradable productivity shock leads to a fall in the nominal interest rate. This decrease of the nominal interest partially stabilizes deflationary pressures in the

domestic nontraded sector and supports an increase in domestic aggregate output and consumption (not shown here). Since the foreign nominal interest rate remains constant, the uncovered interest rate parity induces a nominal exchange rate depreciation followed by an expected appreciation. The initial nominal exchange rate depreciation results in a strong initial increase of CPI inflation, which declines in subsequent periods, reverting to its mean.

The constrained policy is characterized by both CPI targeting and nominal interest rate targeting. To reduce the initial CPI increase (observed under the unconstrained policy), such a policy induces a more muted response of the real exchange rate and a stronger fall in domestic nontraded prices. These two effects are achieved through a more contractionary policy, i.e. a higher nominal interest rate as compared with the unconstrained optimal policy. Such behavior of the nominal interest rate is in line with the nominal interest rate targeting feature of the constrained optimal policy. As a result, an initial increase of the CPI inflation is smaller. Moreover, a higher domestic nominal interest rate leads to a smaller depreciation of the nominal exchange rate through the uncovered interest rate parity.

Summing up, in response to domestic nontradable productivity shocks, the constrained optimal policy leads to smaller fluctuations in all three Maastricht variables than unconstrained optimal monetary policy. However, it must be kept in mind that the constrained optimal policy commits to the inflation rate and the nominal interest rate that are lower than their foreign counterparts which results in substantial welfare costs.

6.5 Discussion - comparison with the exercise for the Czech Republic

Now we compare the results of our exercise with the results of a similar exercise for the Czech Republic economy described in Lipińska (2008). In Table (6.7) we present the welfare losses for both exercises together with the corresponding shifts in the steady state consumption. Absolute values of welfare losses are significantly different between two exercises. At the same time the relative difference between the welfare losses for the constrained and unconstrained policy are very similar for two exercises. Still shift in the steady state consumption is lower for the case of Poland. Below we examine the reasons that lie behind these results.

	UOP	COP
loss (Poland)	1.8288	3.3889
% shift in the ss C		0.0156%
loss (Czech Republic)	7.1533	9.2956
% shift in the ss C		0.0214%
note: Losses are multiplied by 100^2 (in $(\%)^2$)		

Table 6.7: Welfare losses for the unconstrained and constrained optimal policy

Our calibration strategy is based on matching moments of the key macro variables for the Czech Republic and Polish economy (see Table (A.3)). There are some important differences between variances of two datasets that affect our calibration. First of all, nominal exchange rate is much more volatile in Poland. Our calibration

attributes this to differences in monetary regimes between two countries. That means that this does not affect our results. Second, consumption in Czech Republic is much more volatile than output and also consumption in Poland. Our calibration takes this fact into account by assuming a higher persistence and variance of preference shock in the Czech Republic. This leads to much higher absolute values of welfare losses of the unconstrained and constrained policy in the Czech Republic exercise as compared with the exercise for Poland. However the relative difference between two policies is similar for both experiments. Why is that? Variance decomposition of the Maastricht variables (see table (6.6) and Table 6 in Lipińska (2008)) reveals that preference shocks do not explain a lot of variance of the Maastricht variables.⁴⁵ The only exception is the nominal exchange rate. Preference shocks explain 20% of its variability in the Czech Republic exercise. Smaller variability of preferences shocks in the Polish exercise leads to a smaller variability of the nominal exchange rate. This in turn makes it easier to satisfy the Maastricht criteria. As a result percentage shift in the steady state consumption associated with the constrained policy is smaller for the Polish exercise.

7 Conclusions

This paper characterizes the optimal monetary policy for the EMU accession countries, taking into account their obligation to meet the Maastricht convergence criteria. We perform our analysis in the framework of a two-sector small open economy DSGE model.

First, we derive the micro founded loss function which represents the policy objective function of the optimal monetary policy using the second-order approximation method. We find that the optimal monetary policy should not only target inflation rates in the domestic sectors and aggregate output fluctuations, but also domestic and international terms of trade. This results originates from the distortions present in the economy: monopolistic competition that implies inefficient sector outputs, price stickiness in both sectors that leads to an inefficient path of the domestic terms of trade and the international terms of trade externality that can affect the wedge between marginal disutility from labour and utility of consumption. All these distortions lead to the introduction of new elements in the loss function: domestic and international terms of trade.

Second, we reformulate the Maastricht convergence criteria taking advantage of the method developed by Rotemberg and Woodford (1997, 1999) to address the zero bound nominal interest rate problem. We show how the loss function changes when the monetary policy is subject to the Maastricht convergence criteria: the CPI inflation rate criterion, the nominal interest rate criterion and the nominal exchange rate criterion. The loss function of such a constrained policy is characterized by additional elements that penalize fluctuations of the CPI inflation rate, the nominal interest rate and the nominal exchange rate around the new targets different from the steady state of the unconstrained optimal monetary policy.

Under the chosen parameterization (which roughly represents Poland), optimal monetary policy violates the CPI inflation and the nominal interest rate criteria. The optimal policy that instead satisfies these two criteria also satisfies the nominal exchange rate criterion. Both the deterministic component and the stabilization

⁴⁵This is consistent with results of Kolasa (2008).

component of the constrained policy are different from the unconstrained optimal policy. The constrained policy leads to a lower variability of the CPI inflation, the nominal interest rate and the nominal exchange rate. At the same time, this policy targets the CPI inflation rate and the nominal interest rate that are 0.6% lower (in annual terms) than their counterparts in the reference countries. This produces additional welfare costs that amount to 0.016% shift in the steady state consumption. As discussed earlier, these costs do not take into account the credibility gains related to compliance with the Maastricht criteria.

The tools developed in this paper can be used to describe the optimal policy which faces additional constraints that are exogenously decided and do not form part of the structural constraints of an economy. Importantly, the Maastricht Treaty also sets restrictions on the debt and deficit policy of the EMU accession countries. Therefore a natural extension of the analysis involves the introduction of fiscal policy by endogenizing tax and debt decisions. Including all the restrictions faced by the fiscal and monetary policies in the EMU accession countries would enable us to investigate the effects of these restrictions on the interaction between the two policies.

References

- [1] Aiyagari S. R., Marcet A., Sargent T., Seppala J. (2002), Optimal Taxation without State-Contingent Debt, *Journal of Political Economy*
- [2] Altissimo F., Benigno P., Rodriguez - Palenzuela D. (2005), Inflation differentials in a currency area: facts, explanation and policy, *CEPR Discussion Paper*
- [3] Aoki K. (2001), Optimal monetary responses to relative price changes, *Journal of Monetary Economics*
- [4] Arratibel O., Rodriguez - Palenzuela D., Thimann C. (2002), Inflation dynamics and dual inflation in accession countries: a new Keynesian perspective, *ECB Working Paper 132*
- [5] Backus D. K., Kehoe P. J., Kydland F. E. (1992), International Real Business Cycles, *Journal of Political Economy*
- [6] Balassa B. (1964), The purchasing power parity doctrine: a reappraisal, *Journal of Political Economy* 72
- [7] Barro R. J. (1979), On the Determination of Public Debt, *Journal of Political Economy*
- [8] Benigno G., Benigno P. (2006), Designing Targeting Rules for International Monetary Policy Cooperation, *Journal of Monetary Economics*, Vol. 53 Issue 3
- [9] Benigno G., De Paoli B. (2006), Optimal Fiscal and Monetary Policy for a Small Open Economy, mimeo
- [10] Benigno G., Thoenisen C. (2003), Equilibrium exchange rates and supply side performance, *The Economic Journal* 113

- [11] Benigno P. (2001), Price stability with imperfect financial integration, CEPR Working Paper
- [12] Benigno P. (2004), Optimal monetary policy in a currency area, *Journal of International Economics*, Vol. 63, Issue 2
- [13] Benigno P., Lopez - Salido D. (2003), Inflation persistence and optimal monetary policy in the euro area, ECB Working Paper no. 178
- [14] Benigno P., Woodford M. (2003), Optimal Monetary and Fiscal Policy: A Linear Quadratic Approach, NBER Macroeconomics Annual
- [15] Benigno P., Woodford M. (2005), Inflation Stabilisation and Welfare: the case of a distorted steady state, *Journal of the European Economic Association*
- [16] Buiters W., Grafe C. (2002), Anchor, float or abandon the ship: exchange rate regimes for accession countries, CEPR Discussion Paper 3184
- [17] Canzoneri M., Cumby R., Diba B. (1999), Relative labour productivity and the real exchange rate in the long run: Evidence from a panel of OECD countries, *Journal of International Economics* 47
- [18] Chari V.V., Kehoe P., McGrattan E. (2002), Can sticky price models generate volatile and persistent real exchange rates?, *Review of Economic Studies*, Vol. 69, No. 3
- [19] Cipriani M. (2001), The Balassa-Samuelson effect in transition economies, mimeo, IMF, September
- [20] Coricelli F. (2002), Exchange rate policy during transition to the European Monetary Union, *Economics of Transition* 10 (2)
- [21] Coricelli, F., Jazbec B. (2001), Real exchange rate dynamics in transition economies, CEPR Discussion Paper, no 2869
- [22] Corsetti G., Dedola L., Leduc S. (2003), International risk sharing and the transmission of productivity shocks, Federal Reserve Bank of Philadelphia Working Paper No. 03-19
- [23] Corsetti G., Pesenti P. (2001), Welfare and Macroeconomic Interdependence, *The Quarterly Journal of Economics*
- [24] de Broek M., Sloek T.(2001), Interpreting real exchange rate movements in transition countries, IMF Working Paper 01/56
- [25] De Paoli B. (2004), Monetary policy and welfare in a small open economy, CEP Discussion Paper 639
- [26] Devereux M. B. (2003), A macroeconomic analysis of EU accession under alternative monetary policies, *Journal of Common Market Studies* vol.41 no.5

- [27] Domowitz I., Hubbard G., Peterson B. (1988), Market structure and cyclical fluctuations in U.S. manufacturing, *Review of Economics and Statistics*
- [28] Duarte M., Wolman A. (2002), Regional inflation in a currency union: fiscal policy vs. fundamentals, ECB Working Paper No. 180
- [29] ECB (2006), Convergence report, ECB, Frankfurt
- [30] Egert B., Drine I., Lommatzsch K., Rault C. (2002), The Balassa-Samuelson effect in central and eastern Europe: myth or reality, William Davidson Institute working paper, no. 483
- [31] Ferreira Lopes A. (2006), The costs of EMU for transition countries, mimeo
- [32] Ferrero A. (2005), Fiscal and Monetary Rules for a Currency Union, ECB Working Paper No. 502
- [33] Fidrmuc J., Korhonen I. (2003), Similarity of supply and demand shocks between the euro area and the CEECs, *Economic Systems* 27
- [34] Fischer C. (2002), Real currency appreciation in accession countries: Balassa-Samuelson and investment demand, Deutsche Bundesbank discussion paper, no. 19/02.
- [35] Froot K., Rogoff K., (1995), Perspectives on PPP and long - run real exchange rates, *Handbook of International Economics* vol. 3, Gene Grossman and Kenneth Rogoff (eds.)
- [36] Gali J., Gertler M., Lopez - Salido D. (2001), European inflation dynamics, *European Economic Review* 45
- [37] Gali J., Monacelli T. (2005), Monetary policy and exchange rate volatility in a small open economy, *Review of Economic Studies*
- [38] Gali J., Monacelli T. (2008), Optimal Monetary and Fiscal Policy in a Currency Union, mimeo
- [39] Giannoni M., Woodford M. (2003), Optimal inflation targeting rules, NBER Working Paper No. W9939
- [40] Halpern L., Wyplosz C. (2001), Economic transformation and real exchange rates in the 2000s: the Balassa-Samuelson connection, *Economic Survey of Europe*, no. 1, Geneva, United Nations Economic Commission for Europe.
- [41] Kolasa M., (2008), Structural heterogeneity or asymmetric shocks? Poland and the euro area through the lens of a two-country DSGE model, National Bank of Poland Working Paper
- [42] Laxton D., Pesenti P. (2003), Monetary rules for small open, emerging economies, *Journal of Monetary Economics* 50 pp. 1109-1146
- [43] Lipińska A. (2007), The Maastricht Convergence Criteria and the Monetary Regimes in the EMU Accession Countries, mimeo

- [44] Lipińska A. (2008), The Maastricht Convergence Criteria and the Optimal Monetary Policy for the EMU Accession Countries, ECB Working Paper no 896
- [45] Lucas R. (1987), *Models of Business Cycles*, Blackwell, Oxford
- [46] Liu Z., Pappa E. (2005), Gains from international monetary policy coordination: Does it pay to be different?, European Central Bank Working Paper No. 514
- [47] Maastricht Treaty (1994), European Commission webpage
- [48] Marimon R., Zilibotti F. (1998), Actual versus virtual unemployment in Europe: is Spain different?, *European Economic Review* 42
- [49] Martins J., Scarpetta S., Pilat D. (1996), Mark - up pricing, market structure and the business cycle, *OECD Economic Studies* 27
- [50] Mihaljek D., Klau M. (2004), The Balassa - Samuelson effect in central Europe: a disaggregated analysis, BIS Working Paper 143
- [51] Morrison C. (1994), The cyclical nature of markups in Canadian manufacturing: a production theory approach, *Journal of Applied Econometrics* 9
- [52] Natalucci F., Ravenna F. (2003), The road to adopting the Euro: Monetary policy and exchange rate regimes in EU accession countries, Federal Reserve Board International Finance Discussion Paper 741
- [53] National Bank of Hungary (2002), Adopting the Euro in Hungary: Expected Benefits, Costs and Timing, NBH Occasional Paper No. 24
- [54] National Bank of Poland (2004), A Report on the Costs and Benefits of Poland's Adoption of the Euro
- [55] Obstfeld M., Rogoff K. (1999), New directions for stochastic open economy models, NBER Working Paper 7313
- [56] Pappa E., Vassilotos V. (2006), The Unbearable Tightness of Being in a Monetary Union: Fiscal Restrictions and Regional Stability, IGIER working paper series No.294
- [57] Ravenna F. (2005), The European Monetary Union as the commitment device for new EU member states, ECB Working Paper no. 516
- [58] Rotemberg J., Woodford M. (1997), An optimization-based framework for the evaluation of monetary policy, in Ben S. Bernanke and Julio J. Rotemberg (eds.), *NBER Macroeconomics Annual 1997*
- [59] Rotemberg J., Woodford M. (1999), Interest rate rules in an estimated sticky price model, NBER Working Paper

- [60] Smets F., Wouters R. (2004), An estimated stochastic dynamic general equilibrium model of the euro area, Journal of the European Economic Association, Vol. 1, Issue 5
- [61] Stahl H. (2003), Price rigidity in German manufacturing, unpublished manuscript
- [62] Stockman A., Tesar L. (1994), Tastes in a two - country model of the business cycle: explaining international comovement, American Economic Review vol. 85, No. 1
- [63] Schmitt-Grohe S., Uribe M. (2004), Optimal Fiscal and Monetary Policy under Sticky Prices, Journal of Economic Theory
- [64] Sutherland A. (2002), A simple second-order solution method for dynamic general equilibrium models" CEPR Discussion Paper No 3554
- [65] Woodford M. (2001), Fiscal requirements for price stability, Journal of Money, Credit and Banking, 33: 669-728
- [66] Woodford M. (2003), Interest and prices: Foundations of a theory of monetary policy, Princeton University Press

A Model characteristics

A.1 Steady state characterization

We define a deterministic steady state with zero inflation rate. We present a small open economy as the limiting case of a two country model, i.e. $n = 0$ and $\nu = 1 - \lambda$. All variables in the steady state are denoted with a bar. All the shocks take the constant values, in particular: $\overline{A_N} = \overline{A_H} = 1$, $\overline{B} = 1$.⁴⁶

Moreover discount factors are:

$$Q_{t_0,t} = Q_{t_0,t}^* = \beta^{t-t_0} \quad (\text{A.1})$$

First order conditions of the domestic firms are the following:

$$\overline{p}_N = \frac{\sigma_N}{(\sigma_N - 1)(1 - \overline{\tau}_N)} \frac{\overline{W}^N}{\overline{P}}, \quad (\text{A.2})$$

$$\overline{p}_H = \frac{\sigma_H}{(\sigma_H - 1)(1 - \overline{\tau}_H)} \frac{\overline{W}^H}{\overline{P}} \quad (\text{A.3})$$

where $\overline{p}_N = \frac{\overline{P}_N}{\overline{P}}$, $\overline{p}_H = \frac{\overline{P}_H}{\overline{P}}$. We define markups in the domestic nontraded and home traded sector: $\overline{\mu}_N = \frac{\sigma_N}{(\sigma_N - 1)(1 - \overline{\tau}_N)}$, $\overline{\mu}_H = \frac{\sigma_H}{(\sigma_H - 1)(1 - \overline{\tau}_H)}$.

Labour supply optimality conditions are presented below:

⁴⁶Foreign consumption is derived from the steady state relations of the foreign economy.

$$\bar{C}^{-\rho} \frac{\bar{W}^N}{\bar{P}} = (\bar{L})^\eta, \quad (\text{A.4})$$

$$\bar{C}^{-\rho} \frac{\bar{W}^H}{\bar{P}} = (\bar{L})^\eta. \quad (\text{A.5})$$

These two conditions imply that real wages are equalized in the domestic sectors:

$$\frac{\bar{W}^N}{\bar{P}} = \frac{\bar{W}^H}{\bar{P}} = \bar{w}. \quad (\text{A.6})$$

Moreover substituting first order conditions of the firms ((A.2), (A.3)) into the labour supply optimality conditions ((A.4), (A.5)) we obtain the following relation:⁴⁷

$$\bar{p}_N \bar{\mu}_N^{-1} = \bar{p}_H \bar{\mu}_H^{-1}. \quad (\text{A.7})$$

From the market clearing condition in the domestic labour market we know that:

$$\bar{L} = \bar{L}_N + \bar{L}_H. \quad (\text{A.8})$$

Moreover from the production function we get that:

$$\bar{L}_N = \bar{Y}_N, \quad (\text{A.9})$$

$$\bar{L}_H = \bar{Y}_H. \quad (\text{A.10})$$

Demands for domestic traded and nontraded goods are presented below:

$$\bar{Y}_N = \bar{C}_N = \bar{p}_N^{-\phi} \mu \bar{C}, \quad (\text{A.11})$$

$$\bar{Y}_H = \bar{C}_H + \bar{C}_H^* = \bar{p}_H^{-\theta} \bar{p}_T^{\theta-\phi} (1-\lambda)(1-\mu) \bar{C} + \bar{p}_H^{*-\theta} \bar{p}_F^{*\theta-\phi} \lambda (1-\mu^*) \bar{C}^* \quad (\text{A.12})$$

where $\bar{p}_H^* = \frac{\bar{P}_H^*}{\bar{P}^*}$, $\bar{p}_F^* = \frac{\bar{P}_F^*}{\bar{P}^*}$.

We define aggregate output in the following way:

$$\bar{Y} = \bar{p}_N \bar{Y}_N + \bar{p}_H \bar{Y}_H. \quad (\text{A.13})$$

Notice that since the law of one price holds for the traded goods: $\bar{p}_H^* = \bar{p}_H \bar{RS}^{-1}$.

We define the following steady state ratios for the home economy:

⁴⁷Notice that if markups in the nontraded and home traded sector are equal, i.e. $\bar{\mu}_N = \bar{\mu}_H$ then also the domestic relative prices of nontraded and home traded goods are equal, i.e. $\bar{p}_N = \bar{p}_H$.

$$d_{CH} = (1 - \lambda)(1 - \mu) \frac{\bar{C}}{\bar{Y}_H} \bar{p}_H^{-\theta} \bar{p}_T^{-\theta - \phi}, \quad (\text{A.14})$$

$$s_C = \bar{\omega} \frac{\bar{L}}{\bar{C}} = \frac{\bar{p}_N \bar{Y}_N + \bar{Y}_H}{\bar{\mu}_N \bar{C}}, \quad (\text{A.15})$$

$$d_{Y_N} = \frac{\bar{p}_N \bar{Y}_N}{\bar{Y}}, \quad (\text{A.16})$$

$$d_{Y_H} = \frac{\bar{p}_H \bar{Y}_H}{\bar{Y}}, \quad (\text{A.17})$$

$$\tilde{d}_{Y_N} = \frac{\bar{Y}_N}{\bar{Y}_N + \bar{Y}_H}, \quad (\text{A.18})$$

$$\tilde{d}_{Y_H} = \frac{\bar{Y}_H}{\bar{Y}_N + \bar{Y}_H}. \quad (\text{A.19})$$

From the complete asset market assumption and the assumption that the initial wealth distribution is such that $v = RS_0 \left(\frac{C_0}{C_0^*} \right)^\rho = 1$ we obtain:

$$\bar{C} = \overline{RS}^{\frac{1}{\rho}} \bar{C}^*. \quad (\text{A.20})$$

Lastly from the definition of price indices and the assumption that the law of one price holds in the traded sector we get the following relations between relative prices:

$$\bar{p}_T^{1-\theta} \bar{p}_H^{\theta-1} = (1 - \lambda) + \lambda (\overline{RS} \bar{p}_H^{-1} \bar{p}_F^*)^{1-\theta}, \quad (\text{A.21})$$

$$1 = \mu \bar{p}_N^{1-\phi} + (1 - \mu) \bar{p}_T^{1-\phi}. \quad (\text{A.22})$$

The set of the following conditions solves for the steady state values of domestic variables: \bar{p}_N , \bar{p}_H , \bar{p}_T , \bar{C} , \overline{RS} (\bar{C}^* and \bar{p}_F^* are treated as exogenous and are obtained from a similar set of the conditions for the foreign economy):⁴⁸

$$\bar{C} = \overline{RS}^{\frac{1}{\rho}} \bar{C}^*, \quad (\text{A.26})$$

⁴⁸The set of optimality conditions for the foreign economy which determines the steady state values of \bar{C}^* , \bar{p}_F^* , \bar{p}_N^* is the following:

$$\bar{C}^{*- \rho - \eta} \bar{p}_N^* \bar{\mu}_N^{*-1} = \left(\bar{p}_N^{*- \phi} \mu^* + \bar{p}_F^{*- \phi} (1 - \mu^*) \right)^\eta \quad (\text{A.23})$$

$$\mu^* \bar{p}_N^{*1-\phi} + (1 - \mu^*) \bar{p}_F^{*1-\phi} = 1 \quad (\text{A.24})$$

$$\bar{p}_N^* \bar{\mu}_N^{*-1} = \bar{p}_F^* \bar{\mu}_F^{*-1} \quad (\text{A.25})$$

$$\overline{p}_N \overline{\mu}_N^{-1} = \overline{p}_H \overline{\mu}_H^{-1}, \quad (\text{A.27})$$

$$\overline{C}^{-\rho} \overline{p}_H \overline{\mu}_H^{-1} = \left(\overline{p}_N^{-\phi} \mu \overline{C} + \overline{p}_H^{-\theta} \overline{p}_T^{-\phi} (1-\lambda)(1-\mu) \overline{C} + \overline{p}_H^{-\theta} \overline{RS}^{\theta} \overline{p}_F^{*\theta-\phi} \lambda (1-\mu^*) \overline{C}^* \right)^{\eta}, \quad (\text{A.28})$$

$$\overline{p}_T^{1-\theta} = (1-\lambda) \overline{p}_H^{1-\theta} + \lambda (\overline{RS} \overline{p}_F^*)^{1-\theta}, \quad (\text{A.29})$$

$$1 = \mu \overline{p}_N^{1-\phi} + (1-\mu) \overline{p}_T^{1-\phi}. \quad (\text{A.30})$$

A.2 A loglinearized version of the model

We loglinearize the model around the above presented steady state. We present the structural equations that describe dynamics of the domestic economy. All the variables with hat represent the log deviations from the steady state. The system is closed by specifying the monetary rules for each of the economies.

$$\widehat{\pi}_{N,t} = k_N (\rho \widehat{C}_t + \eta \widehat{L}_t - \widehat{A}_{N,t} - \rho \widehat{B}_t - \widehat{p}_{N,t}) + \beta \widehat{\pi}_{N,t+1}, \quad (\text{A.31})$$

$$\widehat{\pi}_{H,t} = k_H (\rho \widehat{C}_t + \eta \widehat{L}_t - \widehat{A}_{H,t} - \rho \widehat{B}_t - \widehat{p}_{H,t}) + \beta \widehat{\pi}_{H,t+1}, \quad (\text{A.32})$$

$$\widehat{L}_t = \widetilde{d}_{Y_N} (\widehat{Y}_{N,t} - \widehat{A}_{N,t}) + \widetilde{d}_{Y_H} (\widehat{Y}_{H,t} - \widehat{A}_{H,t}), \quad (\text{A.33})$$

$$\widehat{Y}_{N,t} = \widehat{C}_t - \phi \widehat{p}_{N,t}, \quad (\text{A.34})$$

$$\begin{aligned} \widehat{Y}_{H,t} = d_{CH} \widehat{C}_t - \theta \widehat{p}_{H,t} + b(\phi - \theta) d_{CH} \widehat{T}_t^d + (1 - d_{CH}) \theta \widehat{RS}_t + \\ + (1 - d_{CH}) \widehat{C}_t^* + b^*(\phi - \theta) (1 - d_{CH}) \widehat{T}_t^{d*}, \end{aligned} \quad (\text{A.35})$$

$$\widehat{Y}_t = d_{Y_N} (\widehat{p}_{N,t} + \widehat{Y}_{N,t}) + d_{Y_H} (\widehat{p}_{H,t} + \widehat{Y}_{H,t}) \quad (\text{A.36})$$

$$\widehat{C}_t = \widehat{B}_t + \frac{1}{\rho} \widehat{RS}_t + \widehat{C}_t^* - \widehat{B}_t^*, \quad (\text{A.37})$$

$$(a-1) \widehat{p}_{H,t} = b \widehat{T}_t^d + a \widehat{RS}_t - b^* a \widehat{T}_t^{d*}, \quad (\text{A.38})$$

$$\widehat{p}_{N,t} = (1 - b)\widehat{T}_t^d, \quad (\text{A.39})$$

$$\widehat{p}_{H,t} = -b\widehat{T}_t^d - a\widehat{T}_t, \quad (\text{A.40})$$

$$\widehat{T}_t = \widehat{\Delta S}_t - \widehat{\pi}_{H,t} + \widehat{T}_{t-1}, \quad (\text{A.41})$$

$$\widehat{T}_t^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{T,t} + \widehat{T}_{t-1}^d, \quad (\text{A.42})$$

$$\widehat{\pi}_{T,t} = \widehat{\pi}_{H,t} + a(\widehat{T}_t - \widehat{T}_{t-1}). \quad (\text{A.43})$$

The Maastricht variables can be derived from the following equations:

$$\widehat{S}_t = \widehat{\Delta S}_t - \widehat{S}_{t-1}, \quad (\text{A.44})$$

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^* + \widehat{R}S_t - \widehat{R}S_{t-1}, \quad (\text{A.45})$$

$$\widehat{R}_t = \rho(\widehat{C}_{t+1} - \widehat{B}_{t+1}) - \rho(\widehat{C}_t - \widehat{B}_t) + \widehat{\pi}_{t+1}, \quad (\text{A.46})$$

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1 - a)(1 - b)\widehat{\pi}_{H,t} + a(1 - b)(\widehat{S}_t - \widehat{S}_{t-1}). \quad (\text{A.47})$$

A.3 Parameterization

We present values of the structural parameters and also values of the stochastic parameters chosen in the numerical exercise.

Note: The policy rule is calibrated following Natalucci and Ravenna (2003): $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $SD(\widehat{\varepsilon}_{R,t}) = 0.45$.

Note: As far as the historical statistics are concerned our data sample for Poland is 1995:1 - 2007:4. CPI inflation rate in the traded and nontraded sector data sample is 1997:1 - 2007:4. All series are logged (except for interest and inflation rates) and Hodrick - Prescott filtered. Rates of change are quarterly.

All data were collected from the Eurostat webpage. Data are seasonally adjusted where appropriate. We present the detailed data series. Output: Gross value added (GVA) at 1995 constant prices in national currency.

The parameter definition	value of the parameter	
inverse of the intertemporal elasticity of substitution	ρ	2
inverse of the labour supply elasticity	η	3
discount factor	β	0.99
intra-temporal elasticity between variety of the goods	σ	10
elasticity of substitution between home and foreign tradables	θ	1.5
elasticity of substitution between tradables and nontradables	ϕ	0.5
share of nontradables	μ	0.38
degree of openness	λ	0.4
price rigidity in the nontradable sector	α_N	0.85
price rigidity in the home tradable sector	α_H	0.8
steady state share of taxes in the nontradable sector	τ_N	0.12
steady state share of taxes in the tradable sector	τ_H	0.12
share of nontradables in the foreign economy	μ^*	0.6

Table A.1: Structural parameters

shocks	autoregressive parameter	standard deviation (in %)
nontradable productivity (A_N)	0.85	1.6
tradable productivity (A_H)	0.85	1.8
preference (B)	0.7	0.4
foreign consumption (C^*)	0.85	0.23
$corr(\widehat{A}_{N,t}, \widehat{A}_{H,t}) = 0.7$ where $corr$ - correlation coefficient		

Table A.2: Stochastic environment

Traded output is an aggregate of sectoral GVA for: Agriculture; Hunting; Forestry and Fishing; Total industry (excluding construction). Nontraded output is an aggregate of sectoral GVA for: Wholesale and retail trade, repair of motor vehicles, motorcycles and personal household goods; Hotels and restaurants; Transport, storage and communication; Financial intermediation, real estate, renting and business activities. Consumption: Final consumption expenditure of households at 1995 constant prices in national currency. Nominal interest rate: 3 months' money market interest rate. Nominal exchange rate: Bilateral Zloty/euro exchange rate (quarterly average). Real exchange rate: CPI based real effective exchange rate (12 trading partners, quarterly average). CPI inflation rate: Harmonized Index of Consumer Prices (HICP). CPI inflation rate in the nontraded sector: HICP - Services. CPI inflation in the traded sector: HICP - Goods.

A.4 Quadratic representation of the optimal loss function

A.4.1 The second order approximation of the welfare function

We present a second order approximation to the welfare function (2.1):

$$W_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z'_v \widehat{v}_t - \frac{1}{2} \widehat{v}'_t Z_v \widehat{v}_t - \widehat{v}'_t Z_{\xi} \widehat{\xi}_t] + tip + O(3) \quad (\text{A.48})$$

where $\widehat{v}'_t = \left[\widehat{C}_t \quad \widehat{Y}_{N,t} \quad \widehat{Y}_{H,t} \quad \widehat{\pi}_{N,t} \quad \widehat{\pi}_{H,t} \right]$; $\widehat{\xi}'_t = \left[\widehat{A}_{N,t} \quad \widehat{A}_{H,t} \quad \widehat{B}_t \quad \widehat{C}_t^* \right]$; tip stands for *terms independent of policy* and $O(3)$ includes terms that are of order higher than the second in the deviations of

Statistics	Poland		Czech Republic	
	Model	Historical	Model	Historical
Standard deviation in %				
Output:	1.54	1.54	1.87	1.68
nontraded sector	1.16	1.69	1.95	1.56
traded sector	3	1.65	3.23	4.32
Consumption	1.04	1.04	1.94	1.93
Nominal interest rate	0.46	0.54	0.52	0.52
Nominal exchange rate	5.82	5.61	2.84	2.59
Real exchange rate	1.88	6.41	2.35	3.62
CPI inflation rate:				
nontraded sector	0.65	0.61	0.82	0.91
traded sector	0.53	0.44	0.59	0.97
	0.76	0.73	0.94	0.74

Table A.3: Matching the moments

variables from their steady state values. The matrices z_v , Z_v , Z_ξ are defined below:

$$z'_v = \begin{bmatrix} 1 & -s_C \tilde{d}_{Y_N} & -s_C \tilde{d}_{Y_H} & 0 & 0 \end{bmatrix}, \quad (\text{A.49})$$

$$Z_v = \begin{bmatrix} \rho - 1 & 0 & 0 & 0 & 0 \\ 0 & s_C \tilde{d}_{Y_N} (1 + \eta \tilde{d}_{Y_N}) & \eta s_C \tilde{d}_{Y_N} \tilde{d}_{Y_H} \hat{Y} & 0 & 0 \\ 0 & \eta s_C \tilde{d}_{Y_N} \tilde{d}_{Y_H} \hat{Y} & s_C \tilde{d}_{Y_H} (1 + \eta \tilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & s_C \tilde{d}_{Y_N} \frac{\sigma}{k_N} & 0 \\ 0 & 0 & 0 & 0 & s_C \tilde{d}_{Y_H} \frac{\sigma}{k_H} \end{bmatrix}, \quad (\text{A.50})$$

$$Z_\xi = \begin{bmatrix} 0 & 0 & -\rho & 0 \\ -s_C \tilde{d}_{Y_N} (1 + \eta \tilde{d}_{Y_N}) & -\eta s_C \tilde{d}_{Y_H} \tilde{d}_{Y_N} & 0 & 0 \\ -\eta s_C \tilde{d}_{Y_H} \tilde{d}_{Y_N} & -s_C \tilde{d}_{Y_H} (1 + \eta \tilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (\text{A.51})$$

A.4.2 Elimination of the linear terms

This section describes in detail how we eliminate the linear terms in the second order approximation to the welfare function in order to obtain a quadratic loss function. Moreover we reduce the number of structural variables that represent the policy problem by appropriate substitutions.

The optimal monetary policy solves the welfare maximization problem with the constraints given by the structural equations of the economy (their loglinearized versions are (A.31) - (A.43)). The matrix representation of the second order approximation to the welfare function is the following:

$$W = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z'_x \hat{x}_t - \frac{1}{2} \hat{x}'_t Z_x \hat{x}_t - \hat{x}'_t Z_\xi \hat{\xi}_t] + tip + O(3). \quad (\text{A.52})$$

Similarly we present a second order approximation to all the structural equations in the matrix form:

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\begin{array}{c} A_1 \hat{x}_t \\ A_2 \hat{x}_t \\ \dots \\ A_{13} \hat{x}_t \end{array} \right] + \frac{1}{2} \left[\begin{array}{c} \hat{x}_t' B_1 \hat{x}_t \\ \hat{x}_t' B_2 \hat{x}_t \\ \dots \\ \hat{x}_t' B_{13} \hat{x}_t \end{array} \right] + \left[\begin{array}{c} \hat{x}_t' C_1 \hat{\xi}_t \\ \hat{x}_t' C_2 \hat{\xi}_t \\ \dots \\ \hat{x}_t' C_{13} \hat{\xi}_t \end{array} \right] + tip + O(3) = 0 \quad (\text{A.53})$$

with

$$\hat{x}_t = \left[\hat{Y}_t \quad \hat{L}_t \quad \hat{C}_t \quad \hat{Y}_{N,t} \quad \hat{Y}_{H,t} \quad \hat{p}_{N,t} \quad \hat{p}_{H,t} \quad \hat{T}_t^d \quad \hat{T}_t \quad \widehat{RS}_t \quad \widehat{\Delta S}_t \quad \widehat{\pi}_{H,t} \quad \widehat{\pi}_{N,t} \quad \widehat{\pi}_{T,t} \right], \quad (\text{A.54})$$

$$\hat{\xi}_t' \equiv \left[\hat{A}_{N,t} \quad \hat{A}_{H,t} \quad \hat{B}_t \quad \hat{C}_t^* \right]$$

where *tip* means terms independent of policy.

Following the methodology of Benigno and Woodford (2005) in order to eliminate the linear terms in the welfare function we solve the system of linear equations:

$$\zeta A = z_x' \quad (\text{A.55})$$

$$\text{where } A_{(13 \times 14)} = \begin{bmatrix} A_1 \\ A_2 \\ \dots \\ A_{13} \end{bmatrix};$$

$$\zeta_{(1 \times 13)} = \left[\zeta_1 \quad \zeta_2 \quad \zeta_3 \quad \zeta_4 \quad \zeta_5 \quad \zeta_6 \quad \zeta_7 \quad \zeta_8 \quad \zeta_9 \quad \zeta_{10} \quad \zeta_{11} \quad \zeta_{12} \quad \zeta_{13} \right] \text{ and } z_x_{(14 \times 1)}.$$

As a result we obtain the loss function:

$$L_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2} \hat{x}_t' L_x \hat{x}_t + \hat{x}_t' L_{\xi} \hat{\xi}_t \right] + tip + O(3) \quad (\text{A.56})$$

where

$$L_x = Z_x + \zeta_1 B_1 + \zeta_2 B_2 + \zeta_4 B_4 + \zeta_6 B_6 + \zeta_9 B_9 + \zeta_{10} B_{10} + \zeta_{11} B_{11} \quad (\text{A.57})$$

$$+ \zeta_{12} B_{12} + \zeta_{13} B_{13}, \quad (\text{A.58})$$

$$L_{\xi} = Z_{\xi} + \zeta_1 C_1 + \zeta_2 C_2 + \zeta_4 C_4 + \zeta_6 C_6 + \zeta_{13} C_{13}. \quad (\text{A.59})$$

A.4.3 Substitution of the variables

We want to represent the loss function (??) and also the whole model just in terms of the following variables:

$$\widehat{y}'_t = \left[\widehat{Y}_t \quad \widehat{T}_t^d \quad \widehat{T}_t \quad \Delta \widehat{S}_t \quad \widehat{\pi}_{H,t} \quad \widehat{\pi}_{N,t} \quad \widehat{\pi}_{T,t} \right]. \quad (\text{A.60})$$

In order to do this we define matrices $N_{x(14 \times 7)}$ and $N_{\xi(14 \times 4)}$ that map all the variables in the vector \widehat{y}'_t in the following way:

$$\widehat{x}_t = N_x \widehat{y}'_t + N_{\xi}' \widehat{\xi}_t \quad (\text{A.61})$$

where:

$$N_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & ltd & lt & 0 & 0 & 0 & 0 \\ 1 & ctd & ct & 0 & 0 & 0 & 0 \\ 1 & yntd & ynt & 0 & 0 & 0 & 0 \\ 1 & yhtd & yht & 0 & 0 & 0 & 0 \\ 0 & pntd & 0 & 0 & 0 & 0 & 0 \\ 0 & phtd & pht & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & rstd & rst & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (\text{A.62})$$

$$N_x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ lan & lah & lb & 0 \\ 0 & 0 & cb & 0 \\ 0 & 0 & ynb & 0 \\ 0 & 0 & yhb & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{A.63})$$

with parameters defined below:

$$pntd = 1 - b \quad (\text{A.64})$$

$$phtd = -b \quad (\text{A.65})$$

$$pht = -a \quad (\text{A.66})$$

$$rstd = -b \quad (\text{A.67})$$

$$rst = 1 - a \quad (\text{A.68})$$

$$cb = (1 - d_{CH})d_{YH} \quad (\text{A.69})$$

$$ct = ad_{YH}(1 - \theta) + d_{YH}(1 - d_{CH})\left(\frac{1}{\rho} - \theta\right)(1 - a) \quad (\text{A.70})$$

$$ctd = d_{YN}(1 - b)(\phi - 1) + bd_{YH}(1 - \theta) + b(\theta - \phi)d_{CH}d_{YH} + \quad (\text{A.71})$$

$$+(1 - d_{CH})\left(\theta - \frac{1}{\rho}\right)d_{YH}b \quad (\text{A.72})$$

$$yntd = ctd - \phi * (1 - b) \quad (\text{A.73})$$

$$ynb = cb \quad (\text{A.74})$$

$$ynt = ct \quad (\text{A.75})$$

$$yhnd = d_{YN}b\theta - d_{YN}d_{CH}b(\theta - \phi) - (1 - d_{CH})\left(\theta - \frac{1}{\rho}\right)bd_{YN} + \quad (\text{A.76})$$

$$-d_{YN} + b + d_{YN}\phi(1 - b) \quad (\text{A.77})$$

$$yht = d_{YN}\theta a - d_{YN}(1 - d_{CH})\left(\theta - \frac{1}{\rho}\right)(a - 1) + d_{YH}a \quad (\text{A.78})$$

$$yhb = -dyn(1 - d_{CH}) \quad (\text{A.79})$$

$$ltd = \tilde{d}_{YN} * yntd + \tilde{d}_{YH} * yhnd \quad (\text{A.80})$$

$$lt = \tilde{d}_{YN} * ynt + \tilde{d}_{YH} * yht \quad (\text{A.81})$$

$$lan = -\tilde{d}_{YN} \quad (\text{A.82})$$

$$lab = -\tilde{d}_{YH} \quad (\text{A.83})$$

$$lb = \tilde{d}_{YN} * ynb + \tilde{d}_{YH} * yhb \quad (\text{A.84})$$

The loss function can be expressed now as:

$$L_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2} \hat{y}'_t L_y \hat{y}_t + \hat{y}'_t L_{\xi,y} \hat{\xi}_t \right] + tip + O(3) \quad (\text{A.85})$$

where:

$$L_y = N'_x L_x N_x, \quad (\text{A.86})$$

$$L_{\xi,y} = N'_x L_x N_{\xi} + N'_x L_{\xi}. \quad (\text{A.87})$$

Since variables $[\Delta S_t, \pi_{T,t}]$ do not appear in the original welfare objective function and in the second order terms of the structural equations we can further reduce the set of the variables which appear in the loss function to:

$$\hat{y}'_t = \left[\hat{Y}_t \quad \hat{T}_t^d \quad \hat{T}_t \quad \hat{\pi}_{H,t} \quad \hat{\pi}_{N,t} \right] \quad (\text{A.88})$$

The final set of the structural equations which represent the constraints of the maximization problem is:

$$\hat{\pi}_{N,t} = k_N \widehat{m}c_t^{N,r} + \beta \hat{\pi}_{N,t+1}, \quad (\text{A.89})$$

$$\hat{\pi}_{H,t} = k_H \widehat{m}c_t^{H,r} + \beta \hat{\pi}_{H,t+1}, \quad (\text{A.90})$$

$$\hat{C}_t^* = \hat{Y}_t + (ctd - \frac{1}{\rho} rstd) \hat{T}_t^d + (ct - \frac{1}{\rho} rst) \hat{T}_t + (cb - 1) \hat{B}_t, \quad (\text{A.91})$$

$$\hat{T}_t^d - \hat{T}_{t-1}^d = \hat{\pi}_{N,t} - \hat{\pi}_{H,t} - a(\hat{T}_t - \hat{T}_{t-1}) \quad (\text{A.92})$$

where:

$$\widehat{m}c_t^{N,r} = (\rho + \eta) \hat{Y}_t + (\rho * ctd + \eta * ltd - pntd) \hat{T}_t^d + \quad (\text{A.93})$$

$$+ (\rho * ct + \eta * lt) \hat{T}_t - (1 + \eta * \tilde{d}_{YN}) \hat{A}_{N,t} + \quad (\text{A.94})$$

$$- \eta \tilde{d}_{YH} \hat{A}_{H,t} + (\rho * (cb - 1) + \eta * lb) \hat{B}_t \quad (\text{A.95})$$

$$\widehat{m}c_t^{H,r} = (\rho + \eta) \hat{Y}_t + (\rho * ctd + \eta * ltd - phtd) \hat{T}_t^d + \quad (\text{A.96})$$

$$+ (\rho * ct + \eta * lt - pht) \hat{T}_t - \eta \tilde{d}_{YN} \hat{A}_{N,t} + \quad (\text{A.97})$$

$$- (1 + \eta * \tilde{d}_{YH}) \hat{A}_{H,t} + (\rho * (cb - 1) + \eta * lb) \hat{B}_t \quad (\text{A.98})$$

with:

$$m_{N,Y} = \rho + \eta \quad (\text{A.99})$$

$$m_{N,T^d} = \rho * ctd + \eta * ltd - pntd \quad (\text{A.100})$$

$$m_{N,T} = \rho * ct + \eta * lt \quad (\text{A.101})$$

$$m_{N,A_N} = -(1 + \eta * \tilde{d}_{YN}) \quad (\text{A.102})$$

$$m_{N,A_H} = -\eta \tilde{d}_{YH} \quad (\text{A.103})$$

$$m_{N,B} = \rho * (cb - 1) + \eta * lb \quad (\text{A.104})$$

$$m_{H,Y} = \rho + \eta \quad (\text{A.105})$$

$$m_{H,T^d} = \rho * ctd + \eta * ltd - phtd \quad (\text{A.106})$$

$$m_{H,T} = \rho * ct + \eta * lt - pht \quad (\text{A.107})$$

$$m_{H,A_N} = -\eta \tilde{d}_{YN} \quad (\text{A.108})$$

$$m_{H,A_H} = -(1 + \eta * \tilde{d}_{YH}) \quad (\text{A.109})$$

$$m_{H,B} = \rho * (cb - 1) + \eta * lb \quad (\text{A.110})$$

$$n_{T^d} = ctd - \frac{1}{\rho} rstd \quad (\text{A.111})$$

$$n_T = ct - \frac{1}{\rho} rst \quad (\text{A.112})$$

$$n_B = cb - 1 \quad (\text{A.113})$$

Structural equations defining the Maastricht variables:

$$\hat{R}_t = \hat{\pi}_{t+1} - \rho(1 - cb)(\hat{B}_{t+1} - \hat{B}_t) + \rho(\hat{Y}_{t+1} - \hat{Y}_t) + \rho * ctd(\hat{T}_{t+1}^d - \hat{T}_t^d) + \rho * ct(\hat{T}_{t+1} - \hat{T}_t), \quad (\text{A.114})$$

$$\hat{\pi}_t = \hat{\pi}_{H,t} + b(\hat{T}_t^d - \hat{T}_{t-1}^d) + a(\hat{T}_t - \hat{T}_{t-1}), \quad (\text{A.115})$$

$$\hat{S}_t = \hat{S}_{t-1} + \hat{\pi}_t + rstd(\hat{T}_t^d - \hat{T}_{t-1}^d) + rst(\hat{T}_t - \hat{T}_{t-1}). \quad (\text{A.116})$$

A.5 Reinterpretation of the Maastricht convergence criteria

We show how to reinterpret each of the Maastricht criteria in order to be able to use the method of Rotemberg and Woodford (1997, 1999).

A.5.1 Exchange rate criterion

We reinterpret the criterion on the nominal exchange rate (4.3) into two inequalities given below:⁴⁹

$$E(\hat{S}_t) - k * SD(\hat{S}_t) \geq -15\%, \quad (\text{A.117})$$

⁴⁹ E stands for the expectation operator and SD stands for the standard deviation operator.

$$E\left(\widehat{S}_t\right) + k * SD(\widehat{S}_t) \leq 15\%. \quad (\text{A.118})$$

where k is large enough to prevent from violating the criterion (4.3) and SD refers to the standard deviation statistic.

These two inequalities can be represented as the following two sets of inequalities (to conform with the welfare measure we use discounted statistics):

$$\left\{ \begin{array}{l} (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(\widehat{S}_t - (-15\%) \right) \geq 0 \\ (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(\widehat{S}_t - (-15\%) \right)^2 \leq K \left((1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(\widehat{S}_t - (-15\%) \right) \right)^2, \end{array} \right. \quad (\text{A.119})$$

$$\left\{ \begin{array}{l} (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(15\% - \widehat{S}_t \right) \leq 0 \\ (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(15\% - \widehat{S}_t \right)^2 \leq K \left((1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(15\% - \widehat{S}_t \right) \right)^2, \end{array} \right. \quad (\text{A.120})$$

where $K = 1 + k^{-2}$.

A.5.2 Inflation criterion

We redefine the condition (4.1). We assume that the average inflation in the domestic economy should be at least k standard deviations smaller than the average inflation in the foreign economy plus a margin summarized by B_π (where $B_\pi = \sqrt[4]{1,015} - 1$):

$$E(\widehat{\pi}_t) \leq E(\widehat{\pi}_t^*) + B_\pi - kSD(\widehat{\pi}_t) \quad (\text{A.121})$$

where $\widehat{\pi}_t, \widehat{\pi}_t^*$ are treated as deviations from the zero inflation steady state in the domestic economy and the foreign one accordingly (i.e. $\bar{\pi} = \bar{\pi}^* = 0$) and k large enough to prevent from violating criterion (4.1). We assume that the foreign economy is in the steady state so $\widehat{\pi}_t^* = 0 \forall t$. As a result our restriction (A.121) becomes:

$$E(\widehat{\pi}_t) \leq B_\pi - kSD(\widehat{\pi}_t). \quad (\text{A.122})$$

Since B_π is a constant we can use the following property of the variance: $Var(\widehat{\pi}_t) = Var(B_\pi - \widehat{\pi}_t)$. Our restriction becomes:

$$kSD(B_\pi - \widehat{\pi}_t) \leq E(B_\pi - \widehat{\pi}_t). \quad (\text{A.123})$$

This restriction can be represented as a set of two restrictions:

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_{\pi} - \pi_t) \geq 0, \quad (\text{A.124})$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_{\pi} - \hat{\pi}_t)^2 \leq K \left((1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_{\pi} - \hat{\pi}_t) \right)^2. \quad (\text{A.125})$$

A.5.3 Nominal interest rate criterion

Similarly to the criterion on the CPI aggregate inflation we interpret the inequality (4.2):

$$E(\hat{R}_t) \leq E(\hat{R}_t^*) + C_R - kSD(\hat{R}_t) \quad (\text{A.126})$$

where k is large enough to prevent from frequent violating the criterion (4.2) and $C_R = \sqrt[4]{1,02} - 1$.

As in the case of the foreign inflation we assume that $\hat{R}_t^* = 0 \forall t$. So the restriction (A.126) becomes:

$$kSD(C_R - \hat{R}_t) \leq E(C_R - \hat{R}_t). \quad (\text{A.127})$$

This inequality can be represented as a set of two inequalities:

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \hat{R}_t) \geq 0, \quad (\text{A.128})$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \hat{R}_t)^2 \leq K \left((1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \hat{R}_t) \right)^2. \quad (\text{A.129})$$

A.6 The constrained loss function

We provide the proof of the Proposition 1 stated in the main text. Since all the sets of the constraints have a similar structure the proof concerns the optimal monetary policy with only one constraint on the CPI inflation rate. The proof is based on the proof of Proposition 6.9 in Woodford (2003).

Proposition 1 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0} \left\{ (1 - \beta) \sum_{t=t_0}^{\infty} \beta^t L_t \right\} \quad (\text{A.130})$$

subject to (4.6) - (4.7). Let $m_{1,\pi}$, $m_{2,\pi}$ be the discounted average values of $(B_{\pi} - \hat{\pi}_t)$ and $(B_{\pi} - \hat{\pi}_t)^2$ associated with the optimal policy. Then the optimal policy also minimizes a modified discounted loss criterion of the form (A.130) with L_t replaced by:

$$\tilde{L}_t \equiv L_t + \Phi_{\pi}(\pi^T - \hat{\pi}_t)^2 \quad (\text{A.131})$$

under constraints represented by the structural equations. Importantly $\Phi_{\pi} \geq 0$ and takes strictly positive value if and only if the constraint (4.7) binds. Moreover if the constraint (4.7) binds the corresponding target value

π^T is negative and given by the following relation:

$$\pi^T = B_\pi - Km_{1,\pi} < 0. \quad (\text{A.132})$$

Proof. Let $m_{1,\pi}$ and $m_{2,\pi}$ be the discounted average values of $(B_\pi - \pi_t)$ and $(B_\pi - \pi_t)^2$ associated with the policy that solves the constrained optimization problem stated in the corollary. Let $m_{1,\pi}^*$ and $m_{2,\pi}^*$ be the values of these moments for the policy that minimizes (A.130) without additional constraints. Notice that since $m_{1,\pi}^* = B_\pi$ the constraint (4.6) does not bind.⁵⁰ We identify the deterministic component of policy, i.e. $m_{1,\pi}$ and also the stabilization component of policy which is: $m_{2,\pi} - (m_{1,\pi})^2$. Moreover we also conclude that $m_{1,\pi} \geq m_{1,\pi}^*$ since there is no advantage from choosing $m_{1,\pi}$ such that: $m_{1,\pi} < m_{1,\pi}^*$ - both constraints set only the lower bound on the value of $m_{1,\pi}$ for any value of the stabilization component of policy. If one chooses $m_{1,\pi}$ such that: $m_{1,\pi} > m_{1,\pi}^*$ then one can relax the constraint (4.7). So $m_{1,\pi} \geq m_{1,\pi}^*$. Based on the above discussion we formulate two alternative constraints to the constraints (4.6, 4.7):

$$(1 - \beta)E_0 \sum_{t=0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t) \geq m_{1,\pi}, \quad (\text{A.133})$$

$$(1 - \beta)E_0 \sum_{t=0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t)^2 \leq m_{2,\pi}. \quad (\text{A.134})$$

Observe that any policy that satisfies the above constraints satisfies also the weaker constraints: (4.6, 4.7). Now we take advantage of the Kuhn – Tucker theorem: the policy that minimizes (A.130) subject to (A.133, A.134) also minimizes the following loss criterion:

$$E_0 \left\{ (1 - \beta) \sum_{t=0}^{\infty} \beta^t L_t \right\} - \mu_{1,\pi} E_0 \left\{ (1 - \beta) \sum_{t=0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t) \right\} + \mu_{2,\pi} E_0 \left\{ (1 - \beta) \sum_{t=0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t)^2 \right\} \quad (\text{A.135})$$

where $\mu_{1,\pi}$ and $\mu_{2,\pi}$ are the Lagrange multipliers which are nonnegative. If (4.7) binds then we obtain the following relation between the multipliers:

$$\mu_{1,\pi} = 2Km_{1,\pi}\mu_{2,\pi} \quad (\text{A.136})$$

since $m_{2,\pi} = Km_{1,\pi}^2$.

Rearranging the terms in (A.135) we can define the new loss function as:

$$\tilde{L}_t \equiv L_t + \mu_{2,\pi} \left((B_\pi - \hat{\pi}_t) - \frac{\mu_{1,\pi}}{2\mu_{2,\pi}} \right)^2 \quad (\text{A.137})$$

⁵⁰Means of all the variables under the unconstrained optimal policy are zero.

where the final term appears only when $\mu_{2,\pi} > 0$. Therefore $\Phi_\pi = \mu_{2,\pi} \geq 0$ and takes a strictly positive value only if (4.7) binds. Moreover for $\Phi_\pi > 0$ we have that:

$$\pi^T = B_\pi - \frac{\mu_{1,\pi}}{2\mu_{2,\pi}} = B_\pi - Km_{1,\pi}. \quad (\text{A.138})$$

Notice that the target value for the CPI inflation is negative (since $K > 1$ and $m_{1,\pi} \geq B_\pi$):

$$\pi^T = B_\pi - Km_{1,\pi} < 0. \quad (\text{A.139})$$

■

A.7 The constrained optimal monetary policy

We derive the first order conditions for the optimal monetary policy that satisfy the additional criteria on the nominal interest and the CPI aggregate inflation.

$$\begin{aligned} \min L_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} & \left[\frac{1}{2} \Phi_Y (\hat{Y}_t - \hat{Y}_t^T)^2 + \frac{1}{2} \Phi_{T^d} (\hat{T}_t^d - \hat{T}_t^{dT})^2 + \right. \\ & \frac{1}{2} \Phi_T (\hat{T}_t - \hat{T}_t^T)^2 + \Phi_{TT^d} \hat{T}_t^d \hat{T}_t + \Phi_{YT^d} \hat{Y}_t \hat{T}_t^d + \Phi_{YT} \hat{Y}_t \hat{T}_t + \\ & \frac{1}{2} \Phi_{\pi_H} \hat{\pi}_{H,t}^2 + \frac{1}{2} \Phi_{\pi_N} \hat{\pi}_{N,t}^2 + \frac{1}{2} \Phi_R (\hat{R}_t - R^T)^2 \\ & \left. + \frac{1}{2} \Phi_\pi (\hat{\pi}_t - \pi^T)^2 \right] + tip + O(3) \quad (\text{A.140}) \end{aligned}$$

subject to :

$$\begin{aligned} \hat{\pi}_{N,t} = k_N (m_{N,Y} \hat{Y}_t + m_{N,T^d} \hat{T}_t^d + m_{N,T} \hat{T}_t + m_{N,A_N} \hat{A}_{N,t} + m_{N,A_H} \hat{A}_{H,t} \\ + m_{N,B} \hat{B}_t) + \beta \hat{\pi}_{N,t+1}, \quad (\text{A.141}) \end{aligned}$$

$$\begin{aligned} \hat{\pi}_{H,t} = k_H (m_{H,Y} \hat{Y}_t + m_{H,T^d} \hat{T}_t^d + m_{H,T} \hat{T}_t + m_{H,A_N} \hat{A}_{N,t} + m_{H,A_H} \hat{A}_{H,t} \\ + m_{H,B} \hat{B}_t) + \beta \hat{\pi}_{H,t+1}, \quad (\text{A.142}) \end{aligned}$$

$$\hat{C}_t^* = \hat{Y}_t + n_{T^d} \hat{T}_t^d + n_T \hat{T}_t + n_B \hat{B}_t, \quad (\text{A.143})$$

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}), \quad (\text{A.144})$$

$$\begin{aligned} \widehat{R}_t = & b\widehat{\pi}_{N,t+1} + (1-b)\widehat{\pi}_{H,t+1} - \rho(1-cb)(\widehat{B}_{t+1} - \widehat{B}_t) \\ & + \rho(\widehat{Y}_{t+1} - \widehat{Y}_t) + \rho * ct d(\widehat{T}_{t+1}^d - \widehat{T}_t^d) + \\ & (\rho * ct + a(1-b))(\widehat{T}_{t+1} - \widehat{T}_t), \end{aligned} \quad (\text{A.145})$$

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1}). \quad (\text{A.146})$$

First order conditions of the minimization problem:

- wrt $\widehat{\pi}_{N,t}$:

$$0 = \Phi_{\pi_N} \widehat{\pi}_{N,t} + \gamma_{1,t} - \gamma_{1,t-1} - \gamma_{4,t} - b\beta^{-1}\gamma_{5,t-1} - b\gamma_{6,t}, \quad (\text{A.147})$$

- wrt $\widehat{\pi}_{H,t}$:

$$0 = \Phi_{\pi_H} \widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} - (1-b)\beta^{-1}\gamma_{5,t-1} - (1-b)\gamma_{6,t}, \quad (\text{A.148})$$

- wrt \widehat{Y}_t :

$$\begin{aligned} 0 = \Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) + \Phi_{YT^d} \widehat{T}_t^d + \Phi_{YT} \widehat{T}_t - k_N m_{N,Y} \gamma_{1,t} \\ - k_H m_{H,Y} \gamma_{2,t} - \gamma_{3,t} + \rho \gamma_{5,t} - \rho \beta^{-1} \gamma_{5,t-1}, \end{aligned} \quad (\text{A.149})$$

- wrt \widehat{T}_t^d :

$$\begin{aligned} 0 = \Phi_{T^d}(\widehat{T}_t^d - \widehat{T}_t^{dT}) + \Phi_{TT^d} \widehat{T}_t + \Phi_{YT^d} \widehat{Y}_t - k_N m_{N,T^d} \gamma_{1,t} \\ - k_H m_{H,T^d} \gamma_{2,t} - n_{T^d} \gamma_{3,t} + \gamma_{4,t} \\ - \beta \gamma_{4,t+1} + \rho ct d \gamma_{5,t} - \rho ct d \beta^{-1} \gamma_{5,t-1}, \end{aligned} \quad (\text{A.150})$$

- wrt \widehat{T}_t :

$$\begin{aligned} 0 = \Phi_T(\widehat{T}_t - \widehat{T}_t^T) + \Phi_{TT^d} \widehat{T}_t^d + \Phi_{YT} \widehat{Y}_t - k_N m_{N,T} \gamma_{1,t} \\ - k_H m_{H,T} \gamma_{2,t} - n_T \gamma_{3,t} + a \gamma_{4,t} - \beta a \gamma_{4,t+1} \\ + (\rho ct + a(1-b)) \gamma_{5,t} - (\rho ct + a(1-b)) \beta^{-1} \gamma_{5,t-1} \\ - a(1-b) \gamma_{6,t} + a(1-b) \beta \gamma_{6,t+1}, \end{aligned} \quad (\text{A.151})$$

- wrt \widehat{R}_t :

$$0 = \Phi_R(\widehat{R}_t - R^T) + \gamma_{5,t}, \quad (\text{A.152})$$

- wrt $\widehat{\pi}_t$:

$$0 = \Phi_\pi(\widehat{\pi}_t - \pi^T) + \gamma_{6,t}. \quad (\text{A.153})$$

A.8 The constrained loss function - general case

We provide the proposition⁵¹ that summarizes the discussion in Section 6 regarding the foreign economy. Since sets of the constraints for the CPI inflation rate and the nominal interest rate have the same structure the proposition concerns the optimal monetary policy with only one constraint on the CPI inflation rate.

Proposition 2 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0} \left\{ (1 - \beta) \sum_{t=t_0}^{\infty} \beta^t L_t \right\} \quad (\text{A.154})$$

subject to

$$(1 - \beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*) \geq 0, \quad (\text{A.155})$$

$$(1 - \beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*)^2 \leq K \left((1 - \beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*) \right)^2. \quad (\text{A.156})$$

Let $n_{1,\pi}$, $n_{2,\pi}$ be the discounted average values of $(B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*)$ and $(B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*)^2$ associated with the optimal policy. Moreover assume that the average of the foreign CPI inflation rate (m_{1,π^*}) satisfies the following inequality: $m_{1,\pi^*} \geq -B_\pi$. Then the optimal policy also minimizes a modified discounted loss criterion of the form (A.154) with L_t replaced by:

$$\widetilde{L}_t \equiv L_t + \Phi_\pi(\pi_t^T - \widehat{\pi}_t)^2 \quad (\text{A.157})$$

under constraints represented by the structural equations. Importantly $\Phi_\pi \geq 0$ and takes strictly positive value if and only if the constraint (A.156) binds. Moreover if the constraint (A.156) binds the corresponding target value π_t^T is given by the following relation:

$$\pi_t^T = B_\pi + \widehat{\pi}_t^* - K n_{1,\pi} < \widehat{\pi}_t^*. \quad (\text{A.158})$$

A.8.1 Discussion

In this subsection we discuss briefly how an inclusion of variable foreign inflation in the criterion can alter the characteristics of the constrained policy and also the loss associated to it. The constraints (A.155) - (A.156) can be represented in the following way:

⁵¹We do not present the proof for this proposition since it resembles to a great extent the proof of Proposition 1.

$$m(\hat{\pi}_t) - m(\hat{\pi}_t^*) \leq B_\pi \quad (\text{A.159})$$

$$\text{var}(\hat{\pi}_t - \hat{\pi}_t^*) \leq (K - 1)(B_\pi - m(\hat{\pi}_t) + m(\hat{\pi}_t^*))^2 \quad (\text{A.160})$$

where m is defined by (4.4) and var is defined by (4.5).

Constraint (A.160) can be further simplified:

$$\text{var}(\hat{\pi}_t) \leq (K - 1)(B_\pi - m(\hat{\pi}_t))^2 - \text{var}(\hat{\pi}_t^*) + 2\text{cov}(\hat{\pi}_t, \hat{\pi}_t^*) - 2m(\hat{\pi}_t)m(\hat{\pi}_t^*) + 2B_\pi m(\hat{\pi}_t^*) \quad (\text{A.161})$$

The nature of the constrained policy depends on the characteristics of the foreign inflation process and also the covariance of domestic inflation with foreign inflation. Not surprisingly, a higher (nonzero) mean of foreign inflation decreases the tightness of the inflation criterion. However a higher variance of foreign inflation increases the tightness of the inflation criterion. A higher variability of foreign inflation means that extreme realisations of foreign inflation happen more frequently. That is why variability of domestic inflation has to be reduced in order to comply with the criterion. At the same time higher correlation of domestic and foreign inflation makes it easier to satisfy the inflation criterion. Correlation of inflation processes depends to a great extent on the variance decomposition of shocks in both countries and also on the degree of synchronisation of shocks between the countries. If the home inflation process is explained primarily by domestic shocks and these shocks are not synchronised with the foreign ones, then the covariance of the inflation processes will be small. Kolasa (2008) finds evidence in favour of heterogeneity of most shock processes for Poland and the euro area. This would imply that the nature of the constrained policy would be influenced by the relative strength of each of the moments of the foreign inflation process. Taking into account the fact that usually, higher mean of inflation is associated with a higher variability (and vice versa) gives an intuition that the nature of the constrained policy should not change dramatically with respect to our benchmark calculation.

B Maastricht Criteria and Data on EMU Accession Countries

B.1 The Convergence Criteria in the Maastricht Treaty

The Article 109j(1) of the Maastricht Treaty lays down the following monetary criteria as a prerequisite for entering the EMU:

- *the achievement of a high degree of price stability* which means that a Member State (of the EU) has a sustainable price performance and an average rate of inflation (the Consumer Price Index (CPI) inflation), observed over a period of one year before the examination, which does not exceed that of the three best performing Member States in terms of price stability by more than 1.5% points (the CPI inflation rate

criterion);

- *the durability of the convergence ... reflected in the long term interest rate levels* which means that, over a period of one year before the examination, a Member State has an average nominal long-term interest rate that does not exceed that of the three best performing Member States in terms of price stability by more than 2% points (the nominal interest rate criterion);
- *the observance of the normal fluctuation margins provided for by the Exchange Rate Mechanism of the European Monetary System ($\pm 15\%$ bound around the central parity), for at least two years, without devaluing against the currency of any other Member State* (the nominal exchange rate criterion).

Importantly, the Maastricht Treaty also imposes the criterion on the fiscal policy, i.e. the sustainability of the government financial position which refers to a government budgetary position without an excessive deficit (Article 104c(6) of the Maastricht Treaty). The treaty stipulates: *The sustainability of the government financial position will be apparent from having achieved a government budgetary position without a deficit that is excessive.* In practice, the European Commission sets out two criteria:

- the annual government deficit: the ratio of the annual government deficit to gross domestic product must not exceed 3% at the end of the proceeding financial year. If this is not the case, the ratio must have declined substantially and continuously and reached a level close to 3% (interpretation in trend terms according to Article 104(2)) or alternatively, must remain close to 3% while representing only an exceptional and temporary excess
- government debt: the ratio of gross government debt to GDP must not exceed 60% at the end of the preceding financial year. If this is not the case, the ratio must have sufficiently diminished and must be approaching the reference value at a satisfactory pace (interpretation in trend terms according to Article 104(2)).

B.2 Data on EMU Accession Countries

We present figures and data regarding the EMU accession countries. All the data were collected from the Eurostat database and the European Commission webpage.

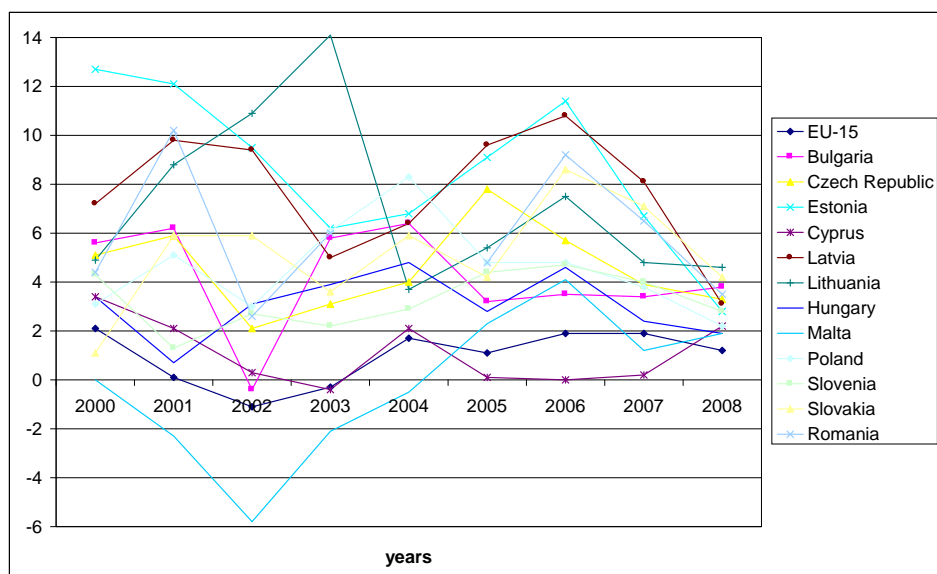


Figure B.1: Total annual labour productivity growth in the EMU accession countries and the EU 15 (annual rates in %) for the period 2000 - 2008. Values for 2007 and 2008 are forecasts.

countries	monetary regime	share of nontradables in consumption*	share of imports in GDP (at current places)#
Czech Republic	managed float	42%	68%
Estonia	peg	39%	86%
Hungary	managed float	44%	71%
Latvia	peg	37%	55%
Lithuania	peg	33%	58%
Poland	CPI targeting	37%	35%
Slovakia	managed float	41%	78%
average in the EU - 15		51%	63%

note: * – average 2000 - 2005; # – average 2000 - 2007; ^ – average 2000 - 2004

Table B.1: Structural characteristics of the EMU accession countries

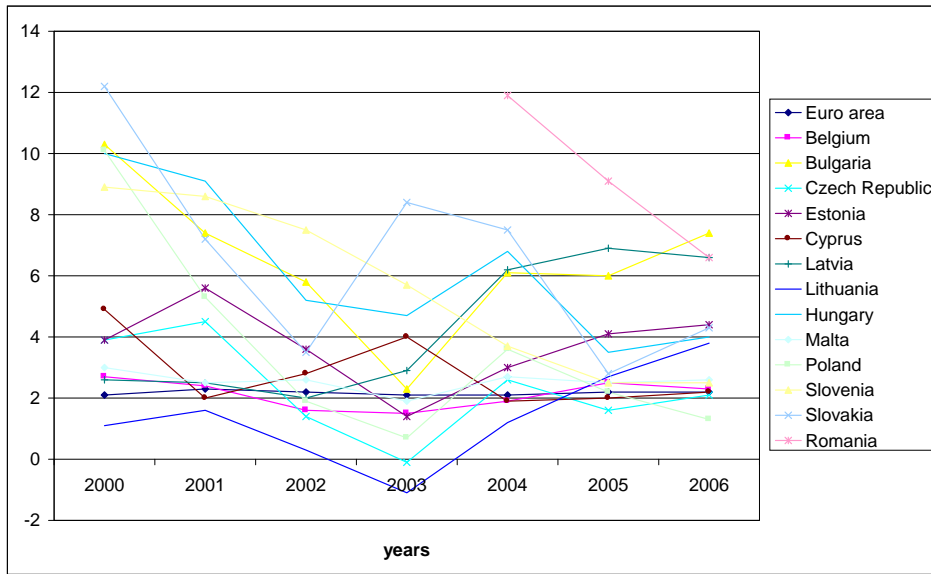


Figure B.2: CPI inflation in the EMU accession countries and the EU - 15 in 2000 - 2006 (annual % rates). For the purpose of clarity, CPI inflation rates in Romania are reported only for 2004 - 2006.

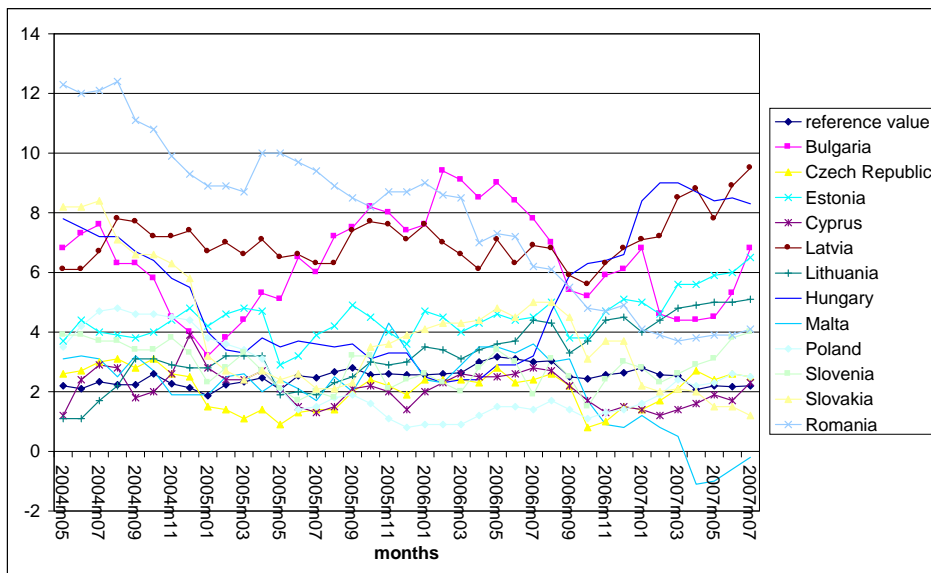


Figure B.3: CPI inflation rates in the EMU accession countries since their accession to the EU (annual rates in %)

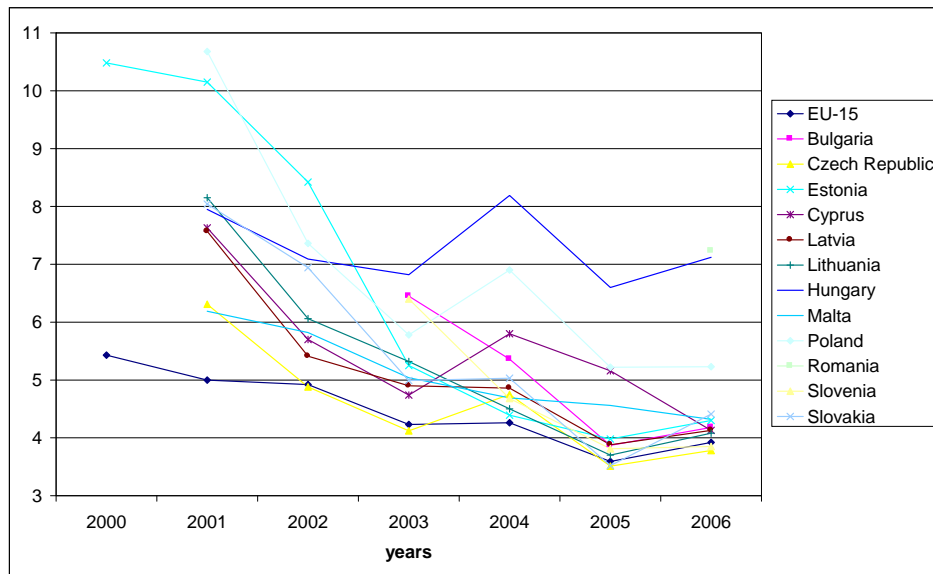


Figure B.4: EMU convergence criterion bond yields for the EMU accession countries and the euro area in 2001 - 2006 (annual % rates)

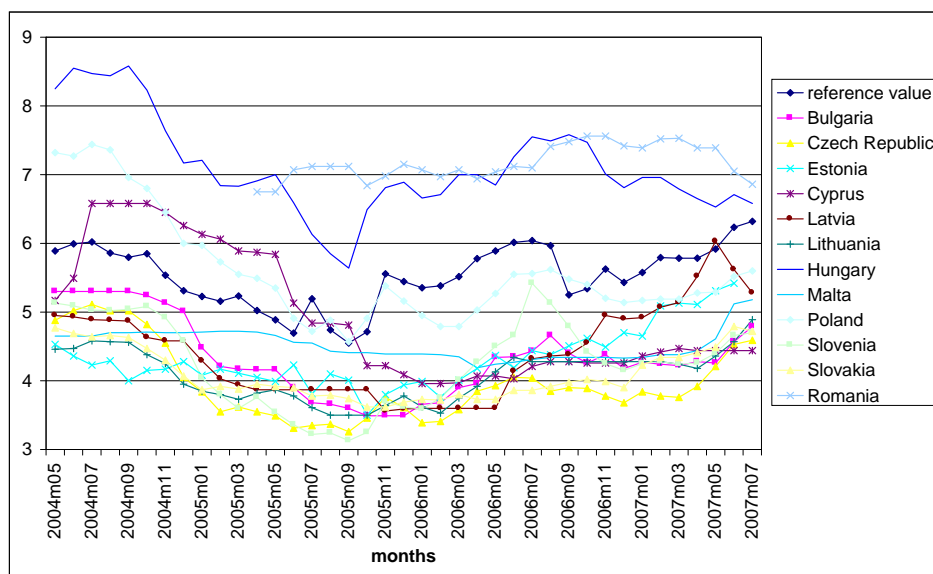


Figure B.5: EMU convergence criterion bond yields for the EMU accession countries since their accession to the EU (annual rates in %)

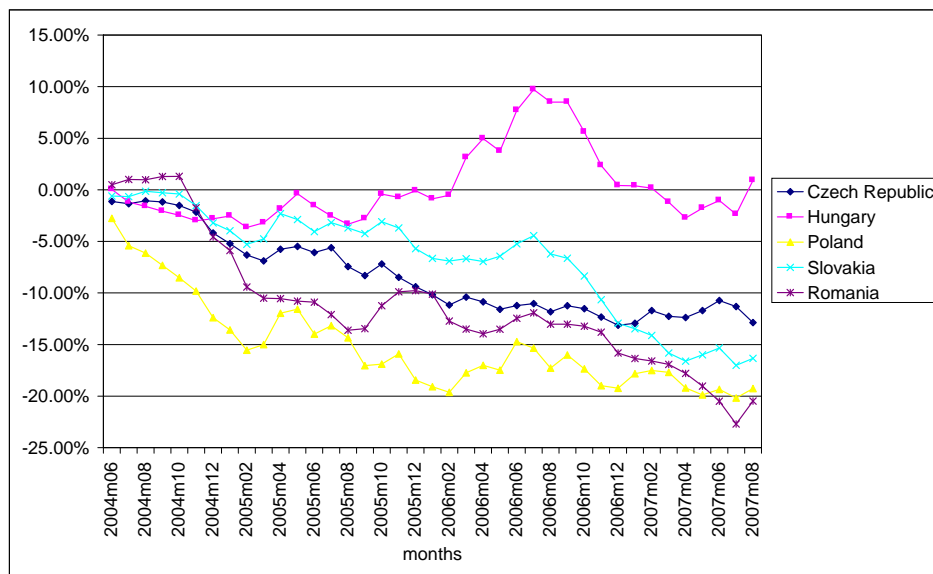


Figure B.6: Nominal exchange rate fluctuations vs. euro of the EMU accession countries since the accession to the EU (average monthly changes since the EU accession date)