Estimating the Natural Rate of Interest: A SVAR Approach

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Abstract

For the successful conduct of monetary policy the central bank needs reliable indicators of the monetary policy stance. A recently often advocated one is the gap between the real, market and the natural rate of interest.

In this article we estimate the historical time series of the natural rate of interest using a structural vector autoregressive model. This method returns plausible results and thus seems to be well designed for the estimation of the natural rate of interest. We show that the natural rate exhibits quite substantial variability over time, of comparable magnitude to the variability of the real interest rate. We also find that it is a procyclical variable. We conclude that the gap between the natural and real market interest rates can be considered a useful, although not perfect, indicator of the stance of monetary policy.

Keywords: natural rate of interest, interest rate gap, monetary policy, SVAR.

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1. Introduction

The idea of defining a neutral level of interest rates is not a new one. Most economists ascribe the term “natural rate of interest (NRI)” to the Swedish economist Knut Wicksell. However, the first descriptions of the economic processes following a mismatch between the market rate and the “equilibrium rate of interest” can be tracked back as far as to T.Joplin and H.Thornton (Humphrey 1993). Today, after almost 100 years of *desinteressement*, economists again show much interest in the idea of the natural rate of interest. The main reason is probably its good applicability to the monetary policy regime of direct inflation targeting, which many countries adopted in the recent decade.

For an inflation targeter it is important to have a good definition of neutral monetary policy, and consequently of the neutral level of its instrument. A reasonable one seems to be defining neutral policy as the one that will stabilize inflation in the horizon of monetary transmission (approximately 2 years). Respectively restrictive and loose policy can be defined as such that will lead to a decrease or increase of the rate of inflation in a comparable time horizon.

These definitions can be extended to the instrument of monetary policy – the interest rate. Although central banks directly control money market nominal rates, it is well known that, in the absence of nominal illusion, real rates govern private expenditure, aggregate demand and inflation. Thus, throughout the article it will be assumed that central bankers are aware of this fact, and that they control nominal rates so as to hit a path for the short-term real interest rate. **Consequently, we will define the natural rate of interest as such level of the real interest rate that will make monetary policy neutral and thus, stabilize inflation. This definition seems to be very close to what is considered as “neutral level of interest rates” in monetary policy rules. In other words we will concentrate on estimating such level of the monetary policy instrument that is compatible with stabilizing the inflation rate in the horizon of monetary transmission.**

This is obviously not the only possible definition. For instance Wicksell (1907) assumed that the real rate would equalize *ex ante* domestic saving with investment and thus stabilize the general price level (i.e. bring inflation to zero). In a recent paper E.Nelson and K.Neiss (2001) defined the natural rate as the flexible price equilibrium level of the real rate. J.Chadha and C.Nolan (2001) estimated the NRI on the basis of a general equilibrium model as equal to the marginal product of capital. On the other hand, M.Woodford (2001) presented forward-looking models, where inflation depends on the discounted sum of all future interest rate gaps and J.Amato (2001) presents a comprehensive overview of the implementation of Wicksellian ideas in New Keynesian models. The definition suggested in our paper has been recently advocated by some economists: A.Blinder (1998), J.Fuhrer and R.Moore (1995) or T.Laubach and J.C.Williams (2001), J.Archibald and L.Hunter (2001) and myself (M.Brzoza-Brzezina 2002) to name a few. Although it definitely differs from the original Wicksellian concept of the NRI, which was based rather on long-term interest rates, in our opinion it is of more practical use for a central bank, which needs reliable indicators respective to its instrument (short-term interest rate).

Estimation of the so-defined NRI could give a powerful tool to monetary policy. Consider a central bank that plans to disinflate. If it knows the current level of the NRI, the only thing the bank has to do is to raise real rates above the NRI and wait until inflation comes down. When the rate of inflation approximates the desired level, monetary authorities simply return interest rates into their neutral position so that inflation stabilizes. In particular the last operation could be significantly simplified by the knowledge of the NRI. It is relatively simple to raise rates above neutral and thus, subdue inflation. On the contrary, it is very difficult to find their neutral level and thus to terminate a disinflation smoothly, without risking reflating the economy.

Certainly the above outlined model is extremely simplified as we can only dream of heaving up-to-date precise estimates of the dynamic relationship between the interest rate gap and inflation and of the NRI itself. This is, however, the problem with quite a number of the monetary policy stance indicators (money gap (P-star), output gap, NAIRU), as they are often based on an unobservable variable. The rate of monetary expansion is a good example. Its usefulness for the central bank is determined by the behavior of money demand, if it is stable (or changes in a predictable way), money growth is a good indicator, if not - it is useless. The NRI represents a similar case. The main precondition for the gap between the real and natural rate, to yield helpful information about the monetary policy stance, is our ability to calculate and predict the future behavior of the NRI. Since it is an unobservable variable, the gap will be easier to forecast and hence more useful for monetary policy purposes the lower the relative variance of the natural rate and the real rate (Neiss, Nelson 2001). Still, even if this would not be the case, the NRI concept can be considered a useful tool for historical analysis and educational purposes.

This study is devoted to estimating the historical time series of the natural rate of interest in the US over the period 1960-2002 using a structural VAR model. A similar exercise has been recently done by T.Laubach and J.C.Williams (2001) by means of a state space model. One can hope that using various techniques to achieve the same goal can help in achieving a consensus about the performance of the NRI. How difficult it may prove, can be seen on the basis of the Neiss, Nelson (2001) and Chadha, Nolan (2001) studies. They both estimated the NRI as the equilibrium real rate within a calibrated GE model of the UK economy, but their results are completely different.

The rest of the paper is structured as follows. The SVAR used to estimate the NRI is described in section 2. Section 3 presents the data, describes and analyzes the results. Section 4 concludes.

2. The SVAR methodology

Structural VAR models have been recently used by many economists to recover the historical time series of unobservable variables. A popular technique is based on the methodology of imposing long-run restrictions proposed by O.J.Blanchard and D.Quah (1989) to estimate potential output[2]. The application of a similar technique to estimating the natural rate of interest will be described in this section. The major innovation to the Blanchard – Quah method is that we replace the orthogonality assumption with respect to the shocks with a short-run restriction. In our view, such a specification is less restrictive and allows for greater flexibility of the system.

Let us start with the definition of the interest rate gap:

(1) $$GAP \equiv r - r^*,$$

where r^* and r are respectively the natural and real rates of interest. This can be transformed to:

(2) $$r = r^* + GAP.$$

[2] For other good descriptions of the method see W.Enders (1995) or I.Claus (1999).

Further we assume that both, the natural rate and the interest rate gap follow stationary, autoregressive processes:

(3) $$r_t^* = \Phi_1(L) r_{t-1}^* + u_{1,t} = \Xi_1(L) u_{1,t}$$

(4) $$GAP_t = \Phi_2(L) \cdot GAP_{t-1} + u_{2,t} = \Xi_2(L) u_{2,t},$$

where $\Phi(L)$ and $\Xi(L)$ are polynomials in the lag operator and $\Xi(L) = (I - \Phi(L) \cdot L)^{-1}$. Hence, the real interest rate is affected by two basic (primitive) shocks, $u_{1,t}$ and $u_{2,t}$:

(5) $$r = \Xi_1(L) u_{1,t} + \Xi_2(L) u_{2,t}.$$

According to the definition of the NRI:

(6) $$\Delta\pi = \psi(r - r^*) = \psi \cdot GAP = \psi \cdot \Xi_2(L) u_{2,t} \qquad \psi < 0,$$

where Δ is the difference operator and π the inflation rate, the $u_{2,t}$ shock also affects inflation. Thus, both the real rate and inflation growth rate can be expressed as a distributed lag of all current and past primitive shocks:

(7) $$\begin{bmatrix} \Delta\pi_t \\ r_r \end{bmatrix} = \begin{bmatrix} S_{11}(L) & S_{12}(L) \\ S_{21}(L) & S_{22}(L) \end{bmatrix} \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix},$$

where $S_{i,j}(L)$ is a polynomial in the lag operator, whose coefficients are denoted as $s_{i,j}(l)$.

Unfortunately, the system of equations (7) is in practice not very helpful in recovering the **u** vector. The standard way to proceed is thus the following. First a standard vector autoregression has to be estimated:

$$\Delta\pi_t = \sum_{l=1}^{k_1} a_{1,1}(l) \Delta\pi_{t-l} + \sum_{l=1}^{k_2} a_{1,2}(l) r_{t-l} + \varepsilon_{1,t},$$

(8)

$$r_t = \sum_{l=1}^{k_3} a_{2,1}(l)\Delta\pi_{t-l} + \sum_{l=1}^{k_4} a_{2,2}(l) r_{t-l} + \varepsilon_{2,t},$$

or in matrix notation:

$$(9) \qquad \begin{bmatrix} \Delta\pi_t \\ r_r \end{bmatrix} = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta\pi_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix},$$

where $A_{i,j}(L)$ is again a polynomial in the lag operator. This VAR model can be estimated by OLS and, equally as in (7), presented in the vector moving average form:

$$(10) \qquad \begin{bmatrix} \Delta\pi_t \\ r_r \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix},$$

where:

$$(11) \qquad C(L)=(I-A(L)L)^{-1}.$$

Unfortunately, the residuals **ε** differ from our innovations **u**. A critical insight is that the VAR residuals are composites of the pure innovations **u** (Enders 1995, p.333):

$$(12) \qquad \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} = \begin{bmatrix} s_{11}(0) & s_{12}(0) \\ s_{21}(0) & s_{22}(0) \end{bmatrix} \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}.$$

Thus, it would be possible to calculate the primitive shocks from the VAR residuals, if the coefficients $s_{i,j}(0)$ were known. This can be achieved by imposing four identifying restrictions on the system (7).

First, the variance of the primitive shocks is assumed to be 1. This is a standard way of normalizing the shocks, which provides two restrictions. Further, since according to equation (6), $u_{1,t}$ does not impact upon $\Delta\pi$, we could basically impose the restriction $S_{1,1}(L)=0$ on the S(L) matrix. However, as (6) is supposed to describe long-run relationships, we will only require the NRI shock to have zero influence upon $\Delta\pi$ in the long run, which means that it will not be allowed to permanently affect inflation:

(13) $S_{1,1}(1)=0$.

The last restriction will be based on economic knowledge. As monetary transmission works only with a substantial lag, we can safely restrict the innovation to the interest rate gap $u_{2,t}$ not to have any impact upon inflation in the current month:

(14) $s_{1,2}(0) = 0$.

At this stage, it is important to note that we did not impose the standard identifying restriction of orthogonality of $u_{1,t}$ and $u_{2,t}$. This means for instance that a shock to the natural rate can only partially affect the real rate. The rest of the impact will be interpreted as a change of the interest rate gap. Taking into account the above-described restrictions, some straightforward calculations, presented in detail in Appendix 1, can be done to recover the remaining elements of the s(0) matrix[3]:

(15) $s_{1,1}(0) = \pm\sqrt{\mathrm{var}(\varepsilon_{1,t})}$,

(16) $s_{2,1}(0) = \mp\left[\frac{C_{1,1}(1)}{C_{1,2}(1)}\right]\cdot\sqrt{\mathrm{var}(\varepsilon_{1,t})}$,

(17) $s_{2,2}(0) = \sqrt{-2\frac{s_{2,1}(0)}{s_{1,1}(0)}\cdot \mathrm{cov}(\varepsilon_{1,t},\varepsilon_{2,t}) + s_{2,1}^2(0) + \mathrm{var}(\varepsilon_{2,t})}$.

Thus, as the VCV matrix of **ε** is known, the elements of the S(0) matrix can be easily calculated. As a consequence, we can calculate the natural rate of interest, as solely affected by $u_{1,t}$ disturbances. This means setting all $S_{2,2}(L)=0$ in (7):

(18) $r_t^* = S_{2,1}(L)u_{1,t}$,

where the coefficients $s_{2,1}(l)$ can be calculated from:

(19) S(L)=C(L) S(0),

which results from substituting (7) and (10) into (12).

3. Estimation results

In this study, monthly US data for the period 01.1960-06.2002 was used. Additional calculations have been done for the shorter sample 01.1980-06.2002. The basic interest rate is the federal funds rate, additional calculations have been conducted on the basis of 12 month T-bills. The inflation measure is the year-on-year change in the consumer price index[4]. Expected inflation, necessary to deflate interest rates, was obtained from the Livingstone survey.

The first step of analysis was related to testing the integration level of the series, because the SVAR procedure restricts the variables to be stationary. The results are reported in Appendix 2. Both measures of real interest rates can be treated as stationary, whereby inflation seems to be integrated of order 1[5]. Thus, our VAR model must consist of the real rate of interest and the change of the inflation rate, exactly as in (5). Table 1 presents the data.

Table 1: *The data series*

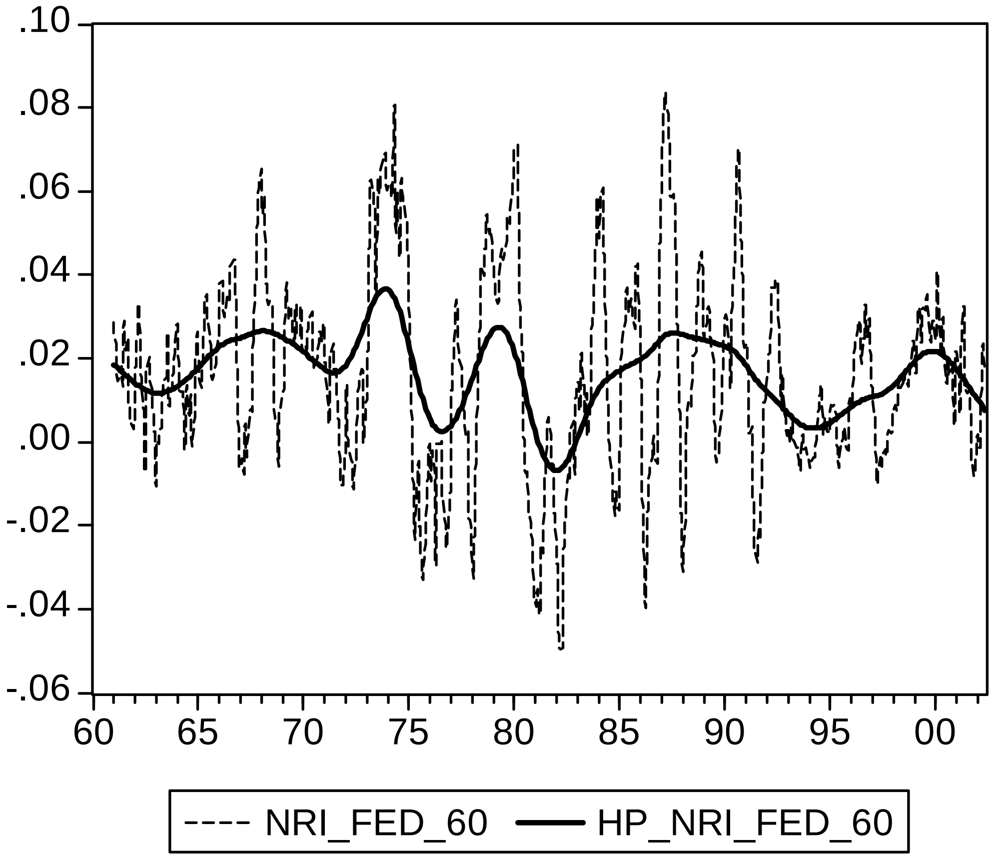
Variable	Description
DLCPI	inflation rate (12 month difference of log CPI)
DDLCPI	change in inflation (first difference of DLCPI)
RFEDFUND	federal funds rate deflated by expected CPI inflation
RTBILL1Y	12-month T-bill rate deflated by expected CPI inflation
NRI_FED_60	natural rate of interest for FEDFUND (period 1960-2002)

[3] It is important to note that in spite of the existence of two solutions for $s_{1,1}(0)$ and $s_{2,1}(0)$ the natural rate of interest in equation (18) is unique.
[4] Calculations performed with the CPI less food and energy index did not give substantially different results and thus have not been reported in detail.
[5] The time series properties of inflation and real interest rates have been debated for years. In a recent publication M.Lanne (2002) argues that inflation is a unit root process, whereas real interest rates are stationary. However, it must be noted that some researchers have come to the conclusion that nominal rates and inflation do not move one for one, which implies a unit root process for the real rate of interest (see J.Bullard 1999).

NRI_TB1_60	natural rate of interest for TBILL1Y (period 1960-2002)
NRI_FED_80	natural rate of interest for FEDFUND (period 1980-2002)
NRI_TB1_80	natural rate of interest for TBILL1Y (period 1980-2002)
HP_NRI_FED_60	HP trend of NRI_FED (period 1960-2002)
HP_NRI_TB1_60	HP trend of NRI_TB1 (period 1960-2002)
HP_NRI_FED_80	HP trend of NRI_FED (period 1980-2002)
HP_NRI_TB1_80	HP trend of NRI_TB1 (period 1980-2002)
GDP_CYCLE	cyclical component of GDP (log GDP minus HP trend)
FED_GAP_60	interest rate gap (NRI_FED-RFEDFUND) (period 1960-2002)
TB1_GAP_60	interest rate gap (NRI_TB1-TBILL1Y) (period 1960-2002)
FED_GAP_80	interest rate gap (NRI_FED-RFEDFUND) (period 1980-2002)
TB1_GAP_80	interest rate gap (NRI_TB1-TBILL1Y) (period 1980-2002)

As the lag order for the VAR could not be unambiguously chosen according to information criteria (Akkaike (AIC), Schwarz (SC) and the sequential modified likelihood ratio (LR) test (Lütkepohl 1995); (Appendix 2)), it was arbitrarily decided to take 12 lags in all models. This ensured lack of autocorrelation in the residuals.

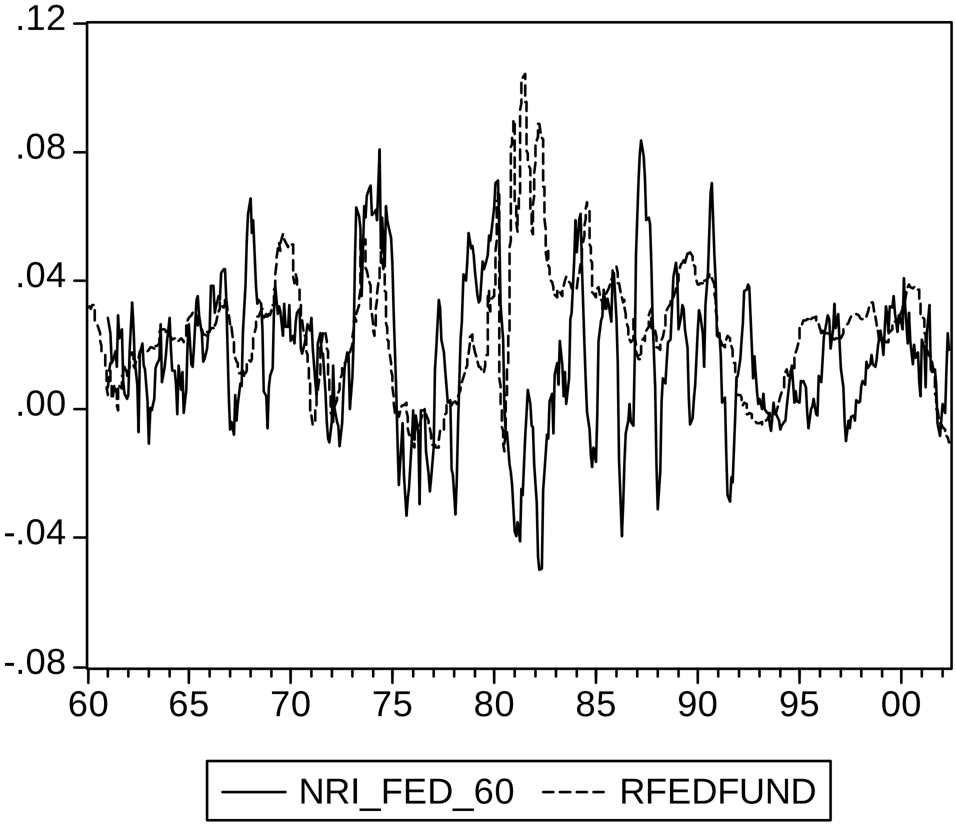
Figure 1: *Natural rate of interest for the federal funds rate and HP-trend*



The first test was calculated on the basis of the federal funds rate. Figure 1 presents the estimate of the natural rate of interest and its trend over the period 1960-2002. On the first sight, substantial variability of the natural rate can be observed. It can also be noted that the NRI reflected an increase in the second half of the 1990's, a phenomenon ascribed by many to high productivity growth over that period. However, as compared to earlier periods, the NRI's absolute level was not exceptionally high.

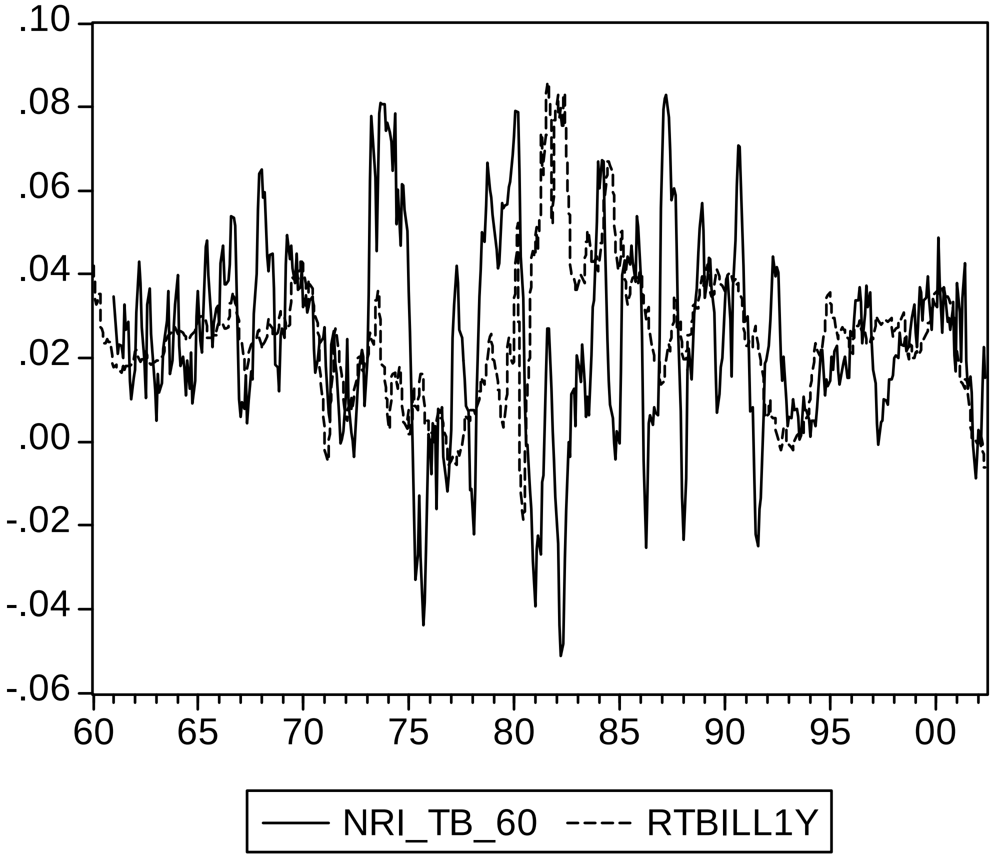
It might be interesting to see how the NRI behaves as related to the actual series of the federal funds rate. This is presented in Figure 2. The standard deviations of NRI_FED_60 and the real federal funds rate are relatively similar (Table 3). This result does not seem in line with the estimates of Laubach and Wiliams (2001) for the US and Neiss and Nelson (2001) for the UK. In both cases lower variance of the natural rate relative to the real rate has been reported. However, other research papers (e.g. J.Rotemberg, M.Woodford 1997) came to similar conclusions of relatively high NRI variance. This shows that there is still much to do in the field of NRI estimation.

Figure 2: *NRI_FED_60 and the real federal funds rate.*



The results for the interest rate on treasury bills are in general quite similar. As previously, the natural rate shows substantial variability as compared to the real rate (Figure 3) and almost equals the natural rate obtained from the federal funds model.

Figure 3: *NRI_TB_60 and the real treasury bill rate.*



It could be also interesting to have a look at the estimated contemporaneous correlation of the NRI and GAP shocks. This ranges from 0.48 in the short sample to 0.87 in the long one (Tab. 2). Such a result can be interpreted as the inability of the Fed to track immediately the NRI shocks with its instrument. This result should not be surprising, as it would be very hard to imagine a central bank having precise estimates of NRI shocks already in the current month.

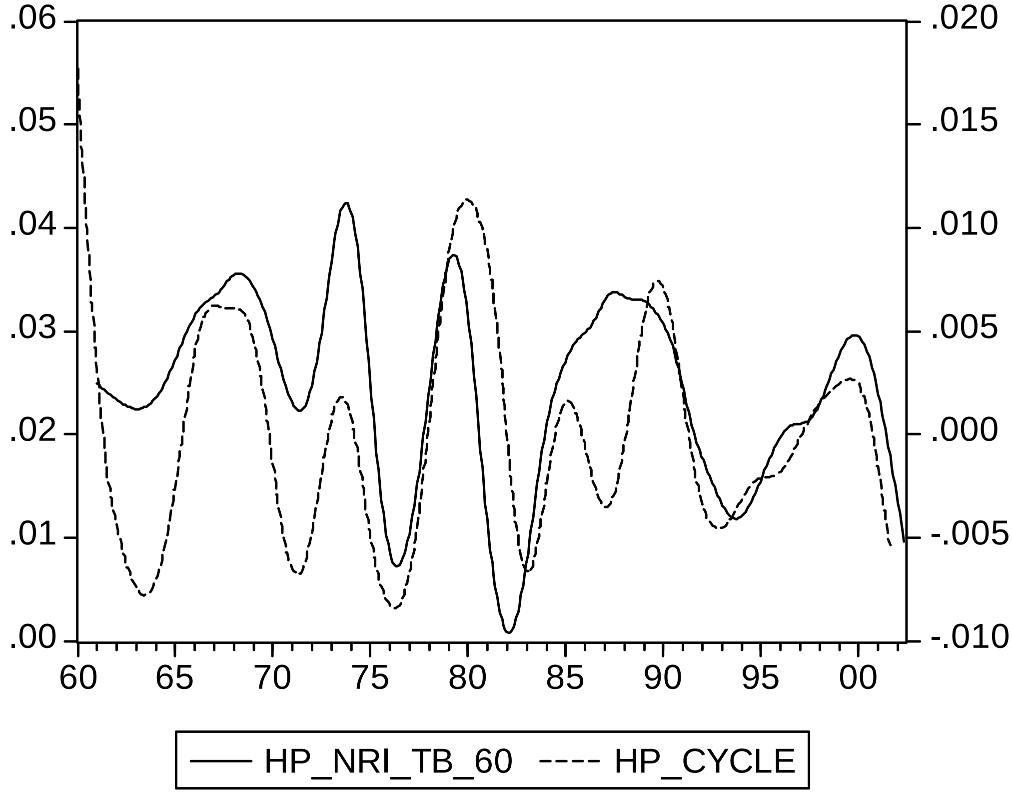
Table 2: Contemporaneous correlation of $u_{1,t}$ and $u_{2,t}$

Model with	RFEDFUND 1960-2002	RTBILL1Y 1960-2002	RFEDFUND 1980-2002	RTBILL1Y 1980-2002
Correlation of $u_{1,t}$ and $u_{2,t}$	0.8	0.87	0.48	0.77

Note: 5% significance level of the correlation coefficients is 0.08.

To estimate how fast the Fed has been able to fully react to NRI shocks we looked at the cross-correlograms of the natural rates and the interest rate gaps. In all four cases significant correlation disappeared after 8-10 month from the initial shock. This is approximately the time the Federal Reserve needed in the past to fully adjust its instrument to the new macroeconomic conditions.

Figure 4: *HP_NRI_TB_60 (left axis) and the cyclical component of output.*

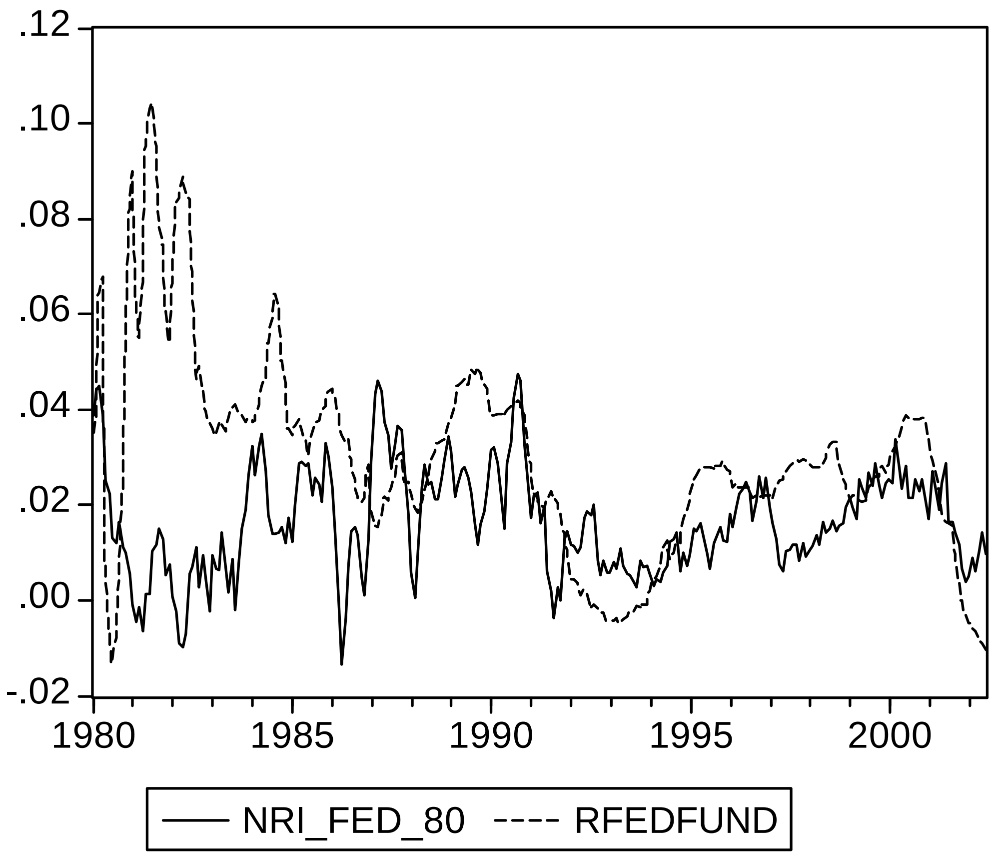


Laubach and Williams state that, in line with the steady state outcome of the Ramsey model, the natural rate of interest is positively correlated with productivity growth. The behavior of the NRI in figure 1, 2 and 3 suggests also that the variable may be correlated with the economic cycle. This seems quite natural, as for instance investment demand is affected not only by real interest rates but also by expected economic performance (S.Bond, T.Jenkinson 1996). Thus, to induce the same growth of aggregate demand in a downturn, the real rate has to be lowered to a lower level than during a boom. This obviously can make the NRI a procyclical variable. To verify this empirically, the correlation between the natural rates and the cyclical component of output were calculated. Positive correlation of 0.25 in the case of the federal funds rate and 0.33 in the case of treasury bills seems to support our supposition. The correlation grows substantially to 0.5 if H-P trends of the NRI and the cyclical component of output are regarded (Figure 4).

All the above-described results find confirmation in the calculations in the short sample 1980-2002, the major difference being the drop in relative variance. In the 1980's and 1990's the standard deviation of the natural rate was only half of the S.D. of the real federal funds rate (Table 3). One can conclude that over the last 20 years the conditions to conduct interest rate based monetary policy have substantially improved.

Another interesting conclusion can be drawn from Figure 5. In the 1990's the Fed almost perfectly followed the movements of the natural rate. The correlation between the natural rate and the federal funds rate increases to 0.62 in the last decade from 0.04 in the whole sample 1960-2002.

Figure 5: NRI_FED_80 and the real federal funds rate 1980-2002.



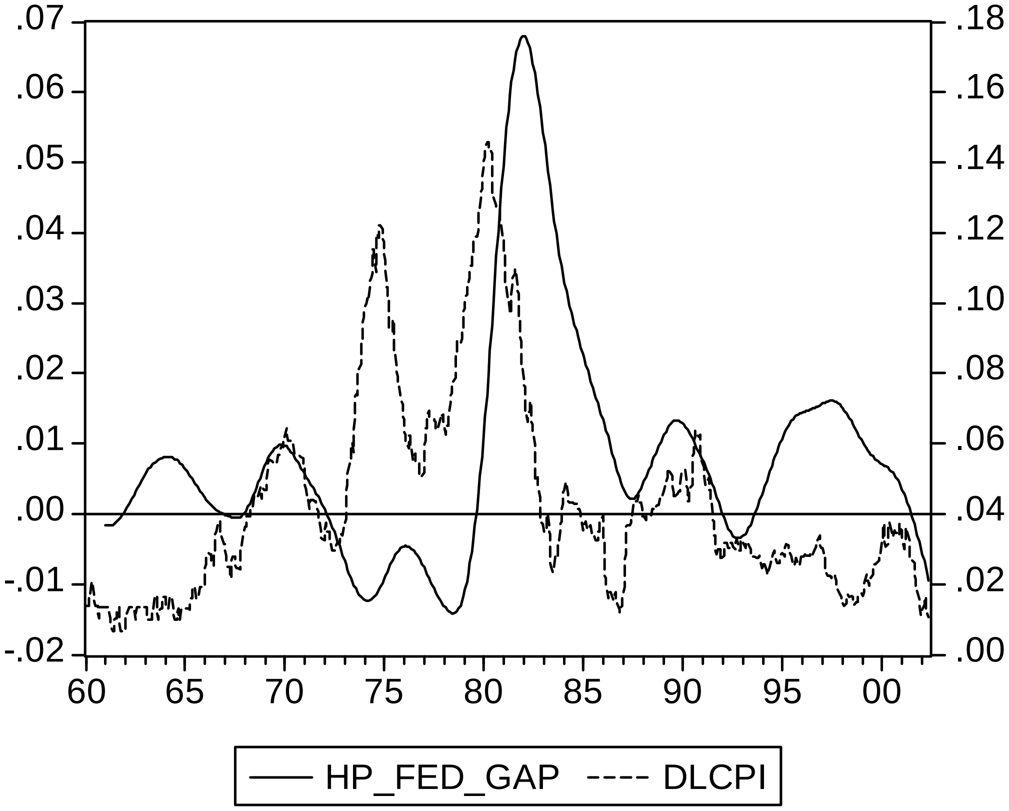
This observation leads us to an important conclusion. It is often argued that the natural rate would be a useful concept in central banking only, if its variability were significantly lower, than the variability of the real rate (Neiss, Nelson 2000). However, one has to note that a successful central bank should make its instrument follow closely the movements of the natural rate of interest in order to avoid strong variability of inflation. Thus, a substantially lower variability of the NRI as compared with the short term real rate can be regarded as a sign of weak forecasting ability of the monetary authorities.

Table 3: *Standard deviations, relative variances and correlations of real and natural rates of interest*

	Federal funds rate 1960-2002	12 month T-bill 1960-2002	Federal funds rate 1980-2002	12 month T-bill 2980-2002
S.D. of the real rate	1,9%	1,6%	2,1%	1,9%
S.D. of the natural rate	2,4%	2,3%	1,1%	2,0%
Correlation NRI and real rate	0.04	-0.02	-0.01	-0.05

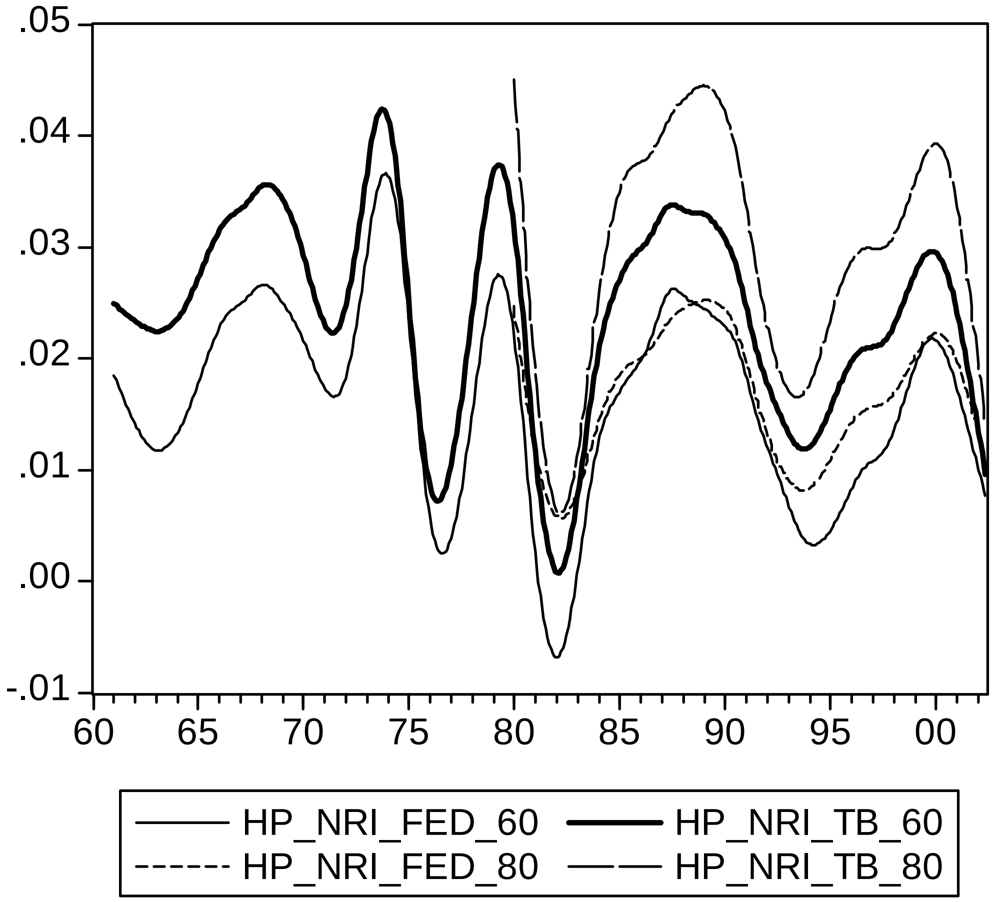
The performance of the interest rate gap (calculated as in equation 1) as indicator of inflation pressure can be seen from Figure 6. It can be observed that in general, a positive interest rate gap resulted in falling inflation and a negative gap in growing inflation. As the variability of the gap decreased substantially in the 1980’s and 1990’s, inflation remained relatively stable.

Figure 6: *H-P filtered interest rate GAP (left axis) and inflation.*



Finally it could be interesting to look at the estimated natural rates together. As they are quite variable, we present only the trend series. As it can be seen from Figure 7, the natural rates show quite high colinearity. Common troughs were reached in the mid-seventies early eighties and early nineties. In all three cases there is substantial growth in the NRIs in the 90's, however, their absolute level does not seem exceptionally high.

Figure 7: *Trend series of the estimated natural rates of interest*



4. Conclusions

For the successful conduct of monetary policy the central bank needs reliable indicators of the monetary policy stance. One recently advocated is the gap between the real, market interest rate and the natural rate of interest. Its obvious advantage against other indicators (output gap, unemployment gap) is the direct indication, how interest rates should be set in order to stabilize/lower inflation. On the contrary, the alternative indicators are useful only if there exists a stable relationship between them and the monetary policy instrument.

In this paper we used a structural vector autoregression to estimate the historical time series of the natural rate of interest in the US over the period 1960-2001. We defined the NRI as the level of the real rate of interest that is compatible with stable inflation. Our econometric model was designed to calculate the NRI by definition, i.e. as such changes in the real rate that do not affect the rate of inflation in the long run. The results seem plausible and thus, confirm that the structural VAR is well designed for the estimation of the natural rate of interest.

Further we studied the statistical properties of the NRI. The estimated NRI shows substantial variability, of comparable magnitude to that of the real interest rate. An interesting finding is that the variability of the natural rate of interest falls in the second half of the sample. This means that it must have been relatively easier to conduct interest rate based monetary policy in the 1980's and 1990's than before.

The time series of the natural rates for the federal funds rate and the 12-month T-bill rate show similar patterns. Substantial declines of the NRIs can be observed in the mid-seventies, early eighties and early nineties. In both cases there was substantial growth in the NRI over the last decade, its absolute level did not, however, seem exceptionally high.

We have also taken a look at the estimated contemporaneous correlation of the NRI and GAP shocks. This ranges from 0.48 in the short sample to 0.87 in the long one. Such a result can be interpreted as the inability of the Fed to track immediately the NRI shocks with its instrument. This result is, however not surprising, as it would be very hard to imagine a central bank having precise estimates of NRI shocks already in the current month.

Finally, we tested the business cycle properties of the estimated series. The natural rate of interest proved to be a procyclical variable. This reflects the possibility (even necessity) of an aggressive lowering of interest rates during a downturn without risking negative inflationary outcomes, provided that monetary policy properly and timely detects macroeconomic situation.

The detected high variability of the natural rate of interest can be regarded as a handicap for the widespread practical use of the interest rate gap concept in central banking. Still some points have to be noticed in favor of the NRI. First, its variability can be partially explained by other factors, like supply side shocks (oil prices), productivity growth and the business cycle. Second, as noted earlier, other indicators of the monetary policy stance suffer from similar problems. Third, the concept of the interest rate gap seems very powerful for theoretical explanation of how monetary policy works. Fourth, as a central bank ought to closely follow the movements of the NRI with the short-term rate, we should not expect the variances to differ much. Thus, the comparable variability of the interest rates can be just o consequence of central bank behavior. Accordingly, in our opinion the natural rate of interest can be considered a useful, although not perfect, indicator of the stance of monetary policy.

Further research could be devoted to estimating the properties of the natural rate calculated on the basis of long-term interest rates. It would be interesting to see, whether the

NRI becomes less volatile, as we move towards the long end of the yield curve. Unfortunately, at least one important problem would have to be overcome - the estimation of long-run inflationary expectations.

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Appendix 1

Given the identifying assumptions (13), (14) and the assumption of unit variance of the $u_{1,t}$ and $u_{2,t}$ shocks, the elements of the s(0) matrix can be calculated in the following way.

From (12) and (14) we have:

(20) $$\mathrm{Var}(\varepsilon_{1,t}) = s_{1,1}(0)^2 + s_{1,2}(0)^2 + 2\,\mathrm{cov}(u_{1,t}u_{2,t}) \cdot s_{1,1}(0) \cdot s_{1,2}(0) = s_{11}^2(0)\,,$$

(21) $$\mathrm{Var}(\varepsilon_{2,t}) = s_{2,1}(0)^2 + s_{2,2}(0)^2 + 2\,\mathrm{cov}(u_{1,t}u_{2,t}) \cdot s_{2,1}(0) \cdot s_{2,2}(0)$$

and

(22) $$\mathrm{cov}(\varepsilon_{1,t}\varepsilon_{2,t}) = s_{1,1}(0) \cdot s_{2,1}(0) + s_{1,1}(0)s_{2,2}(0) \cdot \mathrm{cov}(u_{1,t}u_{2,t})$$

From (20):

(23) $$s_{1,1}(0) = \overset{+}{\underset{-}{}}\sqrt{\mathrm{Var}(\varepsilon_{1,t})}\,,$$

and from (13) and (19) we can write:

(24) $$C_{1,1}(1) \cdot s_{1,1}(0) = -C_{1,2}(1) \cdot s_{2,1}(0)\,.$$

Substituting from (23) into (24) allows us calculate $s_{2,1}(0)$:

(25) $$s_{2,1}(0) = \overset{-}{\underset{+}{}}\left[\frac{C_{1,1}(1)}{C_{1,2}(1)}\right] \cdot \sqrt{\mathrm{Var}(\varepsilon_{1,t})}\,.$$

Substituting for $\mathrm{cov}(u_{1,t}u_{2,t})$ from (21) into (22) yields the following expression for $s_{2,2}(0)$:

(26) $$s_{2,2}(0) = \sqrt{-2\frac{s_{2,1}(0)}{s_{1,1}(0)} \cdot \operatorname{cov}(\varepsilon_{1,t}, \varepsilon_{2,t}) + s_{2,1}^2(0) + \operatorname{var}(\varepsilon_{2,t})},$$

which can be easily calculated by substituting $s_{1,1}(0)$ and $s_{2,1}(0)$ from previous results.

Appendix 2

Table A1: *Unit root tests*

		ADF with const.
DLCPI	8	-2.36
DDLCPI	7	-13.99***
RFEDFUND	5	-4.13***
RTBILL1Y	6	-2.79*

* denotes rejection of H_0 at 10%

** denotes rejection of H_0 at 5%

*** denotes rejection of H_0 at 1%

Table A2: *VAR lag length*

VAR variables	AIC	SC	LR
RFEDFUND, DDLCPI, short sample 1980-2002	17	2	17
RTBILL1Y, DDLCPI, short sample 1980-2002	16	2	12
RFEDFUND, DDLCPI, long sample 1960-2002	17	2	14
RTBILL1Y, DDLCPI, long sample 1960-2002	20	12	20