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Remarks on models of currency crises and the existance of multiple equilibria

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Summary

The aim of this paper is a reconsideration of some features of first and second generation models of currency crises and some conclusions from these models. In particular, theoretical justification for the existence of multiple equilibria is put under discussion in a broader context of uncertainity and expectations.

The role that different assumptions concerning government and monetary authority's behaviour play in the determination of the results of first generation models is analyzed and the deterministic character of these models which does not allow for multiple equilibria is explained. An explanation is suggested why equilibria can be in practice unstable when the rule of central bank's (government's) behaviour is not clear as it is in the models.

The structure of a second generation model is analyzed with an intention to explain that for the existence of multiple equilibria there must be a convincing justification based on the game theory which the second generation lacks. It is also pointed out that an equilibrium may not exist at all.

It is argued in this paper that in the world of rational expectations and perfect foresight a change of an equilibrium is possible only due to an external shock. Vulnerability of a fixed exchange rate to shocks is thus an important issue. Consequently, some features of an economy and economic policy which make fixed exchange rate vulnerable to external shocks are discussed.

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1
Introduction

The aim of this paper is a reconsideration of some features of first and second generation models of currency crises and some conclusions from these models. In particular, theoretical justification for the existence of multiple equilibria is put under discussion in a broader context of uncertainity and expectations. It is argued in this paper that in the world of rational expectations and perfect foresight a change of an equilibrium is possible only due to an external shock. Vulnerability of a fixed exchange rate to shocks is thus an important issue.

In the next part of the paper I analyze the role that different assumptions concerning government and monetary authority's behaviour play in the determination of the results of first generation models. I explain the deterministic character of these models which does not allow for multiple equilibria and I suggest an explanation why equilibria can be in practice unstable.

In the third part of the paper I analyze the structure of a second generation model (Obstfeld, 1994) with an intention to explain that for the existence of multiple equilibria there must be a convincing justification based on the game theory which the second generation lacks. I point out, moreover, that an equilibrium may not exist at all in (Obstfeld, 1994).

In the fourth part I indicate some features of an economy and economic policy which make fixed exchange rate vulnerable to external shocks. This analysis is built upon (Obsfeld, 1995) though its aim and conclusions are different.

The last part is conclusions.

2

2
The deterministic structure of the first generation

In (Krugman, 1979) - a first generation, canonical model - there is no room left for uncertainty. In the model it is assumed that domestic money supply is backed by domestic and foreign assets of the central bank. Domestic assets grow at a constant rate due to monetized fiscal deficits. The exchange rate is fixed and the price level is stable (the price level is governed by purchasing power parity). The interest rate obeys uncovered interest rate parity; at fixed exchange rate the domestic currency interest rate is equal to the foreign currency interest rate and the quantity of international reserves adjusts to balance the money market. This reflects the monetary attitude to the balance of payments.

These assumptions give to the model a fully deterministic character and are crucial for its outcome. The rate of growth of domestic assets of the central bank is determined, so is the rate of depletion of the official reserves. Market participants are able to compute the exchange rate which would balance the money market if the foreign reserves were exhausted in a speculative attack; this is the shadow exchange rate[1]. An attack results in the flotation of the exchange rate and then its gradual depreciation at a rate determined by a growth of domestic assets. Flotation of the exchange rate and perspectives of its consant depreciation translate into inflationary expactations and a rise of the nominal interest rate in the moment of an attack. The shadow exchange rate reflects both a change in money supply due to depletion of the foreign reserves and a change in money demand due to a rise of the (nominal) interest rate.

If an attack on the reserves took place too early (the reserves were still too high) speculators would incur capital loses – the shadow rate would be higher then the actual rate and the domestic currency would appreciate as a result of an attack. An attack should thus occur when shadow and actual exchange rates are equal – this is the first moment when speculators would not experience loses and competition among them does not let them wait longer. This condition is met when the level of the official reserves is still positive so that the resulting reduction of money supply should be equal to a decline in money demand in effect of an interest rate rise.

An exchange rate crisis in the disscused model is perfectly deterministic. Moreover, a crisis does not mean a jump of the exchange rate level but only flotation of the fixed exchange rate regime and an attack on official reserves. This conclusion, however, depends on the bahaviour of the central bank in the moment of an attack – it intervenes untill it has no reserves at all.

Some attention should be paid to the fact that in the model it is the nominal interest rate which balances the money market. This implicit assumption could be reasonably changed so that the equilibrium of the money market should depend on the real interest rate. This change has important consequences. Since flotation of the exchange rate and inflationary expectations don't change the real interest rate there would be no change in money demand due to a speculative attack. The first possible moment for an attack is when it would not diminish money supply – it means the moment when there is no more of the object of an attack, the foreign reserves. Hence, in this case there is no room for any attack and the modified model would describe only an orderly and perfectly foreseeable change of the exchange rate regime. There is little reason to talk about a crisis.

[1] It is the price level which adjusts to balance the money market; the exchange rate must only obey the purchasing power parity condition. This means that there is an implicit assumption of perfect price elasticity.

Let's return to the problem of the central bank's behaviour. If we admit that the central bank decides to float the exchange rate in the moment of an attack without foreign exchange intervention the results of the model change importantly. Let's first stick to the original version of the model where the money market is balanced by the nominal interest rate. In this case exchange rate flotation without precedent intervention induces a decrease in money demand but does not change money supply. An effect is a devaluation of the domestic currency. If market participants expected that the bank would not intervene they should attack earlier: an attack which does not trigger intervention always leads to devaluation before flotation.

However, when official reserves are still higher then the level at which shadow and actual exchange rates are equal the central bank can always defend the rate and even punish the speculators with appreciation of the domestic currency. There is room for uncertainty due to the lack of the central bank's unique strategy which market participants would be aware of (a lack of credible policy rules). In this juncture speculators would possibly apply different - also mixed - strategies. Market participants might be to different degrees inclined to test the central bank's strategy. Strategies could be "shaky", easily changed with even minor developments, particularly those concerning credibility of the policy of the central bank (its dominantly presumed strategy). For an individual trader the strategies of other traders would also be unclear and they might change with evolving conditions in a way which is not really foreseeable. In such a situation even a spontaneous coordination of strategies (herding behaviour) can possibly lead to "successful" results of the test – the central bank revealing its strategy to give uo the exchange rate under a major attack. Such a test under uncertainty concerning the behaviour of other market participants – with the central bank in the first place - is the essence of a **speculative** attack. However, this is neither the case of the first generation, nor of the second one.

In the modified version of the model, when the real interest rate matters for the money market equilibrium, an attack always leads to appreciation if the central bank intervenes. If it decides to float the currency without intervention there is no jump devaluation but only a change of the exchange rate regime followed by gradual depreciation reflecting inflation. In this situation an attack is never reasonable. However, this conclussion seems to depend heavily on some features of the theoretic structure of the model (PPP, flexible prices which can restore money market equilibrium, neglect of the balance of payments equilibrium).

In any case the outcome of the model is precisely determined if only the cost function (rule of behaviour) of the central bank is given and market participants are conscious of it. If this condition is not met there is some policy uncertainty which may incline market participants to apply mixed strategies being a source of possible multiple equilibria (at least in the version of the model with the nominal rate).

In the remarks above, so as in the Krugman canonical model, no attention has been paid to a possible adjustment of the money supply undertaken by the central bank through open market operations. When this possibility is taken into account in the original version of the model with nominal interest rate market participants can still experience a loss even though they attack when shadow and actual exchange rates are equal, given the level of domestic assets. This means that there is even more room for uncertainty and a game between speculators and the central bank.

In (Krugman, 1979) there is implicitly assumed a very simple rule of the central bank's behaviour (cost function). It always intervenes up to complete depletion of its foreign reserves and it never undertakes open market operations. Any different but precise rule of behaviour would also produce fully determined outcomes. In Flood, Marion (1998) for example it is assumed that foreign exchange interventions are always fully sterilized. In this case the behaviour of the central bank is different then in (Krugman, 1979) but it is still a subject to a precise rule. As a consequence the model gives a single solution without any room for uncertainty or a game; an attack would always lead to devaluation, no matter the level of foreign reserves, and a fixed rate could never be succesfully defended. Since this conclusion does not seem realistic Flood, Garber and Kramer (1996) developed a model where interest parity condition contains risk premium. This premium depends on the structure of investors portfolios which is influenced by sterilized interventions. In this model there is also a single solution – an attack would take place when expected depreciation will be equal to risk reduction.

In all the examples above there was an implicit or explicit assumption of a precise cost function (rule of behaviour) of the central bank. This function is known to the market participants who act in accordance with perfect foresight[2]. It is also worth stressing how strongly the outcomes of the models depend on particularities of the assumptions.

An impact of uncertainity on a possibility of a speculative attack is included in (Flood, Marion, 1996). This model has a structure very similar to (Flood, Garber, Kramer, 1996). However, in (Flood, Marion, 1996) risk premium – which determines the shadow exchange rate and the moment of an attack - depends on expectations of exchange rate volatility after flotation. A change in expectations can thus become a reason for a crisis. However, such a change is exogenous to the model. This is the reason why multiple equilibria – even in an infinite number – are possible. The model still has a deterministic character, given a certain state of expectations. Expectation change is not analised although it is a central issue in the model. Indogenous description of expectations is realised only in the models of the second generation.

It must be mentioned, however, that Flood and Garber already in 1984 (Flood, Garber, 1984) introduced devaluation expectations to their model and noticed that "A more complete tratment of expected devaluations would require a model of the government's decision-making process"[3]. These are the seeds of the second generation.

In the same paper Flood and Garber suggested the possibility that arbitrary spaculative behaviour can cause the collapse of a fixed exchange rate regime. Such a behaviour is thus a source of possible multiple solutions. However, also in this case speculative behaviour is arbitrary (external to the model). A change in the speculative behaviour is a factor which also determines post-attack equilibrium exchange rate but it is not clear what is the mechanism behind it and why should it permanently divert the exchange rate from its level determined by the fundamentals.

[2] Flood, Marion (1998) in their comment on Flood, Garber, Kramer (1996) mention that "the introduction of a risk premuim into a perfect foresight model is na anomaly, to say the least". This does not undermine, however, the deterministic character of the model.

[3] Flood and Garber wrightly pointed that the assumption of instantaneous expected rate of change of the exchange rate equal to zero is problematic. If an attack and flotation and depreciation of the exchange rate is inevitable and it is only a matter of time it should be expressed in exchange rate expectations in advance of the attack. The fixed rate might be preserved only at a higher interest rate which is possible at lower foreign reserves in comparison with the situation of no expectations of devaluation. Flood and Garber also introduce to their model a random disturabance of an assumed density. This disturbance influences exchange rate expectations. However, this is redundant for the conclusion concerning the impact of exchange rate expectations on foreign reserves and the uncertainty due to the random disturbance is not analised. In conclusions Flood and Garber notice that "since the disturbance is unpredictable and its origin is outside the bounds of our theories, the only problem remaining for the economist is to determine what its minimum size must be to have such a drastic effect". This problem is the subject of part 4 of this paper.

3
Rational expectations and perfect foresight - – is there any room for indeterminism?

An eminent representative of the second generation models is (Obstfeld, 1994). In the model the cost function of the government (including the monetary authority) is determined; its variables are the tax rate and devaluation/depreciation. Market participants are aware of the cost function of the government and they have rational expectations. It means that for each level of the interest rate they are able to decide upon future devaluation which would minimize the cost function of the government– this relation between the interest rate and devaluation minimising the government's cost is named the government's reaction function. The interest rate and depreciation/devaluation must also obey *ex ante* and *ex post* interest rate parity.

There are thus two functions in the model describing the relation between the interest rate and the exchange rate: the government's reaction function and interest rate parity. Obstfeld argues that "in a perfect foresight equilibrium, the depreciation rate the market expects must equal the depreciation rate the government finds optimal, given market expectations. Thus, intersections of the government's reaction function and the interest parity curve determine possible equilibria of nominal interest rates and currency depreciation".

This set of two equations have according to Obstfeld two solutions. The first one is characterised with low depreciation expectations and as a consequence low interest rates and low actual depreciation. In the other state of equilibrium expected depreciation and interest rates are higher what makes the government cost rise and induces stronger actual depreciation. Thus, there are multiple equlibria. The choise of one of them depends on expectations which are self-fulfilling.

The model suggests the possibility of the existance of multiple equilibria (precisely two equilibria in this case). Obstfeld proves that by giving some values to the variables of the model. Changing these values (for example by giving a higher value to the foreign interest rate) it is also possible to show that the model can have no solution at all – no equilibrium is possible. To find a reasonable interpretation of such an outcome would be of the highest importance for the assesment and interpretation of the model.

Moreover, although the two equations of the model can have two solutions it is questionable whether both are an admissible economic result. Obstfeld writes: "Obviously the government's loss is lower in the low-depreciation equilibrium but there is no way to ensure that the bond market coordinates on the ralatively low (...) interest rate. The government faces a dynamic inconsistency problem: much as it would like to, it cannot credibly promise not to validate expectations if the bond market settles on the high-inflation equilibrium's interest rate". A question arises, however, why market participants should settle the interest rate on the high-inflation level since –according to the model's assumptions – they are fully aware that this equilibrium is worse from the government's point of view (it is connected with a higher cost) and thus they have no reason to expect that the government voluntarily could realize higher inflation (depreciation) what would be irrational. This question draws attention to the problem of game-theoretic, strategic foundations of the alleged existance of multiple equlibria.

Before turning to this issue critical for the subject of this paper it is worth noticing some other features of the model.

Firstly, in the structure of the model's assumptions the government cannot cheat market participants introducing high inflation in spite of low inflation expectations – a possibility of such deception could justify expectations of higher inflation and as a consequence higher interest rates.

Such cheating, even if successful, would not be optimal for the government minimizing its cost function, either.

Secondly, the remarks above pertain to a flexible exchange rate which is also a starting point to concider a fixed rate regime in (Obstfeld, 1994).

Obstfeld argues that at low levels of the interest rate the government's cost at a fixed exchange rate might not surpass the cost at flexible exchange rate by more then a fixed cost of flotation/devaluation. In this case the fixed rate is maintained. If market participants set their exchange rate expectations at a higher level and make the interest rate rise, then the cost at a fixed rate might be higher than the cost at a flexible rate augmented by the cost of devaluation and the exchange rate would be floated. An increase in expectations of devaluation gives rise to the cost of a fixed rate and trigers flotation/devaluation.

This conclusion can be subject to the same reservation which has been made with respect to a flexible rate regime; within the model there is no reason why expectated devaluation should rise and one needs proper strategic foundations to justify the existance of multiple equilibria.

What concerns the strategic foundations Obstfeld (1995) before the presentation of a model discusses the foundations with an example of a game. In the example the government has foreign currency reserves equal 10 and two traders (players) have resources in domestic currency of 6 each. To sell and take a position against the current rate costs 1. None of them is able to attack succesfully alone but attacking together they can deplete the official reserves and force the government to devalue (by assumed 50%) and the traders gain (5/2) – 1 = 3/2 each. This example is illustrated with Figure 1.

Figure 1

		Trader 2	
		Hold	Sell (attack)
Tader 1	Hold	0,0	0,-1
	Sell	-1,0	3/2,3/2

Source: Obstfeld (1995)

In this game there are two Nash equilibria. Obstfeld (1995) notices that "in the first shown in the southheast corner, both traders sell and the currency peg falls. But if neither trader believes the other will attack, the Nash equilibrium in the northwest corner rusults and the currency peg survives. In this game the attack equilibrium has a self-fulfilling element because the exchange rate collapses if attacked, but survives otherwise". Obstfeld neglects that two other solutions, which are not equilibria, in this situation are also possible solutions.

Straffin (2001) states that Nash equilibria have some undesirable features which don't allow treat them as a universal solution. Let's thus analise the equilibria in Figure 1.

Firstly, let's notice that only the solution when both traders attack is Pareto-optimal. This criterion of collective rationality is not in a contradiction with individual rationality – strategy "hold" is not dominant.

Secondly, both equilibria are not equivalent, but southeast equilibrium gives better results for both traders so that there are no contradictory interests which would make traders hesitate which strategy to choose.

Thirdly, the example in Figure 1 does not elliminate a possibility of some sequencing or other strategic actions such as noticing the chosen strategy to the other trader. Only a possibility of sequencing is enough to show that the solution would be an attack. Each trader knows that when he attacs the rational strategy of the other is to attack too.

Fourthly, in the example the resources surpass the government's reserves. This trader who would attack first and manage to get a bigger part of the reserves would gain higher profits. The results of this game depend thus on a sequence the traders attack and both of them are inclined to attack first.

The example in Figure 1 (with some reservations) has two Nash equilibria but it has only one rational and probable solution. The strategic foundations proposed by Obstfeld are not sufficient to back his conclusion from models that multiple equilibria are viable in an economic - and not only mathematic - sense. Allegedly multiple rational solutions of a game and respective different strategies of traders are identified in second generation models with multiple equilibria and different expectations. If there is only one rational solution possible there are no, however, multiple equilibria and self-fulfilling expectations.

Multiple equilibria can be also questioned in another way. It has been accepted in the example and the remarks above that market participants gain when they attack succesfully. Whether an attack of a tipe of second generation models would bring gains is still an open question.

First of all let's notice that in the models of the second generation an attack is a change of exchange rate expectations (and an identical change in exchange rate in effect). This change induces an equal change in interest rate due to interest rate parity. The resources of market participants are "strength of market opinion". This interpretation of a crisis seems to be rather vague. However, whether market participants change during an attack their currency positions (they buy official reserves) or they don't change their positions (so that they don't need other resources then strength of market opinion) they never gain or lose due to simultaneous and equal change of interest and exchange rates. Since the interest rate parity is met *ex ante* and *ex post* for each strategy the result of the game is equal zero for each trader (if we now neglect the cost of the attack which is only a change of market sentiment and it does not necessitate any transactions)[4].

Even though this conclusion means that the game in Figure 1 may not be adequate to second generation models it does not necessarily falsificate the idea of multiple equilibria and possibly it could even paradoxally sustain it. It is not so. To explain it examination of all solutions is necessary.

The arguments above led to the conclusion that there is no gain and no cost both in the case of a successful attack and in the case of no attack since interest rate parity is met *ex ante* and *ex post* and since market participants do not incur transaction costs. It does not mean that they would not incur loses in the case of an unsuccessful attack. Since attacking means creating expectations of devaluation and raising interest rates a trader who attacked and failed (his strenght of market opinion was too weak) had interest rates at excessive levels in comparison with prevailing market rates. The trader was offering too high interest rates and lost some contracts (or decided to borrow at excessive rates) so that there is a cost of an attack which failed. Let it be equal 1. The game is illustrated in Figure 2.

[4] If we interpret an attack as a run on official reserves it creates transaction costs of changing positions and the change can not bring gains – the only Nash equilibrium and solution of the game would be no attack.

Figure 2

		Trader 2	
		Hold	Sell (attack)
Tader 1	Hold	0,0	0,-1
	Sell	-1,0	0,0

In Figure 2 there are two Nash equilibria; sothwest and northeast corners. Both are Pareto-optimal and equivalent. It might suggest that this is a case when there are two acceptable solutions, what would sustain the idea of multiple equilibria in the models of the second generation. However, strategy "hold" is dominant for both traders. This critirion of individual rationality settles the problem – only southwest corner (no attack) is a rational and probable result of the game.

We argued that there is no attack since it brings no gain. In the models of the first generation there is no gain either. However, the situation is different. In the first generation expectations of devaluation are justufied by an objective factor – monetized deficits, and devaluation is inevitable. In the models of the first generation market participants know that devaluation will take place and they are keen to take advantage out of it. If the advantage is equal zero it is only due to a boarder condition according to which an attack takes place as soon as the negative results of an attack turn positive. The decisions of market participants are anyway well motivated with profit seeking. The models of the second generation lack this clarity on the level of strategic foundations which also need more precise interpretation of a crisis.

It has been argued here that a change of exchange rate expectations gives no gain of an attack through a rise in interest rates which mirrors these expectations and it is not a rational strategy of market participants to attack in this situation. An attack can not be interpreted so that it is the government who needs raise interest rates to defend the exchange rate, what makes costs of its policy increase and leads to the decision to devalue. Such an interpretation is inconsistent. If the only reason to devalue is a defence of the exchange (interest rate rise) it is enough not to defend the exchange rate and there would be no devaluation. In other words, if there is no fundamental, objective presure to devalue, an attack can be ignored and it will fail. Of cource, in this juncture there is no reason to attack.

Summing up, there must be positive gains from an attack if market participants are to attack. Moreover, what can be explained with an analysis like in the case of Figure 1, if there are such gains, an attack would certainly take place. The gains stem from the fact that exchange rate expectations happen not to be expressed in interest rates although interest rate parity (possibly with a premium) generally should hold. This is a case of a shock which makes market participants change rapidly their exchange rate expectations but interest rates have not yet reacted equally. A shock can of course lead to some market presures on interest rates – attacking traders try to make their domestic currency open positions shorter which means an extra demand for domestic currency liabilities. Also the government can raise the interest rate to defend the exchange rate (to sustain interest rate parity plus demanded risk premium). This means a rise in the government's cost and may lead to a decision to devalue. If market participants knew the government's cost function they would also know which shock (what change of exchange rate expectations) would be offset by the government through an interest rate rise and which shock would result in devaluation. They would act respectively. Both before a shock and after it, given the assumptions of the second generation, an equilibrium must be unique and determined. In other words there is no room for self-fulfilling expectations and multiple equilibria.

In order to question self-fullfiling expectations and multiple equilibria and to explain the role of a shock I argued that a crisis is possible only when interest rates happen not to met the condition of interest rate parity (allowing for a given risk premium) due to a shock and a change of exchange rate expectations. Such thinking seems to be best reflected in remarks by Masson and Agenor (1996) who pointed out that in a pre-crisis period in Mexico a difference between interest rates of treasury bonds denominated in pesos and dollars did not cover expected devaluation declared by investors in questionaires. In this context it seems also important that Rose and Svensson (1993) found no strong relation between exchange rate expectations measured by interest rate differentials and economic fundamentals what led them to a conclusion that the crisis of ERM was largely unanticipated and that there was little ralation between expectations and fundamentals. They wrote: "The non-response of ERM realignement expectations to (...) important economic factors seems surprising." Having in mind the reasoning presented here it would be rather just to say that interest rates did not accomodate to these factors and exchange rate expectations. In this particular case an alternative hipothesis is probably even more justified; rising interest rate differentials constituted themselves a shock. They were inconsistent with fixed exchange rates (they meant disparities) and were a source of realignement expectations and resulting gains which made an attack reasonable.

A necessity of an external shock as a reason for an attack and devaluation is also perceived by Obstfeld (1995), which is not consistent with main conclusions of the second generation. Obstfeld writes: "(...) some seemingly minor random event (a sunspot) could shift the exchange rate from a position where it is vulnerable to only very bad *u* (i.i.d. mean-zero shock – A.K.) realizations to one where output is so low absent devaluation that even a moderate shocks will induce the authorities to realign". Let's notice that such an attitude gives room for a double external shock or impact – a random event (a sunspot) and some realization of *u*. In the model – Obstfeld (1995) there is no role for sunspots and an equilibrium does not depend on *u* realizations.

What is important in the context of external shocks as a reason for a crisis is fixed exchange rate vulnerability. (Obstfeld, 1995) supplies a very good structure to analyse this issue.

4

Fixed exchange rate vulnerability to shocks

In (Obstfeld, 1995) the government minimizes the cost of its policy. It is a function of a difference between the actual income level and the target level, depreciation (and inflation) and fixed costs of devaluation/revaluation in the case of a fixed rate. A constraint is augmented Philips curve which also contains i.d.d. mean-zero shock u. Exchange rate (and inflation) expectations influence the position of the Philips curve and the cost of the government and determine the rate of depreciation the governmnet actually chooses. The government's reaction is described with equation

1. $$\varepsilon = [\alpha(y^* - y' + u) + \alpha^2\varepsilon^e]/(\alpha^2 + \beta),$$

where ε is exchange rate change, y^* is targeted income, y' is actual income, ε^e is predetermined exchange rate expectations the government faces, α, β are parameters of the Philips curve and the government's reaction function respectively[5].

In the case of a fixed exchange rate equation 1 is operational only for such values of u that, firstly, the cost at a flexible rate augmented by the cost of revaluation c' is lower then the cost at a fixed rate or, secondly, the cost at a flexible rate augmented by the cost of devaluation c'' is lower then the cost at a fixed rate. The values of u such that for $u < u'$ the government would revalue and for $u > u''$ it would devalue are expressed by the following equations;

2. $$u'' = \frac{1}{\alpha}\sqrt{c''(\alpha^2+\beta)} - y^* + y' - \alpha\varepsilon^e$$

3. $$u' = -\frac{1}{\alpha}\sqrt{c'(\alpha^2+\beta)} - y^* + y' - \alpha\varepsilon^e,$$

where y' is the "natural" output level.

Under an assumption that u is uniformly distributed on $[-\mu, \mu]$ the rational expectation of next period's ε, given predetermined expectations ε^e, is[6]

4. $$E\varepsilon = \alpha/(\alpha^2+\beta)\,[(1-\frac{u''-u'}{2\mu})(y^*-y'+\alpha\varepsilon^e) - (u''^2 - u'^2)/4\mu]$$

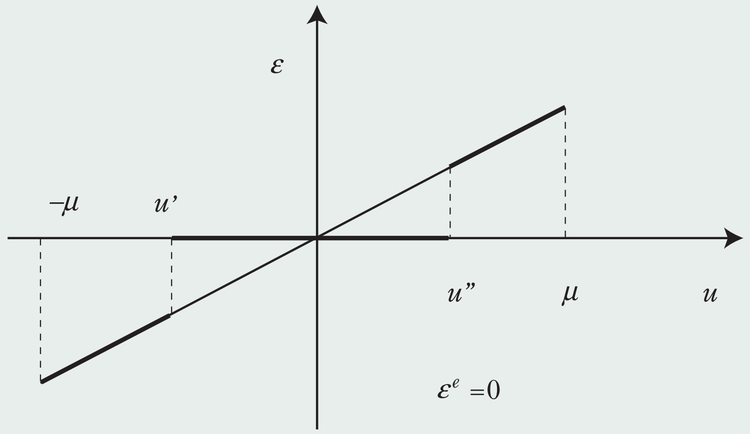
This equation is achieved under an assumption that $-\mu < u'$ and $\mu > u''$; this is important for further analysis of exchange rate expectations.

After substituting eq. 2 and 3 for u' and u'' and after reduction eq. 4 takes a form where $E\varepsilon$ is a linear function of ε^e.

Let's concider a case when $y^* = y$, $c' = c''$ and $\varepsilon^e = E\varepsilon$ at full equilibrium. Thus, $\varepsilon^e = 0$ and actual devaluation is only a function of u. For values of u from the interval $[u', u'']$ exchange rate remains fixed. This situation is presented in Figure 3.

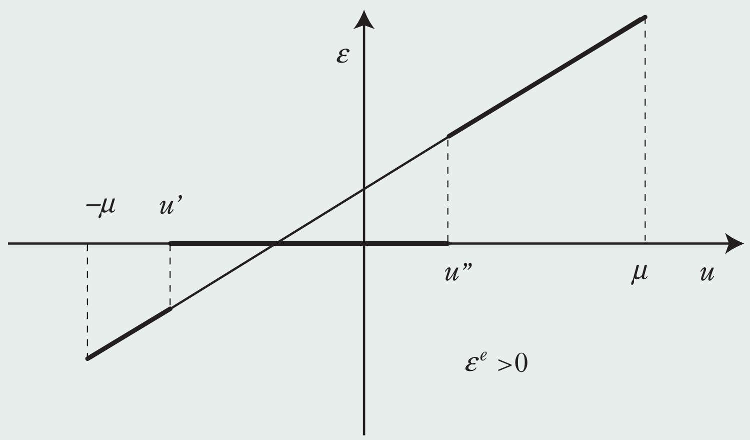
[5] This is equation 3.3 in (Obsfeld, 1995).
[6] This is equation 3.4 in (Obstfeld, 1995).

Figure 3



A situation is different when $y^*>y$ (what is consistent with (Obstfeld, 1995) assumptions) or when $c'>c''$ and as a result $\varepsilon^e > 0$. This is shown in Figure 4.

Figure 4



The higher is the difference between the targeted income and the natural output level the higher is the difference for each u between the targeted income and the actual output. It means that for each u the government's cost minimization demands ralatively high depreciation under a flexible rate and that a shock inducing devaluation of a fixed rate is highly probable why the probability of a shock resulting in revaluation is low. In Figure 4 this is reflected in a shift of the interval $[u', u'']$ to the left inside the interval $[-\mu, \mu]$ in comparison to its central position in Figure 3. Expected exchange rate change is positive, it means it shows expectations of a devaluation. Potentially, with a further rise of the difference between y^* and y', a fixed rate can be mainteined only in the presence of a positive shock.

In the situation in Figure 4 even a relatively weak shock (low values of u) can bring a currency crisis – flotation and devaluation of the currency. In other words this is a situation when the exchange rate regime is prone to negative shocks what is reflected in exchange rate expectations. They determine – together with realizations of u - the government's cost and government's actual decisions on devaluation.

The analysis above shows how some features of an economy and economic policy can influence the vulnerability of an economy to shocks and currency crises.

At some values of y^* and y' the value of u' may be less than $-\mu$ what means that there is no possible shock which could induce revaluation. Its probability is zero. The fixed exchange rate becomes even more vulnerable – it is subject to devaluation under even a weaker shock.

This should be taken into account when eq. 4 is derived; it should be replaced with the equation 5.

4

5. $$E\varepsilon = [\alpha/(\alpha^2+\beta)]\,[(y^*-y'+\alpha\varepsilon^e)\,(\mu-u'')/2\mu + (\mu^2- u''^2)/4\,\mu]$$

It is easy to notice that after substituting eq. 2 for u'' eq. 5 is a quadratic function of ε^e. The values of u' and u'' depend on y^*, y' (and c' and c'') and as a consequence the shape of the function linking Ee to ε^e changes when $u'<-\mu$ and again when $u''<-\mu$.

Obstfeld (1995) in a similar way presents that the shape of this function depends in the same way on the value of e^e which also influences the values of u' and u''; for low values of ε^e $(u'>-\mu$ and $u''<\mu)$ this function is linear, then it becomes quadratic (for $u'<-\mu$) and then it becomes linear again (for $u''<\mu$ the rate is fexible and actual devaluation is a linear function of expectations). This situation is presented in Figure 3 in (Obstfeld, 1995)[7]. However, in (Obstfeld, 1995) all three cases (two linear and one qadratic relation) are treated as different intervals of the same function. This is wrong. The change of linear function to a quadratic one and again to linear is due to diffrent values of y^*, y', c', c'' which depict different economic conditions. For given y^*, y', c', c'' this function is either linear or quadratic. They determine the values of ε^e, u' and u'' as a solution of a set of equations: 2, 3 and 4 or 5 and the condition $E\varepsilon = \varepsilon^e$. The type of the functional relation between ε^e and Ee is thus determined (in particular it is expressed either by eq. 4 or eq. 5). Only when $u'<-m$ what depends on y^*, y', c' and respective value of ε^e, a linear eq.4 must be changed for quadratic eq. 5. In this case there may be two solutions (in the mathematical sense). Any other equilibria are not possible for the same values of y^*, y', c', c''. The idea that there are possible more than two solutions (equilibria) at given conditions which is illustrated in Obstfeld's (1995) Figure 3 is incorrect.

Eq. 5 under the condition $E\varepsilon = \varepsilon^e$ can give two values of ε^e. This should not be interpreted as the existance of multiple equilibria, as argued in part 3 of this paper.

Let's also notice that for some acceptable values of $\alpha, \beta, \mu, c'', y^*$ and y' eq. 5 under the condition $E\varepsilon = \varepsilon^e$ can have no solution at all – this problem was also raised in part 3 of the paper.

Only at even higher difference between y^* and y' or even lower c'' a relation between $E\varepsilon$ and ε^e would be linear again ($u''<\mu$) and give a single value of ε^e and ε for each u. It would be a situation when fixed excange rate can't be sustained no matter the value of u so that the exchange rate would be flexible. High values of ε^e and ε would mirror strong pressures on depreciation.

7 Obsfeld did not obtain eq. 5 and did not applied this equation and eq.1 to find solutions when there is zero probability of shocks resulting in revaluation – due to the value of ee. Instead, Obstfeld to determine in this case the shape of the function expressed by eq. 4 used derivatives du'/dee and du''/dee. However, when u''= -m u' is beyond the interval [-m,m] and thus there is no respective derivative (Obstfeld treated it as equal zero). The results are anyway the same

5
Conclusions

The ideas of multiple equilibria and self-fulfilling expectations have become very popular. However, under *ceteris paribus*, rational expectations and perfect feresight assumptions it is not justified to say that multiple equilibria can exist and that expectations may be self-fullfilling.

This does not mean that an economy must always be fully determined. As this paper suggests it seems possible that some models may have no solution for some reasonable values of parameters and thus possibly stable equilibrium doesn't exists. Moreover, in a situation of uncertainty concerning policy rules, market participants may apply mixed strategies, what could result in multiple equilibria.

Some features of an economy and economic policy may turn the exchange rate more vulnerable to external shocks – this fact seems reflect well the factors behind the ERM crisis of the early 90's.

6
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