

Skill Requirements, Search Frictions and Wage Inequality*

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Abstract

This paper examines wage inequality in the context of a Burdett-Mortensen (1998) model that is extended to incorporate worker heterogeneity through skill requirements in the production process. In this environment, wage dispersion is a consequence of worker and firm production heterogeneity and frictions in the search process. When there are a continuum of workers and firms we provide sufficient conditions for more productive firms to offer higher wages and characterize this equilibrium when it exists. We use the model to examine recent changes in wage inequality and show that the Autor-Levy-Murnane interpretation of skill-biased technical change is qualitatively consistent with both between- and within-group inequality movements observed in the data.

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1 Introduction

There have been significant changes in both between- and within-group wage inequality in the United States in recent decades.¹ Our starting point is that while the competitive model of the labor market can explain changes in between-group inequality as results of technological progress, it has a harder time capturing within-group inequality. The main reason is that explanations of the evolution of within-group inequality in the competitive model need to appeal to changes in the prices or distribution of unobserved skills. Such explanations are not only inherently difficult to test, but they may not capture how movements in within-group inequality are related to movements in between-group inequality through changes in the technological environment. In contrast, a structural-frictional model of the labor market generates formal predictions of how the within-group wage distribution changes when there are changes in exogenous factors such as the productivity distribution of firms or the degree of frictions in an economy. In such a model it is also possible to link movements in within- and between-group inequality, a link that is underdeveloped in the current literature on wage inequality.

To tackle between- and within-group inequality in a unified framework, we develop a model of labor market with search frictions by generalizing the work of Burdett and Mortensen (1998) in Section 2. Their original model has been successful in explaining wage dispersion - that is, why workers with similar skills are paid differently. However, the assumed homogeneity in worker skills makes the standard Burdett and Mortensen model inappropriate to address issues such as wage inequality where between- and within-group inequality are such important features of the data. To extend their model to a situation with heterogeneity in worker productivity we introduce the concept of skill requirements in the production process. In particular, it is assumed that workers require a certain level of skill to be able to work for certain firms: the greater the productivity of a firm, the greater the required skill level. In other respects the basic model remains unchanged. Firms post wages to attract workers while workers search when unemployed and on the job. The equilibrium behavior of workers in this model is similar to that in the original Burdett and Mortensen model. When unemployed, individuals accept wage offers that exceed their reservation value and when employed they accept wage offers that exceed their current wage. On the demand side of the market, firms optimally post wages to maximize profits by attracting workers.

Our main theoretical contribution is to develop a tractable model of heterogeneous workers and firms in an environment with search frictions. We provide sufficient conditions for the existence of an equilibrium in which more productive firms offer higher wages in a setting with a continuum

¹See for example, Katz and Autor (1999), Acemoglu (2002) or Lemieux (2007) for surveys of the literature.

of worker and firm types. Essentially, the productivity differences between firm types must be large enough. Moreover, if an equilibrium of this type does exist, we characterize the wages offered by firms and the steady state distribution of workers across firm types. These results allow us to describe the expected wage and variance of wages conditional upon worker type. Due to skill requirements, workers with limited skills are restricted to be employed in jobs with relatively low productivity while more skilled workers are employed across a broader variety of firms. This difference in economic opportunities generates between-group inequality. Moreover, regardless of skill type, the exact productivity and wage of a worker depends upon luck to some degree. Due to search frictions, some workers are lucky and find jobs in which their skills are well matched while other workers are placed in jobs for which they are over-qualified. This element of luck in a frictional labor market generates a source of within-group wage inequality that is absent from perfectly competitive models of the labor market.

The theoretical model presented in Section 2 is then used to explain movements in between- and within-group inequality in the post-1990 period in Section 3. This period has been described by Autor, Katz and Kearney (Forthcoming) as one in which the labor market has been polarized. Wages of the highest paid workers and the lowest paid workers have increased while there has been a relative decline in the wages of mediocre jobs. Examining groups defined according to education, the 1990s have been characterized by increasing between-group inequality among skilled workers while between-group inequality among less skilled workers has stabilised or even declined. There are also distinctive patterns associated with residual inequality in this period. Lemieux (2006) notes that within-group wage inequality for skilled workers increased during the 1990s, but decreased for unskilled workers.

Motivated by Autor, Levy and Murnane (2003) and Goos and Manning (2007), we model skill-biased technical change as a hollowing out of the productivity distribution of firms, i.e. there are less firm types closer to the median productivity level and more firms at both extremes. To capture such a change we use specific functional forms in a numerical exercise. The impact of skill-biased technical change upon between-group inequality within this model is standard. The most and the least productive firms become relatively more efficient while the productivity of firms in the middle of the distribution remain stagnant. This change in the distribution of productivity flows through to affect wage offers. Since workers differ in their employment opportunities this creates the polarization observed in the labor market and movements consistent with observed changes in between-group inequality.

More surprising is the effect of skill-biased technical change upon within-group inequality. In our example, skill-biased technical change leads to a decrease in within-group inequality for low-

skilled workers while simultaneously increasing within-group inequality for more skilled workers. This complicated behavior arises because different workers are employed across different types of firms. As the degree of polarization increases, the productivity and wage offers of low to medium productivity firms is compressed. Low skilled workers are employed exclusively at these firms and thus within-group wage inequality decreases for this set of workers. On the other hand, skilled workers are employed across both low and high productivity firms and an increase in skill-bias increases the variance of wages offered by more productive firms. This leads to greater within-group inequality among skilled workers capable of working at a wide variety of firms. These results imply that recent interpretations of skill-biased technical change are capable of generating movements in within-group inequality that are qualitatively consistent with the observed patterns noted by Lemieux (2006).

The most similar paper to this one in terms of emphasizing the role of search frictions and wage determination in explaining wage inequality is Shi (2002). Shi examines the wage setting behavior of firms in response to productivity changes however there are some important differences. Most importantly, Shi examines a static environment that generates within-group wage inequality only among low skilled workers. This paper on the other hand, examines a dynamic model that generates within-group inequality among all workers. This is crucial since the increase in within-group inequality observed in the data is concentrated among skilled workers while Shi's result only generates within-group inequality among unskilled workers. Also related is the work of Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). They study how worker production heterogeneity contributes to wage dispersion when firms have complete information about the characteristics of the workers and wages are determined by open bidding between firms. Our paper differs by retaining the original Burdett and Mortensen assumption of wage posting which seems reasonable in an environment where potential employers have imperfect information regarding the quality and employment status of a worker.

2 The Model

This section describes a model of wage inequality in an environment with search frictions. The economy consists of risk neutral, utility maximizing workers who produce output by forming job matches with profit maximizing firms. Output is divided between worker and firm by a wage rate that the firm posts prior to meeting a worker. The labor market is one in which frictions exists so that matching between workers and firms is a time consuming and stochastic process. Workers search for an acceptable wage while unemployed and for a better wage while on the job.

There exists two-sided heterogeneity in the labor market with firms differing in terms of productivity and workers differing in skill levels. To model this, we define $p \in [0, 1]$ the type of a firm and $P(p)$ the corresponding, atomless, strictly increasing, continuous distribution function of firm types. The amount of revenue produced by a firm of type p that employs a worker is denoted as $R(p)$ where R is a strictly increasing, continuously differentiable function and $R(0) = 0$. Heterogeneity among workers is introduced by defining $v \in [0, 1]$ the type of a worker and $V(v)$ the corresponding atomless, strictly increasing continuous distribution function of worker types. To incorporate skill requirements in production we assume that a worker of type v is capable of working for a firm of type p only if $v \geq p$. Note that the productivity of a firm that is matched with a worker does not depend upon worker type but the set of workers that are capable of matching with a firm depends upon the level of productivity.

Using the simplest environment for theoretical tractability, we assume that workers meet firms randomly with an arrival rate of λ which is independent of employment status or skill type. The random nature of the search process implies that when a worker of type v meets a firm, there is a probability of $P(v)$ that the worker will have the necessary skills to form a job match. In this case the worker is offered a wage that may either be accepted or rejected. With a probability of $1 - P(v)$ the worker has an insufficient skill level to work at the firm and will not receive a wage offer. The equilibrium behavior of workers in this environment is easy to describe. Since the rate of matching is independent of job status, unemployed workers will accept job offers that post a wage in excess of their reservation value which for simplicity we will assume to be zero. Employed workers will accept wage offers that exceed their current wage and reject other offers.² Finally, the unit of time is normalized so that the arrival rate of job offers, λ , is equal to one and job matches are destroyed at an exogenous rate of δ .

The focus of our theoretical work is upon determining sufficient conditions for the existence of a monotone equilibrium in which the wage rate of a firm of type p , $w(p)$, is strictly increasing in p and if such an equilibrium exists, it is characterized. Let

$$H_v(w) = Pr(\text{wage} \leq w | \text{type} = v)$$

be the distribution function of the wage (in a stationary equilibrium) of a type v worker. Since we consider equilibrium in which $w(p)$ is strictly increasing, the following notation can be introduced:

$$T_v(p) = Pr(\text{wage} \leq w(p) | \text{type} = v).$$

²A formal discussion of the equilibrium behavior of workers is provided in the original Burdett and Mortensen (1998) and the interested reader is referred to their paper for full details.

This describes the probability that a worker of type v has a wage less than $w(p)$.³ Analogous to the Burdett and Mortensen model, the law of motion is such that

$$\dot{T}_v(p) = (1 - T_v(p))\delta - T_v(p)(P(v) - P(p))$$

when $v \geq p$ and $\dot{T}_v(p) = 0$ otherwise. The first term defines the flow into unemployment of workers of type v receiving a wage greater than $w(p)$, while the second term describes the flow of type v workers from wages below $w(p)$ to wages above. In the steady state of a monotone equilibrium, it follows that

$$T_v(p) = \frac{\delta}{\delta + P(v) - P(p)} \quad (1)$$

if $v > p$ and $T_v(p) = 1$ if $v < p$ since a worker of type v will never obtain an offer above $w(v)$ which is less than $w(p)$. This equation describes the steady state distribution of workers across firm types and follows from the fact that there is always efficient hiring conditional upon frictions.

Note that the probability of being unemployed for a type v worker, $u(v)$ is such that

$$u(v) = H_v(0) = T_v(0) = \frac{\delta}{\delta + P(v)}.$$

which implies that more skilled workers have lower unemployment rates, consistent with empirical evidence. The total level of unemployment in the economy is given by the following,

$$U = \int_0^1 \frac{\delta V'(v)}{\delta + P(v)} dv.$$

The above analysis describes the distribution of workers across different firms and employment states. We now focus upon the wage-setting decision of a firm and begin by describing the amount of labor that a firm will be able to employ in a monotone equilibrium. Note that a firm of type p will be able to hire workers of type $v \in [p, 1]$. In the steady state of a monotone equilibrium, the distribution of workers across firm types is given by $T_v(p)$ from equation (1). This implies that conditional upon meeting an arbitrary worker, a firm of type p that offers a wage of $w(\hat{p})$, will hire a worker with probability

$$\int_p^1 \frac{V'(v)\delta}{\delta + \max\{P(v) - P(\hat{p}), 0\}} dv.$$

Conditional upon accepting employment, the expected time that a worker of type v remains with a firm offering a wage of $w(\hat{p})$ depends upon the separation rate. Job matches are exogenously destroyed at a rate of δ and are endogenously destroyed when a worker receives a better offer which occurs at a rate of $P(v) - P(\hat{p})$ if $v > \hat{p}$ and zero otherwise. This implies that conditional upon

³Note, that $H_v(w)$ and $T_v(p)$ incorporate unemployed workers and are *not* distribution functions associated with wages conditional upon employment.

employment, the expected time that a worker of type v remains employed at a firm that offers a wage of $w(\hat{p})$ is

$$\frac{1}{\delta + \max\{P(v) - P(\hat{p}), 0\}}.$$

Combining these implies that a firm of type p that offers a wage $w(\hat{p})$ in a monotone equilibrium will expect to hire workers for the time length,

$$M(p, \hat{p}) = \begin{cases} \int_p^1 \frac{V'(v)\delta}{(\delta + P(v) - P(\hat{p}))^2} dv & \text{for } p > \hat{p} \\ \int_{\hat{p}}^1 \frac{V'(v)\delta}{(\delta + P(v) - P(\hat{p}))^2} dv + \int_p^{\hat{p}} \frac{V'(v)\delta}{\delta^2} dv & \text{for } p < \hat{p}. \end{cases}$$

Using this result the profit function of a firm with type p who offers a wage $w(\hat{p})$ can be written as:

$$\pi(p, \hat{p}) = (R(p) - w(\hat{p}))M(p, \hat{p}),$$

where we have assumed for simplicity that the discount rate of future profits is zero.

The first order condition implies that in equilibrium

$$(R(p) - w(p))M^{(2)}(p, p) - w'(p)M(p, p) = 0$$

or equivalently,

$$w'(p) = (R(p) - w(p)) \frac{M^{(2)}(p, p)}{M(p, p)} \quad (2)$$

where $M^{(2)}(p, \hat{p})$ is the derivative of M with respect to the second variable. As in the standard Burdett and Mortensen model, search frictions imply that firms have a degree of market power and set their wages as in a monopolistically competitive market.⁴ In particular, wages are set to balance the ability to attract workers with the desire to retain as much surplus as possible from match formation. Changes in the wages offered by firms arise due to changes in the underlying distribution of workers or firms, or via changes in the revenue function.

If a monotone equilibrium exists, profit maximization implies $w(p)$ must satisfy the differential equation implied by the first order condition in equation (2). Furthermore, since $R(0) = 0$ it is also known that $w(0) = 0$. Defining $\tau(p) = \frac{M^{(2)}(p, p)}{M(p, p)}$, the solution to this first order linear differential equation with accompanying boundary condition is given by the following,

$$w(p) = \frac{\int_0^p R(x)\tau(x) \text{Exp}\left(\int_0^x \tau(u)du\right) dx}{\text{Exp}\left(\int_0^p \tau(u)du\right)}. \quad (3)$$

⁴Perhaps the easiest way to see this is to note that the first order condition for wages can be rewritten as

$$\frac{R(p) - w(p)}{w(p)} = \frac{w'(p)}{w(p)} \frac{M(p, p)}{M^2(p, p)}$$

which is the standard mark-up pricing formula associated with monopolistic competition.

This describes the equilibrium wages posted by firms as a function of productivity when a monotone equilibrium exists. The sufficient second-order conditions that ensure existence are

$$\begin{aligned}\pi^{(2)}(p, \hat{p}) &< 0 \quad \text{if } p < \hat{p} \\ \pi^{(2)}(p, \hat{p}) &> 0 \quad \text{if } p > \hat{p}.\end{aligned}$$

The first condition implies that a firm of type p that deviates to a higher wage, $w(\hat{p})$, has an incentive to reduce the wage offer towards $w(p)$. The second condition implies that a firm of type p that deviates to a lower wage has an incentive to increase the offered wage. To facilitate the analysis, it is assumed that $V(x) = P(x)$, that is the distributions of worker skill levels coincides with the distribution of firm skill requirements. Then, one may assume that $V(x) = P(x) = x$, without loss of generality.⁵ The appendix proves the following proposition.

Proposition 1. *When types are normalized such that $V(x) = P(x) = x$ and $R(0) = 0$, a sufficient condition for the existence of a unique monotone equilibrium is that $R(p)$ is a (weakly) convex function.*

It can be verified in some cases that a monotone equilibrium will not exist. This potential lack of existence arises because different firms face a different set of potential employees when making wage offers. Firms operate in an environment with imperfect competition and their offered wage depends upon their productivity and the elasticity of labor supply. Since firms are hiring different sets of workers, the elasticity of labor varies across firms and more productive firms may face a smaller elasticity. This may generate an incentive for more productive firms to offer lower wages, an effect which is absent from models where the form of the ex post competition is Bertrand (as in the work of Postel-Vinay and Robin (2002)) or in models where workers are homogenous (as in the original work of Burdett and Mortensen (1998)). In the above proposition, convexity of $R(p)$ ensures that the productivity differences between firms are large enough so that a monotone equilibrium exists.

When a monotone equilibrium exists, our previous results characterize the equilibrium outcome. Workers are distributed across firm types according to equation (1) and the equilibrium wage offered

⁵To see this start with a model with general (but equal) distribution functions. Then the distribution function of the maximum production a worker is capable of is

$$\Pr(R(v) \leq y) = V(R^{-1}(y)).$$

Then consider a model where the distributions are uniform and the production function is

$$\tilde{R}(p) = R(V^{-1}(p)).$$

Then

$$\Pr(\tilde{R}(v) \leq y) = \tilde{R}^{-1}(y) = V(R^{-1}(y)),$$

which implies that the two specifications describe the same model just rescaling variables.

by firm type is defined by equation (3). Combining these results implicitly defines the multivariate distribution of worker types across wages. It follows that changes in the distribution of wages arise either due to a change in the manner in which workers are distributed across firms or alternatively due to a change in the wage setting policies of firms or a combination of both of these factors.

The introduction of worker heterogeneity in the form of skill requirements leads to both between- and within-group inequality in this economy. Groups are naturally defined according to worker types. In this model, more skilled workers are more likely to be employed at firms with a higher productivity and hence wage rate. This generates a source of between-group inequality. Furthermore, due to frictions, workers of identical skill level differ in the quality of firms that they meet and the wages they receive. This produces a source of within-group inequality that varies depending upon worker type. Unlike perfectly competitive models, this within-group inequality does not represent unobserved heterogeneity but rather reflects the importance of luck in the labor market.

3 Skill-Biased Technical Change and Wage Inequality

The previous section derives an equilibrium wage distribution in the presence of a continuum of heterogeneous firms and workers. Wage dispersion arises as a consequence of labor market heterogeneity and the existence of search frictions. This section examines the comparative statics associated with skill-biased technical change. Special attention is devoted to understanding how changes in productivity can generate the observed changes in between- and within-group inequality.

We begin by briefly reviewing the evolution of between-group wage inequality since 1980.⁶ Table 1 summarizes wage differentials by education relative to high school graduates over time. These differentials are constructed from the Current Population Survey data by using a log wage regression with education levels represented by dummy variables and controls included for experience, marital status and race.⁷ A number of features are apparent. Firstly, the changes in wage differentials have been large; the male college wage premium (relative to high school graduates) has increased from 0.29 in 1980 to 0.49 in 2000. It is also notable that much of the rise in inequality is concentrated in the 1980s. Of the 0.2 increase in log wage points of the male college wage premium, about 0.15 or 75 per cent occurred during the 1980s. The changes in between-group wage inequality in the 1990s, in addition to being smaller, were less pervasive and focused upon more skilled workers.

⁶Changes in wage inequality have been the focus of a large empirical literature. More complete surveys are provided by Katz and Autor (1999) and more recently by Lemieux (2007).

⁷The underlying CPS data is from the Merged Outgoing Rotation Groups and is adjusted to take account of top-coding and the sample is restricted to exclude outliers. See Lemieux (2006) for specific details of the adjustment process.

Table 1: Between-Group Wage Inequality

	Males			Females		
	1980	1990	2000	1980	1990	2000
High School Dropouts	-0.26	-0.30	-0.32	-0.22	-0.28	-0.28
Some College	0.09	0.15	0.12	0.12	0.19	0.16
College Graduates	0.29	0.44	0.49	0.31	0.49	0.53
Post-graduates	0.37	0.58	0.65	0.50	0.68	0.76

Measures of between-group inequality relative to high school graduates. Controls for experience, race and marital status.

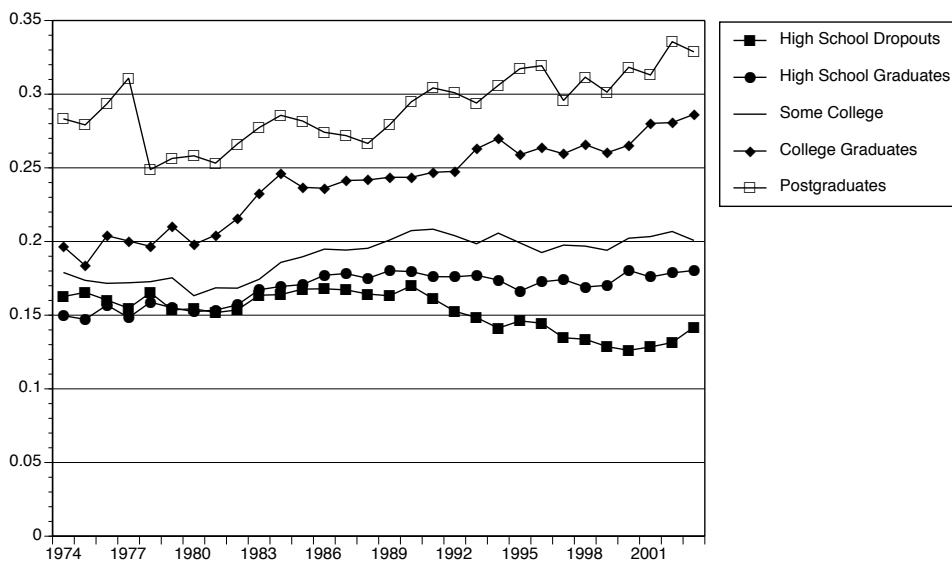


Figure 1: Within-group inequality with groups defined according to education. Graph constructed from data and programs made available by Thomas Lemieux (2006).

Although the education wage premium increased for both male and female college graduates and post-graduates in the 1990s, there was a decrease in the wage premium associated with “some college” education. Finally, the magnitude of the wage differential between high school dropouts and high school graduates remained stable for females and increased marginally for males.

Lemieux (2006) details the changing pattern of within-group wage inequality. His results regarding the evolution of male within-group inequality are reproduced here in Figure 1. After a period of stability during the 1970s, within-group inequality increased rapidly for all groups during the early 1980s. From 1990 a new pattern emerges; within-group inequality for skilled workers (college or postgraduate education) has steadily increased while it has generally remained constant or decreased for less skilled workers (high school dropouts, high school graduates and some college).⁸

⁸Although Lemieux also examines the evolution of total residual inequality in his paper, we focus on within-group inequality on a group-by-group basis since this is a more informative measure of the evolution of wage inequality.

We focus upon explaining wage inequality in the post-1990 period.⁹ During this period the wage premium for college education increases while wage differentials associated with low levels of education remained relatively constant or decreased in some cases. Simultaneously, there is a divergence in within-group inequality across education groups. We show within our model that these patterns of changing wage inequality are consistent with the recent view of skill-biased technical change proposed by Autor, Levy and Murnane (2003). They suggest that computers are complementary to workers performing nonroutine cognitive tasks, substitutable with workers performing manual routine tasks and have little effect upon workers involved in nonroutine manual tasks. In an approach similar to Autor, Katz and Kearney (2006) we view that recent technological change has increased the productivity of the most productive jobs, reduced the productivity of midrange jobs and had little impact upon low productivity jobs. Considering a productivity change of this sort within our model generates an increase in within-group inequality of skilled workers and a decrease in the within-group inequality of low skilled workers; consistent with the empirical evidence presented by Lemieux (2006).

It is difficult to derive general results in our framework so we focus upon specific functional forms. Our starting point, in formally relating our model to the work of Autor, Levy and Murnane (2003) is to note that changes in the revenue function, $R(p)$, represent changes in productivity.¹⁰ An improvement in technology for firm type p is represented as an increase in $R(p)$. Skill-biased technological change is loosely thought of, as a change in which $R(p)$ becomes more convex. With this in mind, we capture a large degree of complementarity between technology and skills with the following functional form,

$$R(p) = \frac{1 + (1 - \alpha)p^{1/2} + (1 + \alpha)p^2}{\int_0^1 (1 + (1 - \alpha)p^{1/2} + (1 + \alpha)p^2) dp} \quad (4)$$

where $-1 < \alpha < 1$. As α increases, greater weight is placed upon the convex term p^2 and less weight is placed upon the concave term, $p^{1/2}$. It is natural to think of higher values of α corresponding to situations in which technology is more complementary with skills. Also note, that regardless of the value of α , the average productivity of firms is normalized to equal unity. Finally to help ensure a monotone equilibrium exists we make the additional assumption that the reservation wage of a worker is equal to $R(0)$.

To proceed, we numerically approximate the above model. In particular, we discretize the distribution of workers and firms so that there are n types of firms and n types of workers. The distribution

⁹Our model is also able to provide results qualitatively consistent with the growth in between- and within-group inequality over the period from 1980-1990. This is easy to achieve with a proportional increase in productivity of all firms. We focus upon the post-1990 period since it provides more novel results.

¹⁰More generally, we can think of changes in $R(p)$ as representing changes in productivity or changes in demand that affect the price of output of different firms.

of workers across firm types is provided by (1). A numerical approximation of wages offered by each firm type is derived using equation (2). Combining the distribution of workers across firm types and the wages offered by firms allows us to approximate the expected wage and the variance of wages associated with each type or group of worker. In our simulations, we consider a fine grid of $n = 1000$. Attention is restricted to cases in which $P(x) = V(x) = x$ for $x \in [0, 1]$. The matching rate is normalized to one and we set $\delta = 0.1$. In these examples the revenue function is nonconvex and hence does not satisfy the requirements of Proposition 1; we verify a monotone equilibrium exists by considering deviations of each type of firm. In particular, we calculate the profit that a firm of type j would earn if it offered the wage of a firm type j' . In our calculations, we find that no firm has an incentive to deviate to an alternative wage policy.

To examine the effects of skill-biased technological change we compare the equilibrium wage distribution in an economy with a low degree of skill-bias ($\alpha = -1/2$) to one with a high degree of skill-bias ($\alpha = 1/2$). The productivity and wages offered by firms, as a function of α , are presented in Figure 2. The left-hand panel shows the productivity of each firm type for both values of α . As α increases, there is a polarization of productivity in the economy. Firms at the upper and the lower end of the productivity distribution experience productivity increases while firms towards the middle of the distribution face productivity declines. The right-hand panel shows the corresponding equilibrium wage offered by firm type. Unsurprisingly, there is a close relationship between the revenue function and wages offered.

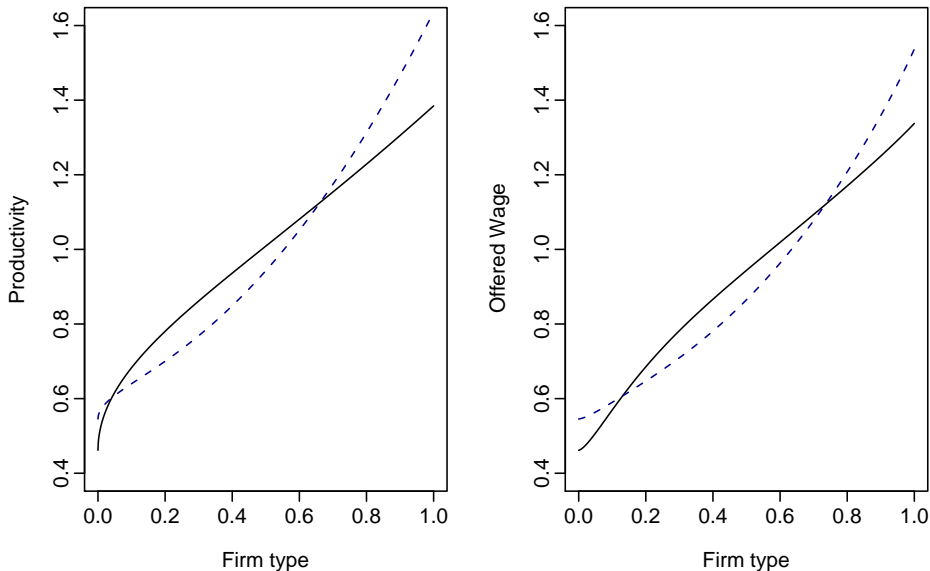


Figure 2: Productivity and wages offered by firm type. Solid line displays case of $\alpha = -0.5$. Dashed line displays case of $\alpha = 0.5$.

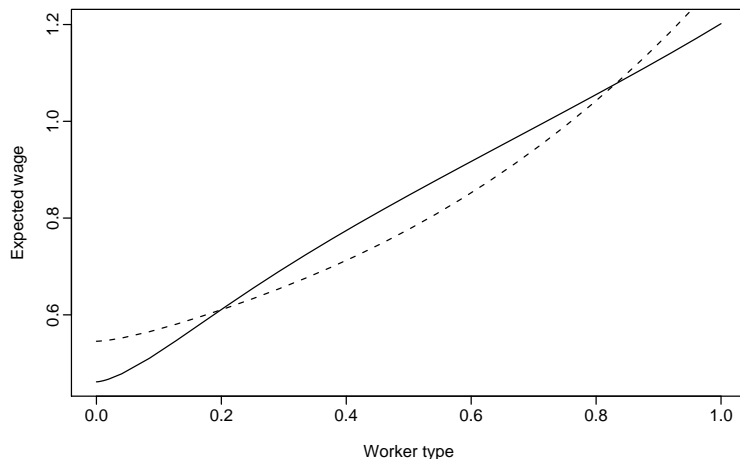


Figure 3: Skill-Biased Technological Change and Expected Wages: Solid line represents $\alpha = -0.5$. Dashed line is for $\alpha = 0.5$.

At least within this model, skill-biased technical change alters the wages offered by firms without altering the distribution of workers across firm types.¹¹ This change in the equilibrium wages offered has an effect upon the distribution of wages received and consequently affects both between- and within-group inequality. Between-group inequality reflects differences in the average wage across worker types. The expected wage conditional upon worker type for different values of α are presented in Figure 3. As α increases the revenue function and consequently, the expected wage as a function of worker type becomes more convex. This results in an increase in the expected wage of the most and least skilled workers and a decrease in the expected wages of mediocre workers. In this model with a continuum of types, no simple measure of between-group inequality exists and whether between-group inequality increases or decreases depends upon the exact worker groups considered. However, the pattern of changes in expected wages is consistent with the recent literature documenting the polarization of the labour force as discussed in Autor, Katz and Kearney (2006) and Lemieux (2007).

We also examine the impact of skill-biased technological change on within-group inequality. Figure 4 displays the variance of wages conditional upon worker type which provides a measure of within-group inequality. There is a complicated interaction between the degree of capital-skill complementarity and within-group inequality. An increase in α leads to a decrease in within-group inequality for low-skilled workers while simultaneously leading to an increase in within-group inequality for more skilled workers. This complicated behavior arises because different workers are employed across different types of firms. As α increases, the productivity and wage offers of low

¹¹The distribution of workers across firms could be reasonably altered by adding an endogenous response of vacancies using a zero profit condition. See Mortensen (2003) for a discussion of endogenizing firm entry.

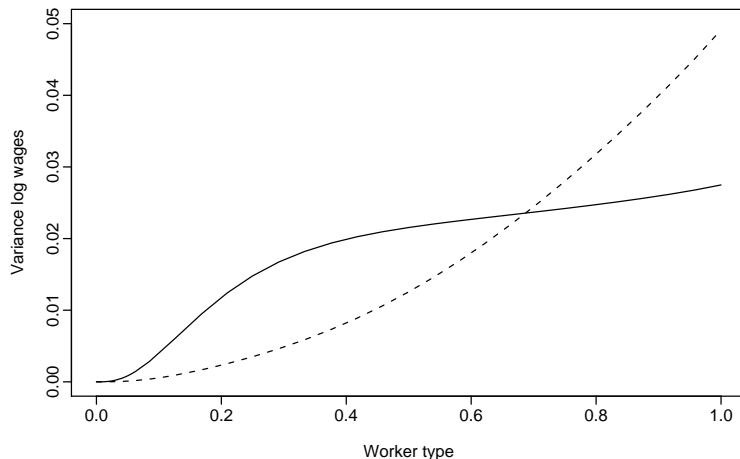


Figure 4: Skill-Biased Technological Change and the Within-Group Inequality: Solid line represents $\alpha = -0.5$. Dashed line is for $\alpha = 0.5$.

productivity firms is compressed. Low skilled workers find employment at these firms and thus within-group wage inequality decreases for this set of workers. Skilled workers are employed across both low and high productivity firms and an increase in α increases the variance of wages offered by more productive firms. This leads to greater within-group inequality among skilled workers capable of working at these productive firms. Hence, at least in this illustrative example, skill-biased technological change tends to increase the average wage of more skilled workers and have a differential impact upon within-group inequality that depends upon skill type. These movements of within-group inequality implied by skill-biased technical change are qualitatively consistent with the changes noted by Lemieux (2006) and reproduced here in Figure 1.

The above example illustrates in a manner similar to Autor, Katz and Kearney (2006) that the recent interpretations of skill-biased technical change are able to generate between-group inequality movements consistent with the observed data. Going beyond their work, this paper also shows that skill-biased technical change is also able to explain detailed movements in within-group wage inequality. Crucial to our results is the presence of heterogeneity which allows discussion of inequality based upon groups, but also search frictions which provide a source of luck. A key aspect of our frictional model of wages, is that changes in technology have an impact upon both between-group inequality but also within-group inequality in a well-defined manner.

4 Conclusion

This paper demonstrates that incorporating a reasonable mechanism for wage determination in a labor market with frictions helps provide a plausible explanation of changes in the wage distribution in the United States. This is shown by extending the Burdett and Mortensen model to incorporate heterogeneity among workers in the form of skill requirements, enabling discussion of within- and between-group wage inequality. This environment allows firms to adjust their wages optimally to productivity shocks and creates rich comparative statics for both between- and within-group inequality. Unlike standard competitive models of the labor market, frictions in the matching process generate within-group inequality that is a consequence of luck rather than unobserved heterogeneity.

From a theoretical perspective, this paper incorporates worker production heterogeneity via the introduction of skill requirements into the Burdett and Mortensen model. We provide sufficient conditions for the existence of a monotone equilibrium in this model. Furthermore, when a monotone equilibrium does exist we characterize the properties of the steady state equilibrium. In particular, the distribution of workers across firm types and employment states is derived and the equilibrium wages posted by firms is described as the solution to a first-order linear differential equation with appropriate boundary condition. These results allow us to discuss in detail the distribution of wages and examine both between- and within-group inequality.

The model is then used to provide insights into the recent evolution of wage inequality. An illustrative example demonstrates how a recent view of skill-biased technological change may increase the within-group inequality of skilled workers while decreasing that of unskilled workers. Motivated by the work of Autor, Levy and Murnane (2003), skill-biased technical change is modeled as a “hollowing out” of the productivity distribution of firms and generates the qualitative changes in between- and within-group wage inequality that are consistent with the patterns observed in the data and discussed by Lemieux (2006).

5 Appendix

Proof of Proposition 1

The proposed monotone equilibrium is characterized by a function $w(p)$, that solves the differential equation (2) with boundary condition $w(0) = 0$. This section shows that when $V(x) = P(x) = x$

and $R(p)$ is convex, that a monotone equilibrium exists.

All we need to do is to check that global second order conditions are satisfied for this candidate equilibrium. For checking the second order conditions it is sufficient to prove that

$$\begin{aligned}\pi^{(2)}(p, \hat{p}) &< 0 \quad \text{if } p < \hat{p} \\ \pi^{(2)}(p, \hat{p}) &> 0 \quad \text{if } p > \hat{p}.\end{aligned}$$

For example, the second condition means that a high type always has a marginal incentive to increase a bid from $w(\hat{p})$, which is sufficient to rule out that there is an incentive to deviate downwards. The first condition works in the opposite direction.

Let us check the first condition. Then,

$$\begin{aligned}\pi^{(2)}(p, \hat{p}) &= -w'(\hat{p})M(p, \hat{p}) + (R(p) - w(\hat{p}))M^{(2)}(p, \hat{p}) \\ &= (R(p) - R(\hat{p}))M^{(2)}(p, \hat{p}) + w'(\hat{p}) \left(\frac{M(\hat{p}, \hat{p})}{M^{(2)}(\hat{p}, \hat{p})} M^{(2)}(p, \hat{p}) - M(p, \hat{p}) \right)\end{aligned}$$

using the first order condition to eliminate $w(\hat{p})$.

Then for $\hat{p} > p$,

$$M(p, \hat{p}) = \int_p^{\hat{p}} \frac{1}{\delta} dv + \int_{\hat{p}}^1 \frac{\delta V'(v)}{(\delta + P(v) - P(p))^2} dv > M(\hat{p}, \hat{p})$$

and

$$M^{(2)}(p, \hat{p}) = \int_{\hat{p}}^1 \frac{2\delta P'(p)V'(v)}{(\delta + P(v) - P(p))^3} dv + \int_p^{\hat{p}} 0 dv = M^{(2)}(\hat{p}, \hat{p})$$

implies,

$$\left(\frac{M(\hat{p}, \hat{p})}{M^{(2)}(\hat{p}, \hat{p})} M^{(2)}(p, \hat{p}) - M(p, \hat{p}) \right) < 0$$

Note that $w'(\hat{p}) > 0$, $M^{(2)}(p, \hat{p}) > 0$ and $(R(p) - R(\hat{p})) < 0$ implies $\pi^{(2)}(p, \hat{p}) < 0$

Now, consider the second condition,

$$\pi^{(2)}(p, \hat{p}) = (R(p) - R(\hat{p}))M^{(2)}(p, \hat{p}) + w'(\hat{p}) \left(\frac{M(\hat{p}, \hat{p})}{M^{(2)}(\hat{p}, \hat{p})} M^{(2)}(p, \hat{p}) - M(p, \hat{p}) \right)$$

Showing $\pi^{(2)}(p, \hat{p})$ is positive is equivalent to showing that $\frac{\pi^{(2)}(p, \hat{p})}{M(p, \hat{p})}$ is positive since $M^{(2)}(p, \hat{p})$ is always positive.

Note that

$$\begin{aligned}\frac{\pi^{(2)}(p, \hat{p})}{M(p, \hat{p})} &= (R(p) - w(\hat{p})) \frac{M^{(2)}(p, \hat{p})}{M(p, \hat{p})} - w'(\hat{p}) \\ &= (R(p) - R(\hat{p})) \frac{M^{(2)}(p, \hat{p})}{M(p, \hat{p})} + w'(\hat{p}) \left(\frac{M^{(2)}(p, \hat{p})}{M(p, \hat{p})} \frac{M(\hat{p}, \hat{p})}{M^{(2)}(\hat{p}, \hat{p})} - 1 \right)\end{aligned}$$

At this point we use the assumption that $V(x) = P(x) = x$ for all x . Then after calculating integrals we obtain:

$$\frac{M^{(2)}(p, \hat{p})}{M(p, \hat{p})} = \frac{\int_p^1 \frac{2\delta P'(v)P'(\hat{p})}{(\delta+P(v)-P(\hat{p}))^3} dv}{\int_p^1 \frac{\delta P'(v)}{(\delta+P(v)-P(\hat{p}))^2} dv} = \frac{2\delta + 1 + p - 2\hat{p}}{(\delta + 1 - \hat{p})(\delta + p - \hat{p})}.$$

Also,

$$\frac{M(\hat{p}, \hat{p})}{M^{(2)}(\hat{p}, \hat{p})} = \frac{\delta(\delta + 1 - \hat{p})}{(2\delta + 1 - \hat{p})}$$

Using these in the above implies

$$\begin{aligned}\frac{\pi^{(2)}(p, \hat{p})}{M(p, \hat{p})} &= (R(p) - R(\hat{p})) \frac{2\delta + 1 + p - 2\hat{p}}{(\delta + 1 - \hat{p})(\delta + p - \hat{p})} - w'(\hat{p})(p - \hat{p}) \frac{\delta + 1 - \hat{p}}{(2\delta + 1 - \hat{p})(\delta + p - \hat{p})} \\ &\geq \frac{1}{\delta + p - \hat{p}} \left((R(p) - R(\hat{p})) \frac{2\delta + 1 - \hat{p}}{\delta + 1 - \hat{p}} - w'(\hat{p})(p - \hat{p}) \frac{\delta + 1 - \hat{p}}{2\delta + 1 - \hat{p}} \right)\end{aligned}$$

since we assume $p > \hat{p}$. Thus it is sufficient to show that for all $p > \hat{p}$, that

$$\left((R(p) - R(\hat{p})) \frac{2\delta + 1 - \hat{p}}{\delta + 1 - \hat{p}} - w'(\hat{p})(p - \hat{p}) \frac{\delta + 1 - \hat{p}}{2\delta + 1 - \hat{p}} \right)$$

is positive.

Condition (2) implies that for all p ,

$$w'(p) = (R(p) - w) \frac{2\delta + 1 - p}{\delta(\delta + 1 - p)}$$

Substituting for $w'(\hat{p})$ into the above expression yields the following:

$$(R(p) - R(\hat{p})) \frac{2\delta + 1 - \hat{p}}{\delta + 1 - \hat{p}} - (p - \hat{p}) \frac{R(\hat{p}) - w(\hat{p})}{\delta}$$

Thus to show that $\pi^{(2)}(p, \hat{p})$ is positive, it is sufficient to show that

$$\frac{R(p) - R(\hat{p})}{p - \hat{p}} - \frac{\delta + 1 - \hat{p}}{2\delta + 1 - \hat{p}} \frac{R(\hat{p}) - w(\hat{p})}{\delta} \geq 0$$

We concentrate on the case when firms with higher type have sufficient incentives to bid more than firms with lower types, i.e. a monotone equilibrium exists. From the analysis of the two-type case one suspects that this holds when the productivity of higher type firms are much higher than that of the lower types. In a continuous type space model this condition can be captured by assuming that the marginal productivity from increasing p is increasing, i.e. that R is a convex function. Under that assumption

$$\frac{R(p) - R(\hat{p})}{p - \hat{p}} \geq R'(\hat{p})$$

and thus it is sufficient to show that

$$R'(\hat{p}) \geq \frac{\delta + 1 - \hat{p}}{2\delta + 1 - \hat{p}} \frac{R(\hat{p}) - w(\hat{p})}{\delta}.$$

To prove that this condition holds, first note that equation (2) implies that

$$R(x) - w(x) \leq R'(x)\delta$$

for all x .¹² Then it follows that

$$R'(\hat{p}) > \frac{\delta + 1 - \hat{p}}{2\delta + 1 - \hat{p}} R'(\hat{p}) \geq \frac{\delta + 1 - \hat{p}}{2\delta + 1 - \hat{p}} \frac{R(\hat{p}) - w(\hat{p})}{\delta}$$

which shows that our condition is satisfied for any δ .

¹²To see this observe that

$$R(0) - w(0) = 0$$

and whenever

$$R(x) - w(x) = \delta R'(x)$$

holds it follows that

$$w'(x) = R'(x) \frac{2\delta + 1 - x}{\delta + 1 - x} > R'(x).$$

Then at that point $R - w$ is decreasing, while the right hand side, $\delta R'$ is increasing because R is convex.

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